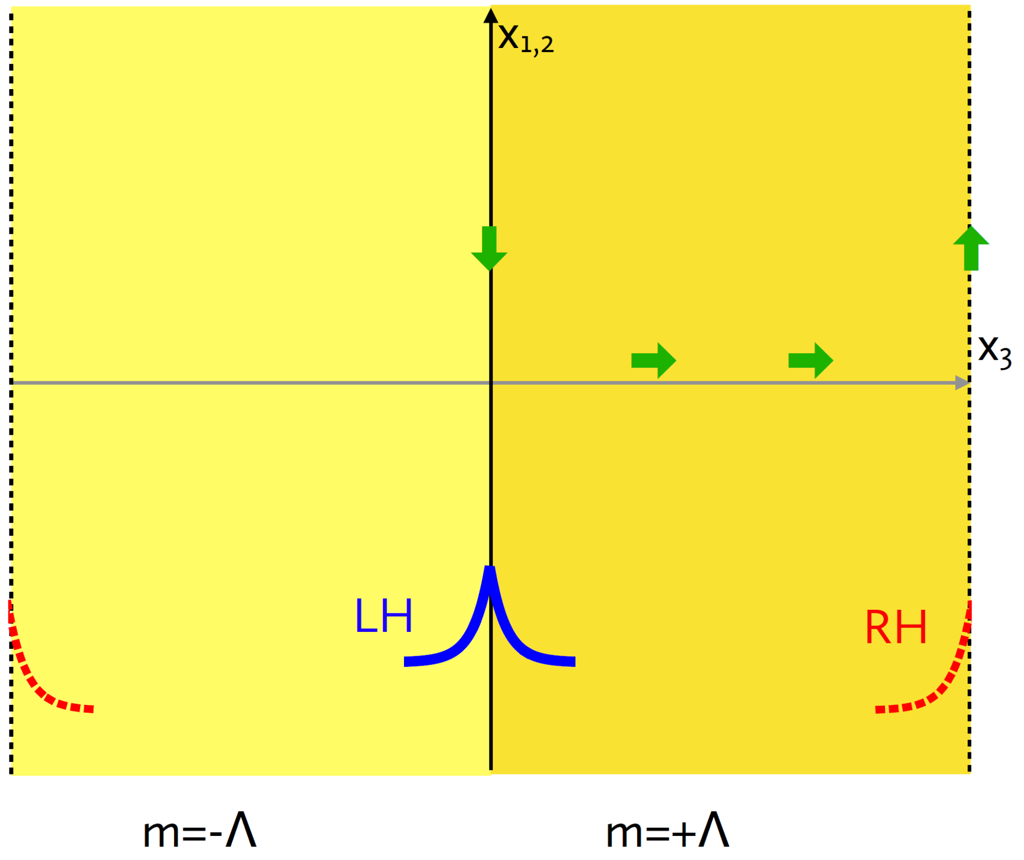
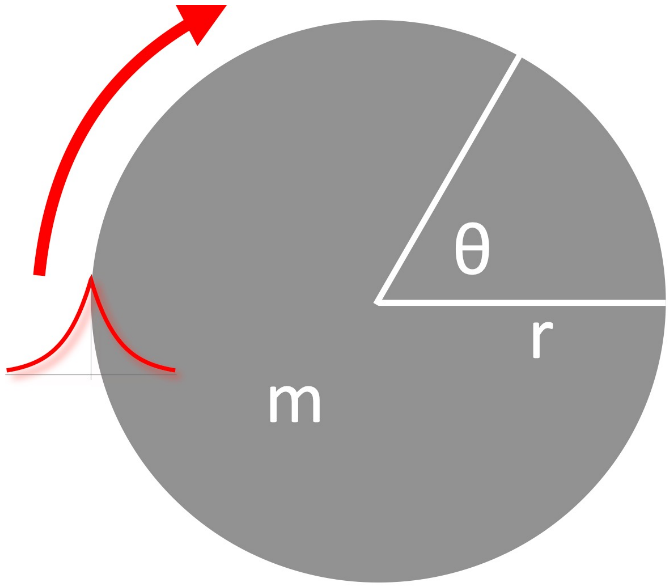
Weyl fermion on a lattice: A path to lattice chiral gauge theory





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Based on ***Phys.Rev.Lett.*** **132 (2024) 14, 141604**

with **David Kaplan**, University of Washington

XVIth quark confinement and Hadron spectrum conference, Cairns, Australia

Takeaway message before I begin

Regulating the standard model (or chiral gauge theories) on the lattice has remained one of the nagging unsolved problems over the last forty years.

Recent developments provide major breakthroughs that has brought us very close to solving this problem.

But, some challenges remain.

That's the whole talk!

Plan of the talk

- What are chiral gauge theories?
- Why is it hard to formulate them on the lattice?
- A few of the past attempts, that are yet to work or don't work.
- What is new and why is it exciting?

Chiral gauge theories

Even dimensional world with massless fermions and gauge field.

Chiral symmetry is gauged.

Fermion mass terms transform under gauge transformation.

So, a simple mass term is disallowed.

Example: The standard model.

Good chiral symmetry is essential

Why is chiral symmetry hard?

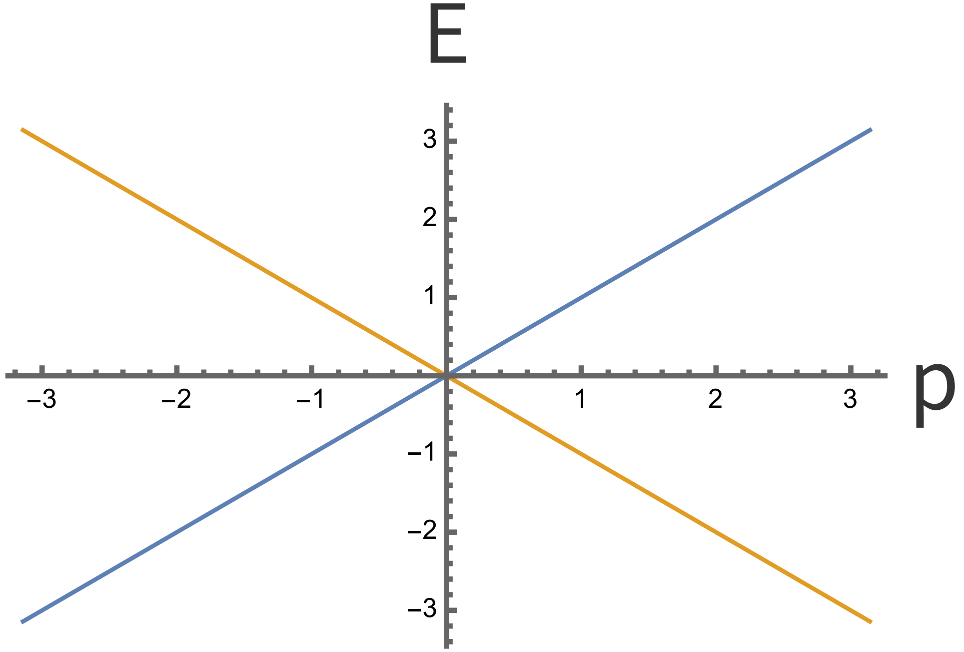
Nielsen-Ninomiya theorem is one of the major reasons.

Nielsen Ninomiya (1981): Cannot formulate Dirac fermion with exact chiral symmetry without an unwanted doubling of all fermion species.

Arbitrary number of massless Dirac fermions hard: global chiral symmetry

Odd number of Weyl fermions needed for gauging chiral symmetry

Why chiral symmetry is hard: Dispersion (1 spatial dimension)



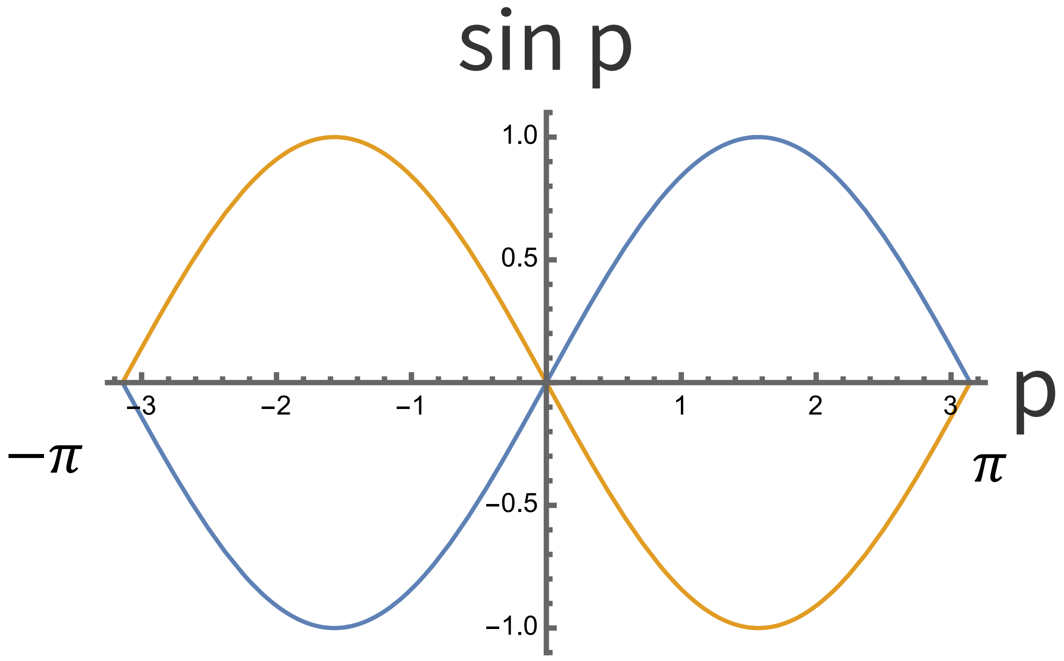
Continuum dispersion for a Dirac Hamiltonian

The no-go is better visualized using dispersion relation in Minkowski space-time (time continuous).

