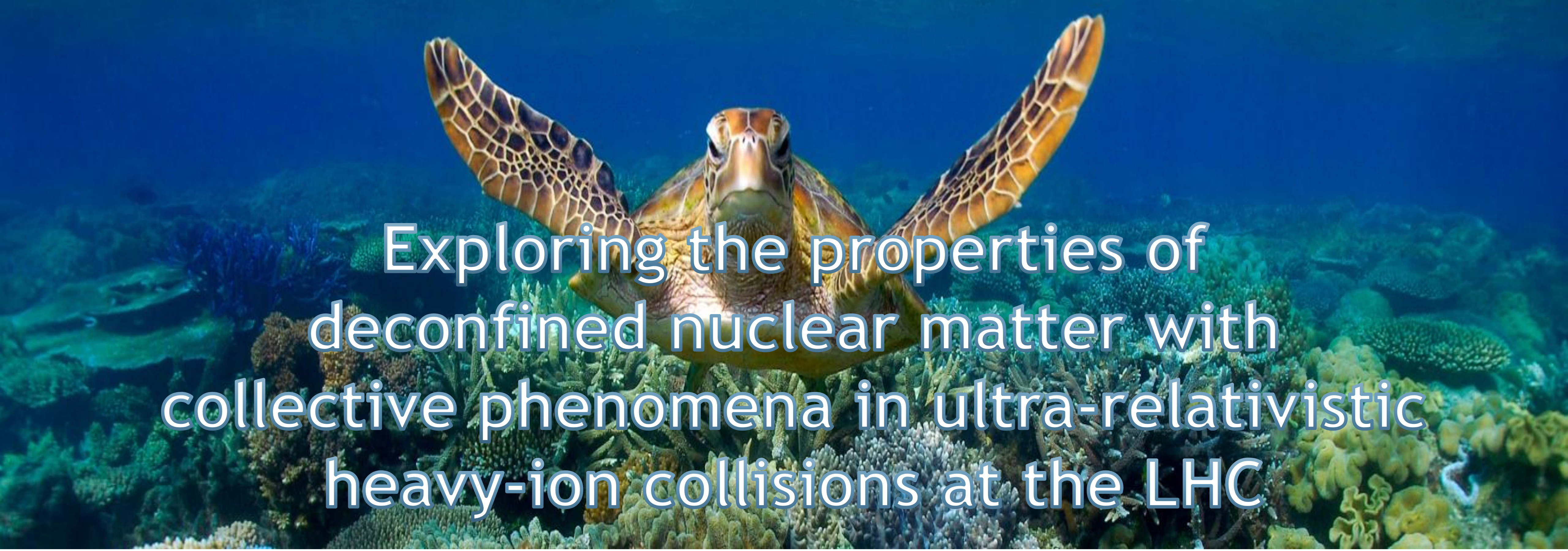


# Exploring the properties of deconfined nuclear matter with collective phenomena in ultra-relativistic heavy-ion collisions at the LHC

Ante Bilandzic
Technical University of Munich

**"XVIth Quark Confinement and Hadron Spectrum"**, Cairns
19/08/2024

# Outline

- QCD phase diagram and Quark-Gluon Plasma
- Heavy-ion collisions
  - Collective phenomena
  - Multi-particle correlations
- Multi-harmonic observables
  - Symmetric Cumulants (SC)
  - Asymmetric Cumulants (AC)

C. Mordasini, AB, D. Karakoç, F. Taghavi, Phys. Rev. C **102**, 024907 (2020),
e-Print 1901.06968

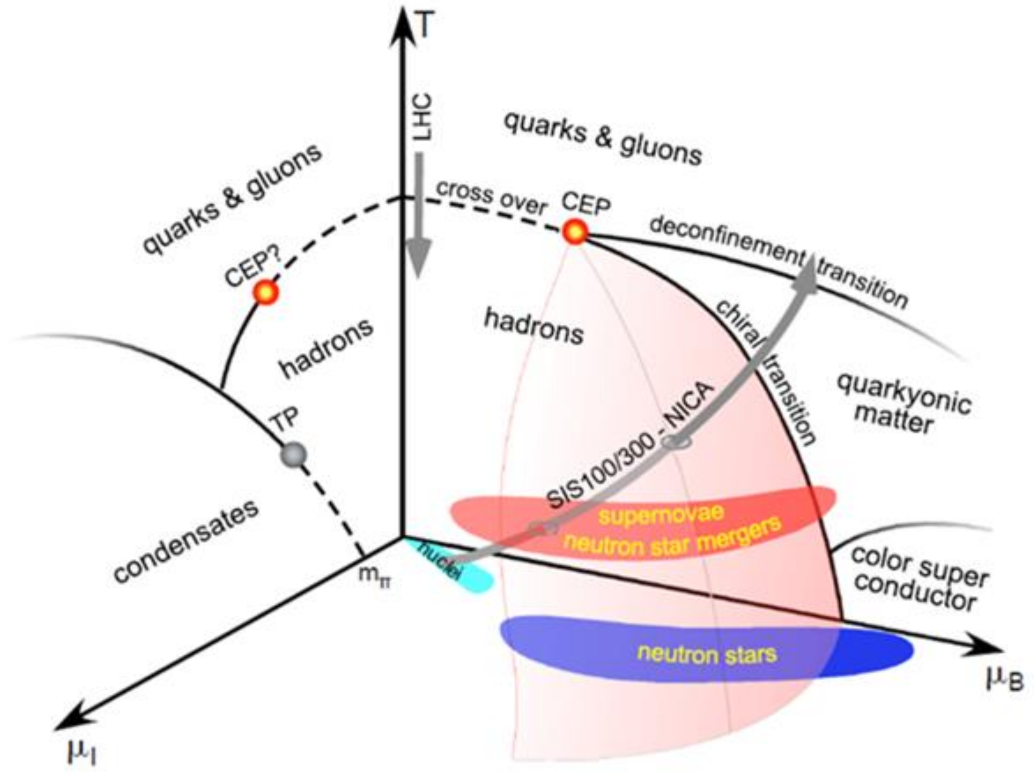
**ALICE Collaboration**, Phys. Rev. Lett. 127 (2021) 9, 092302,
e-Print: 2101.02579

AB, M. Lesch, C. Mordasini, F. Taghavi, Phys. Rev. C **105** (2022) 2, 024912,
e-Print: 2101.05619

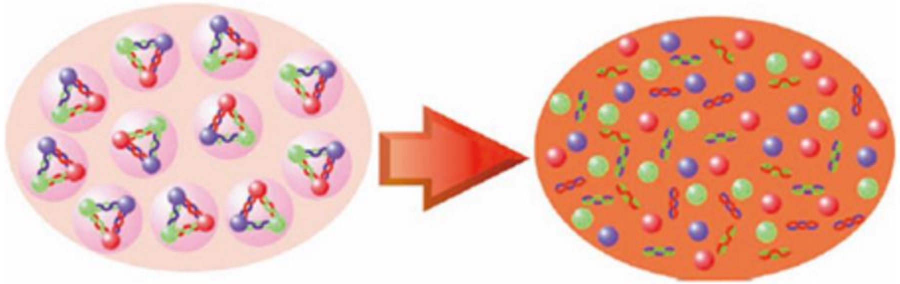
**ALICE Collaboration,** Phys. Rev. C **108** (2023) 5, 055203,
e-Print: 2303.13414

# QCD phase diagram and Quark-Gluon Plasma

- Phase diagram of Quantum Chromodynamics (QCD)

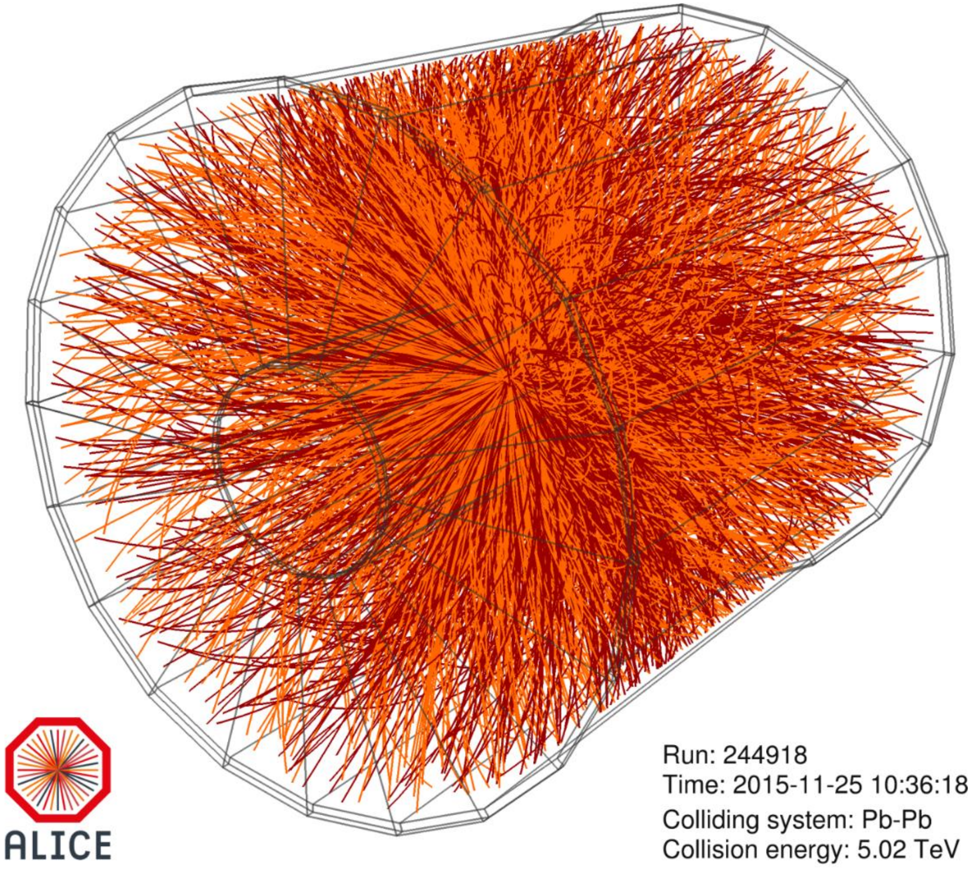


- Quark-Gluon Plasma (QGP): Extreme state of matter in which quarks and gluons can move freely over distances comparable to the size of hadrons

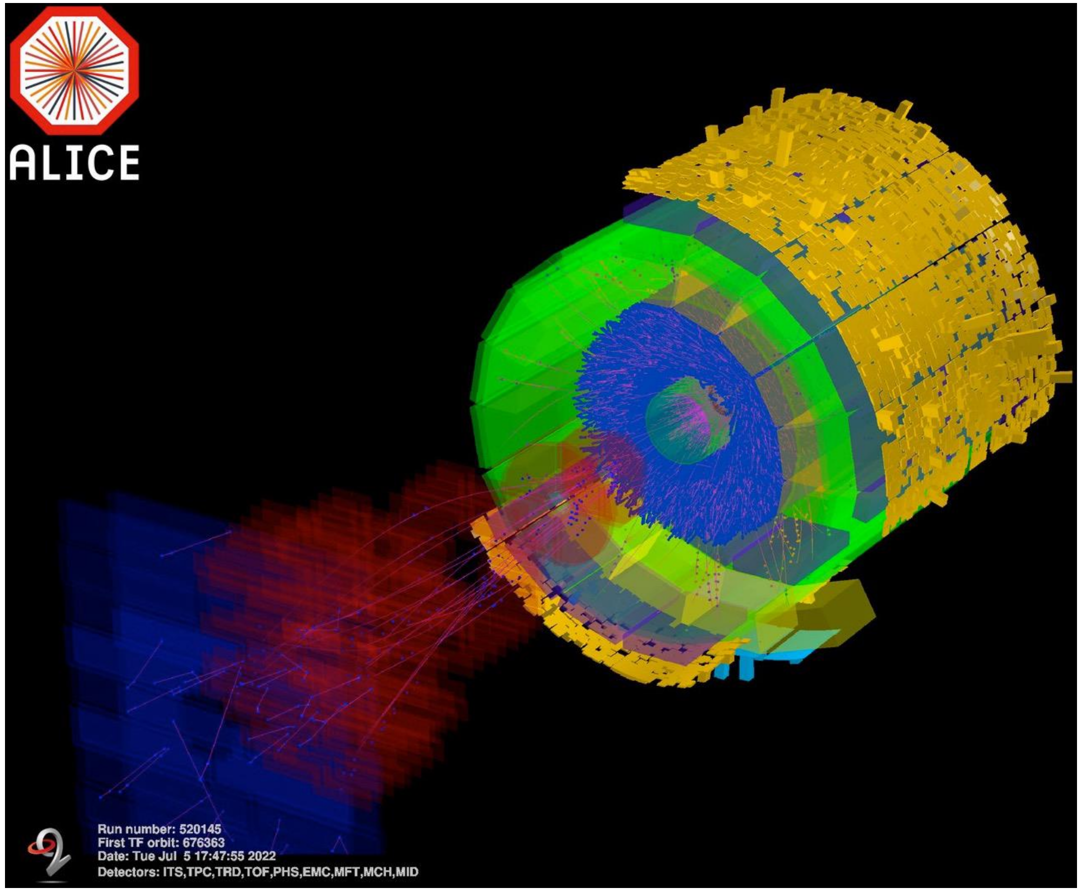


# Heavy-ion collisions

- How to constrain the properties of QGP out of this mess?



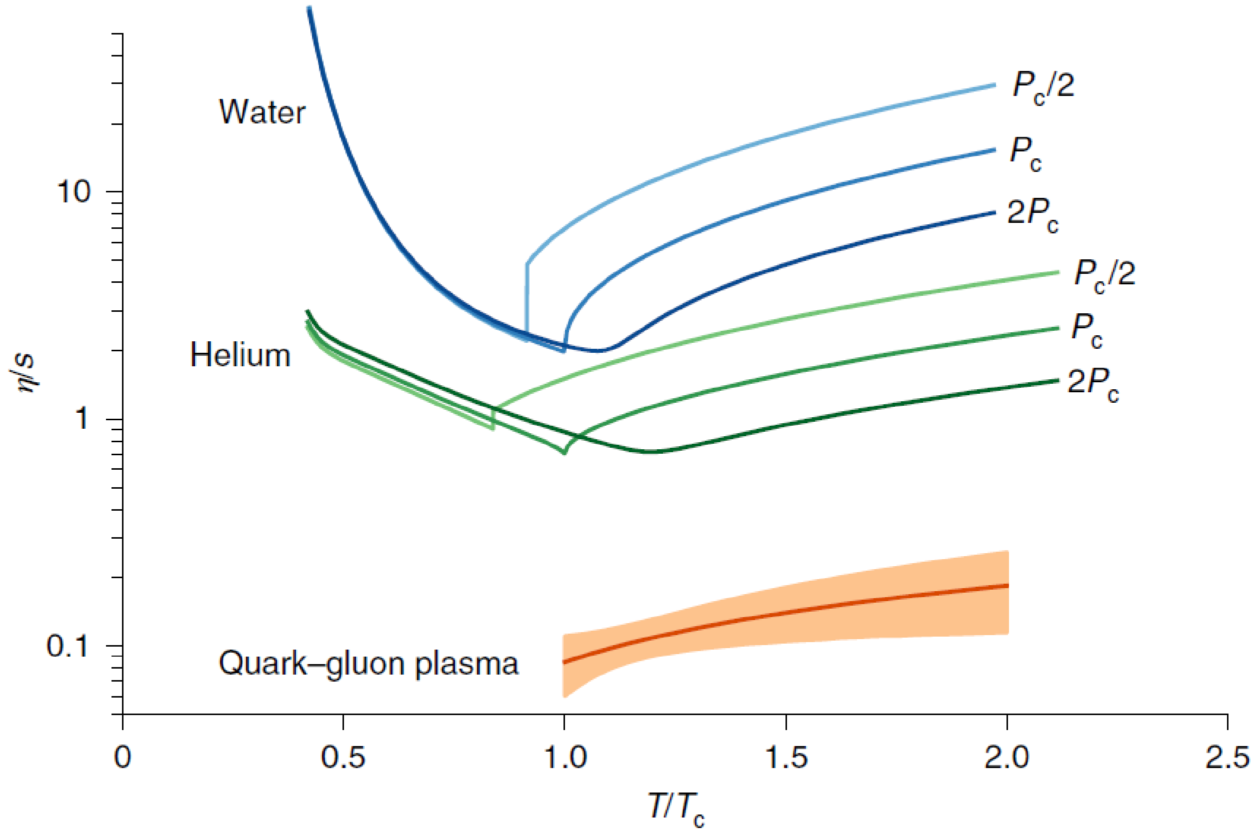
Run: 244918
Time: 2015-11-25 10:36:18
Colliding system: Pb-Pb
Collision energy: 5.02 TeV



Run number: 520145
First TF orbit: 676363
Date: Tue Jul 5 17:47:55 2022
Detectors: ITS,TPC,TRD,TOF,PHS,EMC,MFT,MCH,MID

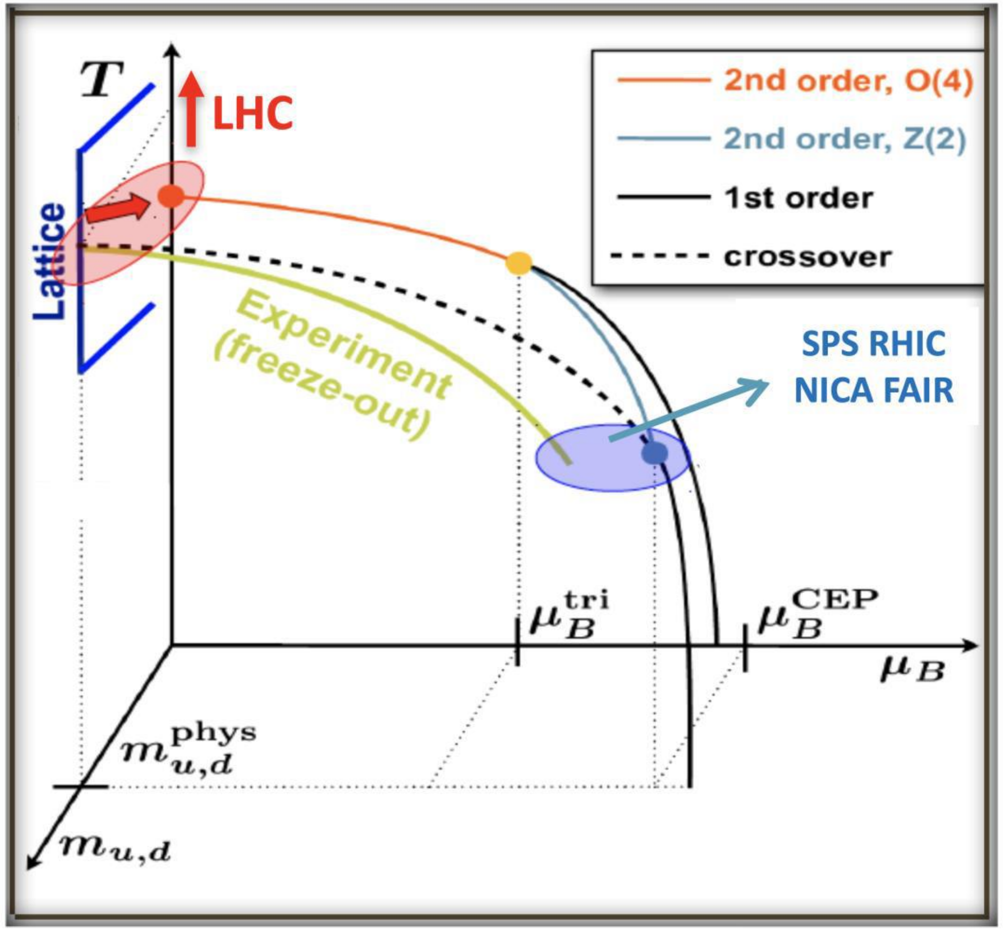
# Example #1: Transport properties of QGP

- Temperature dependence of QGP's specific shear viscosity (*η/s*) is smallest of all known substances



Bernhard, J.E., Moreland, J.S. & Bass, S.A. *Nat. Phys.* **15**, 1113–1117 (2019),

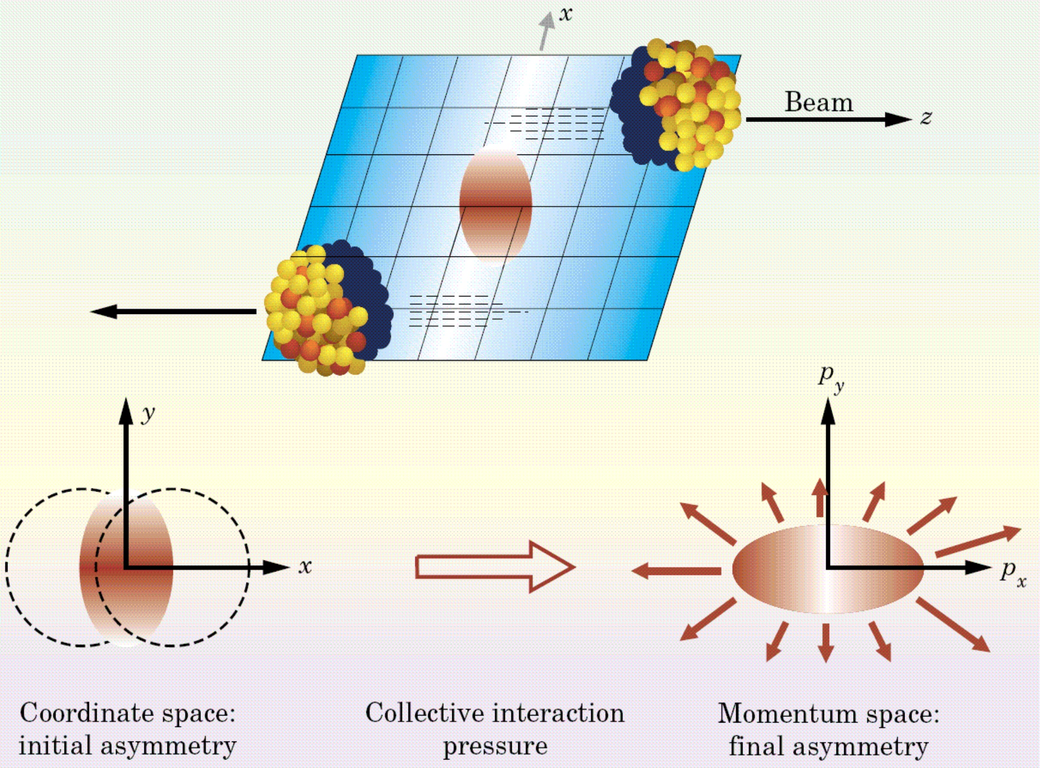
# Example #2: Nature of QCD phase transitions



- QCD phase diagram of strongly interacting nuclear matter can be explored in ultrarelativistic heavy-ion collisions
- QGP phase is probed as a function of temperature and baryon chemical potential
- What is the nature of phase transitions in QCD phase diagram (smooth cross over, 1st or 2nd order phase transition, etc.)?
- Existence of critical point?

