Does the Z boson have a light cousin?

and other adventures in the Higgs phase

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Distinction between Higgs and confinement?

What is the physical distinction (if any) between the Higgs and confinement phases of, e.g., an SU(2) gauge Higgs theory?

We know that both phases have C confinement: all asymptotic particle states are color neutral. Yet there would *appear* to be some qualitative differences.

In the Higgs phase, as we know from ordinary perturbation theory:

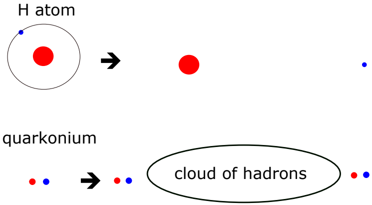
1. There are only Yukawa forces.
2. There are no linear Regge trajectories.
3. There is no flux tube formation, even as metastable states.

Differs from the situation in the confined region of the phase diagram.

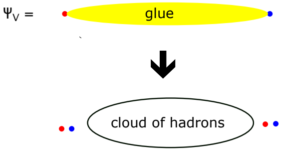
Can we make the distinction precise?

What is the binding energy of the proton?

Or the J/ψ, or any hadron. Obviously we cannot ionize a proton, or quarkonium, at least experimentally. Instead of an isolated quark and antiquark, we get a bunch of color neutral hadrons.



But there *are* physical states in the Hilbert space which *do* correspond to isolated ("ionized") quarks (e.g. of electric charge $\pm\frac{2}{3}e$), separated by a large distance. Even if these states are hard to realize experimentally.



Unscreened charge states

For a $q\overline{q}$ system, they have this form

$$\Psi_V \equiv \overline{q}^a(\mathbf{x})V^{ab}(\mathbf{x},\mathbf{y};A)q^b(\mathbf{y})\Psi_0$$

where $V(\mathbf{x},\mathbf{y};A)$ is a gauge bi-covariant operator which is a functional *of only the gauge field*, transforming as

$$V(\mathbf{x},\mathbf{y};A) \rightarrow g(\mathbf{x})V(\mathbf{x},\mathbf{y};A)g^\dagger(\mathbf{y})$$

In QCD there would be, e.g., a $+\frac{2}{3}$ electric charge at $\mathbf{x}$, and a $-\frac{2}{3}$ electric charge at $\mathbf{y}$. Of course the system would rapidly decay into integer-charged hadrons.

But the question is: *what is the energy $E_V(R)$* of such states of isolated quarks, as separation $R \rightarrow \infty$?

Separation-of-charge ("S_c") confinement

Let

$$E_V(R) = \langle \Psi_V | H | \Psi_V \rangle - \mathcal{E}_{vac}$$

A gauge theory has the property of separation-of-charge confinement if the following condition is satisfied:

S_c confinement

$$\lim_{R\to\infty} E_V(R) = \infty$$

for ***ANY*** choice of bi-covariant $V(\mathbf{x}, \mathbf{y}; A)$.

Again $V(\mathbf{x}, \mathbf{y}; A)$ depends only on the gauge field, not on any matter fields.

This is a much stronger condition than C confinement. It holds for QCD. What about gauge Higgs theories?

Existence of S_c-confinement

Consider a lattice SU(2) gauge-Higgs theory with fixed modulus Higgs $|\phi| = 1$:

$$S = -\beta \sum_{\text{plaq}} \frac{1}{2}\text{Tr}[UUU^\dagger U^\dagger] - \gamma \sum_{\text{links}} \phi^\dagger U \phi$$

1. Does S_c-confinement exist *anywhere* in the $\beta - \gamma$ phase diagram, apart from pure gauge theory ($\gamma = 0$)?

   Yes. We can show that SU(2) gauge-Higgs theory is S_c-confining at least in the region

   $$\gamma \ll \beta \ll 1 \quad \text{and} \quad \gamma \ll \frac{1}{10}$$

   This is based on strong-coupling expansions and a theorem (Gershgorim) in linear algebra. *Matsuyama & JG, PRD 98 (2018) 074504*.

