Probing the photon emissivity of the quark-gluon plasma without an inverse problem in lattice QCD

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Overview

- Introduction and motivation
- Theory
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  - Photon emissivity
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- Results
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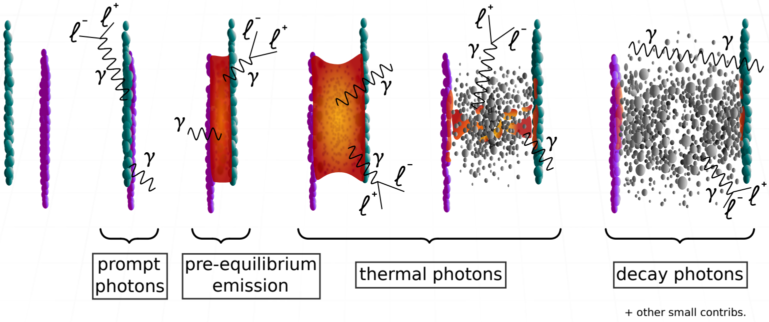
    - first Matsubara sector
  - Continuum extrapolation in the high temperature phase $(N_f = 2)$
    - first and second Matsubara sector

Introduction



direct photons = total - [decay photons]

- colorless nature of photons makes them penetrating probes of the QGP
  → on-going experimental research (RHIC, LHC, GSI)
- for $1\,\text{GeV} \lesssim p_T \lesssim 3\,\text{GeV}$: dominant contribution from thermal photons (both QGP and hadronic phase)

Introduction

- PHENIX and ALICE collaborations show a direct photon excess at low $p_T \lesssim 3\,\mathrm{GeV}$ w.r.t. theoretical predictions [1509.06738, 2106.11216, 2308.16704]

  $\hookrightarrow$ **thermal photons**
- large measured anisotropy [1108.2131]
- one possible explanation: (hydro) models underestimate the photon emissivity around the phase transition [1308.2440]
- in the later stages of the collision, the photons tend to 'inherit' the anisotropic flow of the strongly-interacting medium [1907.08893]

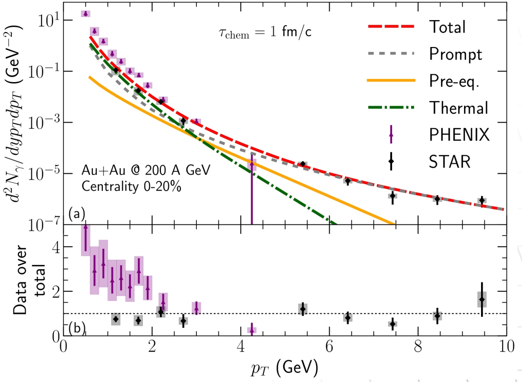


Figure 1: Direct photon yield in central Au+Au collisions at maximum RHIC energies. The plot is taken from [2106.11216].

Motivation

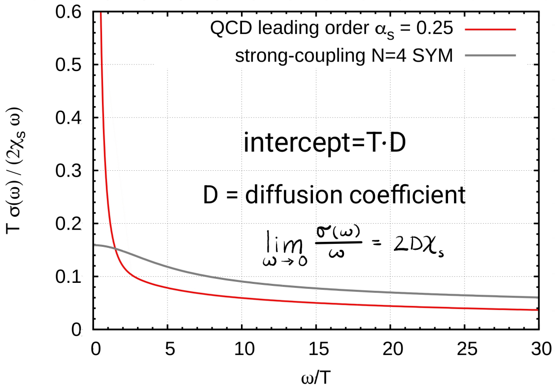


Figure 2: The spectral function (normalized by $2\chi_s\omega/T$) at lightlike kinematics in QCD at complete leading order according to [hep-ph/0111107], and in the strongly-coupled $\mathcal{N} = 4$ SYM theory [hep-th/0607237]. The plot is taken from [2309.09884].

