Color-magnetic correlation in SU(3) lattice QCD

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in collaboration with A. Tokutake, K. Tohme (Kyoto U.)

Abstract (largely changed):

Motivated by color-magnetic instability in QCD, we numerically calculate spatial color-magnetic correlation $\langle H_z(s)H_z(s+r)\rangle$ in SU(2) and SU(3) lattice QCD in the Landau gauge.

Curiously, this correlation is found to be always negative for r on xy-plane, apart from the same-point correlation.

We analyze Quadratic, Cubic and Quartic terms of the gluon field A.

- The negative behavior of Quadratic term is explained with Yukawa-type Landau-gluon propagator $\langle A_\mu(s)A_\mu(0)\rangle \propto e^{-mr}/r$.
- Quadratic and Cubic terms tend to cancel and this cancellation makes the total color-magnetic correlation small.
- Phenomenologically, the color-magnetic correlation $\langle H_z(s)H_z(s+x)\rangle$ seems to be expressed with a squared Yukawa function e^{-Mr}/r^2.
- Parallel-type magnetic correlation $\langle H_z(s)H_z(s+z)\rangle$ is always positive and tends to be opposite sign to perpendicular-type one in the Landau gauge.

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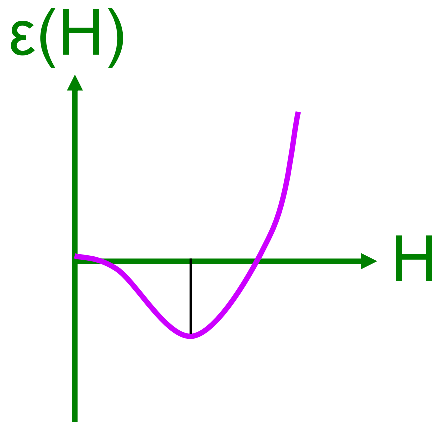
Motivation: Color-Magnetic Instability in QCD Vacuum

In 1977, Savvidy calculated the effective potential of YM theory in the presence of constant color-magnetic field at the one-loop level, and he showed spontaneous generation of color-magnetic field in YM vacuum.

Energy density ε(H) of SU(2) YM theory at the one loop level

$$\varepsilon(H) - \varepsilon(0) = \frac{1}{2}H^2 + \frac{11(gH)^2}{\underline{48\pi^2}} \ln\frac{gH}{\mu^2} - i\frac{(gH)^2}{8\pi}$$

— β-function coefficient



The energy minimum is achieved at non-zero color-magnetic field

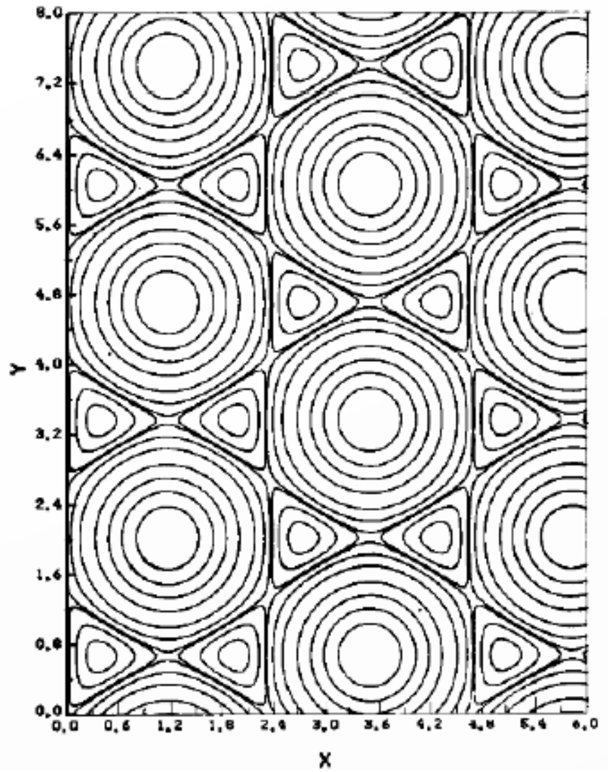
$$gH = \mu^2 \exp\left[-\left(\frac{24\pi^2}{11g^2} + \frac{1}{2}\right)\right]$$

← Asymptotic Freedom
negative coefficient of β-function

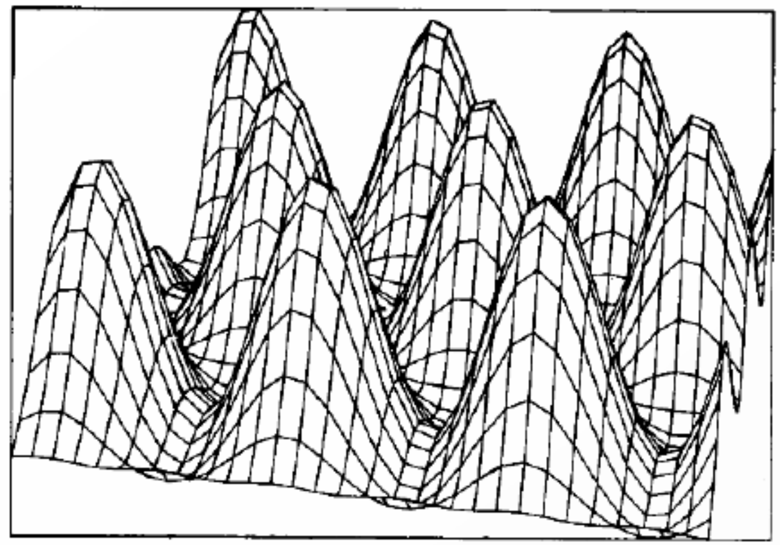
Asymptotic Freedom → Spontaneous Generation of Color-Magnetic Field
~ Color-Magnetic Instability of QCD vacuum
Actually, the gluon condensate is positive $\langle G_{\mu\nu}G^{\mu\nu}\rangle > 0$ in Minkowski metric.

Color-Magnetic instability of QCD ~ Copenhagen vacuum

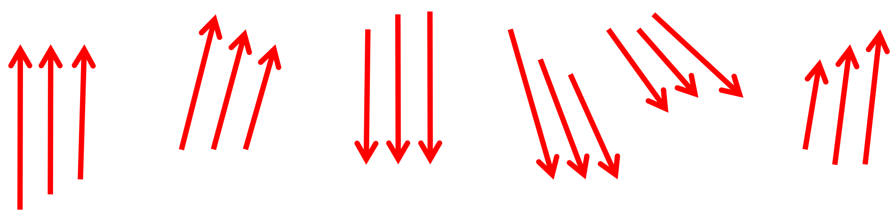
In 1980, Ambjorn-Olesen found that true YM solution at loop-level effective action is inhomogeneous vortex-like color-magnetic distribution.～Copenhagen vacuum



J. Ambjorn, P. Olesen, Nucl.Phys.B 170 (1980) 265.



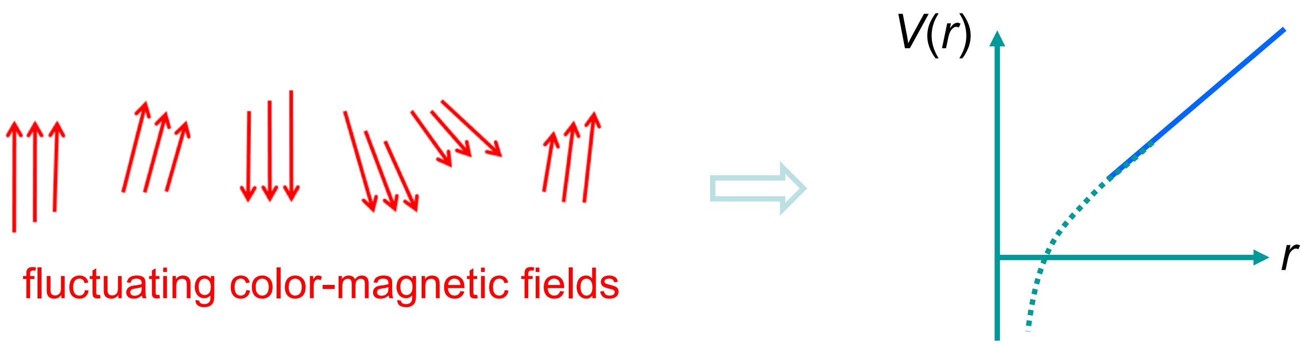
At a large scale, the vortex-like systems form a fluctuating stochastic domain structure.