2. Then does S_c-confinement hold *everywhere* in the $\beta - \gamma$ phase diagram?

   No. We can construct V operators which violate the S_c-confinement criterion when γ is large enough. *Matsuyama & JG, PRD 96 (2017), 094510*

So there must exist a transition between S_c and C confinement.
Does it have anything to do with symmetry?

Spontaneous Breaking

of Gauge Symmetry

Charged states and gauge symmetry

One often hears that physical states are always gauge invariant. This is demonstrably untrue.

Consider a quantized Maxwell field coupled to a static charge (e.g. infinite mass fermion). The ground state of the system in an infinite volume is (Dirac, 1955)

$$
\begin{aligned}
\Psi_{\text{chrg}} &= \overline{\psi}(x)\rho(x;A)\Psi_0 \\
\rho(x;A) &= \exp\left[-i\frac{e}{4\pi}\int d^3z\; A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right] \\
\Psi_0[A] &= \exp\left[-\int d^3x \int d^3y\; \frac{\nabla\times\mathbf{A}(x)\cdot\nabla\times\mathbf{A}(y)}{16\pi^3|x-y|^2}\right]
\end{aligned}
$$

Consider a U(1) gauge transformation

$$g(x) = e^{i\theta(x)} = e^{i\theta_0}e^{i\tilde{\theta}(x)}$$

where θ_0 is the zero mode of $\theta(x)$.

The Global Center Subgroup

Under this transformation

$$\overline{\psi}(x) \to \overline{\psi}(x)e^{-i\theta(x)} \ , \ \rho(x) \to e^{i\tilde{\theta}(x)} \ , \ \Psi_0 \to \Psi_0$$

And therefore

$$\Psi_{\text{chrg}} \to e^{-i\theta_0}\Psi_{\text{chrg}}$$

Although the Gauss Law is satisfied, and Ψ_{chrg} is physical state, it is nevertheless *not invariant* under constant U(1) gauge transformations

$$g(x) = e^{i\theta_0}$$

These transformations form the *Global Center Subgroup (GCS)* of the U(1) gauge group.

Add a dynamical scalar field ϕ, and we construct fully gauge-invariant neutral states

$$\Psi_{\text{neutral}} = \overline{\psi}(x)\phi(x)\Psi_0 \ ,$$

Unless the GCS is spontaneously broken, there is a sharp distinction, in infinite volume, between charged and neutral states

$$\langle \Psi_{\text{neutral}} | \Psi_{\text{chrg}} \rangle = 0$$

Pseudomatter Fields

In SU(N) gauge theories the GCS is $g(x) = z\mathbb{1}$ with $z \in Z_N$.

Definition

A *pseudomatter field* $\xi(x; A)$ is a functional of the gauge field on a time slice which transforms like a field in the fundamental representation of the gauge group, *except* under transformations in the GCS.

Examples:

- $$\rho(x; A) = \exp\left[-i\frac{e}{4\pi}\int d^3z\, A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right]$$
- Gauge transformations to a physical gauge ($F[A] = 0$), e.g. Coulomb or axial gauge, can be decomposed into pseudomatter fields.
- Eigenstates ξ_n of the covariant lattice Laplacian

  $$-D^{ab}_{xy}[U]\xi^b_n(y; U) = \lambda_n \xi^a_n(x; U)$$

  where

  $$D^{ab}_{\mathbf{xy}} = \sum_{k=1}^{3}\left[2\delta^{ab}\delta_{\mathbf{xy}} - U^{ab}_k(\mathbf{x})\delta_{\mathbf{y},\mathbf{x}+\hat{k}} - U^{\dagger ab}_k(\mathbf{x}-\hat{k})\delta_{\mathbf{y},\mathbf{x}-\hat{k}}\right]$$

