Fractional instantons, anomalies, and the gaugino condensate

Erich Poppitz, U. of Toronto

with Mohamed Anber, Durham U.

2210.13568, 2307.07495, and 240*.xxxxx

the big picture:

problems of confinement and dynamical mass generation are difficult, strong coupling

(incl. recent) interest in various toroidal compactifications of 4d theories, which allow for

calculable semiclassical studies of confinement on $\mathbb{R} \times \mathbb{T}^3, \mathbb{R}^2 \times \mathbb{T}^2, \mathbb{R}^3 \times \mathbb{S}^1 \ldots$

García Perez, González-Arroyo…1990+; Ünsal…2007+; Tanizaki-Ünsal…2020+

not the real world… but argue for /shown/ continuous connection to $\mathbb{R}^4$

… this talk:

particular focus on $\mathbb{T}^4$**, fractional instantons, and the gaugino condensate**

I realize this is not a SUSY conference, so apologies!

∃ *raison d'être* **to speak here: many things I use are relevant for non-SUSY theories**

e.g. fractional instantons ≃ center vortices/monopole-instantons

“Quanta” Spring ’23 —>

MATHEMATICAL PHYSICS

A New Kind of Symmetry Shakes Up Physics

23

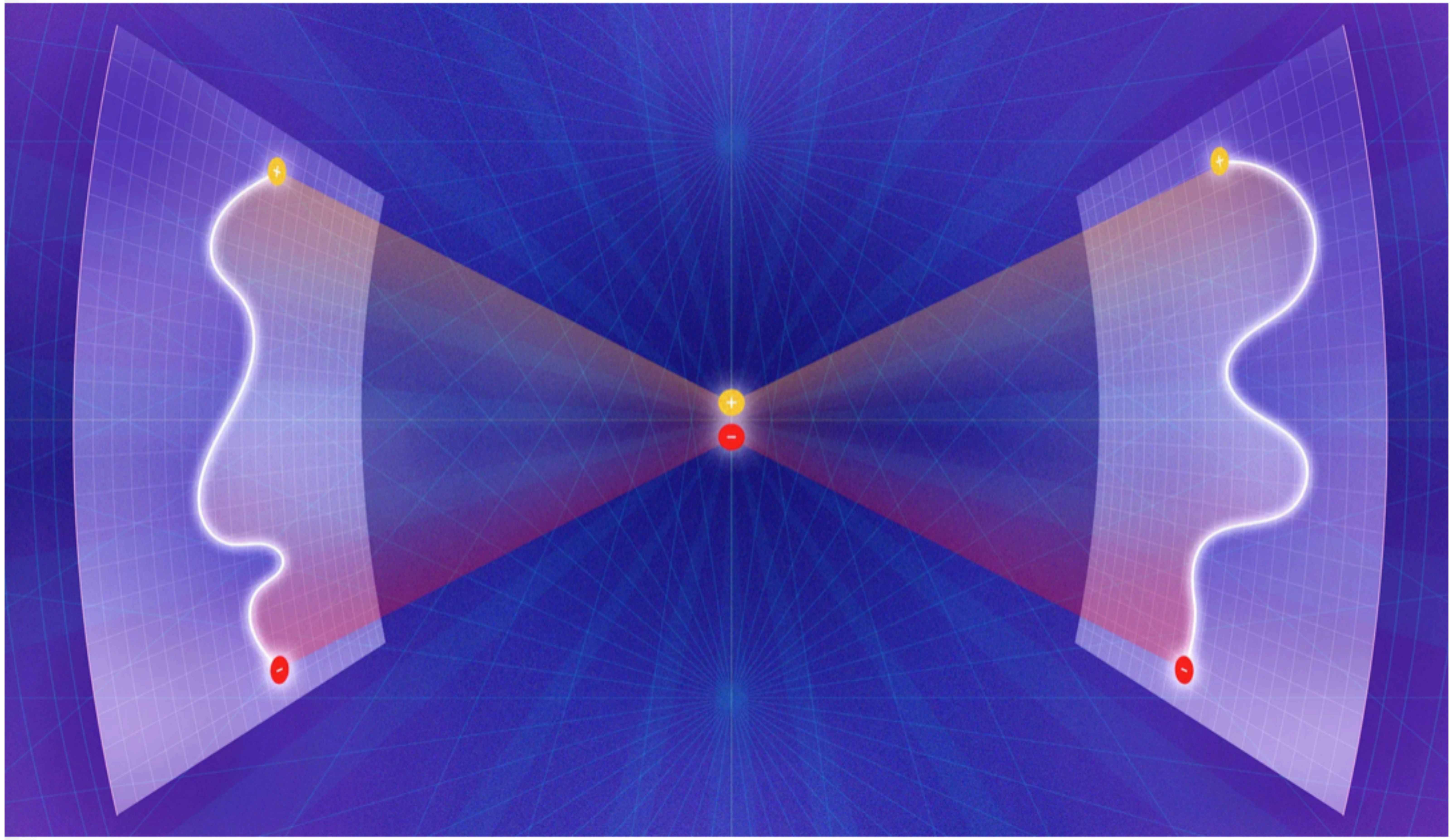
So-called “higher symmetries” are illuminating everything from particle decays to the behavior of complex quantum systems.

