Finite temperature hadronic spectral properties

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On behalf of the FASTSUM collaboration

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Deconfinement
Thursday 22nd August 2024

The XVIth Quark Confinement and the Hadron Spectrum Conference

Spectral `Functions`

$$G(\tau) = \int_0^\infty \frac{d\omega}{2\,\pi}\, e^{-\omega\,\tau}\,\rho(\omega)$$

Euclidean Correlation Function

- From lattice QCD

Known `Kernel` Function

- Here Laplace transform

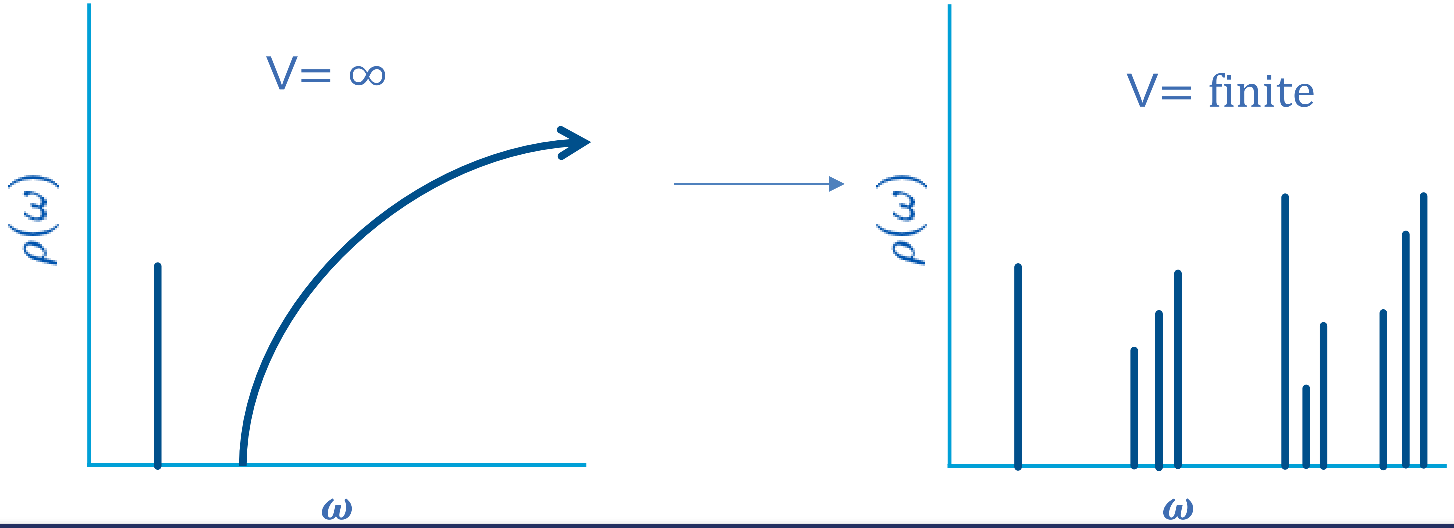
Spectral Density

- Contains information we want!

Spectral Functions

Systematics

- Effect of finite volume
- Uncertainty in Euclidean correlator
- Finite number of samples in Euclidean correlator
- Unphysical (heavy) quark mass



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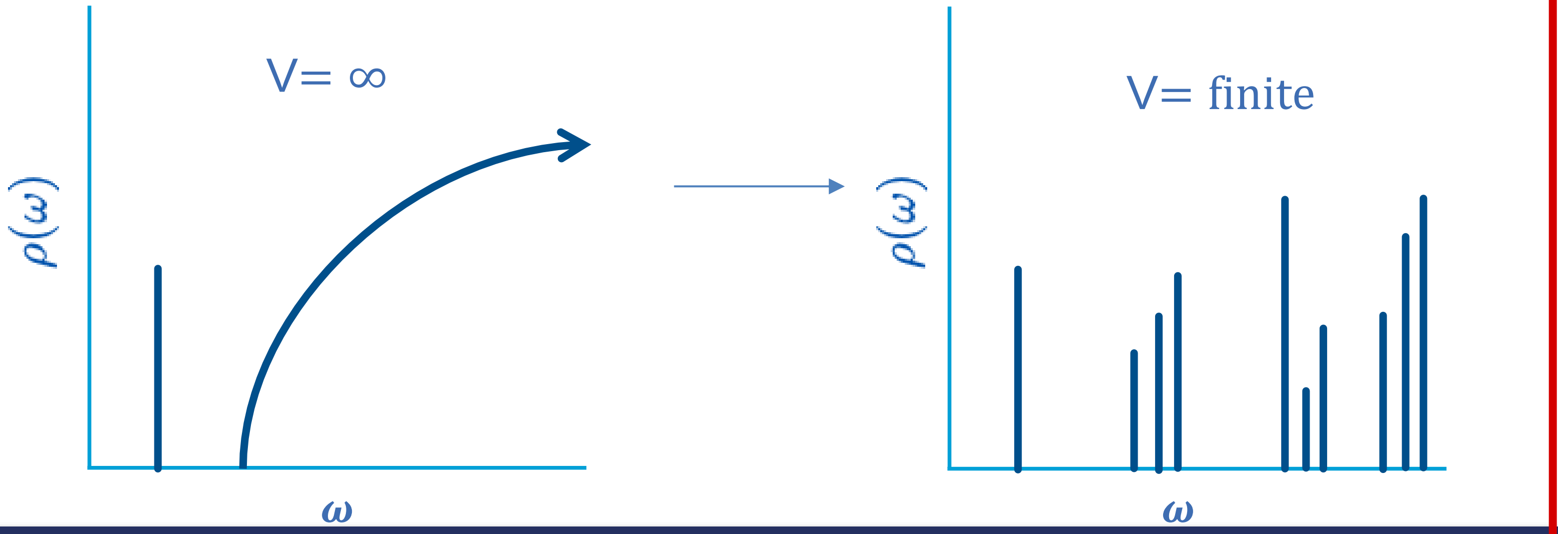


Bottomonium spectra

- Heavy-quark bound states dissociation in deconfined medium
  - Contributes to suppression of quarkonium yield in heavy ion collisions
- Suppression pattern may provide a thermometer for quark-gluon plasma
  - Which bound states dissociate first?
- Lattice QCD aims to provide first principles non-perturbative data
- Information contained in spectral function $\rho(\omega)$

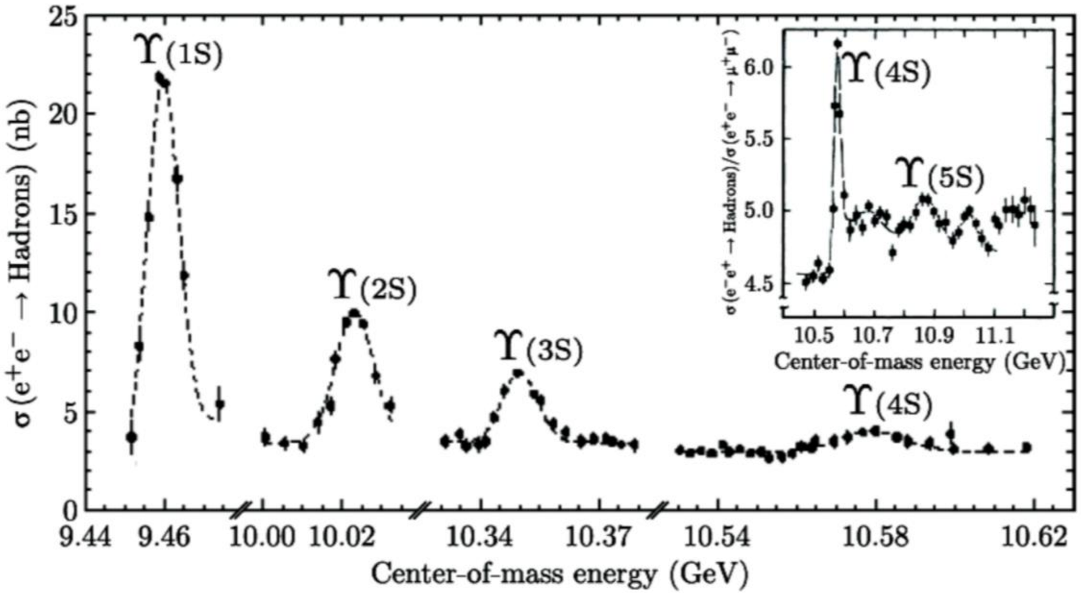
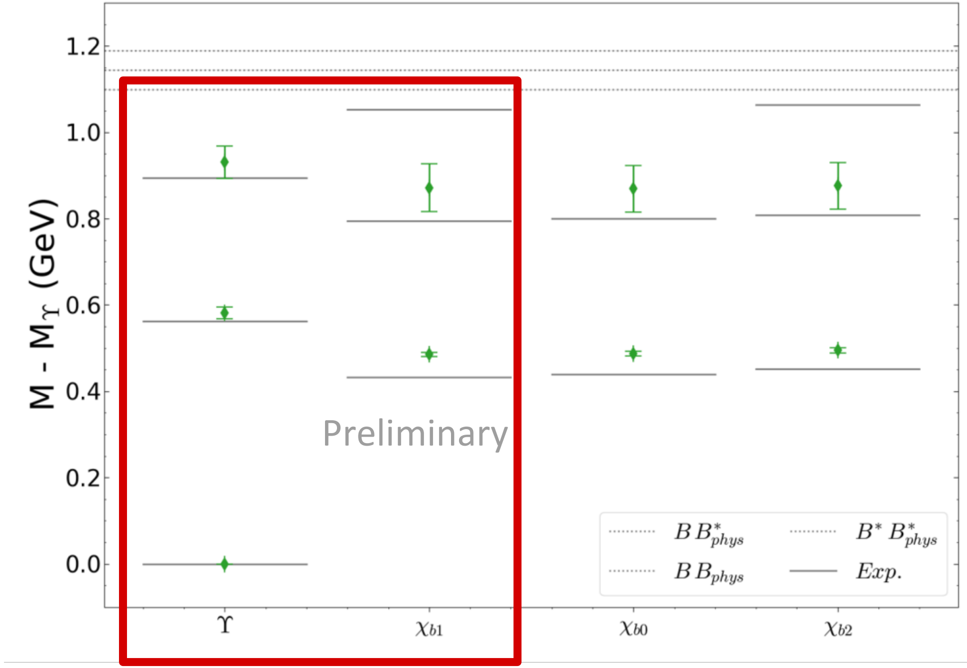


Figure modified by Stottler of CUSB data:
http://hdl.handle.net/10919/109723

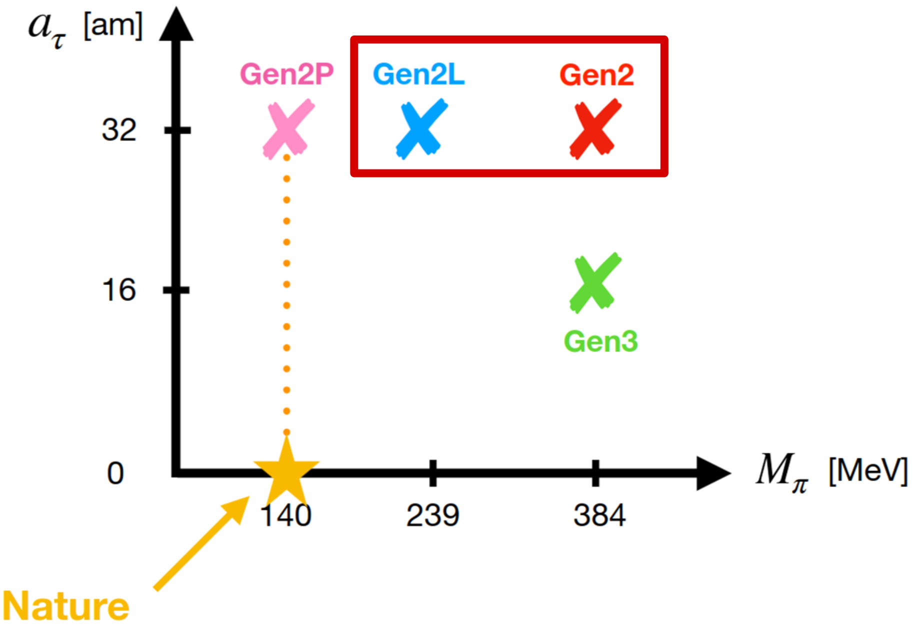
Bottomonium spectrum @ zero temperature

- Easily computable
  - via lattice NRQCD
  - statistically well-behaved due to scale separation
- NRQCD action for bottom quarks
  - Incorporating $O(v^4)$ corrections
  - Tree-level matching coefficients



FASTSUM Approach

- Anisotropic lattices
  - Temporal spacing is ~3.5x finer than spatial
  - Allows fine temperature dependence to be elucidated
  - Many points aid inverse problem methods
- Multiple quark (pion) masses to examine non-physical pion mass systematic effect
- T ∈ [~0, 760] MeV
- See also Skullerud Wed. 15:30



Maximum Entropy Method

Bayesian approach to spectral function reconstruction

Bayes theorem:

$$P(p|DI) = \frac{P(D|pI)P(p|I)}{P(D|I)}$$

Parameterise prior probability

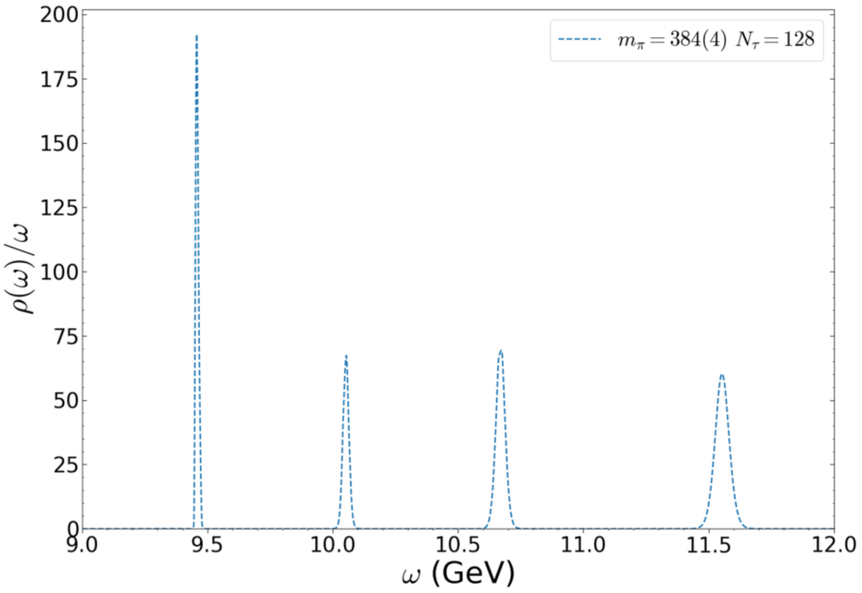
$$\rho(p|I) \propto e^{aS[p]} \Rightarrow p(p|DI) \propto \mathrm{e}^{-L[D,p]+aS[p]}$$

L is standard likelihood (χ^2)