Gauge Higgs theory as a spin glass I

So in non-abelian theories, for arbitrary pseudomatter fields $\xi(x;\mathbf{U})$, we can construct charged physical states, and neutral states in which charge is screened by matter

$$\begin{aligned}\Psi_{\text{chrg}} &= \overline{\psi}(x)\xi(x;\mathbf{U})\Psi_0 \\ \Psi_{\text{neutral}} &= \overline{\psi}(x)\phi(x)\Psi_0\end{aligned}$$

Charged states in SU(N) gauge theory transform covariantly under the GCS

$$\Psi_{\text{chrg}} \rightarrow z\Psi_{\text{chrg}} \quad , \quad z \in Z_N$$

providing the GCS is not spontaneously broken, and in that case $\langle\Psi_{\text{neutral}}|\Psi_{\text{chrg}}\rangle = 0$ for *all* neutral and all charged states. This distinction is lost if the GCS is spontaneously broken.

The special role of the GCS, and an order parameter for that symmetry, appears when we write the usual gauge Higgs partition function as a sum of "spin glass" partition functions

$$Z = \int D\mathbf{U}\; Z_{SG}[\mathbf{U}]$$

Gauge Higgs theory as a spin glass II

Model	"Spin"	random coupling	global symmetry
Edwards-Anderson	s_i	J_{ij}	Z_2
gauge Higgs	$\phi(\mathbf{x})$	$U_k(\mathbf{x})$	GCS

Let $\mathbf{U}, \boldsymbol{\phi}$ denote U, ϕ on a timeslice, say $t = 0$. Integrate out all other d.o.f.

$$\begin{aligned} e^{-H_{SG}[\mathbf{U},\boldsymbol{\phi}]} &= \int DU_0 (DU D\phi)_{t\neq 0} e^{-S} \\ Z_{SG}[\mathbf{U}] &= \int D\boldsymbol{\phi}\, e^{-H_{SG}[\mathbf{U},\boldsymbol{\phi}]} \\ Z &= \int D\mathbf{U}\, Z_{SG}[\mathbf{U}] \end{aligned} \tag{1}$$

$H_{SG}[\mathbf{U}, \boldsymbol{\phi}]$ is the "spin glass" Hamiltonian, regarding ϕ as the dynamical variable, and $\mathbf{U}$ is fixed. It is clearly invariant under GCS transformations $\boldsymbol{\phi} \rightarrow z\boldsymbol{\phi}$. In the statistical system defined by $Z_{SG}[U]$, this global symmetry can be spontaneously broken.

The order parameter is constructed as in the Edwards-Anderson model.

We then define

$$
\begin{aligned}
Z_{\rm SG}(\mathbf{U}) &= \int D\phi(\mathbf{x})\, e^{-H_{\rm SG}(\phi,\mathbf{U})/kT} \\
\overline{\phi}(\mathbf{x};\mathbf{U}) &= \frac{1}{Z_{\rm SG}(\mathbf{U})} \int D\phi\, \phi(\mathbf{x}) e^{-H_{\rm SG}(\phi,\mathbf{U})/kT} \\
\Phi(\mathbf{U}) &= \frac{1}{V} \sum_{\mathbf{x}} |\overline{\phi}(\mathbf{x};\mathbf{U})| \\
\langle \Phi \rangle &= \int D\mathbf{U}_i(\mathbf{x})\, \Phi(\mathbf{U}) P(\mathbf{U})
\end{aligned}
$$

where

$$
P(\mathbf{U}) = \frac{Z_{\rm SG}(\mathbf{U})}{Z} = \frac{1}{Z} \int DU_0 (DU_k)_{t\neq 0} D\phi\, e^{-S}
$$

is the probability distribution for a give $\mathbf{U}$ configuration on a time slice (obtained after integrating over all other d.o.f).