renewed interest in $\mathbb{T}^4$ due to generalized anomalies

missed in the 1980s

to see need spacetime with noncontractible 2-cycles

Gaiotto,Kapustin,Komargodski,Seiberg 2014-



The symmetries of 20th-century physics were built on points. Higher symmetries are based on one-dimensional lines.

Samuel Velasco/Quanta Magazine

disclaimer: no way to review generalized anomalies, or many details, will only give flavour

so, get to the point:

SYM in 4d: SU(N) + 1 massless adjoint Weyl fermion λ_α^a (SUSY emergent when $m_\lambda = 0$)

chiral U(1) broken to $\mathbb{Z}_{2N}$ by anomaly

$\mathbb{Z}_{2N}$ spontaneously broken to $\mathbb{Z}_2$ by bilinear gaugino condensate ($\lambda^2(x) \equiv \text{tr}\, \lambda^\alpha(x)\lambda_\alpha(x)$)

$$\langle \lambda^2 \rangle = e^{i\frac{2\pi k}{N}}\, c\, \Lambda^3,\ k = 1,...,N,\quad c = 16\pi^2$$

the "mother" of all exact results in SUSY

1983-1999: Novikov, Shifman, Vainshtein, Zakharov; Amati, Konishi, Rossi, Veneziano; Affleck, Dine, Seiberg; Cordes; Finnell, Pouliot (SQCD —> SYM on R^4); Davies, Hollowood, Khoze, Mattis;… 2014 Anber, Teeple, EP (SYM on $R^3 \times S^1$ —> SYM on R^4)

semiclassical weakly-coupled instanton calculations + power of SUSY

recent independent large-N lattice determination!

2406.08955
Bonnano, García Perez, González-Arroyo, Okawa et al

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here, I will discuss the calculation of the condensate on $\mathbb{T}^4$

why, if all agree so well?

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why, if all agree so well?

1. because we can: new developments allow us to do the $\mathbb{T}^4$calculation

- first attempt in 1984, Cohen and Gomez, *could not and did not* compute “c”

2. the semiclassical objects (instantons on twisted torus) are closely related to both center vortices and monopoles, argued to be responsible for confinement/mass gap/chiral symmetry breaking -

García Perez-González-Arroyo et al, more recent: Wandler-EP ’22; Hayashi-Tanizaki; Güvendik-Schäfer-Ünsal; Wandler ‘24

as opposed to BPST/ADHM instantons used in $\mathbb{R}^4$ calculation

$\mathbb{R} \times \mathbb{T}^3, \mathbb{R}^2 \times \mathbb{T}^2, \mathbb{R}^3 \times \mathbb{S}^1 \ldots$: *all argue for continuity to* $\mathbb{R}^4$