Spectral function in terms of default model $m(\omega)$

$$p(w) = m(w)\exp\left[\sum_{k=1}^{Nb} b_k u_k(w)\right]$$

S is the Shannon-Jaynes entropy & solve via SVD



Maximum Entropy Method

Regularises via additional information

Bayesian approach to spectral function reconstruction

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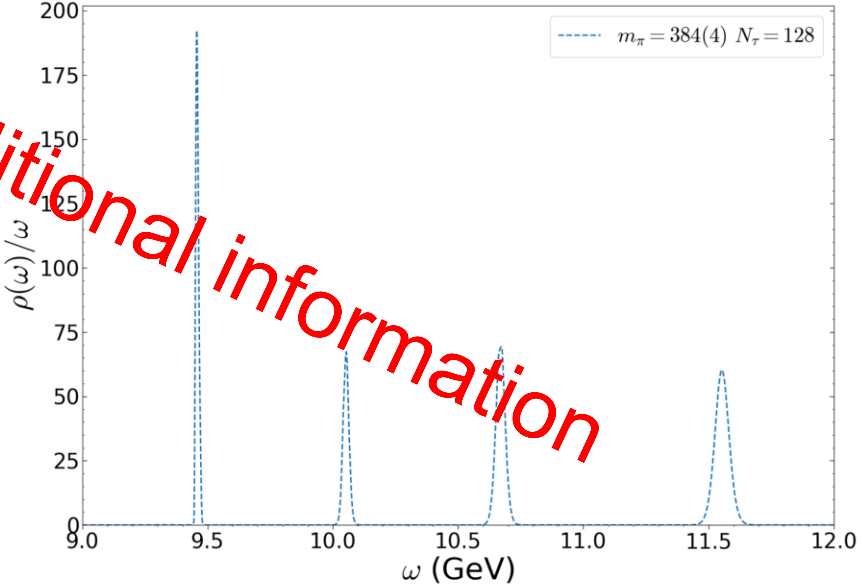
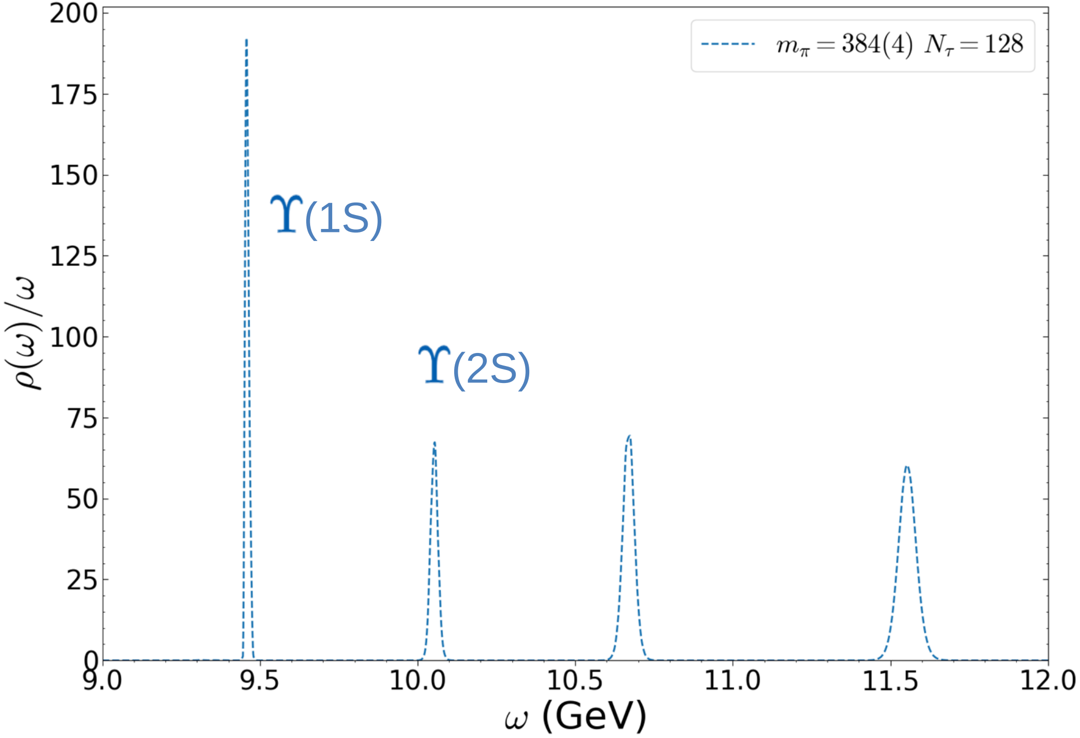


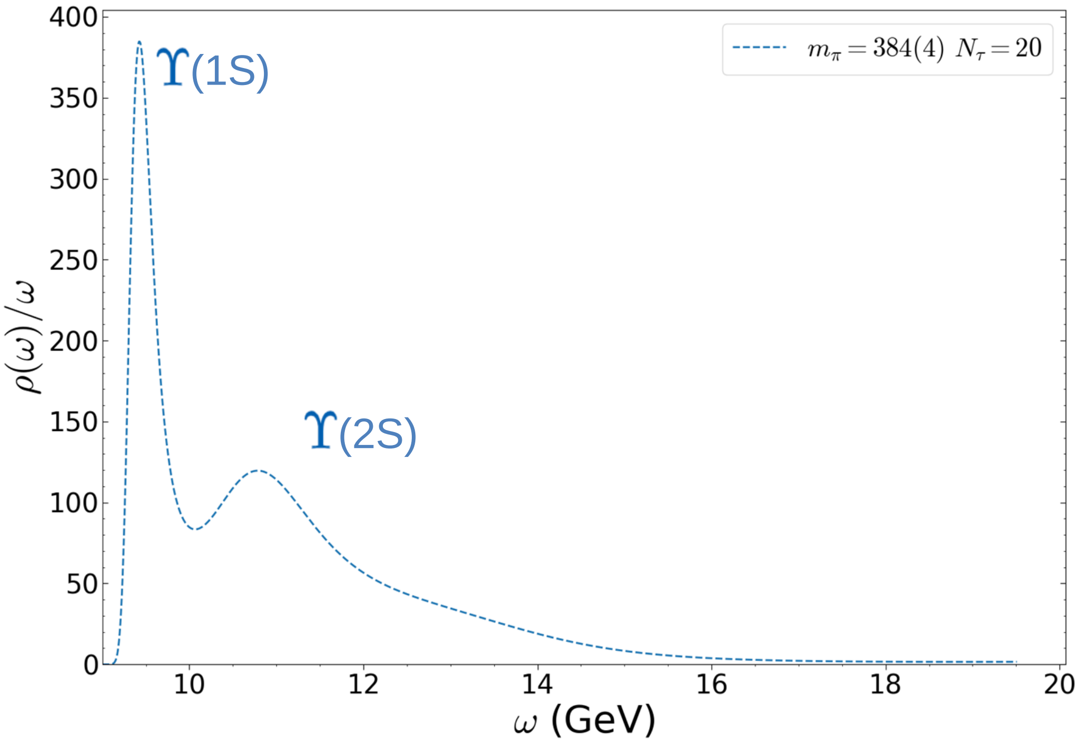
MEM - Zero Temperature

Generation 2 only



MEM - Finite Temperature (hot!)

Generation 2 only



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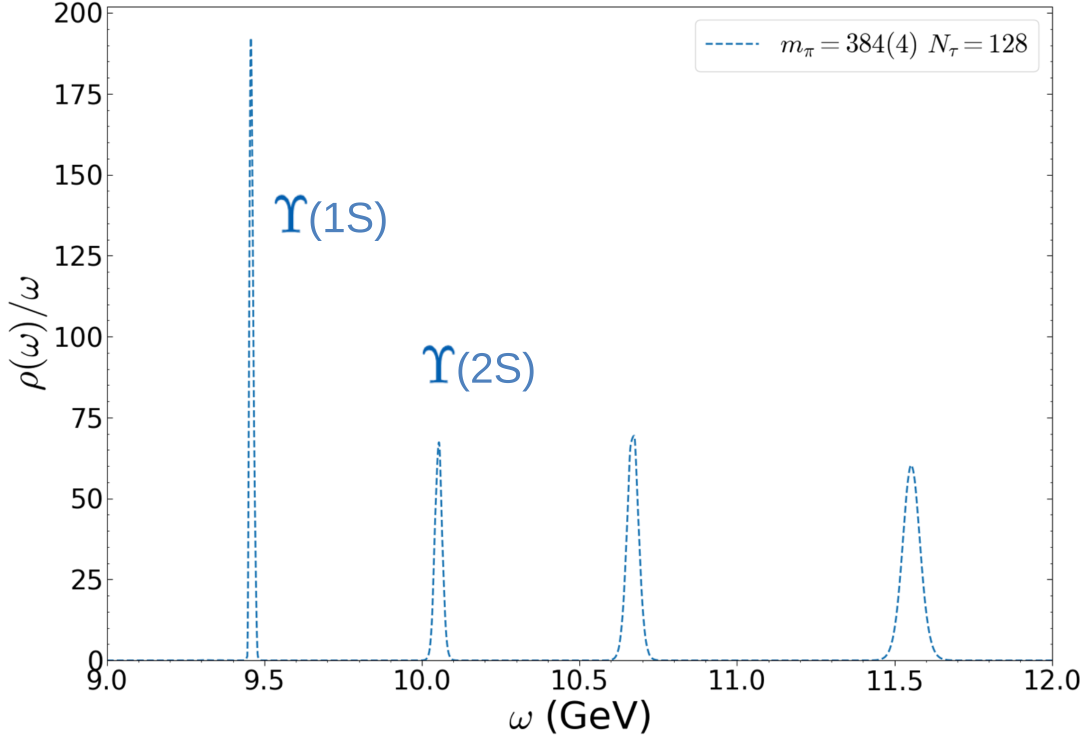
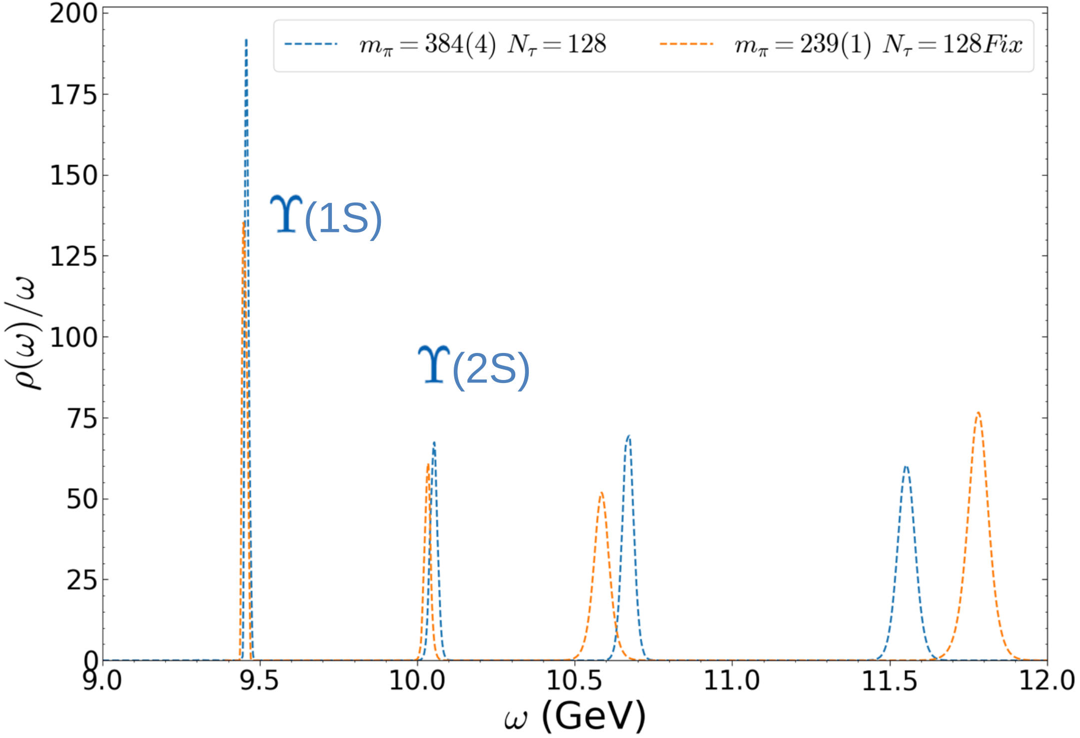


MEM - Zero Temperature

Generation 2 & 2L



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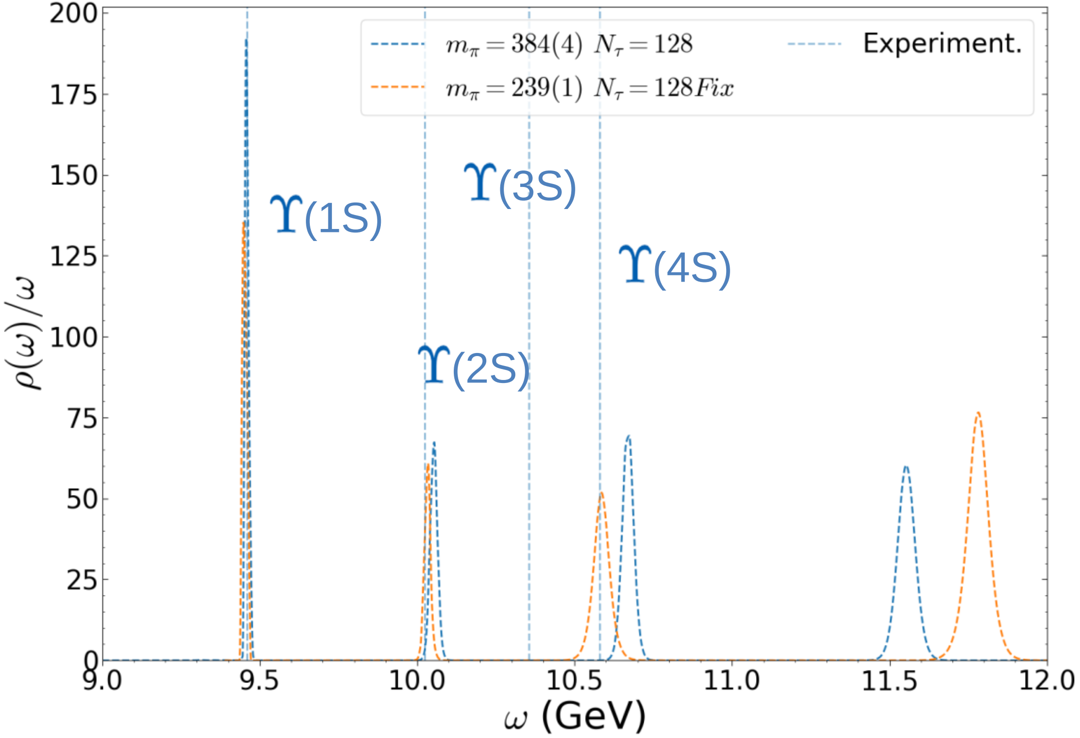

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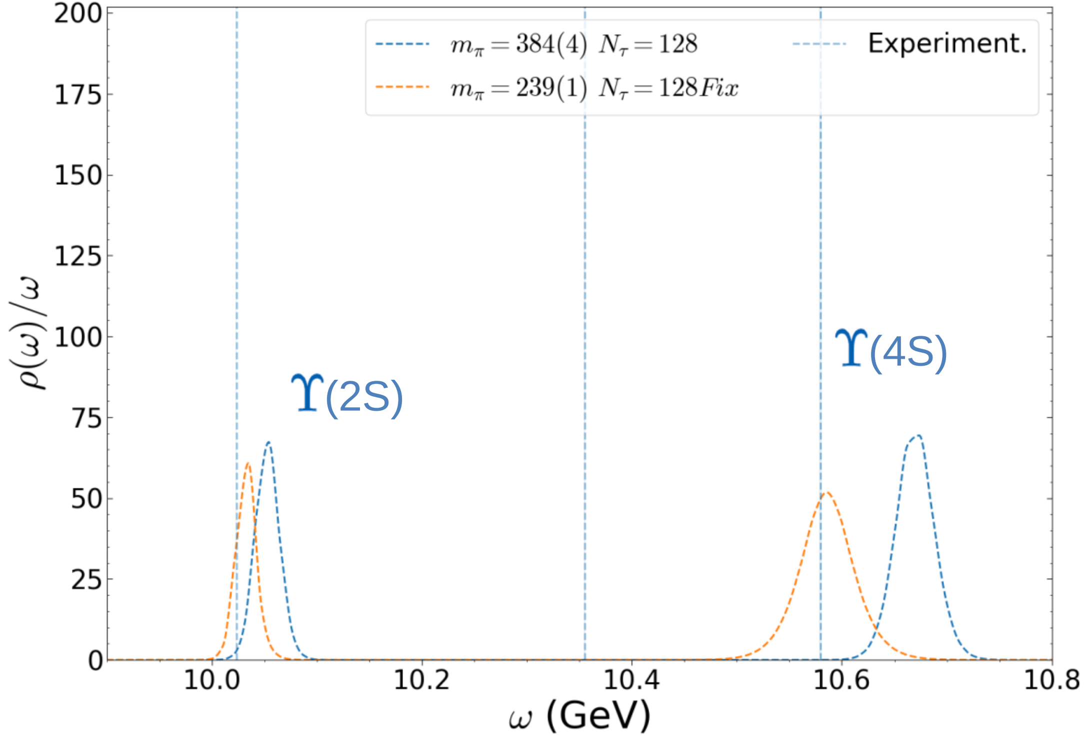