We now have a gauge invariant criterion for the spontaneous breaking of *GCS symmetry:*

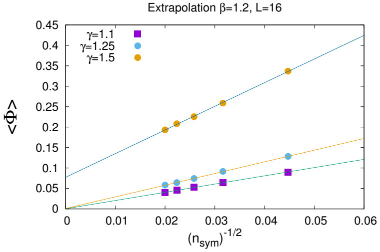
$$\lim_{h\to 0}\lim_{V\to\infty}\langle\Phi\rangle \begin{cases} = 0 & \text{unbroken symmetry} \\ > 0 & \text{broken symmetry} \end{cases}$$

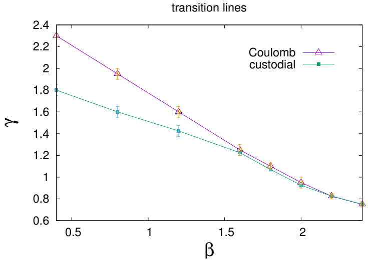
which is entirely analogous to the Edwards-Anderson criterion for the spontaneous symmetry breaking of global Z_2 symmetry in a spin glass:

$$\lim_{h\to 0}\lim_{V\to\infty}\langle q\rangle \begin{cases} = 0 & \text{non-spin glass phase} \\ > 0 & \text{spin glass phase} \end{cases}$$

Goldstone modes in the gauge Higgs case are absent, for reasons known since 1965 (Guralnik et al).

$\langle \Phi \rangle$ can be evaluated numerically by lattice Monte Carlo. Each data-taking sweep is itself a Monte Carlo for n_{sym} sweeps, holding link variables constant at $t = 0$, and evaluating $\overline{\phi}(\mathbf{x}, 0)$. Then one extrapolates to $n_{sym} \to \infty$.





The essential result

Stated here without proof:

The transition to the spin-glass phase of a gauge Higgs theory coincides with the transition from S_c confinement to C confinement.

See ***Matsuyama & JG, PRD 101, 054508 (2020), arXiv: 2001.03068***

Higgs and Confinement phases

$$\begin{aligned} |\text{charged}_{\mathbf{xy}}\rangle &= \overline{q}^a(\mathbf{x})V^{ab}(\mathbf{x},\mathbf{y};U)q^b(\mathbf{y})|\Psi_0\rangle = |\Psi_V\rangle \\ |\text{neutral}_{\mathbf{xy}}\rangle &= (\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x}))(\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y}))|\Psi_0\rangle \end{aligned}$$

Higgs phase

$$\exists\, V \text{such that} \lim_{|x-y|\to\infty} \langle \text{neutral}_{\mathbf{xy}}|\text{charged}_{\mathbf{xy}}\rangle \neq 0$$

No firm distinction, in the Higgs phase, between Z_N charged and neutral states.
$E_V(R)$ of Ψ_V is finite as $R \to \infty$. No S_c confinement.

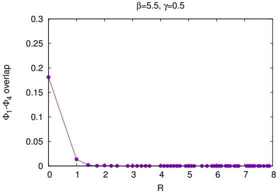
Confinement phase

$$\text{for all } V\ , \lim_{|x-y|\to\infty} \langle \text{neutral}_{\mathbf{xy}}|\text{charged}_{\mathbf{xy}}\rangle = 0$$

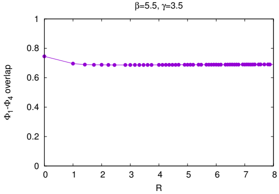
There is a sharp distinction between charged and neutral states.

Charged state of finite energy $\Longrightarrow$ massless phase.
Charged state of infinite energy $\Longrightarrow$ S_c confinement phase.

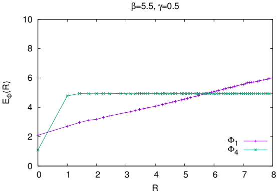
Look at $E(R,1)$ for the Higgs $\Phi_4(R)$ and pseudomatter $\Phi_1(R)$ states, as well as the overlap of these (normalized) states at $\beta = 5.5$ in the confinement phase ($\gamma = 0.5$) and Higgs phase ($\gamma = 3.5$) respectively.