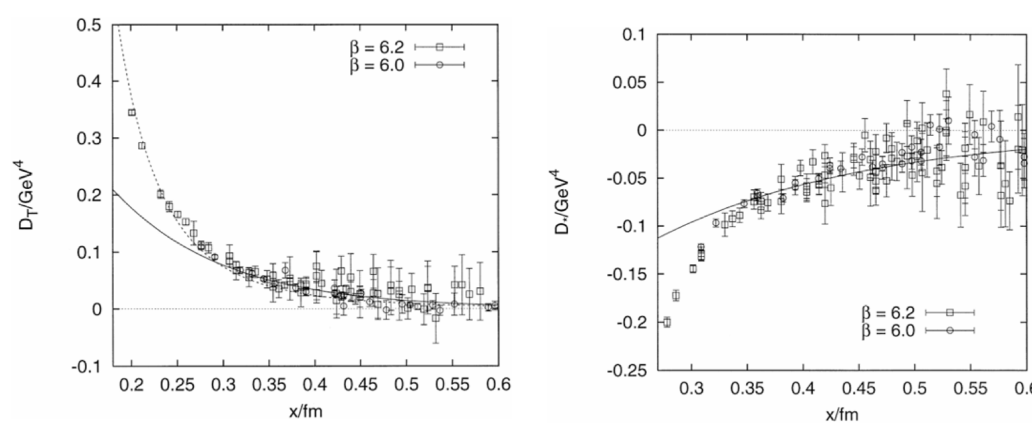
fluctuating color-magnetic fields

Stochastic Vacuum Model

In 1987, considering the fluctuating color fields in the QCD vacuum, Dosch and Simonov proposed stochastic vacuum model for gauge-invariant field-strength correlators and demonstrated that its infrared exponential damping leads to an asymptotic linear potential.



Giacomo, Bali, Brambilla, Vairo found that the gauge-invariant field-strength correlator shows infrared exponential damping in lattice QCD.



Bali, Brambilla, Vairo, Phys. Lett. B 421 (1998) 265.

Motivated by these studies, we reconsider the field-strength correlation in QCD. Here, we are interested in not only infrared behavior but also its whole behavior.

In this study, using lattice QCD, we mainly investigate color-magnetic correlation in the Landau gauge, which has many merits on Lorentz and color symmetry and minimal gauge-field fluctuations.

In Euclidean QCD, the Landau gauge has a global definition to minimize the "total amount of the gauge-field fluctuation",

$$R \equiv \int d^4x \, \mathrm{Tr}\{A_\mu(x)A_\mu(x)\} = \frac{1}{2}\int d^4x A_\mu^a(x)A_\mu^a(x)$$

by the gauge transformation.

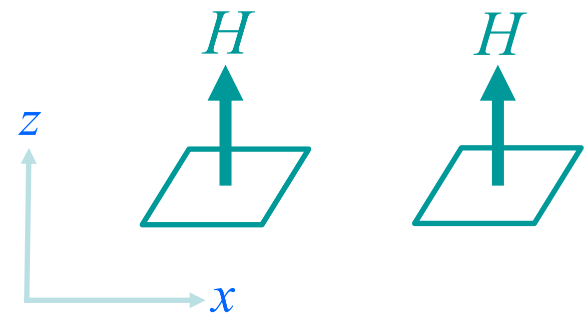
In the global definition, the Landau gauge has a clear physical interpretation that it maximally suppresses artificial gauge-field fluctuations relating to the gauge degrees of freedom.

Color-magnetic correlation in lattice QCD

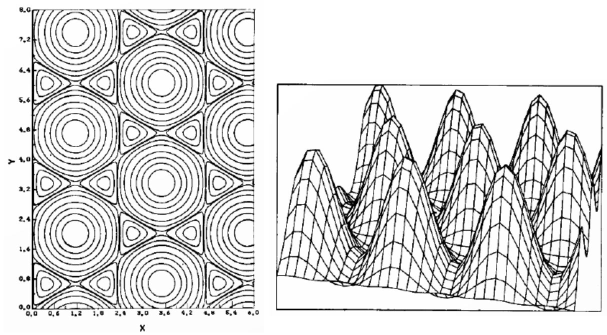
We investigate the following type color-magnetic correlation in lattice QCD in the Landau gauge.

1) Perpendicular-type magnetic correlation

$$C(r) = \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$

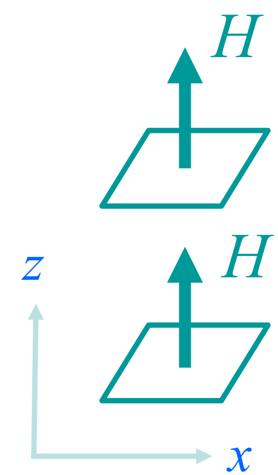


appropriate to examine vortex-like structure



2) Parallel-type magnetic correlation

$$C_{\parallel}(r) = \langle H_z^a(s) H_z^a(s + r\hat{z}) \rangle$$



In Euclidean QCD, because of symmetries, all the two-point field-strength correlations $\langle G_{\mu\nu}^a(s) G_{\alpha\beta}^a(s') \rangle$ can be expressed with these correlations.

SU(3) Lattice QCD

For the calculation of color-magnetic correlation, we use SU(3) quenched lattice QCD with standard plaquette action.

We use the following three lattices:

β=5.7, 16^4 i.e. a = 0.186fm, La = 3fm

β=5.8 ,16^4 i.e. a = 0.152fm, La = 2.4fm

β=6.0, 24^4 i.e. a = 0.104fm, La = 2.5fm

For each β, 200 configurations are used.
(thermalization: 20,000 sweeps, interval: 1,000 sweeps)

Landau gauge fixing, we use ordinary iterative maximization algorithm with over-relaxation parameter of 1.6.

We define SU(3) gluon fields with the link-variable in the Landau gauge:

$$A_\mu(x) \equiv \frac{1}{2ia}\left[U_\mu(x) - U_\mu^\dagger(x)\right] - \frac{1}{2iaN_c}\mathrm{Tr}\left[U_\mu(x) - U_\mu^\dagger(x)\right]$$

SU(2) Lattice QCD

For the calculation of color-magnetic correlation, we use SU(2) quenched lattice QCD with standard plaquette action.

We use the following three lattices:

- β=2.3, 16^4 i.e. a = 0.18fm, La = 2.9fm
- β=2.4, 24^4 i.e. a = 0.127fm, La = 3.0fm
- β=2.5, 24^4 i.e. a = 0.09fm, La = 2.2fm

For each β, 200 configurations are used.
(thermalization: 2,000 sweeps, interval: 2,000 sweeps)

Landau gauge fixing, we use ordinary iterative maximization algorithm with over-relaxation parameter of 1.6.

