Limiting fragmentation in the dilute Glasma

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August 21st, 2024

XVIth Quark Confinement and the Hadron Spectrum Conference
Track D: Deconfinement
Cairns, Queensland, Australia

Dilute Glasma in A+A collisions

Ingredients

· Nuclear model

· Realization of two full nuclei color fields
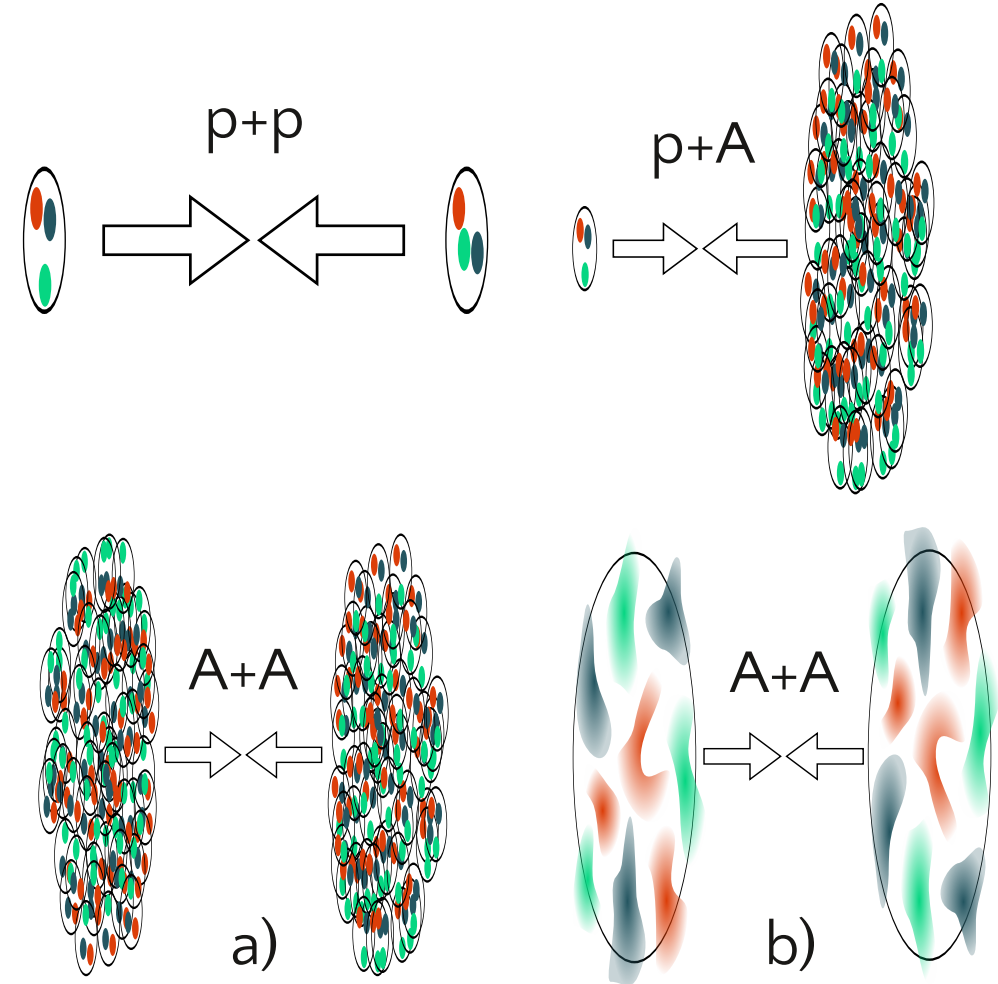
· Solution of dilute Glasma $f^{\mu\nu}$ integrals

Nuclear models for A:

a) Nucleonic and subnucleonic structure:

$A \sim N \times p$

b) Charge fluctuations for whole nucleus:

$A \propto \langle \rho\rho \rangle$

p+p

p+A

A+A

A+A

a)

b)

Dilute Glasma in A+A collisions

Ingredients

· Nuclear model

· Realization of two full nuclei color fields
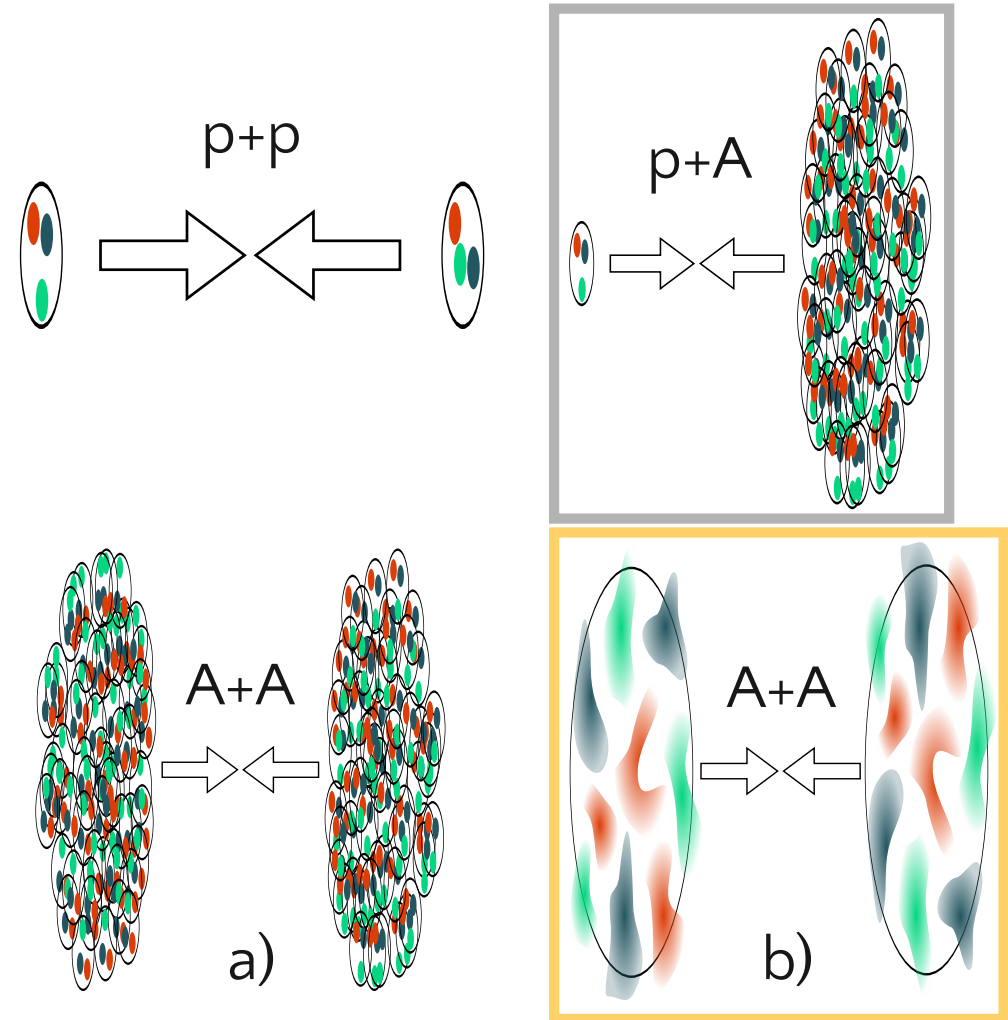
· Solution of dilute Glasma $f^{\mu\nu}$ integrals

Nuclear models for A:

a) Nucleonic and subnucleonic structure:

A $\sim$ N×p   Previous talk by M. Leuthner

b) Charge fluctuations for whole nucleus:

A $\propto \langle \rho\rho \rangle$ **This talk**

Color charges are drawn from Gaussian probability functional defined by 1-pt. function

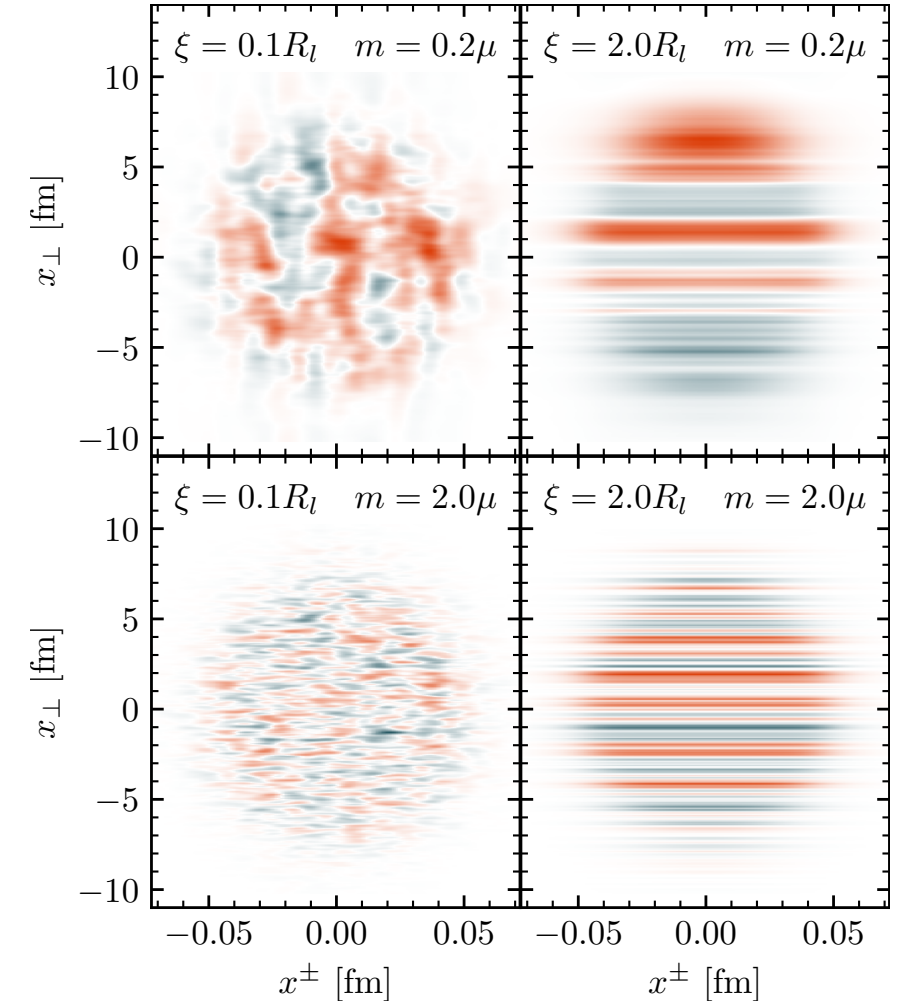$$\langle \rho^a(x^{\pm}, \mathbf{x}) \rangle = 0$$

and 2-pt. function

$$\langle \rho^a(x^{\pm}, \mathbf{x}) \rho^b(y^{\pm}, \mathbf{y}) \rangle = g^2 \mu^2 \delta^{ab} \sqrt{T(x^{\pm}, \mathbf{x})} \sqrt{T(y^{\pm}, \mathbf{y})}$$
$$\times U_{\xi}(x^{\pm} - y^{\pm}) \delta^{(2)}(\mathbf{x} - \mathbf{y})$$

with

$$T(x^{\pm}, \mathbf{x}) = \frac{c}{1 + \exp\left(\frac{\sqrt{2(\gamma\, x^{\pm})^2 + \mathbf{x}^2} - R}{d} \right)}$$

$$U_{\xi}(x^{\pm} - y^{\pm}) = \frac{1}{\sqrt{2\pi\xi^2}} e^{\frac{(x^{\pm} - y^{\pm})^2}{8R_l^2}} e^{-\frac{(x^{\pm} - y^{\pm})^2}{2\xi^2}}$$
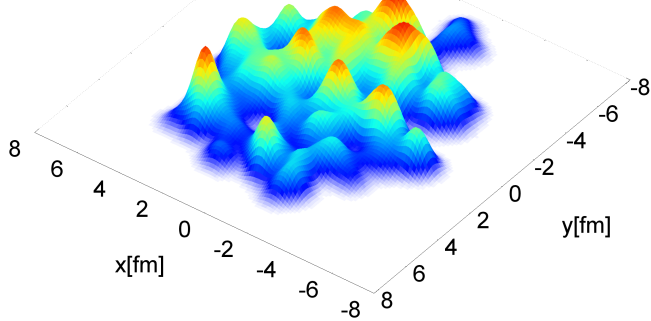
Initial energy density comparison

