Topological structures with twisted boundary conditions and adjoint mode filtering

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1. Introduction: fractional instanton models
2. Techniques: analyzing and filtering of gauge configurations
3. Results: fractional instantons in lattice simulations of Yang-Mills theory

Fractional instanton liquid model

- final main goal
  - analysis of the $SU(N_c)$ Yang-Mills (super YM, QCD) vacuum structure based on semiclassics
  - model for microscopic confinement mechanism
- conditions for a viable approach:
  - try to find most elementary building blocks
  - get to arbitrary precision in certain parameter limits
  - works at large N_c
  - no gauge fixing
  - avoid phase transitions when going away from the limiting cases

$\Rightarrow$ fractional instanton liquid model

Review: [A. Gonzales-Arroyo (2023)]

Fractional instantons and twisted boundary conditions

$$Q = \nu - \frac{\vec{k} \cdot \vec{m}}{N_c}; \quad k_i = n_{0,i},\ n_{ij} = \epsilon_{ijk} m_k$$

- natural non-trivial (anti-) self-dual solution for appropriate twisted torus
- carry flux and contribute to confinement (area law)
- relates to several ideas: vortices, instantons, constituent monopoles of calorons
- exponential localization
- analytic solutions known only in special cases
- generalization: multi-fractional-instantons

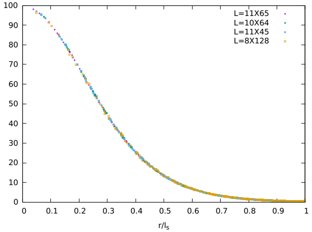
Approaches based on compactified spaces

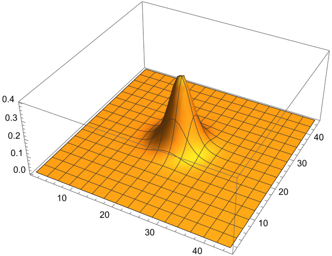
Smallest dimension controls semiclassical limit:

$$l_s << 1/\Lambda \xrightarrow[\text{continuous ?}]{} l_s > 1/\Lambda$$

- $S^1 \times R^2$: requires modification of theory to avoid phase transition (e. g. super YM)
- $T^3 \times R$: quantum mechanical description (1D effective model) (T-directions with 't Hooft flux)
- $T^2 \times R^2$: controlled semiclassical regime more similar to YM, Wilson loops and string tension in R^2 plane

Fractional instantons on $T^2 \times R^2$





- numerical solution on the lattice with twist in R^2 and T^2 planes

  [A. Gonzales-Arroyo, A. Montero (1998)]
- smooth vortex-like configurations
- scaling with the size of the small torus l_s
- main investigations with twist only in T^2 plane: configurations of multiple fractional (anti) instantons

Analyzing the Yang-Mills vacuum

Goal:

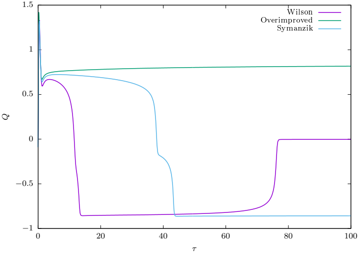
- identify underlying semiclassical structure of configurations
- filter UV noise
- preserve important structures

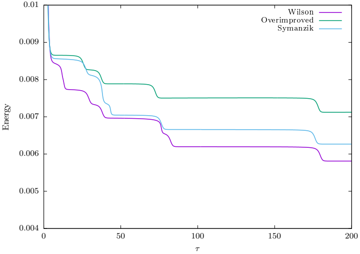
Methods:

- gradient flow
- adjoint mode filtering
- factional instanton identification

Gradient flow

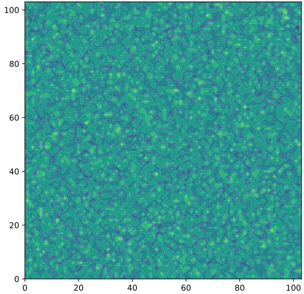
- fractional instanton configuration: stable under GF
- lattice artefacts tend to remove high density (instantons): avoided with overimproved GF
- pair annihilation: underestimation of number of objects, without changing Q_{top}