Hamiltonian formulation.

$$E = \pm p$$

Brilliuoin zones (Dirac)



Two Dirac fermions

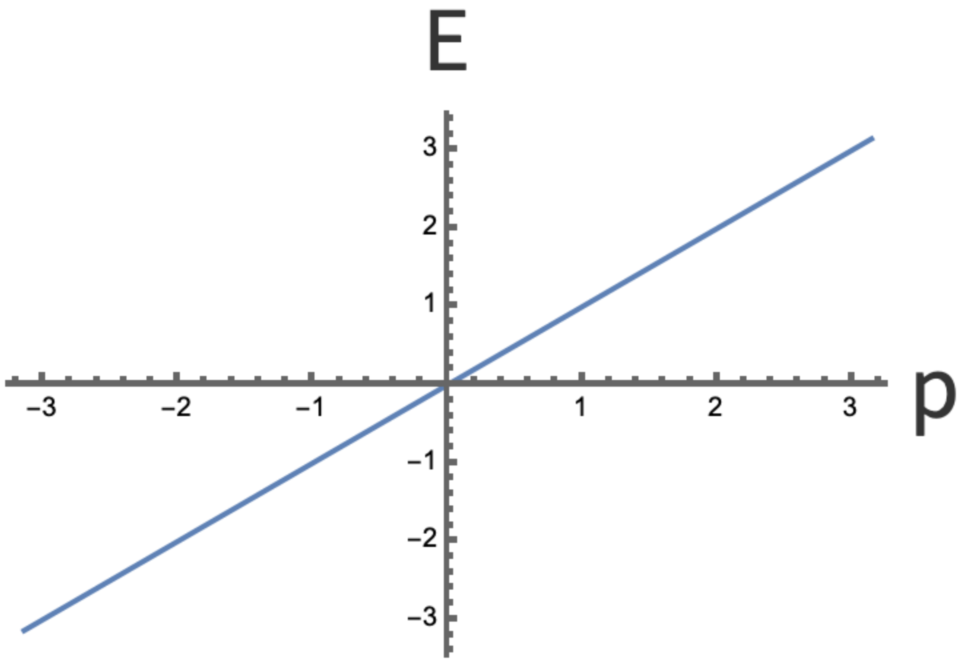
Lattice in space.

Time not discretized.

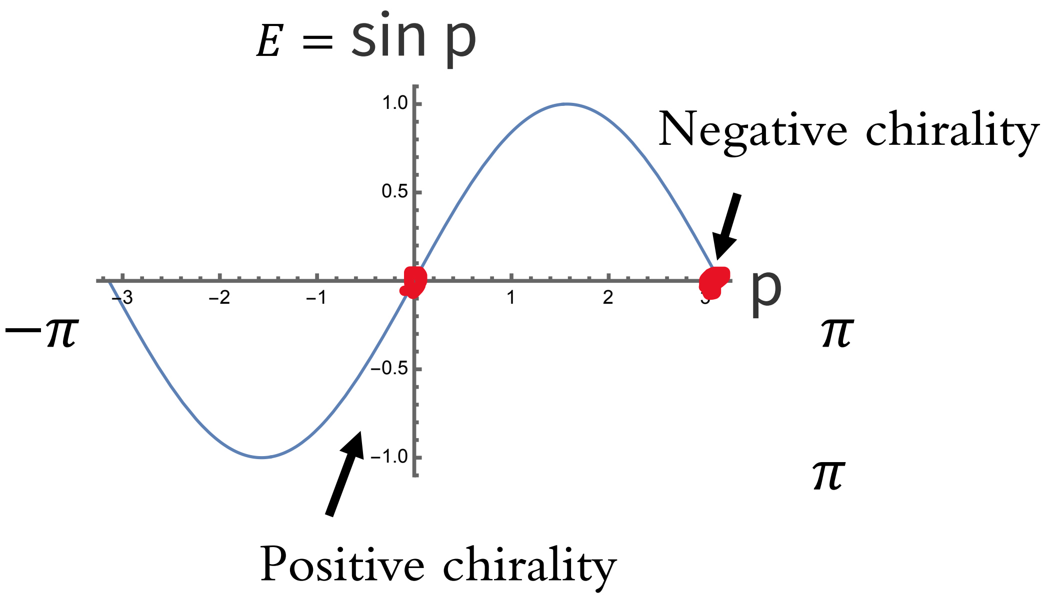
Solving the naively discretized Dirac Hamiltonian with eigenvalues $\pm\sin p$

$$E = \pm \sin p$$

Brilliuoin zones(Weyl)



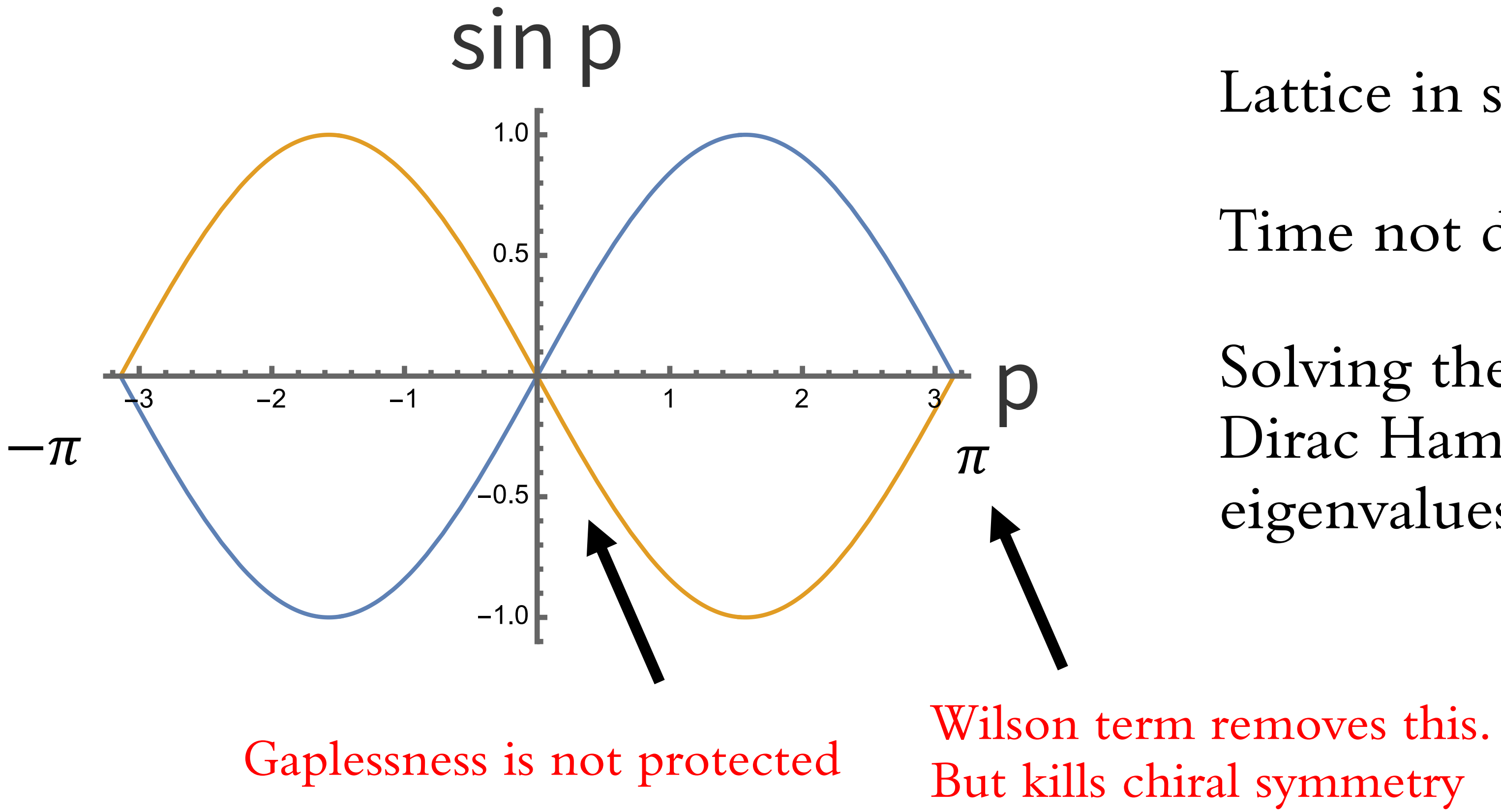
Continuum



Lattice

Even number of zero crossing of periodic functions

Wilson term for Dirac



Lattice in space.

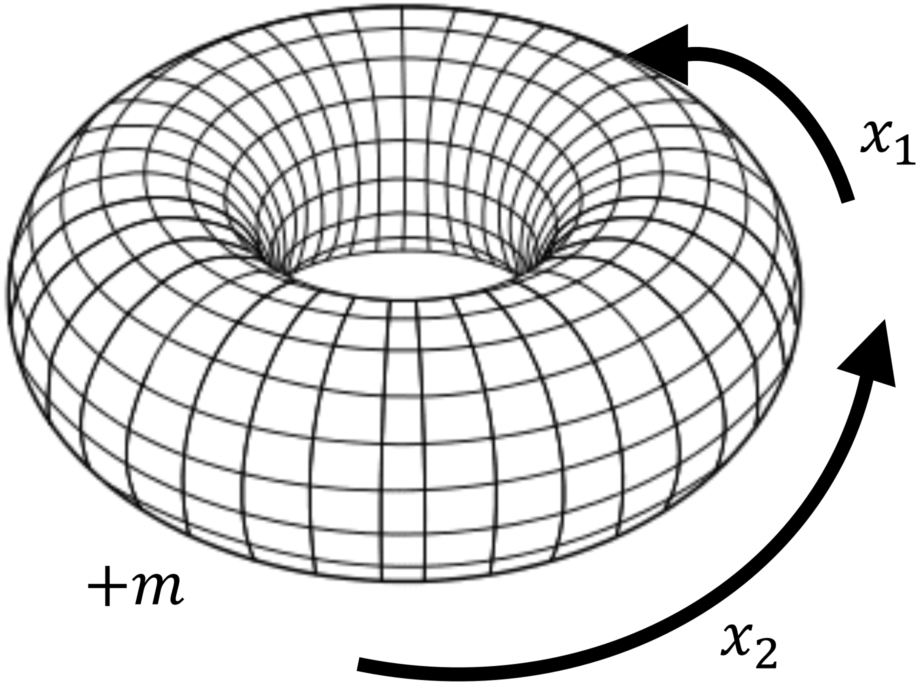
Time not discretized.