# Collective anisotropic flow

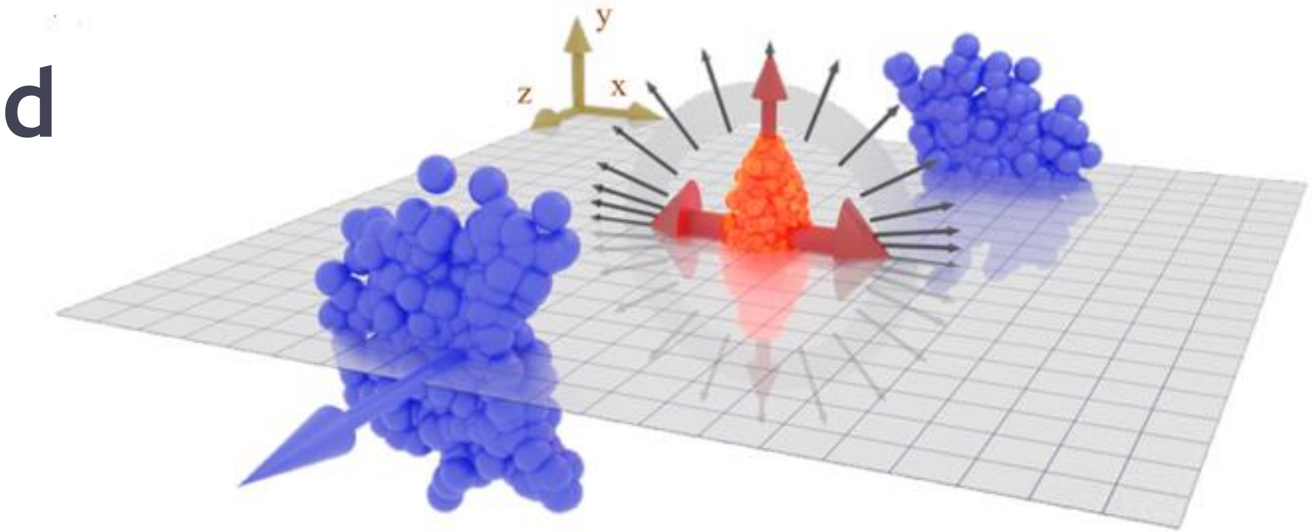
- Transfer of anisotropy from the initial coordinate space into the final momentum space via the thermalized medium:



- Anisotropic flow will develop in heavy-ion collisions only if both of the following two requirements are met:
  - Initial anisotropic volume in coordinate space
  - Thermalized medium

J.Y. Ollitrault, Phys. Rev. D **46** (1992) 229

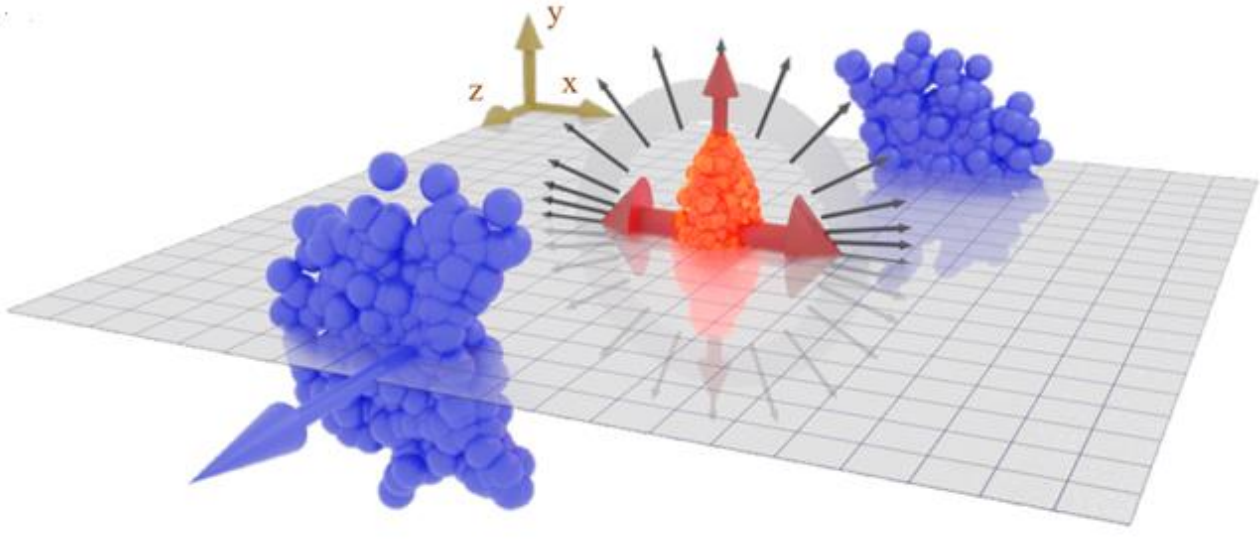
# Heavy-ion collisions and collective phenomena



Credits: D.D. Chinellato, ICHEP 2020

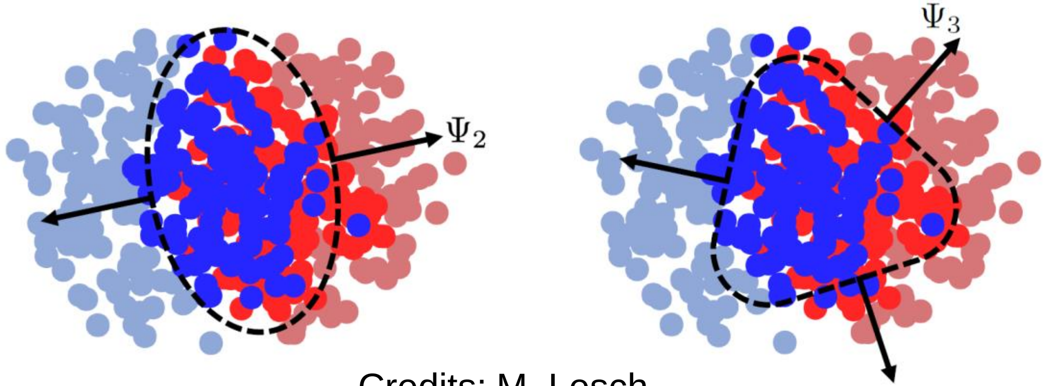
# Initial anisotropic volume and thermalized medium

- **Non-central** heavy-ion collision is a prime example
  - Trivially (#1): Due to geometry of collision the resulting volume containing interacting matter is anisotropic in coordinate space
  - Trivially (#2): To leading order this anisotropic volume is ellipsoidal



Credits: D.D. Chinellato, ICHEP 2020

- More subtle cases of initial anisotropic volume can occur due to **fluctuations of participating nucleons**

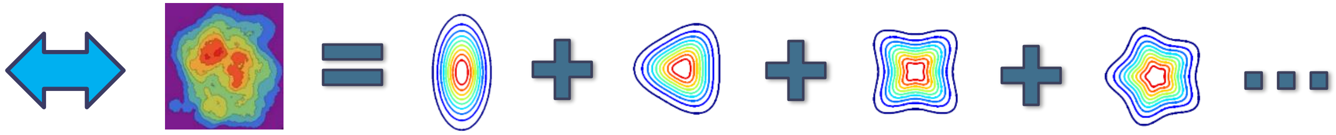


Credits: M. Lesch

# Fourier series

- In flow analysis, anisotropic emission of particles in the transverse plane after heavy-ion collision is described by:

$$f(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]\right]$$

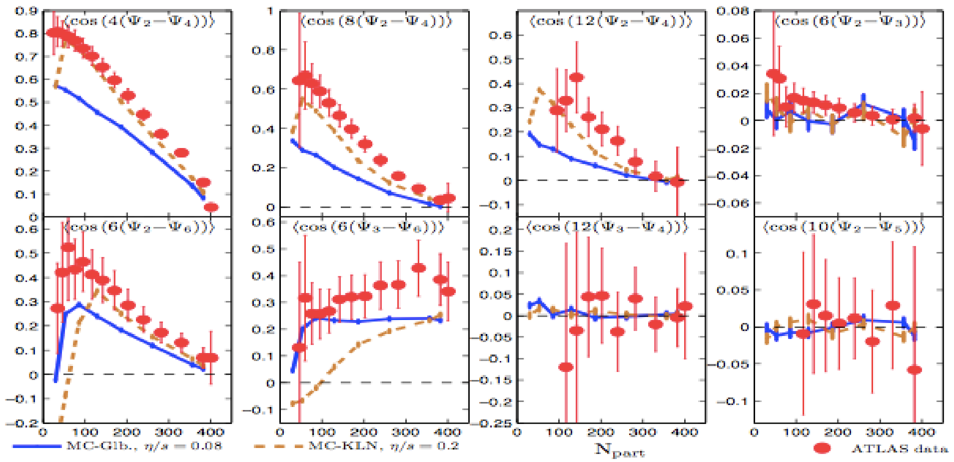


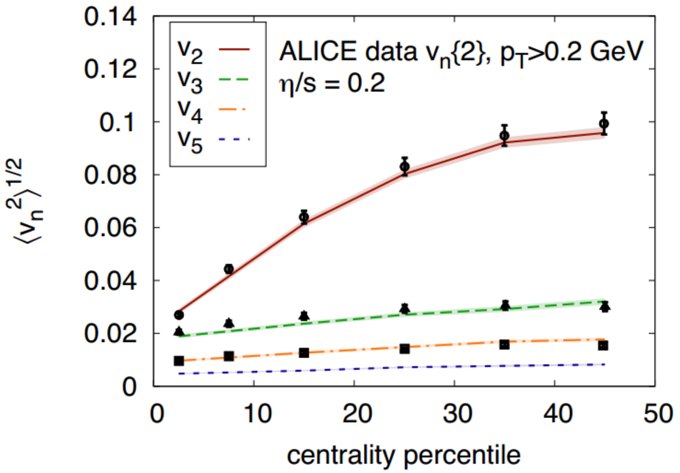
  - $v_n$ : flow amplitudes
  - $\Psi_n$ : symmetry planes
  - Anisotropic flow is quantified with $v_n$ and $\Psi_n$
    - ○ $v_1$ is directed flow
    - ○ $v_2$ is elliptic flow
    - ○ $v_3$ is triangular flow
    - ○ $v_4$ is quadrangular flow, etc.

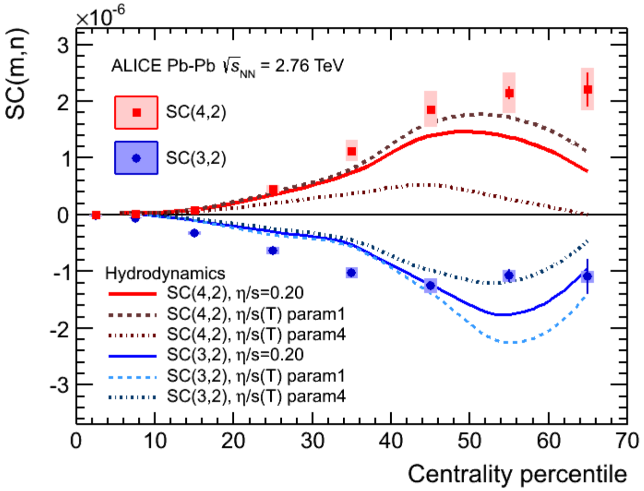
S. Voloshin and Y. Zhang, Z.Phys. C70 (1996) 665-672

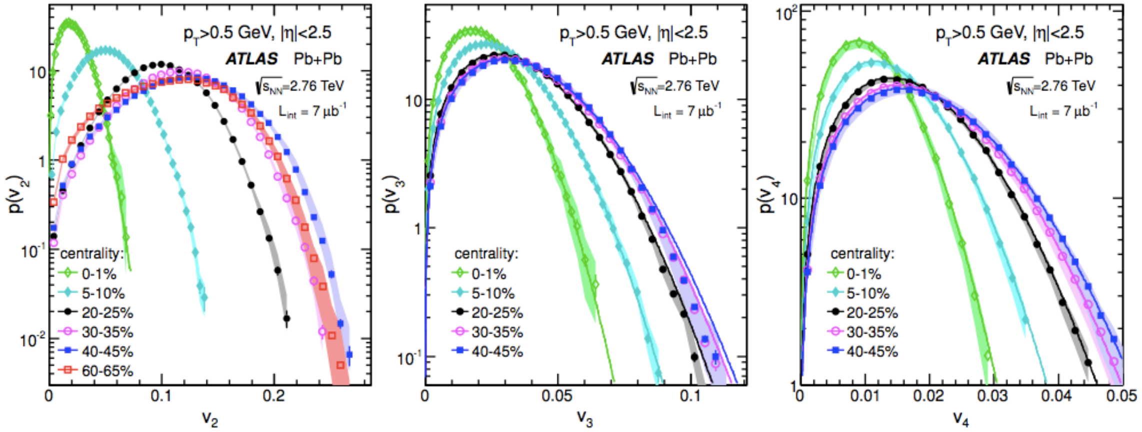
# Flow observables

- Individual flow harmonics: $\boldsymbol{v_1, v_2, v_3, v_4, \ldots}$
- Correlations between harmonics: $\langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle$
- Symmetry plane correlations: $\langle\cos[mn(\Psi_m - \Psi_n)]\rangle$
- Probability density function: $\mathbf{P}(v_n)$
- …



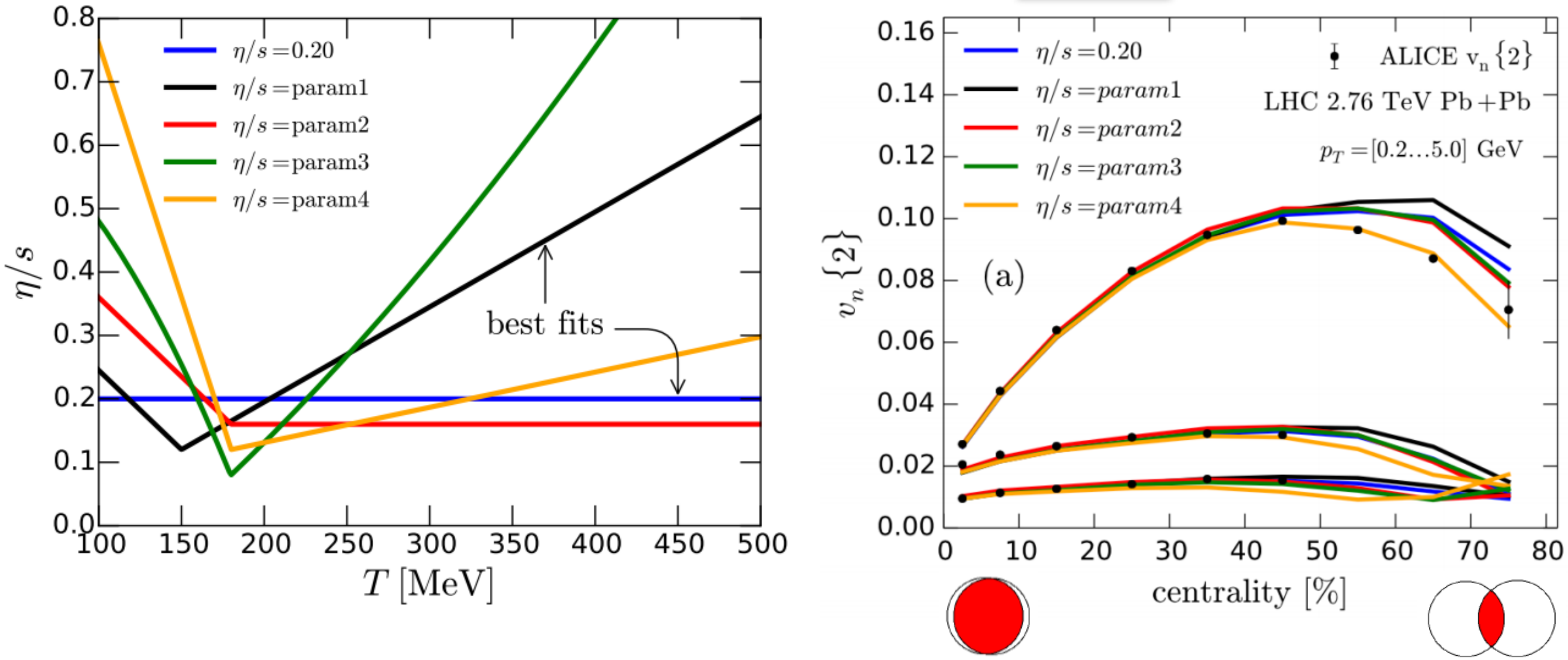






# ‘Classical’ flow observables

- Individual $v_n$ harmonics are sensitive only to average values of η/s
  - Insensitivity to the differential temperature dependence of η/s



H. Niemi, K. J. Eskola, R. Paatelainen, Phys. Rev. C 93, 024907 (2016), e-Print: 1505.02677

# Historical account

- New observables were introduced in 2012, independently by theorists and experimentalists:
  - Theorists: $p_\mathrm{T}$ dependence of covariance between $v_m$ and $v_n$
  - Experimentalists: even moments of $v_n^{2k}$ and their intercorrelations => Symmetric Cumulants (SC)
- Two sets of observables were proposed on exactly the same day 5th of December 2012!