MC–Glauber

IP–Glasma

MC–KLN

- Fluctuating domains on similar scales compared to IP–Glasma

- Q_s equivalent parameter m (IR regulator)

dilute Glasma

ϵ_{LRF} [GeV/fm^3]

$\sqrt{s_{\text{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c with impact parameter $b = R$

$\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c with impact parameter $b = R$
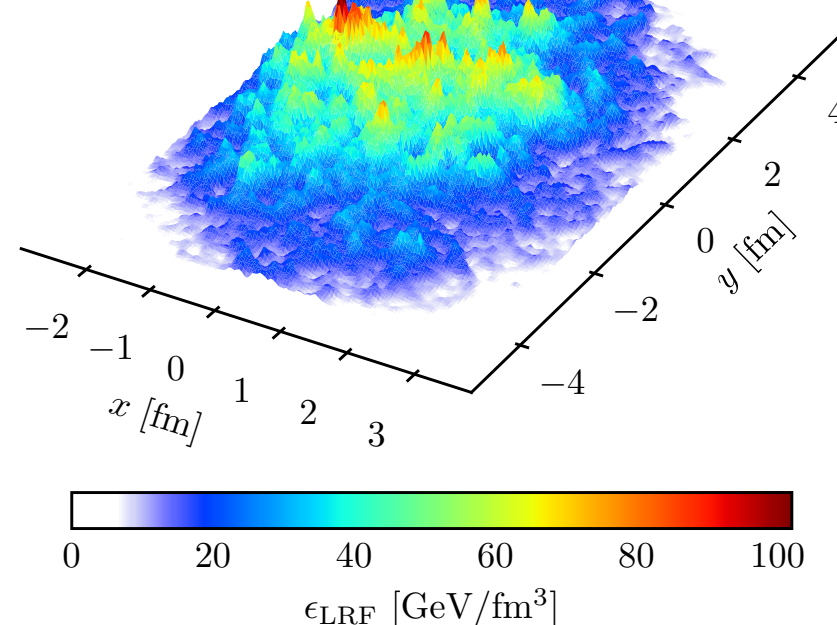
Almond shape
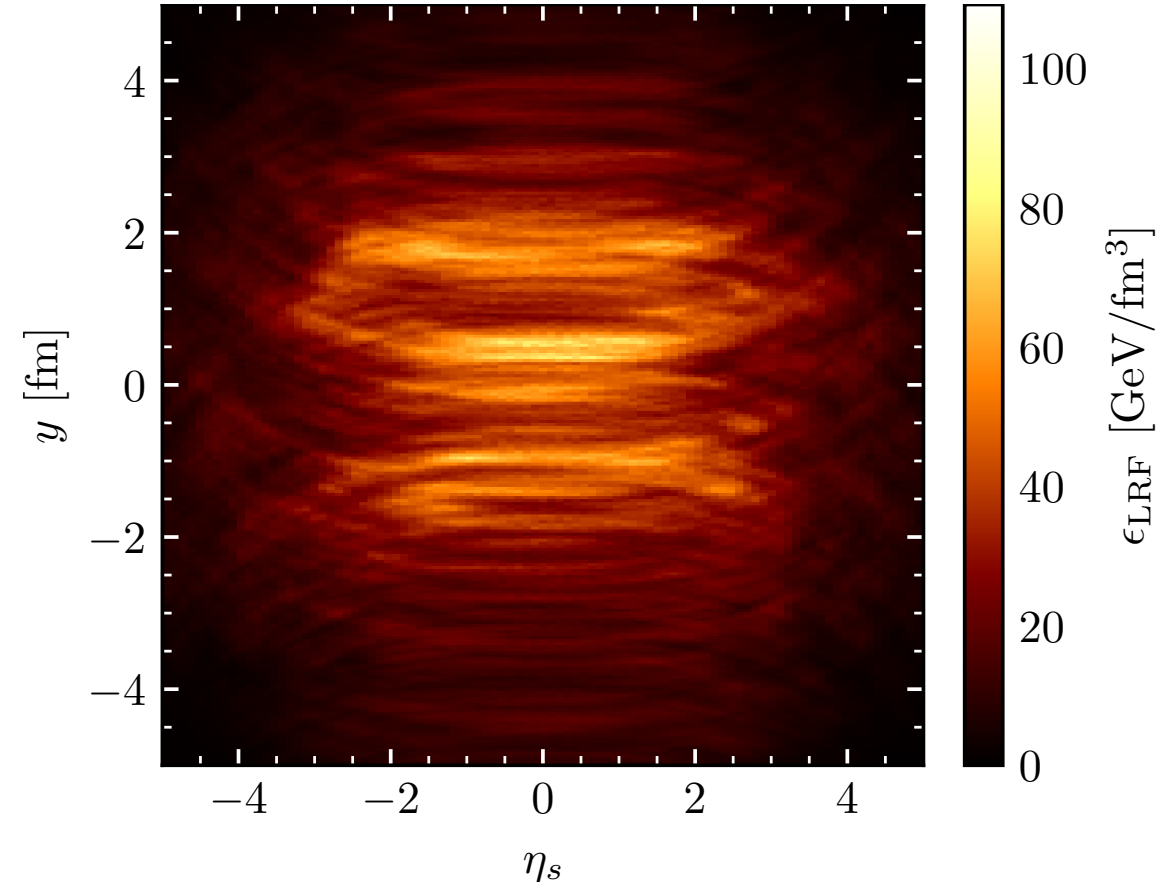
"Flux tube" structure

$\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c with impact parameter $b = R$
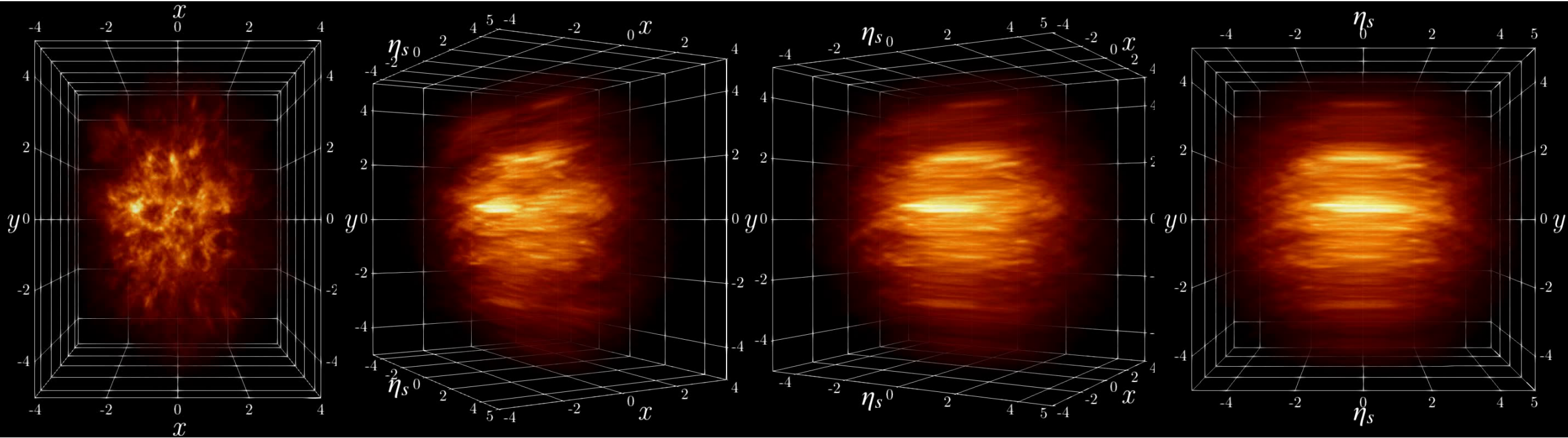
Full video available online: PhysRevD.109.094040 or [2401.10320]

TECHNISCHE
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WIEN

- Transverse structure

$$\varepsilon_n(\tau, \eta_s) = \frac{\int_{\mathbf{x}} \gamma \epsilon_{\mathrm{LRF}} \, \bar{r}^n \exp(in\bar{\phi})}{\int_{\mathbf{x}} \gamma \epsilon_{\mathrm{LRF}} \, \bar{r}^n}$$

- Connected to flow coefficients v_n
  $\Rightarrow$ Hydro evolution

10 events with impact parameter $b = R$ at $\tau = 0.4$ fm/c

RHIC: $\sqrt{s_{\mathrm{NN}}} = 200$ GeV, LHC: $\sqrt{s_{\mathrm{NN}}} = 2700$ GeV
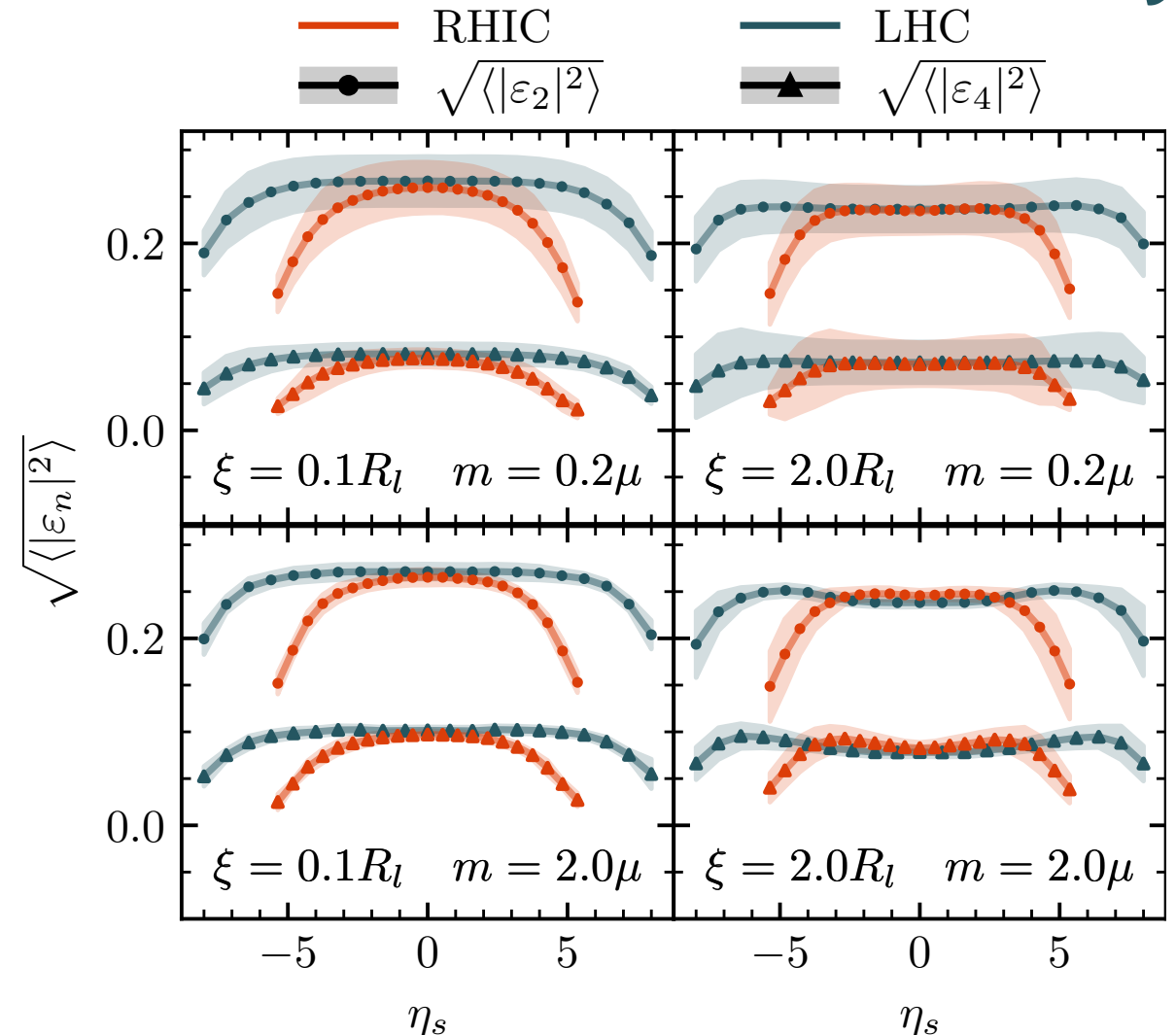
- Transverse integrals