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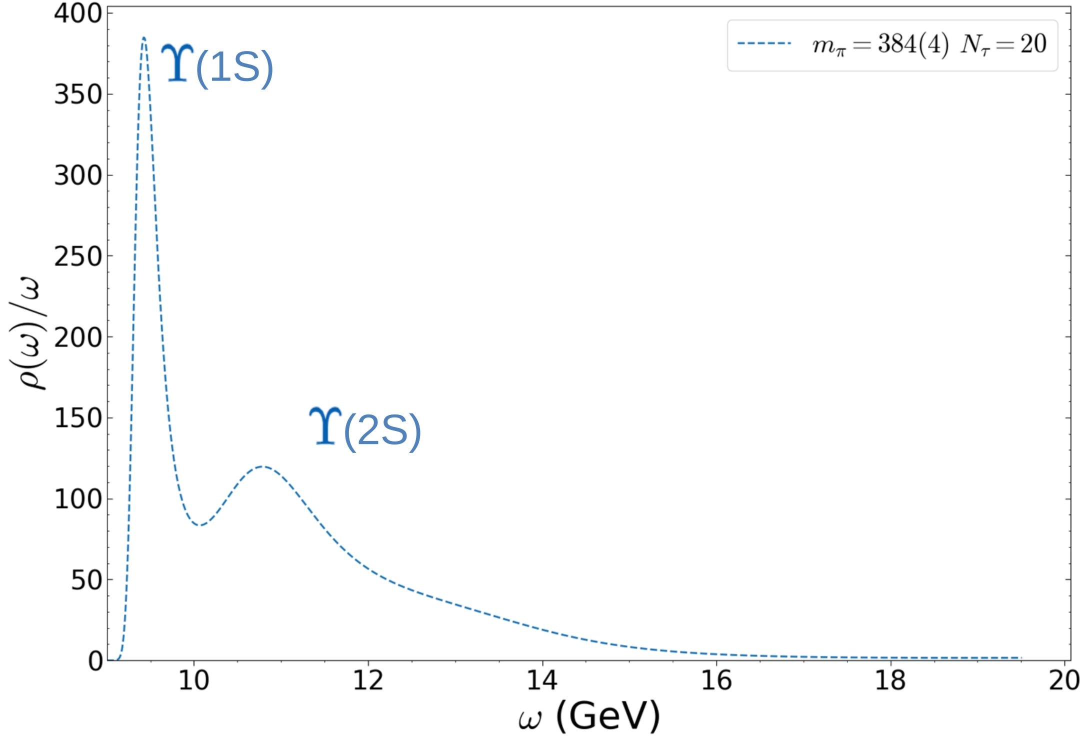
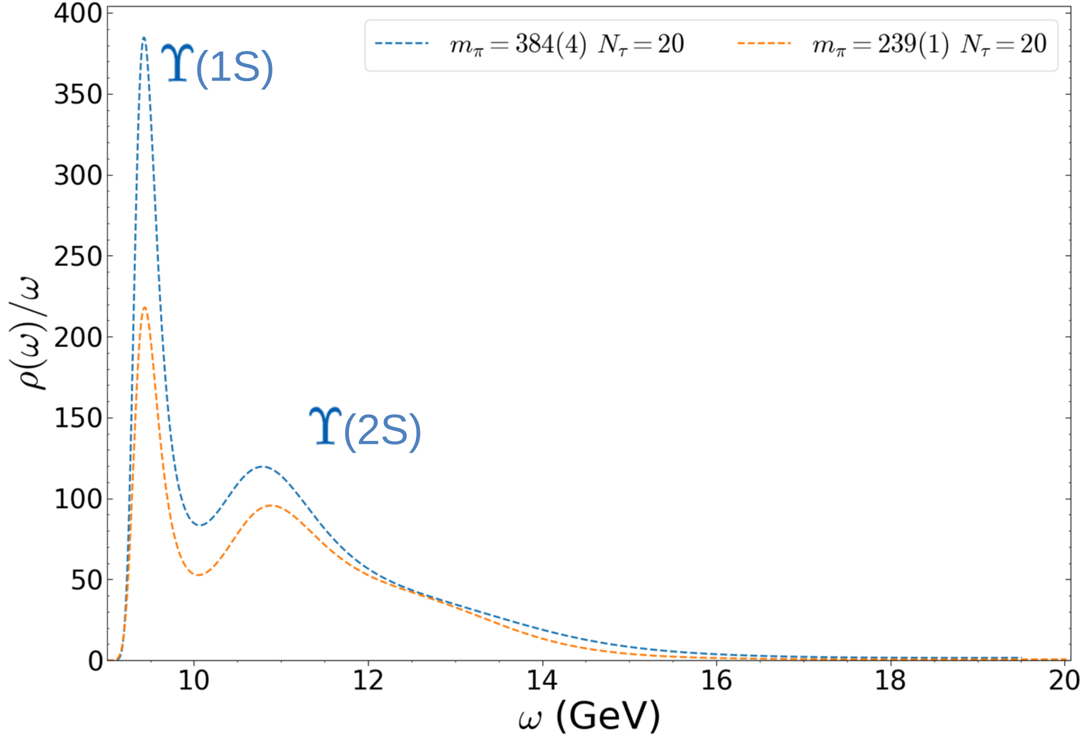


MEM - Finite Temperature (hot!)

Generation 2 and 2L

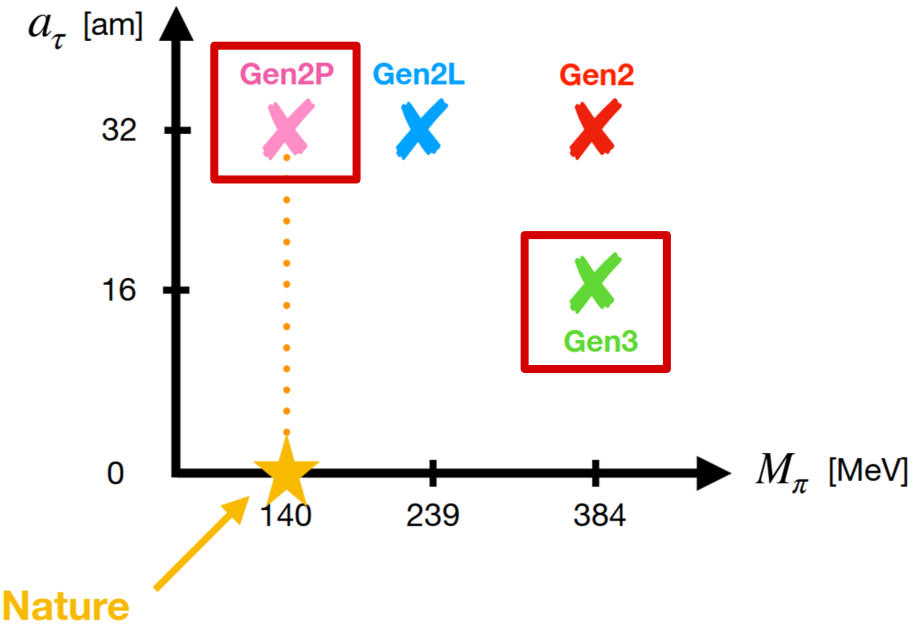


FASTSUM Approach

New Ensembles

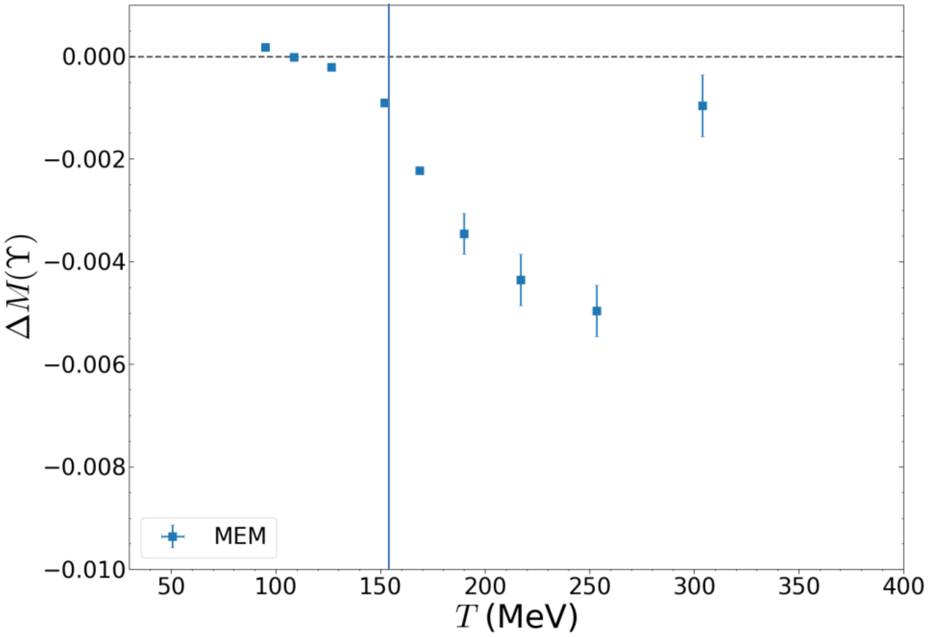
- Aim to examine systematics
  - Number of data points
  - Quark (pion) mass

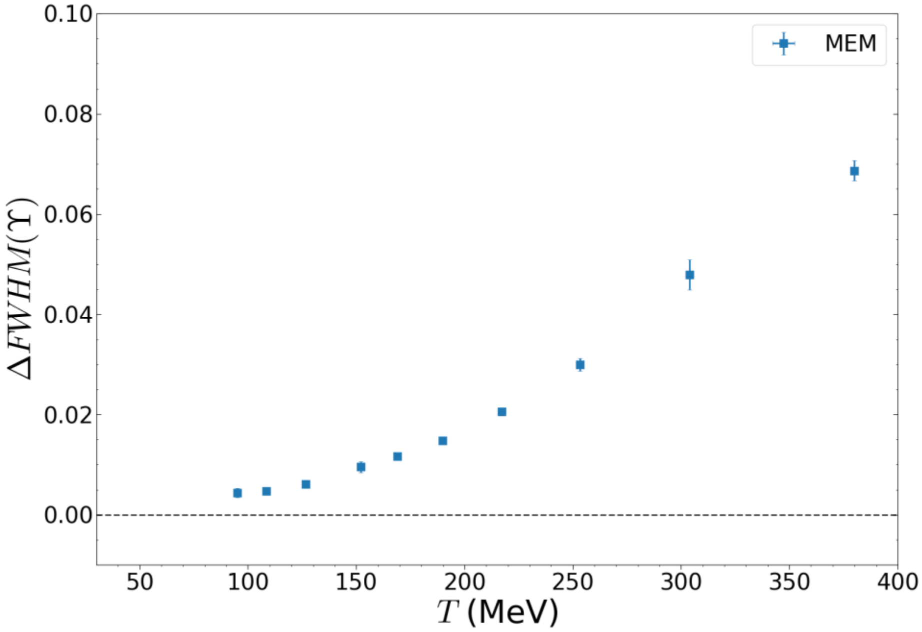
- More data points
  - Generation 3!
  - Same parameters as Gen 2
  - Twice number temporal data points
- Pion mass
  - Generation 2P will have physical pion mass



Upsilon (1S) - MEM

As a function of temperature





Spectral Representation

of NRQCD correlator

$$G(\tau) = \int_0^\infty \frac{d\omega}{2\,\pi}\, \mathrm{e}^{-\omega\,\tau}\, \rho(\omega)$$

Model spectral function $\rho(\omega)$ using a delta-function of the ground state.

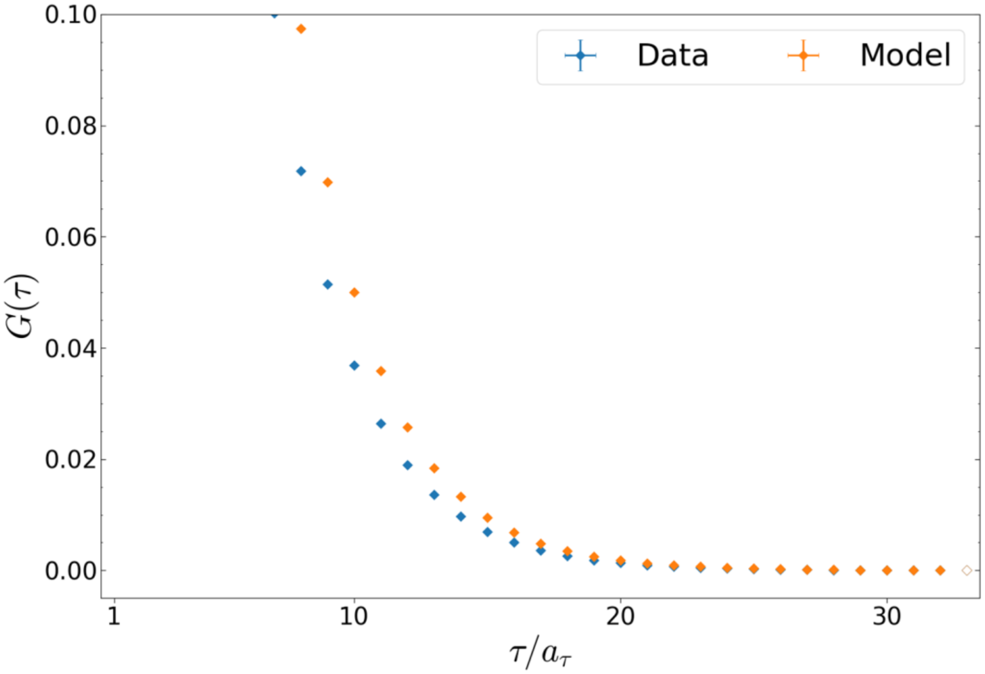
Construct single ratio

$$r(\tau; T, T_0) = \frac{G(\tau;T)}{G_{\mathrm{model}}(\tau;T,T_0)}$$

And hence double ratio

$$R(\tau; T, T_0) = \frac{r(\tau;T,T_0)}{r(\tau;T_0,T_0)}$$

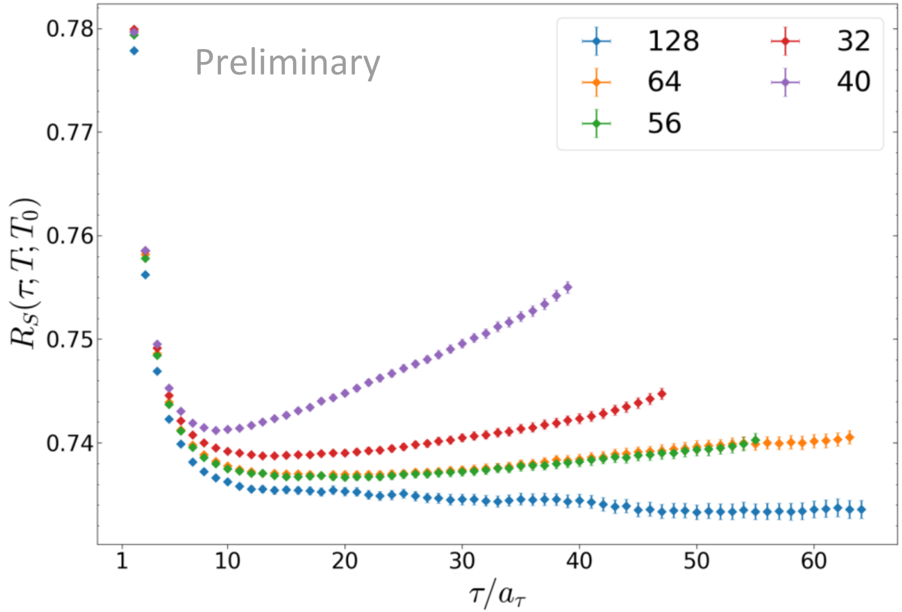
Describes the `change` in spectral function $\rho(\omega)$