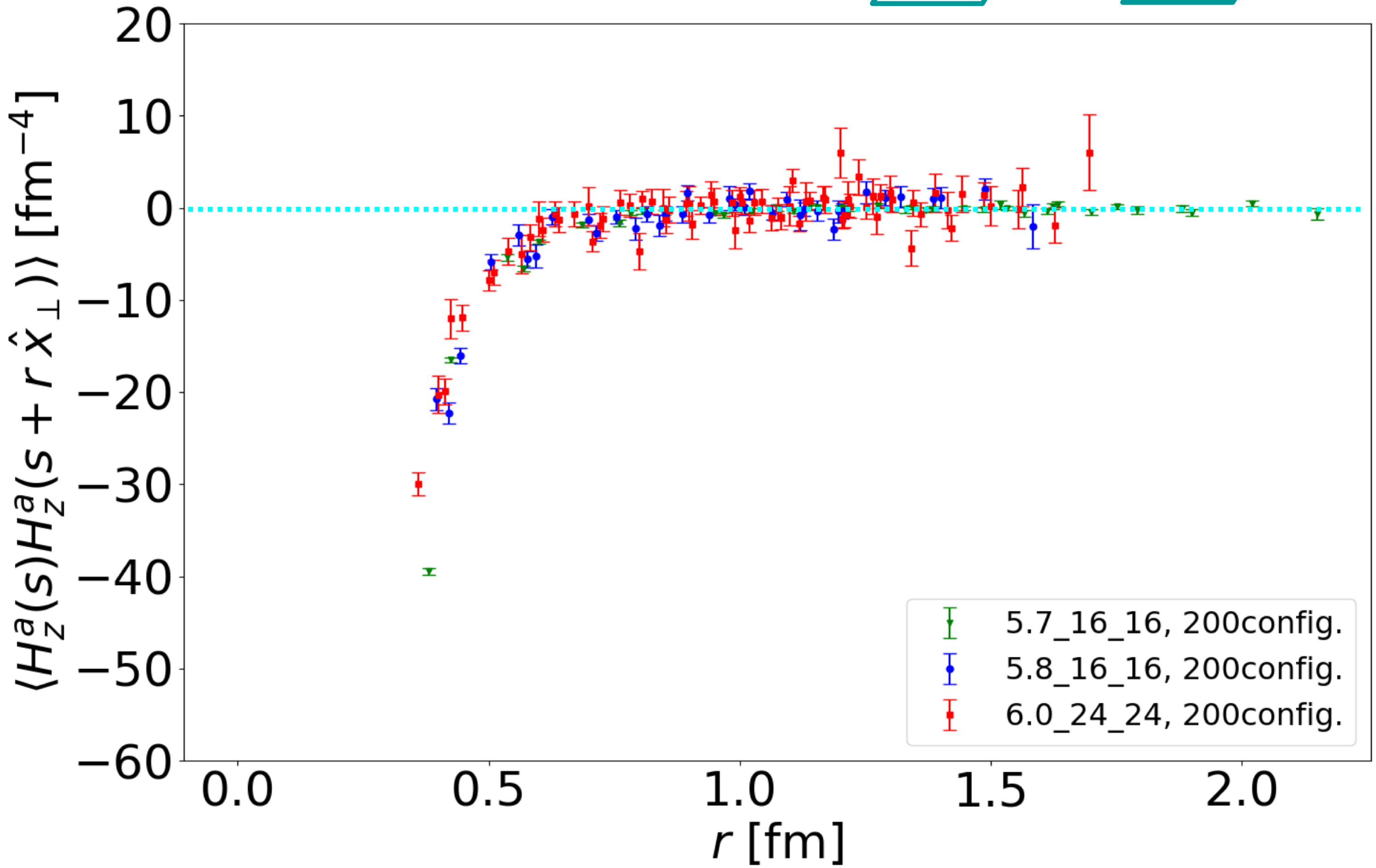
For SU(2), gluon fields *A* are directly obtained from the link-variable *U*:

$$U = e^{-agi\tau^a A^a} = \cos(agA) + i\tau^a \hat{A}^a \sin(agA)$$

Color-magnetic correlation in SU(3) lattice QCD

Perpendicular-type

$$C(r) = \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$

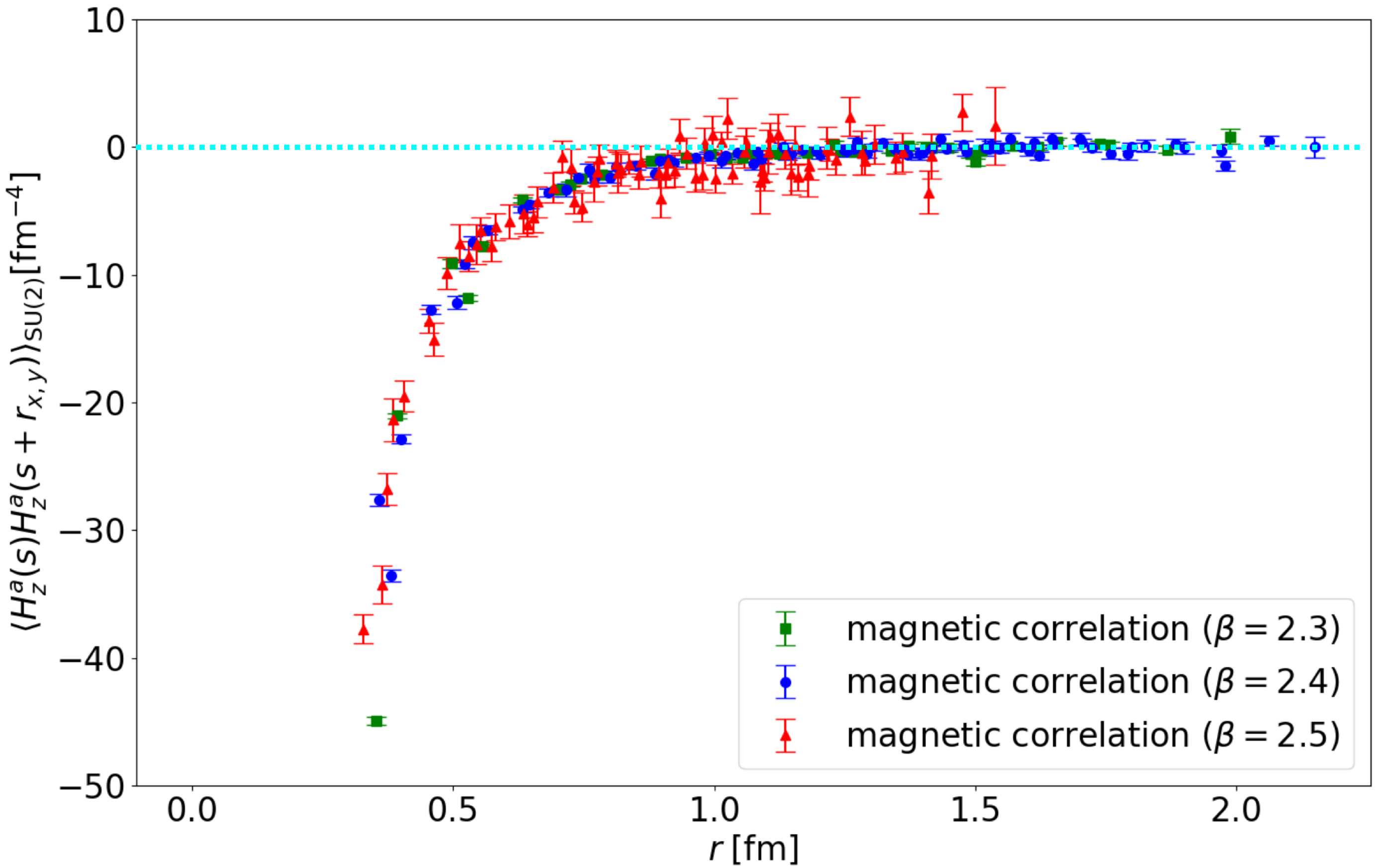


Curiously, perpendicular-type color-magnetic correlation is found to be always negative, apart from the same-point correlation of r = 0.

Color-magnetic correlation in SU(2) lattice QCD

Perpendicular-type

$$C(r) = \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$



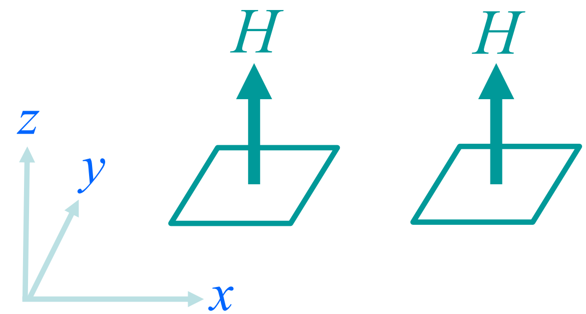
Curiously, perpendicular-type color-magnetic correlation is found to be always negative, apart from the same-point correlation of $r = 0$.

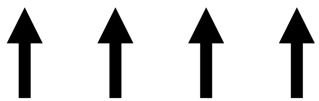
Why “always negative” ?

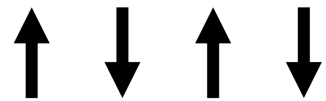
Perpendicular-type

$$C(r) = \langle H_z^a(s) H_z^a(s + r\,\hat{\perp}) \rangle < 0$$

$$(\perp = x, y)$$







“Always positive correlation” and “alternating correlation” have been observed in various fields of physics.
However, “always negative” correlation is not popular.

One may suspect that the gauge fixing gives some unphysical effect.
We investigate also gauge-invariant field-strength correlators and obtain the similar result. In fact, the corresponding correlation is always negative.