SYM the only theory where exact agreement should hold -and one should get that one case straight

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3. we'll see that calculation raises interesting questions about semiclassics, boiling down to the basic definition of path integrals … not quite understood!

in fact, on $\mathbb{T}^4$ we'll be able to do more than

$$\langle \lambda^2 \rangle = c\,\Lambda^3$$ (taking one particular vacuum)

SUSY Ward identities: $\langle \lambda^2(x_1)\lambda^2(x_2)\ldots\lambda^2(x_k)\ \rangle \equiv \langle \lambda^{2k} \rangle = (c\,\Lambda^3)^k$

=> x-independence / + clustering /

verified in weak-coupling calculation of $\langle \lambda^{2k} \rangle$ in SQCD on $\mathbb{R}^4$ using ADHM

Dorey, Hollowood, Khoze, Mattis 2002

we will also calculate $\langle \lambda^{2k} \rangle$ on small $\mathbb{T}^4$, gcd(N,k)=1; agrees with $\mathbb{R}^4$

$$\langle (\mathrm{tr}\,\lambda^2)^k \rangle \Big|_{\text{also } \mathbb{R}^4} = \mathcal{N}^{-1}\, N^2 \left(16\pi^2\Lambda^3\right)^k$$ for $\mathcal{N}^{-1} = N^2$ → rationale… / → nagging point

thus, the main points of my talk:

1. **semiclassical objects contributing to gaugino condensate on the torus are related to center vortices and monopoles, argued responsible for chiral symmetry breaking and confinement**

(just state… won't describe relation…other talks?)

2.

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→ rationale…

→ nagging point

3. ↓ nagging point

the main points relevant for the $\mathbb{T}^4$ calculation of $\langle \lambda^{2k} \rangle$

1. Hamiltonian: $\mathbb{T}^3$ with 't Hooft twist $m_3 = n_{12} = -k$, $gcd(N, k) = 1$

$\mathbb{Z}_{2N}$ chiral- $\mathbb{Z}_N$ center anomaly:

$$\hat{T}_3 \, \hat{X}_{2N} \, \hat{T}_3^{-1} = e^{i \frac{2\pi}{N} k} \hat{X}_{2N}$$

exact degeneracies on $\mathbb{T}^3$!

$\hat{T}_3 \,|\, E, e_3, \ldots \rangle = |\, E, e_3, \ldots \rangle \; e^{i\frac{2\pi}{N} e_3}$ $\quad$ e_3 flux-states N-fold degenerate $\quad$ Cox, Wandler, EP 2021

lowest E (=0, SUSY) degenerate flux states —> N clustering vacua in $\mathbb{R}^4$ limit

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e_3 flux-states N-fold degenerate

Cox, Wandler, EP 2021

$$\langle \mathcal{O} \rangle \equiv \mathcal{N}^{-1} \, \mathrm{tr}_{\,\mathcal{H}_{m_3}} \left[\mathcal{O} e^{-\beta H} \hat{T}_3 (-1)^F \right]$$

$\hat{T}_3$ inserts $n_{34} = 1$, $\mathcal{N}$ - convenience

anomaly => torus trace above adds absolute values $\mathcal{O} = \lambda^{2k}$ in degenerate states:

$$\langle (\mathrm{tr}\lambda^2)^{\mathrm{k}} \rangle \Big|_{\beta \to \infty} = \mathcal{N}^{-1} \, \mathrm{N} \, \langle \mathrm{E} = 0, \mathrm{e}_3 = 0 \,|\, (\mathrm{tr}\lambda^2)^{\mathrm{k}} \,|\, \mathrm{E} = 0, \mathrm{e}_3 = 0 \rangle$$