Solving the naively discretized Dirac Hamiltonian with eigenvalues $\pm\sin p$

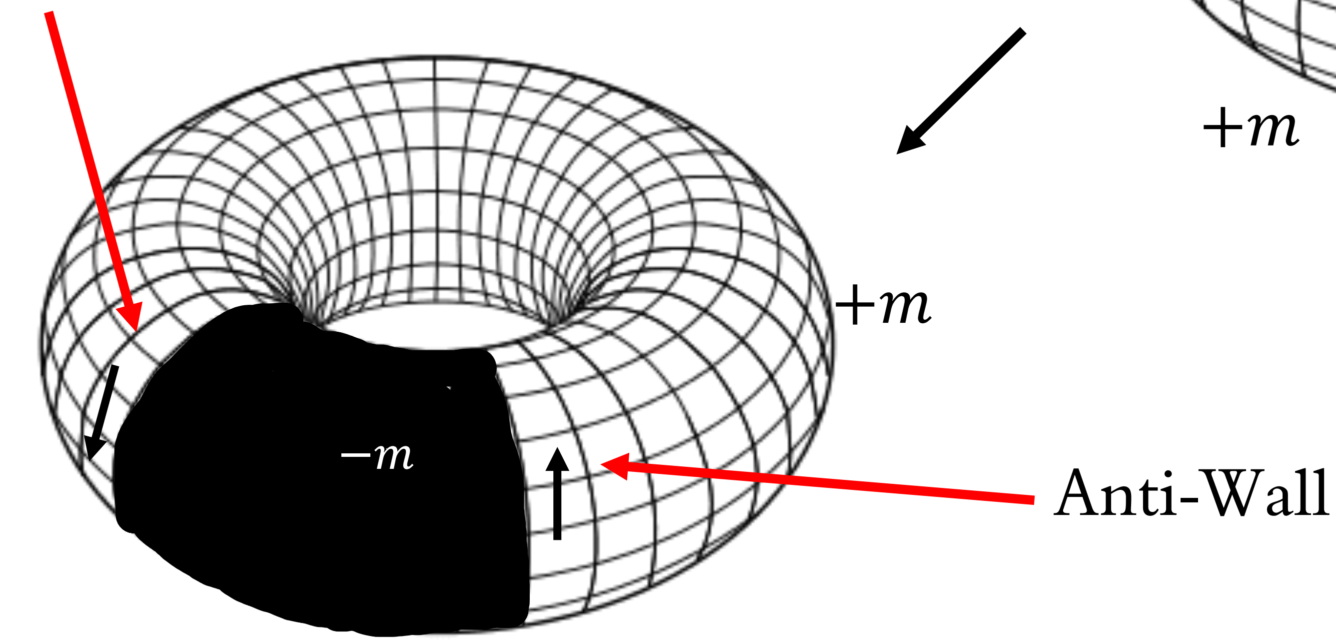
$$E = \pm \sin p$$

Domain wall fermion for global chiral symmetry

Higher dimensional Dirac fermion theory with mass defect



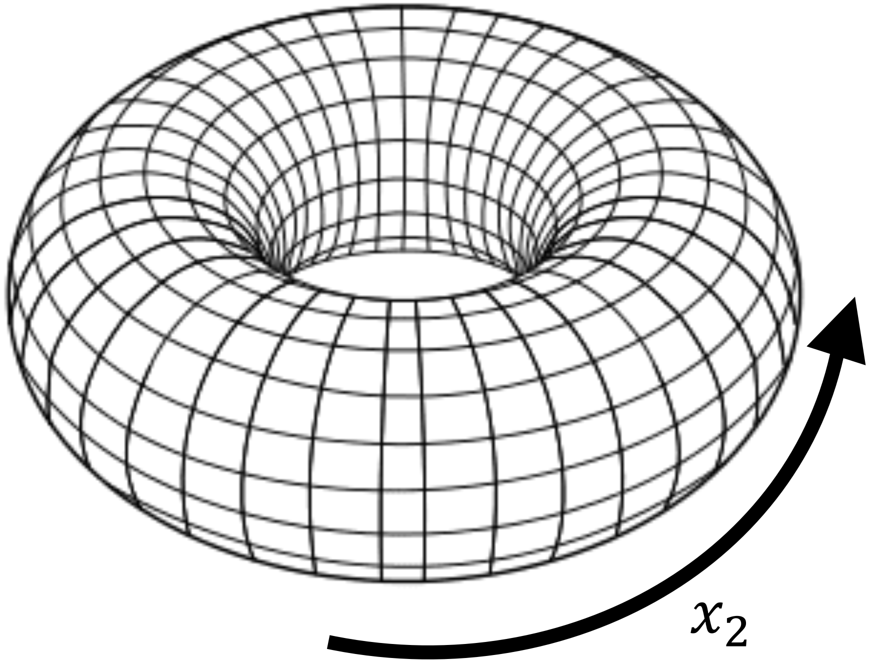
Standard domain wall fermion defect



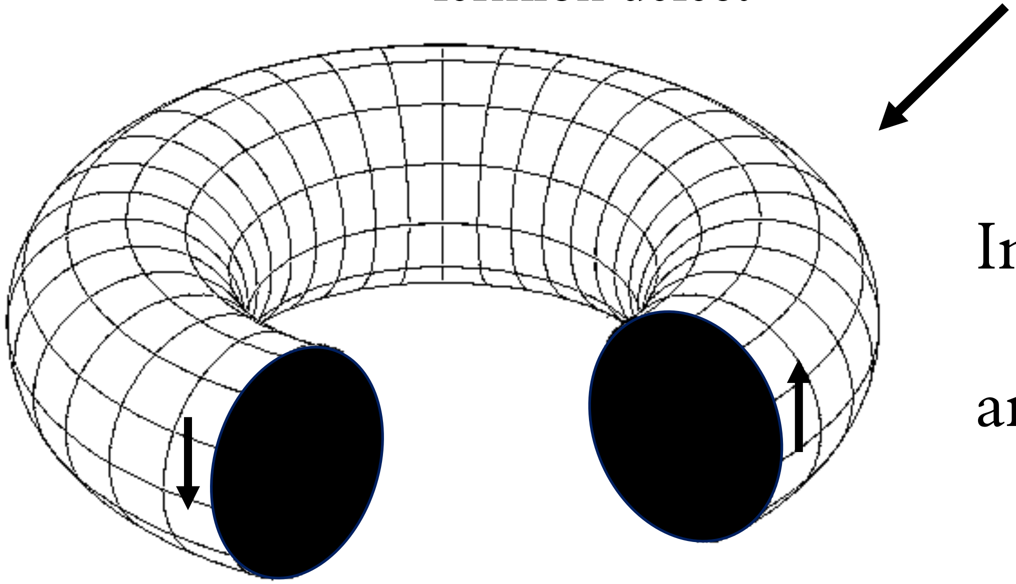
Periodic boundary condition in all directions

Kaplan 1992

Domain wall fermion



Standard domain wall fermion defect



In fact, we can get rid of the negative mass region

and work with open boundary condition in x_2

(OBC)

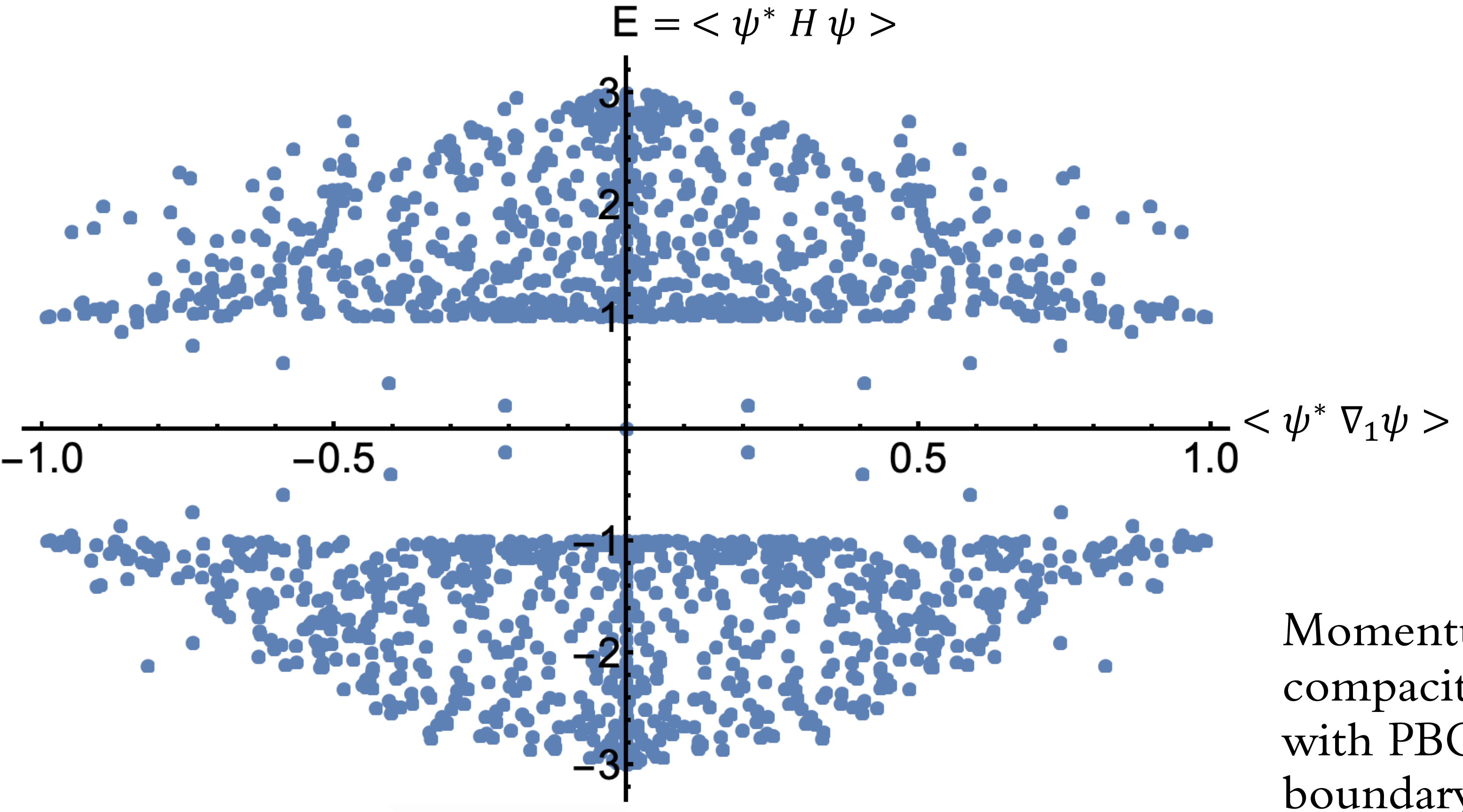
Towards the spectrum: the DW Hamiltonian

It’s the Wilson fermion with no discretization in time.