Single & Double Ratio

$$r(\tau; T, T_0) = \frac{G(\tau;T)}{G_{\mathrm{model}}(\tau;T,T_0)}$$

- Single Ratio shows how similar to zero-temperature
  - Excited states still present
  - Constant if $\rho(\omega)$ is a delta-function
- Double Ratio
  - Removes excited state effect
  - Differences from one show difference in correlator



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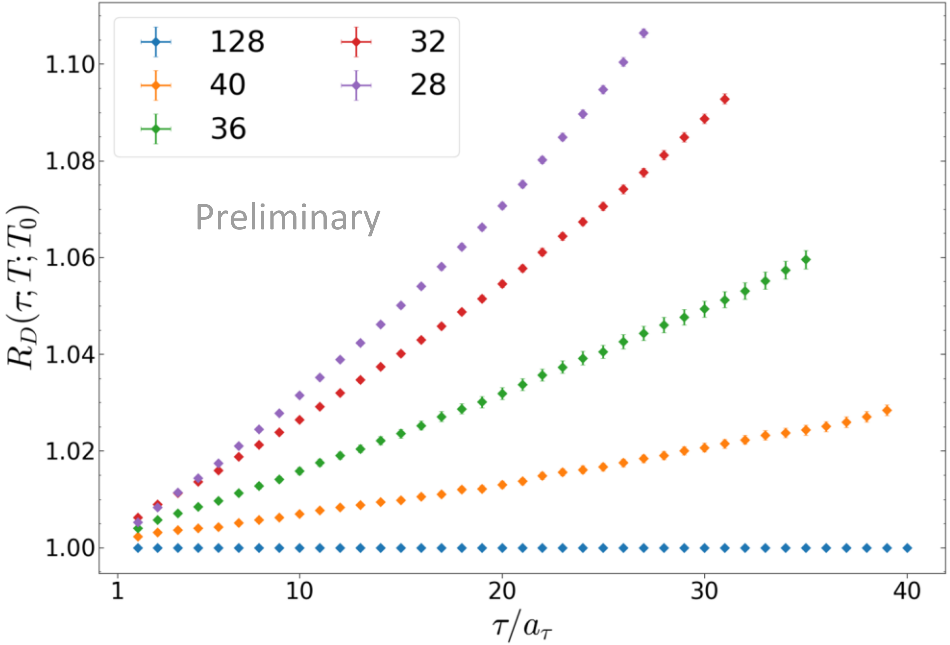
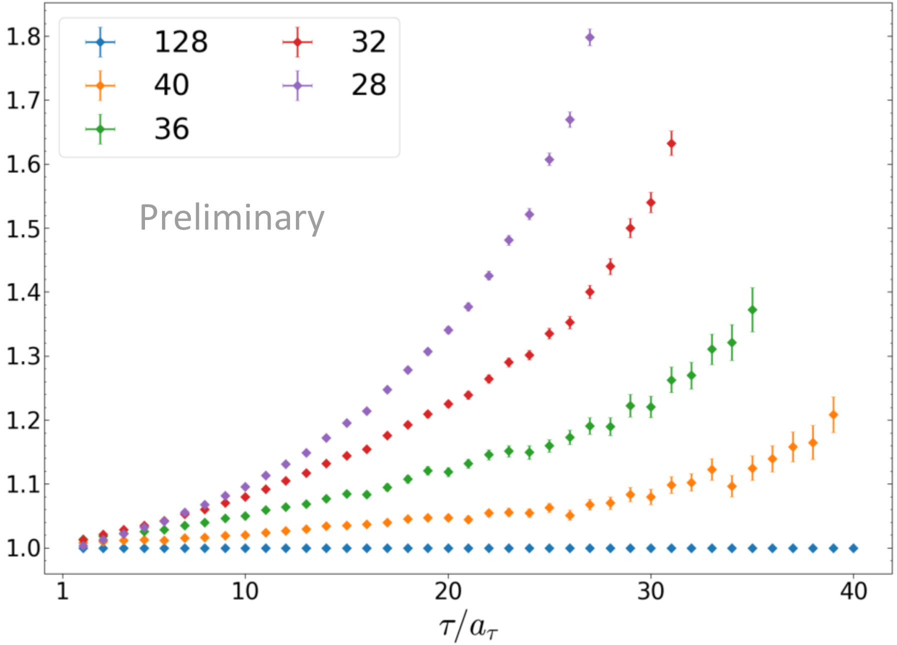


$\Upsilon(1S)$



$\Upsilon(2S)$

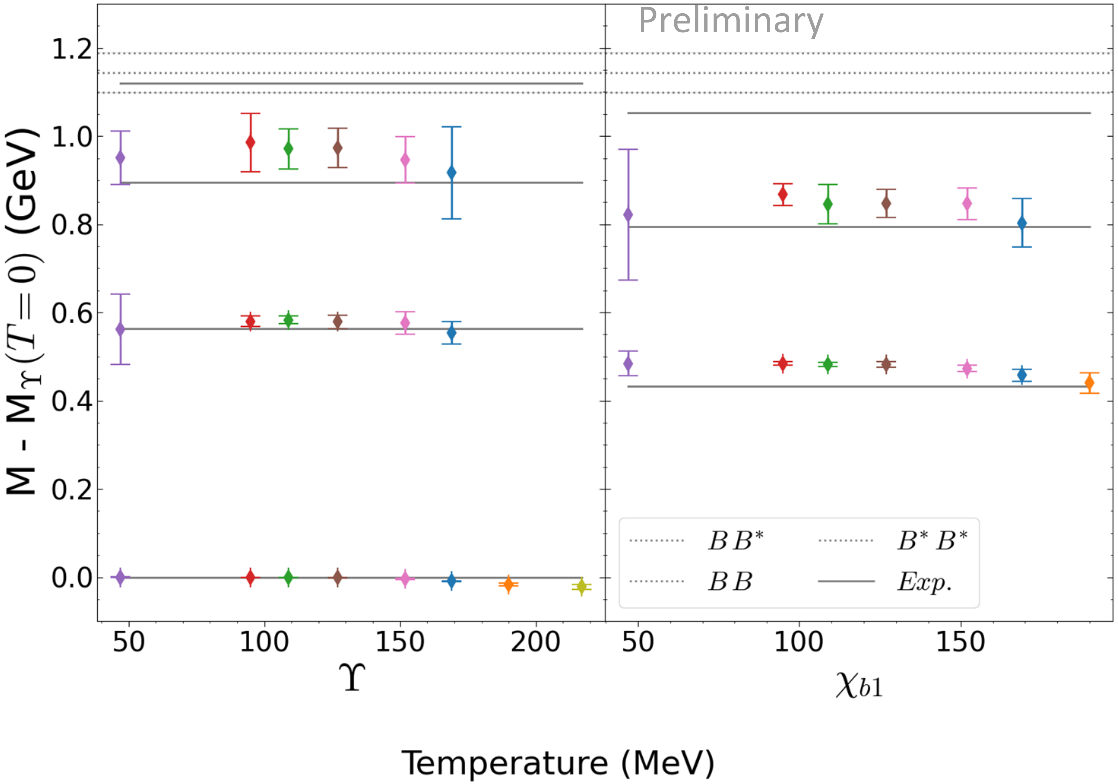
Mass Spectrum Results

Subtract zero-temperature $\Upsilon(1S)$

- Double Ratio informs trust in standard (multi-) exponential fits

$$\sum_i A_i e^{-E_i \tau}$$

- Model averaging techniques used to give robust determination of energy.



Mass Spectrum Results

Subtract zero-temperature $\Upsilon(1S)$ or $\chi_{b1}(1P)$

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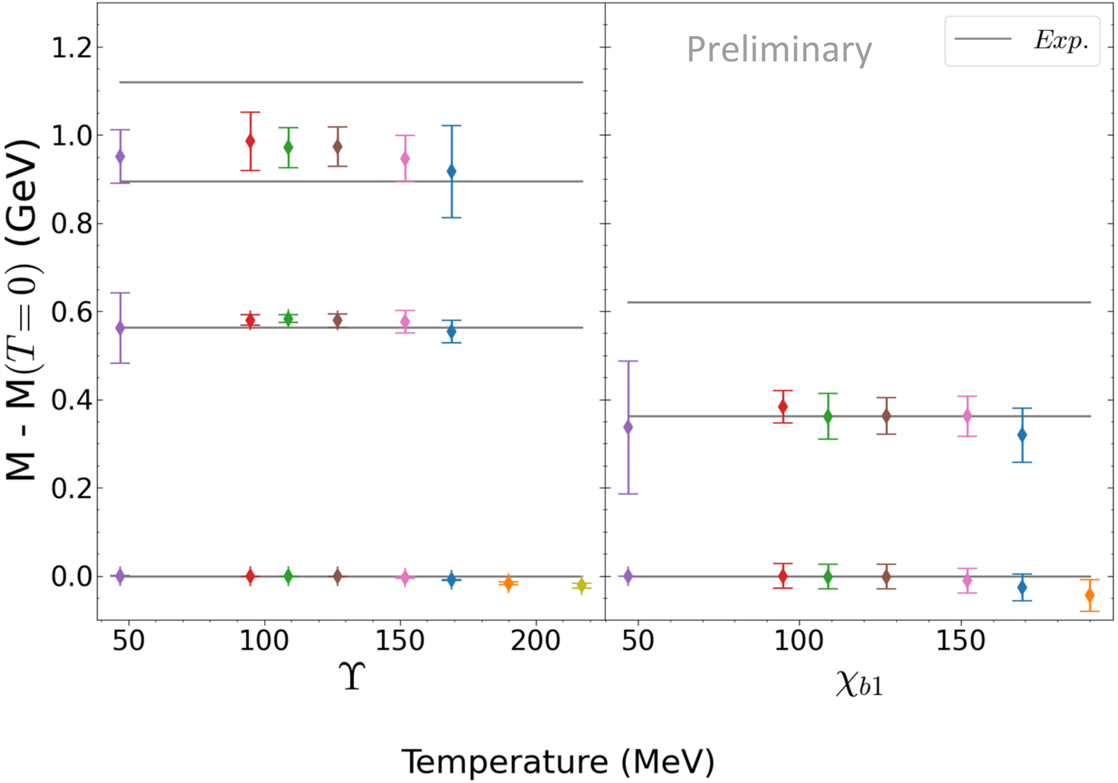

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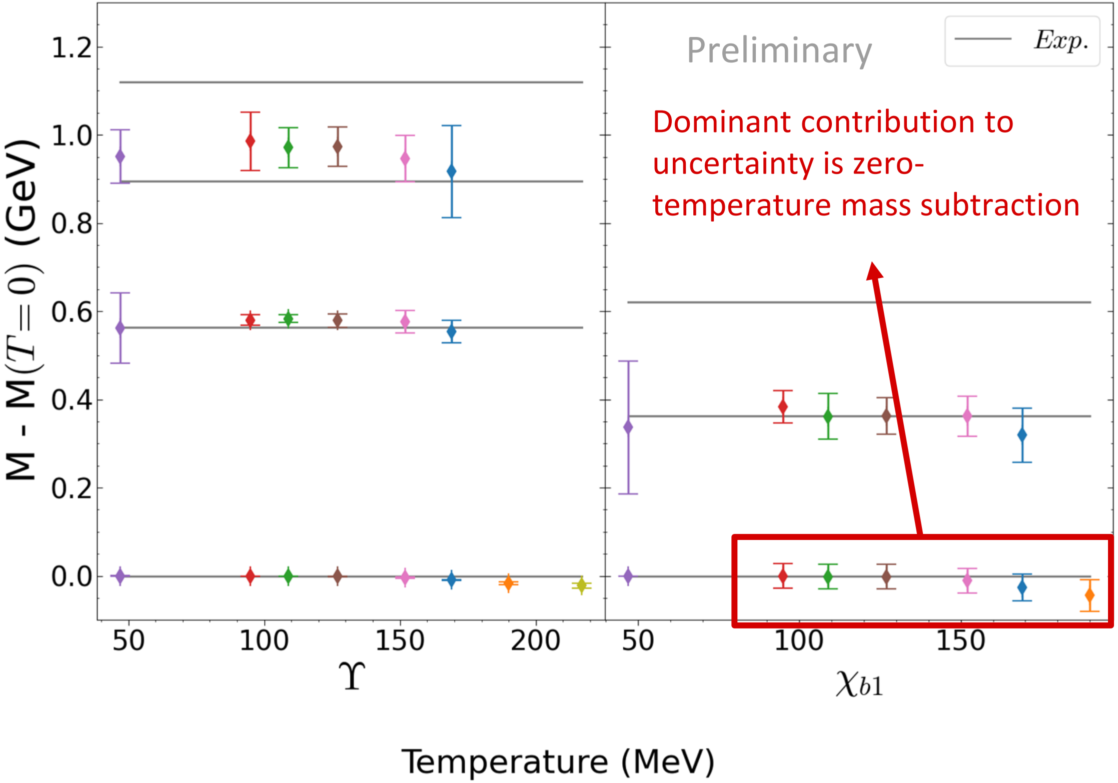
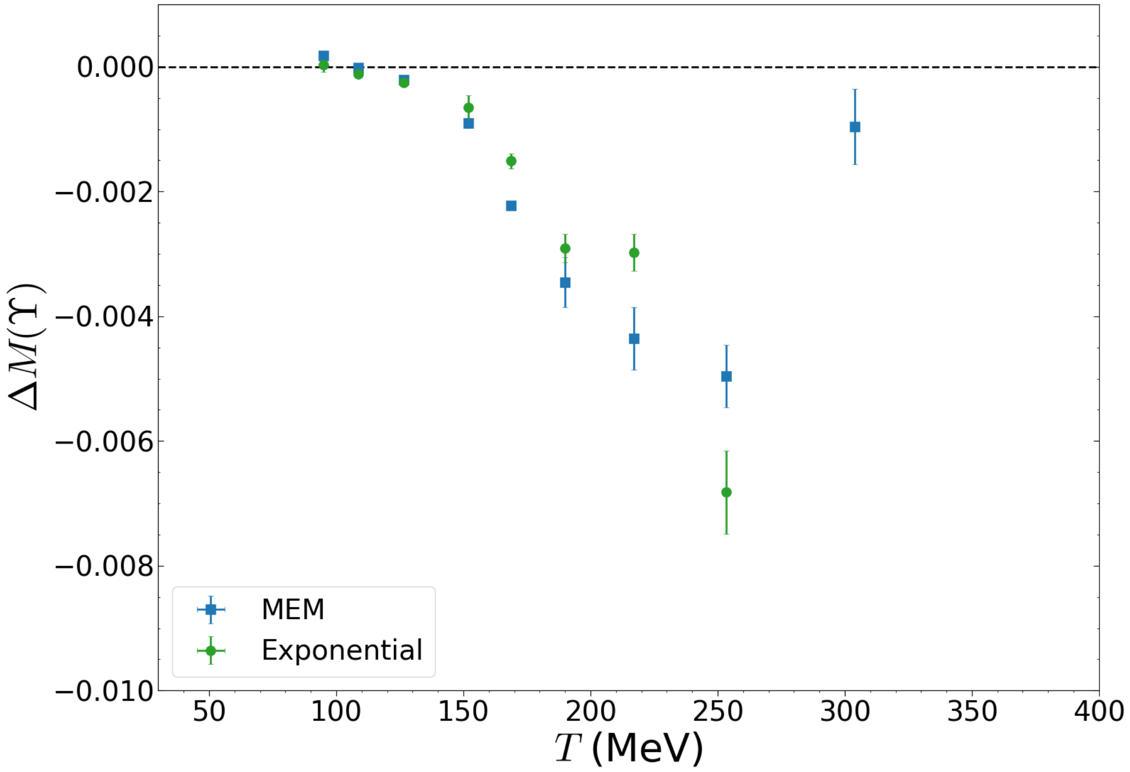