N times value in one of the e_3 flux-states

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1. Hamiltonian: $\mathbb{T}^3$ with 't Hooft twist $m_3 = n_{12} = -k$, $gcd(N,k) = 1$

upshot:

$$\langle (\mathrm{tr}\lambda^2)^{\mathrm{k}} \rangle \Big|_{\beta \to \infty} = \mathcal{N}^{-1} \, \mathrm{N} \, \langle \mathrm{E} = 0, \mathrm{e}_3 = 0 \,|\, (\mathrm{tr}\lambda^2)^{\mathrm{k}} \,|\, \mathrm{E} = 0, \mathrm{e}_3 = 0 \rangle$$

SUSY Ward identity implies β, L - independence

$$\langle (\mathrm{tr}\lambda^2)^{\mathrm{k}} \rangle \Big|_{\beta,\mathrm{L} \to \infty}^{\mathrm{Hamilt.}} = \mathcal{N}^{-1} \, \mathrm{N} \, \langle (\mathrm{tr}\lambda^2)^{\mathrm{k}} \rangle \Big|_{1 \text{ vacuum on } \mathbb{R}^4}$$

$\mathcal{N}$ - convenience, to cancel N to get single-vacuum value

- next: calculate semiclassically at small $\mathbb{T}^4$

2. Hilbert space -> path integral

$$\langle \mathcal{O} \rangle \equiv \mathcal{N}^{-1} \, \mathrm{tr}_{\mathcal{H}_{m_3}} \left[\mathcal{O} e^{-\beta H} \hat{T}_3 (-1)^F \right]$$ $\hat{T}_3$ inserts $n_{34} = 1$

with $\mathcal{O} = \prod_{i=1}^{k} \mathrm{tr}(\lambda\lambda)(x_i)$:

$$\langle (\mathrm{tr}\,\lambda^2)^k \rangle = \mathcal{N}^{-1} \sum_{\nu \in \mathbb{Z}} \int [DA_\mu][D\lambda][D\bar{\lambda}] \left[\prod_{i=1}^{k} \mathrm{tr}(\lambda\lambda)(x_i) \right] e^{-S_{SYM} - i\theta(\nu + \frac{k}{N})} \Big|_{n_{12}=-k\,,n_{34}=1}$$

twists + index theorem imply only $Q_{top.} = \dfrac{k}{N}$ contribute: what are these?