Decomposition of Field Strength in Landau gauge

To consider the negativity the color-magnetic correlation, we decompose field-strength correlator into three parts: quadratic, cubic and quartic terms of the gluon field *A*.

$$\langle \mathrm{Tr}\, G_{\mu\nu}(s) G_{\alpha\beta}(s') \rangle$$

Quadratic term

$$= \langle \mathrm{Tr}\, \{\partial_\mu A_\nu - \partial_\nu A_\mu\}(s)\{\partial_\alpha A_\beta - \partial_\beta A_\alpha\}(s') \rangle$$

$$+ig\langle \mathrm{Tr}\{\partial_\mu A_\nu - \partial_\nu A_\mu\}(s)[A_\alpha, A_\beta](s')\rangle + ig\langle \mathrm{Tr}[A_\mu, A_\nu](s)\{\partial_\alpha A_\beta - \partial_\beta A_\alpha\}(s')\rangle$$

$$+g^2\langle \mathrm{Tr}[A_\mu, A_\nu](s)[A_\alpha, A_\beta](s')\rangle$$

Cubic term

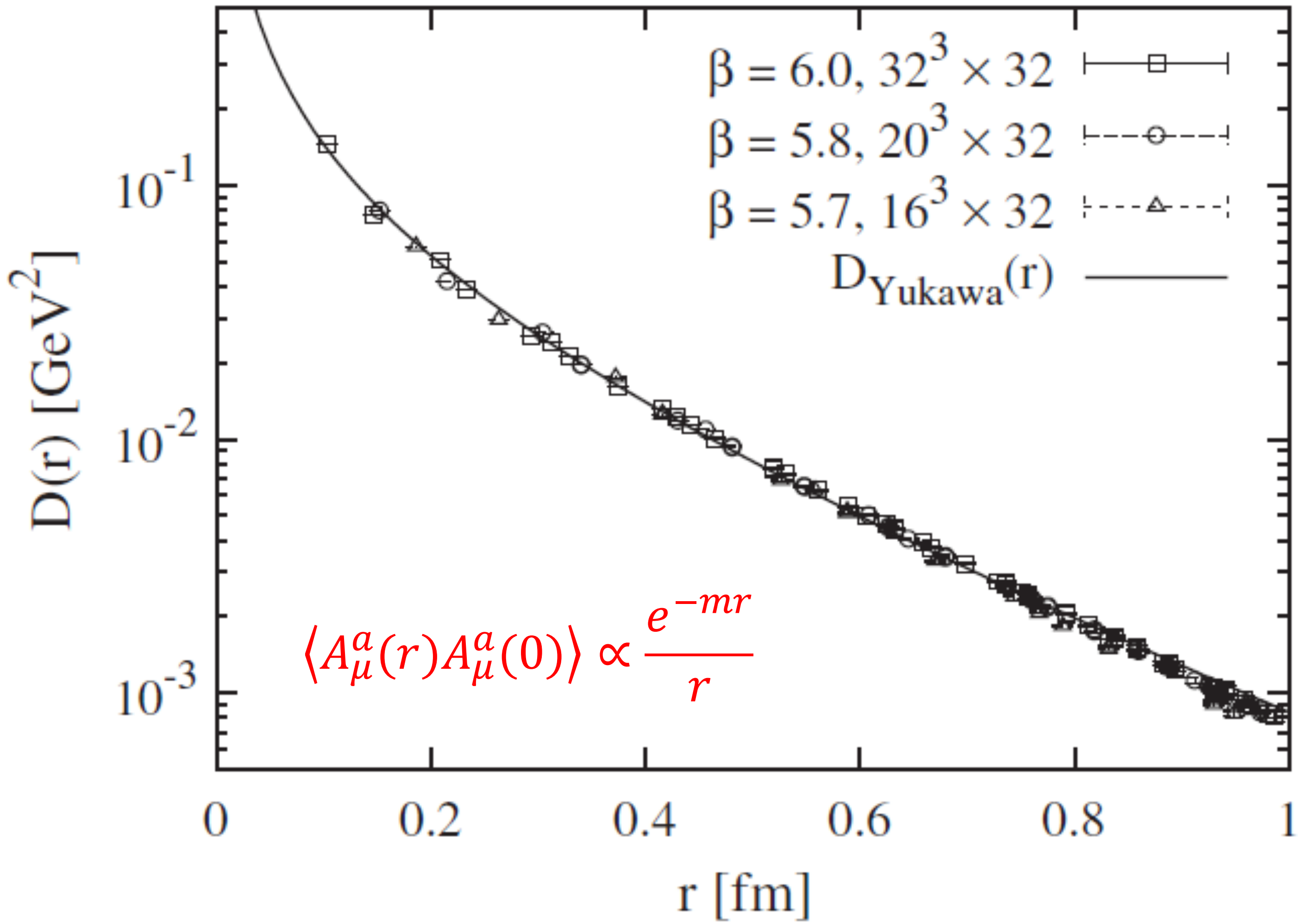
Quartic term

Among them, the Quadratic term can be directly expressed with the gluon propagator $\langle \mathrm{Tr} A_\mu(s) A_\nu(s') \rangle$

In the Laudau gauge, due to the Lorentz symmetry, we only have to consider scalar combination of the gluon propagator $\langle A_\mu^a(r) A_\mu^a(0) \rangle$ as a function of four-dimensional Euclidean distance *r* .

Landau-gauge gluon propagator in SU(3) lattice QCD

T. Iritani, H.S. H. Iida , Phus. Rev. D80, 114505 (2009).

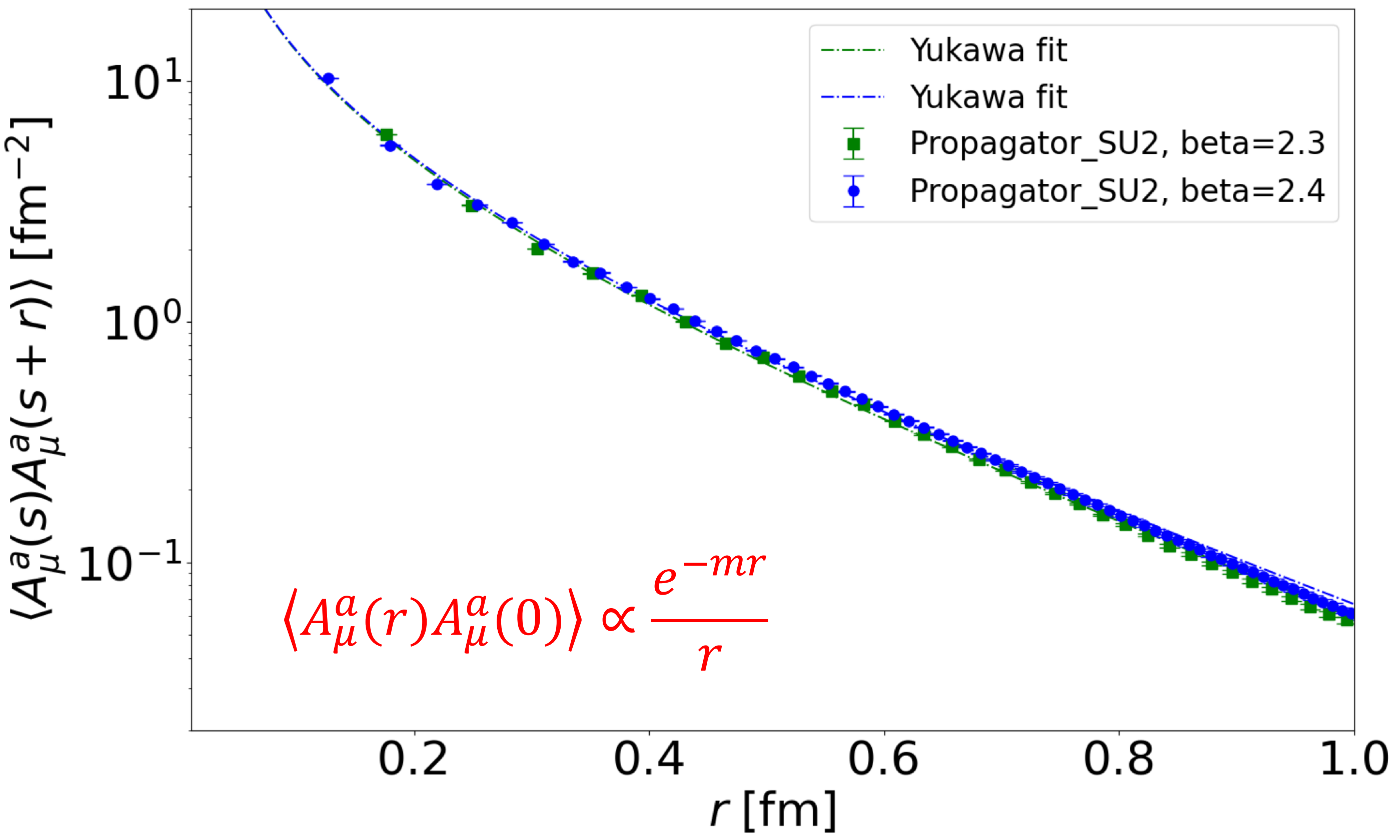


Yukawa-damping mass parameter

$m = 0.624$ GeV

In a wide region of r = 0.1~1.0 fm, the *Landau-gauge gluon propagator* is well described with *Yukawa-type function* of four-dimensional Euclidean space-time distance.

