Thermal chromoelectric field correlators and loop integrals

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@ Cairns on 22.08.2024

congregation of correlators

theoretical motivations

What can we compute in a thermal (near-)equilibrium QFT?

Correlators / N-point functions! [1]

- Usual Yang–Mills suspects: $\langle AA \rangle$, $\langle AAA \rangle$, $\langle AAAA \rangle$...
- Correlators involving $F_{\mu\nu}$ ($\langle FF \rangle$...) are also interesting: Field strength is a very ”tangible” concept from a classical pov
- Can also split $\langle FF \rangle$ into $\langle EE \rangle$ and $\langle BB \rangle$

[1] And the free energy

computation methods

Weak-coupling expansion: For large enough temperatures T, expand everything for small coupling g_s. Precise (near-)analytic control, but only works at large temperatures.

- Imaginary-time: Compactify time-axis and discretise p_0, calculate in Euclidean space, and analytically continue to get transport and other near-equilibrium quantities
- Real-time: Complexify time-axis, double field content but obtain time-dependent quantities directly

Euclidean calculations allow direct comparison with lattice simulations! Also very neatly defined: No contour ambiguity, countably many distinct correlators.

EFTs and physical motivation

What do we actually use these correlators for?

$\langle EE \rangle$ in fundamental representation: Heavy quark effective theory, enters as a diffusion transport coefficient to a Langevin equation

$\langle EE \rangle$ in adjoint representation: appears in the matching coefficients of potential nonrelativistic QCD, generates transport coefficients responsible for quarkonium diffusion and mass shifts

$\langle BB \rangle$ is subleading for small velocities (Lorentz force!) but possibly interesting when corrections become important

explicit construction

Wilson lines needed to connect distinct spacetime points in a gauge-invariant way

Fundamental correlator (imaginary-time NLO in Burnier & al. [JHEP 08 (2010) 094])

$$\langle EWEW \rangle_F \equiv \langle \mathrm{Tr}\left[E_i(0)U(0,t)E_i(t)U(t,1/T)\right]\rangle$$

(Only) three linearly independent gauge-invariant adjoint correlators with two E-fields in imaginary-time:

$$\begin{aligned}
\langle EE \rangle_U &\equiv \left\langle E^i_a(0) W^{ab}(0,t) E^i_b(t) \right\rangle , \\
\langle EE \rangle_L &\equiv \left\langle E^i_b(0) E^i_a(t) W^{ab}(t,1/T) \right\rangle , \\
\langle EE \rangle_S &\equiv \left\langle E^{ab}_i(0) W^{bc}(0,t) E^{cd}_i(t) W^{da}(t,1/T) \right\rangle ,
\end{aligned}$$

Real-time correlators considered by eg. Binder & al. [JHEP01 (2022) 137] — ”true” time dependence but contour ambiguity, no direct lattice contact.

spectral function and transport coefficients

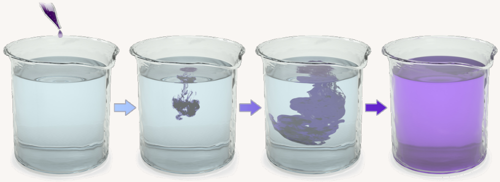
Insight on near-equilibrium quantities from imaginary time by looking at the spectral function ρ:

$$\rho_{\langle EE\rangle}(\omega) = \mathrm{Im}\mathcal{F}_t\left[\langle EE\rangle(t)\right](\omega_n)\,|_{\omega_n\to -i\omega+\eta}$$

ρ gives the momentum diffusion coefficient $\kappa \propto \lim_{\omega\to 0^+} \rho(\omega)/\omega$.

For the fundamental correlator, mass shift $\gamma \propto \int_0^\beta \mathrm{d}t\langle EE\rangle_F$ [Eller & al., Phys. Rev. D 99, 094042 (2019)]. What does this integral mean in the adjoint for the different correlators?