Upsilon (1S) Mass - Exponential Cfn Fits

As a function of temperature

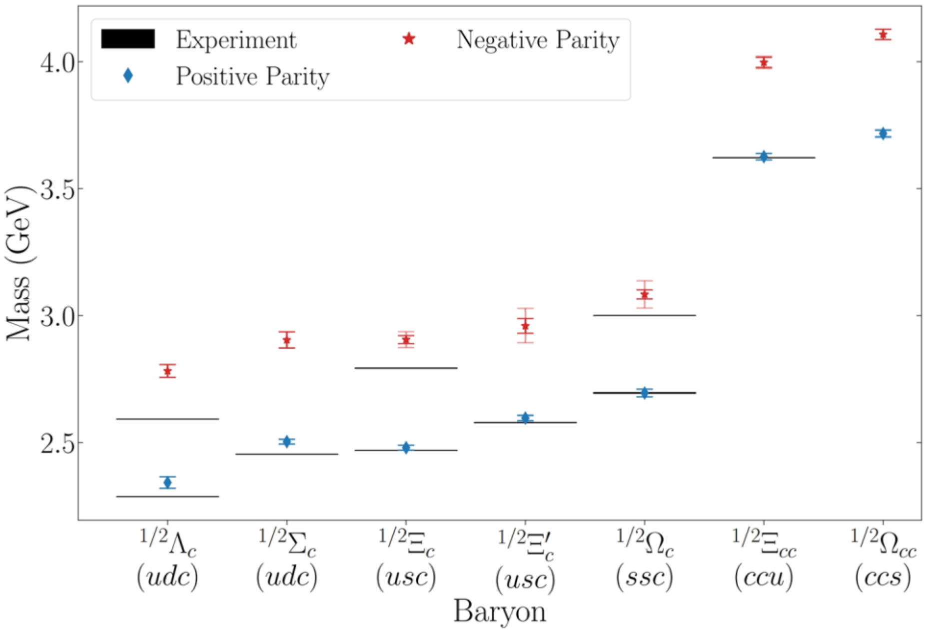
- Good qualitative agreement between MEM & (Multi-)Exponential fits
- Results suggest small decrease in mass as temperature increases



Charm Baryons

At finite temperature: **2308.12207**

- Charm hadrons also important probes of the QGP
- Charm baryons are experimentally accessible (i.e. $\Xi_{cc}(ccu)$ **1807.01919**)
- This extends our previous work on light and strange baryons

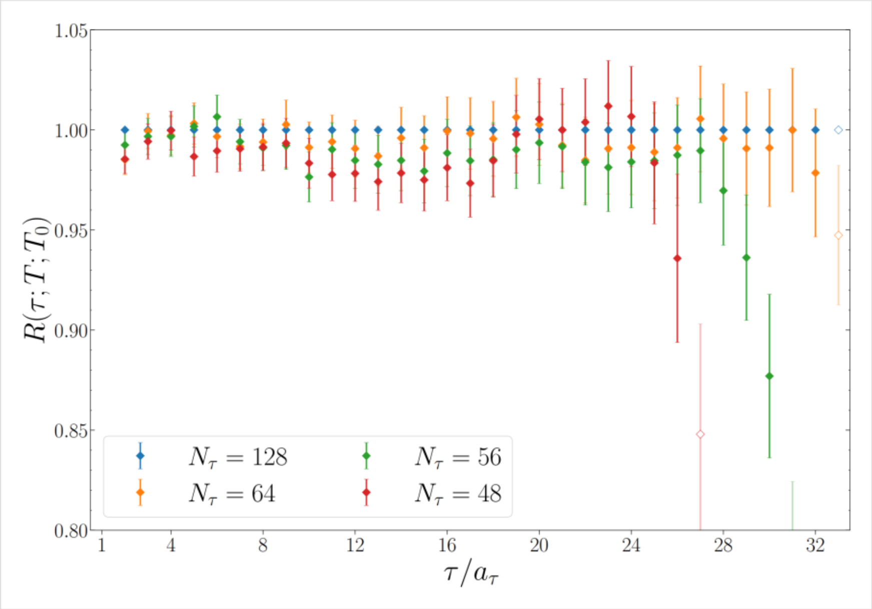


- Examine:
  - Mass change due to temperature
  - `Parity Doubling` (Chiral Symmetry Restoration)

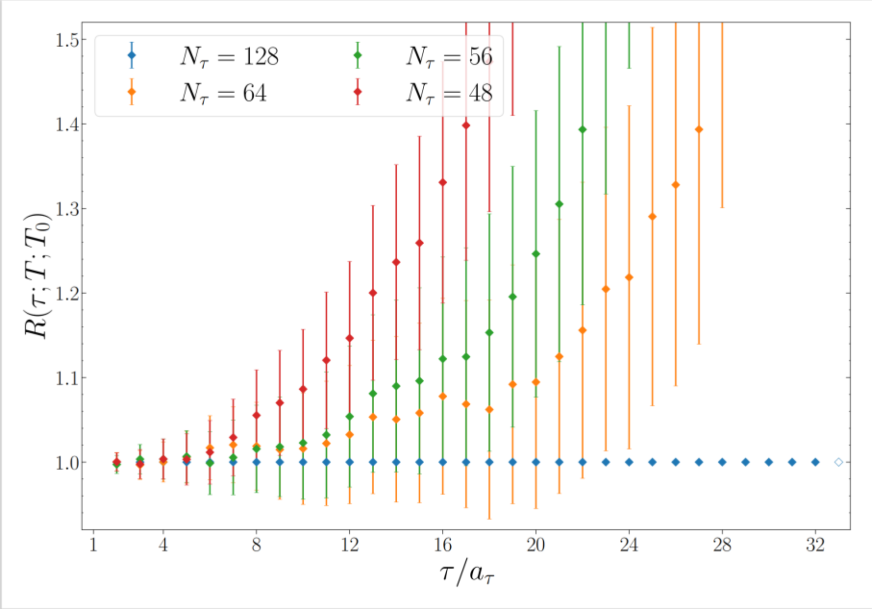
Ratios -$\Sigma_c(udc)$

$$R(\tau; T, T_0) = \frac{r(\tau;T,T_0)}{r(\tau;T_0,T_0)}$$

A very similar approach to that used for the NRQCD Bottomonia already



Positive Parity



Negative Parity

Ratios -$\Sigma_c(udc)$

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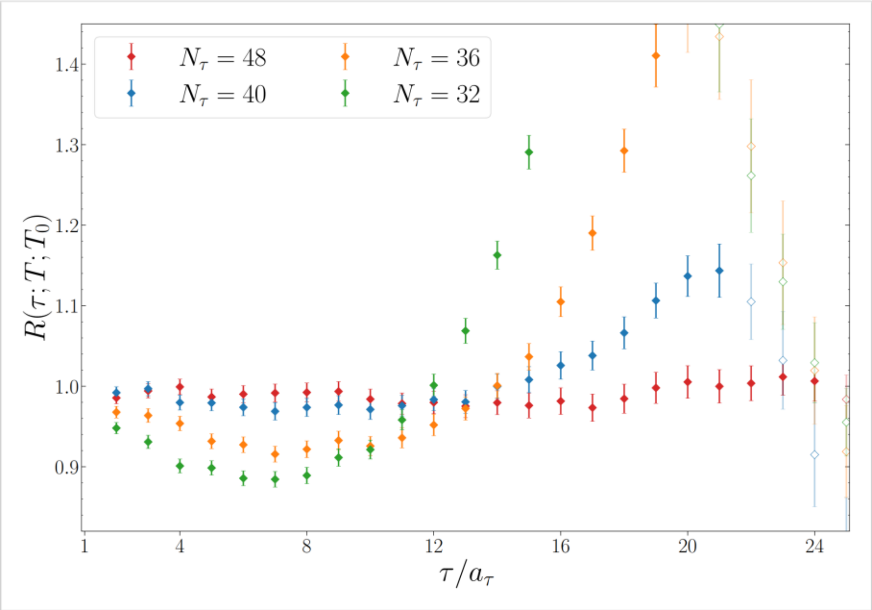

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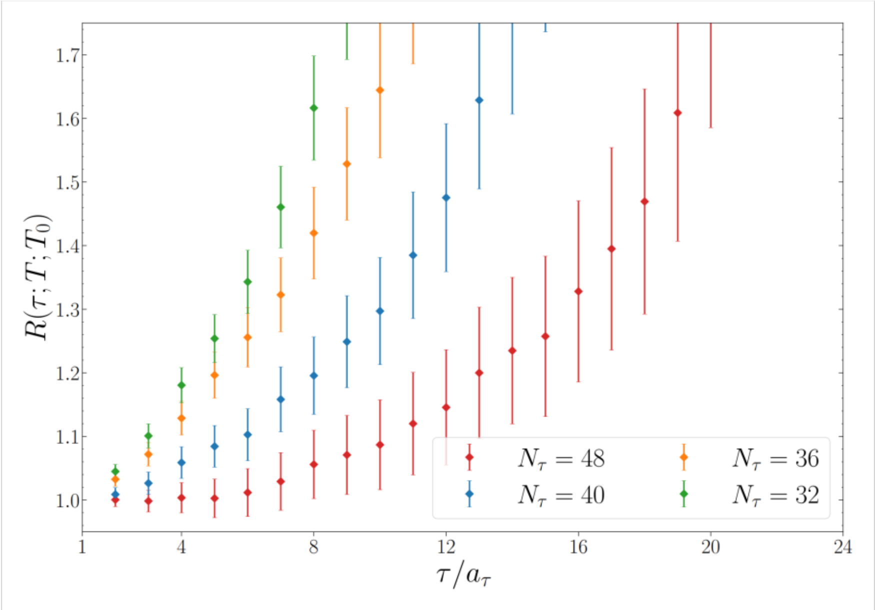
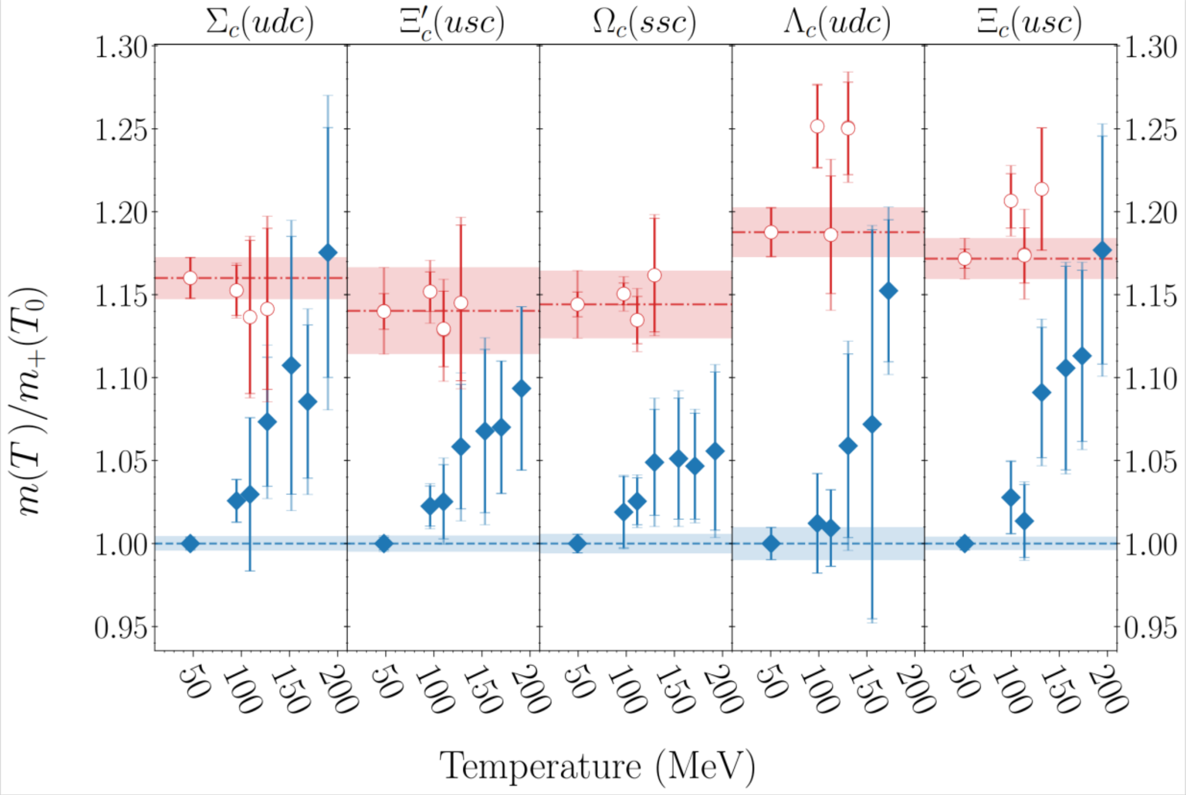


Negative Parity

Mass as function of temperature

Drawing from ratio analysis for insight into fits



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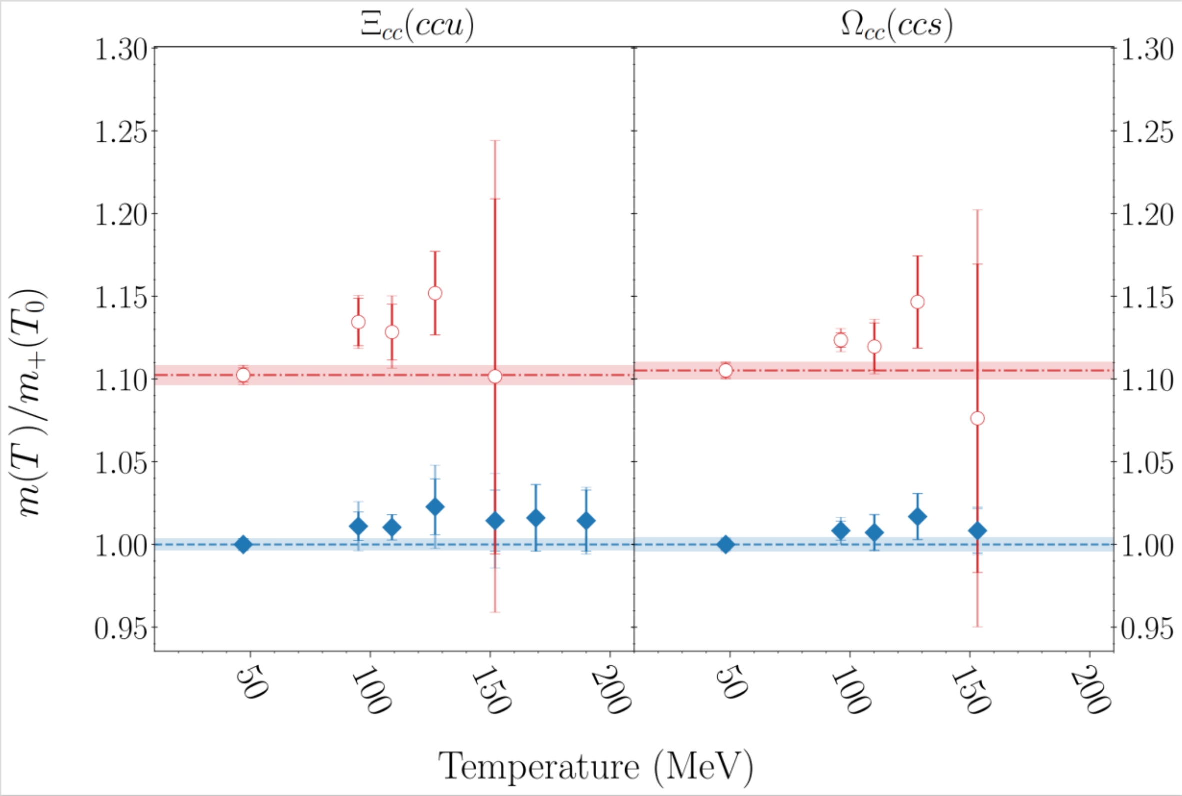


Parity Doubling

- Parity Doubling
  - +ve and –ve states become degenerate
  - Linked to chiral symmetry restoration
  - Examine via summed difference Ratio
    - Expect near 1 when non-degenerate
    - Expect near 0 when degenerate