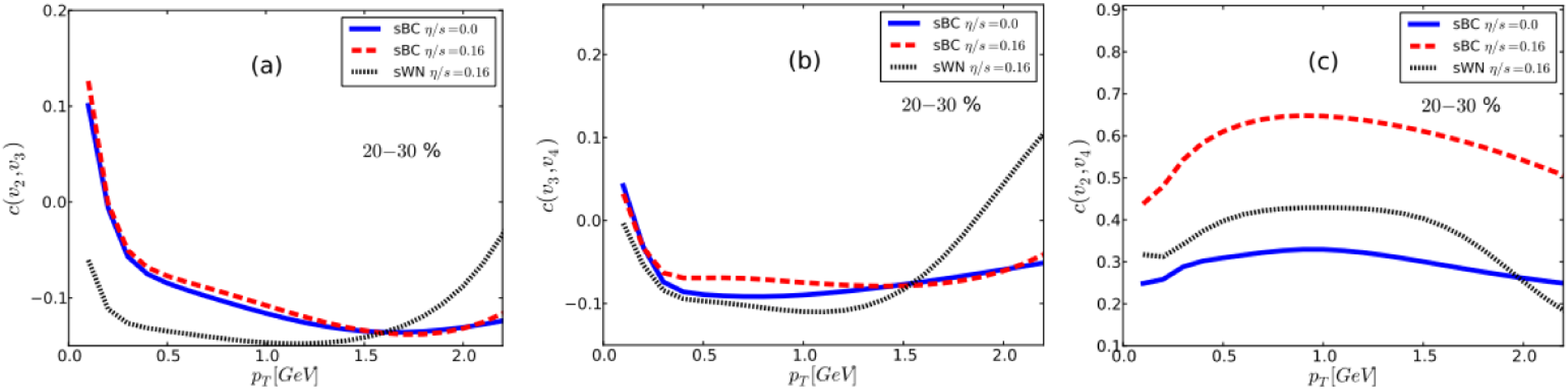
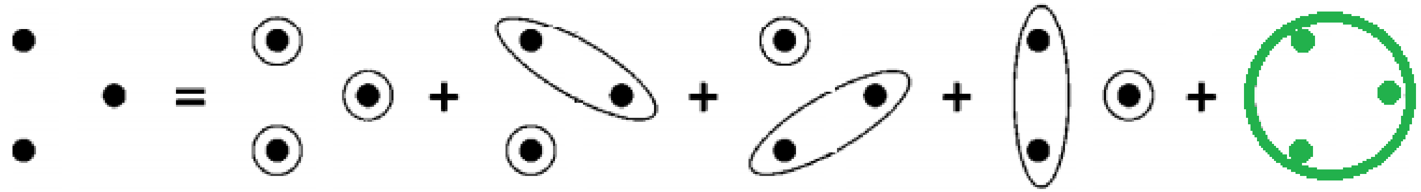


Figure 9 from H. Niemi, G. S. Denicol, H. Holopainen and P. Huovinen,
Phys. Rev. C **87** (2013) 5, 054901 313, e-Print: 1212.1008

# Multiparticle correlations and cumulants

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, Phys. Rev. C **84** 034910 (2011), e-Print: 1104.4740

Kubo, Journal of the Physical Society of Japan, Vol. 17, No. 7, (1962)



# Independent information

- To constrain the functional form of $f(v_1, \ldots, v_n, \Psi_1, \ldots, \Psi_n)$, we have to measure all its **valid moments (or cumulants)**

- The most general mathematical result, which relates multiparticle azimuthal correlators and flow degrees of freedom:

$$\left\langle e^{i(n_1\varphi_1+\cdots+n_k\varphi_k)}\right\rangle = v_{n_1}\cdots v_{n_k}e^{i(n_1\Psi_{n_1}+\cdots+n_k\Psi_{n_k})}$$

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, Phys. Rev. C **84** 034910 (2011), e-Print: 1104.4740

**Open questions:**

- What are the smallest collision systems and energies at which QGP can be formed?
- How to extract new and independent constraints from the available heavy-ion data?
- Is the observed universality of flow measurements in vastly different collision systems physical, or just a subtle artifact of using correlation techniques in the randomized data set?

# 2-particle cumulants in general

- Cumulants are alternative to moments to describe stochastic properties of variable
- If 2 p.d.f.'s have same moments, they will also have same cumulants, and vice versa
  - True both for univariate and multivariate case

- $X_i$ denotes the general $i$-th stochastic variable
- The most general decomposition of 2-particle correlation is:

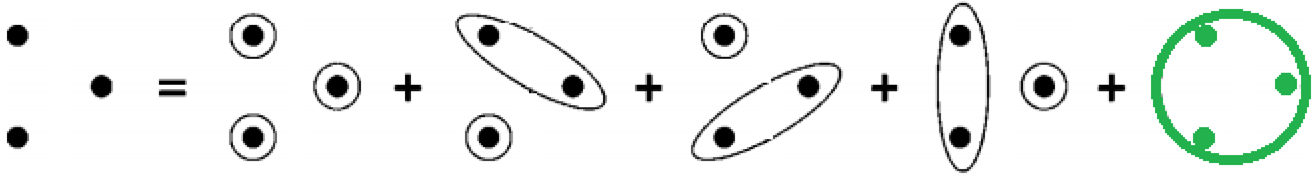
$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2nd term on RHS is 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

# 3-particle cumulants in general

- The most general decomposition of 3-particle correlation is:



- Or written mathematically:

$$\begin{aligned}\langle X_1X_2X_3\rangle &= \langle X_1\rangle\langle X_2\rangle\langle X_3\rangle\\ &+ \langle X_1X_2\rangle_c\langle X_3\rangle+\langle X_1X_3\rangle_c\langle X_2\rangle+\langle X_2X_3\rangle_c\langle X_1\rangle\\ &+ \langle X_1X_2X_3\rangle_c\end{aligned}$$

- The key point: 2-particle cumulants were expressed independently in terms of measured correlations in previous step!

$$\langle X_1X_2\rangle_c=\langle X_1X_2\rangle-\langle X_1\rangle\langle X_2\rangle$$

# 3-particle cumulants in general

- Working recursively from higher to lower orders, we eventually have 3-particle cumulant expressed in terms of measured 3-, 2-, and 1-particle averages

$$\begin{aligned}\langle X_1X_2X_3\rangle_c &= \langle X_1X_2X_3\rangle \\ &- \langle X_1X_2\rangle\langle X_3\rangle - \langle X_1X_3\rangle\langle X_2\rangle - \langle X_2X_3\rangle\langle X_1\rangle \\ &+ 2\langle X_1\rangle\langle X_2\rangle\langle X_3\rangle\end{aligned}$$

- In the same way, cumulants can be expressed in terms of measurable averages for any number of particles
  - The number of terms grows rapidly

# A bit of math

- The mathematical foundation of cumulants is well established!

**Theorem:** A cumulant $\langle X_i X_j \ldots \rangle_c$ is zero if the elements $X_i, X_j$ , ... are devided in two or more groups which are statistically independent.

**Collorary:** A cumulant is zero if one of the variables in it is independent of the others. Conversely, a cumulant is not zero if and only if the variables in it are statistically connected.

Kubo, Journal of the Physical Society of Japan, Vol. 17, No. 7, (1962)

- Careful reading is mandatory, one statement is not covered:

$$\kappa = 0 \Leftarrow \text{variables are independent}\,,$$ ✓

$$\kappa = 0 \not\Rightarrow \text{variables are independent}\,,$$ ✗

$$\kappa \neq 0 \Leftrightarrow \text{variables are not independent}\,.$$ ✓

Cumulant can be trivially zero due to underlying symmetries!

# ‘Multivariate cumulants in flow analyses: The Next Generation’

AB, M. Lesch, C. Mordasini, F. Taghavi, Phys. Rev. C **105** (2022) 2, 024912, e-Print: [2101.05619](2101.05619)

# Choice of fundamental observable

- Cumulants as used in flow analyses in the last ~25 years:
  1. Cumulant expansion is performed on azimuthal angles
  2. Azimuthal correlators which are not isotropic are dropped
  3. The final result is merely re-expressed in terms of flow degrees of freedom $v_n$ and $\Psi_n$ via the analytic relation

$$\left\langle e^{i(n_1\varphi_1+\cdots+n_k\varphi_k)}\right\rangle = v_{n_1}\cdots v_{n_k}e^{i(n_1\Psi_{n_1}+\cdots+n_k\Psi_{n_k})}$$

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, Phys. Rev. C **84** 034910 (2011), e-Print: 1104.4740

- Few additional remarks:
  - Cumulants of $v_n$ and $v_n^2$ are in general different
  - $v_n$ and $\Psi_n$ have different properties (e.g. with respect to Lorentz invariance)

# The root of the problem

- General 2-particle cumulant

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

  - **Old paradigm:** fundamental observable is an angle

$$X_1 \rightarrow e^{in\varphi_1}\,, \quad X_2 \rightarrow e^{-in\varphi_2}$$

  - **New paradigm:** fundamental observable is a flow amplitude

$$X_1 \rightarrow v_n^2\,, \quad X_2 \rightarrow v_m^2$$

- Both choices yielded **accidentally the same results** for SC($m$,$n$) observables
- But results for SC($k$,$l$,$m$), SC($k$,$l$,$m$,$n$), etc., are in general different
  - Which paradigm is correct in general?

C. Mordasini, AB, D. Karakoç, F. Taghavi, Phys. Rev. C **102**, 024907 (2020), e-Print 1901.06968

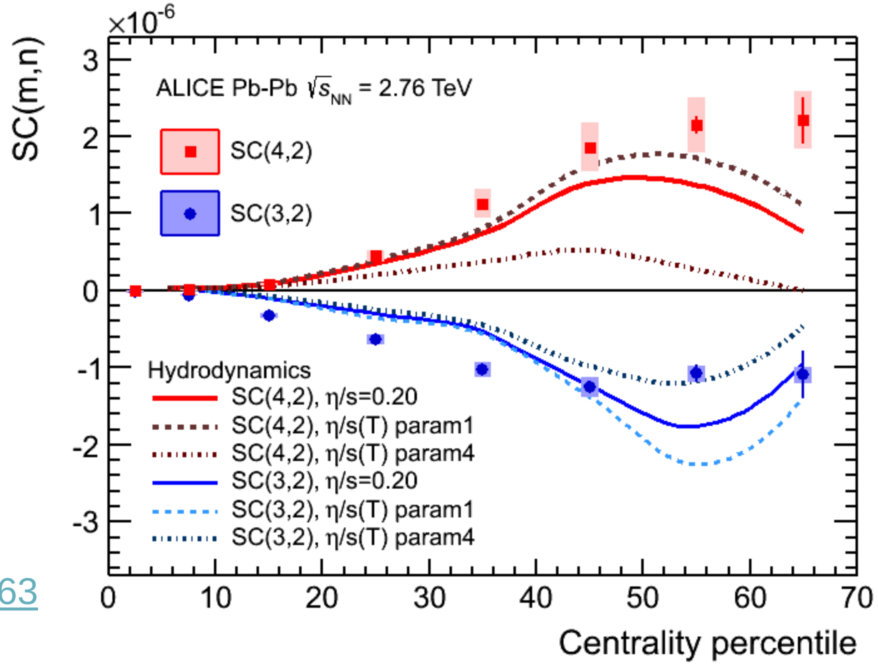
# Symmetric Cumulants SC(*m*,*n*)

- How to quantify experimentally the correlation between two different $v_n$ amplitudes?
  - Symmetric Cumulants (Section IVC in Phys. Rev. C **89** (2014) no.6, 064904, e-Print: 1312.3572)

$$\begin{aligned}\mathrm{SC}(m,n) \equiv & \langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle_c \\ = & \langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle \\ & - \langle\langle\cos[m(\varphi_1 - \varphi_2)]\rangle\rangle\langle\langle\cos[n(\varphi_1 - \varphi_2)]\rangle\rangle \\ = & \left\langle v_m^2 v_n^2\right\rangle - \left\langle v_m^2\right\rangle\left\langle v_n^2\right\rangle\end{aligned}$$

**Old paradigm definition**

- SC observables are sensitive to differential η/s(T) parametrizations
- Individual flow amplitudes are dominated by averages ⟨η/s(T)⟩
- Independent constraints both on initial conditions and QGP properties



**ALICE Collaboration**, Phys. Rev. Lett. 117, 182301 (2016), e-Print: 1604.07663

# Symmetric Cumulants SC($m$,$n$)

- How to quantify experimentally the correlation between two different $v_n$ amplitudes?
  - Symmetric Cumulants (Section IVC in Phys. Rev. C **89** (2014) no.6, 064904, e-Print: 1312.3572)

$$
\begin{aligned}
\mathrm{SC}(m,n) \equiv\ & \langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle_c \\
&= \langle\langle\cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4)\rangle\rangle \\
&\quad - \langle\langle\cos[m(\varphi_1 - \varphi_2)]\rangle\rangle\langle\langle\cos[n(\varphi_1 - \varphi_2)]\rangle\rangle \\
&= \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle
\end{aligned}
$$

← **Old paradigm definition**

$$
\begin{aligned}
\mathrm{SC}(m,n) &\equiv \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle \\
&= \langle\langle e^{i(m\varphi_1+n\varphi_2-m\varphi_3-n\varphi_4)}\rangle\rangle - \langle\langle e^{im(\varphi_1-\varphi_2)}\rangle\rangle\langle\langle e^{in(\varphi_1-\varphi_2)}\rangle\rangle
\end{aligned}
$$

← **New paradigm definition**

C. Mordasini, AB, D. Karakoç, F. Taghavi, Phys. Rev. C **102**, 024907 (2020), e-Print 1901.06968

AB, M. Lesch, C. Mordasini, F. Taghavi, Phys. Rev. C **105** (2022) 2, 024912, e-Print: 2101.05619

# Generalization: $SC(k,l,m)$, $SC(k,l,m,n)$, …

- New paradigm:
  - Cumulant expansion directly on flow amplitudes $v^2$:

$$\mathrm{SC}(k,l,m) \equiv \langle v_k^2 v_l^2 v_m^2 \rangle - \langle v_k^2 v_l^2 \rangle \langle v_m^2 \rangle - \langle v_k^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_l^2 v_m^2 \rangle \langle v_k^2 \rangle + 2 \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle$$

  - Azimuthal angles are used merely to build an estimator for the above expression, term-by-term:

$$\begin{aligned}\mathrm{SC}(k,l,m) &= \langle\langle\cos[k\varphi_1 + l\varphi_2 + m\varphi_3 - k\varphi_4 - l\varphi_5 - m\varphi_6]\rangle\rangle \\ &- \langle\langle\cos[k\varphi_1 + l\varphi_2 - k\varphi_3 - l\varphi_4]\rangle\rangle \langle\langle\cos[m(\varphi_5 - \varphi_6)]\rangle\rangle \\ &- \langle\langle\cos[k\varphi_1 + m\varphi_2 - k\varphi_5 - m\varphi_6]\rangle\rangle \langle\langle\cos[l(\varphi_3 - \varphi_4)]\rangle\rangle \\ &- \langle\langle\cos[l\varphi_3 + m\varphi_4 - l\varphi_5 - m\varphi_6]\rangle\rangle \langle\langle\cos[k(\varphi_1 - \varphi_2)]\rangle\rangle \\ &+ 2\,\langle\langle\cos[k(\varphi_1 - \varphi_2)]\rangle\rangle \langle\langle\cos[l(\varphi_3 - \varphi_4)]\rangle\rangle \langle\langle\cos[m(\varphi_5 - \varphi_6)]\rangle\rangle\end{aligned}$$

C. Mordasini, AB, D. Karakoç, F. Taghavi, Phys. Rev. C **102**, 024907 (2020), e-Print 1901.06968

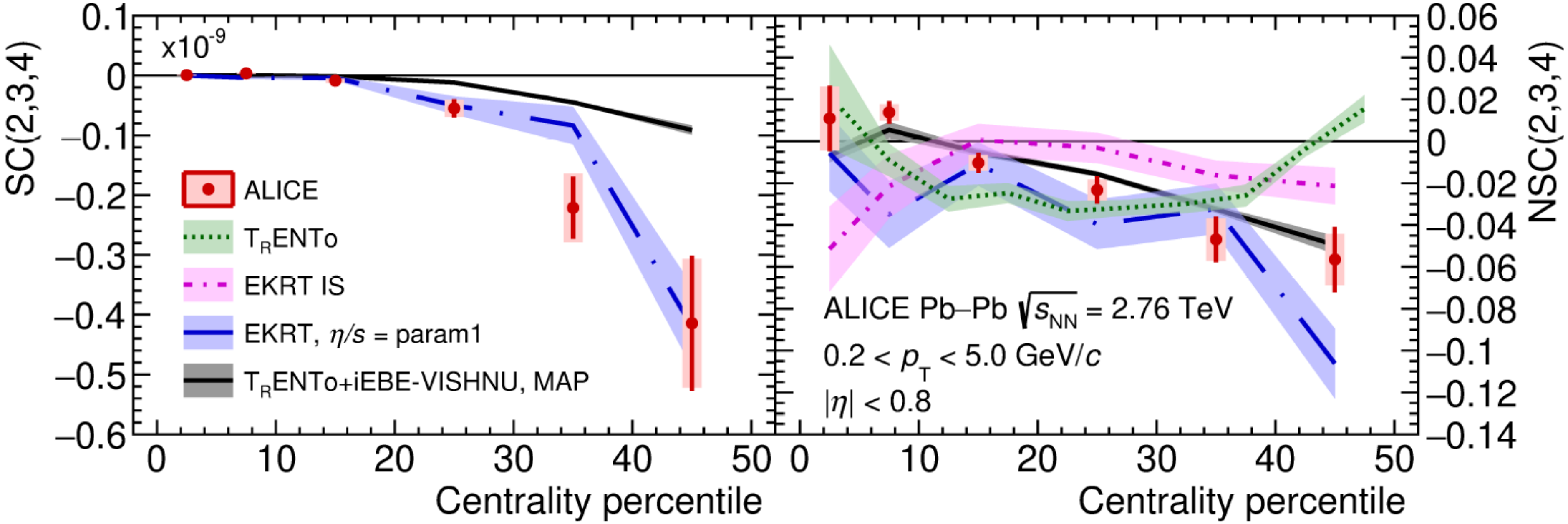
AB, M. Lesch, C. Mordasini, F. Taghavi, Phys. Rev. C **105** (2022) 2, 024912, e-Print: 2101.05619

# Higher-order Symmetric Cumulants SC($k$,$l$,$m$)

- Generalization to more than two $v_n$ harmonics is not trivial
- New definition for higher-order SC observables is always based on a **new paradigm** approach

  AB, Marcel Lesch, Cindy Mordasini, Seyed Farid Taghavi, Phys. Rev. C **105** (2022) 2, 024912, e-Print: 2101.05619

- All fundamental properties of cumulants are satisfied => independent input for Bayesian studies