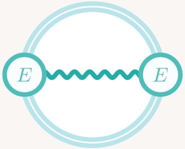
General definitions in terms of real-time correlators. Any way to avoid analytic continuation?



lots of loops

leading-order result

At leading order, all EE-correlators coincide (up to Casimir scaling):



$$= g_s^2 i^2 \delta_a^a \sum\!\!\!\!\!\!\!\int_P e^{ip_0 t}\left[p_0^2 G_{ii}(P) + p^2 G_{00}(P) - 2p_0 p_i G_{0i}(P)\right]$$

$$= d_A g_s^2 (d-1) \sum\!\!\!\!\!\!\!\int_P \frac{p_0^2}{P^2} e^{ip_0 t},$$

where the one-loop integral

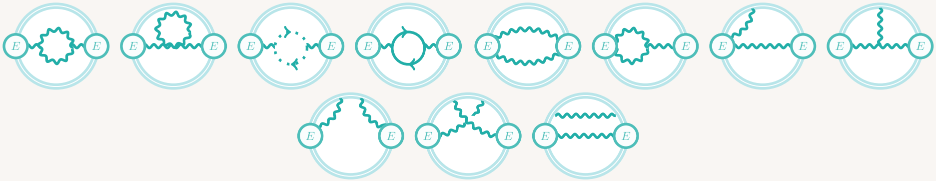
$$\mathcal{E}_m^a(t) \equiv \sum\!\!\!\!\!\!\!\int_P e^{ip_0 t}\frac{p_0^a}{P^{2m}} = \frac{T^{4-2m+a}}{(2\pi)^{2m-a}} \frac{\Gamma\left(m-\frac{d}{2}\right)}{\Gamma\left(m\right)} \pi^{\frac{3}{2}} \left(\frac{e^{\gamma_E}\bar{\Lambda}^2}{4\pi^2 T^2}\right)^{\frac{3-d}{2}}$$

$$\times \left[\mathrm{Li}_{2m-a-d}\left(e^{2\pi i T t}\right) + (-1)^a \mathrm{Li}_{2m-a-d}\left(e^{-2\pi i T t}\right)\right].$$

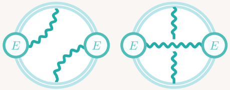
NLO diagrams

Straightforward (if messy): Draw all diagrams w/ E-field and Wilson line insertions and standard Feynman rule.

Eleven topologies shared by all correlators at NLO



Two additional topologies for the symmetric/fundamental correlator



applying IBPs

Integration-by-part (IBP) is a standard method at $T = 0$, recently applicable also at $T > 0$ (see Schröder et al., [JHEP 12 (2022) and 02 (2024)])

Major result: All two-loop integrals with an "additive mass" are factorisable → spatial part of the sum-integral can be factorised. For $I_n(m) \equiv \int_p (p^2 + m^2)^{-n}$,

$$\int_{\mathbf{pq}} \frac{1}{(p_0^2 + p^2)(q_0^2 + q^2)\left[(p_0 + q_0)^2 + (\mathbf{p} + \mathbf{q})^2\right]}$$

$$= -\frac{d-2}{2(d-3)}\left[\frac{I_1(p_0)\, I_1(p_0+q_0)}{p_0(p_0+q_0)} + \frac{I_1(q_0)\, I_1(p_0+q_0)}{q_0(p_0+q_0)} - \frac{I_1(p_0)\, I_1(q_0)}{p_0 q_0}\right],$$

Trivialises most of the NLO computation: Almost everything is a product of $\mathcal{E}_m^n$s.

zero-modes

Bosonic zero-modes $p_0 = 0$ special: integrals become massless, scaleless, and IR-sensitive

Scaleless $\to$ simple, since $\sum\!\!\!\!\!\!\int f(0,p) \equiv 0$ and IBPs let us factorise almost everything, right?

No! Only shifts of three-momenta are allowed ($\sum\!\!\!\!\!\!\int_{PK} f(P+K)g(P) = \sum\!\!\!\!\!\!\int_{PK} f(K)g(P)$ only for the non-zero modes, even though $\int_{pk} f(p+k)g(p) = \int_{pk} f(k)g(p)$ always)

Properly factorisable diagrams still vanish for the zero-modes, but everything with a three-point vertex does not—extra contributions, but simpler structure:

$$\mathcal{Z}^{a}_{nml}(t) \equiv \sum\!\!\!\!\!\!\!\int_{P} \int_{\mathbf{q}} \frac{p_0^a e^{ip_0 t}}{(\mathbf{p}+\mathbf{q})^{2n}(p_0^2+p^2)^m(p_0^2+q^2)^l},$$

more limitations of IBP

Even when the factorisation works, the remaining sums can become coupled. Simpler to compute integrals than sums $\to$ rewind back a few steps