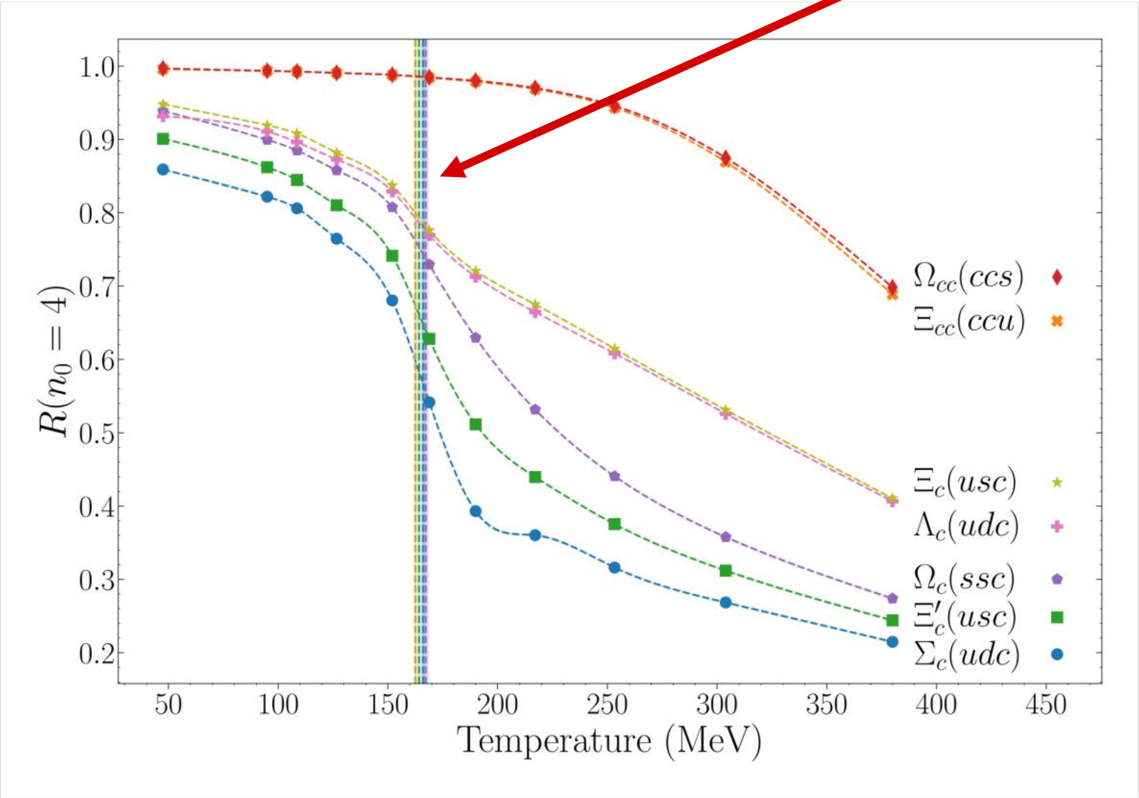
$$R(\tau) = \frac{\boxed{G^+(\tau) - G^-(\tau)}}{G^+(\tau) + G^-(\tau)}$$

$$R(\tau_n) = \frac{\sum_n^{1/2 N_\tau - 1} R(\tau_n) \Big/ \sigma_R^2(\tau_n)}{\sum_n^{N_{1/2}\tau - 1} 1/\sigma_R^2\,(\bar{c}_n)}$$

Parity Doubling

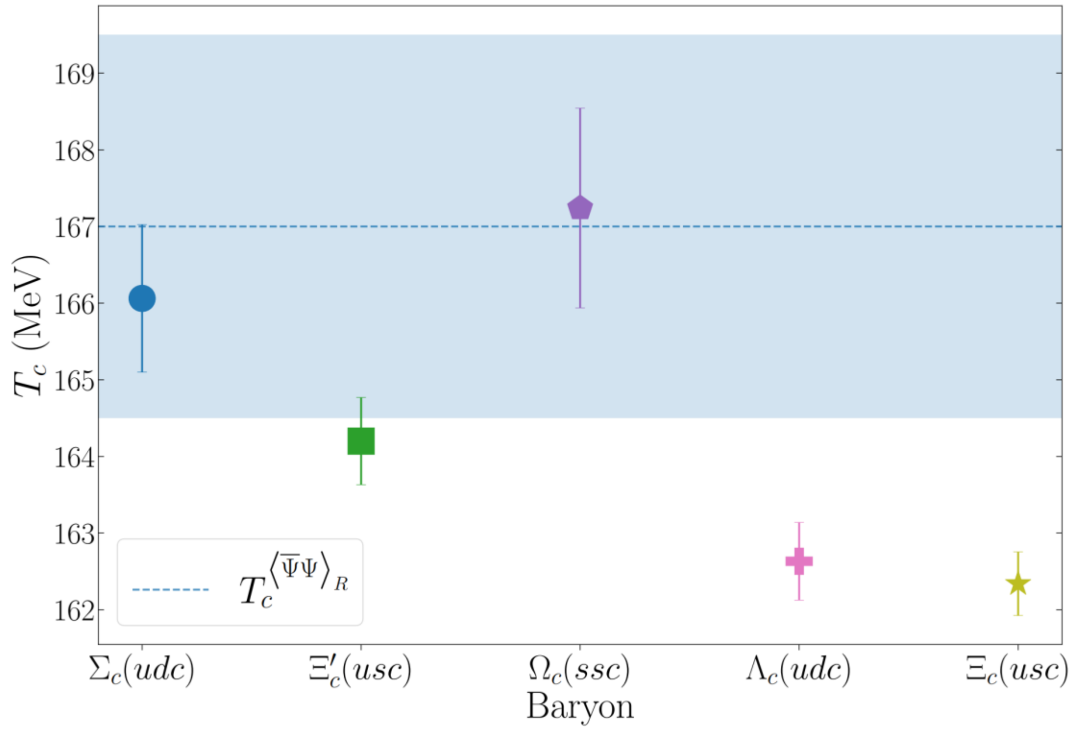
Summed Difference Ratio

Inflection point is near T_c



Parity Doubling

Inflection points



Summary

- **Presented Bayesian (MEM) spectral function results from two ensembles**
  - Showed how this may be improving systematics
  - Discussed how future ensembles will also improve systematics
- **Used Double-Model Ratio method to examine *change* in spectral function and used (multi-)Exponential fits to determine the mass as temperature increases**
  - For both charm baryons and bottomonia
    - Bottomonia suggests small decrease in mass
    - Some charm hadrons remain stable past T_c
- **Discussed parity doubling for charm baryons**

EXTRA SLIDES

Ensemble Details

Generation 2L FASTSUM

N_T	128	64	56	48	40	36	32	28	24	20	16
Temperature (MeV)	47	95	109	127	152	169	190	217	253	304	380
# Wall Sources	16	16	16	20	24	24	32	28	24	20	16

Action details:

- **Gauge: Symanzik-improved, tree-level tadpole**
- **Fermion: Wilson-clover, tree-level tadpole, stout-links**
- **Same parameters as HadSpec Collaboration**
- **Approx. 1000 configurations at each temperature**

- **NRQCD action for bottom quarks**
  - Incorporating O(v^4) corrections
  - Tree-level matching coefficients

$m_\pi \sim 236$ MeV, $\xi \sim 3.5$, $T_c \sim 167$ MeV

Excited State spectroscopy

Generalised EigenValue Problem - GEVP

- Build correlation matrix of two point functions

$$G_{ij}(\tau) = \left\langle \Omega \middle| \mathcal{O}_i \mathcal{O}_j{}^{\dagger} \middle| \Omega \right\rangle = \sum_\alpha \frac{Z_i^\alpha \, Z_j^{\alpha\,\dagger}}{2\,E_\alpha} \, \mathrm{e}^{-E_\alpha\,\tau}$$

- Solve generalised eigenvalue problems

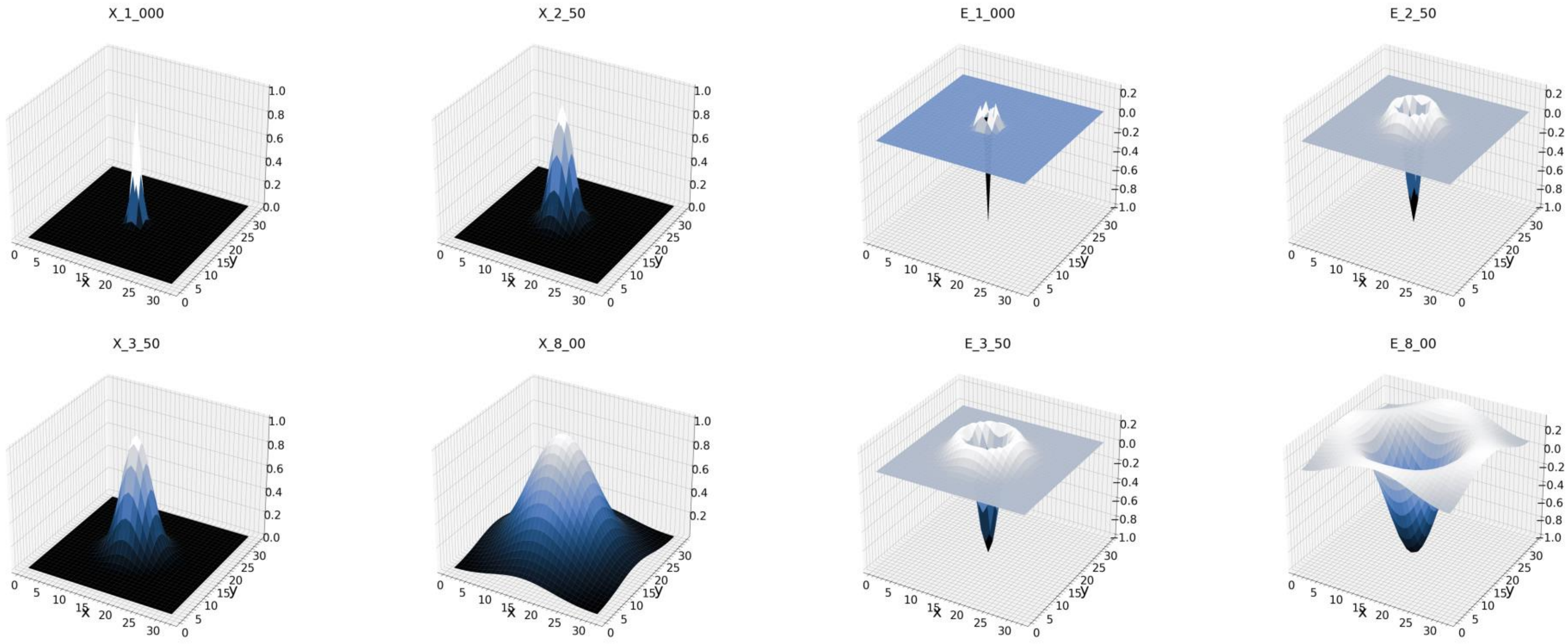
$$G_{ij}(\tau_0 + \delta_\tau)\, u_j^\alpha = \mathrm{e}^{-E_\alpha \delta_\tau}\, G_{ij}(\tau_0)\, u_j^\alpha$$
$$v_i^\alpha \, G_{ij}(\tau_0 + \delta_\tau) = \mathrm{e}^{-E_\alpha \delta_\tau}\, v_i^\alpha \, G_{ij}(\tau_0)$$

- Construct Projected Correlator

$$G_\alpha(\tau) = v_i^\alpha \, G_{ij}(\tau)\, u_j^\alpha$$

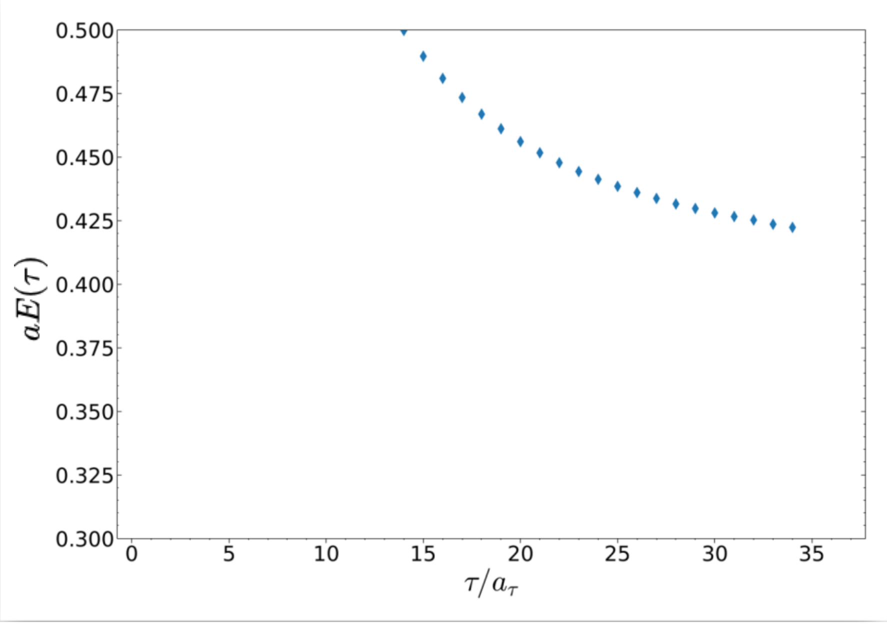
GEVP - Operator Basis

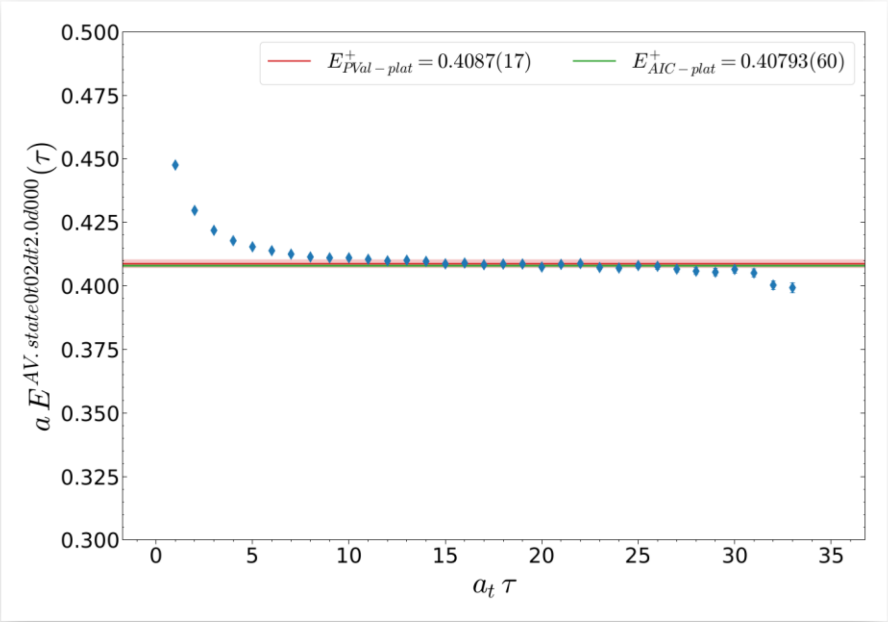
Four widths of Gaussian and `excited` operator