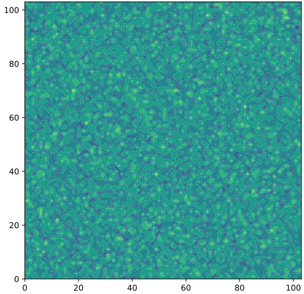


Improved gradient flow

Wilson gradient flow



Overimproved gradient flow

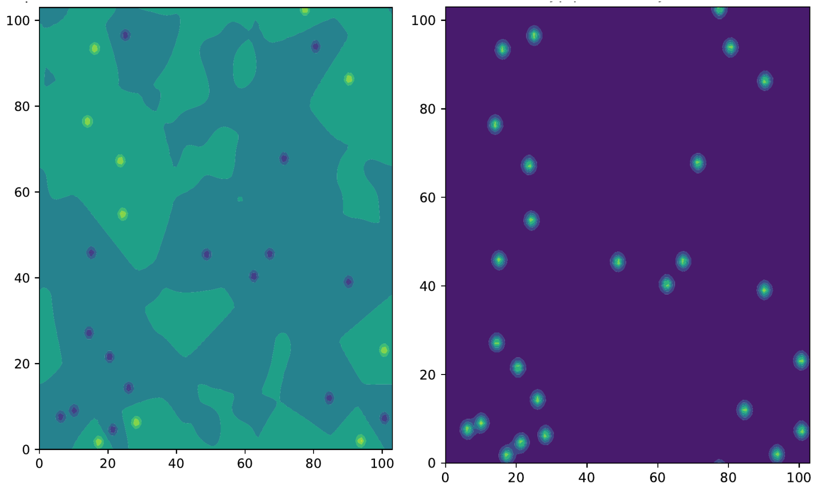


topological charge density on R^2 plane

Parameter: $N_s = 4$, $\beta = 2.6$

Signals in other quantities

topological charge density Polyakov loop



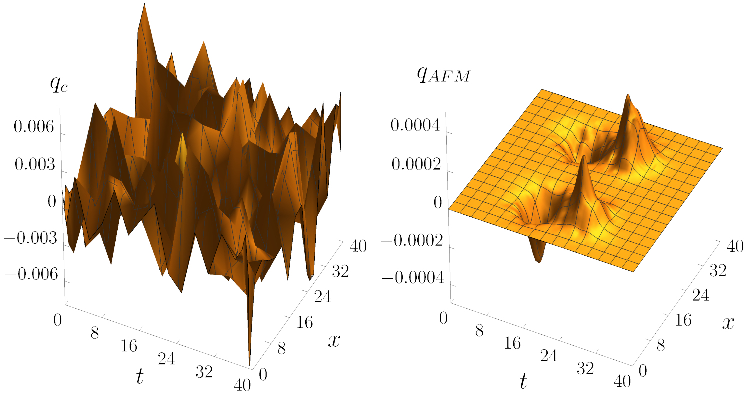
Adjoint mode filtering

- adjoint fermion zero mode [A. Gonzalez-Arroyo, R. Kirchner (2006)]: $\Psi = \frac{1}{8} F_{\mu\nu}[\gamma_\mu, \gamma_\nu] V$, with A_μ solution of eom
- construct energy and topological charge density from Weyl-components of zero mode

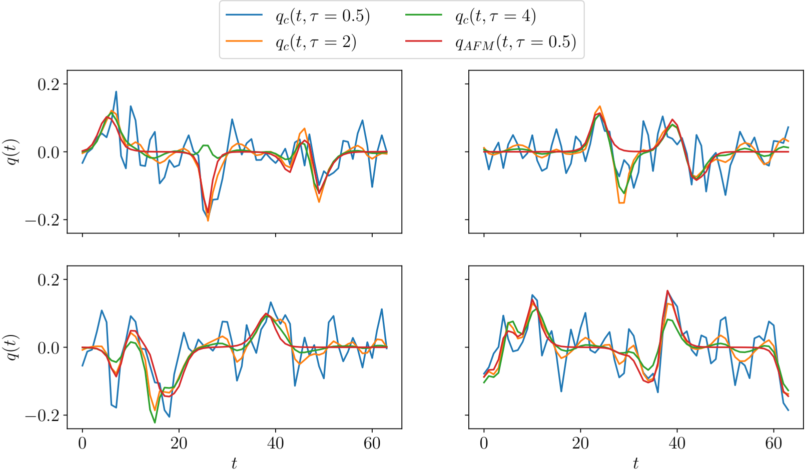
$$S_{AFM}(x) = \frac{1}{g^2}(|\psi_+|^2 + |\psi_-|^2) = \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$q_{AFM}(x) = \frac{1}{8\pi^2}(|\psi_+|^2 - |\psi_-|^2) = \frac{1}{32\pi^2} F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} \ .$$

Tests with know fractional instanton background



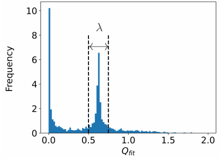
AFM on $T^3 \times R$

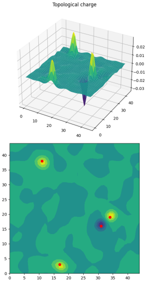


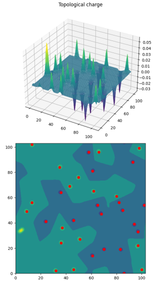
[GB, I. Soler-Calero, A. Gonzalez-Arroyo (2023)]