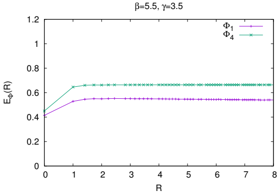
(c) Conf: $\langle \Psi_{neutral} | \Psi_{chrg} \rangle \to 0$



(d) Higgs: $\langle \Psi_{neutral} | \Psi_{chrg} \rangle > 0$



(e) Conf: S_c confinement



(f) Higgs: C confinement

Against the common view...

1. What is meant by the word *confinement*, in a theory (such as QCD) with matter in the fundamental representation of the gauge group?

   It means S_c (separation-of-charge) confinement. Roughly speaking, an infinite "ionization" energy to isolated quark states.

2. What is meant by the phrase *spontaneously broken gauge symmetry*?

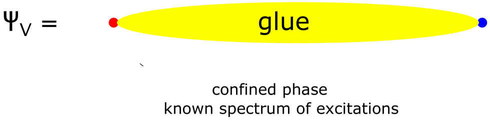
   It means that the global center subgroup of the gauge symmetry group is spontaneously broken. The gauge-invariant order parameter is the same as for a spin glass.

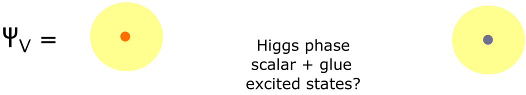
> The S_c to C confinement transition **coincides** with the transition to the (Higgs/spin glass) phase of spontaneously broken gauge symmetry.

New Particles in the Higgs Phase?

Composite systems (molecules, atoms, nuclei, hadrons...) generally have a spectrum of excitations. What about non-composite systems: charged “elementary” particles like quarks and leptons?

If the particle is charged, then by Gauss’s Law it is accompanied by a surrounding gauge (and possibly other) fields. If these surrounding fields interact with themselves, *could they not also exhibit a spectrum of excitations?* This would look like a mass spectrum of the isolated elementary particle.





SU(3) gauge Higgs theory

J.G., PRD 102 (2020) 5, 054504

$$
\begin{aligned}
S &= -\frac{\beta}{3}\sum_{plaq} \mathrm{ReTr}[U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)] \\
&\quad -\gamma\sum_{x,\mu} \mathrm{Re}[\phi^\dagger(x)U_\mu(x)\phi(x+\hat{\mu})]
\end{aligned}
$$

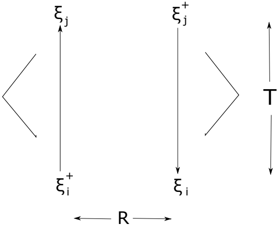
We construct a set of four states, coupling pseudomatter fields, or the Higgs field, to a static quark and antiquark of separation R

$$
\begin{aligned}
\Phi_n(R) &= [\overline{q}^a(\mathbf{x})\xi_n^a(\mathbf{x})] \times [\xi_n^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\,\Psi_0 \quad (n=1,2,3) \\
\Phi_4(R) &= [\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})] \times [\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\,\Psi_0
\end{aligned}
$$

We then diagonalize the transfer matrix in this 4-dimensional subspace, denoting the eigenstates $\Psi_n(R)$. These states then propagate for Euclidean time T, and we compute the energy expectation values $E_n(R,T)$.

We have computed $E_n(R,T)$ for SU(3) gauge theory with a unimodular Higgs field on a $14^3 \times 32$ lattice volume, at $\beta = 5.5$ with $\gamma = 0.5$ and $\gamma = 3.5$, in the confinement and Higgs phases respectively.

