The NNNLO pressure of cold dense pressure with hard thermal loops

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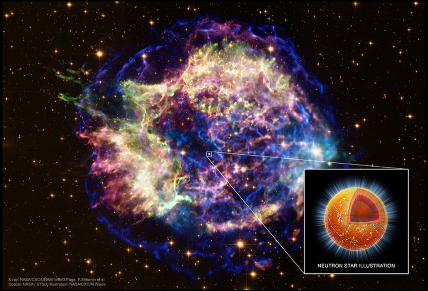
@ Cairns on 21.8.2024 (hopefully around 2 pm after a good lunch)

neutron stars and motivation

what is finite density?

neutron stars are celestial labs for dense (= lots of baryons in a small volume) strongly interacting matter—far denser than anything terrestrial, and constantly running



Cooling neutron star [Nasa, PD]

Finite density $n > 0$
$\longleftrightarrow$ excess of stuff
$\longleftrightarrow$ chemical potential $\mu > 0$

Excess conserved charge
$\longleftrightarrow$ finite (number) density
$\longleftrightarrow$ noether current

why perturbation theory

(subjectively, because it's interesting and fun!)

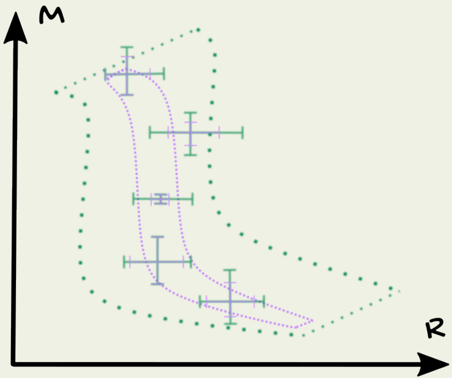
Perturbation theory works well at sufficiently large μ (several GeVs), but not at realistic NS densities

Finite-density lattice QCD suffers from the sign problem $\rightarrow$ unusable unlike at $T > 0$

Otoh, perturbation theory is theoretically ”clean”: No Linde problem, just need very high-order calculations

Other options: Holography (Järvinen), functional approaches (Rennecke), many models ,...

improving the EoS



pQCD PoV: Decrease theoretical uncertainty to shrink the Eos band

observation PoV: plenary (Dexheimer) explained it much better

Lofty end-goal: Incompatible observations and theory
= New Physics
= Lots of grant money

N3LO pressure with perturbation theory

thermal perturbation theory

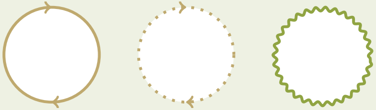
Perturbation theory in Euclidean space at finite μ: Shift p_0 by $i\mu$

- New inherent scale
- Broken Lorentz invariance
- Many integration methods break

(cf. Finite $T \rightarrow$ compactify time factor to a thermal circle, discretise p_0 to Matsubara modes $2n\pi T$, $(2n+1)\pi T$)

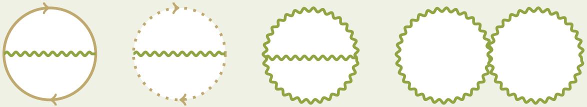
In principle straightforward: Expand $\ln Z = \ln \int \mathcal{D}A\bar{\psi}\psi e^{-S}$ for small g_s, pick up all bubble graphs that contribute at a given order:

$$\Omega_0 =$$



$$\overset{d=3}{=} -\frac{\pi^2}{45}\left(d_A + \frac{7}{4}N_c N_f\right)T^4 - \frac{1}{12\pi^2}N_c \sum_{i=1}^{N_f}\left(\mu_i^2 + 2T^2\right)\mu_i^2,$$

$$g_s^2\Omega_1 =$$

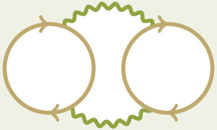


$$\overset{\xi=1,d=3}{=} \frac{g_s^2}{144}d_A\left[\left(N_c + \frac{5}{4}N_f\right)T^4 + \frac{9}{4\pi^4}\sum_{i=1}^{N_f}\left(\mu_i^2 + 2\pi^2T^2\right)\mu_i^2\right]$$

Everything is fine for LO and NLO (even finite T, μ is trivial)