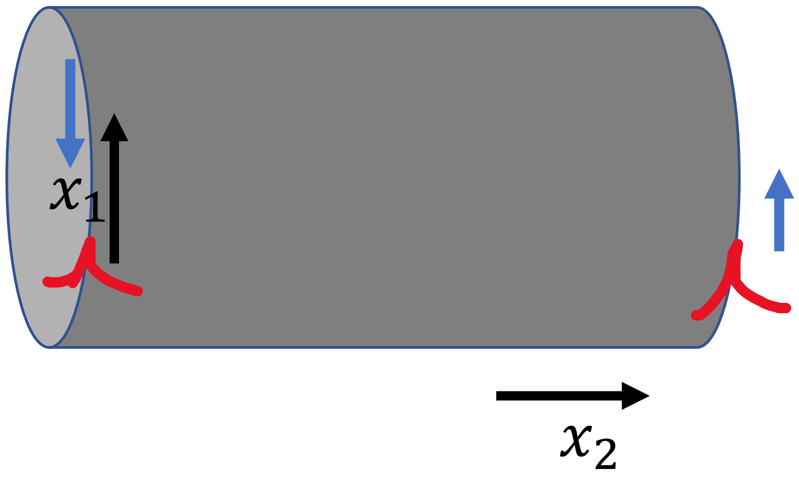
Single particle Hamiltonian: $\mathrm{H} = -i\gamma^i\nabla_i + m + \frac{R}{2}\nabla$

∇_i= Symmetric finite difference in space
∇= symmetric discrete spatial Laplacian

Spectrum



Opposite chiralities



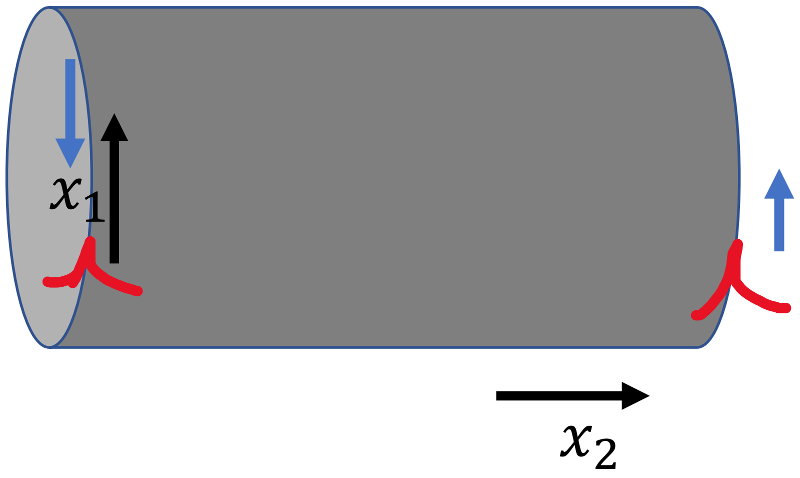
Momentum along the compacitified dimension with PBC (periodic boundary condition)

Solved using domain wall fermions

- Right and left moving modes separated in space. So, any quantum correction to mass exponentially suppressed.

- Allow gauge fields to talk to both walls in the same way producing a vector gauge theory.

- Very useful in QCD simulations.



Other work related to domain wall fermions: Neuberger, Narayanan, Luscher, Shamir, Ginsparg-Wilson

Doesn't work for chiral gauge theories

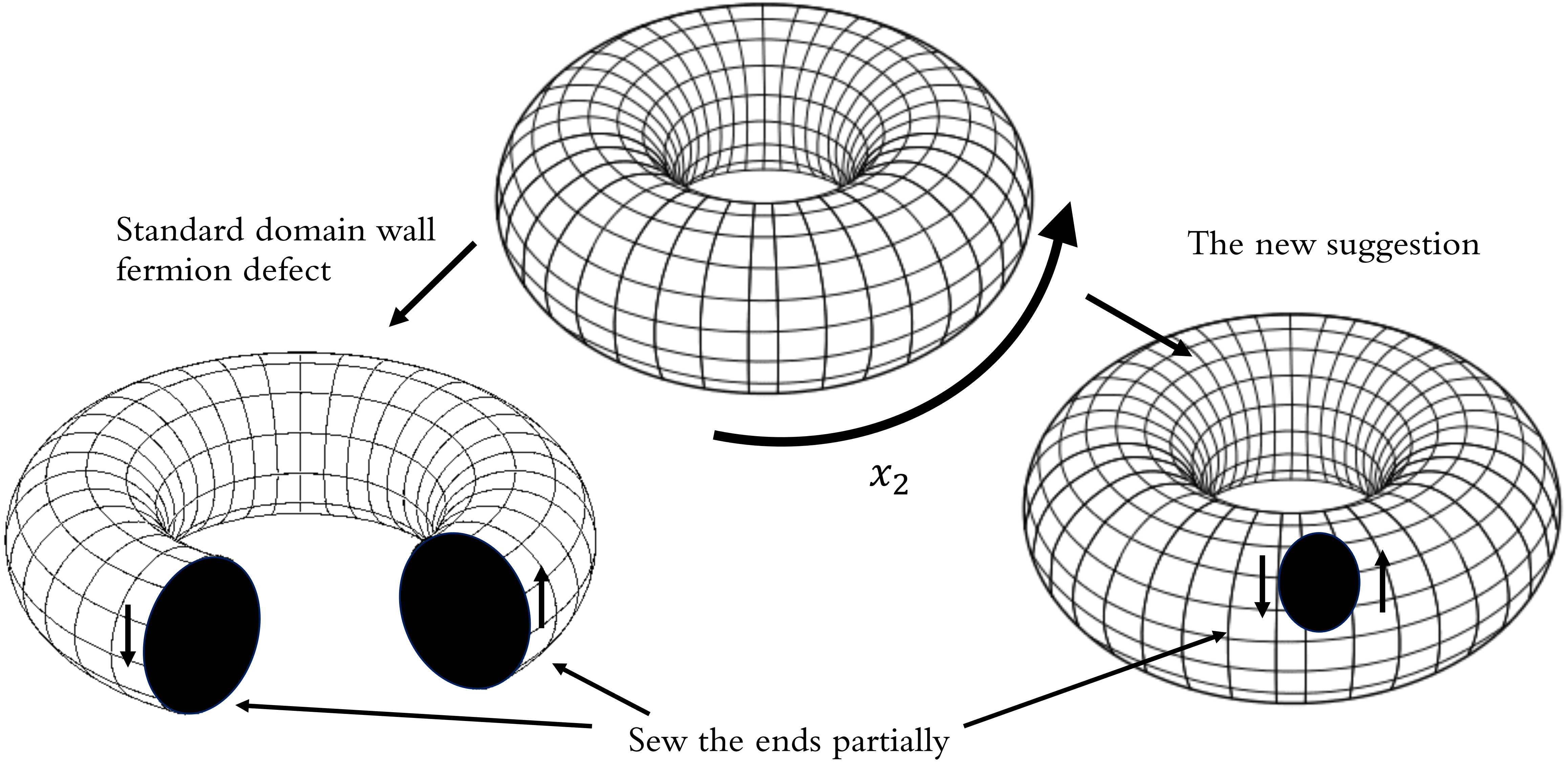
The idea does not work for chiral gauge theories though.

The construction in finite volume necessarily has two defects.

Two defects lead to opposite chiralities producing vector theory.

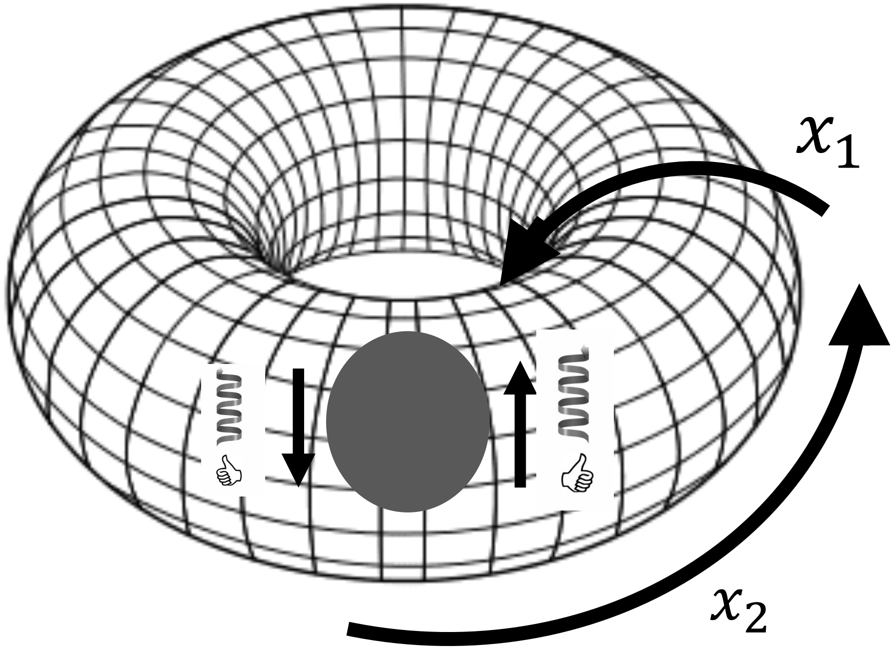
We need to isolate Weyl fermions of a particular chirality --- impossible with the standard domain wall setup.

How about a single disk-like defect?



Opposite chirality on the two sides..

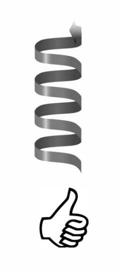
Maybe the problem is that we are keeping the definition of chirality position independent.



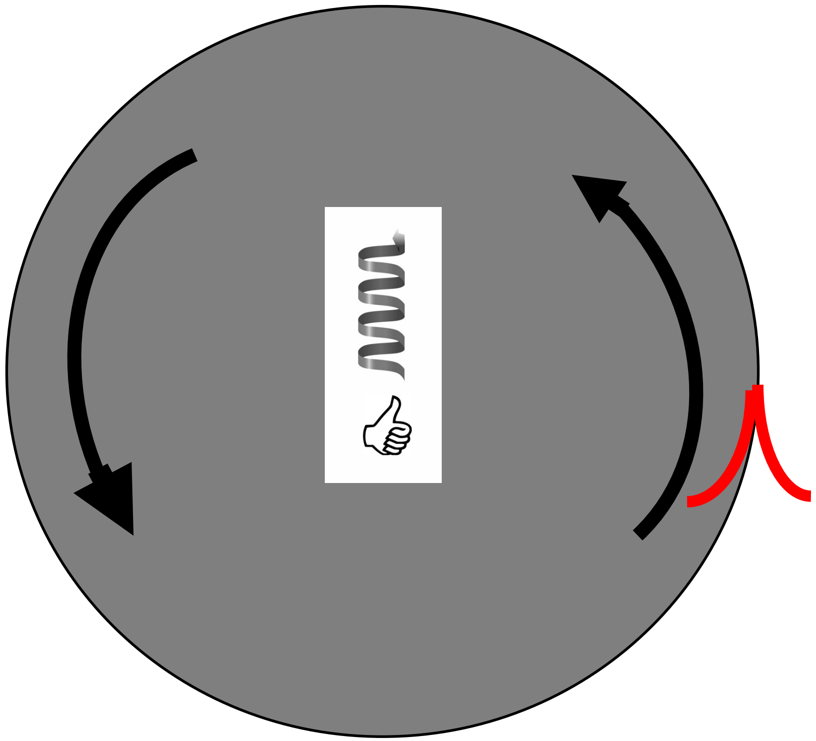
Define chirality in a position dependent manner