Landau-gauge gluon propagator in SU(2) lattice QCD



Yukawa-damping mass parameter

m = 0.66-0.68 GeV

The Landau-gauge gluon propagator seems to be well reproduced with the Yukawa-type function for the range of r = 0.1 - 1 fm

Among the color-magnetic correlation

$$\langle \mathrm{Tr}\, H_z(s) H_z(s') \rangle$$

$$= \langle \mathrm{Tr}\, \{\partial_1 A_2 - \partial_2 A_1\}(s) \{\partial_1 A_2 - \partial_2 A_1\}(s') \rangle$$ Quadratic term

$$+2ig \langle \mathrm{Tr} \{\partial_1 A_2 - \partial_2 A_1\}(s) [A_1, A_2](s') \rangle$$ Cubic term

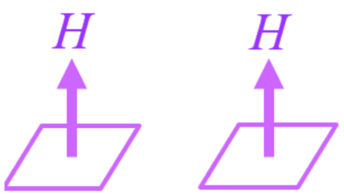
$$+g^2 \langle \mathrm{Tr} [A_1, A_2](s) [A_1, A_2](s') \rangle$$ Quartic term

we first consider the Quadratic term with the gluon propagator.

Using the *Yukawa-type gluon propagator*, we find that the quadratic term of the perpendicular-type color-magnetic correlation becomes *always negative*:

$$\langle \mathrm{Tr}\, H_z(r\hat{x}) H_z(0) \rangle_{quad} \equiv \langle \mathrm{Tr}\, \{\partial_1 A_2 - \partial_2 A_1\}(r\hat{x}) \{\partial_1 A_2 - \partial_2 A_1\}(0) \rangle$$

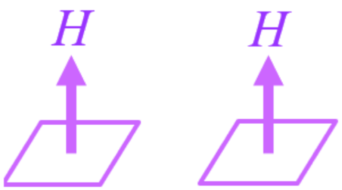
$$= -(N_c^2 - 1) A m^4 \frac{e^{-mr}}{mr} \left[1 + \frac{1}{mr} + \frac{1}{m^2 r^2} \right] < 0$$

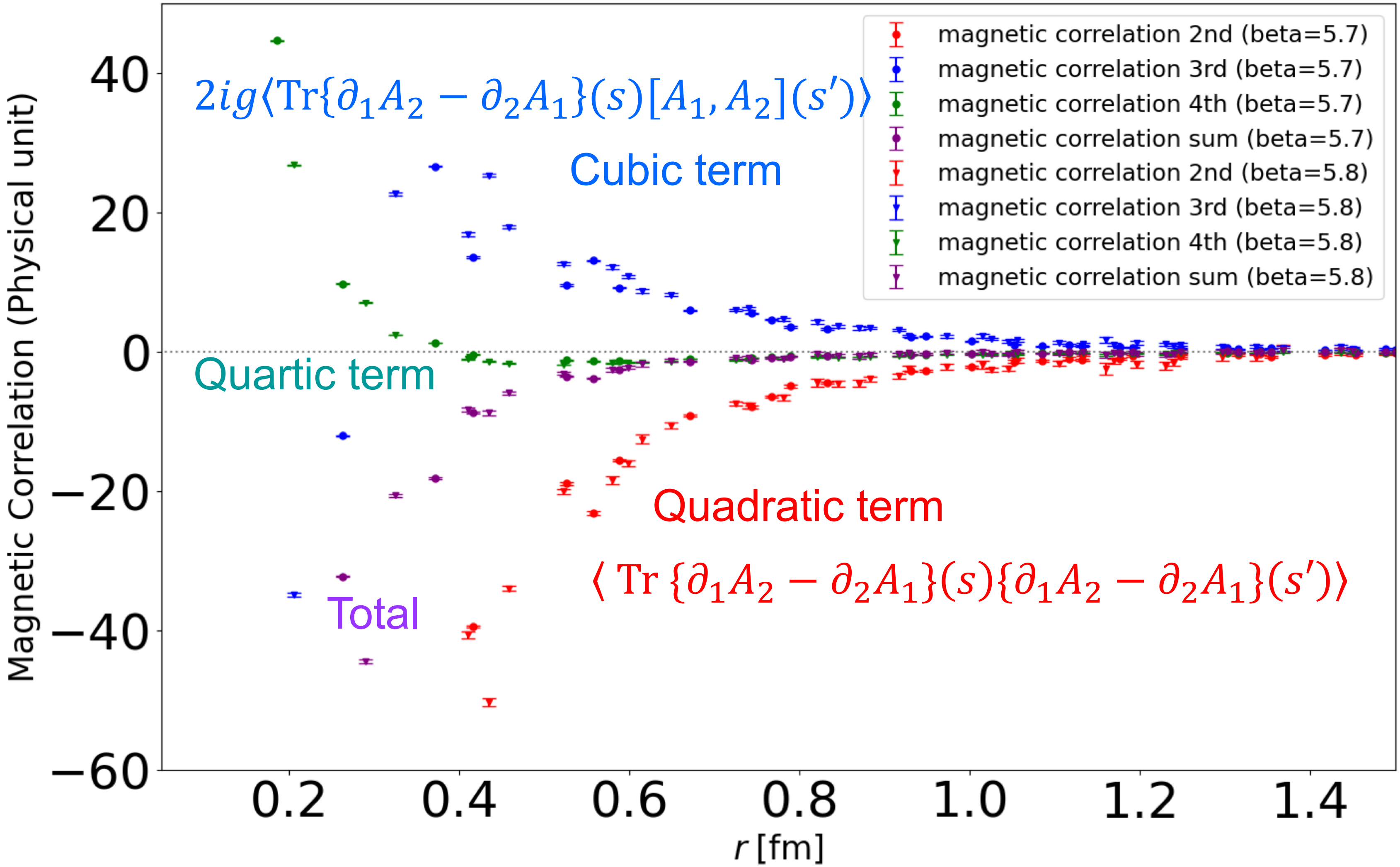


If the quadratic term is dominant,
the negative behavior of the color-magnetic correlation is explained.
However, the real situation is not so simple.

Color-magnetic correlation in SU(3) lattice QCD

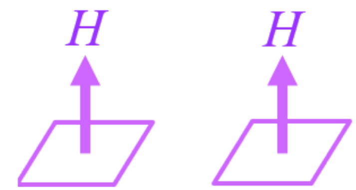
$$C(r) = \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$