Spectral function of the electromagnetic current $j_\mu^{\text{em.}}(x) = \sum_f Q_f \bar{\psi}(x)\gamma_\mu\psi(0)$

$$\rho_{\mu\nu}(\omega, \mathbf{k}) = \int \mathrm{d}^4x\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x}))} \langle [j_\mu^{\text{em.}}(x), j_\nu^{\text{em.}}(0)^\dagger] \rangle$$

$$\sigma(\omega) \equiv \rho_T(\omega, |\mathbf{k}| = \omega)$$

$\hookrightarrow$ we focus on the isovector vector current $J_\mu(x) = \bar{\psi}(x)\frac{\tau_3}{\sqrt{2}}\gamma_\mu\psi(x)$

- $\sigma(\omega)$ vanishes in the vacuum
- $\sigma(\omega)$ vanishes for thermal, non-interacting quarks (at $\omega \neq 0$)

$\Rightarrow$ optimal probe of the medium

Thermal photon rate

- differential photon emissivity (per unit volume) [McLerran, Toimela, PRD '85]:

$$\frac{d\Gamma_\gamma}{d\omega} = \frac{\alpha_{\mathrm{em}}}{\pi} \frac{2\omega}{e^{\beta\omega} - 1} \sigma(\omega) + \mathcal{O}(\alpha_{\mathrm{em}}^2)\,, \tag{1}$$

where

$$\sigma(\omega) \equiv \rho_T(\omega, |\mathbf{k}| = \omega) = \frac{1}{2}\left(\delta^{ij} - \frac{k^i k^j}{k^2}\right) \rho_{ij}(\omega, \mathbf{k}) \tag{2}$$

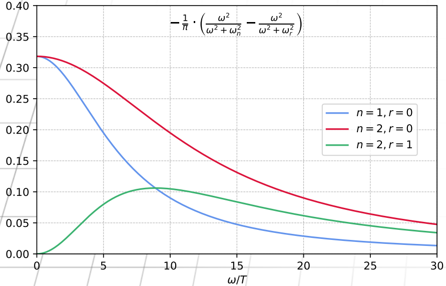
- computing the full energy-differential photon emissivity of a medium at thermal equilibrium from lattice QCD involves a numerically ill-posed inverse problem. However, energy-integrated information on the photon emissivity in the form of $H_E(\omega_n)$ can be obtained without confronting an inverse problem

Motivation

- define the momentum-space Euclidean correlator $H_E(\omega_n) \equiv G_E(\omega_n, k = i\omega_n)$ with Matsubara frequency $\omega_n = 2\pi T n$ and *imaginary spatial momentum* $k = i\omega_n$
- this correlator is related to the transverse channel spectral function via a once-subtracted dispersion relation [1807.00781]:

Dispersion relation

$$H_E(\omega_n) - H_E(\omega_r) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\,\omega \left[\frac{1}{\omega^2+\omega_n^2} - \frac{1}{\omega^2+\omega_r^2}\right]\sigma(\omega) \tag{3}$$



- energy-moments of $\sigma(\omega)$ are *directly* accessible on the lattice without an inverse problem
- $\sigma(\omega) \propto$ differential photon emissivity
- $H_E(\omega_1)$ and $H_E(\omega_2)$ receive a sizeable contribution from *soft* photons
  ↪ the difference $H_E(\omega_2) - H_E(\omega_1)$ is only sensitive to *hard* photon emission

Formulation on the lattice

Lattice subtracted correlator (standard representation)

$$H_E(\omega_n) = -\int_0^\beta \mathrm{d}x_0 \int_{\mathrm{R}^3} \mathrm{d}^3x \left(e^{i\omega_n x_0} - e^{i\omega_n x_2}\right) e^{-\omega_n x_1} \langle J_3(x) J_3(0)\rangle$$
$$= 2\int_0^\infty \mathrm{d}x_1 \cosh(\omega_n x_1) \cdot \left[G^T_{\mathrm{ns}}(\omega_n, x_1) - G^T_{\mathrm{st}}(\omega_n, x_1)\right] \quad (4)$$

- the subtracted static contribution vanishes in the continuum
- with this subtraction $H_E(\omega_n)$ vanishes exactly in the vacuum even at finite lattice spacing
- problem: exponential signal to noise problem $[p_T = \mathcal{O}(1 - 3\,\mathrm{GeV})]$
  $\hookrightarrow$ need to model the tail or use a bounding method

Numerical setup of the projects with $\mathcal{O}(a)$–improved Wilson fermions

Table 1: Parameters and lattice spacing of the $N_f = 2$ ensembles at a fixed aspect ratio $L/N_\tau = 4$ corresponding to a temperature $T = 254\,\text{MeV}$.