$$\tau \int_{\mathbf{x}} T^{\mu\nu}$$

$$\tau \int_{\mathbf{x}} \epsilon_{\mathrm{LRF}}$$

- Normalized to $T^{\tau\tau}(\eta_s = 0)$

- τ and η tensor components are problematic

- Tracelessness $T^{\mu}_{\ \mu} = 0$
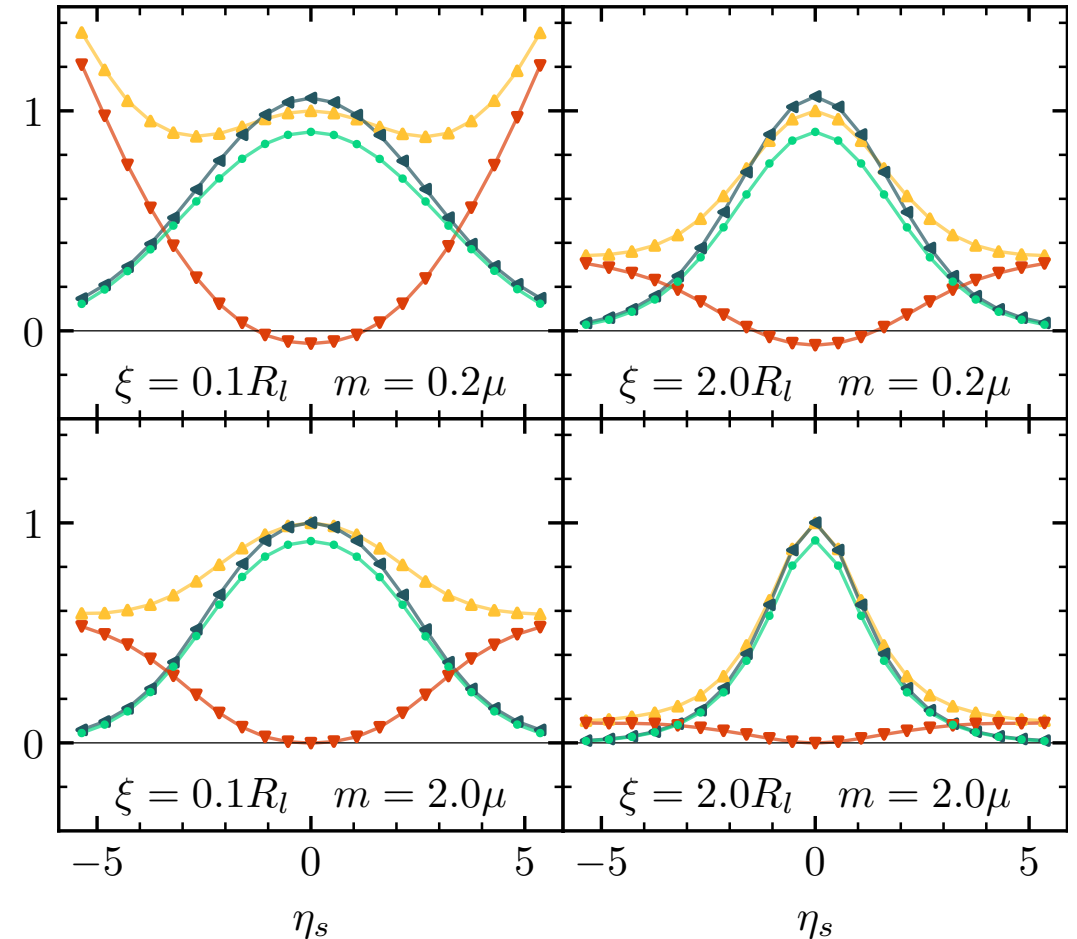
10 central events at $\tau = 0.4$ fm/c at $\sqrt{s_{\mathrm{NN}}} = 200$ GeV

Legend:
- ϵ_{LRF}
- $\tau^2 T^{\eta\eta}$
- $T^{\tau\tau}$
- $T^{xx} + T^{yy}$

Panels:
- $\xi = 0.1 R_l \quad m = 0.2\mu$
- $\xi = 2.0 R_l \quad m = 0.2\mu$
- $\xi = 0.1 R_l \quad m = 2.0\mu$
- $\xi = 2.0 R_l \quad m = 2.0\mu$

x-axis: η_s

Limiting fragmentation

- Differential transverse energy

$$\frac{dE_\perp}{d\eta_s} = \tau \int\limits_{\mathbf{x}} \left(T^{xx}(\tau, \eta_s, \mathbf{x}) + T^{yy}(\tau, \eta_s, \mathbf{x}) \right)$$

- Normalized to LHC at $\eta_s = 0$

- Shifted by beam rapidity Y_{beam}

$\Rightarrow$ Curves at large η_s overlap

10 central events at $\tau = 0.4$ fm/c

RHIC: $\sqrt{s_{\mathrm{NN}}} = 200$ GeV, LHC: $\sqrt{s_{\mathrm{NN}}} = 2700$ GeV

RHIC — LHC — $dE_\perp/d\eta_s$

$\xi = 0.1 R_l \quad m = 0.2\mu$

$\xi = 2.0 R_l \quad m = 0.2\mu$

$\xi = 0.1 R_l \quad m = 2.0\mu$

$\xi = 2.0 R_l \quad m = 2.0\mu$

$\eta_s + Y_{\mathrm{beam}}$

$\eta_s + Y_{\mathrm{beam}}$
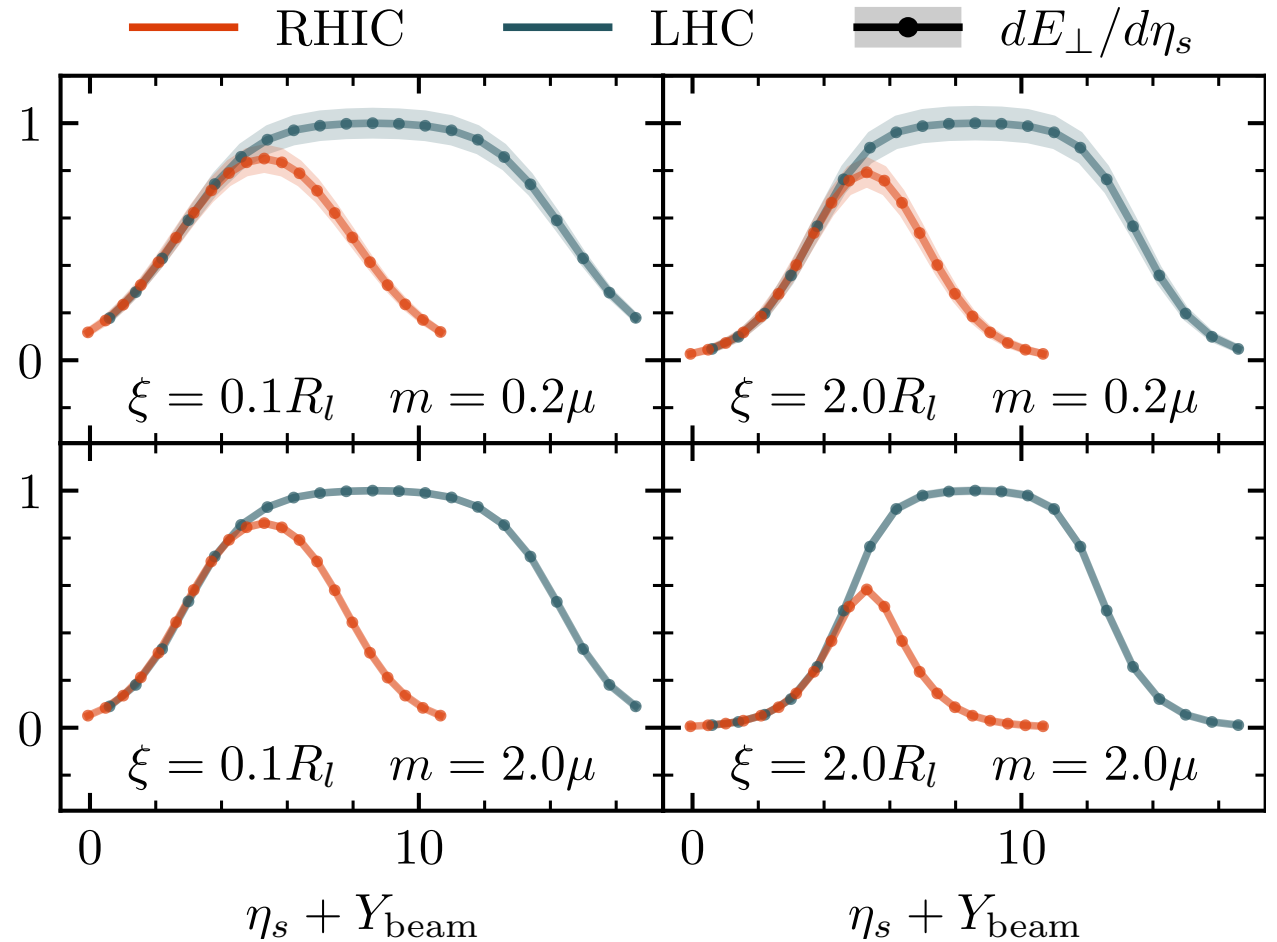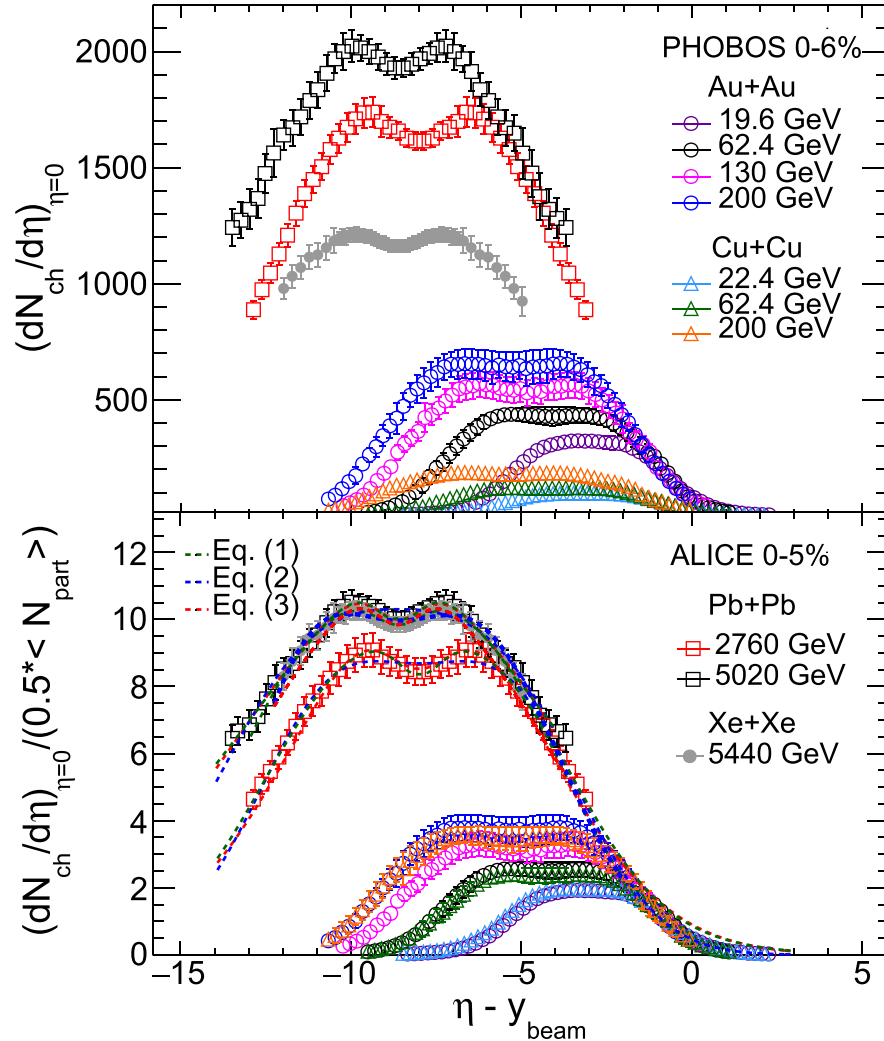
Limiting fragmentation