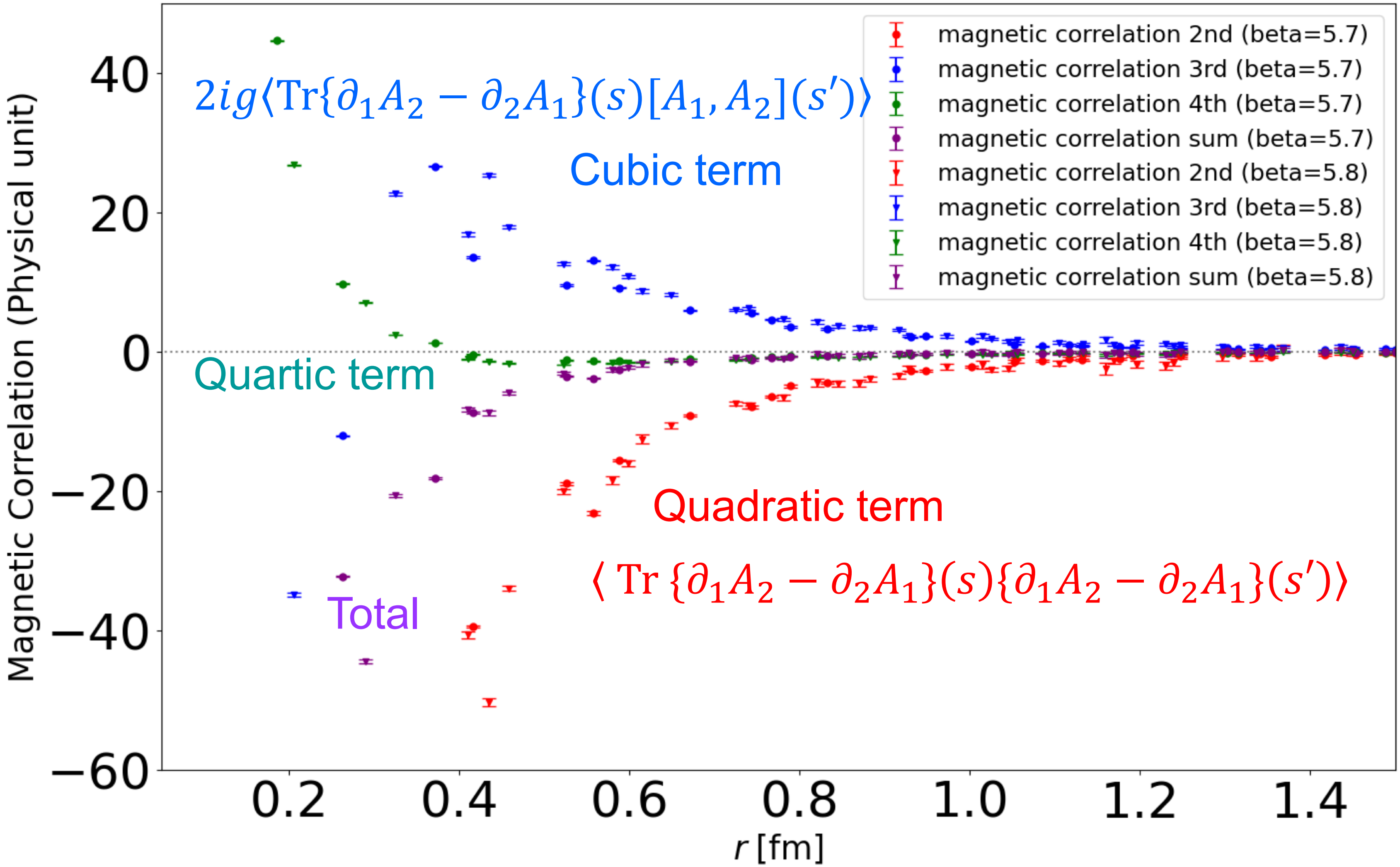


Quartic term is small, but Cubic term is *comparable* to the Quadratic term.

Color-magnetic correlation in SU(3) lattice QCD



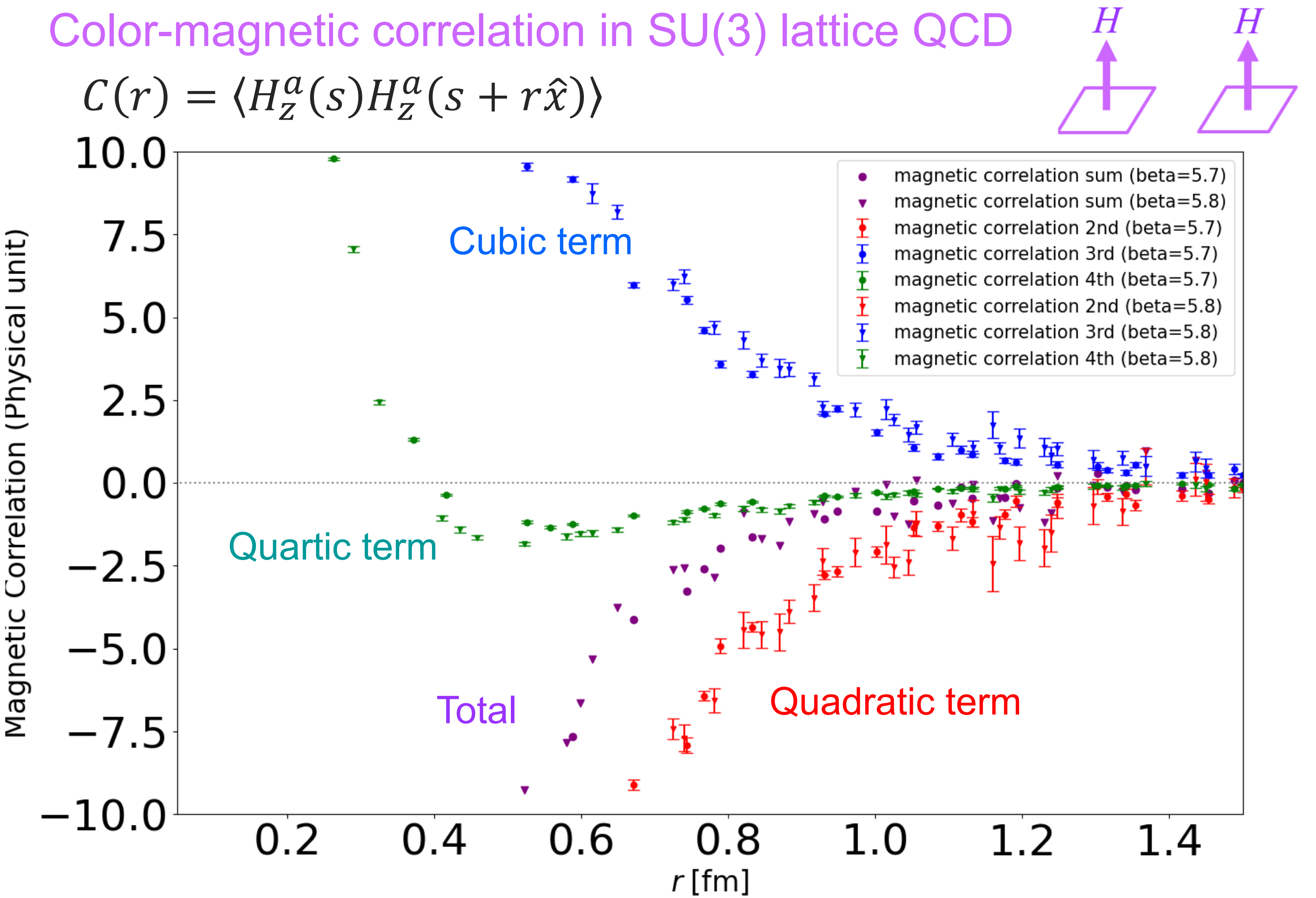
$$C(r) = \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$



In the infrared region, Quadratic term and Cubic term tend to cancel, and this cancellation makes the total value of color-magnetic correlation small.