N_τ/a	L/a	$6/g_0^2$	κ	a [fm]	N_{conf}	Label
24	96	5.827160	0.136544	0.033	1500	X7
20	80	5.685727	0.136684	0.039	1600	W7
16	64	5.5	0.13671	0.049	1400	O7

- in vacuo pion mass $m_\pi = 270\,\text{MeV}$
- quark mass effects suppressed in chirally symmetric phase ($\bar{m}^{\overline{\text{MS}}} \approx 13\,\text{MeV}$)

Table 2: Parameters of the $N_f = 2+1$ ensembles with lattice spacing $a = 0.06426(76)$ fm and $\kappa_l = 0.137232867$. The lattice spacing determination is from Ref. [Bruno et al. PRD '17].

N_τ/a	L/a	$6/g_0^2$	N_{conf}	Label	T [MeV]
24	96	3.55	1200	E250Nt24	127.9(1.5)
20	96	3.55	1200	E250Nt20	153.5(1.8)
16	96	3.55	1500	E250Nt16	191.9(2.3)

- 3 gauge ensembles at quasi-physical quark masses
- $\{T_{24}, T_{20}, T_{16}\}/T_{pc} \approx \{0.82, 0.98, 1.23\}$ with $T_{pc} = 156.5(1.5)$ MeV [HotQCD collab PRL '19]

Reducing cut-off effects on the lattice

- since the cut-off effects get exponentiated through the cosh-kernel, we modify the kernel via

$$\cosh(\omega_2) \longrightarrow \cosh\left[\frac{1}{a}\sin(a \cdot \omega_2)\right] \tag{5}$$

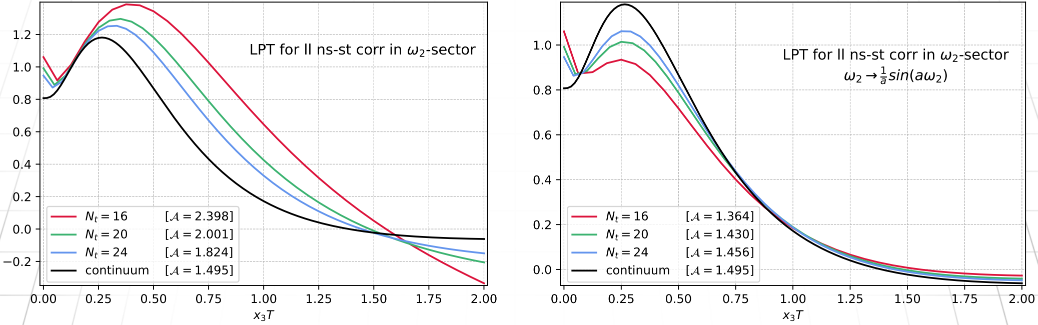
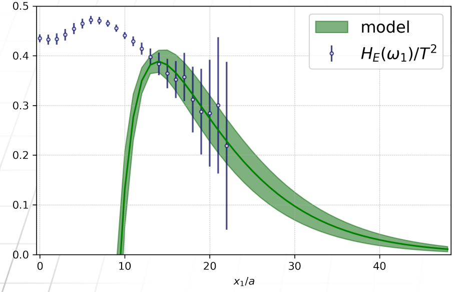


Figure 4: Lattice perturbation theory prediction for the standard subtracted correlator (**Left panel**) and with the modified kernel (**Right panel**) in the free theory [second Matsubara sector].

$H_E(\omega_1)$ results in $N_f = 2+1$ at $T = 154\,\text{MeV}$



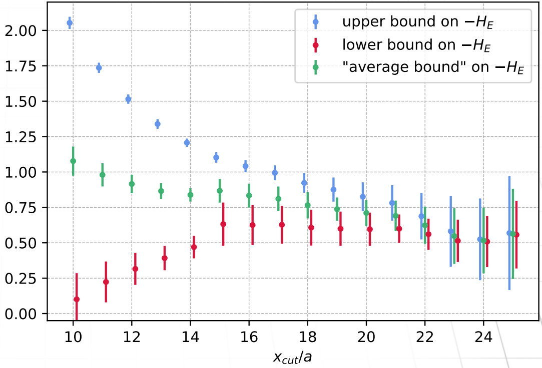
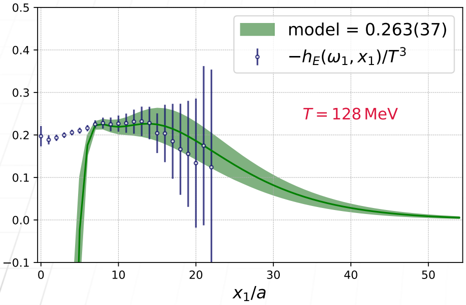
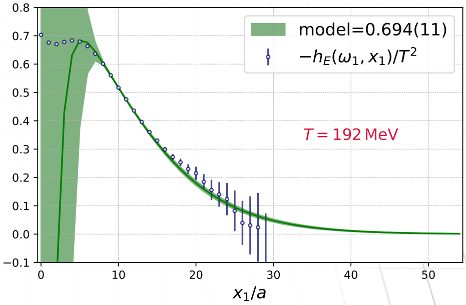