At NLO, problem appears in exactly one unique integral:

$$I_X^R \equiv \frac{i}{2}\int_0^t \mathrm{d}t' \oint\!\!\!\!\!\!\sum_{PQ} {}'' \frac{e^{ip_0\left(t-t'\right)} q_0 e^{iq_0 t}}{\left(P+Q\right)^2 Q^2} + \frac{1}{2} \oint\!\!\!\!\!\!\sum_{PQ} {}'' \frac{e^{ip_0 t}}{P^2 Q^2}$$
$$= \frac{i}{2}\int_0^t \mathrm{d}t' \mathcal{E}_1^1\left(t'\right) \mathcal{E}_1^0\left(t-t'\right) - \frac{t}{2} iT\mathcal{Z}_{011}^1(t) + \frac{1}{2}\mathcal{E}_1^0\left(t\right)\mathcal{E}_1^0\left(0\right).$$

The "Wilson loop integral" makes the p_0-weight negative and the IBP-sum integral nonfactorisable, and divergent to boot

Divergent at the domain edges, but divergences can be subtracted and converted to dimreg by expanding the integral appropriately …

… In the end closed form $O(1)$. $O(\varepsilon)$ is a slightly messy but doable numerical integral

gauge-invariance

Everything computed in the R_ξ-gauge: for nonzero modes, gauge-dependence cancels algebraically

The cancellation is quite delicate, and depends on the choice of the correlator, but in principle straightforward

For zero-modes, cancellation is not algebraic: Gauge-dependence is merely pushed to higher-orders (and presumably cancelled by contributions there), but the physical $d = 3$-correlator is gauge-invariant

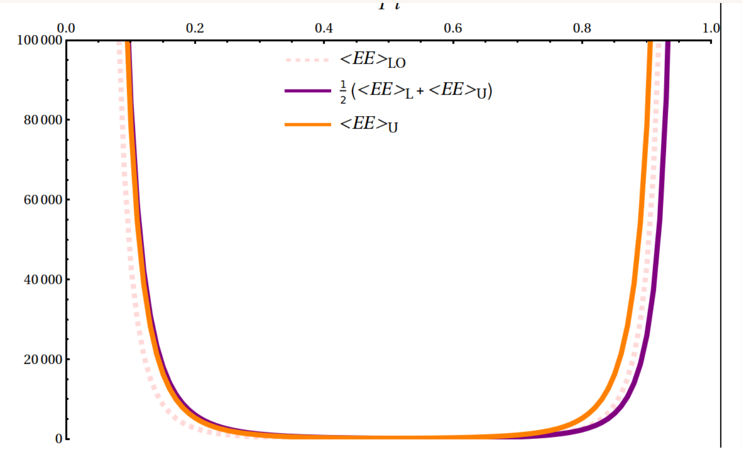
plenty of PRELIMINARY plots

NLO correlator

$$
\begin{aligned}
\langle EE\rangle_I = {} & d_A g_s^2 (d-1)\mathcal{E}_1^2(t) + d_A N_c g_s^4 \Bigg\{ 2\left(d + \frac{3}{d-3}\right)\left[\mathcal{E}_1^0(t)\right]^2 \\
& - \frac{1}{2}\left[d\left(d^2 - 4d + 7\right) + \frac{8}{d-3}\right]\mathcal{E}_1^0(t)\mathcal{E}_1^0(0) - 4\left(\frac{d-3}{d-2} + \frac{4}{d-3}\right)\mathcal{E}_1^1(t)\mathcal{E}_2^1(t) \\
& - 2\left(\frac{d-3}{d-2} + \frac{4}{d-3}\right)\left[\mathcal{E}_2^0(t) - \mathcal{E}_2^0(0)\right]\mathcal{E}_1^2(t) - 2\left(d - 1 + \frac{2}{d-3}\right) I_X \Bigg\} \\
& + d_A N_c g_s^4 \Bigg\{ \frac{19/4}{d-3} T\mathcal{Z}_{011}^0(t) + \frac{8}{d-3} T\mathcal{Z}_{011}^0(0) - \frac{2}{d-3}\lambda(W_I) i T\mathcal{Z}_{011}^1(t) \Bigg\} \\
& + d_A N_f g_s^4 \Bigg\{ (d-1)\left[\mathcal{E}_1^0(t)\right]^2 + (d-1)4^{2-d}\left[\mathcal{E}_1^0\left(\frac{t}{2}\right)\right]^2 \\
& - (d-1)2^{3-d}\mathcal{E}_1^0\left(\frac{t}{2}\right)\mathcal{E}_1^0(t) + \left(2^{2-d} - 1\right)\left[(d-3)d + 4 + \frac{4}{d-3}\right]\mathcal{E}_1^0(t)\,\mathcal{E}_1^0(0) \\
& - \left(1 + \frac{2}{d-3}\right)\left[2\mathcal{E}_1^1(t) - 2^{2-d}\mathcal{E}_1^1\left(\frac{t}{2}\right)\right]\left[\mathcal{E}_2^1(t) - 2^{3-d}\mathcal{E}_2^1\left(\frac{t}{2}\right)\right] \Bigg\} \\
& - d_A N_f g_s^4 \frac{2}{d-3}\left(4^{2-d} - 1\right) T\mathcal{Z}_{011}^0(0)\,.
\end{aligned}
$$