Observed in experiment, e.g. for

Charged particle multiplicity: $\dfrac{\mathrm{d}N_{ch}}{\mathrm{d}\eta}$

Studied in theory

- J. Jalilian–Marian,
  Limiting fragmentation from the color glass condensate.
  Phys. Rev. C 70 (2004), 027902

- F. Gelis, A. M. Stasto and R. Venugopalan,
  Limiting fragmentation in hadron–hadron collisions at
  high energies.
  Eur. Phys. J. C 48 (2006) 489–500

- …

Glasma field strength tensor

$$f^{+-} = -\frac{g}{2\pi} \int_{\mathbf{v}} \int_{-\infty}^{\infty} d\eta' \, V$$

$$f^{\pm i} = \frac{g}{2\pi} \int_{\mathbf{v}} \int_{-\infty}^{\infty} d\eta' \, \left(V^{ij} \mp \delta^{ij} V\right) \frac{v^j}{|\mathbf{v}|} \frac{e^{\pm \eta'}}{\sqrt{2}}$$

$$f^{ij} = -\frac{g}{2\pi} \int_{\mathbf{v}} \int_{-\infty}^{\infty} d\eta' \, V^{ij}$$

$$V := f_{abc} t^c \partial^i \mathcal{A}_A^{-a}(x^+ - \frac{|\mathbf{v}|}{\sqrt{2}} e^{+\eta'}, \mathbf{x} - \mathbf{v}) \partial^i \mathcal{A}_B^{+b}(x^- - \frac{|\mathbf{v}|}{\sqrt{2}} e^{-\eta'}, \mathbf{x} - \mathbf{v})$$

$$V^{ij} := f_{abc} t^c \left(\partial^i \mathcal{A}_A^{-a}(\dots) \partial^j \mathcal{A}_B^{+b}(\dots) - \partial^j \mathcal{A}_A^{-a}(\dots) \partial^i \mathcal{A}_B^{+b}(\dots) \right)$$

- Pick a spacetime point $x = (x^+, x^-, \mathbf{x})$

- Integrate over past lightlike trajectories

$$v = \left(\tfrac{\tau'}{\sqrt{2}} e^{+\eta'}, \tfrac{\tau'}{\sqrt{2}} e^{-\eta'}, \mathbf{v}\right), \quad \tau' = |\mathbf{v}|$$

- The integrand depends on $x - v$

$$V := f_{abc} t^c \partial^i \mathcal{A}_A^{-a}\left(x^+ - \frac{|\mathbf{v}|}{\sqrt{2}} e^{+\eta'}, \mathbf{x} - \mathbf{v}\right) \partial^i \mathcal{A}_B^{+b}\left(x^- - \frac{|\mathbf{v}|}{\sqrt{2}} e^{-\eta'}, \mathbf{x} - \mathbf{v}\right)$$
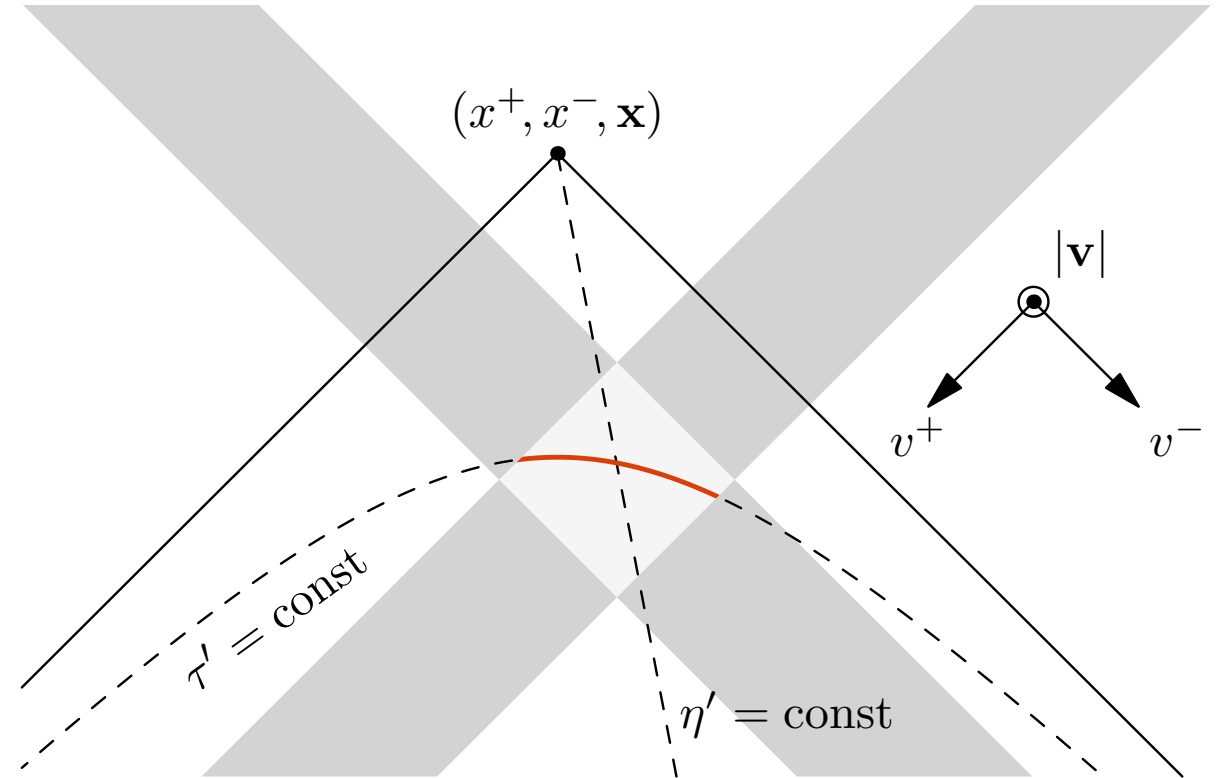
$$V^{ij} := f_{abc} t^c \left(\partial^i \mathcal{A}_A^{-a}(...) \partial^j \mathcal{A}_B^{+b}(...) - \partial^j \mathcal{A}_A^{-a}(...) \partial^i \mathcal{A}_B^{+b}(...) \right)$$

Limiting fragmentation approximation

In the fragmentation region

- Contributions only where $\eta' \gg 1$

- For fixed $\tau' (= |\mathbf{v}|)$:

  – $\mathcal{A}_B$ only evaluated along $v^- = \text{const.}$

  $$\beta_B^i(x, \mathbf{v}) = \partial^i \mathcal{A}_B^+\left(x^-\left(1 - \frac{\mathbf{v}^2}{\tau^2}\right), \mathbf{x} - \mathbf{v}\right)$$

  – $\mathcal{A}_A$ enters via v^+ average

  $$\zeta_A^i(\mathbf{x} - \mathbf{v}) = \partial^i \int \mathrm{d}y^+ \mathcal{A}_A^-(y^+, \mathbf{x} - \mathbf{v})$$
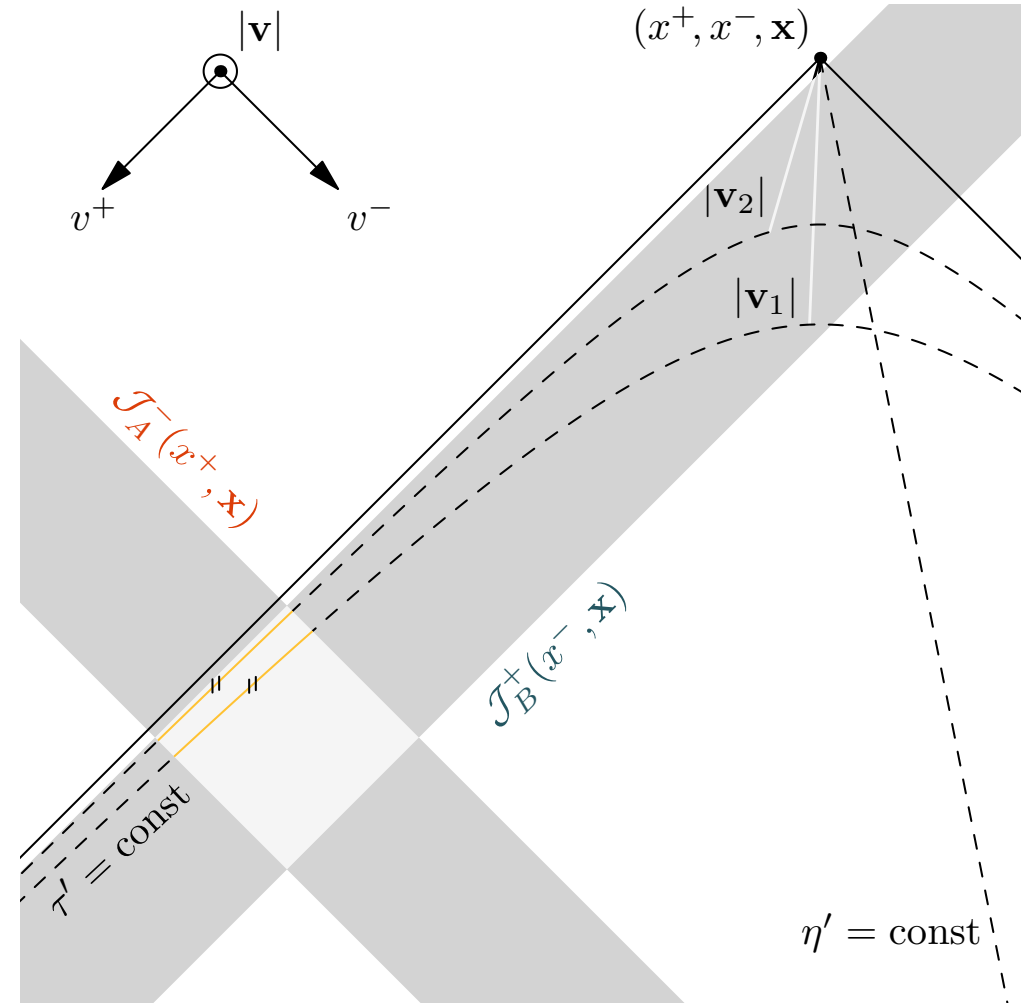
Limiting fragmentation approximation

$$f^{+-} = -\frac{g}{2\pi}\frac{1}{x^+}\int_{\mathbf{v}} Y(x,\mathbf{v})$$

$$f^{ij} = -\frac{g}{2\pi}\frac{1}{x^+}\int_{\mathbf{v}} Y^{ij}(x,\mathbf{v})$$

$$f^{+i} = \frac{g}{2\pi}\int_{\mathbf{v}} \left(Y^{ij}(x,\mathbf{v}) - \delta^{ij}Y(x,\mathbf{v})\right)\frac{v^j}{\mathbf{v}^2}$$