Figure 5: **Left**: Plot of the x_1-symmetrized integrand of Eq. (4) together with the result from modelling the tail. **Right**: Lower and upper bounds on $H_E(\omega_1)$ that converge toward that quantity for $x_{cut} \to \infty$

$$G_T^i(\omega_1, x) = \sum_{n=1}^{2} \left|A_n^i\right|^2 \cosh\left[m_n \cdot \left(x_1 - \frac{L}{2}\right)\right] \tag{6}$$

Summary results on $H_E(\omega_1)$ in $N_f = 2+1$

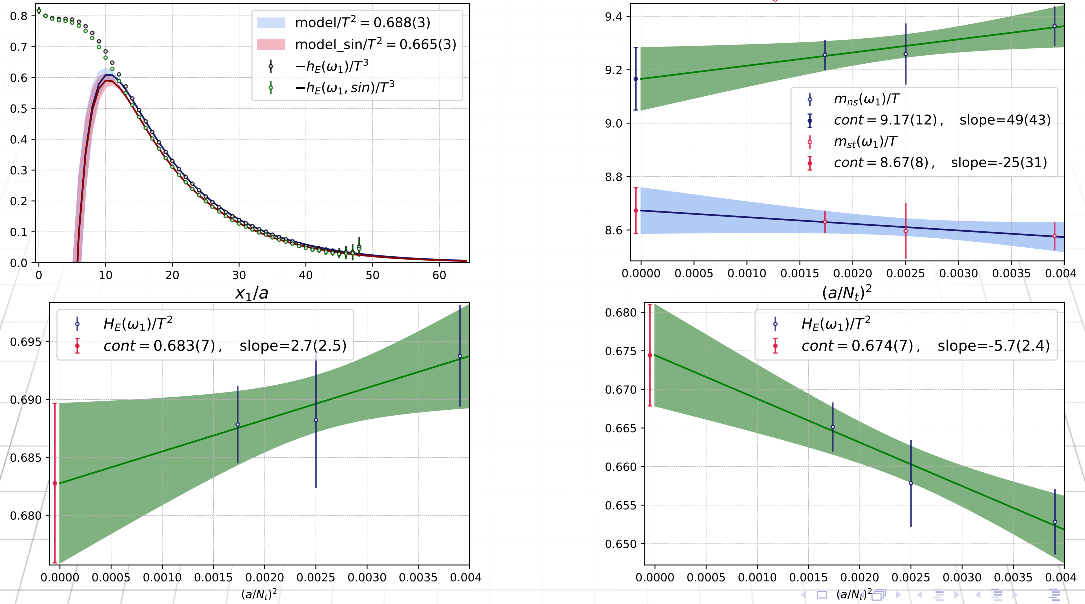




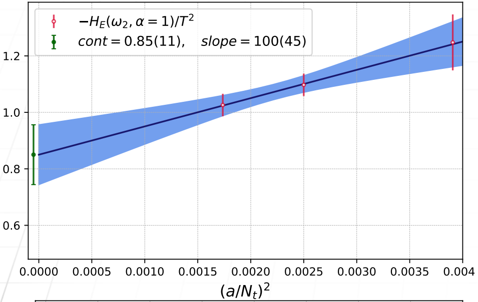
N_τ/a	N_f	T [MeV]	$-H_E(\omega_1)/T^2$
24	2+1	127.9(1.5)	0.263(37)
20	2+1	153.5(1.8)	0.62(13)
16	2+1	191.9(2.3)	0.69(1)
Cont.	2	254	0.683(7)