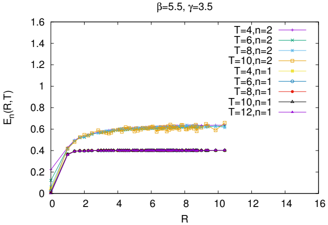
The calculation requires computing matrix elements of the transfer matrix $\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle$, which are expectation values of products of Wilson lines, terminated by matter or pseudomatter fields:



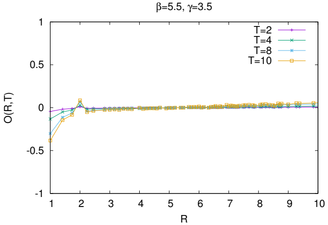
This part is done by lattice Monte Carlo. The rest is algebra.

Higgs phase

Now we show $E_n(R,T)$ and the overlap for $\Psi_1(R)$, $\Psi_2(R)$ and $T = 4 - 12$.



(a) Energies



(b) Overlap

There seems to be clear evidence of a metastable excited state in the spectrum, orthogonal to the ground state.

The energy gap is far smaller than the threshold for vector boson creation.

New gauge bosons?

(in progress)

Lattice formulation of the electroweak sector without quarks and leptons:

$$\begin{aligned} S &= -\beta \sum_{plaq} \left[\frac{1}{2} \text{Tr}[UUU^\dagger U^\dagger] + \frac{1}{\tan^2(\theta_W)} \text{Re}[VVV^\dagger V^\dagger] \right] \\ & -2 \sum_{x,\mu} \text{Re}[\phi^\dagger(x) U_\mu(x) V_\mu(x) \phi(x+\hat{\mu})] - \sum_x \{-(\gamma - 8)\phi^\dagger(x)\phi(x) + \lambda(\phi^\dagger(x)\phi(x))^2\} \end{aligned}$$

with SU(2) gauge field $U_\mu(x)$, U(1) gauge field $V_\mu(x) = e^{i\theta_\mu(x)}$, and Higgs field $\phi(x)$, with θ_W the Weinberg angle. Phenomenology (tree level) gives

$$\sin^2\theta_W = 0.231 \ , \ \beta = 10.1 \ , \ \lambda = 0.13$$

The Z mass, in lattice units, is proportional to $\sqrt{\gamma}$. Get the lattice spacing by fixing to the physical Z mass; then only the lattice spacing, but not physical masses, should depend on γ.

Create a subspace of Hilbert Space

Let $\tilde{U} = UV$ be the SU(2)× U(1) gauge field, and $D^{ab}_{xy}[\tilde{U}]$ the lattice Laplacian.

Denote $\zeta_1(x) = \phi(x)$ the Higgs field, and $\{\zeta_{n+1}(x), n = 1, 2, ..., 10\}$ the first ten Laplacian eigenstates.

We create a subspace of excited states spanned by (non-orthogonal) states $\{|\Phi^n_\mu\rangle, n = 1, 2, ..., 11\}$

$$
\begin{aligned}
\eta(\mathbf{x})e^{i\mathcal{A}^n_\mu(\mathbf{x})} &= \zeta^\dagger_n(x)\tilde{U}_\mu(\mathbf{x},t)\zeta_n(\mathbf{x}+\hat{\mu}) \\
A^n_\mu(\mathbf{x}) &= \sin(\mathcal{A}^i_\mu(\mathbf{x})) \\
Q^n_\mu &= \frac{1}{L^3}\sum_{\mathbf{x}} A^n_\mu(\mathbf{x}) \\
|\Phi^n_\mu\rangle &= Q^n_\mu|\Psi_0\rangle
\end{aligned}
$$

Diagonalize the transfer matrix in the subspace

Compute

$$\begin{aligned} O_{ab} &= \langle \Phi^a_\mu | \Phi^b_\mu \rangle = \langle Q^{a\dagger}_\mu(t) Q^b_\mu(t) \rangle \\ T_{ab} &= \langle \Phi^a_\mu | \tau | \Phi^b_\mu \rangle = \langle Q^{a\dagger}_\mu(t+1) Q^b_\mu(t) \rangle \end{aligned}$$