$$f^{-i} = \frac{g}{2\pi}\frac{1}{(x^+)^2}\int_{\mathbf{v}} \left(Y^{ij}(x,\mathbf{v}) + \delta^{ij}Y(x,\mathbf{v})\right)\frac{v^j}{2}$$
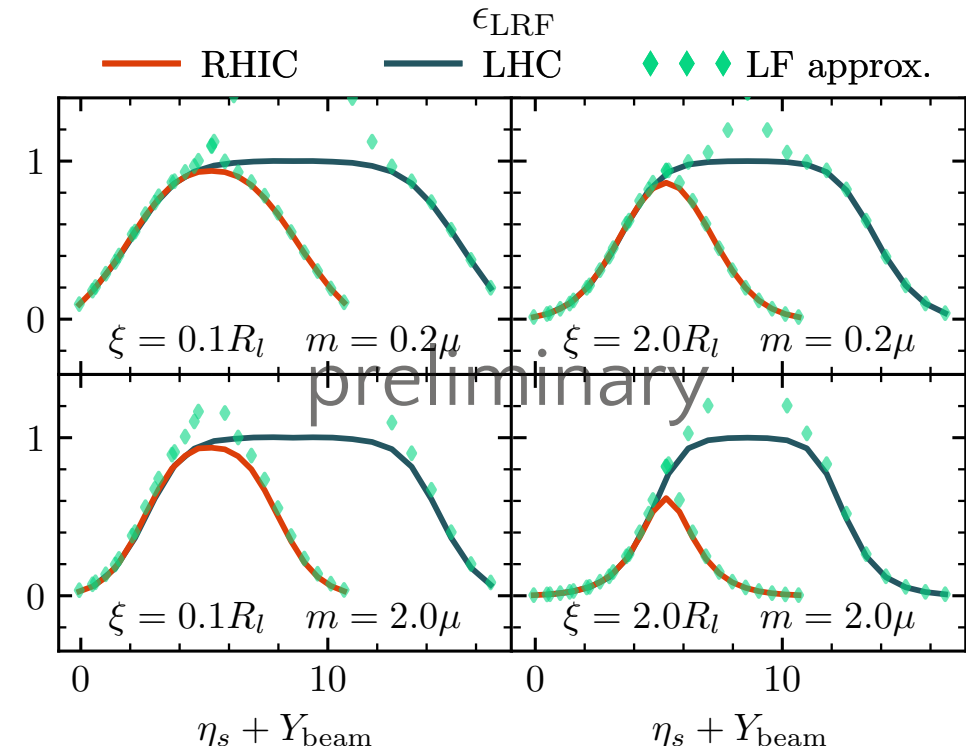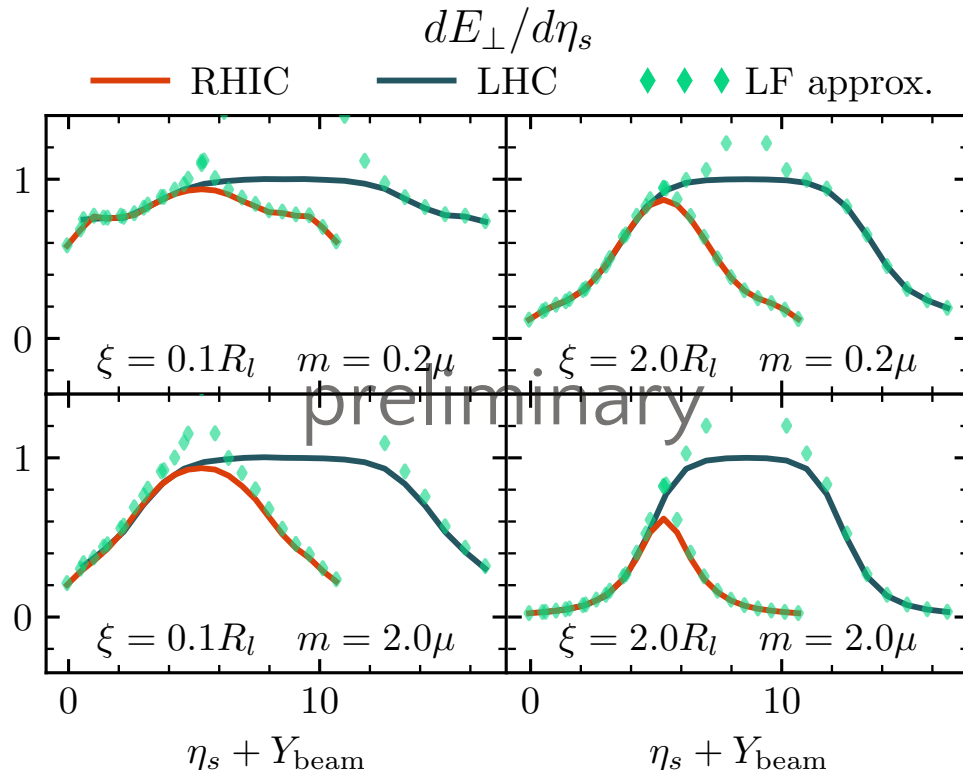
$$Y(x,\mathbf{v}) := f_{abc}t^c\,\zeta_A^{i,a}(\mathbf{x}-\mathbf{v})\,\beta_B^{i,b}(x,\mathbf{v})$$

$$Y^{ij}(x,\mathbf{v}) := f_{abc}t^c\left(\zeta_A^{i,a}(...)\,\beta_B^{j,b}(...) - \zeta_A^{j,a}(...)\,\beta_B^{i,b}(...)\right)$$
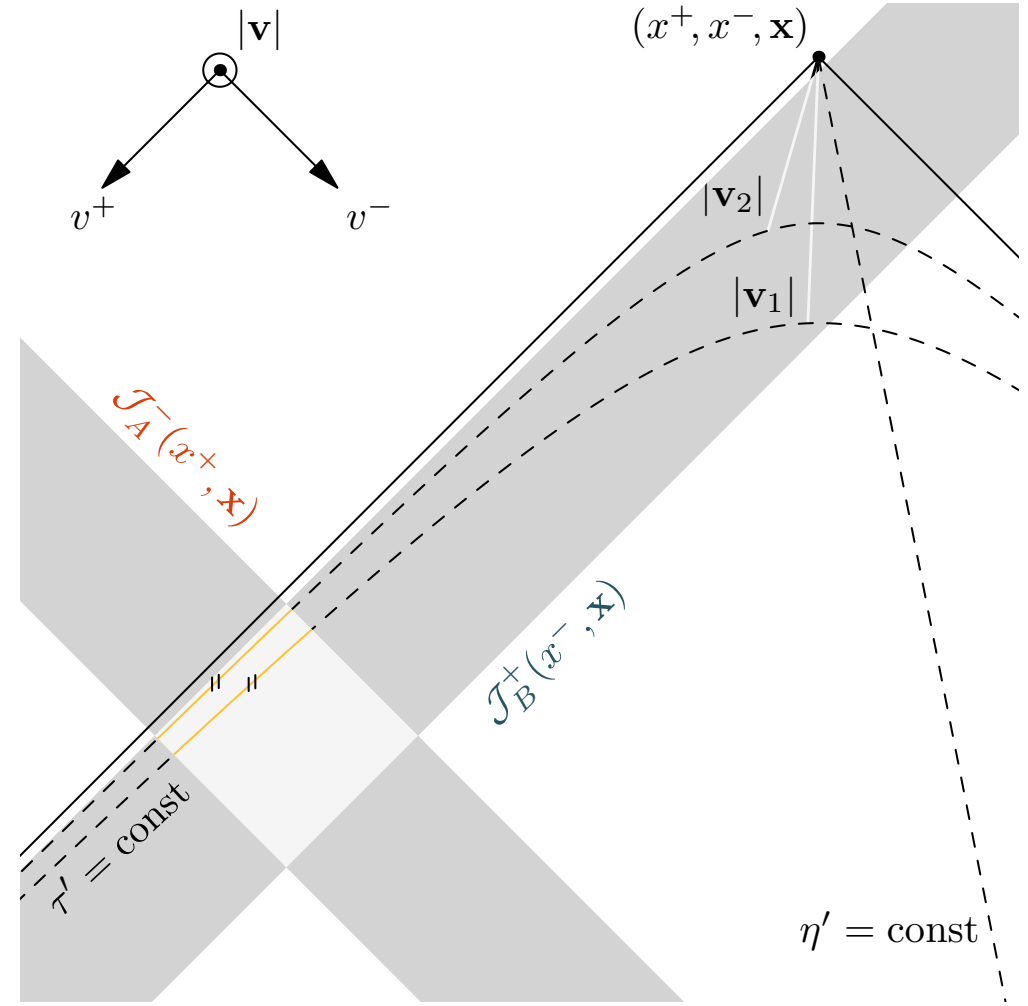
Limiting fragmentation approximation

Comparison to full dilute Glasma results

$dE_\perp/d\eta_s$

—— RHIC   —— LHC   ◆ ◆ ◆ LF approx.

ϵ_{LRF}

—— RHIC   —— LHC   ◆ ◆ ◆ LF approx.

$\xi = 0.1R_l \quad m = 0.2\mu$

$\xi = 2.0R_l \quad m = 0.2\mu$

$\xi = 0.1R_l \quad m = 2.0\mu$

$\xi = 2.0R_l \quad m = 2.0\mu$

preliminary

$\eta_s + Y_{\mathrm{beam}}$

$\eta_s + Y_{\mathrm{beam}}$

$\xi = 0.1R_l \quad m = 0.2\mu$

$\xi = 2.0R_l \quad m = 0.2\mu$

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preliminary

$\eta_s + Y_{\mathrm{beam}}$

$\eta_s + Y_{\mathrm{beam}}$

$$\frac{dE_\perp}{d\eta_s} = \tau \int_{\mathbf{x}} \left(T^{xx}(\tau, \eta_s, \mathbf{x}) + T^{yy}(\tau, \eta_s, \mathbf{x}) \right)$$

* not covering the entire $\mathbf{x}$ plane

· Remarkable agreement over broad η_s intervall
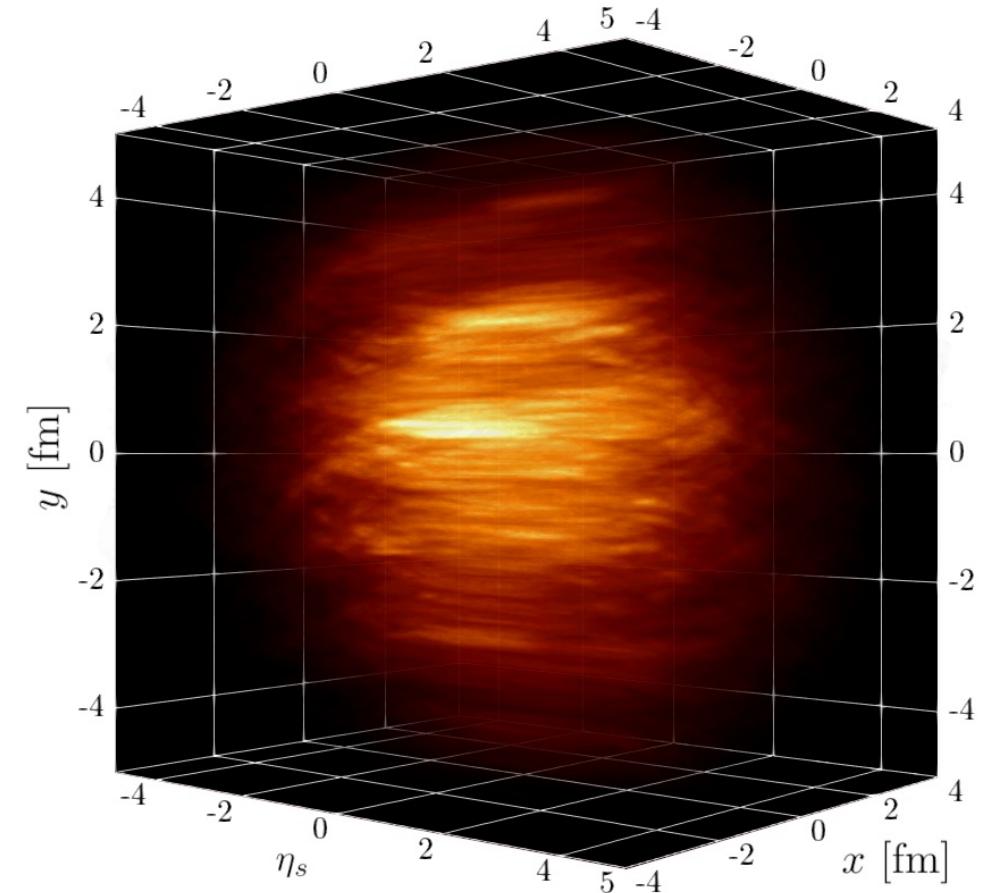
· Prediction of plateau generally fails

Nuclear model

- 3D nuclear model with longitudinal correlations

- Collider energy enters via Lorentz γ

Our results

- 3D energy-momentum tensor of the Glasma

- Rich longitudinal and transverse structure

- Limiting fragmentation

ϵ_{LRF} for $\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c

Contact information

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Don't hesitate to find me and ask questions!

Acknowledgements

The presenter acknowledges funging from the Austrian Science Fund (FWF) P 34764–N.
The travel expenses are paid in part by the TU Wien International office.
The computational results presented have been achieved in part using the Vienna Scientific Cluster (VSC).