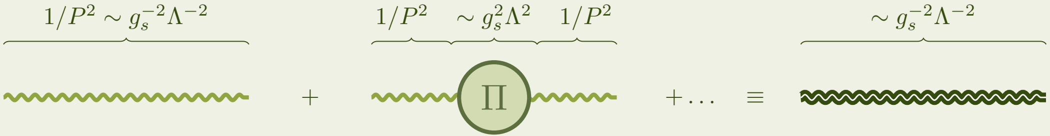
infrared issues at NNLO

Try to do the same with three-loop diagrams: Leftover IR divergence traceable back to exactly one diagram



$$= +\frac{d_A m_E^4}{(8\pi)^2}\left(\frac{\mu_B}{\overline{\Lambda}}\right)^{-2\varepsilon}\left[\frac{1}{2\varepsilon} + O(\varepsilon^0)\right].$$

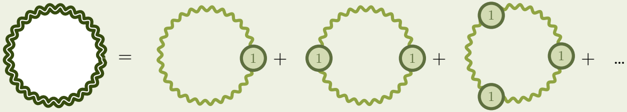
Turns out the quark loop contributes equally to a bare line when the "ring momentum" is soft ($O(g_s\mu)$): Need to resum (gives a "mass" to the diagram)



(Freedman & McLerran 1977): Resummation with full self-energies and NNLO pressure of cold dense matter by Freedman & McLerran—but too difficult to get to N3LO

Hard thermal loops (Braaten & Pisarski, 1992) are much simpler: Simplified propagator for soft momenta = only region where resummation is really needed. Was not used for a long time at finite μ!

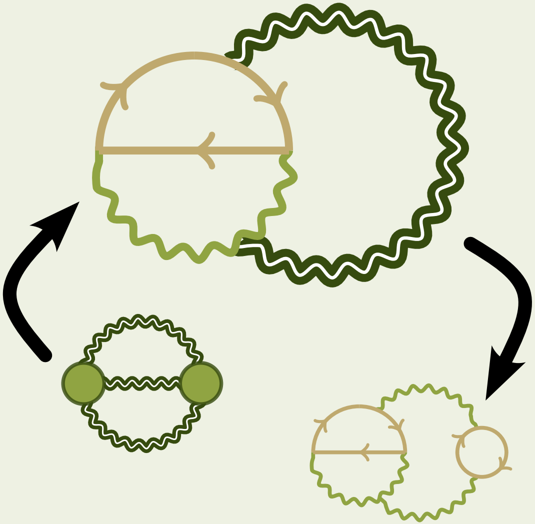
$$g_s^4 \Omega_2^s =$$



$$\approx -\frac{d_A m_E^4}{(8\pi)^2} \left(\frac{m_E}{\overline{\Lambda}}\right)^{-2\varepsilon} \left[\frac{1}{2\varepsilon} + 1.17201\right]$$

Divergences cancel and turn into $\ln g_s$.

N3LO and sectors



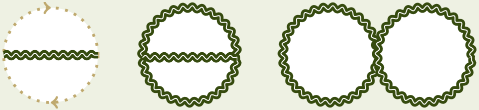
N3LO realisation: Classify diagrams based on number of soft (HTL-resummed) lines

Resum / re-expand lines to move between sectors—at N3LO, there are hard, mixed, and soft diagrams.

Key to understanding conceptual differences between dense and hot perturbation theory!

soft contributions

Soft contributions by evaluating two-loop resummed diagrams (PRL 127 & PRD 104, 2021, leading log in PRL 121, 2018)

$\Omega_3^s =$ 

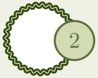
$$\approx -g_s^2 N_c \frac{d_A m_E^4}{4(4\pi)^3} \left(\frac{m_E}{\Lambda_h}\right)^{-4\varepsilon} \left[\frac{11/6\pi}{(2\varepsilon)^2} + \frac{1.50731(19)}{2\varepsilon} + 2.2125(9)\right]$$

Lots of numerical and analytical work. Two-loop HTL diagrams can have double divergences $\sim$ double logs, obtained directly from the leading divergence:

$$g_s^6 \ln^2 g_s \Omega_{3,2}^s = -\frac{d_A N_c}{4(4\pi)^3} \frac{11}{6\pi} g_s^2 m_E^4 \ln^2 g_s$$

mixed contributions

$g_s^6\Omega_3^m =$ 



$= \frac{1}{2}\int_K \mathrm{Tr}\left[G_{\mathrm{HTL}}(K)\Pi^{\mathrm{Pow}}(K) + G_{\mathrm{HTL}}(K)\Pi^{\mathrm{NLO}}(K)\right]$

$$= -\frac{g_s^2 m_E d_A}{(4\pi)^4}\left(\frac{m_E}{\Lambda_h}\right)^{-2\varepsilon}\left(\frac{\mu_B/3}{\Lambda_h}\right)^{-2\varepsilon}\left[-\frac{11}{(2\varepsilon)^2} + \frac{9\ln\left(\frac{3\bar{\Lambda}}{2\mu_B}\right) - 4.8095}{2\varepsilon} - \frac{9}{2}\ln^2\left(\frac{3\bar{\Lambda}}{2\mu_B}\right) + 2.0598\ln\left(\frac{3\bar{\Lambda}}{2\mu_B}\right) - 5.6316\right]$$