Define chirality as clockwise travel vs anticlockwise travel:

counter-clockwise



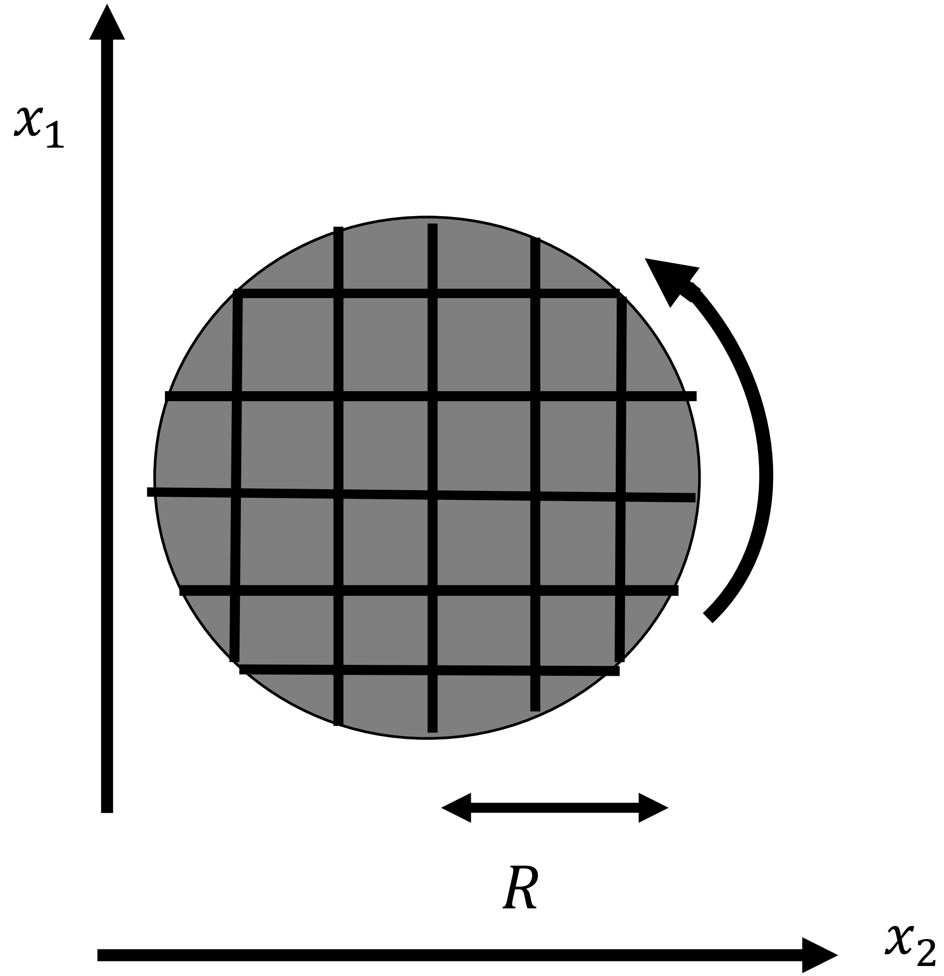
clockwise



Single chirality: Weyl mode

Phys.Rev.Lett. 132 (2024) 14, 141603 (Kaplan)

Disc



Check the dispersion.

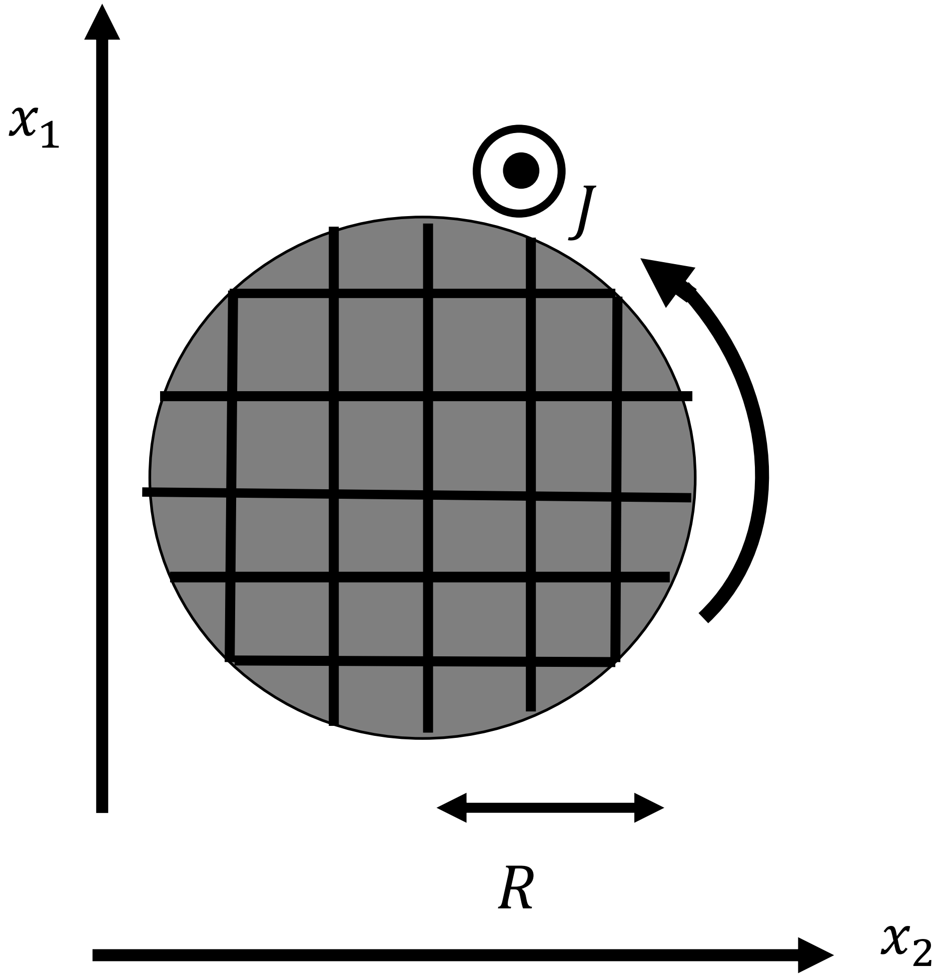
How?

Broken translation invariance along both x_1 and x_2

Does not make sense to plot E vs p_1

Kaplan, Sen, *Phys.Rev.Lett.* 132 (2024) 14, 141604

Disc



We have rotational invariance (approx).

Diagonalize the lattice Hamiltonian.

Compute expectation values of angular momentum J

Plot E vs J

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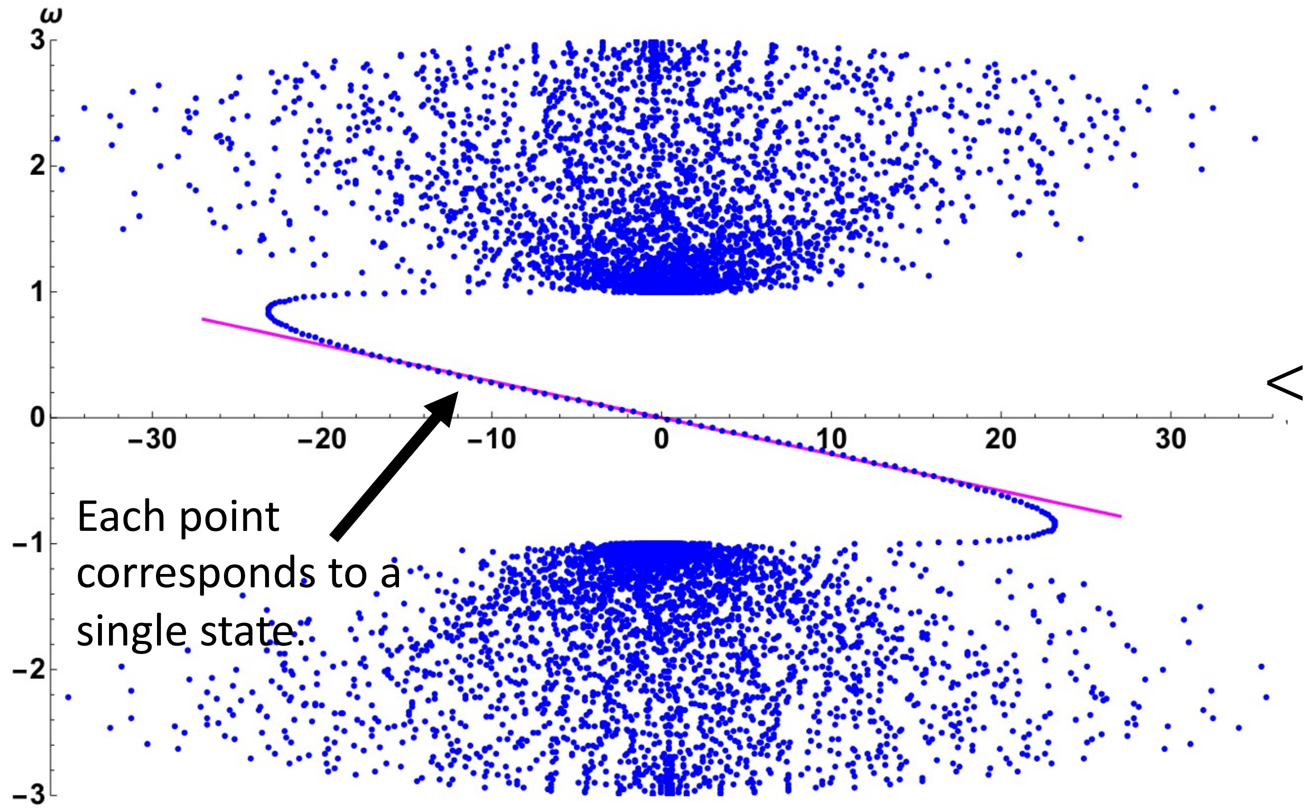
Dispersion for the disk

$E = < \psi^* H \psi >$



$< \psi^* J \psi >$

Exactly as expected from the continuum

Disk of radius $R = 34$ in lattice units.