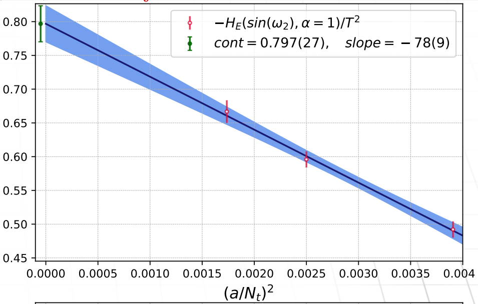
- $H_E(\omega_1)$ at $T \approx T_{pc}$ as a measure of the integrated photon emissivity is compatible (in units of temperature) with $T = 192\,\mathrm{MeV}$ result
- significantly bigger than the result in the hadronic phase
- continuum extrapolated result on $H_E(\omega_1)/T^2$ at $T \approx 1.2\,T_{pc}$ agrees with $N_f = 2+1$ result

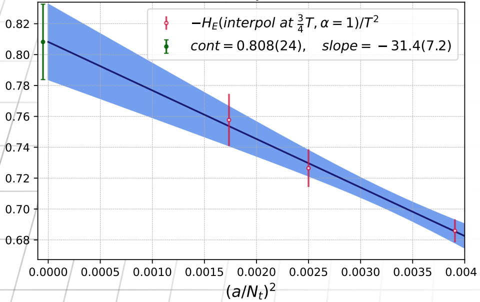
(Continuum extrapolated) results on $H_E(\omega_1)$ in $N_f = 2$ at $T = 254\,\mathrm{MeV}$

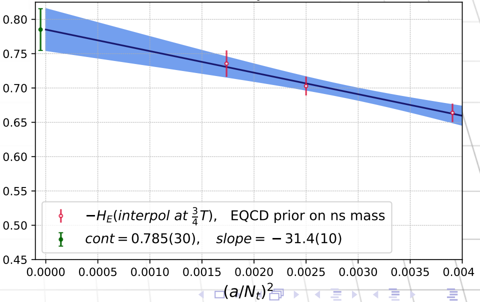


Continuum extrapolated results on $H_E(\omega_2)$ in $N_f = 2$ at $T = 254\,\mathrm{MeV}$









Summary $H_E(\omega_2)$ and comparison with AMY (PRELIMINARY)

Model	$-H_E(\omega_2)/T^2$	$-\left[H_E(\omega_2) - H_E(\omega_1)\right]/T^2$
standard subtraction	0.85(11)	0.17(11)
modified kernel	0.797(27)	0.114(27)
interpolation	0.808(24)	0.125(24)
interpolation EQCD prior	0.785(30)	0.102(30)

$\Longrightarrow$ **QGP emits *hard* photons**

Comparison with AMY

$$-\left[H_E(\omega_2) - H_E(\omega_1)\right]/T^2 = 0.125(24)_{\text{stat.}}(20)_{\text{syst.}} \quad (7)$$

$$-\left[H_E(\omega_2) - H_E(\omega_1)\right]_{\text{LO},0.2<\omega/\text{T}<50}/T^2 \approx 0.25 \quad (8)$$

$\Longrightarrow$ AMY prediction appears to overestimate the emission of hard photons at $T = 254\,\text{MeV}$

Back-Up Slides

Bounding Method

- screening correlators have a representation in terms of energies and amplitudes of screening eigenstates:

$$G^{T}_{E,i}(\omega_r, x) \overset{x\neq 0}{=} \sum_{n=0}^{\infty} \left|A^{(r)}_{i,n}\right|^2 e^{-E^{(r)}_{i,n}|x|}\,,\; i \in \{\mathrm{st},\mathrm{ns}\}\,,\; \omega_r = 2r\pi T \tag{9}$$

- idea: set upper and lower bound for Euclidean correlators [Borsanyi et al., PRL, 2018 Blum et al., PRL, 2018 Gerardin et al., PRD, 2019]

$$0 \leq G^{T}_{E,i}(\omega_1, x_{\mathrm{cut}}) e^{-m_{\mathrm{eff}}(x_{\mathrm{cut}})\cdot(x-x_{\mathrm{cut}})} \tag{10}$$

$$\leq G^{T}_{E,i}(\omega_1, x) \leq G^{T}_{E,i}(\omega_1, x_{\mathrm{cut}}) e^{-E^{(1)}_{i,0}\cdot(x-x_{\mathrm{cut}})}\,,\quad x \geq x_{\mathrm{cut}}$$