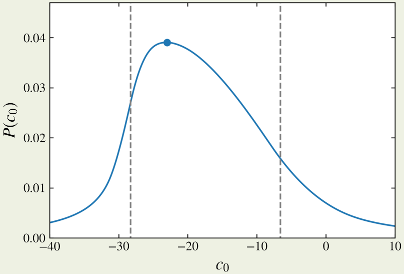
Mixed contributions in 2023 (PRL 131 & JHEP 08) — lots of work to get there, including two papers for just QED. Needed two-loop HTLs (see also Ekstedt, Carignano et al...). They are known at finite T, μ in arbitrary gauge, still working on full NLO including resummed SE; going into this would be a separate talk, but the end result gives the subleading log:

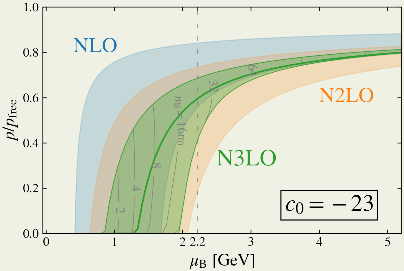
$$g_s^6 \ln g_s \Omega_{3,1} = \left[-22.6431(24) - 6\ln\left(\frac{3\bar{\Lambda}}{2\mu_B}\right)\right] \times \frac{9}{64\pi^6} \times g_s^6 \ln(g_s) \times \Omega_0$$

state of the art

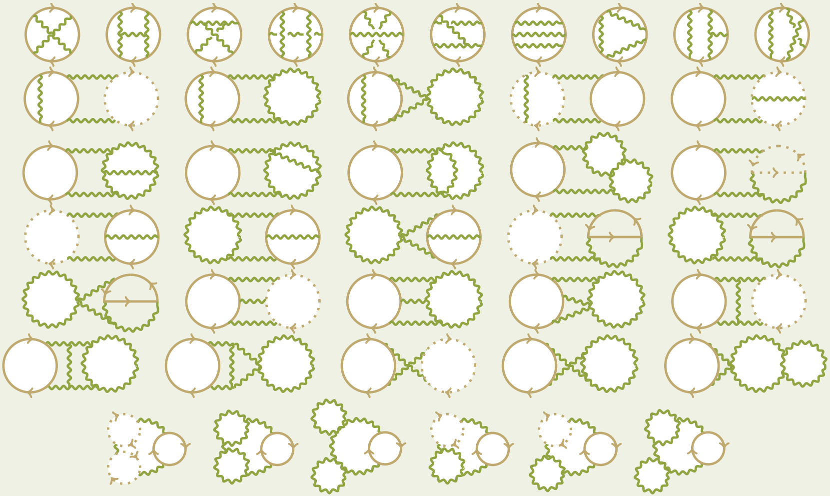
All logarithms known, like at finite T and $\mu = 0$ (Kajantie et al. 2003)

$$p \approx \frac{3}{4\pi^2}\left(\frac{\mu_B}{3}\right)^4 \left\{1 - 2\frac{\alpha_s}{\pi} - 3\left(\frac{\alpha_s}{\pi}\right)^2 \left[\ln\left(3\frac{\alpha_s}{\pi}\right) + 3\ln X + 5.0\right]\right.$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3 \left[\frac{11}{12}\ln^2\left(3\frac{\alpha_s}{\pi}\right) - (-6.6 + 3\ln X)\ln\left(3\frac{\alpha_s}{\pi}\right)\right.$$
$$\left.\left. + 5.1 - 18.\ln X - \frac{9}{2} + \frac{2}{3}c_0\right]\right\} + O(\alpha_s^4)$$





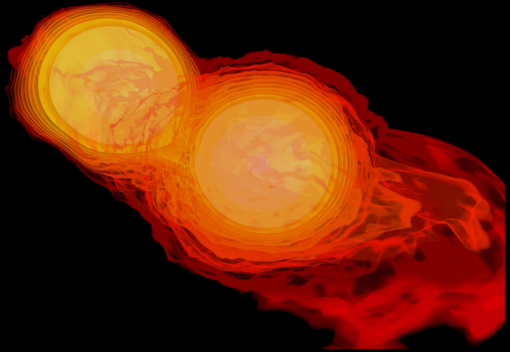
hard contributions



Stay tuned for the next two talks!

what about dynamics?

neutron star collisions



Colliding neutron stars are dynamical systems: Static tricks don't workt

extra scales...