Linear dispersion:

$$E = -J/R$$

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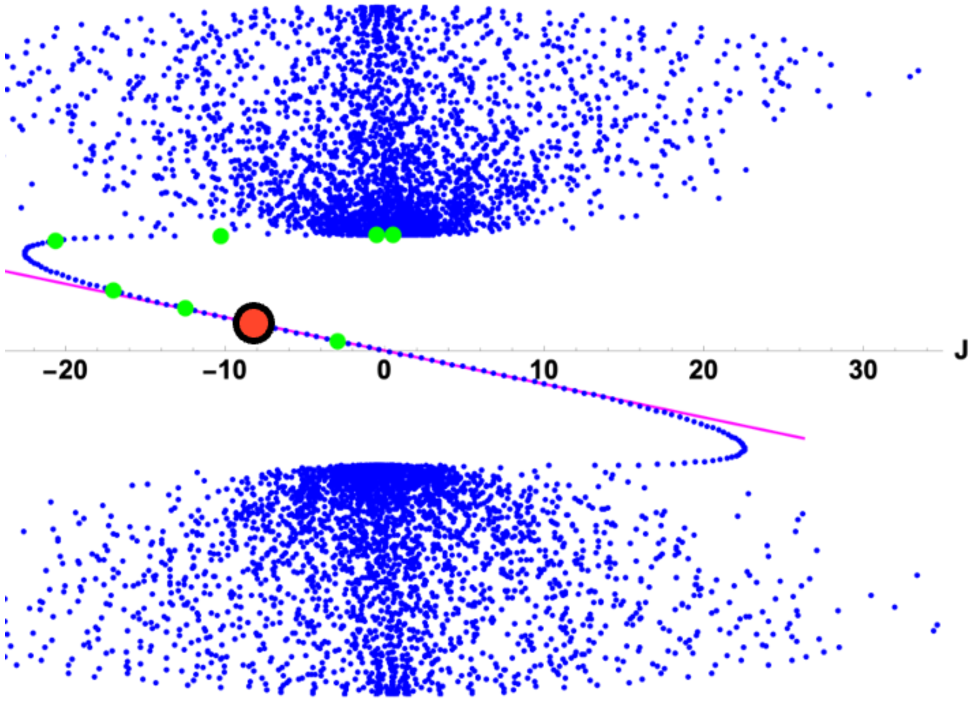
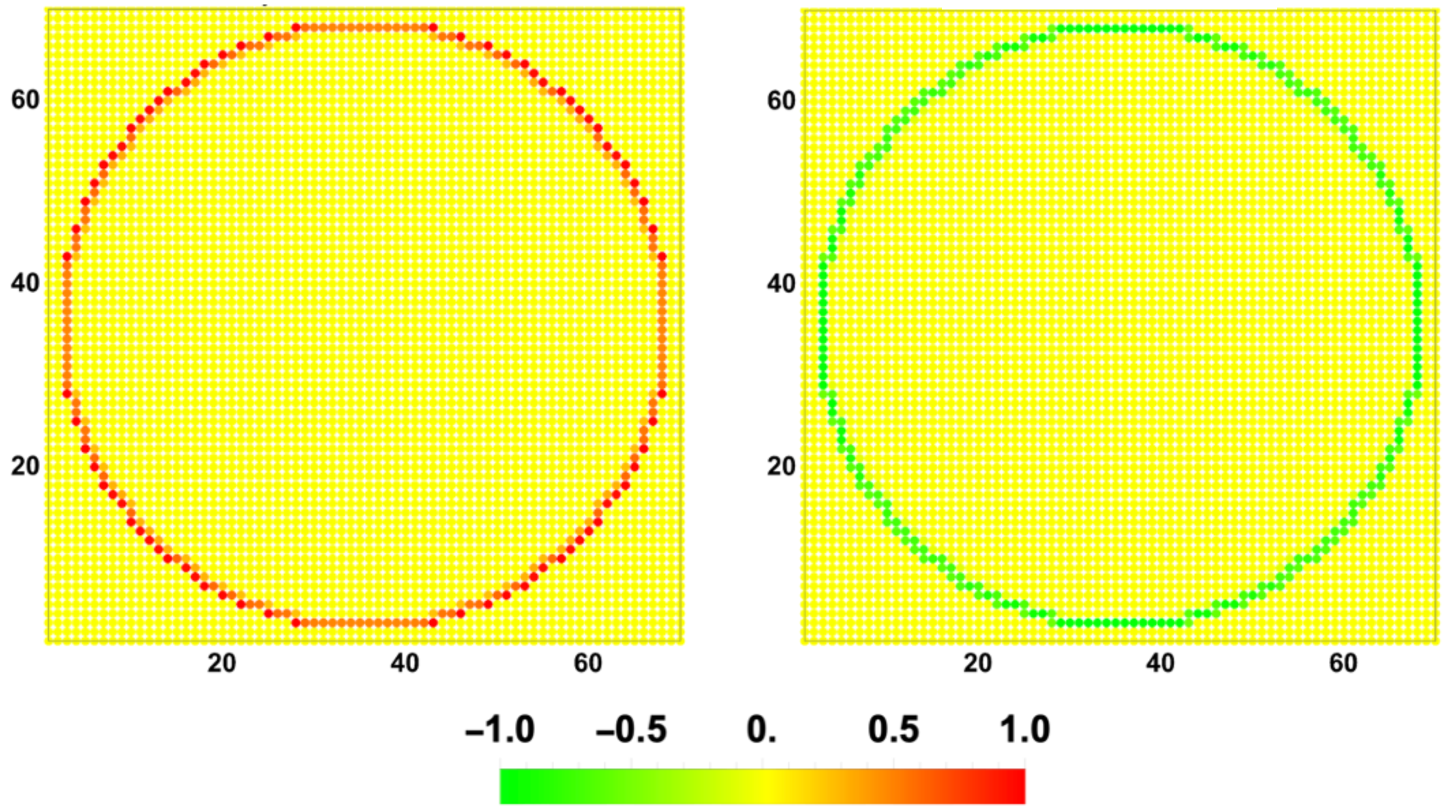


charge density ρ

current density jθ



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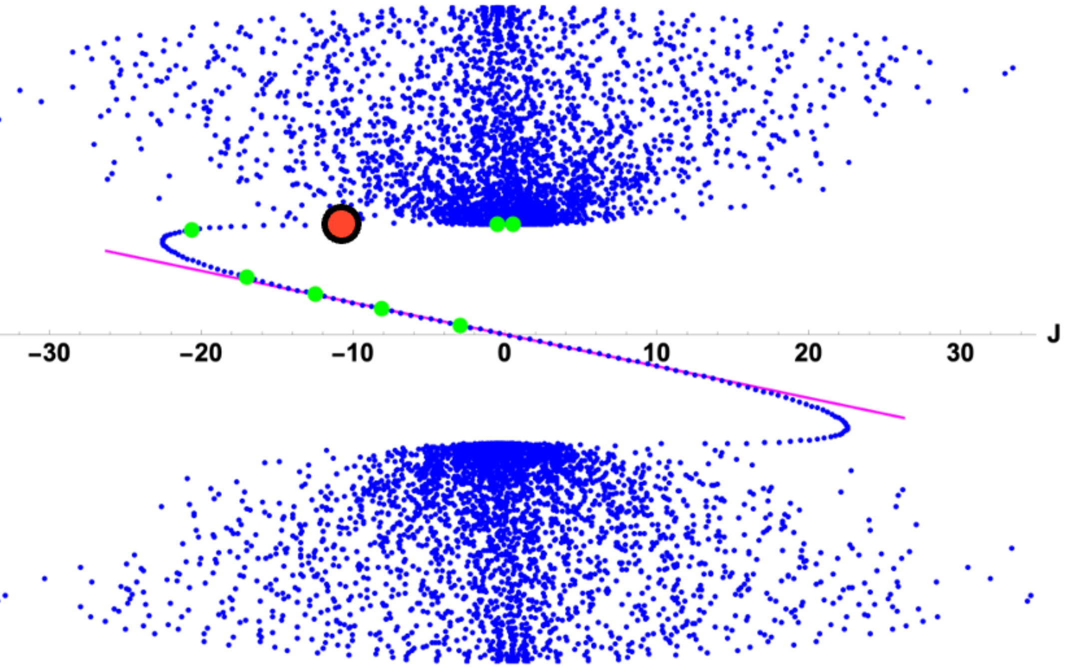

charge density ρ

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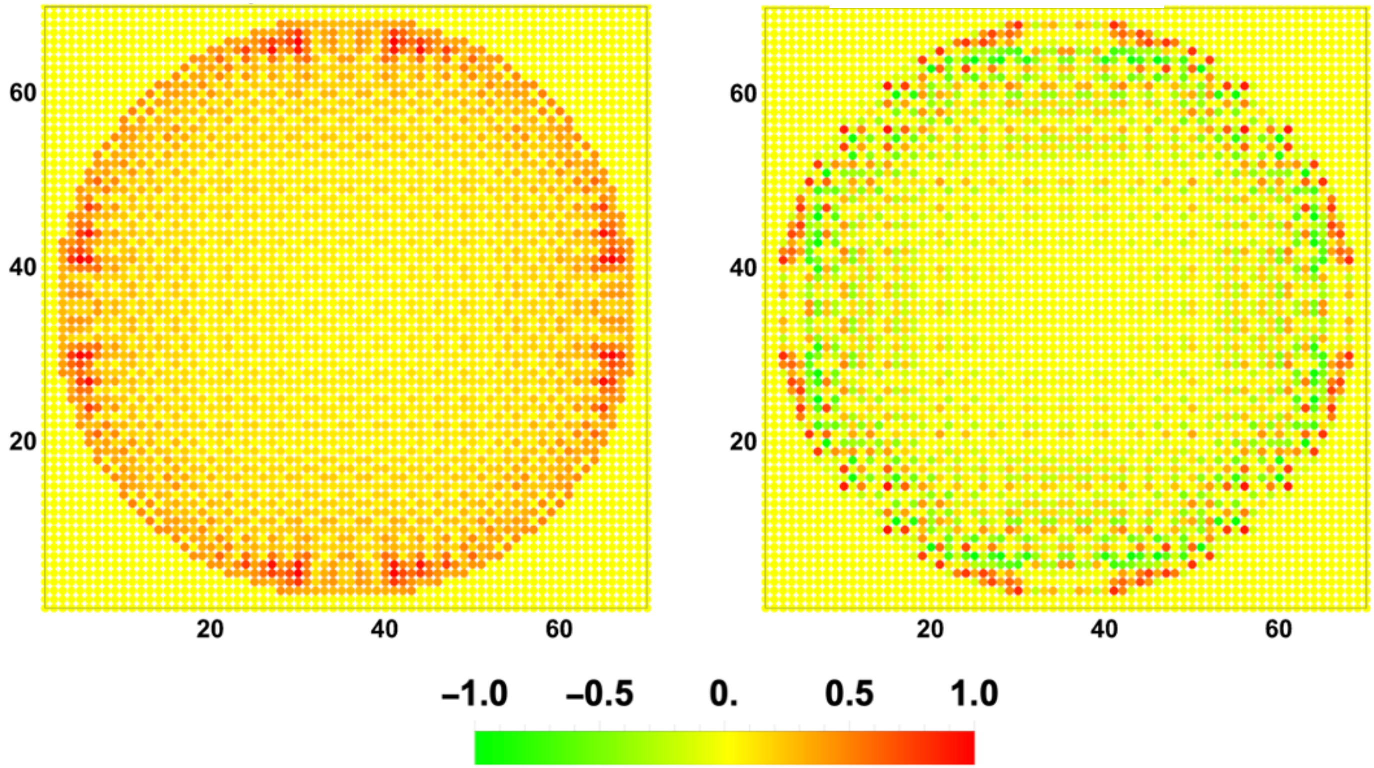