Improved state isolation

N_t = 36, $\chi_{b1}(1P)$





`Moments`

`Time-Derivative Moments`

$$G(\tau) = \int_0^\infty \frac{d\omega}{2\pi} e^{-\omega\tau} \rho(\omega)$$

If $\rho(\omega)$ is Gaussian with width Γ and mean E, second log-derivative is

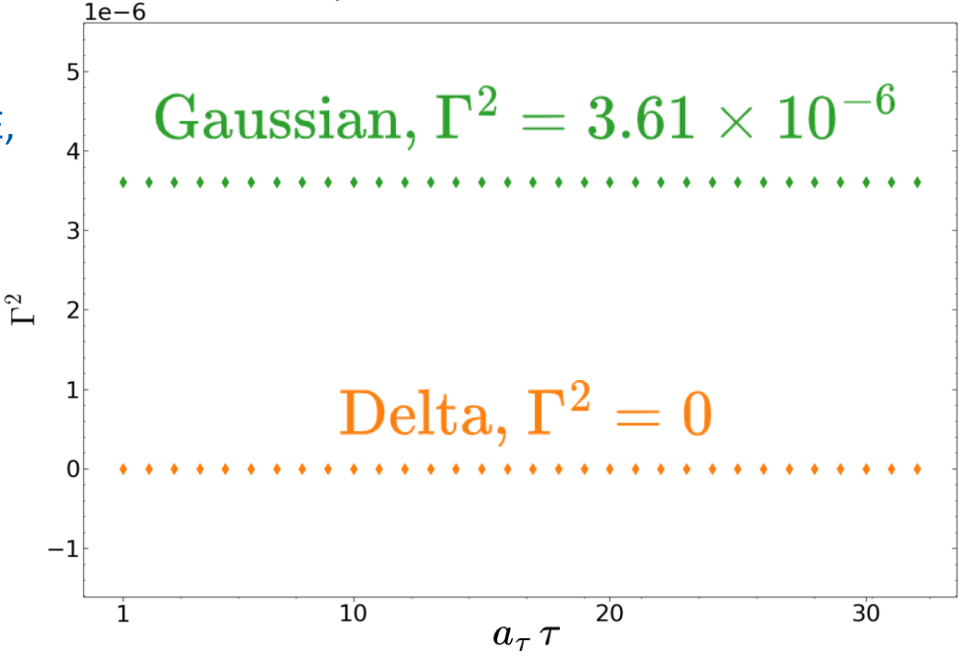
$$\frac{d^2 \log(G(\tau))}{d\tau^2} = \frac{G''(\tau)}{G(\tau)} - \left(\frac{G'(\tau)}{G(\tau)}\right)^2$$

$$= E^2 + \Gamma^2 - (E)^2$$

$$= \Gamma^2$$

This is the difference between 2nd and 1st non-central moments of a Gaussian

Model/Mock Correlator Data



`Moments`

Point-Point

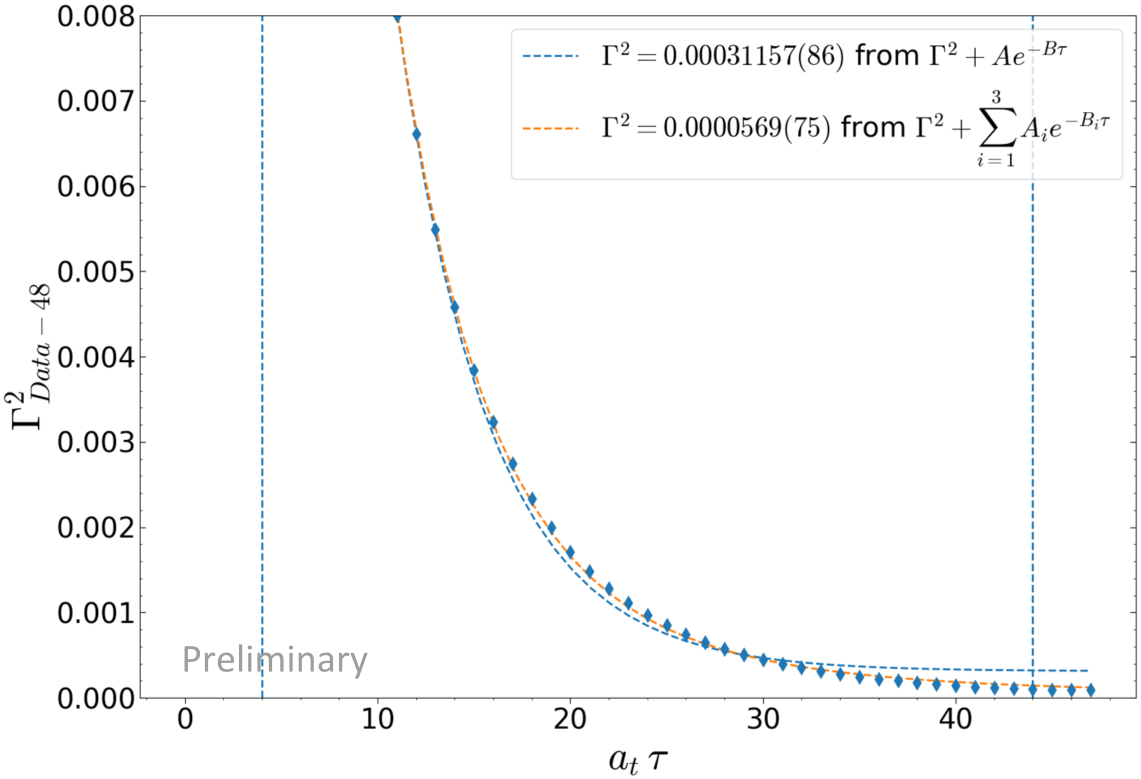
- Excited states shift form
- Fit with function

$$\Gamma^2 + \sum_{i=1}^{N} A_i e^{-B_i \tau}$$

- Easier at higher temperatures as Γ^2 becomes larger

- This is an upper bound only

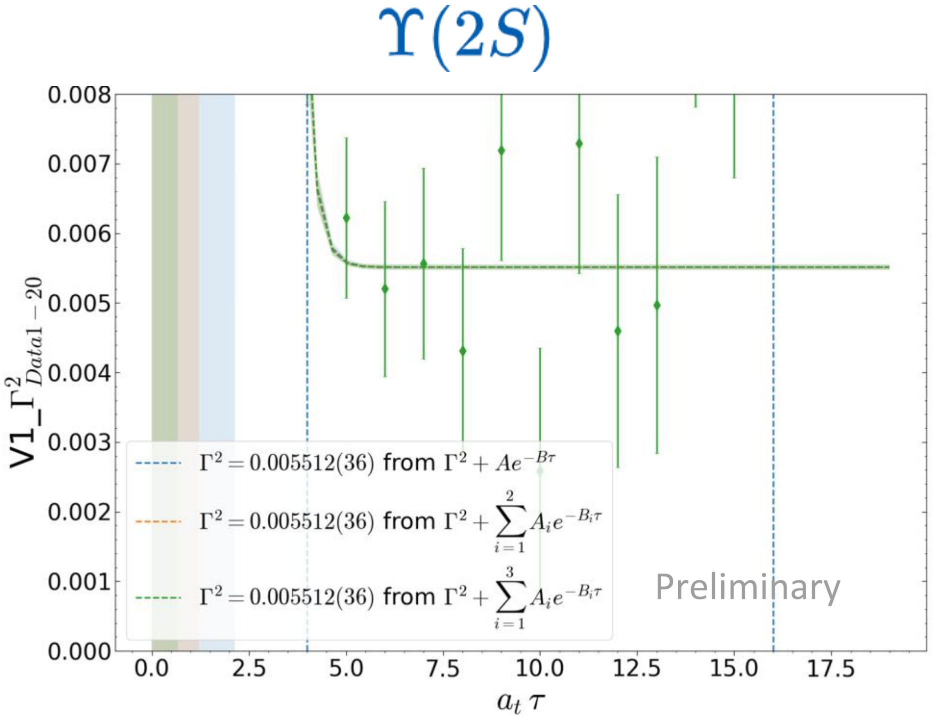
$\Upsilon(1S)$ Point-Point Correlator



`Moments`

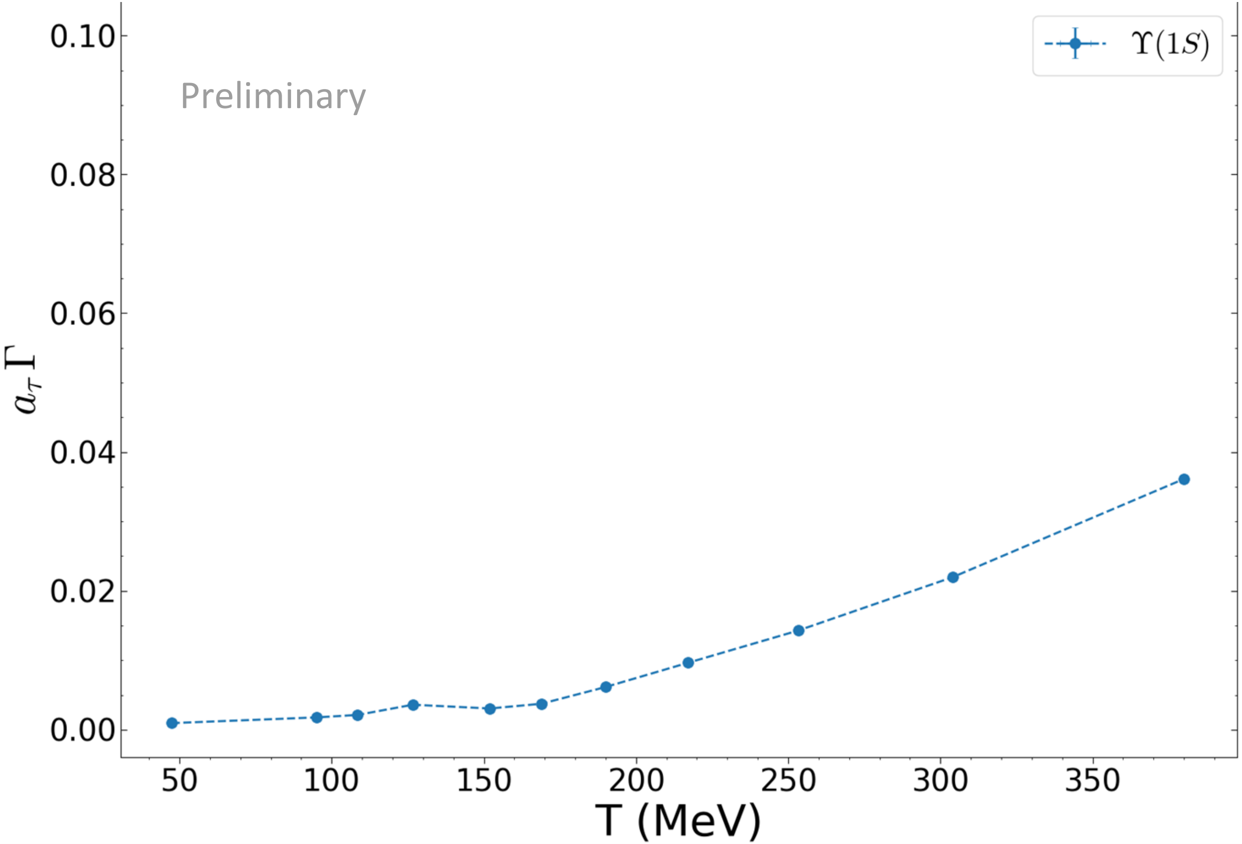
GEVP

- Apply `moments` method to GEVP projected correlators
- GEVP essential for access to excited states for moments
- Method is fairly robust against noise
  - Constant Γ^2 term helps
  - Exponential terms not well constrained
  - More statistics ongoing