Backup slides

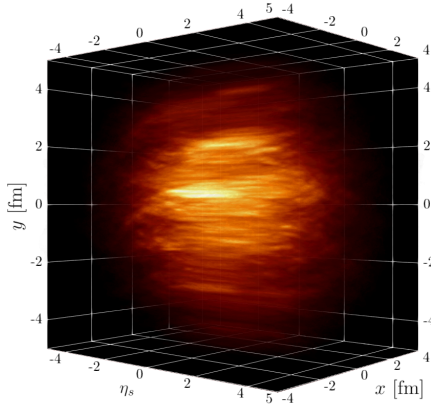$$\mathcal{T}^{\mu\nu}_{\mathrm{hydro}} = \mathcal{T}^{\mu\nu}_{\mathrm{ideal}}$$
$$+ \pi^{\mu\nu}$$
$$- (g^{\mu\nu} - u^\mu u^\nu)\Pi$$

$$\mathcal{T}^{\mu\nu}_{\mathrm{ideal}} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$$

Dilute Glasma	→	Landau matching	→	Equilibrium QGP	→	Hadron gas
$\tau \lesssim 0.2\,\frac{\mathrm{fm}}{c}$		$\tau = \tau_{\mathrm{switch}}$		$1\,\frac{\mathrm{fm}}{c} \lesssim \tau \lesssim 10\,\frac{\mathrm{fm}}{c}$		$10\,\frac{\mathrm{fm}}{c} \lesssim \tau$
Classical Yang-Mills		$T^\mu_{\ \nu}u^\nu = \epsilon u^\mu$		Rel. viscous hydrodynamics		Kinteic theory
Our code		$\pi^{\mu\nu} = \mathcal{T}^{\mu\nu}_{\mathrm{glasma}} - \mathcal{T}^{\mu\nu}_{\mathrm{ideal}}$ $\Pi = \frac{\epsilon}{3} - P_{\mathrm{EOS}}(\epsilon)$		MUSIC		UrQMD
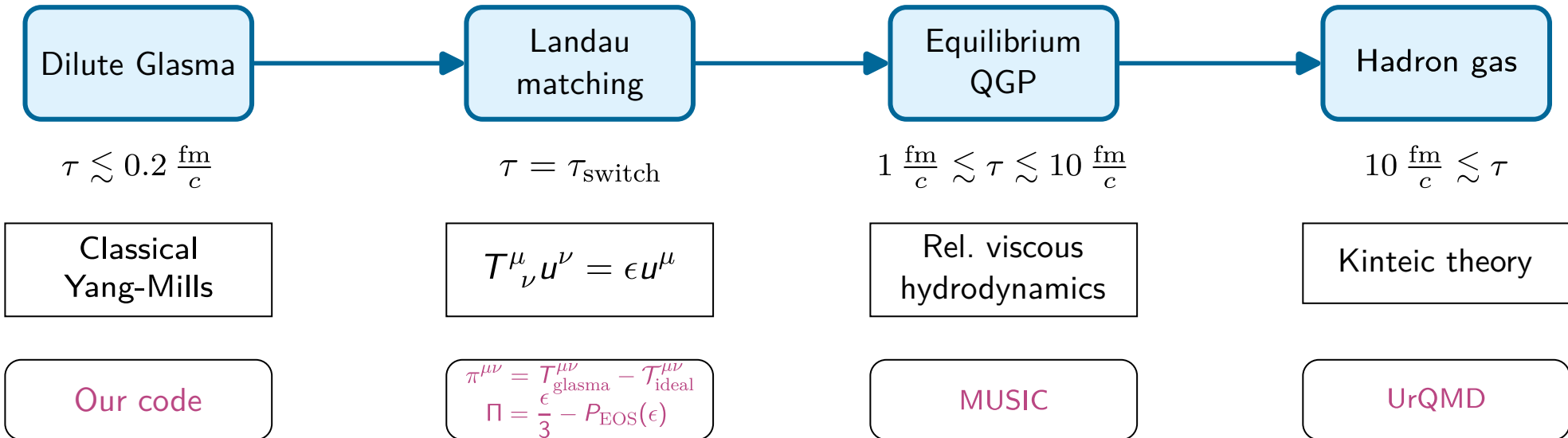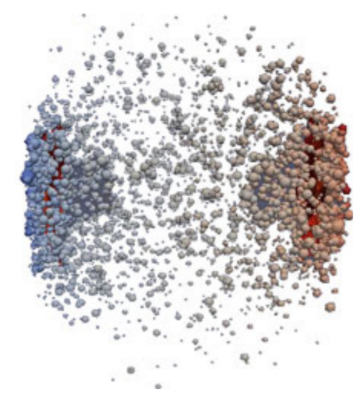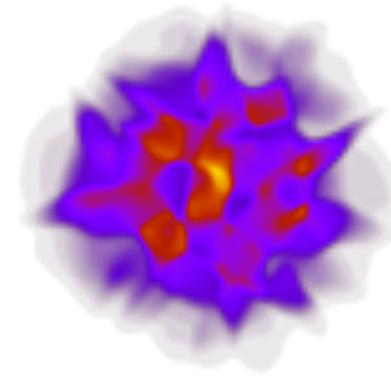
Time evolution at mid-rapidity
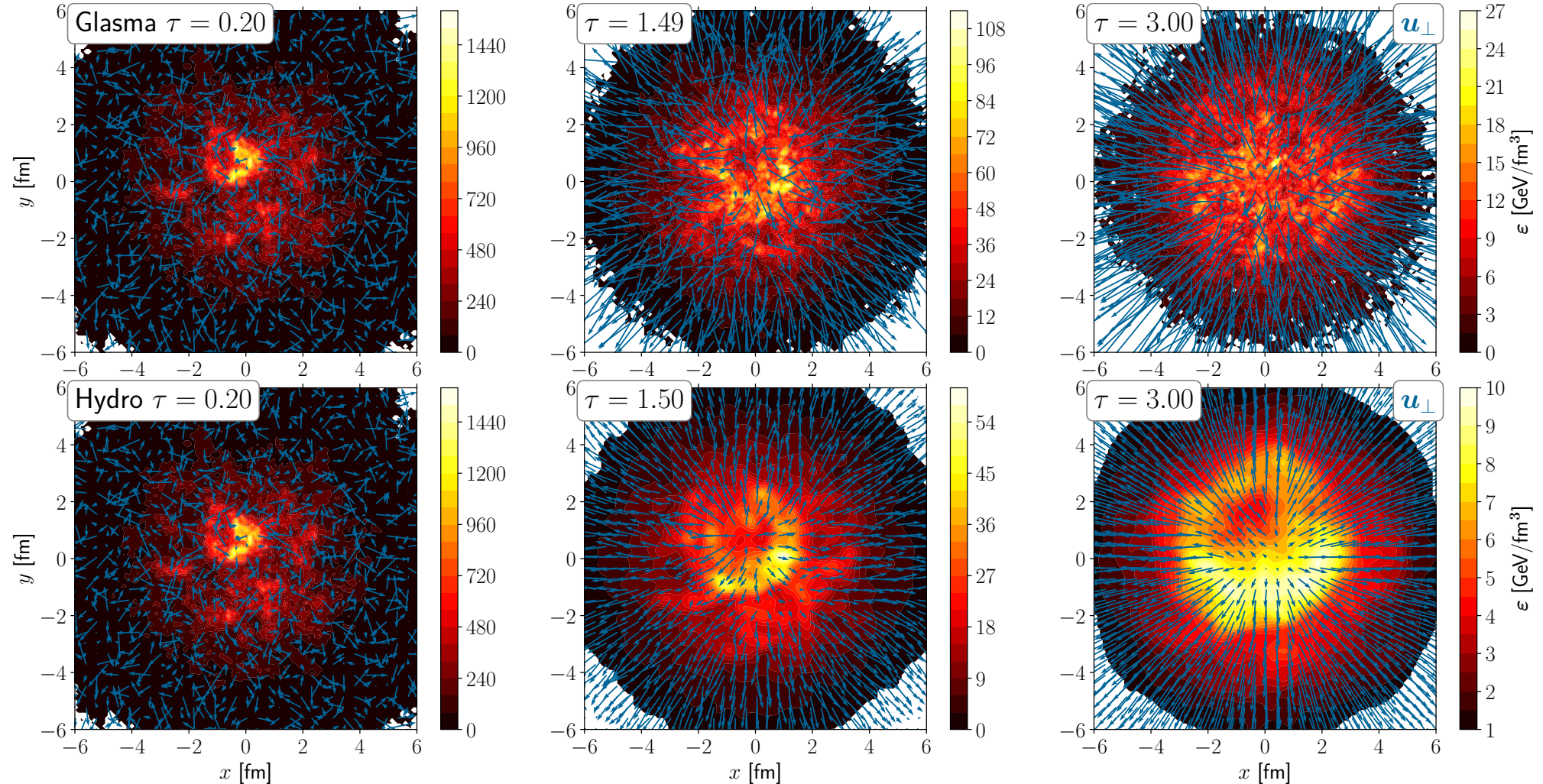
Param.	Name	Value(s)	Unit
N_c	No. of colors	3	–
γ	Lorentz factor	100 (R), 2700 (L)	–
$\sqrt{s_{NN}}$	c.m. energy	200 (R), 5400 (L)	GeV
R	WS radius	6.38 (R), 6.62 (L)	fm
d	WS skin depth	0.535 (R), 0.546 (L)	fm
g	YM coupling	1	–
μ	MV scale	1	GeV
m	IR cutoff	0.2, 2.0	GeV
Λ_{UV}	UV cutoff	10	GeV
ξ	correlation length	0.1, 0.5, 2.0	R_l
b	impact parameter	0, 1	R
τ	proper time	0.2, 0.4, 0.6, 0.8, 1.0	fm/c

Gaussian distribution set by expectation values

$$\langle \rho(x^\pm, \mathbf{x}) \rangle = 0$$

$$\langle \rho^a(x^\pm, \mathbf{x})\, \rho^b(y^\pm, \mathbf{y}) \rangle = \delta^{ab} g^2 \mu^2(x^\pm) \delta(x^\pm - y^\pm) \delta^{(2)}(\mathbf{x} - \mathbf{y})$$

Extension to non–trivial longitudinal structure

$$\langle \rho^a(x^\pm, \mathbf{x})\, \rho^b(y^\pm, \mathbf{y}) \rangle =$$

$$\delta^{ab} \; \underbrace{g^2 \mu^2}_{\substack{\text{strength of} \\ \text{color charges}}} \; \underbrace{T_R \left(\frac{x^\pm + y^\pm}{2} \right)}_{\substack{\text{longitudinal profile} \\ \text{of width } R}} \underbrace{U_\xi \left(x^\pm - y^\pm \right)}_{\substack{\text{correlations} \\ \text{of width } \xi}} \underbrace{T_S \left(\mathbf{x} - \mathbf{y} \right)}_{\substack{\text{transverse profile} \\ \text{of width } S}} \underbrace{\delta^{(2)} \left(\mathbf{x} - \mathbf{y} \right)}_{\text{uncorrelated}}$$

$$\langle \rho^{a,-}(x^{\pm}, \mathbf{x}) \rangle = 0$$

$$\langle \rho^{a,-}(x^{\pm}, \mathbf{x}) \, \rho^{b,-}(y^{\pm}, \mathbf{y}) \rangle =$$

$$\delta^{ab} \, g^2 \mu^2 \, T_R \left(\frac{x^{\pm} + y^{\pm}}{2} \right) U_{\xi} \left(x^{\pm} - y^{\pm} \right) T_S \left(\mathbf{x} - \mathbf{y} \right) \delta^{(2)} \left(\mathbf{x} - \mathbf{y} \right)$$