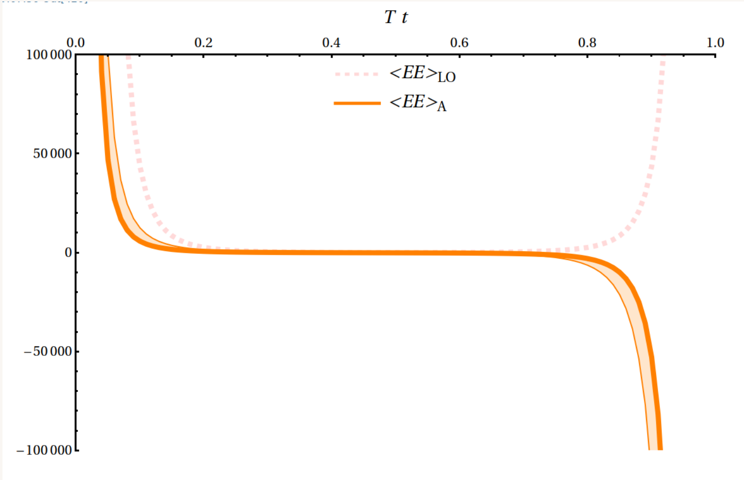
$\langle EWE \rangle$ vs. $\langle EWE + EEW \rangle/2$

Plot at $T = 2.0\,\mathrm{GeV}$, PMS scale, $N_f = N_c = 3$ (includes HTL)



asymmetry

Adjoint correlator is *asymmetric* (band shows RG variation)



perturbative asymmetry

Naïvely, bosonic loop integrals should be symmetric over the thermal circle:

$$\sum\!\!\!\!\!\!\!\int_{P_1\ldots P_L} \frac{e^{ip_0^1 t}\ldots e^{ip_0^L t}}{P_1^{a_1}\ldots P_L^{a_L}} = \sum\!\!\!\!\!\!\!\int_{P_1\ldots P_L} \frac{e^{ip_0^1(1/T-t)}\ldots e^{ip_0^L(1/T-t)}}{P_1^{a_1}\ldots P_L^{a_L}}$$

And fermionic terms don't help, because fermionic lines are always internal...

But zero-modes of Wilson lines explicitly probe the length of the Wilson line (ie, t) and differ for the *EWE* and *EEW*: LO asymmetry from the zero-mode of the triangle diagram

$$(\langle EE\rangle_U(t) - \langle EE\rangle_L(\beta - t))/2 = d_A N_c g_s^4 t T \varepsilon^{-1} \mathcal{Z}_{011}^1(t) + O(\varepsilon, g_s^6)$$

Disappears in the symmetric (and fundamental) correlator thanks to an interplay between overall coefficients of *EWE* and *EWE*

proceeding to NNLO

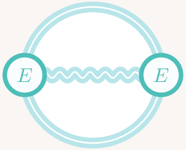
For now, results like transport coefficients are little more than double-checks— already known, just checking the Euclidean derivation and the analytic continuation

Symbolic automation lets us manage the messy higher orders, and modern IBP methods help with computing sum-integrals

Zero-modes are particularly interesting: Relevant for evaluating κ to higher orders as well as improving convergence by computing corrections to asymmetry.

extra: HTL corrections

In the infrared, massless bosons require resummation. At NLO, first resummed contribution appears in the form of the HTL-resummed loop



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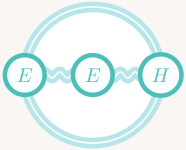