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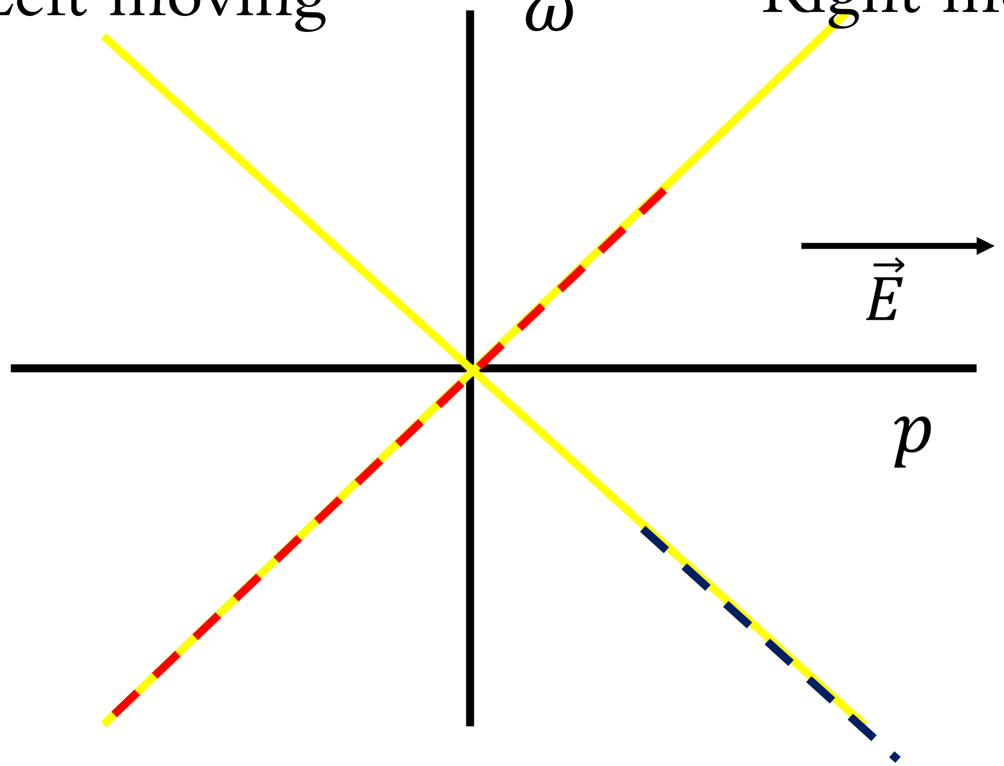
Gauging

Can engineer any number of Weyl fermions on the boundary.

We can gauge any subgroup of the available global symmetry of the free fermion theory. ---- Makes sense only if the theory is anomaly free.

1+1 D massless Dirac fermion spectrum

Left moving Right moving



$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

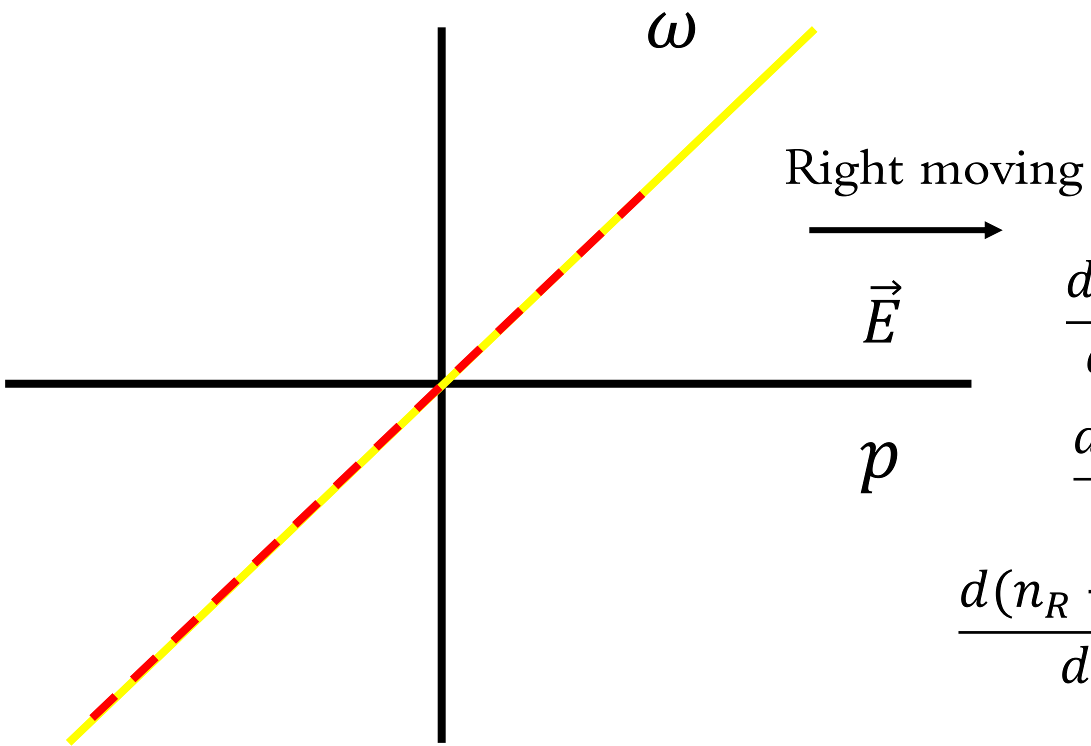
$$\frac{dn_L}{dt} = -\frac{eE}{2\pi}$$

$$\frac{d(n_R - n_L)}{dt} = \frac{eE}{\pi}$$

Current $\propto E$

Vector current or charge $n_R + n_L$ conserved, axial not so.

Edge world: Anomaly, Weyl fermion



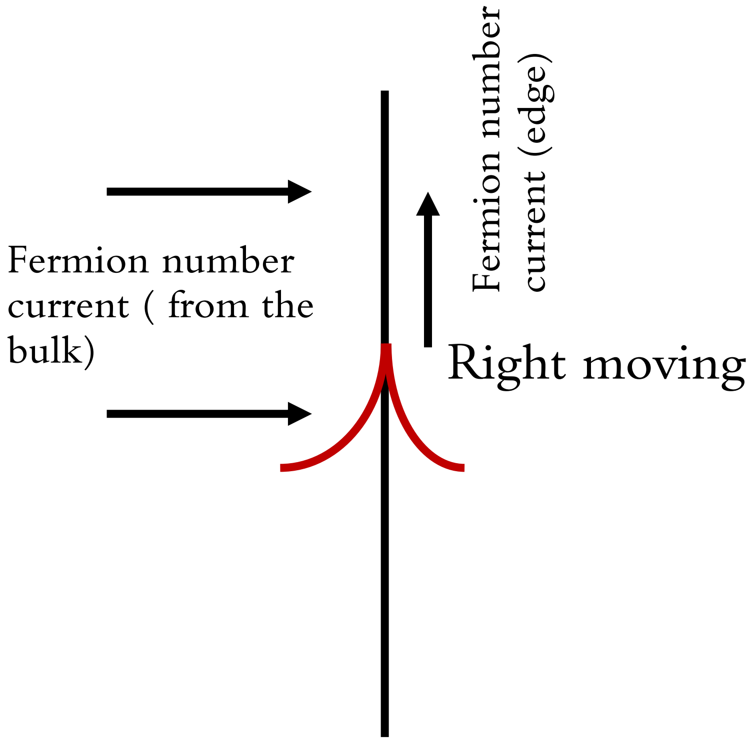
$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

$$\frac{dn_L}{dt} = 0$$

$$\frac{d(n_R + n_L)}{dt} = \frac{eE}{\pi}$$

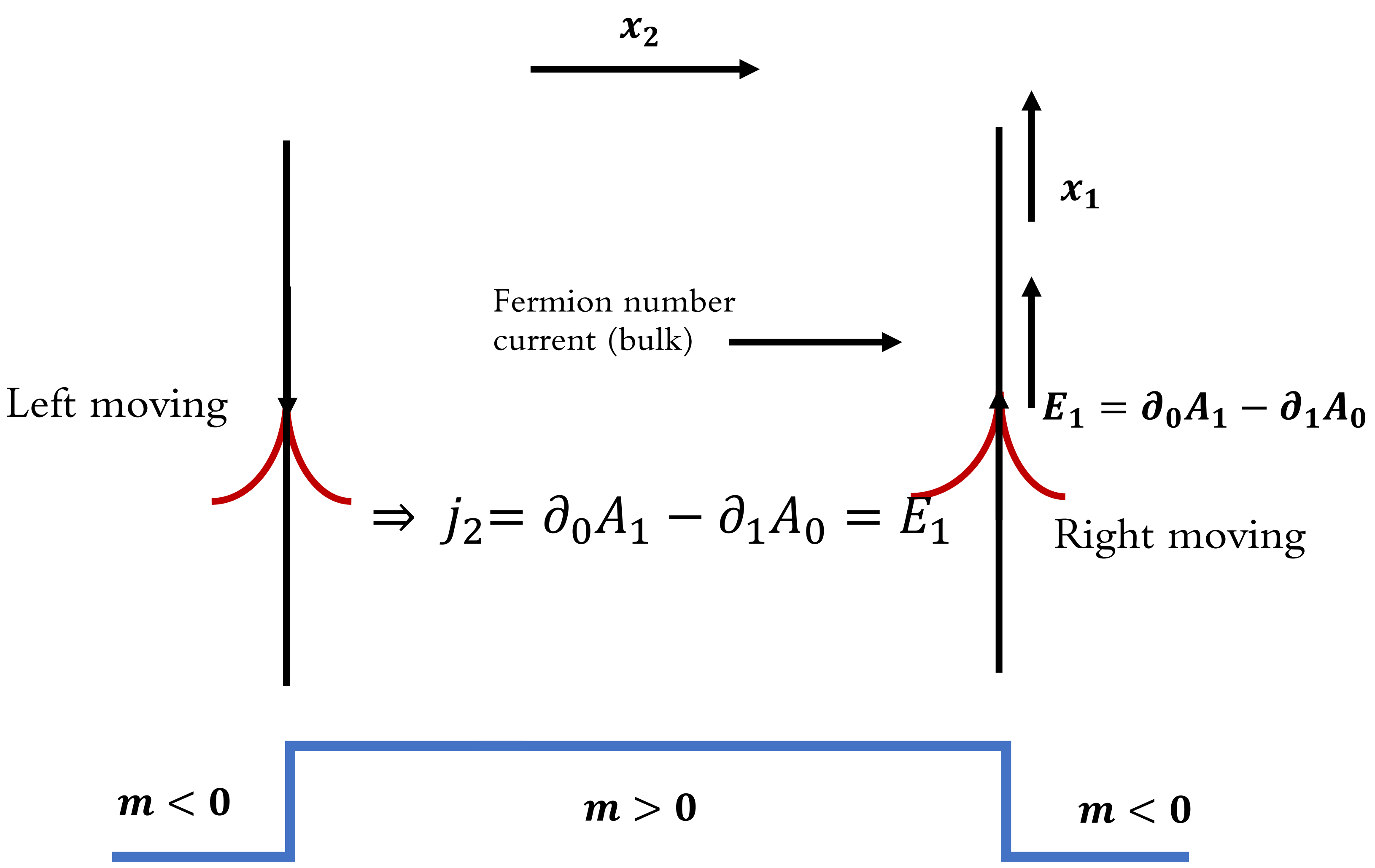
Current $\propto E$

Vector current not conserved, by itself is sick in an electric field.