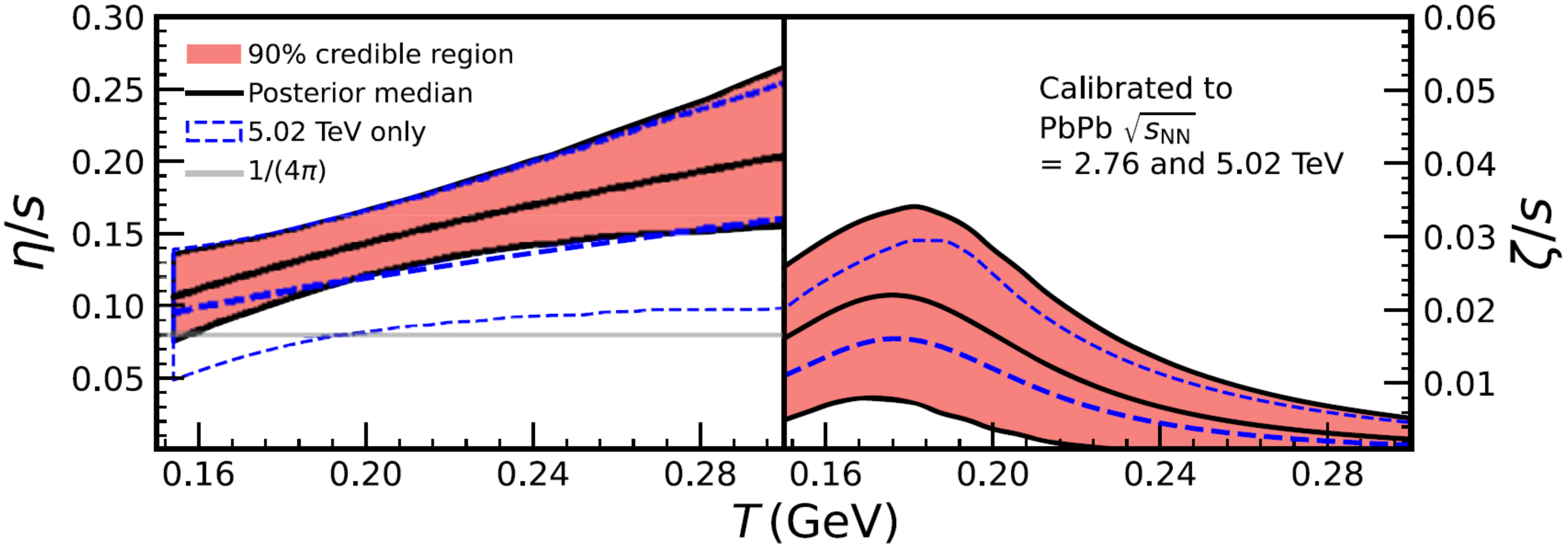


ALI-DER-479271

**ALICE Collaboration**, Phys. Rev. Lett. 127 (2021) 9, 092302, e-Print: 2101.02579

# Bayesian studies

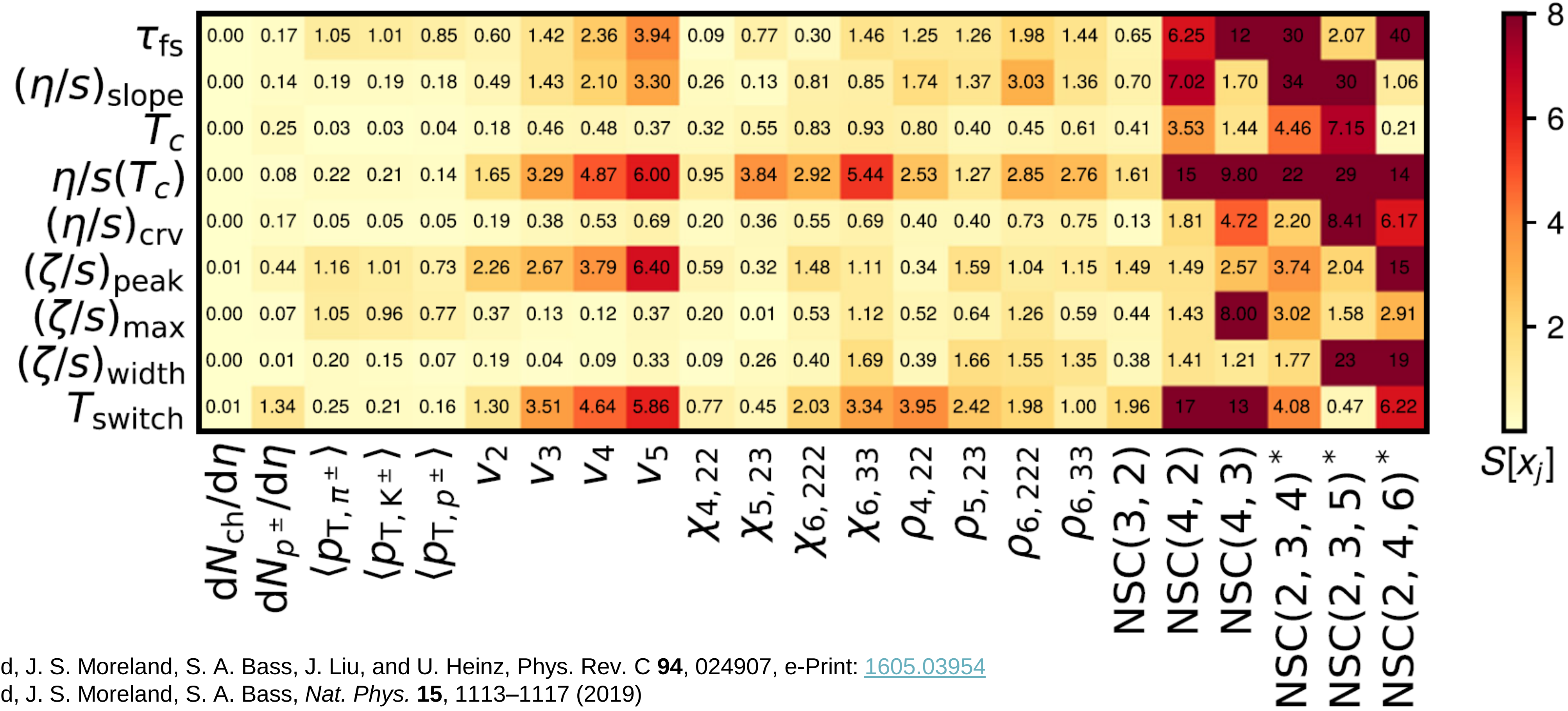
- Constraints on the temperature dependence of specific shear and bulk viscosity change significantly, after including higher-order $v_n$ observables in Bayesian studies



J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu, and U. Heinz, Phys. Rev. C **94**, 024907, e-Print: 1605.03954
J. E. Bernhard, J. S. Moreland, S. A. Bass, *Nat. Phys.* **15**, 1113–1117 (2019)
J.E.Parkkila, A.Onnerstad, S.F.Taghavi, C.Mordasini, AB, M.Virta, D.J.Kim, Phys. Lett. B **835** (2022), 137485, e-Print: 2111.08145

# Bayesian studies

- The highest sensitivity on model parameters comes from higher-order SC observables!

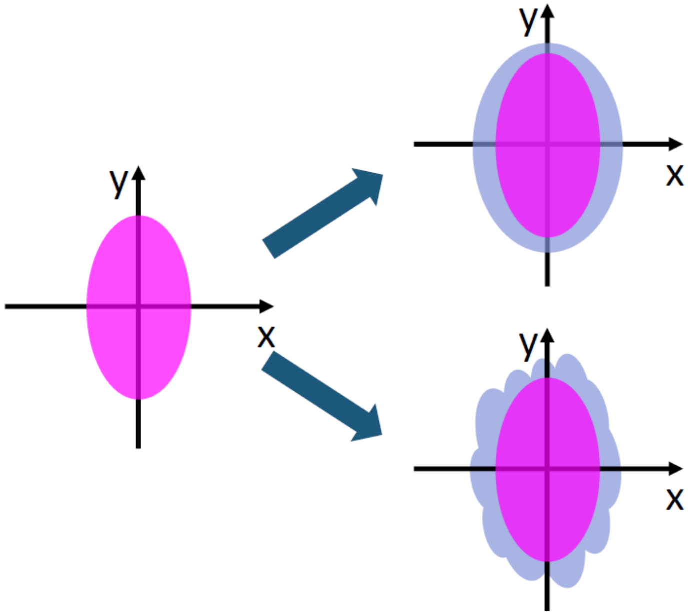


| | $dN_{ch}/d\eta$ | $dN_{p^{\pm}}/d\eta$ | $\langle p_{T,\pi^{\pm}}\rangle$ | $\langle p_{T,K^{\pm}}\rangle$ | $\langle p_{T,p^{\pm}}\rangle$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $\chi_{4,22}$ | $\chi_{5,23}$ | $\chi_{6,222}$ | $\chi_{6,33}$ | $\rho_{4,22}$ | $\rho_{5,23}$ | $\rho_{6,222}$ | $\rho_{6,33}$ | NSC(3, 2) | NSC(4, 2) | NSC(4, 3) | NSC(2, 3, 4)* | NSC(2, 3, 5)* | NSC(2, 4, 6)* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\tau_{fs}$ | 0.00 | 0.17 | 1.05 | 1.01 | 0.85 | 0.60 | 1.42 | 2.36 | 3.94 | 0.09 | 0.77 | 0.30 | 1.46 | 1.25 | 1.26 | 1.98 | 1.44 | 0.65 | 6.25 | 12 | 30 | 2.07 | 40 |
| $(\eta/s)_{slope}$ | 0.00 | 0.14 | 0.19 | 0.19 | 0.18 | 0.49 | 1.43 | 2.10 | 3.30 | 0.26 | 0.13 | 0.81 | 0.85 | 1.74 | 1.37 | 3.03 | 1.36 | 0.70 | 7.02 | 1.70 | 34 | 30 | 1.06 |
| $T_c$ | 0.00 | 0.25 | 0.03 | 0.03 | 0.04 | 0.18 | 0.46 | 0.48 | 0.37 | 0.32 | 0.55 | 0.83 | 0.93 | 0.80 | 0.40 | 0.45 | 0.61 | 0.41 | 3.53 | 1.44 | 4.46 | 7.15 | 0.21 |
| $\eta/s(T_c)$ | 0.00 | 0.08 | 0.22 | 0.21 | 0.14 | 1.65 | 3.29 | 4.87 | 6.00 | 0.95 | 3.84 | 2.92 | 5.44 | 2.53 | 1.27 | 2.85 | 2.76 | 1.61 | 15 | 9.80 | 22 | 29 | 14 |
| $(\eta/s)_{crv}$ | 0.00 | 0.17 | 0.05 | 0.05 | 0.05 | 0.19 | 0.38 | 0.53 | 0.69 | 0.20 | 0.36 | 0.55 | 0.69 | 0.40 | 0.40 | 0.73 | 0.75 | 0.13 | 1.81 | 4.72 | 2.20 | 8.41 | 6.17 |
| $(\zeta/s)_{peak}$ | 0.01 | 0.44 | 1.16 | 1.01 | 0.73 | 2.26 | 2.67 | 3.79 | 6.40 | 0.59 | 0.32 | 1.48 | 1.11 | 0.34 | 1.59 | 1.04 | 1.15 | 1.49 | 1.49 | 2.57 | 3.74 | 2.04 | 15 |
| $(\zeta/s)_{max}$ | 0.00 | 0.07 | 1.05 | 0.96 | 0.77 | 0.37 | 0.13 | 0.12 | 0.37 | 0.20 | 0.01 | 0.53 | 1.12 | 0.52 | 0.64 | 1.26 | 0.59 | 0.44 | 1.43 | 8.00 | 3.02 | 1.58 | 2.91 |
| $(\zeta/s)_{width}$ | 0.00 | 0.01 | 0.20 | 0.15 | 0.07 | 0.19 | 0.04 | 0.09 | 0.33 | 0.09 | 0.26 | 0.40 | 1.69 | 0.39 | 1.66 | 1.55 | 1.35 | 0.38 | 1.41 | 1.21 | 1.77 | 23 | 19 |
| $T_{switch}$ | 0.01 | 1.34 | 0.25 | 0.21 | 0.16 | 1.30 | 3.51 | 4.64 | 5.86 | 0.77 | 0.45 | 2.03 | 3.34 | 3.95 | 2.42 | 1.98 | 1.00 | 1.96 | 17 | 13 | 4.08 | 0.47 | 6.22 |

J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu, and U. Heinz, Phys. Rev. C **94**, 024907, e-Print: 1605.03954
J. E. Bernhard, J. S. Moreland, S. A. Bass, *Nat. Phys.* **15**, 1113–1117 (2019)
J.E.Parkkila, A.Onnerstad, S.F.Taghavi, C.Mordasini, AB, M.Virta, D.J.Kim, Phys. Lett. B **835** (2022), 137485, e-Print: 2111.08145

# Shear vs. bulk viscosities

- Can we separate the effects of shear (η) and bulk (ξ) viscosities?



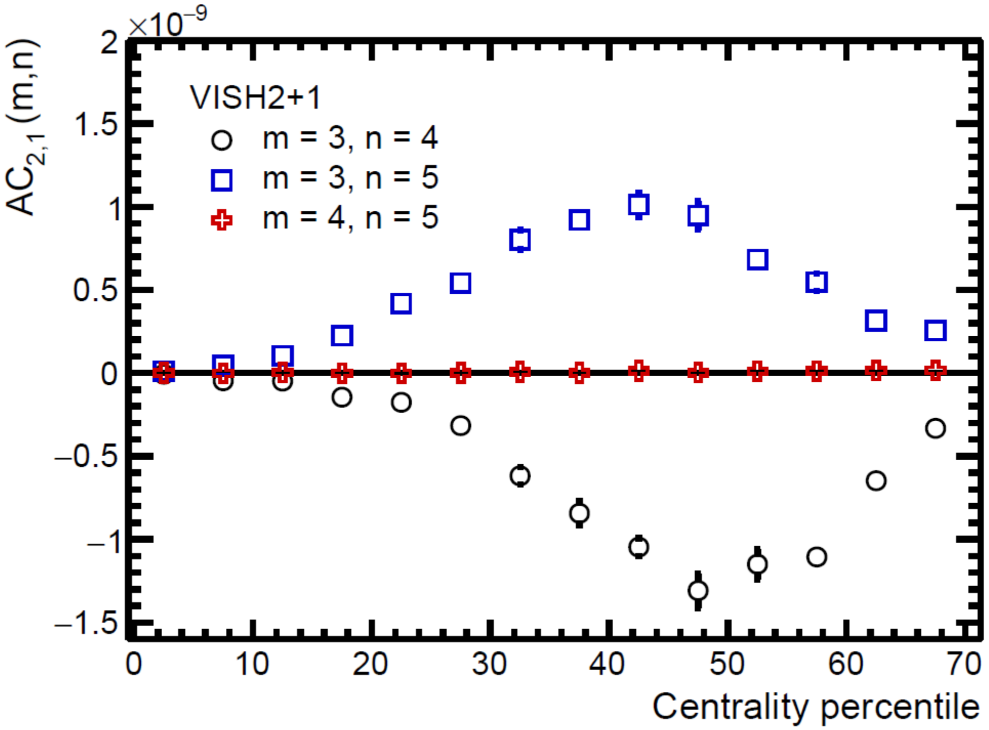
Credits: C. Mordasini

- Isotropic fluctuations
  - Neighbouring layers move at equal velocities
  - Generally preserves the ellipse shape
  - Main sensitivity to $\xi/s$
- Shape fluctuations
  - Neighbouring layers move at different velocities
  - Sensitivity to $\eta/s$

- We need new observables which can separate these different sources of fluctuations

# Asymmetric Cumulants $\mathrm{AC}_{k,l}(m,n)$

- Generalization of Symmetric Cumulants: $\mathrm{SC}(m,n) = \mathrm{AC}_{1,1}(m,n)$
- Fundamental stochastic observable is $v^2$
- Each of these observables is insensitive to lower-order correlations, because they satisfy all mathematical properties of cumulants



$$\mathrm{AC}_{2,1}(m,n) = \langle v_m^4 v_n^2\rangle - \langle v_m^4\rangle\langle v_n^2\rangle - 2\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle + 2\langle v_m^2\rangle^2\langle v_n^2\rangle,$$

$$\begin{aligned}\mathrm{AC}_{3,1}(m,n) = {} & \langle v_m^6 v_n^2\rangle - \langle v_m^6\rangle\langle v_n^2\rangle - 3\langle v_m^2 v_n^2\rangle\langle v_m^4\rangle - 3\langle v_m^4 v_n^2\rangle\langle v_m^2\rangle \\ & + 6\langle v_m^4\rangle\langle v_m^2\rangle\langle v_n^2\rangle + 6\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle^2 - 6\langle v_m^2\rangle^3\langle v_n^2\rangle,\end{aligned}$$

$$\begin{aligned}\mathrm{AC}_{4,1}(m,n) = {} & \langle v_m^8 v_n^2\rangle - \langle v_m^8\rangle\langle v_n^2\rangle - 4\langle v_m^2 v_n^2\rangle\langle v_m^6\rangle - 6\langle v_m^4 v_n^2\rangle\langle v_m^4\rangle \\ & + 6\langle v_m^4\rangle^2\langle v_n^2\rangle - 4\langle v_m^6 v_n^2\rangle\langle v_m^2\rangle + 8\langle v_m^6\rangle\langle v_m^2\rangle\langle v_n^2\rangle \\ & + 24\langle v_m^2 v_n^2\rangle\langle v_m^4\rangle\langle v_m^2\rangle + 12\langle v_m^4 v_n^2\rangle\langle v_m^2\rangle^2 \\ & - 36\langle v_m^4\rangle\langle v_m^2\rangle^2\langle v_n^2\rangle - 24\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle^3 + 24\langle v_m^2\rangle^4\langle v_n^2\rangle,\end{aligned}$$

$$\begin{aligned}\mathrm{AC}_{2,1,1}(k,l,m) = {} & \langle v_k^4 v_l^2 v_m^2\rangle - \langle v_k^4 v_l^2\rangle\langle v_m^2\rangle - \langle v_k^4 v_m^2\rangle\langle v_l^2\rangle - \langle v_k^4\rangle\langle v_l^2 v_m^2\rangle \\ & + 2\langle v_k^4\rangle\langle v_l^2\rangle\langle v_m^2\rangle - 2\langle v_k^2 v_l^2\rangle\langle v_k^2 v_m^2\rangle - 2\langle v_k^2 v_l^2 v_m^2\rangle\langle v_k^2\rangle \\ & + 4\langle v_k^2 v_l^2\rangle\langle v_k^2\rangle\langle v_m^2\rangle + 4\langle v_k^2 v_m^2\rangle\langle v_k^2\rangle\langle v_l^2\rangle \\ & + 2\langle v_k^2\rangle^2\langle v_l^2 v_m^2\rangle - 6\langle v_k^2\rangle^2\langle v_l^2\rangle\langle v_m^2\rangle.\end{aligned}$$

Sec. V in Phys. Rev. C **105** (2022) 2, 024912, e-Print: 2101.05619

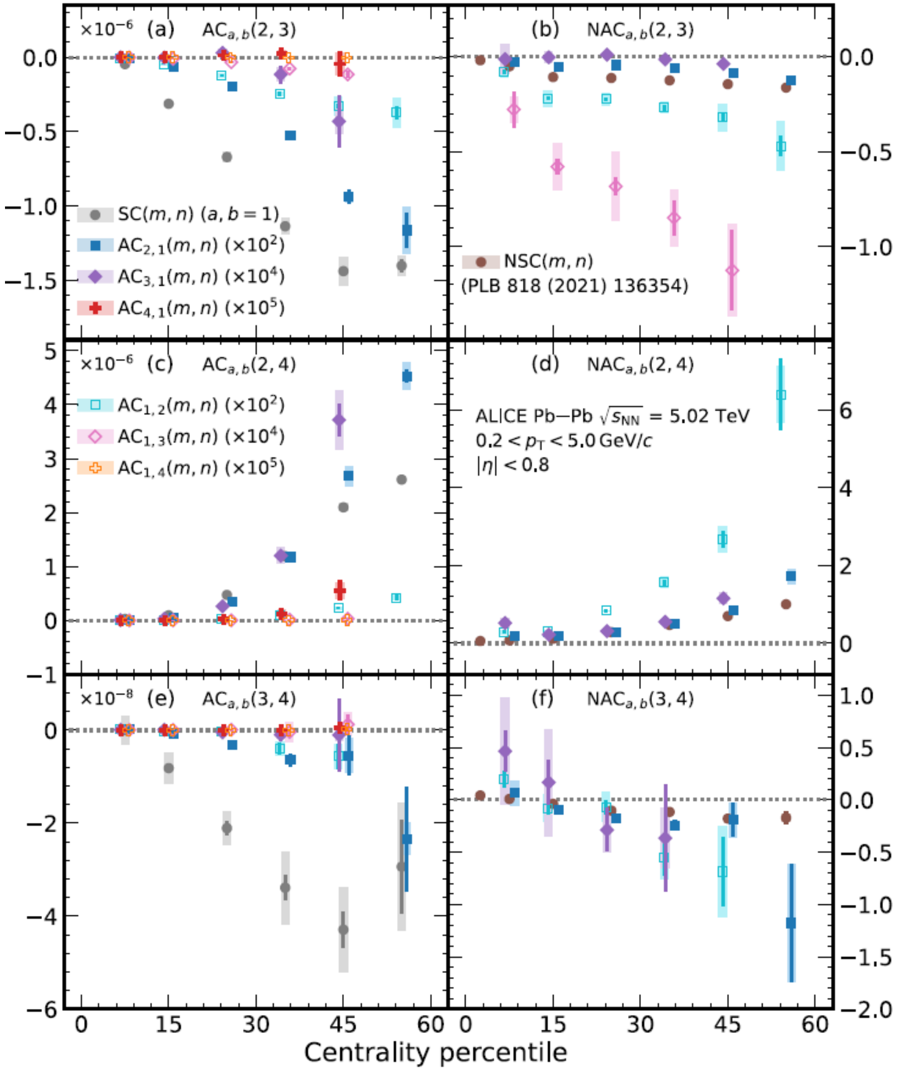
# Asymmetric Cumulants $\mathrm{AC}_{k,l}(m,n)$ in ALICE

- A new set of multiharmonic observables involving different moments of $v_n$ amplitudes
  - Independent information about event-by-event $v_n$ fluctuations, nonlinear response, etc. which is inaccessible to the SC observables

$$\mathrm{AC}_{a,b}(m,n) = \mathrm{AC}_{b,a}(n,m)$$

- The magnitudes of the measured AC observables show a dependence on the different moments as well as on the collision centrality, indicating the presence of nonlinear response in all even moments up to the eighth

- The higher-order asymmetric cumulants show different signatures than the symmetric and lower-order asymmetric cumulants