Single nuclei separation ansatz for gaussian T_R, T_S:

$$\langle \rho^{a,-}(x^{\pm}, \mathbf{x}) \, \rho^{b,-}(y^{\pm}, \mathbf{y}) \rangle = \delta^{ab} \, g^2 \mu^2 \, \sqrt{T_R \left(x^{\pm} \right)} \sqrt{T_S \left(\mathbf{x} \right)} \sqrt{T_R \left(y^{\pm} \right)} \sqrt{T_S \left(\mathbf{y} \right)} \times$$

$$U_{\xi}^{\mathrm{mod}} \left(x^{\pm} - y^{\pm} \right) \delta^{(2)} \left(\mathbf{x} - \mathbf{y} \right)$$

$$U_{\xi}^{\mathrm{mod}} \left(x^{\pm} - y^{\pm} \right) = \frac{1}{\sqrt{2\pi \xi^2}} \exp \left[-(x^{\pm} - y^{\pm})^2 \left(\frac{1}{2\xi^2} - \frac{1}{8R^2} \right) \right]$$

$$\langle \rho^a(x^\pm, \mathbf{x}) \rangle = 0$$

$$\langle \rho^a(x^\pm, \mathbf{x}) \rho^b(y^\pm, \mathbf{y}) \rangle = g^2 \mu^2 \delta^{ab} \sqrt{T(x^\pm, \mathbf{x})} \sqrt{T(y^\pm, \mathbf{y})} U_\xi(x^\pm - y^\pm) \delta^{(2)}(\mathbf{x} - \mathbf{y})$$

$$T(x^\pm, \mathbf{x}) = \frac{c}{1 + \exp\left(\frac{\sqrt{2(\gamma\, x^\pm)^2 + \mathbf{x}^2} - R}{d}\right)}$$

$$U_\xi(x^\pm - y^\pm) = \frac{1}{\sqrt{2\pi \xi^2}} \mathrm{e}^{\frac{(x^\pm - y^\pm)^2}{8 R_l^2}} \mathrm{e}^{-\frac{(x^\pm - y^\pm)^2}{2\xi^2}}$$

$$\mathcal{A}_{A/B}^{\mp a}(x^\pm, \mathbf{x}) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\tilde{\rho}_{A/B}^a(x^\pm, \mathbf{k})}{\mathbf{k}^2 + m^2} \mathrm{e}^{-\mathbf{k}^2/(2\Lambda_{\mathrm{UV}}^2)} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}},$$

3D Woods–Saxon envelope profile

$$T(x^\pm, \mathbf{x}) = \frac{c}{1 + \exp\left(\frac{\sqrt{2(\gamma\,x^\pm)^2 + \mathbf{x}^2} - R}{d}\right)}$$
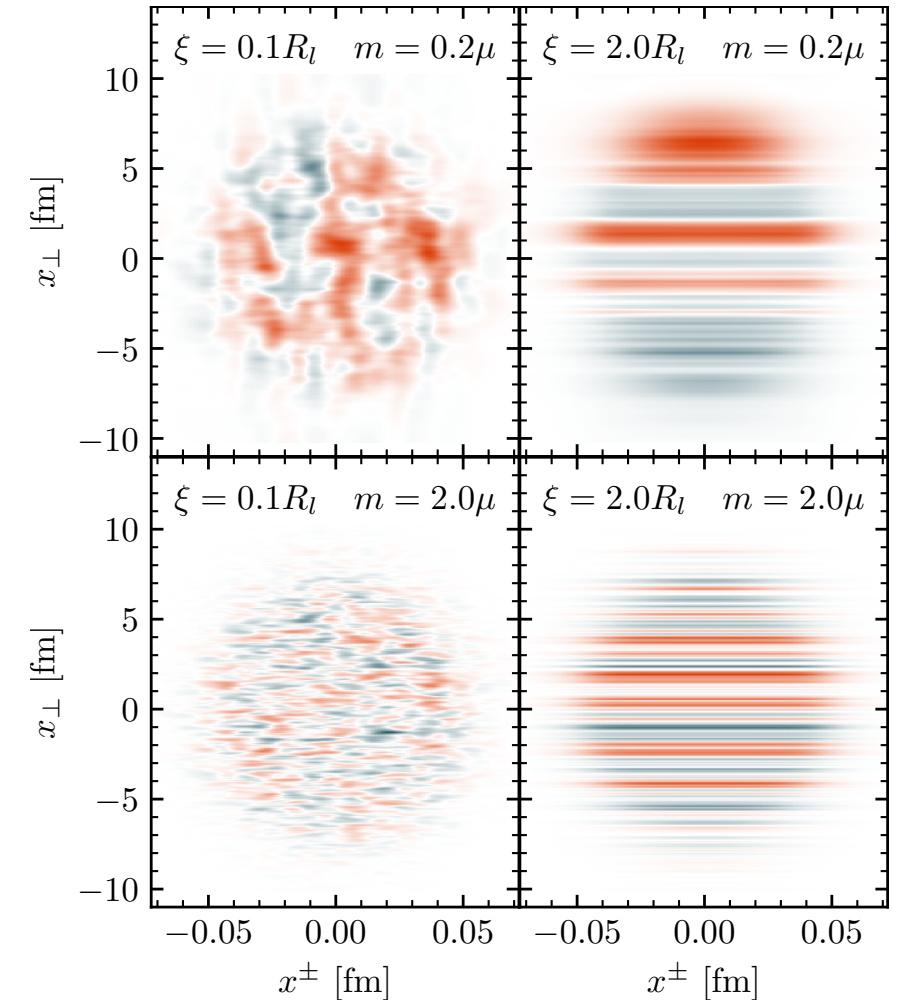
with parameters

Nuclear radius: R

Skin depth: d

Lorentz gamma: γ

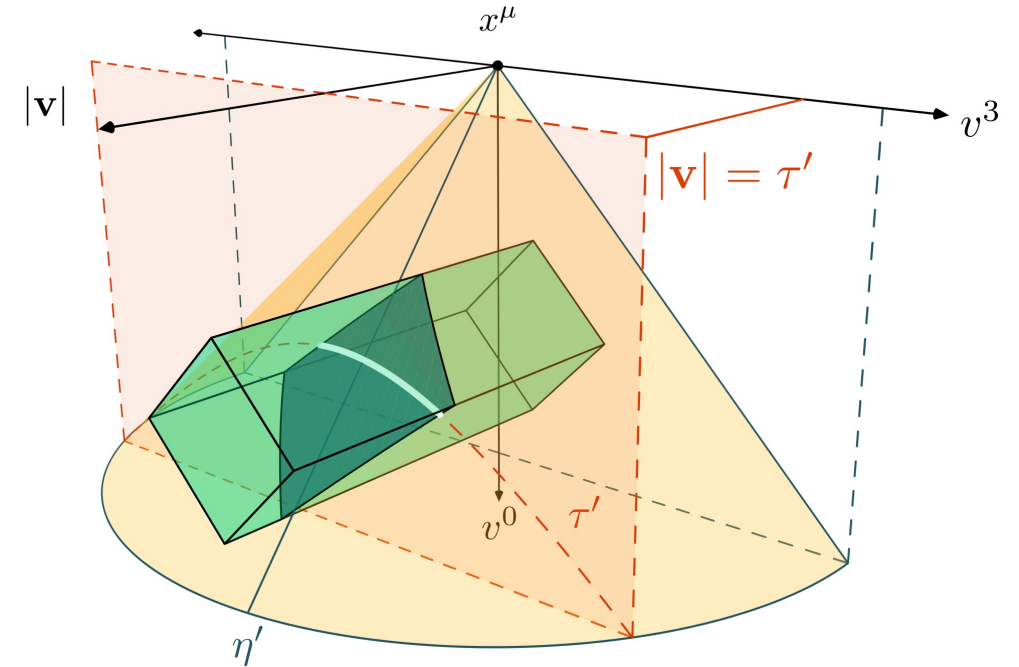that depend on collider energy and nucleus species.

- Pick a spacetime point $x = (x^+, x^-, \mathbf{x})$

- Integrate over past lightlike trajectories

$$v = \left(\frac{\tau'}{\sqrt{2}} e^{+\eta'}, \frac{\tau'}{\sqrt{2}} e^{-\eta'}, \mathbf{v}\right), \ \tau' = |\mathbf{v}|$$

- The integrand depends on $x - v$

$$V := f_{abc} t^c \partial^i \mathcal{A}_A^{-a}\left(x^+ - \frac{|\mathbf{v}|}{\sqrt{2}} e^{+\eta'}, \mathbf{x} - \mathbf{v}\right) \partial^i \mathcal{A}_B^{+b}\left(x^- - \frac{|\mathbf{v}|}{\sqrt{2}} e^{-\eta'}, \mathbf{x} - \mathbf{v}\right)$$

$$V^{ij} := f_{abc} t^c \left(\partial^i \mathcal{A}_A^{-a}(\ldots) \partial^j \mathcal{A}_B^{+b}(\ldots) - \partial^j \mathcal{A}_A^{-a}(\ldots) \partial^i \mathcal{A}_B^{+b}(\ldots)\right)$$

Limiting fragmentation approximation

Inserting nuclei at higher $e^w\sqrt{s}$ with $\gamma \sim e^w$:

$$\mathcal{A}^{\mp}_{A/B}(x^{\pm}, \mathbf{x}) \to e^{+w}\mathcal{A}^{\mp}_{A/B}(e^{+w}x^{\pm}, \mathbf{x})$$

$$\zeta^i_A(\mathbf{x} - \mathbf{v}) = \partial^i \int dy^+ \mathcal{A}^-_A(y^+, \mathbf{x} - \mathbf{v})$$

$$\to \partial^i \int d(y^+ e^{+w}) \, \mathcal{A}^-_A(y^+ e^{+w}, \mathbf{x} - \mathbf{v}) = \zeta^i_A(\mathbf{x} - \mathbf{v})$$

$$\beta^i_B(x, \mathbf{v}) = \partial^i \mathcal{A}^+_B\left(x^-\left(1 - \frac{\mathbf{v}^2}{\tau^2}\right), \mathbf{x} - \mathbf{v}\right)$$

$$\to \partial^i e^{+w} \mathcal{A}^+_B\left(e^{+w}x^-\left(1 - \frac{\mathbf{v}^2}{\tau^2}\right), \mathbf{x} - \mathbf{v}\right) = e^{+w}\beta^{i,b}_B(e^{+w}x^-, ...)$$

$$Y(x, \mathbf{v}) := f_{abc}t^c \, \zeta^{i,a}_A(\mathbf{x} - \mathbf{v}) \, \beta^{i,b}_B(x, \mathbf{v})$$

$$\to f_{abc}t^c \, \zeta^{i,a}_A(\mathbf{x} - \mathbf{v}) \, e^{+w}\beta^{i,b}_B(e^{+w}x^-, \tau, \mathbf{x} - \mathbf{v})$$

$$Y^{ij}(x, \mathbf{v}) := f_{abc}t^c \left(\zeta^{i,a}_A(...) \, \beta^{j,b}_B(...) - \zeta^{j,a}_A(...) \, \beta^{i,b}_B(...) \right)$$

$$\to f_{abc}t^c e^{+w} \left(\zeta^{i,a}_A(...) \, \beta^{j,b}_B(e^{+w}x^-, ...) - \zeta^{j,a}_A(...) \, \beta^{i,b}_B(e^{+w}x^-, ...) \right)$$

$$f^{+-} = -\frac{g}{2\pi}\frac{1}{x^+}\int_{\mathbf{v}} Y(x, \mathbf{v})$$

$$\to f^{+-}(e^{-w}x^+, e^{+w}x^-, \mathbf{x}_\perp)$$

$$f^{ij} = -\frac{g}{2\pi}\frac{1}{x^+}\int_{\mathbf{v}} Y^{ij}(x, \mathbf{v})$$

$$\to f^{ij}(e^{-w}x^+, e^{+w}x^-, \mathbf{x}_\perp)$$

$$f^{+i} = \frac{g}{2\pi}\int_{\mathbf{v}} \left(Y^{ij}(x, \mathbf{v}) - \delta^{ij}Y(x, \mathbf{v}) \right)\frac{v^j}{\mathbf{v}^2}$$

$$\to e^{+w}f^{+i}(e^{-w}x^+, e^{+w}x^-, \mathbf{x}_\perp)$$

$$f^{-i} = \frac{g}{2\pi}\frac{1}{(x^+)^2}\int_{\mathbf{v}} \left(Y^{ij}(x, \mathbf{v}) + \delta^{ij}Y(x, \mathbf{v}) \right)\frac{v^j}{2}$$