Solve numerically the Generalized Eigenvalue Problem (GEP)

$$[T]\vec{v}^n = \lambda_n [O]\vec{v}^n$$

There will be $N+1$ vectors $\vec{v}^n$ which satisfy this equation, and then

$$|\Psi^n_\mu\rangle = \sum_a v^n_a |\Phi^a_\mu\rangle$$

with energies

$$M_n = -\log(\lambda_n)$$

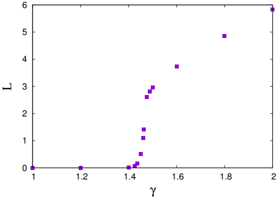
We are free to vary the number $n_{ev} \leq 10$ included in the calculation, to see whether M_n reaches a plateau with increasing n_{ev}.

Transition, time correlator

We can also study numerically

$$G_n(t) = \langle \Psi_\mu^n | \tau^t | \Psi_\mu^n \rangle = \sum v_a^{n*} v_b^n \langle \Phi_\mu^a | \tau^t | \Phi_\mu^b \rangle$$

At fixed $\beta, \lambda, \sin\theta_W$, there is a transition at $\gamma \approx 1.45$

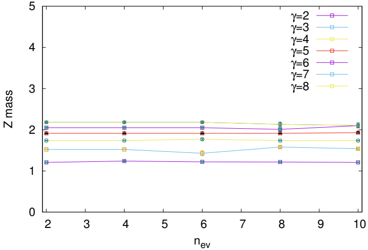


where

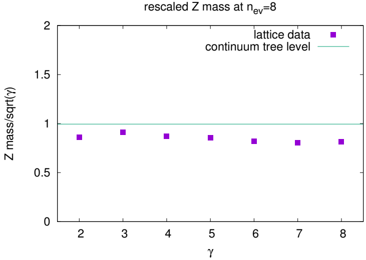
$$L = \langle \phi^\dagger(x) \tilde{U}_\mu(x) \phi(x+\hat{\mu}) \rangle$$

We carry out the numerical calculations at $\gamma = 2, 3, ..., 8$.

At tree level $m_Z = \frac{1}{\cos\theta_W}\sqrt{\frac{\gamma}{\beta}}$.



Figur: Mass of the Z boson in lattice units vs. the number of Laplacian eigenstates used in the GEP calculation, for $\gamma = 2-8$. This an all subsequent calculations were carried out on a $16^3 \times 36$ lattice volume.

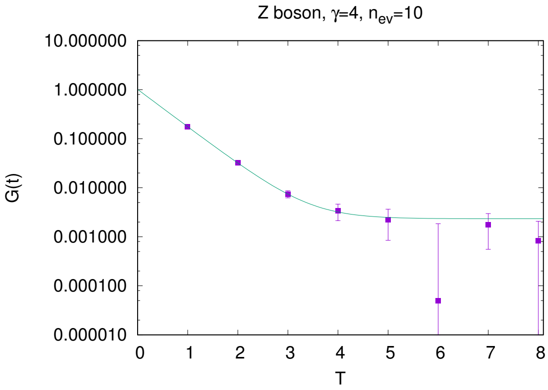


Figur: Evidence of the proportionality of the Z mass, in latttice units, with $\sqrt{\gamma}$. In this case the calculation was carried out for $n_{ev} = 8$. Error bars are smaller than the symbol size. The horizontal line is the perturbative value for the dimensionless ratio $m_Z/\sqrt{\gamma}$.