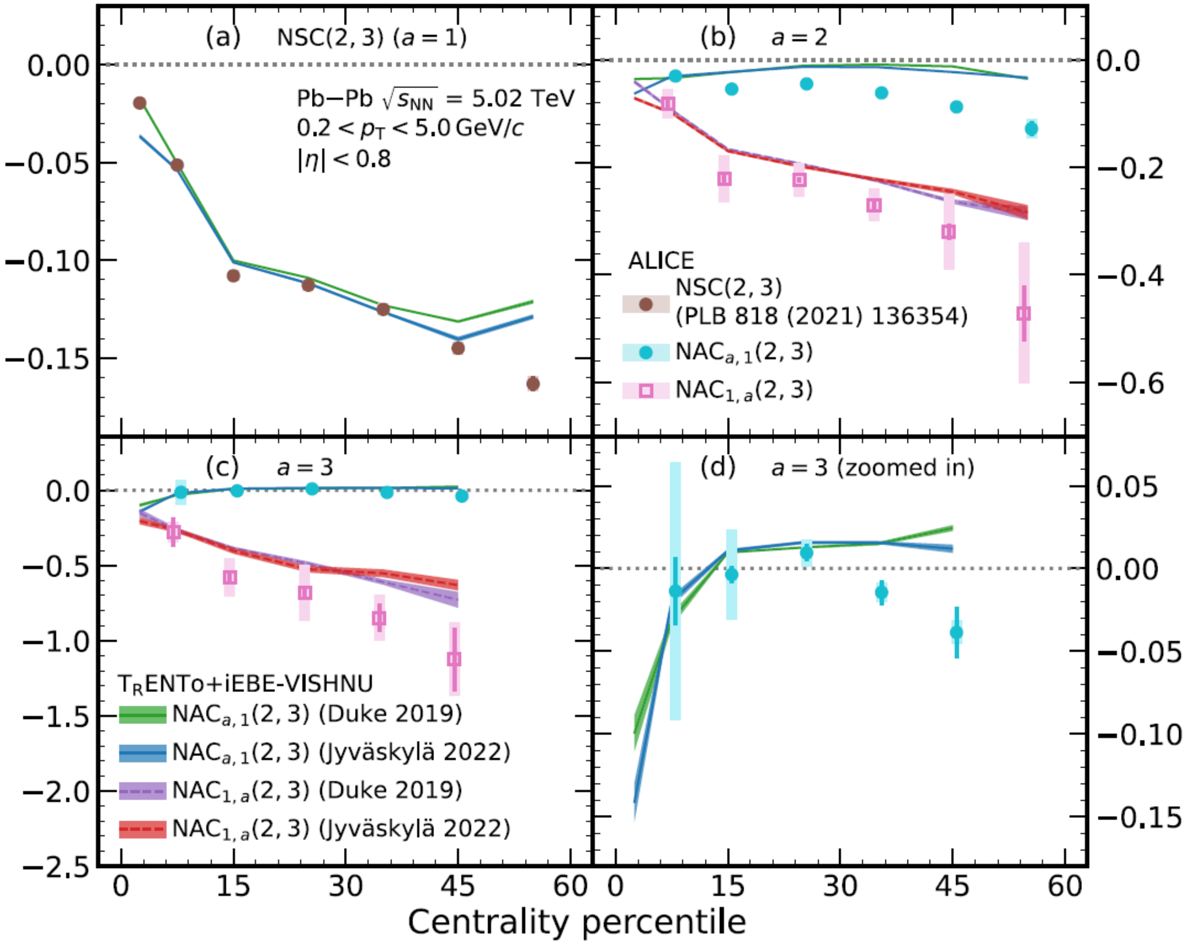
**ALICE Collaboration,** Phys. Rev. C **108** (2023) 5, 055203, e-Print: 2303.13414

# Asymmetric Cumulants $\mathrm{AC}_{k,l}(m,n)$ in ALICE

- Splitting between 'mirror' AC observables $\mathrm{NAC}_{a,1}(2,3)$ and $\mathrm{NAC}_{1,a}(2,3)$ ($a$ = 2, 3) is confirmed by the models

- The biggest discrepancy is for $\mathrm{NAC}_{3,1}(2,3)$: close to zero in data while predicted to be positive by model parameterizations

- No sizable difference between predictions from the two model parameterizations
  - Correlations are dominated by initial state, and in both cases $\mathrm{T_RENTo}$ is used to model it



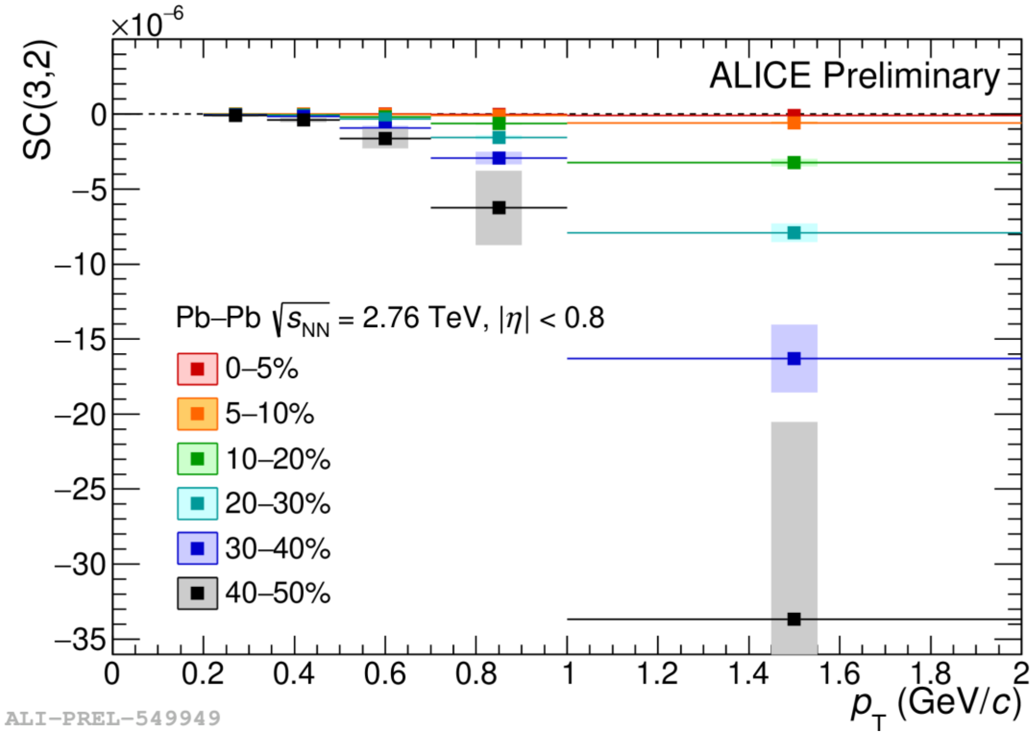
**ALICE Collaboration,** Phys. Rev. C **108** (2023) 5, 055203, e-Print: 2303.13414

# Latest SC/AC results and outlook

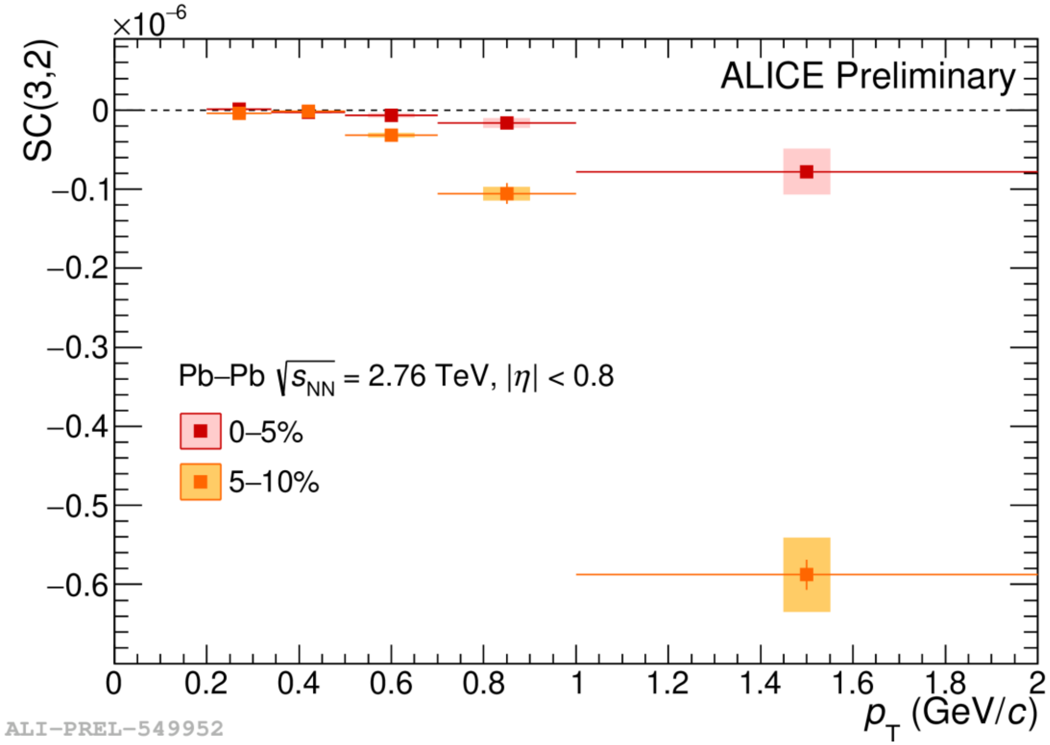
- Differential studies of SC and AC observables

# Differential measurements of SC observables

- First differential measurements of multiharmonic correlations as a function of $p_{\mathrm{T}}$, using SC($k$,$l$) and SC($k$,$l$,$m$) observables
- Further independent input to Bayesian studies



ALI-PREL-549949

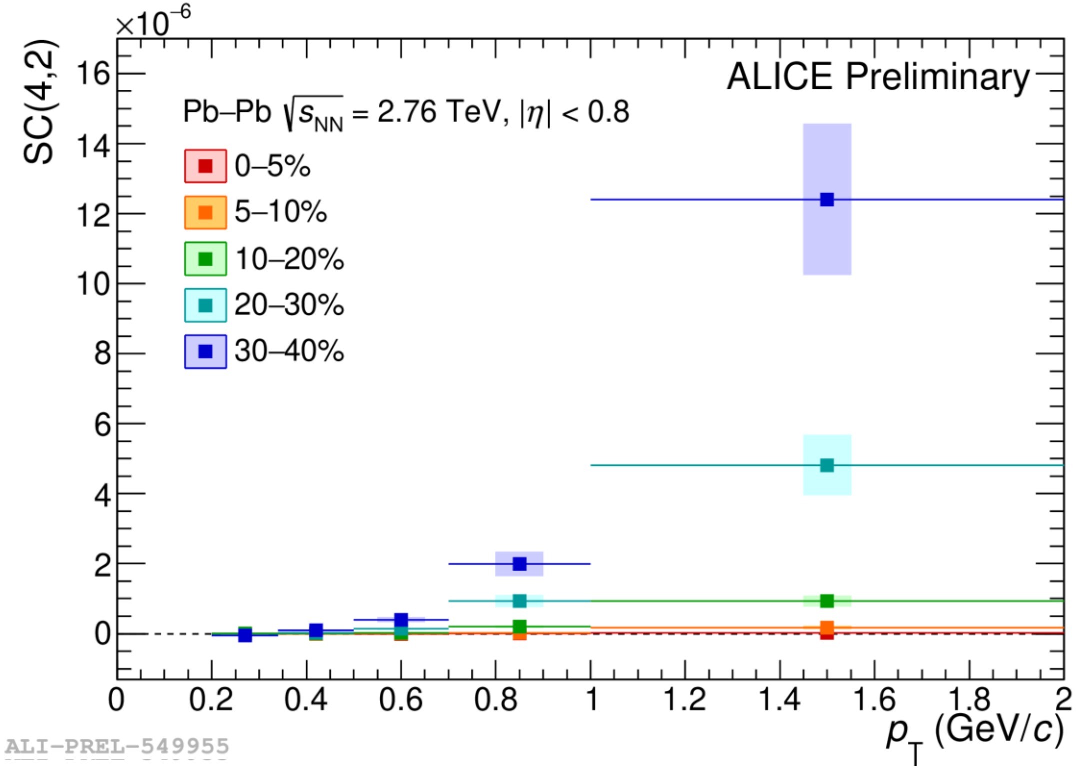


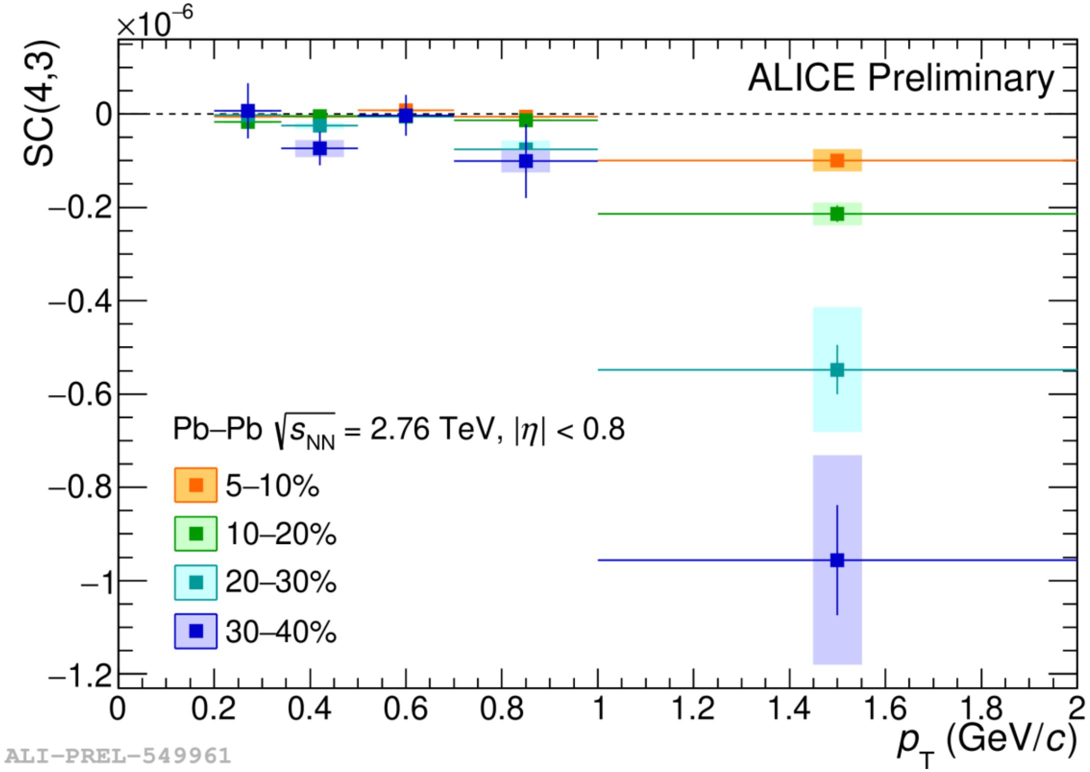
ALI-PREL-549952

AB for **ALICE Collaboration**, QM23 poster

# Differential measurements of SC observables

- First differential measurements of multiharmonic correlations as a function of $p_{\mathrm{T}}$, using SC$(k,l)$ and SC$(k,l,m)$ observables
- Further independent input to Bayesian studies





AB for **ALICE Collaboration**, QM23 poster

# Outlook

- New measurements of multiharmonic correlations using SC and AC observables in new high-statistics Run 3 datasets at LHC:
    - More harmonics
    - Higher orders
    - Energy dependence
    - Differential studies as a function of kinematic variables

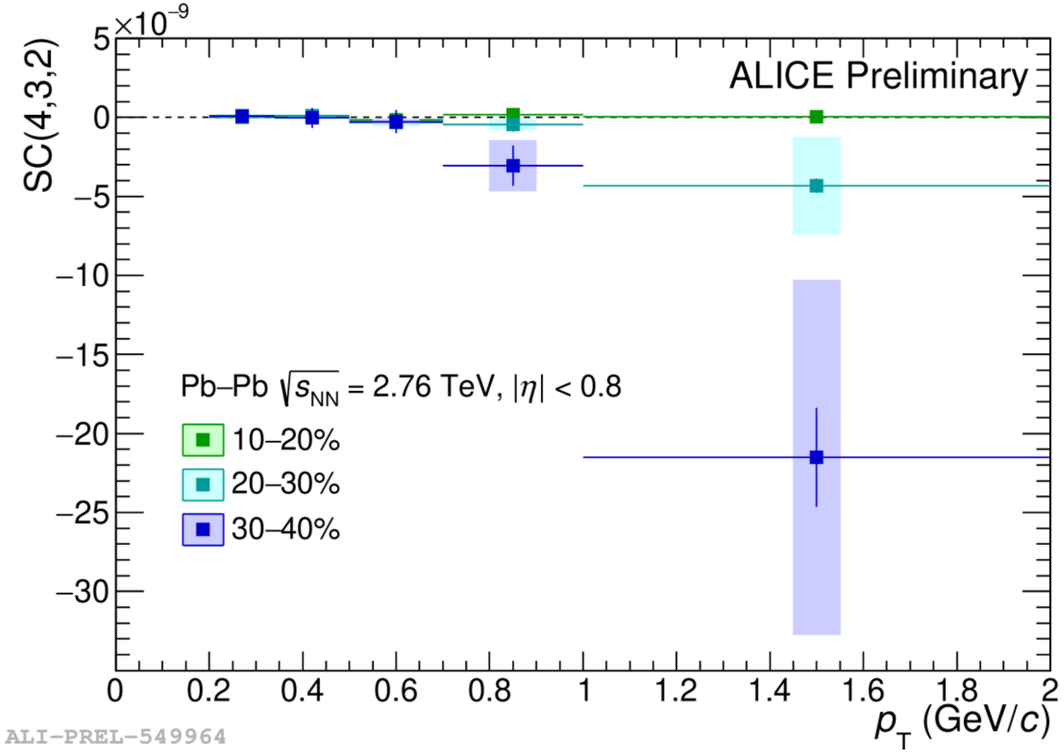
- Separating shape (shear viscosity) and volume fluctuations (bulk viscosity)
- Further independent input to Bayesian studies to constrain all stages in heavy-ion collision

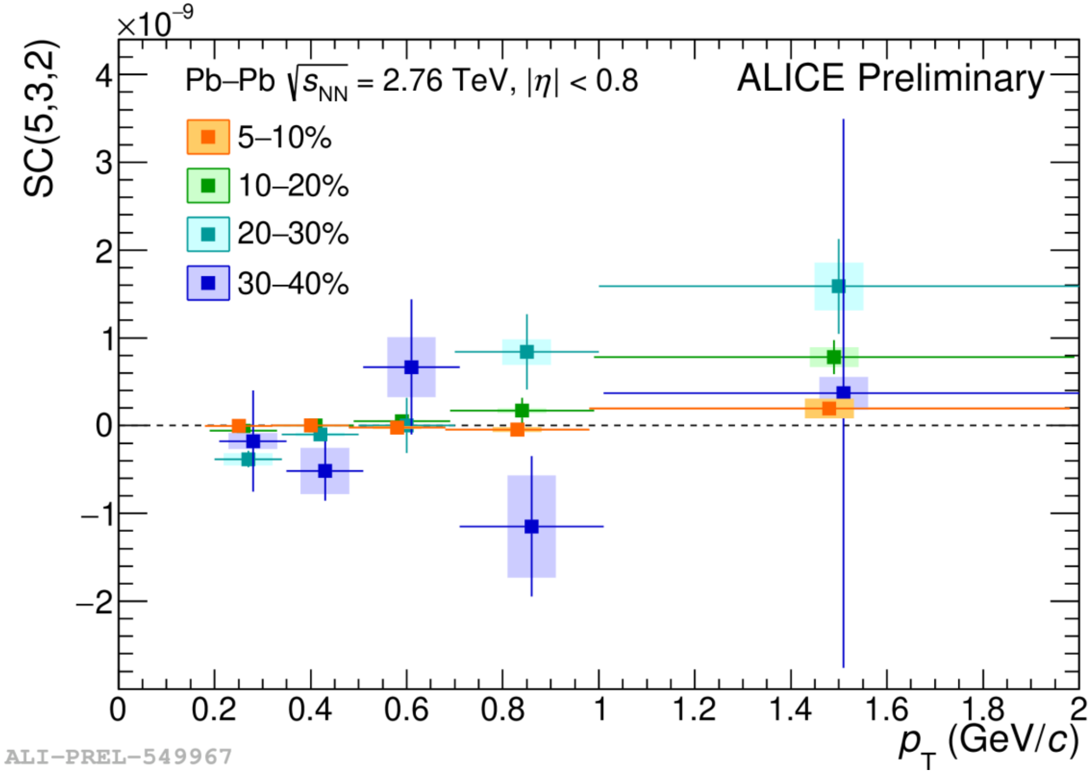
# Thanks!