For quiescent EoS, temperature and masses irrelevant. Not so much for collisions! Need at least small ($O(100)$ MeV) temperatures, and effects of the strange quark mass

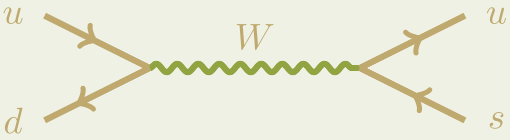
Masses are particularly tough: Even one-loop massive thermal integrals don't admit a closed form... (Gorda & Säppi, PRD 105 2022): Expanding loop-integrals for small masses $m \sim g^r \mu$

$\rightarrow$ simple results for massive thermal equilibrium systems

electroweak process

Bulk viscosity ζ = how well a fluid resists deformation under compression

In NS mergers, primarily driven by the electroweak process $u + d \longleftrightarrow u + s$



The rate of this process enters ζ, we take a very simple approximation:

$$\lambda_1 \approx \frac{64}{5\pi^3} G_F^2 \sin^2\theta_c \cos^2\theta_c \mu_d^5 T^2$$

This is a serious limitation and should be improved (in numerous ways: pairing, QCD corrections, EW corrections, proper T,m_s,$\mu_u - \mu_d$,$\mu_s - \mu_d$-dependence ... — in progress)

bulk viscosity formula

If $u + d \longleftrightarrow d + s$ is the only process, then

$$\zeta = \lambda_1 \frac{A_1^2}{\omega^2 + \lambda_1^2 C_1},$$

where A_1, C_1 are now static quantities determined directly from pressure — standard tricks are usable again!

Holographic collaboration

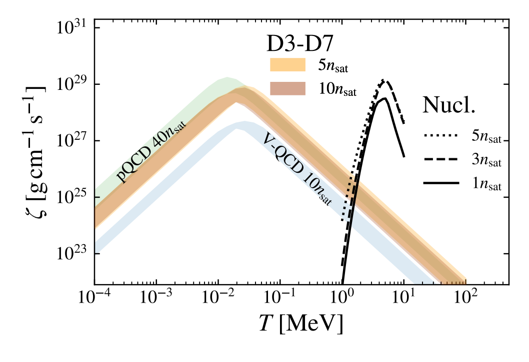
A, B can also be determined from holography to get a complimentary viewpoint—turns out this gives a nice robust results, and a simple formula for ζ from holographic "D3–D7".

$$\zeta = \frac{4\lambda_1\,\mu_d^6\,(M_s^2 - M_d^2)^2}{K_d^2 K_s^2 \omega^2 + \pi^4 \lambda_1^2\,(K_d + K_s)^2}\,,$$

$$K_f = 3\mu_f^2 - M_f^2,$$

$$\zeta_{\text{peak}} = \frac{2}{\pi^2\omega}\,\frac{\mu_d^6(M_s^2 - M_d^2)^2}{K_d K_s (K_d + K_s)},$$

$$T_{\text{peak}}^2 = \frac{\omega}{\pi^2 \Lambda_1}\,\frac{K_d K_s}{K_d + K_s}$$



Brand new PRL 133 (2024), J. Cruz Rojas et al.

fin.

- Perturbation theory gives us a well-defined first-principles way to understand finite density
- Need to understand soft gluons properly using hard thermal loops $\rightarrow$ need to understand HTL better
- Current state-of-the-art pressure is $g_s^6 \ln g_s$
- Starting to move towards full N3LO and transport quantities

Thanks to organisers and collaborators: J. Cruz Rojas (APCTP Pohang), T. Gorda (Frankfurt U.), C. Hoyos (Oviedo U.), N. Jokela (U. of Helsinki), M. Järvinen (APCTP Pohang), A. Kurkela (Stavanger U.), J. Österman (U. of Helsinki), R. Paatelainen (U. of Helsinki), P. Romatschke (U of Boulder, Colorado), P. Schicho (GU Frankfurt), K. Seppänen (U. of Helsinki), A. Vuorinen (U. of Helsinki),
+ WIPs w/ a proper subset of the former and L. Fernandez (U. of Helsinki), M. Heikinheimo (U. of Helsinki) H. Lempiäinen (U. of Helsinki), T. Ruosteoja (U. of Helsinki), K. Tuominen (U. of Helsinki)

And anyone who might have listened!