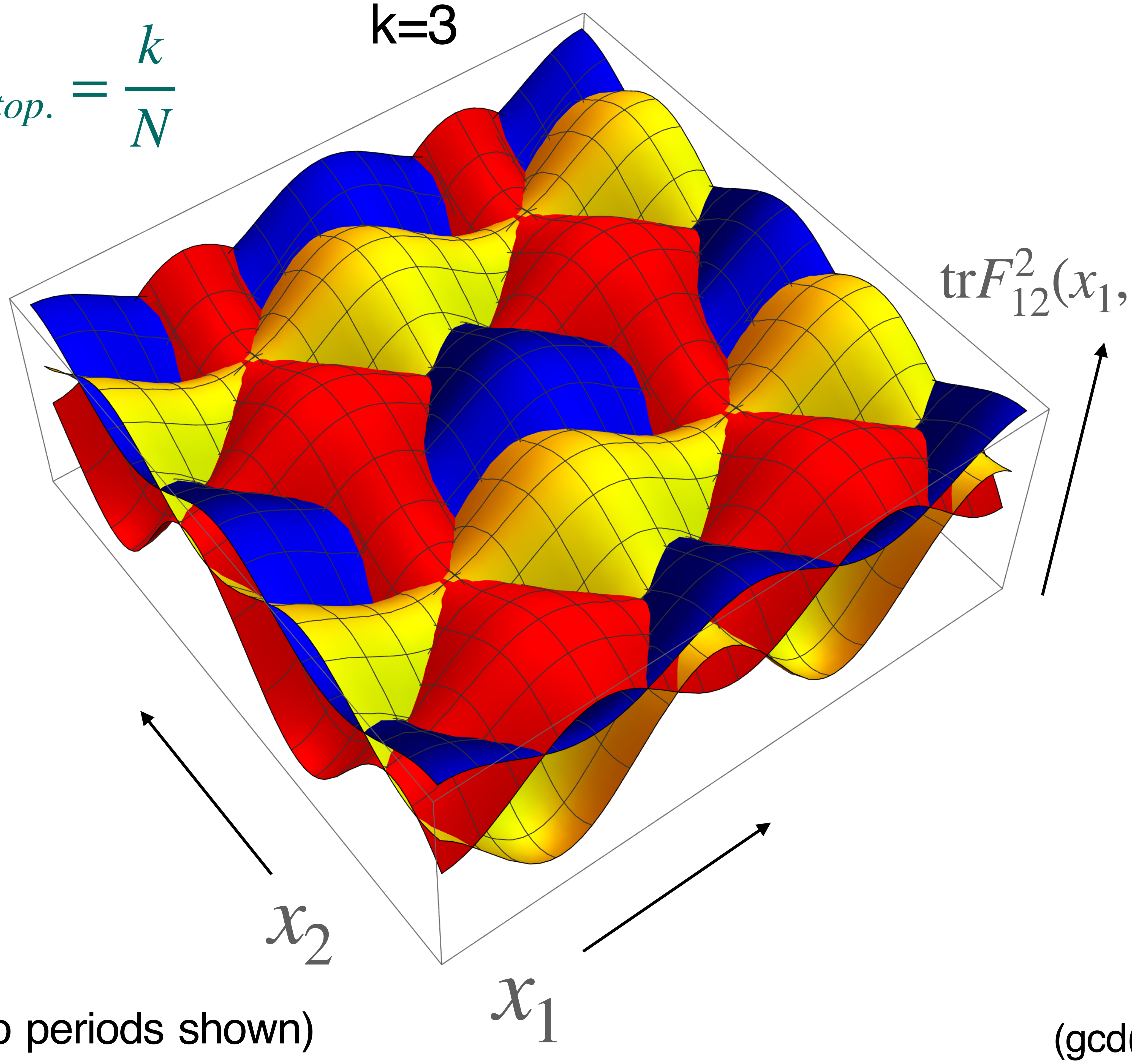
't Hooft, van Baal 1980s

3. Multi-fractional instantons on the twisted $\mathbb{T}^4$

Anber, EP 2307.07495, 240*.xxxxx based on and extending

't Hooft 1982
García Perez, González-Arroyo, Pena 2000
González-Arroyo 2018

$Q_{top.} = \frac{k}{N}$



(two periods shown)

- k lumps, $Q_{top.} = \frac{k}{N}$
- strongly overlapping, liquid-like
- each lump carries 2 gaugino zero modes

(see 2307.07495)

$4k$ bosonic moduli, as per index thm.:

center of mass motion + relative motion ($\Gamma_r^{SU(k)}$ = $SU(k)$ root lattice)

$$\Gamma = \left(\prod_{\mu=1}^{4} \frac{\mathbb{S}^1_\mu \times \Gamma_r^{SU(k)}}{\mathbb{Z}_k} \right) / S_k$$

(in SUSY: $\int_\Gamma$ only!!!)