# Backup slides

# Differential measurements of SC observables

- First differential measurements of multiharmonic correlations as a function of $p_\mathrm{T}$, using SC($k,l$) and SC($k,l,m$) observables
- Further independent input to Bayesian studies





# Fundamental properties of cumulants

- We reviewed everything from scratch and supported proofs for:
    - Statistical independence
    - Reduction
    - Semi-invariance
    - Homogeneity
    - Multilinearity
    - Additivity
    - …

For all technical details, see Section II and Appendix A in e-Print: 2101.05619

- The main strategy in this technical paper is divided into two steps:
    - Confront all existing observables in the field named cumulants with these fundamental properties
    - For the ones which fail to satisfy them, provide the alternative definitions which do satisfy all fundamental properties of cumulants

AB, M. Lesch, C. Mordasini, F. Taghavi, Phys. Rev. C **105** (2022) 2, 024912, e-Print: 2101.05619

# Main conclusions

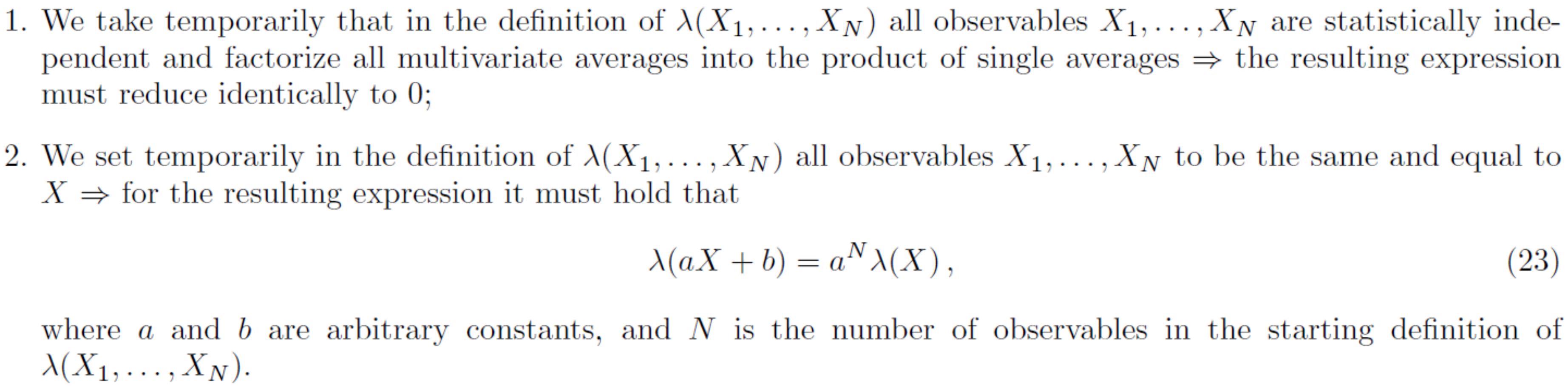
- **The main conclusion #1:** One cannot perform cumulant expansion in one set of stochastic observables, then in the resulting expression perform the transformation to some new set of observables, and then claim that the cumulant properties are preserved in the new set of observables
  - After such transformation, the fundamental properties of cumulants are lost in general

- **The main conclusion #2:** The formal properties of cumulants are valid only if there are no underlying symmetries due to which some terms in the cumulant expansion would vanish identically
  - Due to symmetries, $\left\langle\left\langle e^{in\varphi_i}\right\rangle\right\rangle = 0$, $\left\langle\left\langle e^{in(\varphi_i+\varphi_j)}\right\rangle\right\rangle = 0$, etc., all vanish
  - There are no obvious symmetries for $\langle v_k^2\rangle$, $\langle v_k^2 v_l^2\rangle$, etc., to vanish

AB, M. Lesch, C. Mordasini, F. Taghavi, Phys. Rev. C **105** (2022) 2, 024912, e-Print: 2101.05619

# Necessary conditions for cumulants

- From the fundamental properties of cumulants (statistical independence, reduction, semi-invariance, homogeneity, multilinearity, additivity, etc.), we have established the following two simple necessary conditions:



1. We take temporarily that in the definition of $\lambda(X_1,\ldots,X_N)$ all observables $X_1,\ldots,X_N$ are statistically independent and factorize all multivariate averages into the product of single averages $\Rightarrow$ the resulting expression must reduce identically to 0;
2. We set temporarily in the definition of $\lambda(X_1,\ldots,X_N)$ all observables $X_1,\ldots,X_N$ to be the same and equal to $X$ $\Rightarrow$ for the resulting expression it must hold that

$$\lambda(aX+b) = a^N\lambda(X)\,, \tag{23}$$

where $a$ and $b$ are arbitrary constants, and $N$ is the number of observables in the starting definition of $\lambda(X_1,\ldots,X_N)$.

- A multivariate random variable is a multivariate cumulant only if it satisfies both requirements above

AB, M. Lesch, C. Mordasini, F. Taghavi, Phys. Rev. C **105** (2022) 2, 024912, e-Print: 2101.05619

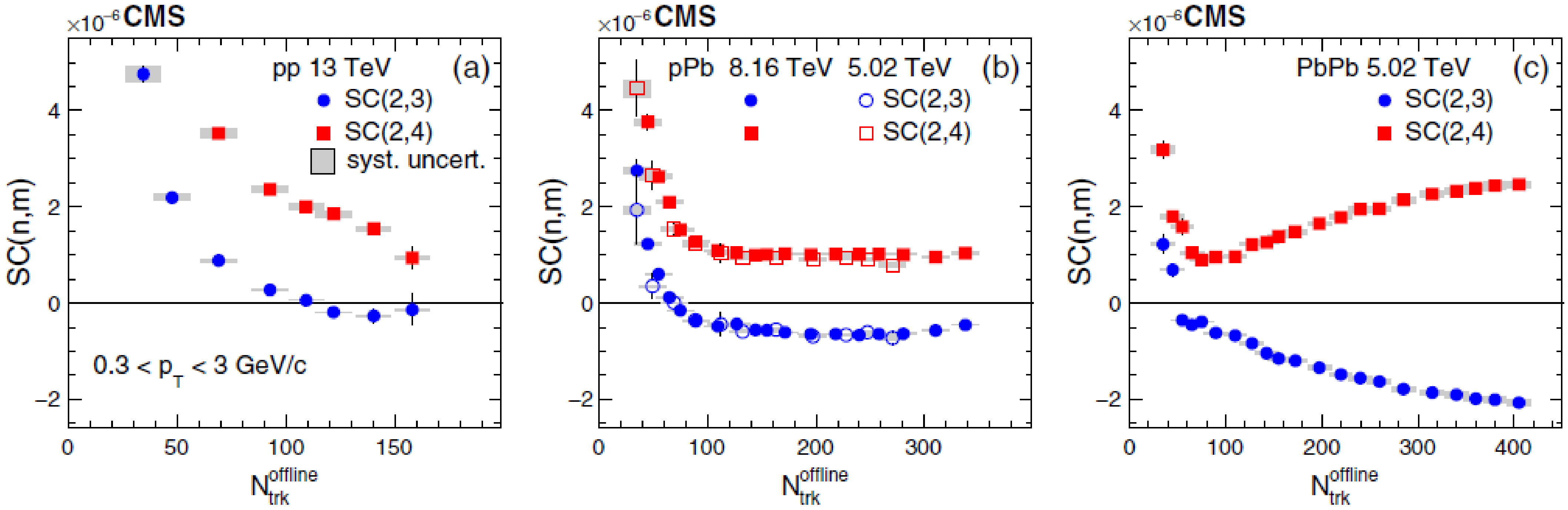
# Reconciliation

- New flow observables (‘The Next Generation’) which do satisfy all formal mathematical properties of cumulants:
  - **‘Symmetric and Asymmetric Cumulants’** (genuine multiharmonic correlations of flow amplitudes)
    - See arXiv:1901.06968 and Sec. V in arXiv:2101.05619
  - **‘Cumulants of symmetry plane correlations’**
    - See Sec. VI in arXiv:2101.05619
  - **‘Event-by-event cumulants of azimuthal angles’**
    - See Sec. IV in arXiv:2101.05619 and arXiv: 2106.05760

- **Open question:** Is it possible to build cumulants of complex flow vectors $v_n e^{in\Psi_n}$?
  - The underlying problem: $\langle v_n e^{in\Psi_n}\rangle_{\text{events}} = 0$
  - Properties of cumulants are lost at any order

# Intermezzo: SC in small collision systems

- There is a big open question in the field: What is the smallest collision system in which QGP can be produced?

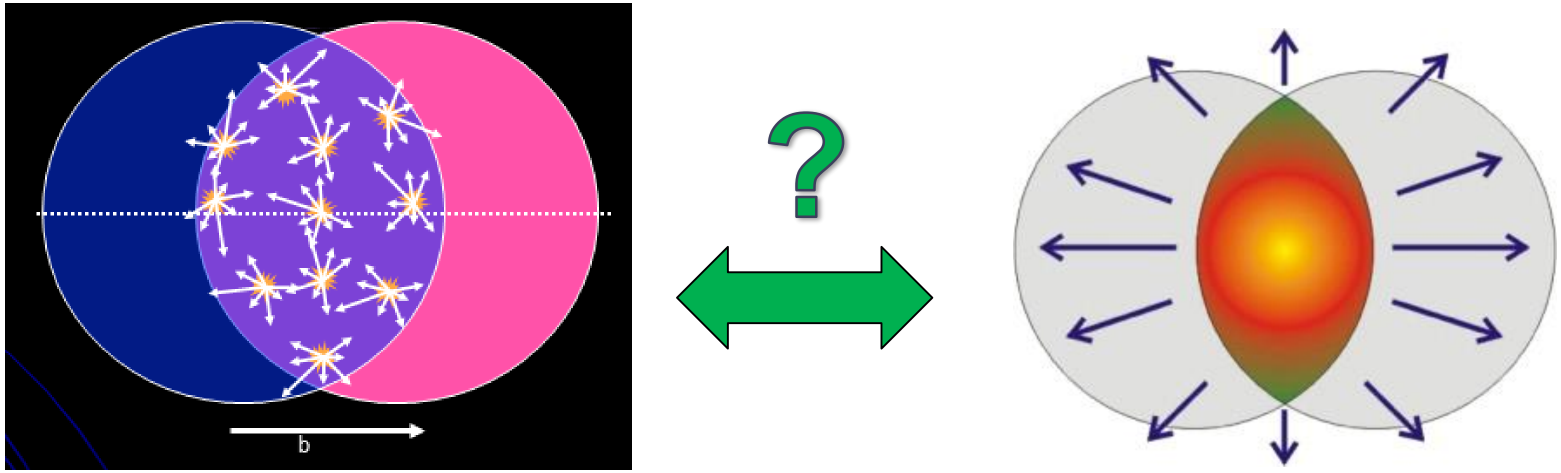


CMS Collaboration, Phys. Rev. Lett. 120, 092301 (2018), e-Print: 1709.09189

SC observables couldn't be more different in pp, pPb and PbPb!

# Transfer of anisotropy

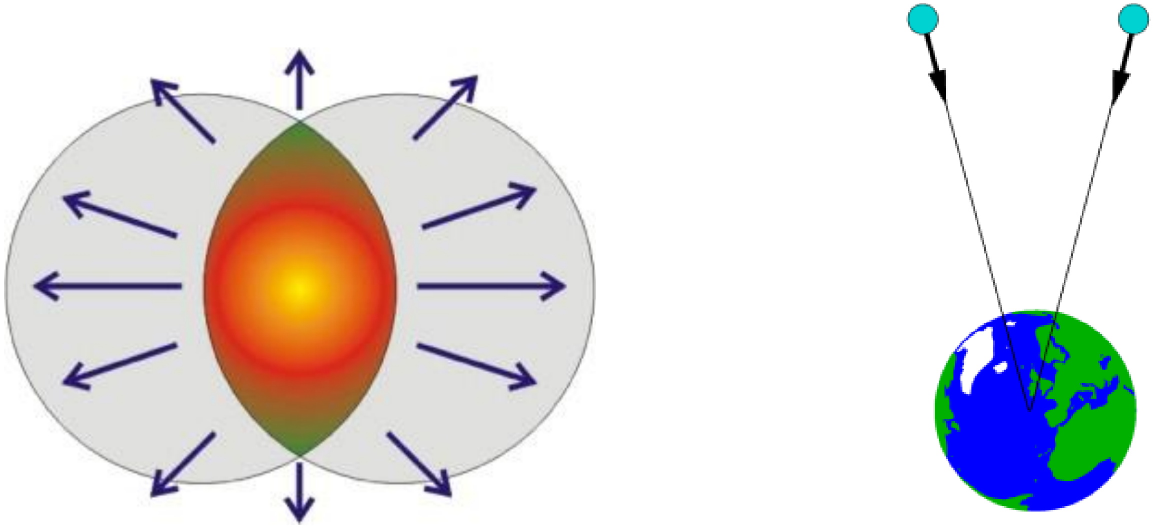
- Two conceptually different notions of anisotropy:
  - **Coordinate space anisotropy:** Is the volume containing the interacting particles produced in a heavy-ion collision anisotropic or not?
  - **Momentum space anisotropy:** Is the final-state azimuthal distribution of resulting particles recorded in the detector anisotropic or not?
- A priori these anisotropies unrelated, in practice **correlated**



made by Mike Lisa

# Flow measurements and observables

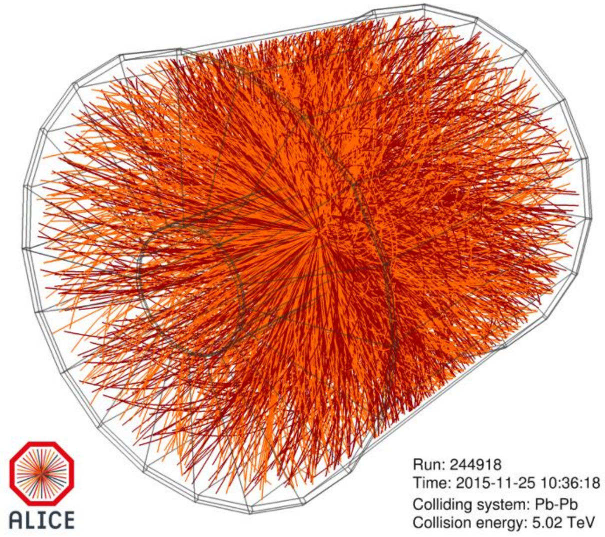
- The ‘flow principle’: Correlations among all produced particles are induced solely by correlation of each single particle to the collision geometry



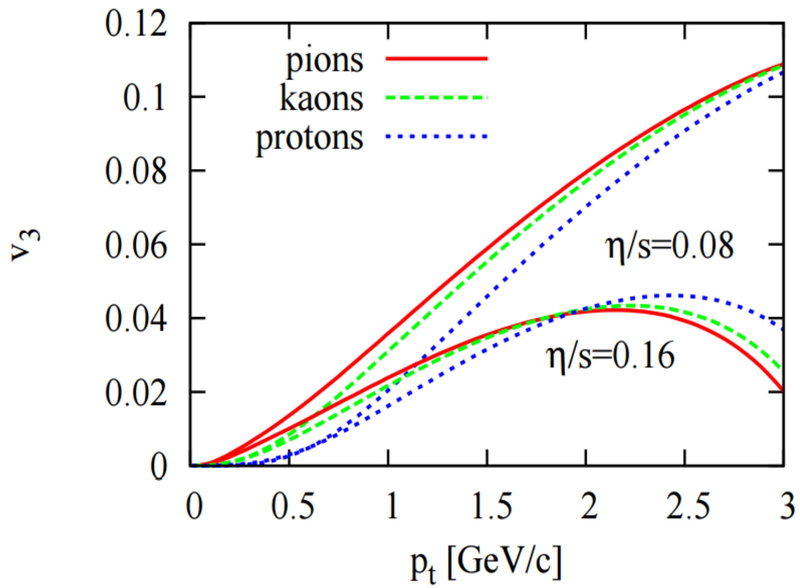
- Analogy with the falling bodies in gravitational field (rhs)
- Whether or not particles are emitted simultaneously, or one by one, trajectories are the same

# Why so many flow observables?

- All **independent** flow observables are welcome!
  - Heavy-ion collision is a rather complex system and we cannot describe everything only with few parameters
- Different observables exhibit different sensitivities to QGP properties
  - Example from theory: Transverse momentum dependence of triangular flow is different for different values of QGP's shear viscosity



Run: 244918
Time: 2015-11-25 10:36:18
Colliding system: Pb-Pb
Collision energy: 5.02 TeV

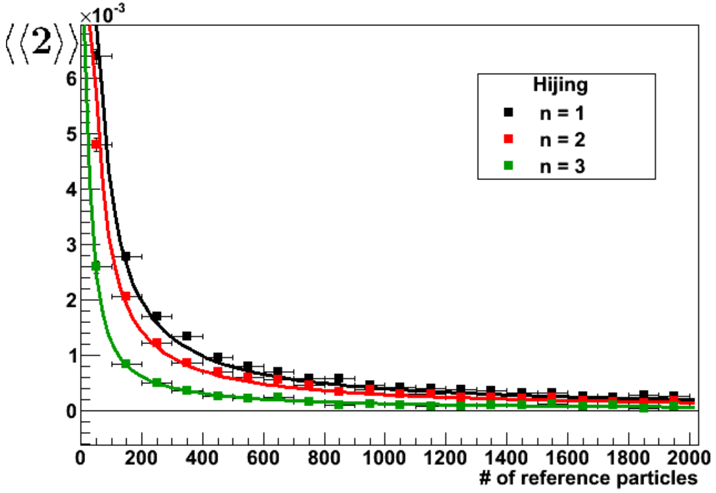


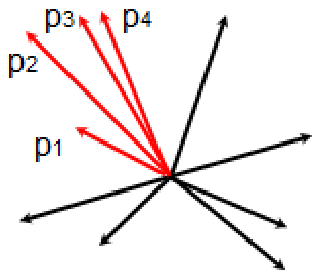
# Individual flow harmonics $v_n$

- Elliptic flow coefficient $v_2$ of inclusive charged particles as a function of centrality, measured with the two- and multi-particle cumulant methods

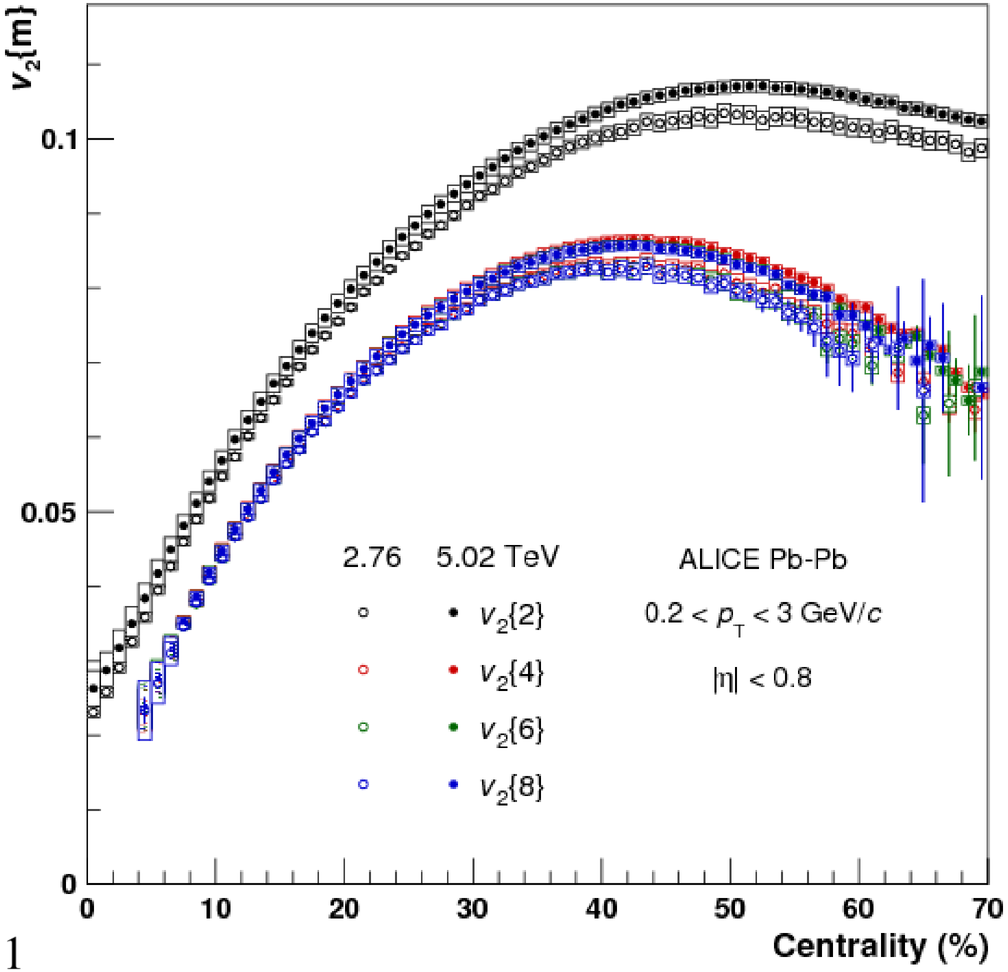
- Proof of nontrivial collective effects in heavy-ion collisions

- Multiplicity scaling of few-particle correlations:





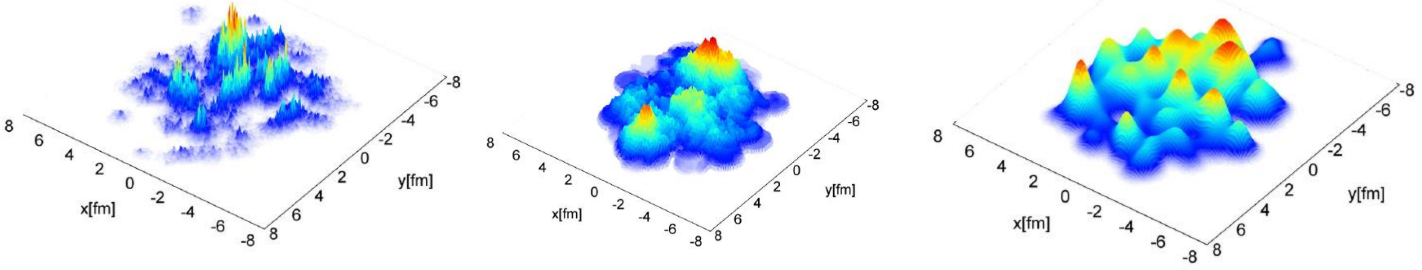
$$\delta_4 \sim \frac{3}{M-1}\frac{2}{M-2}\frac{1}{M-3} \approx \frac{1}{M^3}$$



ALICE, JHEP 1807 (2018) 103

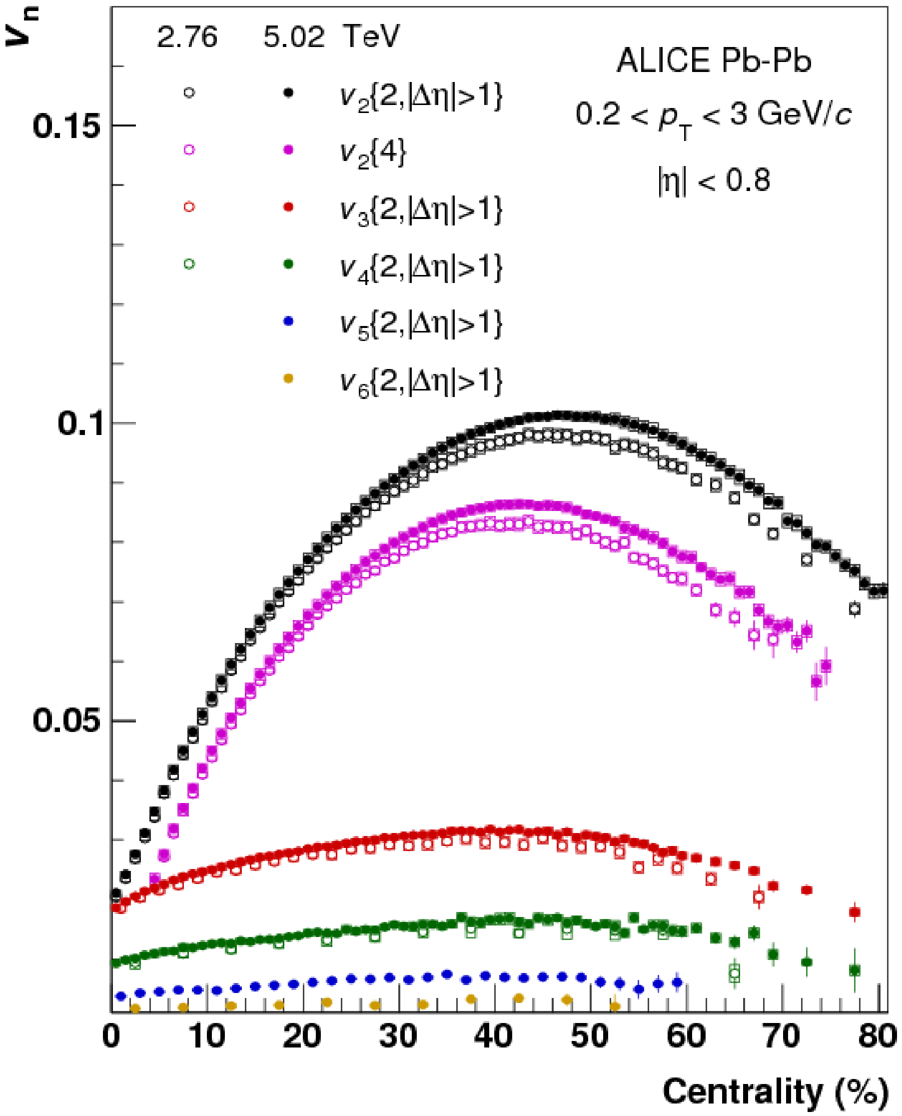
# Individual flow harmonics $v_n$

- Anisotropic flow coefficients $v_n$ of inclusive charged particles as a function of centrality, for the two-particle and four-particle cumulant methods
- Different centrality dependence of geometry-dominated harmonics ($v_2$) and fluctuations-dominated harmonics ($v_3$, $v_4$, $v_5$, and $v_6$)
- Constraints on modelling of initial conditions



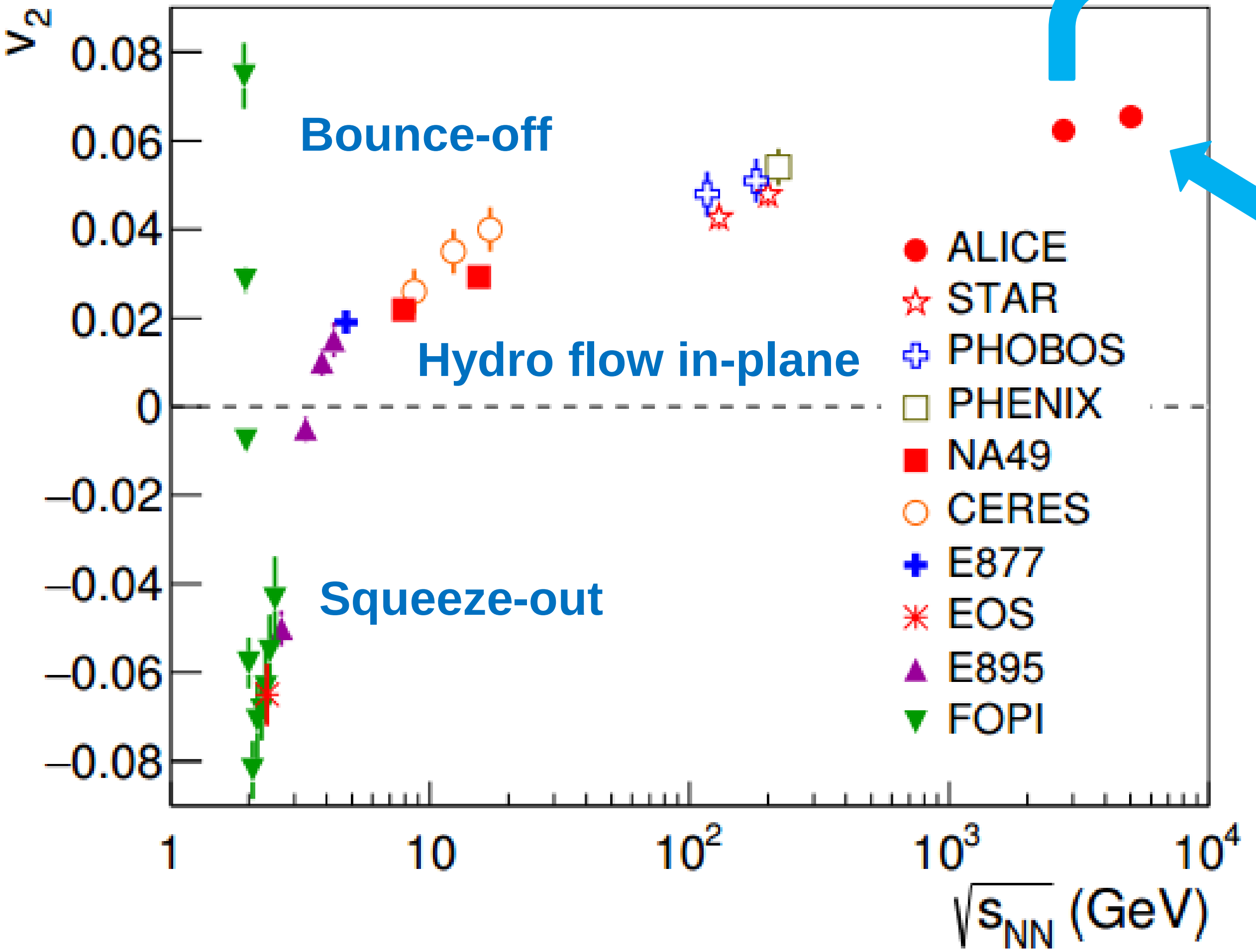
B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. 108 (2012) 252301

ALICE, JHEP 1807 (2018) 103



# Historical snapshot

- Non-trivial dependence on collision energy:



**LHC Run 1**

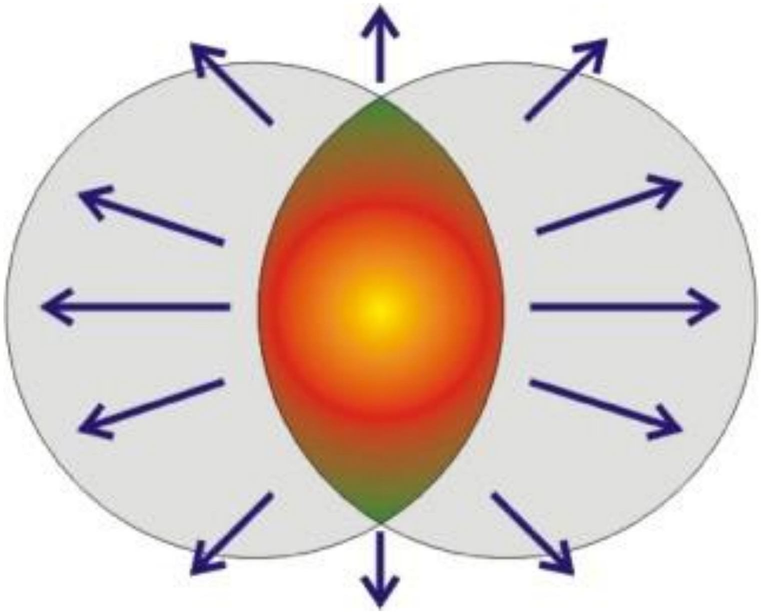
ALICE Collaboration
Phys. Rev. Lett. 105 (2010) 252302

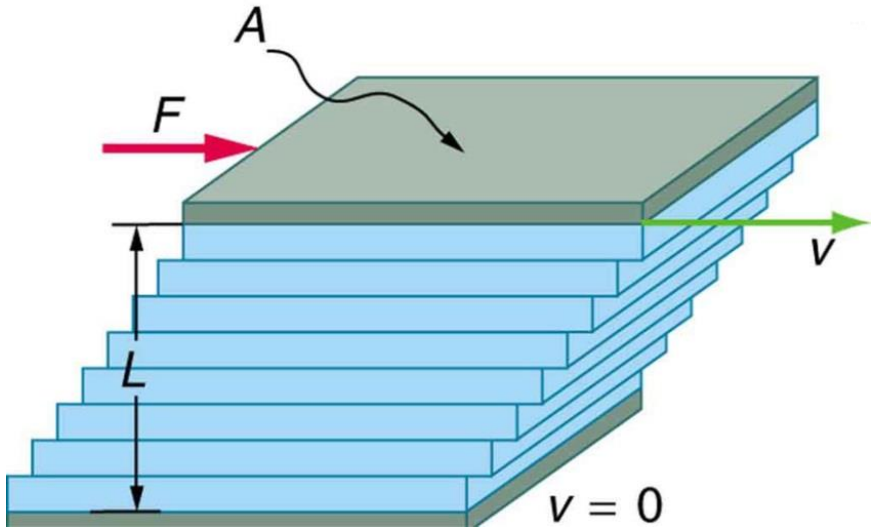
**LHC Run 2**

ALICE Collaboration
Phys. Rev. Lett. 116 (2016) 132302

# Hydro flow in-plane

- Non-trivial effect which is sensitive to transport coefficients of QGP (e.g. its shear viscosity)



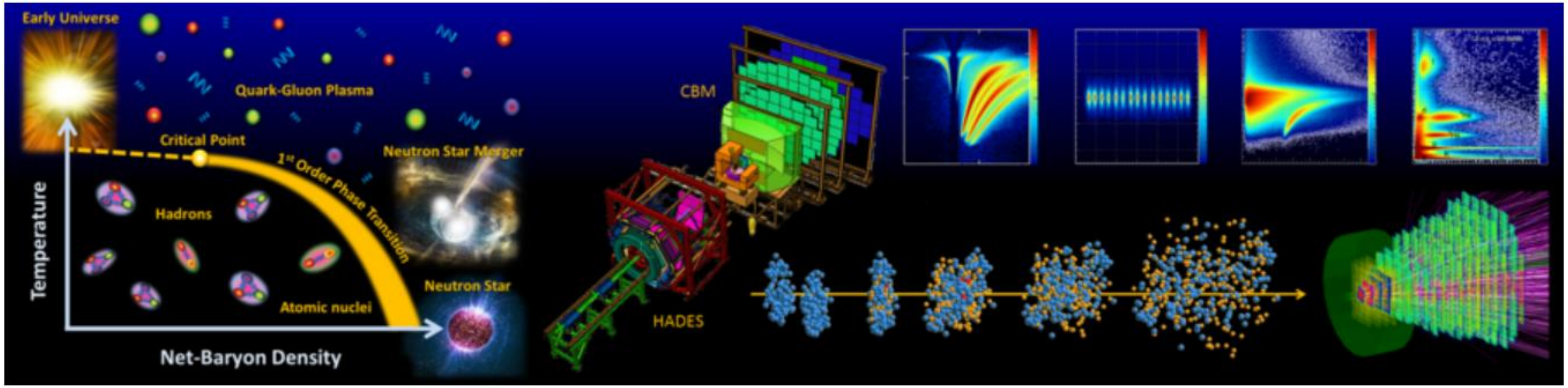


If anisotropic flow has developed, neighboring layers are moving at different relative velocities, parallel displacement is opposed by shear viscosity

**large anisotropic flow ⇔ small shear viscosity**

# How can we produce Quark-Gluon Plasma?

- Alternatively, we can focus on the region of high baryon densities using fixed-target nucleus-nucleus collisions at GSI



- **Compressed Baryonic Matter (CBM)** experiment will be one of the major scientific pillars of the future Facility for Antiproton and Ion Research (FAIR) at GSI in Darmstadt
  - It is expected to start data taking in 2028

# CBM: The physics case

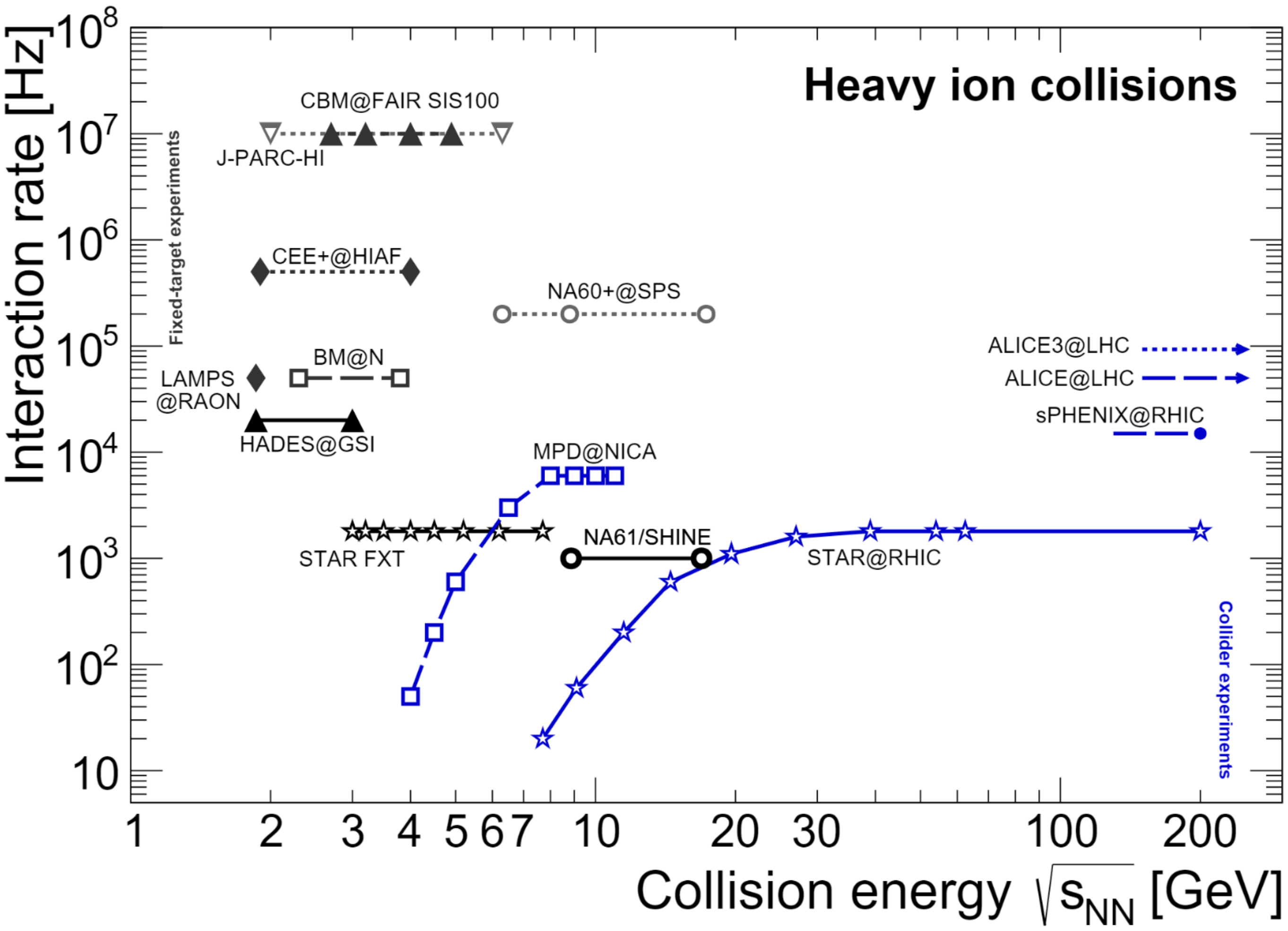
- CBM is a unique high-intensity experiment with the capability of taking data at event rates up to 10 MHz
  - Unprecedentedly large statistics
  - Full mid-rapidity coverage
- Collision energies: $2.9 < s_{NN} < 4.9\ \mathrm{GeV}$
- Experimental investigation of region in QCD phase diagram with baryon chemical potential $500 < \mu_B < 850\ \mathrm{MeV}$
- Collision of a variety of nuclei, including isobaric species with varying isospin content

| | $\sqrt{s_{NN}}$ [GeV] | $\mu_B$ [MeV] |
|---|---|---|
| SIS 18 | 2 – 2.5 | 830 – 760 |
| SIS 100 | 2.3 – 5.3 | 785 – 520 |
| SPS | 5.1 – 17.3 | 530 – 220 |
| STAR Collider | 7.7 – 200 | 400 – 22 |
| STAR FXT | 3 – 13.7 | 700 – 265 |

$\mu_B(\sqrt{s_{NN}})$ from A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, no. 7723, 321 (2018)

D. Almaalol et al, ‘QCD Phase Structure and Interactions at High Baryon Density: Continuation of BES Physics Program with CBM at FAIR’, e-Print: 2209.05009

# CBM: The physics case



- CBM is ideal for studies of rare probes
- Systematic investigation of dependence on energy, size/centrality

Claudia Höhne, Quark Matter 2023, ‘The status of the CBM experiment at FAIR’