Color-magnetic correlation in SU(3) lattice QCD

$$C(r) = \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$



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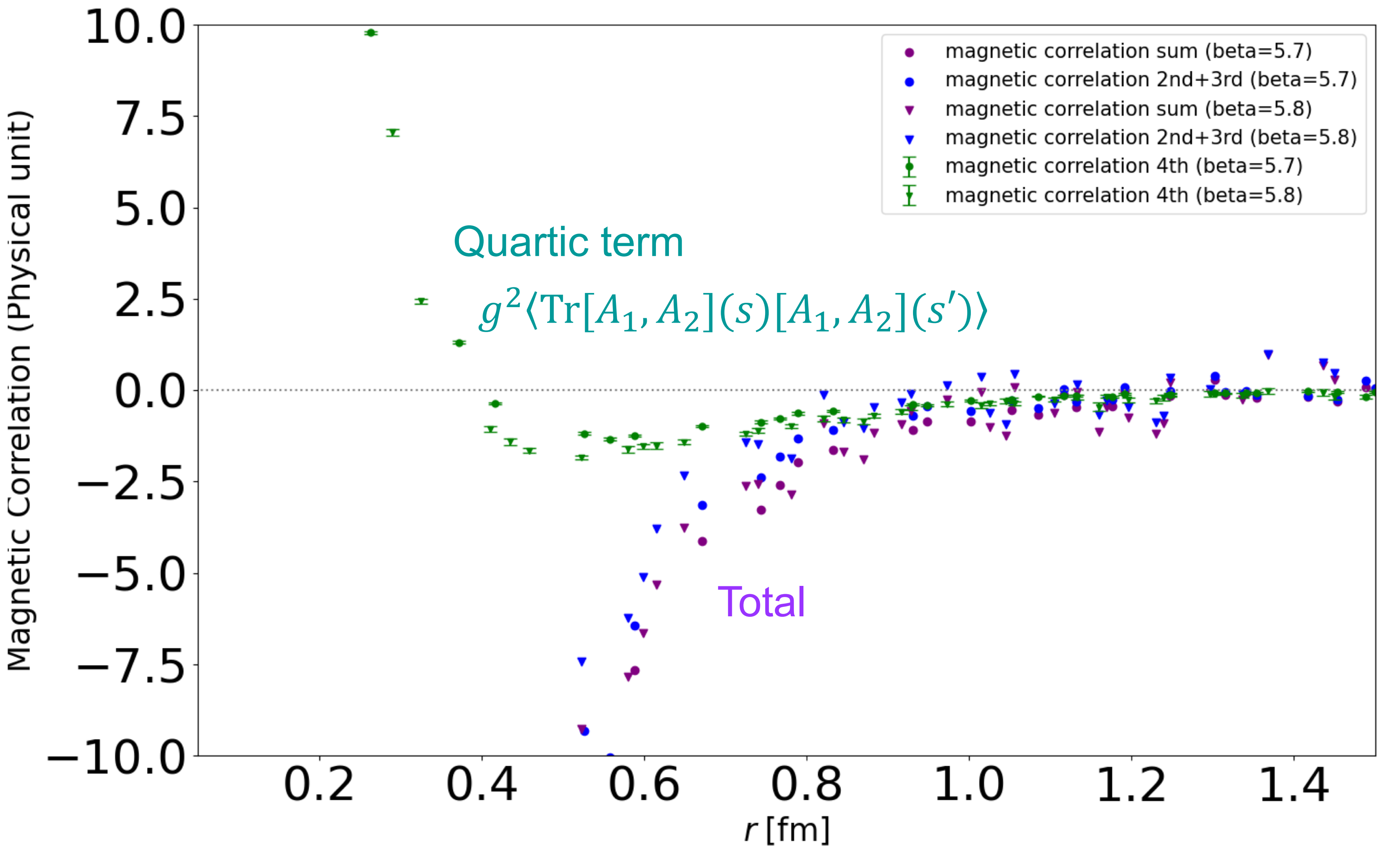
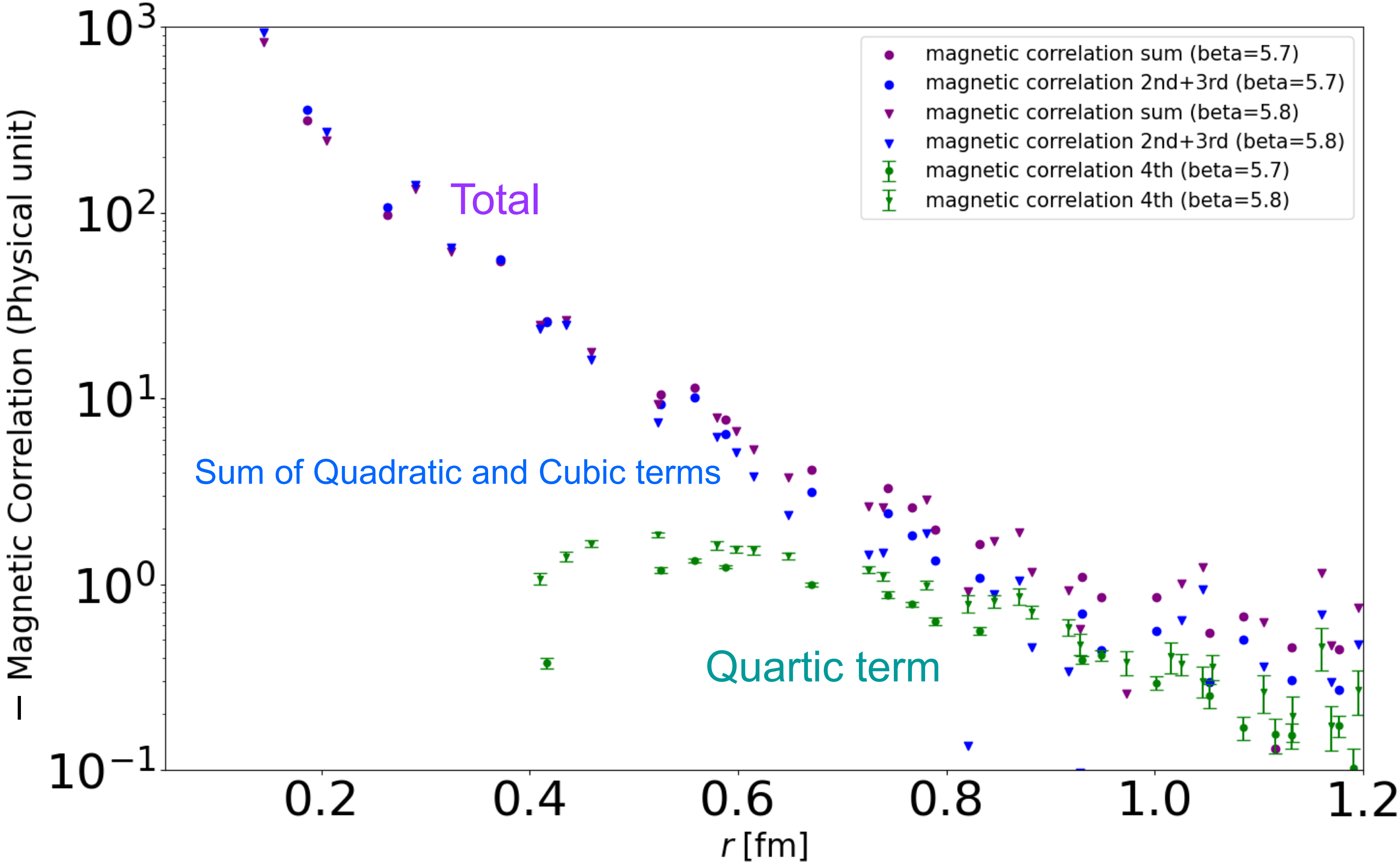
Color-magnetic correlation in SU(3) lattice QCD



Reflecting the cancelation between quadratic and cubic terms, the total color-magnetic correlation seems to behave like Quartic term in the infrared region.

Color-magnetic correlation in SU(3) lattice QCD



The Sum of Quadratic and Cubic terms also seems to behave as Quartic term in the infrared region.

Crude estimation of quartic term

For the Quartic term

$$g^2\langle \mathrm{Tr}[A_1,A_2](s)[A_1,A_2](s')\rangle = \frac{g^2}{2} f^{abc} f^{ade} \langle A_1^b(s)A_2^c(s)A_1^d(s')A_2^e(s')\rangle$$

we try to estimate it based on the Yukawa-type gluon propagator in the Landau gauge.

$$\langle A_\mu^a(x)A_\mu^a(0)\rangle \propto \frac{e^{-mr}}{r}$$

Assuming the Yukawa-type reduction factor of the gluon propagator,
we estimate the quartic term using a crude mean-field-like approximation:

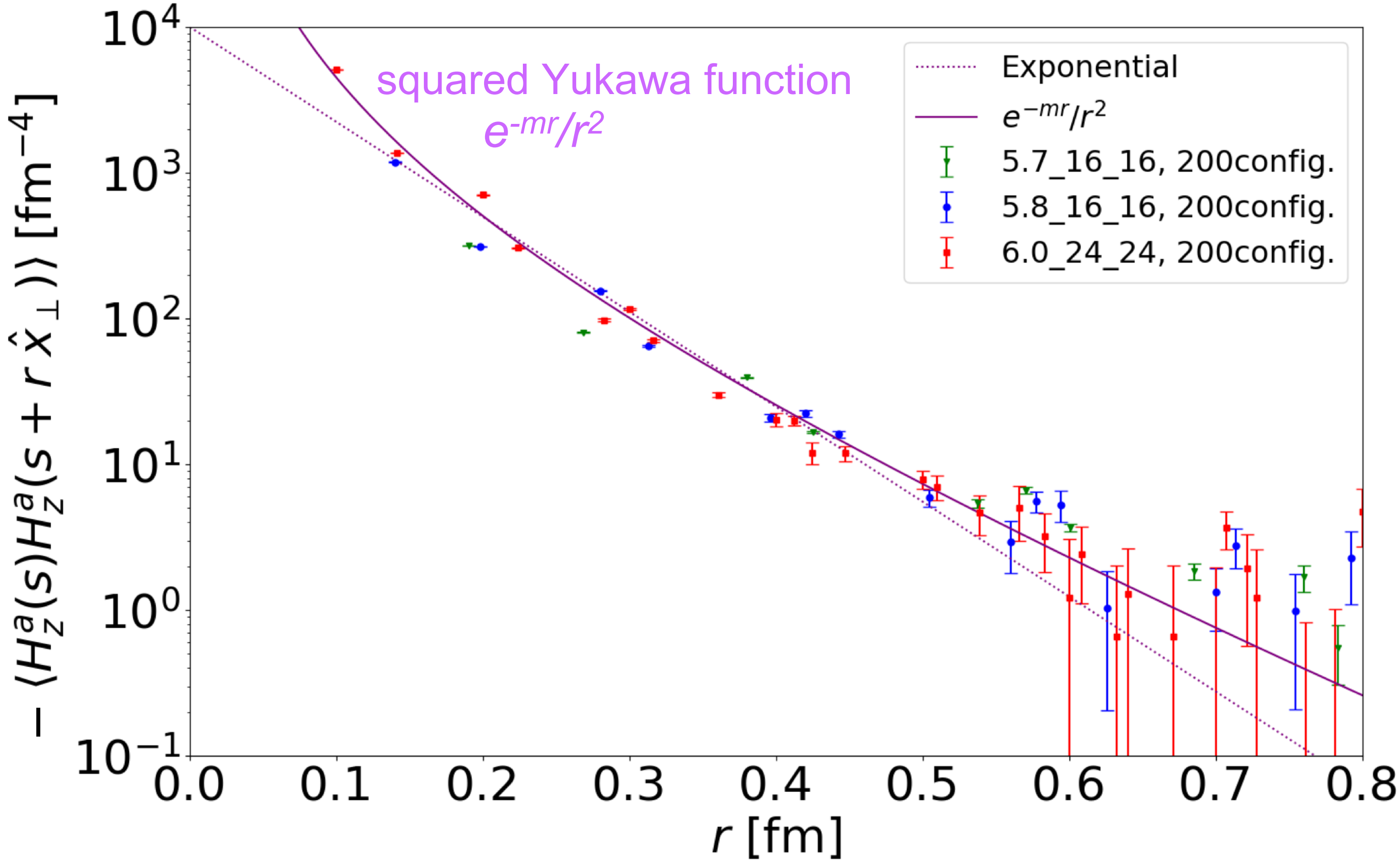
$$\begin{aligned}
g^2\langle \mathrm{Tr}[A_1,A_2](s)[A_1,A_2](0)\rangle &= \frac{g^2}{2} f^{abc} f^{ade} \langle A_1^b(s)A_2^c(s)A_1^d(0)A_2^e(0)\rangle \\
&\sim \frac{g^2}{2} f^{abc} f^{ade} \{\langle A_1^b(s)A_1^d(0)\rangle\langle A_2^c(s)A_2^e(0)\rangle + \langle A_1^b(s)A_2^e(0)\rangle\langle A_2^c(s)A_1^d(0)\rangle\} \\
&= \frac{3}{16} g^2 \{\langle A_1^a(s)A_1^a(0)\rangle\langle A_2^b(s)A_2^b(0)\rangle - \langle A_1^a(s)A_2^a(0)\rangle\langle A_2^b(s)A_1^b(0)\rangle\} \\
&\propto \left(\frac{e^{-mr}}{r}\right)^2 = \frac{e^{-2mr}}{r^2}
\end{aligned}$$

$$\left(\begin{array}{l}
\langle A_\mu^a(s)A_\nu^b(s')\rangle = \dfrac{1}{N_c^2-1}\delta^{ab}\langle A_\mu^c(s)A_\nu^c(s')\rangle \\
f^{abc}f^{abc} = 24
\end{array}\right)$$

Then, the Quartic term becomes squared Yukawa function.
So, we examine a squared Yukawa fit for the color-magnetic correlation.

Color-magnetic correlation in SU(3) lattice QCD

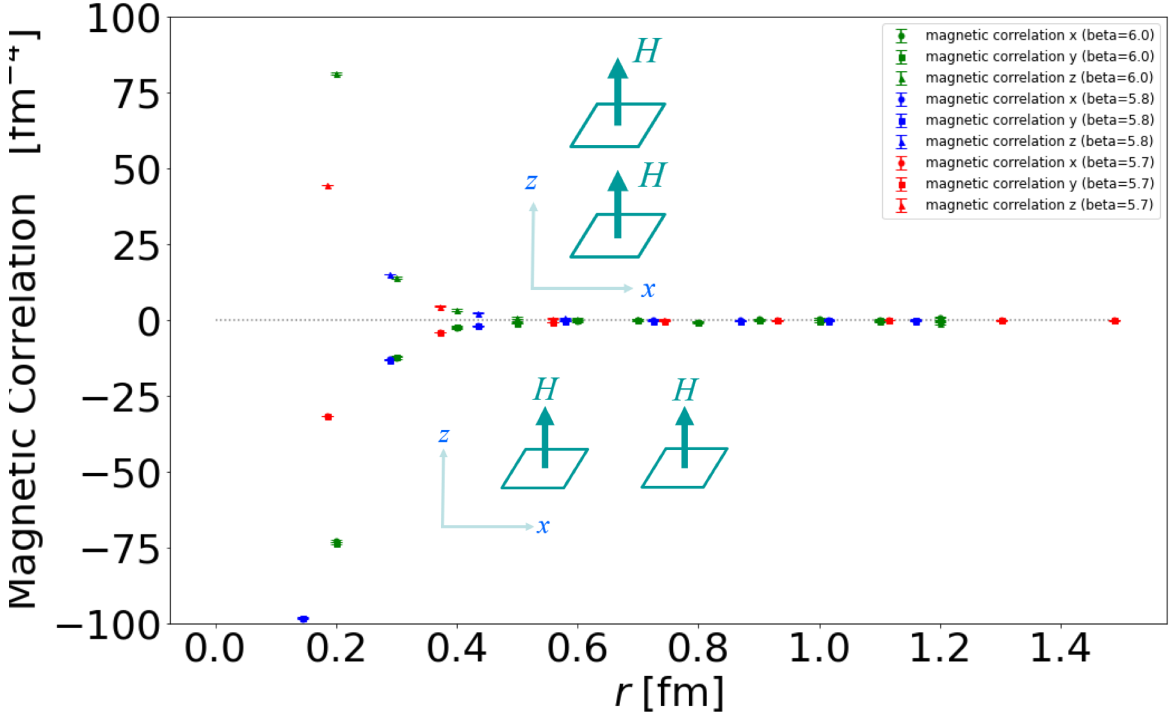
$$C(r) = \langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$



As a phenomenological fit, the color-magnetic correlation seems consistent with the squared Yukawa function e^{-mr}/r^2 with m ~1.6GeV in the range from 0.1fm to 0.7fm.

Parallel-type Color-magnetic correlation in SU(3) lattice QCD

Parallel-type magnetic correlation $C_{\parallel}(r) = \langle H_z^a(s) H_z^a(s + r\hat{z}) \rangle$



Parallel-type magnetic correlation is always positive and tends to be opposite sign to perpendicular-type one in the Landau gauge.

$$\langle H_z^a(s) H_z^a(s + r\hat{z}) \rangle \cong -\langle H_z^a(s) H_z^a(s + r\hat{x}) \rangle$$

This leads to approximate cancelation for the sum of field-strength correlators.

$$\sum_{\mu,\nu} \langle G_{\mu\nu}^a(s) G_{\mu\nu}^a(s') \rangle \cong 0$$

Summary

Motivated by color-magnetic instability in QCD, we have numerically calculate spatial color-magnetic correlation $\langle H_z(s)H_z(s+r)\rangle$ in SU(2) and SU(3) lattice QCD in the Landau gauge.

Curiously, this correlation is found to be always negative for r on xy-plane, apart from the same-point correlation.

We have analyzed Quadratic, Cubic and Quartic terms of the gluon field A.

- The negative behavior of Quadratic term is explained with Yukawa-type Landau-gluon propagator $\langle A_\mu(s)A_\mu(0)\rangle \propto e^{-mr}/r$.

- Quadratic and Cubic terms tend to cancel, and this cancellation makes the total color-magnetic correlation small.

- Phenomenologically, the color-magnetic correlation $\langle H_z(s)H_z(s+r)\rangle$ seems to be expressed with a squared Yukawa function e^{-Mr}/r^2.

- Parallel-type magnetic correlation $\langle H_z(s)H_z(s+z)\rangle$ is always positive and tends to be opposite sign to perpendicular-type one in the Landau gauge.


Thank you!