- correlator difference in Eq. (4) $\Rightarrow$ correct bounds:

$$H_E(\omega_1)|_{\mathrm{ub}} \propto G^{T}_{\mathrm{ns}}(\omega_1, x_1)\Big|_{\mathrm{ub}} - G^{T}_{\mathrm{st}}(\omega_1, x_1)\Big|_{\mathrm{lb}} \tag{11}$$

$$H_E(\omega_1)|_{\mathrm{lb}} \propto G^{T}_{\mathrm{ns}}(\omega_1, x_1)\Big|_{\mathrm{lb}} - G^{T}_{\mathrm{st}}(\omega_1, x_1)\Big|_{\mathrm{ub}} \tag{12}$$

Bounding Method

- Assuming two-pion ground states:

static case

$$E_{st,0}^{(1)}(p) = 2\sqrt{\left(\frac{p}{2}\right)^2 + m_\pi^2 + \left(\frac{2\pi}{L}\right)^2} \tag{13}$$

non-static case

$$E_{ns,0}^{(1)}(\omega_1) = \sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2} + \sqrt{E_\pi(\omega_1)^2 + \left(\frac{2\pi}{L}\right)^2} \tag{14}$$

- $p = \omega_1 = 2\pi T$
- m_π: effective pion mass at zero-momentum
- $\frac{2\pi}{L}$: momentum in direction of the vector index
- $E_\pi(\omega_1)$: effective pseudoscalar mass in the non-static sector

Spatially transverse Euclidean correlators

Euclidean screening vector correlator $[x_\perp = (x_2, x_3)]$

$$G_{E,\mu\nu}(\omega_n, p_2, p_3, x_1) = \int_0^\beta \mathrm{d}x_0 e^{i\omega_n x_0} \int \mathrm{d}^2 x_\perp e^{i(p_2 x_2 + p_3 x_3)} \langle J_\mu(x) J_\nu(0) \rangle \tag{15}$$

Restriction to transverse channel and $\omega_1 = 2\pi T$

$$\begin{aligned} G_E^T(\omega_1, p_2, x_1) &\equiv G_{E,33}(\omega_1, p_2, 0, x_1) \\ &= -\int_0^\beta \mathrm{d}x_0 e^{i\omega_1 x_0} \int \mathrm{d}^2 x_\perp e^{ip_2 x_2} \langle J_3(x) J_3(0) \rangle \end{aligned} \tag{16}$$

Definition of static and non-static transverse channel

$$G_{\mathrm{st}}^T(p, x_1) \equiv G_E^T(\omega_1 = 0, p_2 = p, x_1) \tag{17}$$

$$G_{\mathrm{ns}}^T(\omega_1, x_1) \equiv G_E^T(\omega_1, p_2 = 0, x_1) \tag{18}$$

Spatially transverse Euclidean correlators

Fourier transform of non-static corr. eval. at imaginary momentum

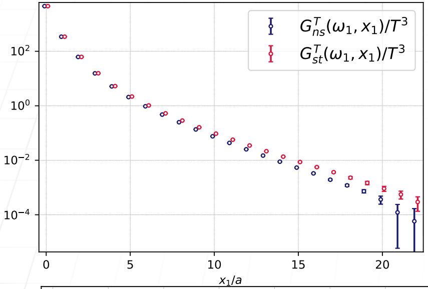
$$G^T_{\text{ns}}(\omega_1, k) = \int_{\mathbb{R}} \mathrm{d}x_1 e^{ikx_1} G^T_{\text{ns}}(\omega_1, x_1) \overset{k=i\omega_1}{\equiv} H_E(\omega_1) \quad (19)$$

H_E-quantity [Meyer, Eur.Phys.J.A, 2018]

$$H_E(\omega_1) = -\int_0^\beta \mathrm{d}x_0 \int \mathrm{d}^3x\, e^{i\omega_1 x_0} e^{-\omega_1 x_1} \langle J_3(x) J_3(0)\rangle < 0 \quad (20)$$

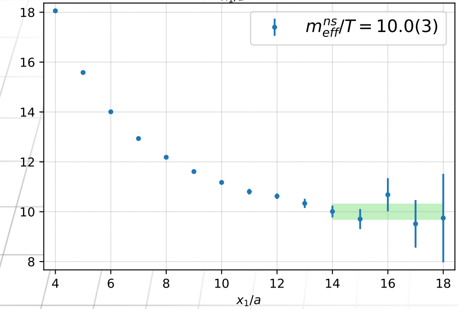
- continuum: $H_E(\omega_1)$ should vanish in the vacuum
- problem: lattice regulator breaks Lorentz symmetry
  $\hookrightarrow$ ultraviolet divergence