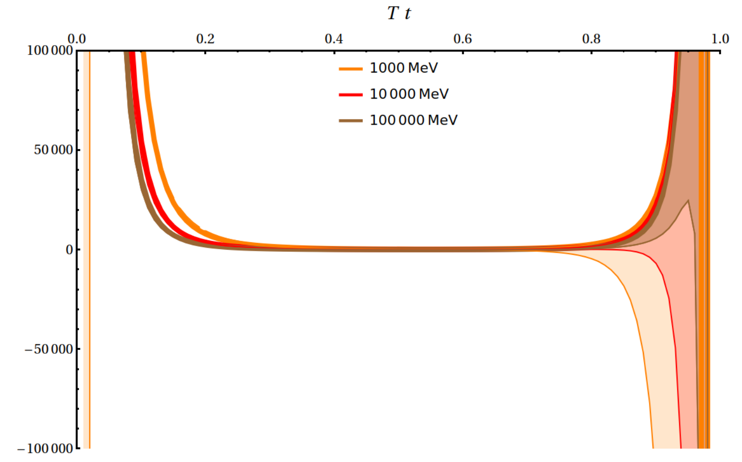
$$\approx \frac{d_A g_s^2 m_E^2}{4} T^2 \csc^2(\pi T t) \left[\left(\frac{4\pi T}{m_E^2} \right) - 1 \right]$$

Won't go into details, since this is just the Casimir-scaled version of the fundamental result

Nothing drastic happens yet—no leftover IR divergences—but very interesting at NNLO

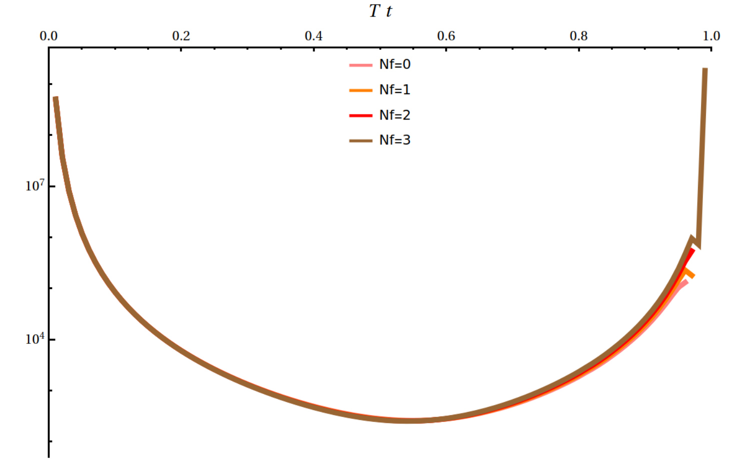
bonus plot: temperature variation

RG-variation of the full result has a misleadingly drastic dip because of the antisymmetric part and the delicateness of I_X^R numerics—to be improved



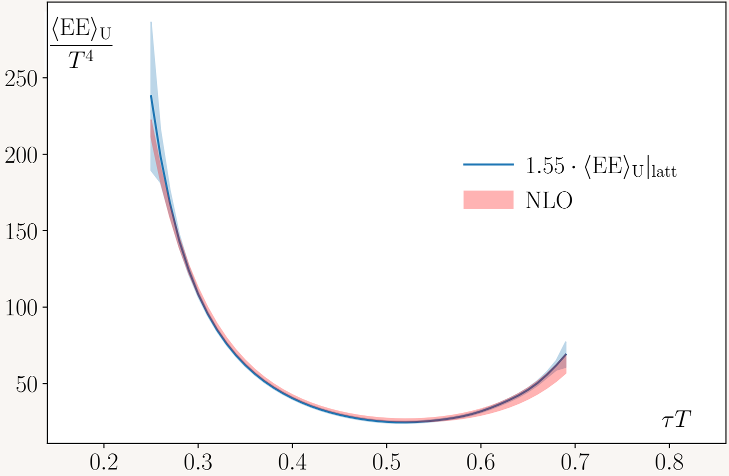
bonus plot: N_f-variation

Increasing N_f ameliorates the dip



bonus plot: vs lattice

At stupendously high T ($10000T_c$), agreement with $N_f = 0$ lattice... up to a mystery coefficient



bonus plot: sym vs lattice

Extra-preliminary, but the symmetric correlator also seems to work — huge errors in comparison, though?