Z Boson II

As an alternative to solving the generalized eigenvalue problem, we can also fit the corresponding $G(t)$ to

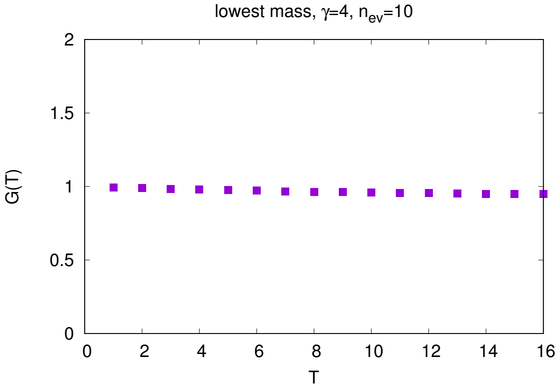
$$G_{fit}(t) = a(e^{-bt} + e^{-b(36-t)}) + c$$



Figur: An example of $G(t)$ for the excitation identified as the Z boson at $\gamma = 4$ and $n_{ev} = 10$. The solid is a best fit to eq. (30) of the first six data points.

The result for $b = m_Z$ is consistent with the GEP.

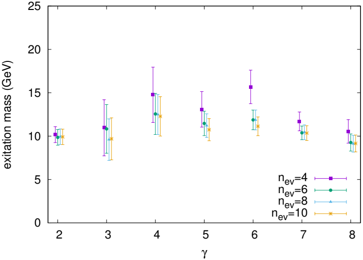
For $n_{ev} \geq 2$, the lowest mass state is massless.



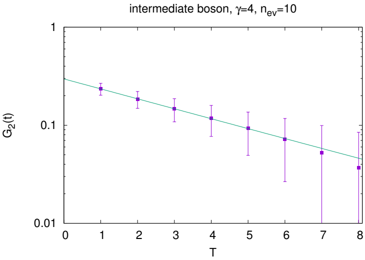
Figur: $G(T)$ for the lowest mass excitation. The nearly T-independent result is consistent with zero mass, hence the identification with the photon.

Something new I

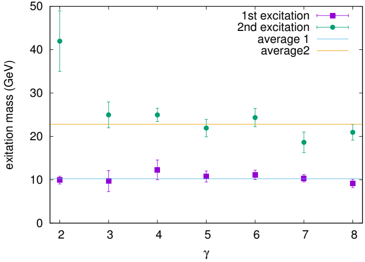
The second excitation:



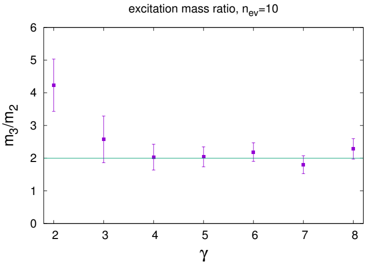
(a) from GEP, various γ



(b) sample $G_2(t)$ correlator



(c) m_2 and m_3



(d) mass ratio

The second excitation looks like a new particle state, at rest, of mass $m_2 \approx 10.2$ GeV. The third excitation is consistent with two particles, at rest, each of mass m_2.

Where is it?

There are a lot of peaks in e^+e^- scattering in the 10-11 GeV range. This is the region of the Υ particles. Could the new "light Z" be mixing with one of them? Some of them are not consistent with pure $b\bar{b}$. Possible room for mixing with a light Z?

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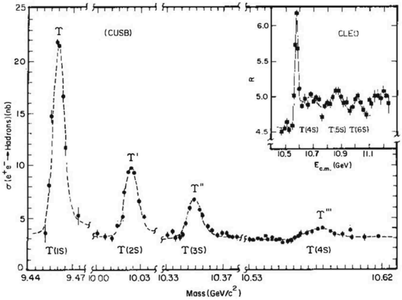

Fig. 5.2 Four Υ peaks at both CLEO and CUSB [25]. The three first peaks may be explained in terms of $b\bar{b} \to ggg$, $b\bar{b} \to \gamma gg$ or $b\bar{b} \to \gamma* \to q\bar{q}/l^+l^-$. In the fourth peak, one should also consider $b\bar{b} \to B\bar{B}$ where $B = b\bar{q}$ ($\bar{B} = \bar{b}q$)

Thanks for your attention!

Any questions?