T. Galatyuk, NPA 982 (2019), update Aug 2023, GitHub, CBM, EPJA 53 3 (2017) 60

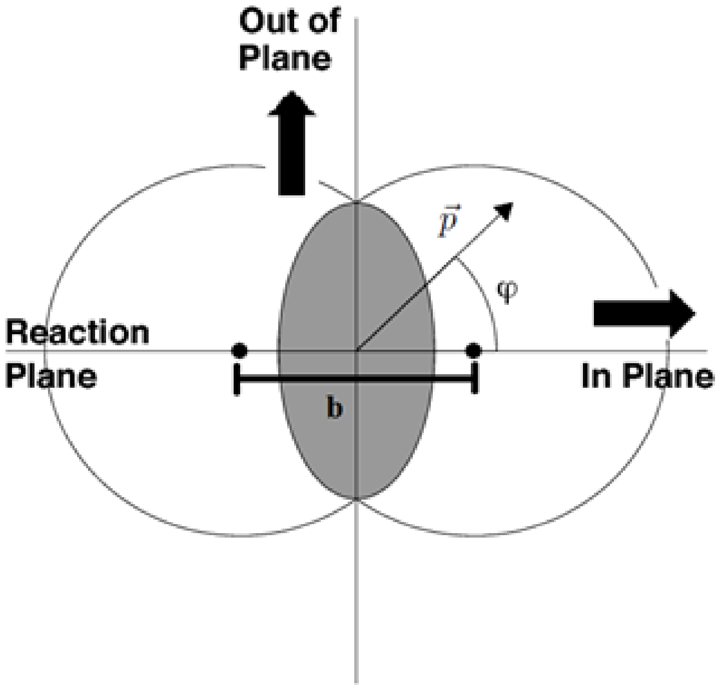
# CBM: The physics case

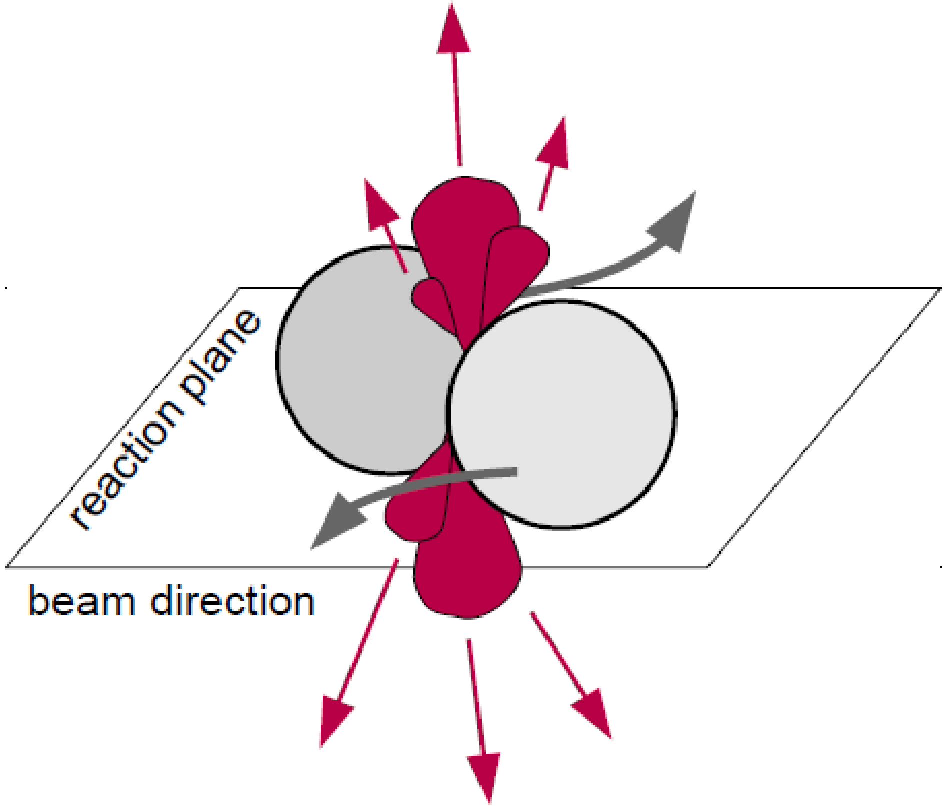
- Decisively confirm or disapprove of the existence of the QCD critical point
- Interactions in a dense baryonic matter — understanding of the inner structure of compact stars and dynamics of neutron star mergers
  - Dynamics and the equation of state of dense nuclear matter
- High-statistics measurements of spectra, flow, and femtoscopic correlations from a systematic scan of beam energies and target-projectile combinations
  - Hyperon-nucleon and hyperon-hyperon interaction
- New phases in the QCD diagram?
  - Quarkyonic phase

D. Almaalol et al, 'QCD Phase Structure and Interactions at High Baryon Density: Continuation of BES Physics Program with CBM at FAIR', e-Print: 2209.05009

# Squeeze-out flow

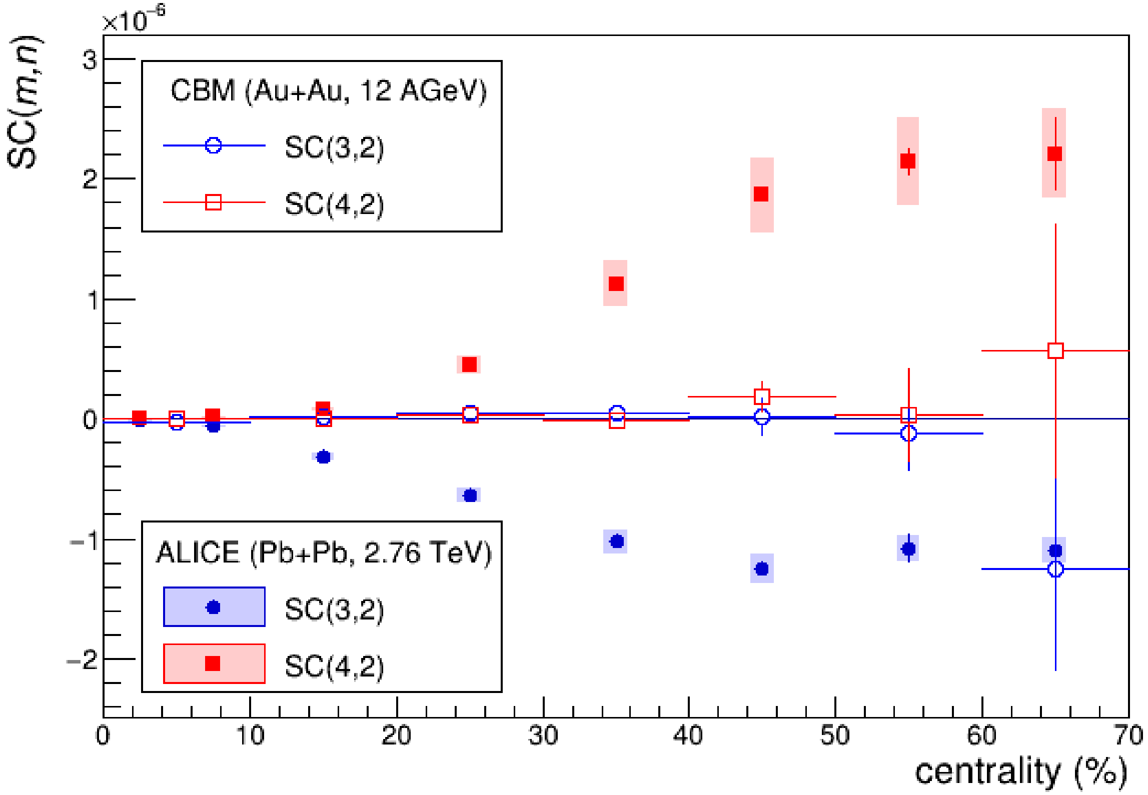
- 'Squeeze-out' a.k.a. elliptic flow 'out-of-plane'
  - Can be both trivial (shadowing) and non-trivial (hydro)
  - Tipically develops at intermediate energies: $\sqrt{s_{NN}} \sim 2-4\,GeV$
  - Negative flow values

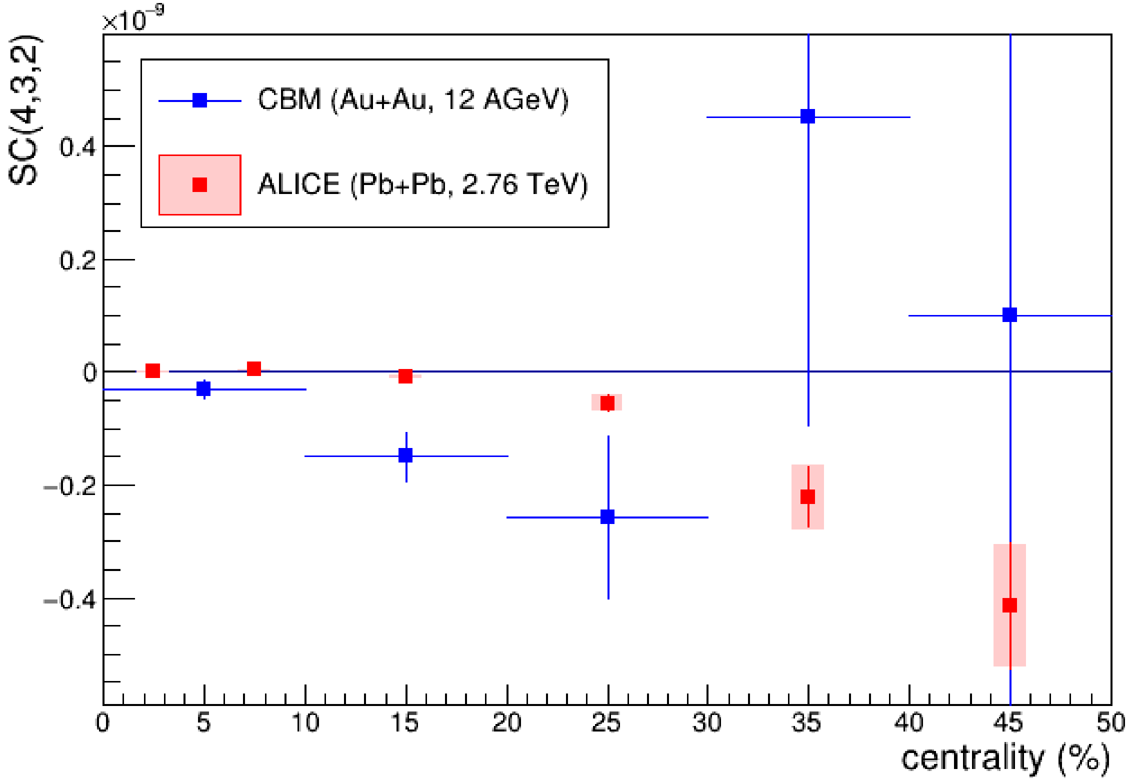




# Multivariate flow observables in CBM

- Multiharmonic correlations estimated with Symmetric Cumulants (SC):





AB for **CBM Collaboration**, QM22 (Krakow) poster

# Multivariate flow observables in CBM

- Multiharmonic correlations estimated with Asymmetric Cumulants (AC):

$$
\begin{aligned}
\mathrm{AC}_{2,1}(m,n) &= \langle v_m^4 v_n^2\rangle - \langle v_m^4\rangle\langle v_n^2\rangle - 2\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle + 2\langle v_m^2\rangle^2\langle v_n^2\rangle,\\
\mathrm{AC}_{3,1}(m,n) &= \langle v_m^6 v_n^2\rangle - \langle v_m^6\rangle\langle v_n^2\rangle - 3\langle v_m^2 v_n^2\rangle\langle v_m^4\rangle - 3\langle v_m^4 v_n^2\rangle\langle v_m^2\rangle\\
&\quad + 6\langle v_m^4\rangle\langle v_m^2\rangle\langle v_n^2\rangle + 6\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle^2 - 6\langle v_m^2\rangle^3\langle v_n^2\rangle,
\end{aligned}
$$

- New higher-order flow observables, which quantify correlations of different higher-order moments of $v_n$, and satisfy all fundamental properties of multivariate cumulants
- By definition, AC extract new and independent information when compared to the previous lower-order flow observables
- In general, AC are not invariant under permutations of different $v_n$, therefore $\mathrm{AC}_{1,2}(2,3)$ and $\mathrm{AC}_{1,2}(3,2)$ are independent of each other

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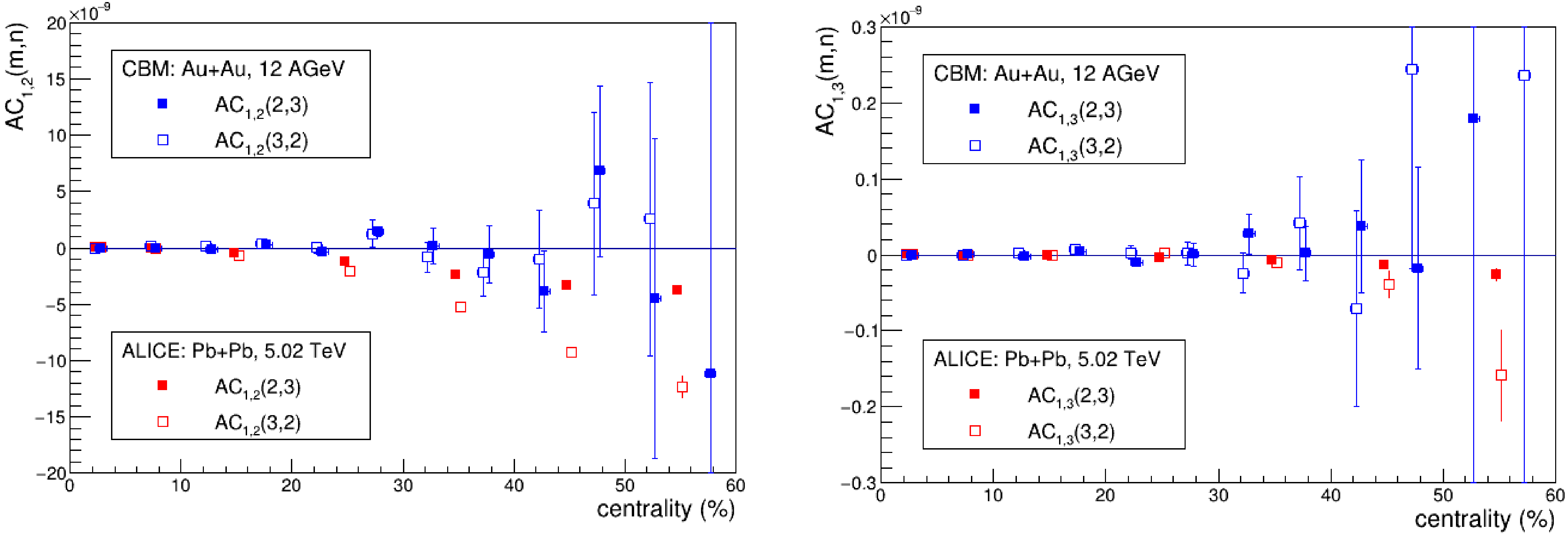

AB for **CBM Collaboration**, QM23 (Houston) poster

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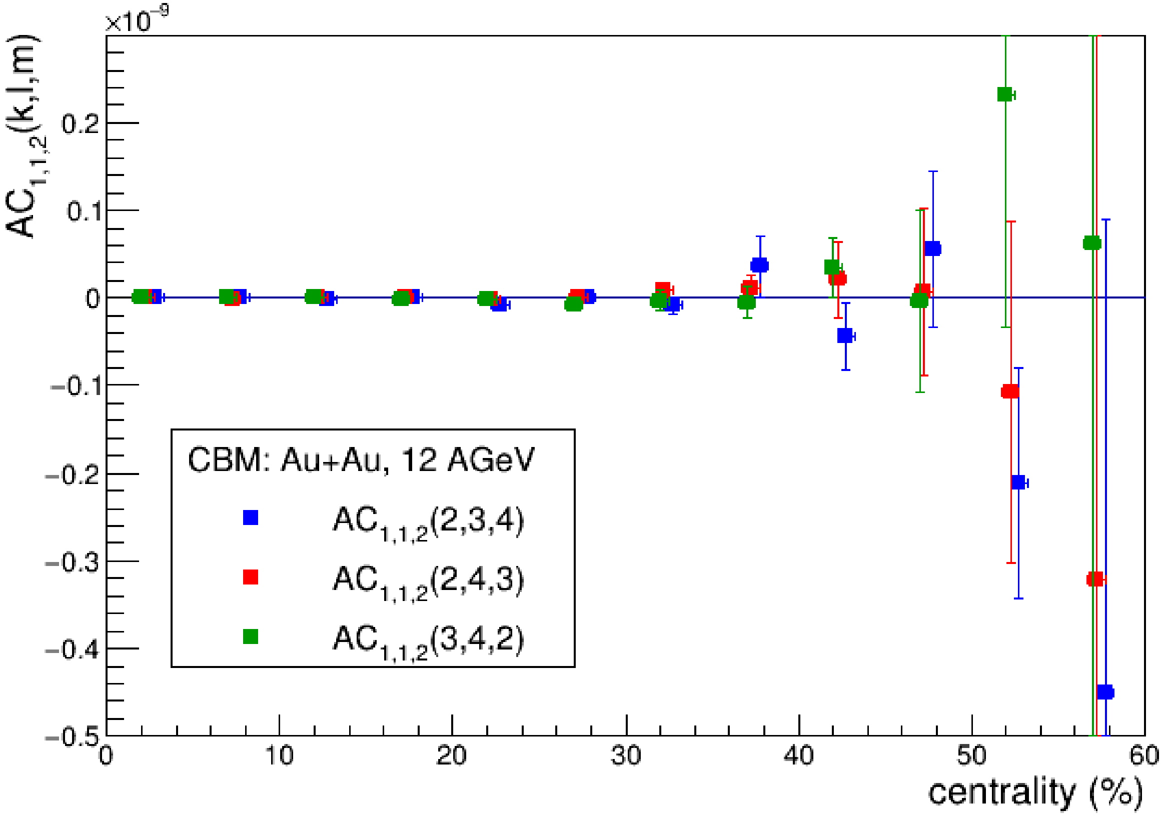

$$
\begin{aligned}
\mathrm{AC}_{2,1,1}(k,l,m) = {} & \langle v_k^4 v_l^2 v_m^2\rangle - \langle v_k^4 v_l^2\rangle\langle v_m^2\rangle - \langle v_k^4 v_m^2\rangle\langle v_l^2\rangle - \langle v_k^4\rangle\langle v_l^2 v_m^2\rangle \\
& + 2\langle v_k^4\rangle\langle v_l^2\rangle\langle v_m^2\rangle - 2\langle v_k^2 v_l^2\rangle\langle v_k^2 v_m^2\rangle - 2\langle v_k^2 v_l^2 v_m^2\rangle\langle v_k^2\rangle \\
& + 4\langle v_k^2 v_l^2\rangle\langle v_k^2\rangle\langle v_m^2\rangle + 4\langle v_k^2 v_m^2\rangle\langle v_k^2\rangle\langle v_l^2\rangle \\
& + 2\langle v_k^2\rangle^2\langle v_l^2 v_m^2\rangle - 6\langle v_k^2\rangle^2\langle v_l^2\rangle\langle v_m^2\rangle .
\end{aligned}
$$

AB for **CBM Collaboration**, QM23 (Houston) poster

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\mathrm{AC}_{2,1}(m,n) &= \langle v_m^4 v_n^2\rangle - \langle v_m^4\rangle\langle v_n^2\rangle - 2\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle + 2\langle v_m^2\rangle^2\langle v_n^2\rangle,\\
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&\quad + 6\langle v_m^4\rangle\langle v_m^2\rangle\langle v_n^2\rangle + 6\langle v_m^2 v_n^2\rangle\langle v_m^2\rangle^2 - 6\langle v_m^2\rangle^3\langle v_n^2\rangle,
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# Scaling of stat. and sys. errors

- Scaling of the statistical uncertainty (*N* is a number of events, *M* is multiplicity, *v* is strength of Fourier harmonic, *k* is the order of correlator):

$$\sigma_v \sim \frac{1}{\sqrt{N}}\frac{1}{M^{k/2}}\frac{1}{v^{k-1}}$$

- Scaling of non-collective contribution in *k*-particle correlator:

$$\delta_k \sim \frac{1}{M^{k-1}}$$

- For both reasons, multiparticle correlations are a precision technique only for:
  - a) large multiplicities
  - b) large values of Fourier harmonics $v_n$

# Fluctuations, p.d.f., moments, cumulants

- Properties of random (stochastic) observable $v$ of interest are specified by functional form of probability density function (p.d.f.) $f(v)$
- Different moments carry by definition independent information about the underlying p.d.f. $f(v_n)$

$$\left\langle v_n^k \right\rangle \equiv \int v_n^k f(v_n)\, dv_n$$

- Two completely different p.d.f.'s $f(v_n)$ can have first moment $\langle v_n \rangle$ to be the same, and all higher-order moments different
- Is it mathematically equivalent to specify functional form $f(v_n)$ and all its moments $\langle v_n^k \rangle$?
- A priori it is not guaranteed that a p.d.f. $f(v_n)$ is uniquely determined by its moments $\langle v_n^k \rangle$
  - Necessary and sufficient conditions have been worked out only recently

$$K[f] \equiv \int_0^\infty \frac{-\ln f(x^2)}{1+x^2}\, dx \quad \Rightarrow \quad K[f] = \infty$$

**Krein-Lin conditions** (1997)

$$L(x) \equiv -\frac{x f'(x)}{f(x)} \quad \Rightarrow \quad \lim_{x\to\infty} L(x) = \infty$$

J. Stoyanov, Section 3 in *'Determinacy of distributions by their moments'*, Proceedings 2006