$$\to e^{-w}f^{-i}(e^{-w}x^+, e^{+w}x^-, \mathbf{x}_\perp)$$

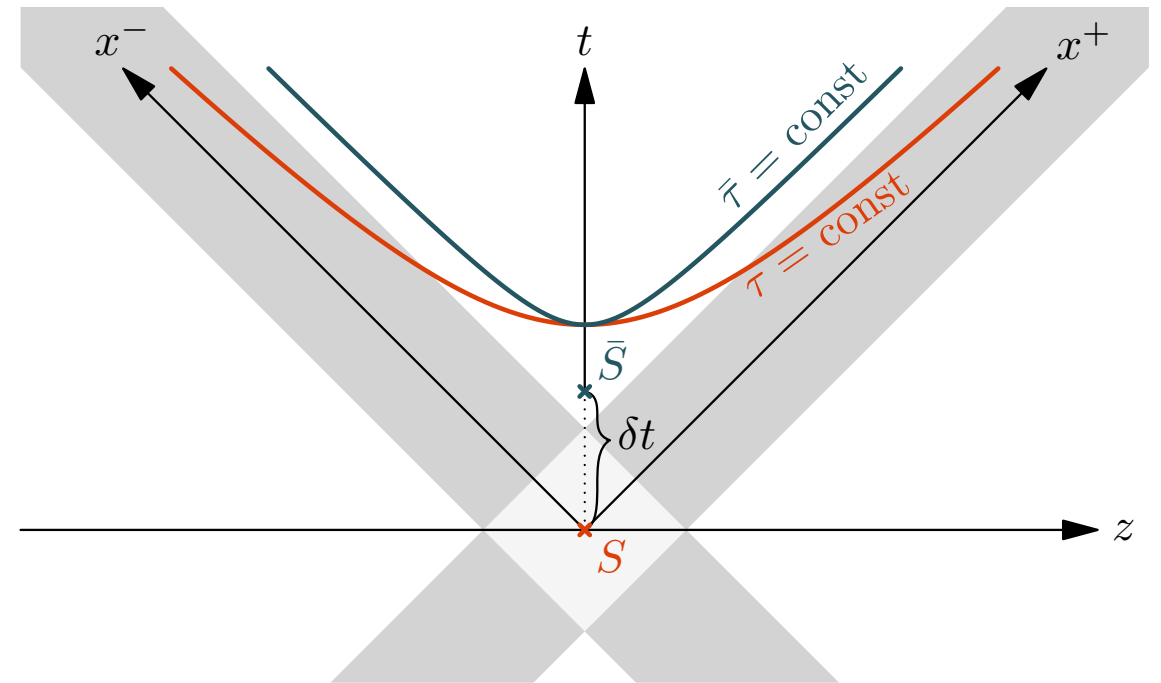$$f^{\mu\nu} \to \Lambda^\mu_{\ \rho}(w)\Lambda^\nu_{\ \sigma}(w)f^{\rho\sigma}(\Lambda^{-1}(w)x)$$

Milne coordinates

$$\tau = \sqrt{2x^+ x^-} = \sqrt{t^2 - z^2}$$

$$\eta_s = \frac{1}{2} \ln \left(\frac{x^+}{x^-} \right) = \mathrm{artanh} \left(\frac{z}{t} \right)$$

are used to parametrize observables of the Glasma.

For extended collision region it is not obvious where to put the origin!

We shift the origin to avoid $\bar{\tau} = \mathrm{const}$ hyperbolas entering the nuclei!

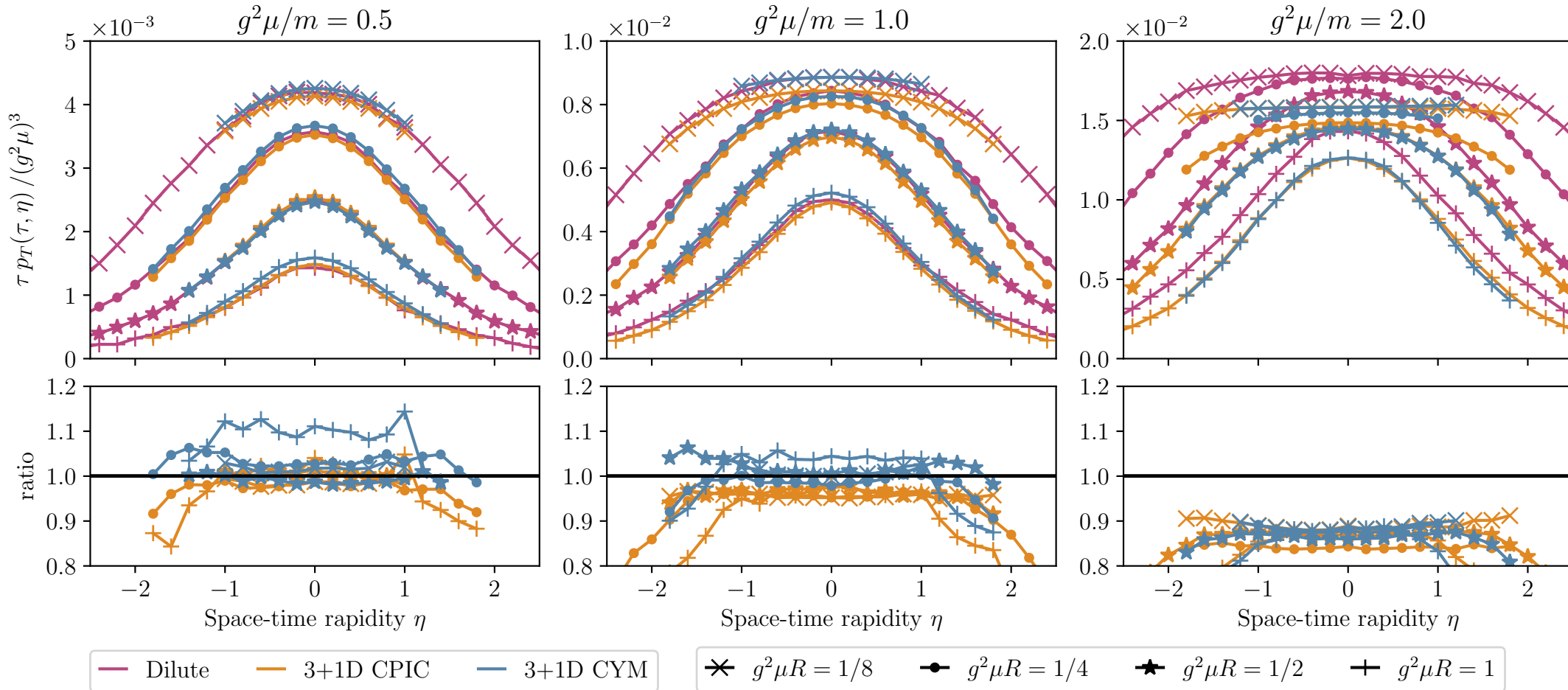Using the dilute Glasma field strength tensor $f^{\mu\nu}$

- Energy–momentum tensor

$$T^{\mu\nu} = 2\,\mathrm{Tr}\left[f^{\mu\rho}f_\rho^{\ \nu} + \frac{1}{4}g^{\mu\nu}f^{\rho\sigma}f_{\rho\sigma}\right]$$

- Local rest frame energy density ϵ_{LRF}

$$T^\mu_{\ \nu}u^\nu = \epsilon_{\mathrm{LRF}}u^\mu$$

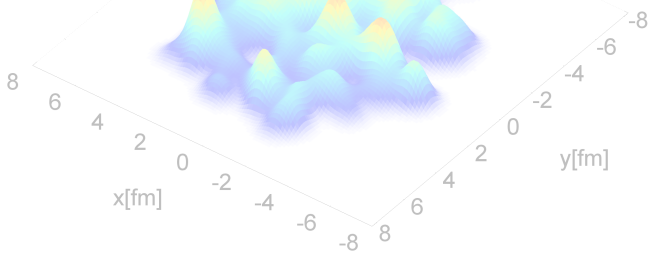(u^μ is the only timelike eigenvector of $T^\mu_{\ \nu}$ with $u^\mu u_\mu = 1$)

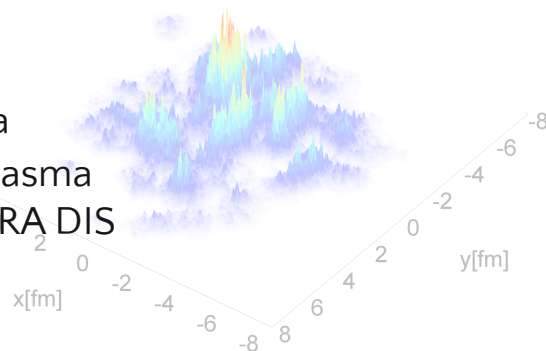# Comparison to lattice simulations

Initial energy density comparison

MC–Glauber   Monte Carlo Glauber-type
Fixed Gaussian at each participant

IP–Glasma

Impact parameter dependent Glasma
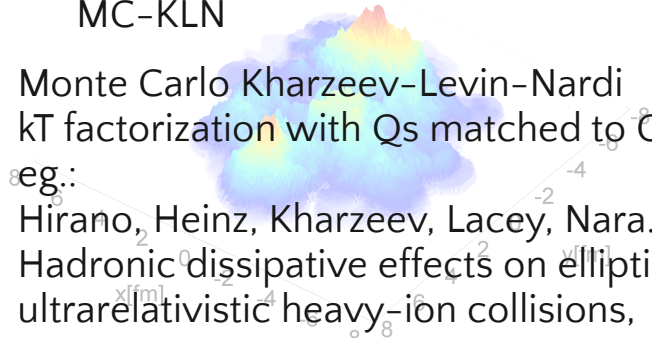IP-Saturation model fitted to HERA DIS

MC–KLN

Monte Carlo Kharzeev–Levin–Nardi
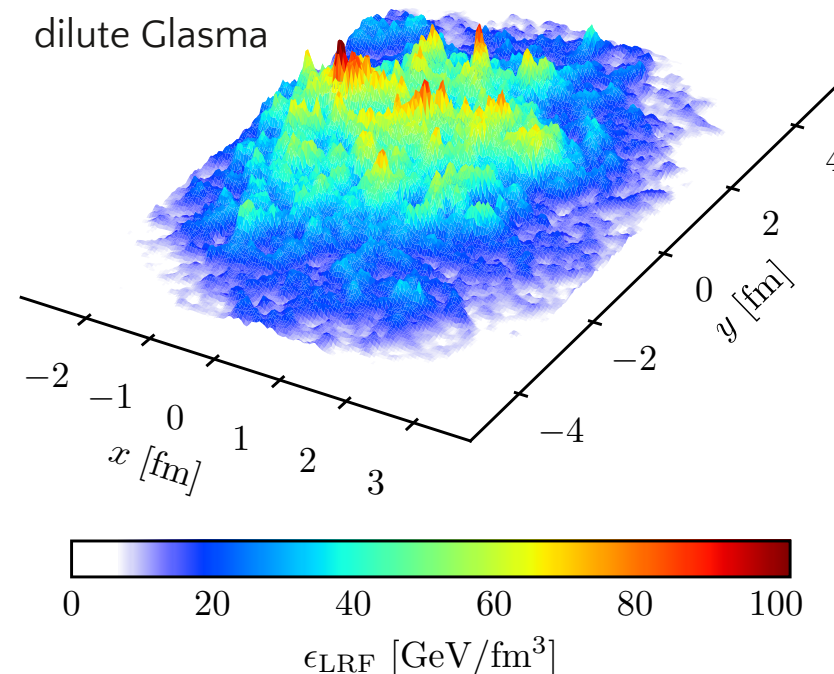kT factorization with Qs matched to G(x, Qs)
eg.:
Hirano, Heinz, Kharzeev, Lacey, Nara.
Hadronic dissipative effects on elliptic flow in
ultrarelativistic heavy-ion collisions,
Physics Letters B Volume 636, Issue 6 (2006), 299–304

- Fluctuating domains on similar scales compared to IP-Glasma

- Q_s equivalent parameter m (IR regulator)

dilute Glasma

ϵ_{LRF} [GeV/fm^3]

$\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au+Au at $\tau = 0.4$ fm/c with impact parameter $b = R$

- $\mathbf{x} = (x, y)$ ... transverse plane

- z ... beam axis

- $\phi \in [0, 2\pi)$ ... azimuthal angle

- $\theta \in [0, \pi)$ ... polar angle

- η ... pseudorapidity
  $\eta = -\ln[\arctan(\theta/2)]$

- y ... rapidity
  $\mathrm{y} = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} \approx \eta$ for $p \gg m$