Fractional instanton identification

- identification of maxima/minima: positions of possible (anti) fractional instantons
- fit of few points around maximum provides estimate of size and top. charge
- compare to expected properties/find clusters



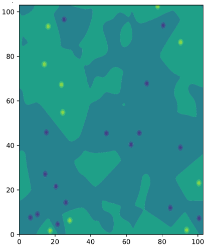




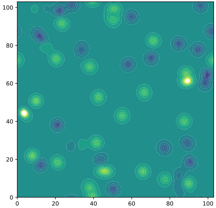
Simulation parameters

- Wilson gauge action
- $N_s = 4, \ldots 13$; $N_r = 64, 108$
- $\beta = 2.4, \ldots, 2.7$; $a = 0.115, \ldots, 0.0434$ fm
- fixed flow time in physical units
- current analysis mainly based on Wilson gradient flow and fractional instanton identification

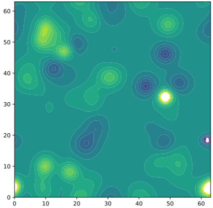
Snapshots of topological charge density



$l_s = 0.236$ fm

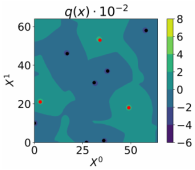


$l_s = 0.472$ fm

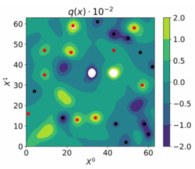


$l_s = 0.59$ fm

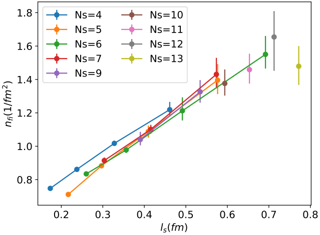
$l_s \sim 0.2$

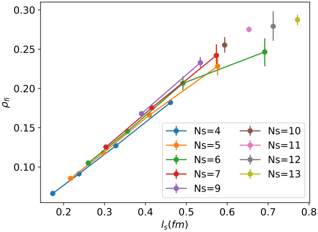


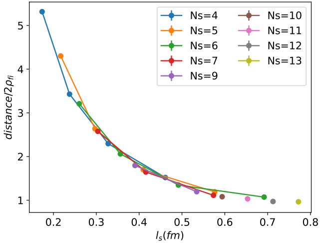
$l_s \sim 0.75$



Validity of semiclassical regime



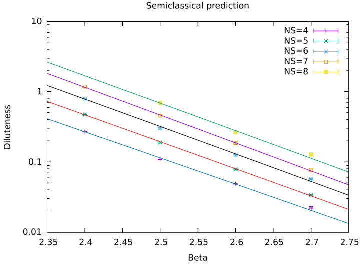


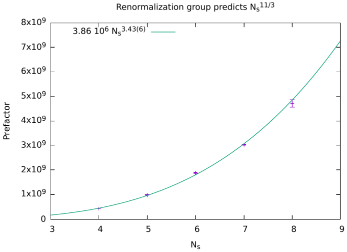


- up to $l_s \sim 0.7$fm good agreement with dilute gas approximation
- $l_s \geq 0.7$fm: decoupling of size from l_s size given by distance

Distribution of fractional charge objects

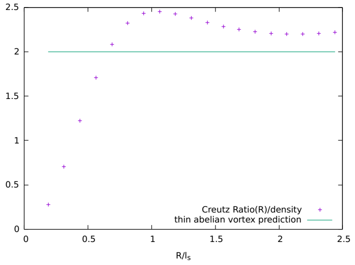
- diluteness: number of objects / (R^2 volume in units of l_s)
- decays with Boltzmann factor: $A\beta^2 \exp(-\beta\pi^2)$
- prefactor A scaling as RG scaling of l_s

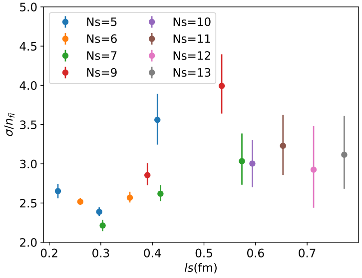




String tension from semiclassical approximation

- thin abelian vortex approximation: $\sigma \sim 2n_{fi}$
- deviation due to size of vortex: $\sigma \sim 2.6n_{fi}$
- no large deviations even at larger l_s





Conclusions and outlook

Semiclassical picture on $T^2 \times R^2$:

- fractional instanton model provides an attractive description of confinement
- several methods provide insights in the underlying semiclassical vacuum structure
- up to $l_s \sim 0.7$fm dilute gas provides reasonable approximation

Further steps:

- complement picture by other methods like adjoint mode filtering
- get beyond dilute gas regime
- continuity: How is the full Yang-Mills theory approached from semiclassics?