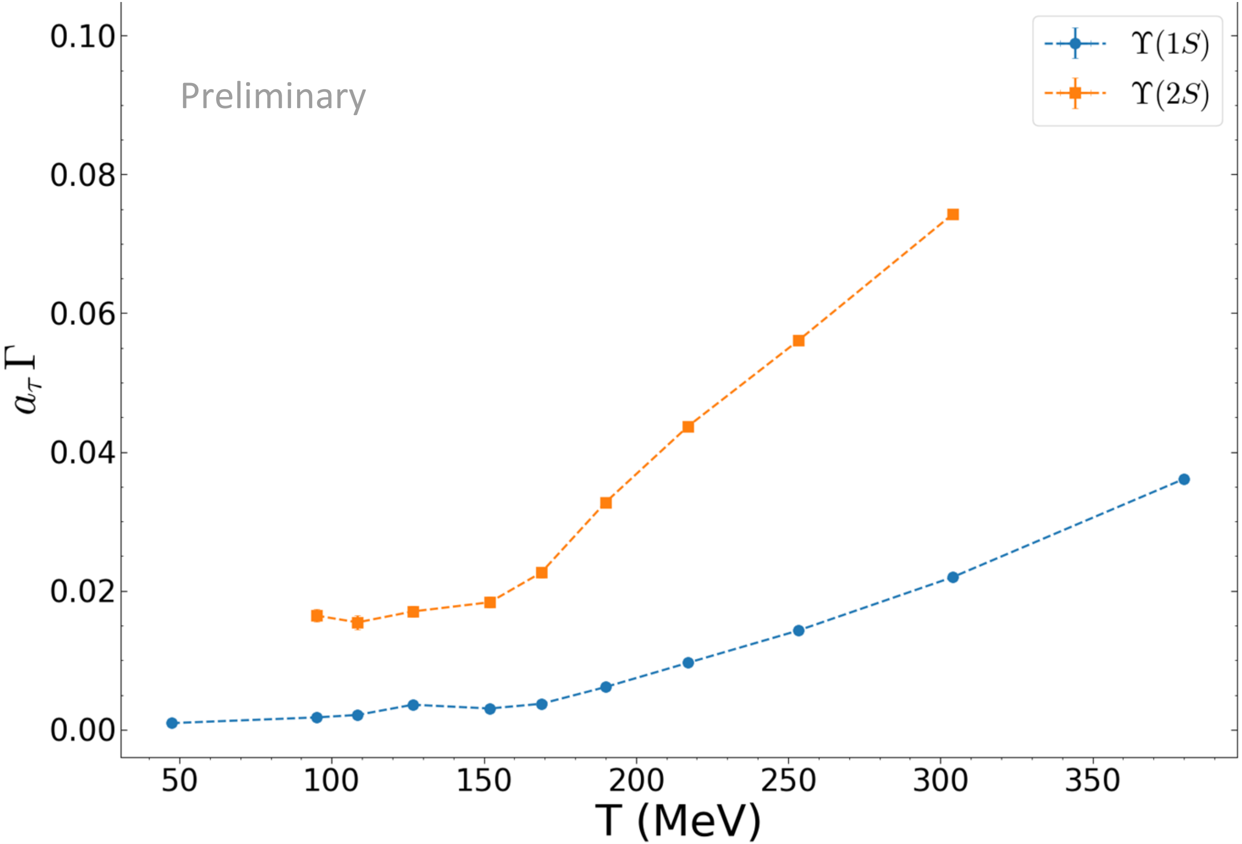
`Moments`

GEVP



`Moments`

GEVP



`Moments`

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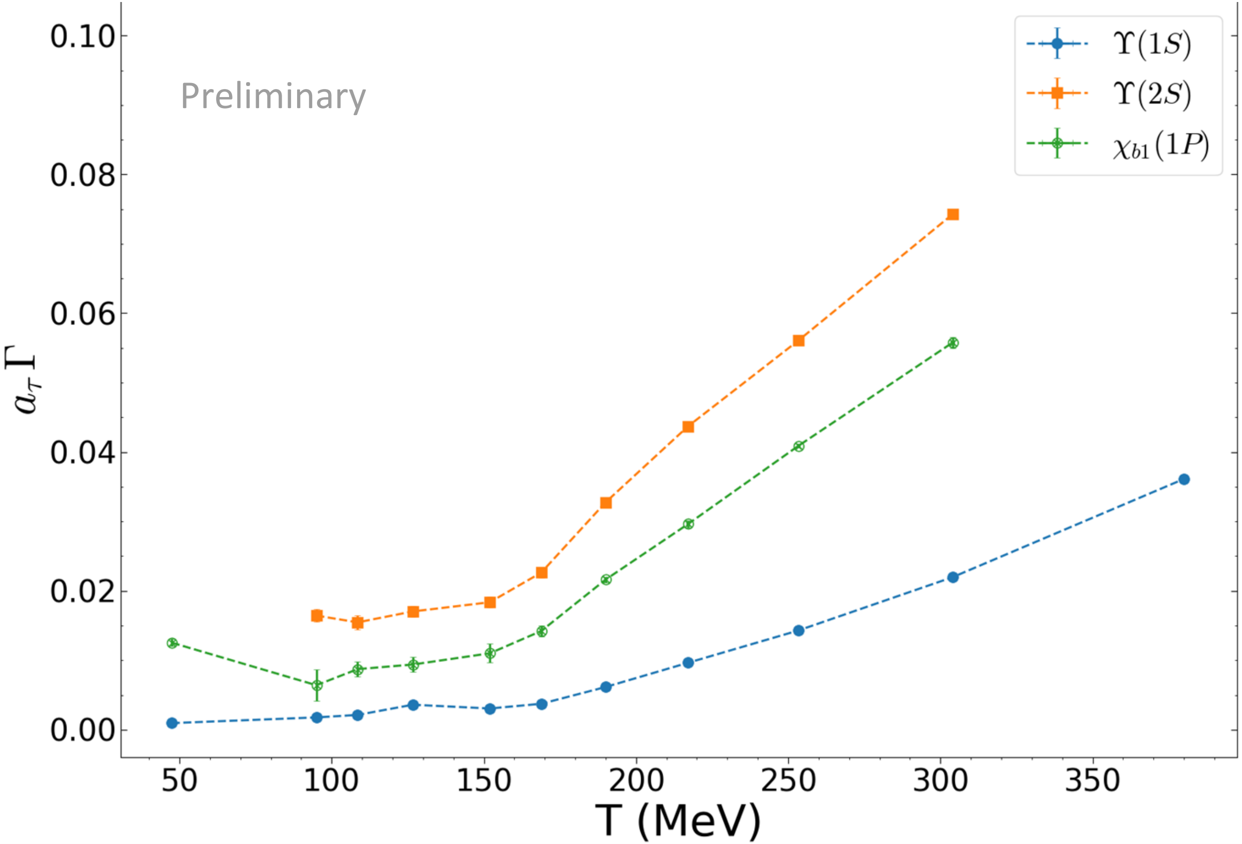
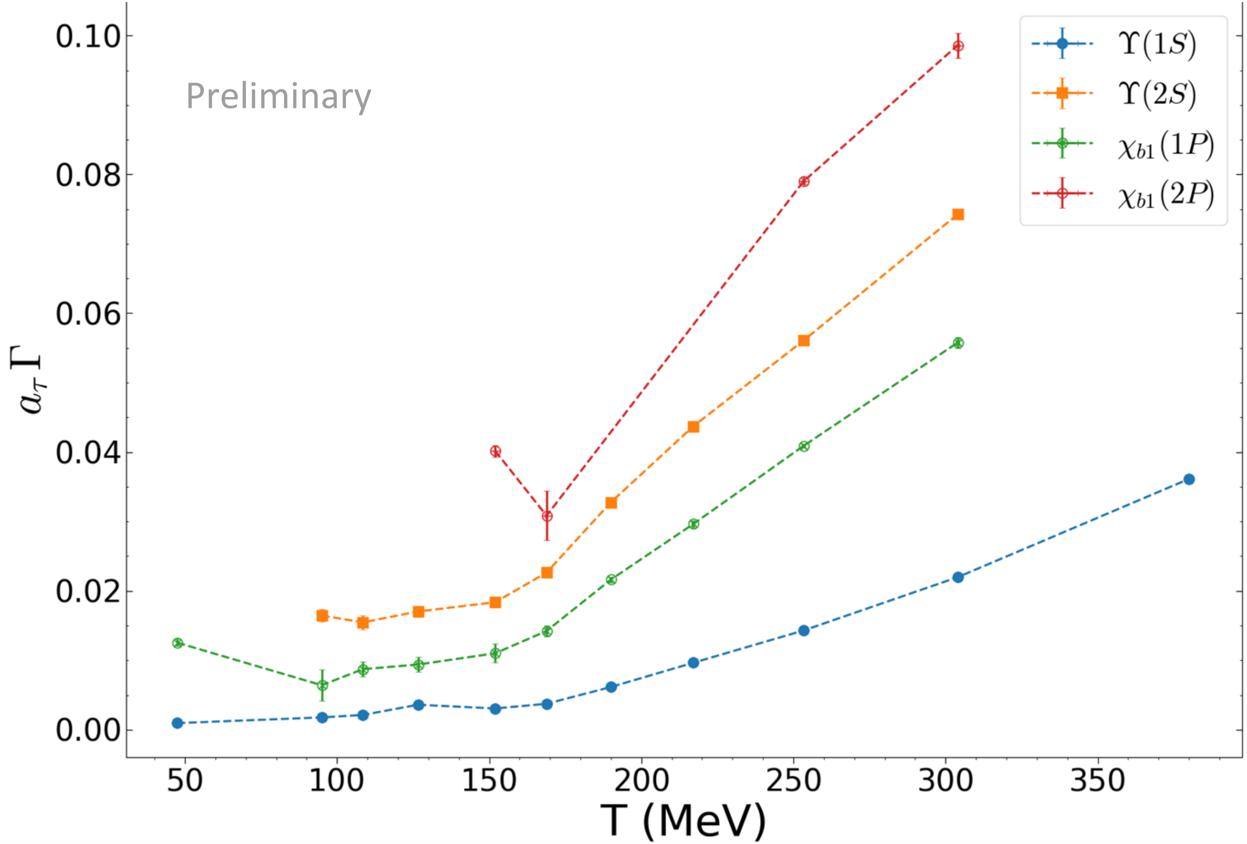


`Moments`

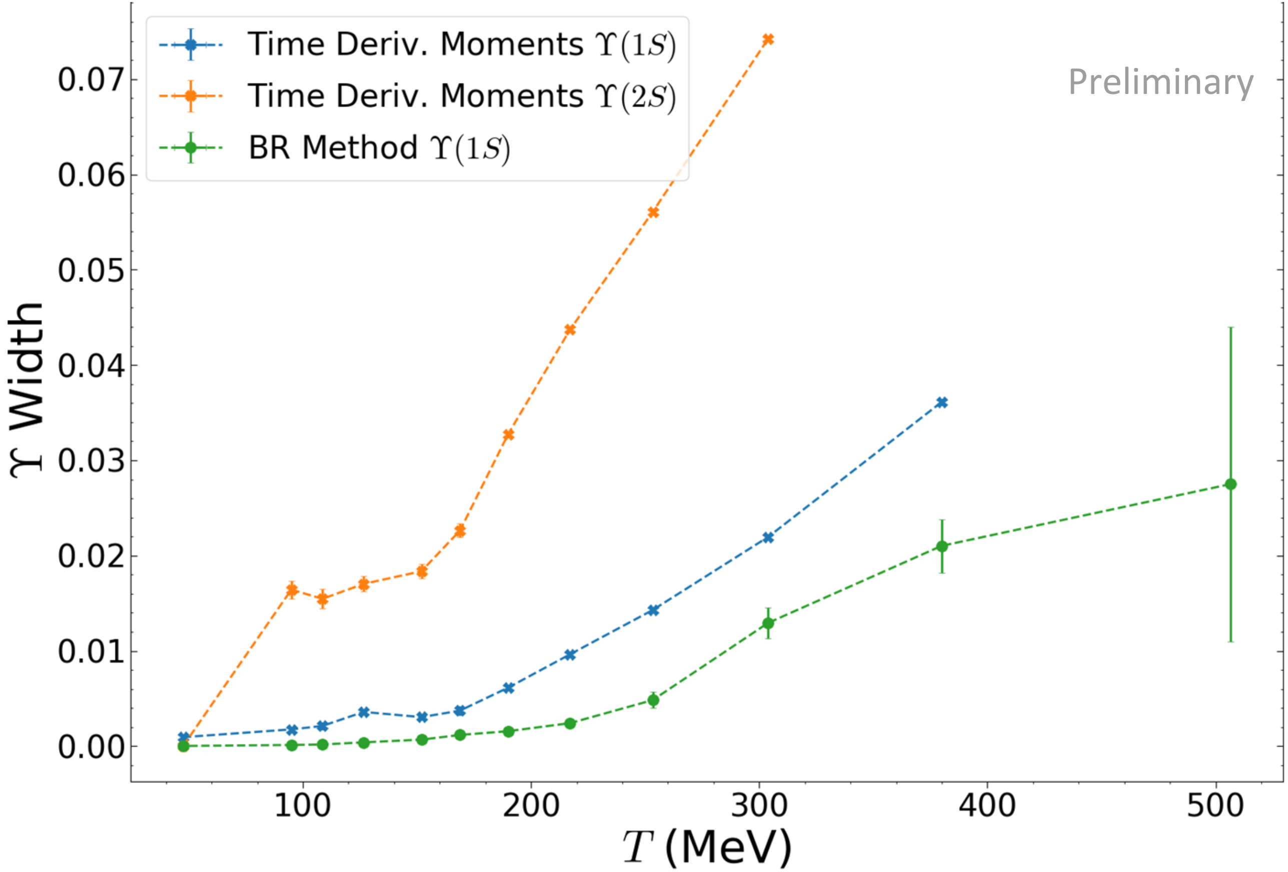
GEVP



`Moments`

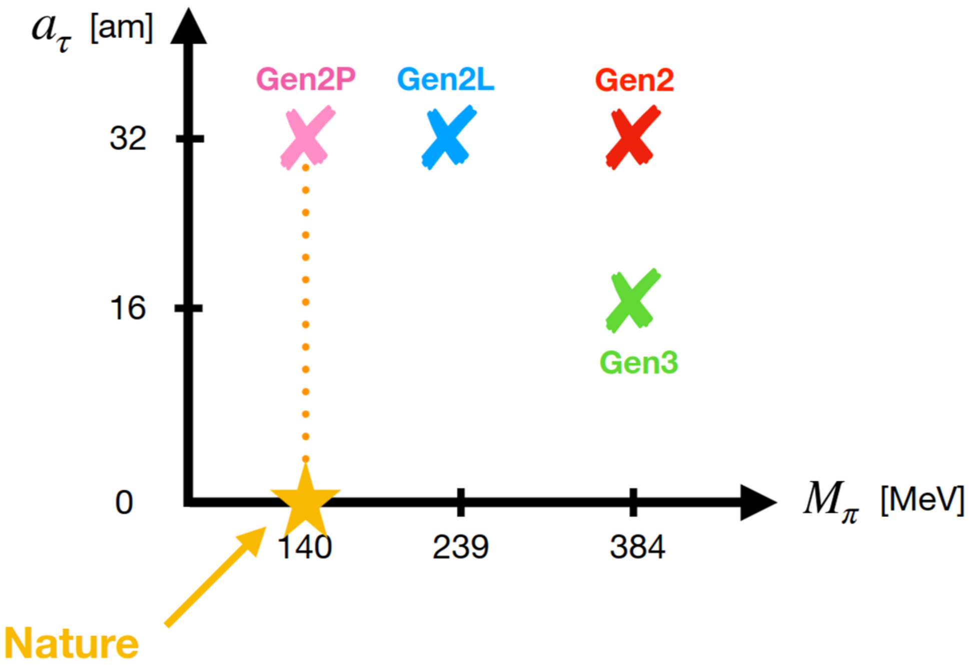
Comparison

- Bayesian Reconstruction method
- Moments method for ground & excited states
- Encouraging similarity between methods
- Excited state is broader than ground state



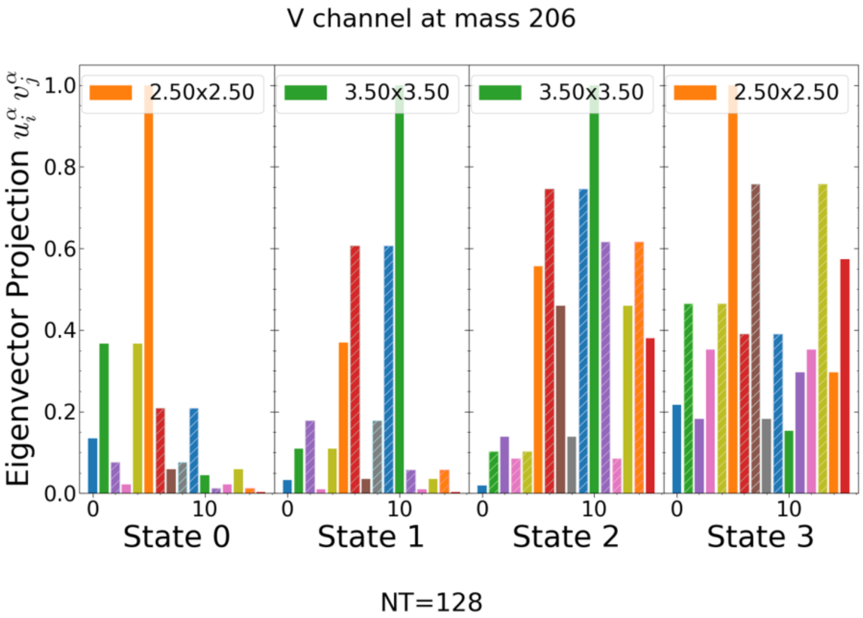
Summary

- Presented results for the mass of Υ and χ_{b1} excited states using a basis of 'smeared' operators
  - At zero and finite temperature
- (Re-)introduced `moments` method to examine 'widths' of ground state (Gaussian) spectral functions
- Applied `moments` to GEVP projected correlators
- GEVP of smeared operators was successful in allowing use of the `moments` method for excited states
- Systematics of method not fully explored for this study (GEVP correlators)

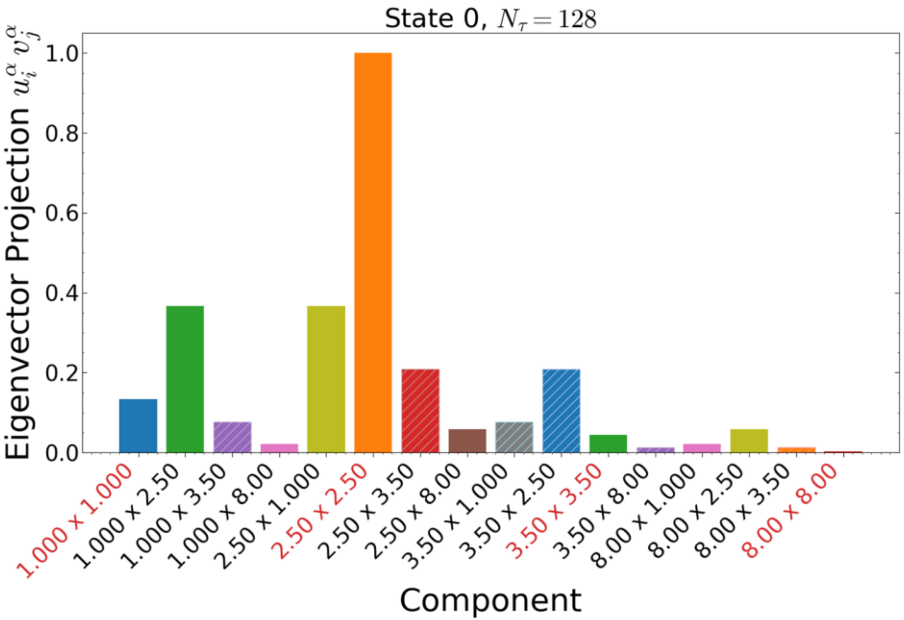


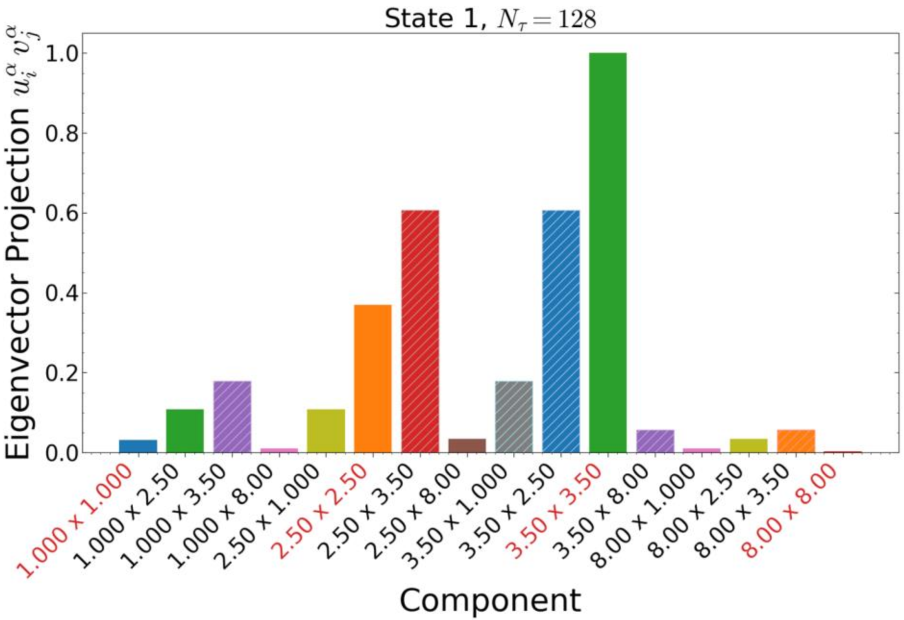
Eigenvectors

- Related to overlap of each ‘operator’ with each state
- Examine eigenvectors to see how they change as temperature increases
  - Plots have the largest contribution is normalised to one, and negative contributions are ‘hashed’



Eigenvectors





Eigenvectors