Can exist on the boundary of a higher dimensional theory

Domain wall + anomaly

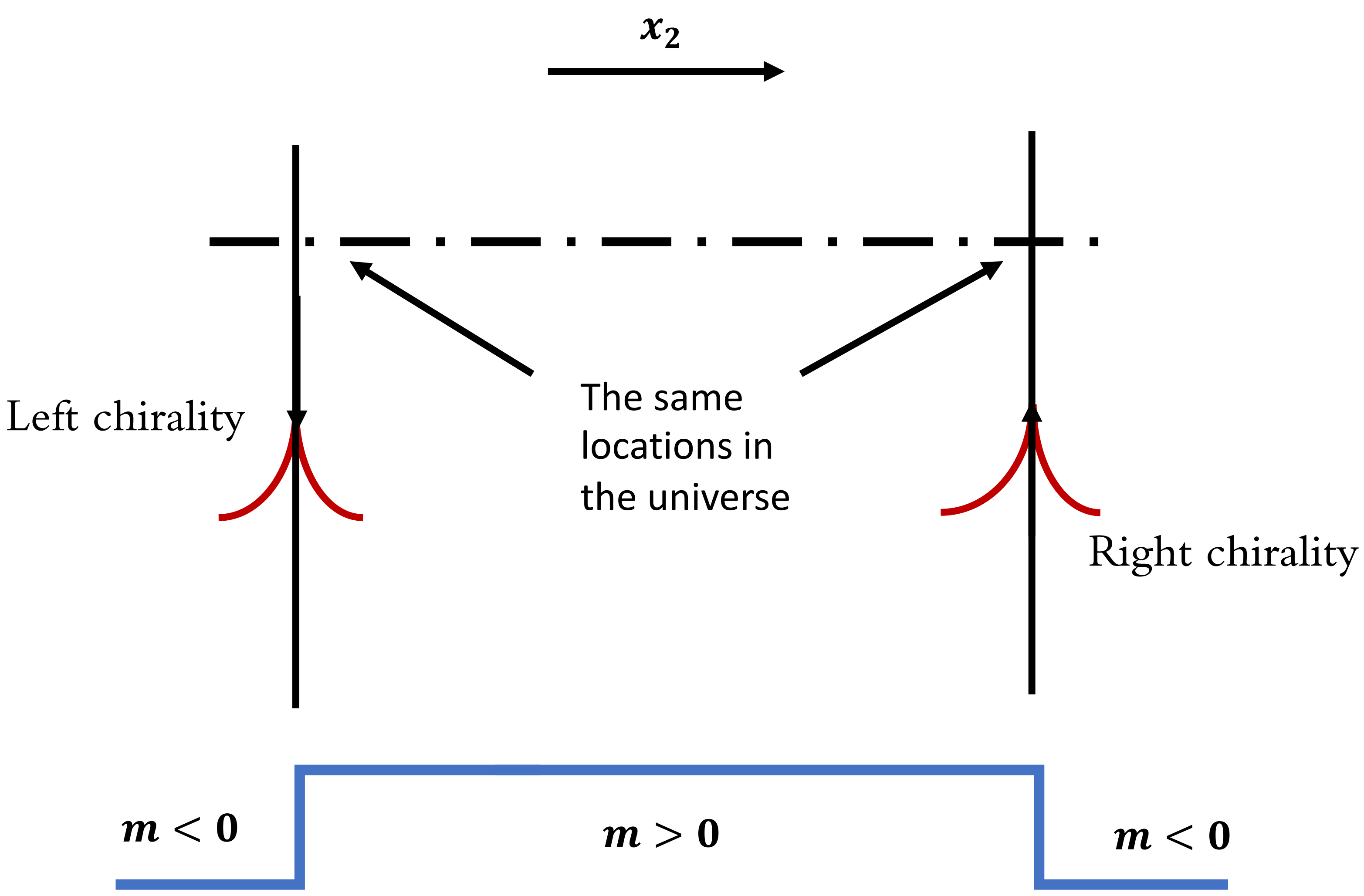


2 + 1 D Dirac fermion setup with domain walls

Gauge the Dirac fermion

Bulk talks to the boundary, great for QCD

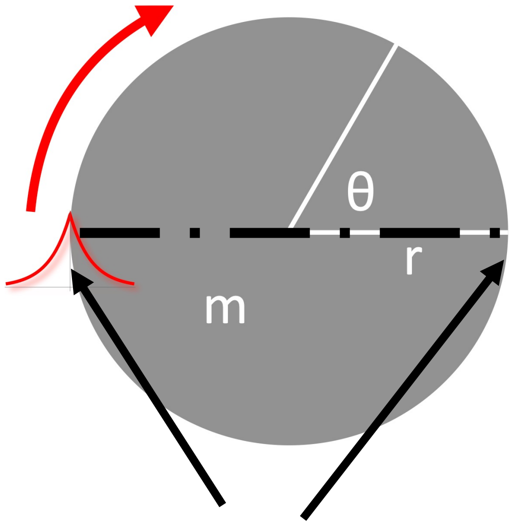
Domain wall + anomaly + QCD



Bulk talks to the boundary, great for QCD

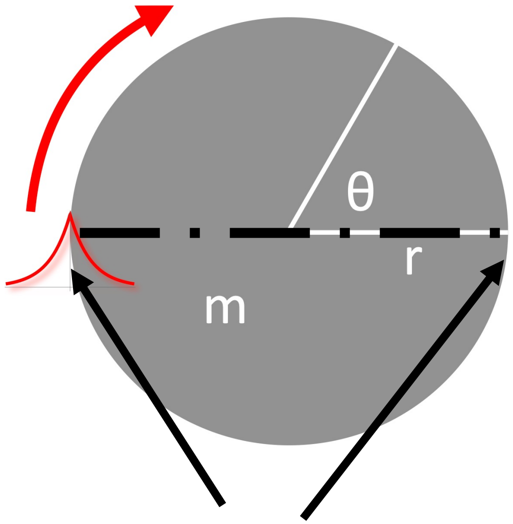
The two walls talk to each other

disk + anomaly



Different locations in the universe, shouldn't communicate across the defect

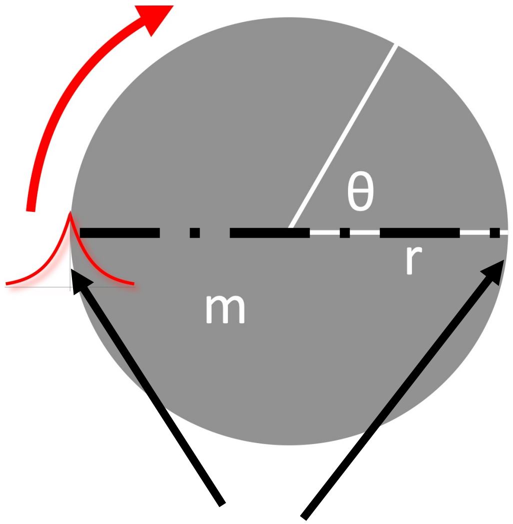
disk + anomaly



Different locations in the universe, shouldn’t communicate across the defect

But they will if the boundary theory has gauge anomaly

disk + anomaly



Different locations in the universe, shouldn't communicate across the defect

But they will if the boundary theory has gauge anomaly

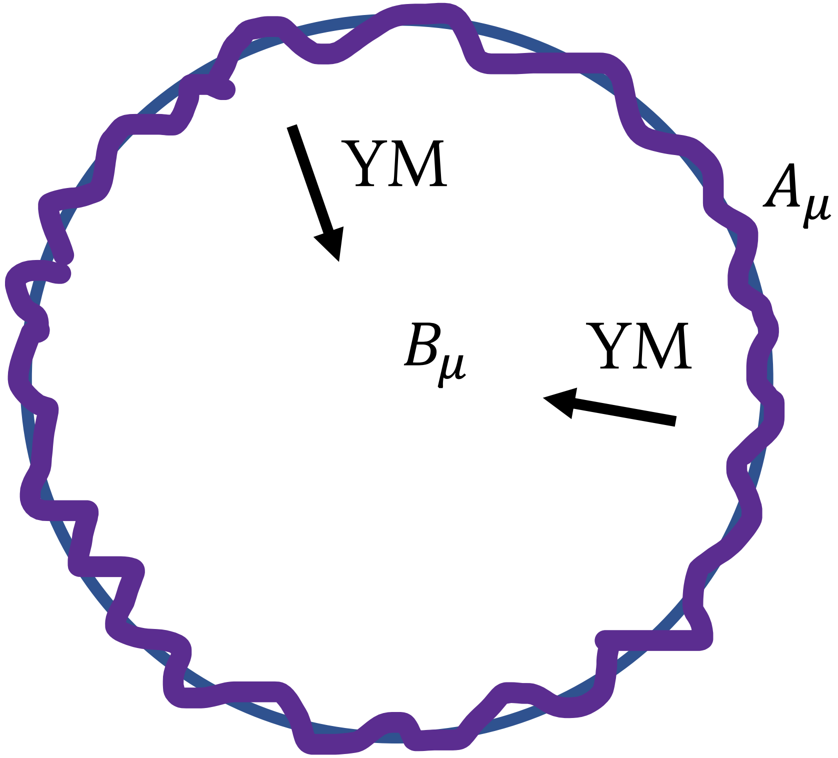
Thankfully the standard model is anomaly free.

So, the disk construction makes sense and the boundary theory is local.

Gauging

Want a $d = 2$, dimensional gauge field A_μ on the edge..

$d = 2$ dimensional gauge field B_μ in $d + 1$=3 dimensional bulk.



Integrate over the boundary gauge field A_μ

Bulk gauge field satisfies equations of motion (e.g. YM) while matching A_μ on the boundary.

Summary

We have a sensible microscopic theory of a Dirac fermion which at low energy produces a single Weyl fermion on the lattice.

Nielsen Ninomiya is not an obstacle. We were fixated on the wrong kind of defect.

Removes one of the most significant obstacles of realizing a chiral gauge theory.

There is more to do though!

Future work

What's the overlap operator for this setup?

How does the latticized version of the overlap operator (lattice boundary theory) realize a Weyl fermion?

Gauge this theory on a small lattice and compute the path integral exactly.

What's the ideal way to simulate this theory? (gauging the full theory or the overlap operator?)