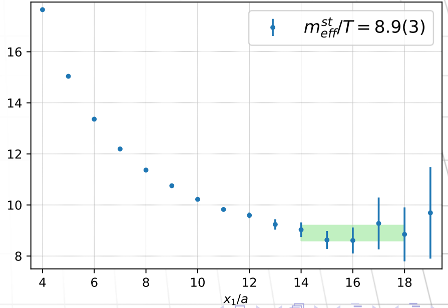
Transverse screening correlators and effective masses



- exponential signal-to-noise problem:

$$\frac{\Delta G(x_1)}{G(x_1)} = \exp[(m_V - m_{PS}) \cdot x_1]$$





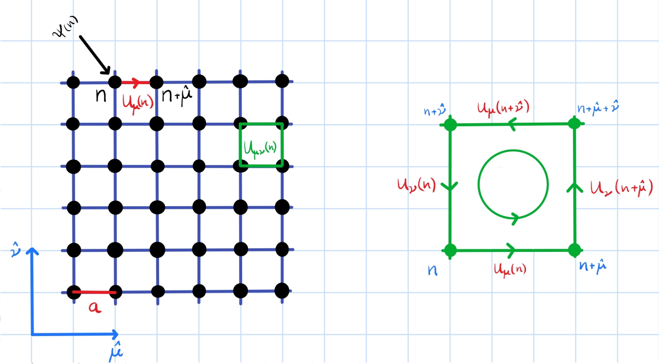
Two-point correlators: Minkowski vs. Euclidean

$$\langle 0|\, \mathcal{O}(x)\bar{\mathcal{O}}(0)\,|0\rangle = \underbrace{\langle 0|\, e^{iH\tau}}_{e^{iE_0\tau}\langle 0|} \mathcal{O}(0,\mathbf{x})e^{-iH\tau}\bar{\mathcal{O}}(0)\,|0\rangle \overset{t=i\tau}{=} \langle 0|\, \mathcal{O}(0,\mathbf{x})e^{-Ht}\bar{\mathcal{O}}\,|0\rangle$$

Pion two-point correlator $\mathcal{O} = \bar{u}\gamma_5 d \equiv P$

$$\begin{aligned} G(t,\mathbf{p}=0) &= \sum_{\mathbf{x}} \langle 0|\,(\bar{u}\gamma_5 d)(0,\mathbf{x})e^{-Ht}\,(\bar{d}\gamma_5 u)(0,\mathbf{0})\,|0\rangle \\ &= L^3 \cdot \sum_n \langle 0|\, P(0,\mathbf{p}=0)e^{-Ht}|n\rangle\,\langle n|P(0,\mathbf{p}=0)\,|0\rangle \\ &= L^3 \cdot |\langle 0|\, P(0,\mathbf{0})\,|\pi\rangle|^2 \cdot e^{-m_\pi t} + \cdots \end{aligned}$$

Lattice QCD



- formulate the theory
  - in a finite volume
  - on a finite grid
  - as a Boltzmann distribution

  and solve using importance sampling Monte Carlo methods
- lattice spacing a is of $\mathcal{O}(10^-16)$ m

- 4D lattice $\Lambda := \{n = (n_1, n_2, n_3, n_4) | n_1, n_2, n_3 = 0, 1, \ldots, L-1\,;\, n_4 = 0, 1, \ldots, T-1\}$
- **quarks**: described by Dirac spinor $\psi(n)$ on lattice sites
  - Grassmann variables $\Rightarrow \psi_1\psi_2 = -\psi_2\psi_1$
- **gluons**: $U_\mu(n) = e^{iaA_\mu(n)} \in SU(3)$ on lattice links

Lattice QCD

- main task: solve Dirac equation $D^{-1}\eta = \psi$
  - D is a sparse matrix with $O(10^{16})$ entries (most elements zero)
  - D is ill-conditioned at physical quark masses and close to the continuum
- for this project: $\mathcal{O}(1000)$ solves on $\mathcal{O}(1000)$ gauge configurations on a single gauge ensemble
- average over configurations, error $\propto 1/\sqrt{N_{conf}}$