(gcd(N,k)=1)

combined weight-lattice/c.m. shifts

permutation of lumps = $SU(k)$ Weyl

(Γ includes images of instanton under global center symmetry + modding by gauge equivalences)

combining all… SUSY -> nonzero modes cancel, only $\int_\Gamma$ remains

$$\langle (\mathrm{tr}\,\lambda^2)^k \rangle = \mathcal{N}^{-1} \sum_{\nu\in\mathbb{Z}} \int [DA_\mu][D\lambda][D\bar\lambda] \left[\prod_{i=1}^{k} \mathrm{tr}(\lambda\lambda)(x_i)\right] e^{-S_{SYM} - i\theta(\nu + \frac{k}{N})}\Big|_{n_{12}=-k\,, n_{34}=1}$$

$$= \mathcal{N}^{-1}\, N^2 \left(\frac{16\pi^2 M_{\mathrm{PV}}^3}{g^2} e^{-\frac{8\pi^2}{Ng^2}} \right)^k \int \prod_{C'=1}^{k} d\zeta_1^{C'}\, d\zeta_2^{C'}\, \zeta_2^{C'}\, \zeta_1^{C'}$$

↑ integral over bosonic and fermionic moduli; $M_{PV}^{n_B - \frac{1}{2} n_F} = M_{PV}^{4k-k} = M_{PV}^{3k}$

<u>path integral on twisted $\mathbb{T}^4$ w/ gcd(N,k)=1:</u> $$\langle (\mathrm{tr}\,\lambda^2)^k \rangle = \mathcal{N}^{-1}\, N^2 \left(16\pi^2 \Lambda^3\right)^k$$

<u>+ SUSY ward identities</u> $$\langle (\mathrm{tr}\,\lambda^2)^k \rangle\Big|_{\text{also } \mathbb{R}^4} = \mathcal{N}^{-1}\, N^2 \left(16\pi^2 \Lambda^3\right)^k$$

generalizes our 2210 calculation for N=2, k=1 to all N,k with gcd(N,k)=1

combining all… SUSY:

SU(2) -> SU(N) same N^2 for all k!

$$\langle (\mathrm{tr}\,\lambda^2)^k \rangle \Big|_{\text{also } \mathbb{R}^4} = \mathcal{N}^{-1}\, N^2 \left(16\pi^2 \Lambda^3\right)^k$$

compared to the k>1 ADHM calculation on $\mathbb{R}^4$, this is (to us) infinitely simpler generalizes our 2210 calculation for N=2, k=1 to all N,k with gcd(N,k)=1

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time to pay the piper:

$\mathbb{R}^4$ weak-coupling instanton/lattice (exact): $\langle \mathrm{tr}\lambda^{2\mathrm{k}} \rangle_{\mathrm{exact}} = (16\pi^2\Lambda^3)^{\mathrm{k}}$

clearly, there is a consistent choice for all our N,k: $\mathcal{N}^{-1} = N^2$

- this is satisfying - and we will end accepting it, as it works for all our N,k

(will give rationale)

- along the way, one nagging point remains that would be nice to understand…

combining all… SUSY:

generalizes our 2210 calculation for N=2, k=1 to all N,k with gcd(N,k)=1

SU(2) -> SU(N) same N^2 for all k!

$$\langle (\mathrm{tr}\,\lambda^2)^k \rangle\Big|_{\text{also } \mathbb{R}^4} = \mathcal{N}^{-1}\, N^2\, (16\pi^2\Lambda^3)^k$$

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from the Hamiltonian perspective, $\mathcal{N}^{-1} = N$ is the natural value to take; recall due to anomaly, we had

$$\langle \mathrm{tr}\lambda^{2k} \rangle\Big|^{\text{Hamilt.}}_{\beta, L \to \infty} = \mathcal{N}^{-1}\, \mathrm{tr}_{m_3}\left(e^{-\beta H} \hat{T}_3 (-1)^F \lambda^{2k} \right) = \mathcal{N}^{-1}\, N\, \langle \lambda^{2k} \rangle\Big|_{1 \text{ vacuum on } \mathbb{R}^4}$$

thus, taking $\mathcal{N}$ to be the Witten index, $\mathcal{N} = \mathrm{tr}_{m_3} e^{-\beta H}(-1)^F = N$ we cancel overall N.

Now, my final story: let $\mathcal{N}$ be the Witten index, $\mathcal{N} = \mathrm{tr}_{m_3=n_{12}} e^{-\beta H}(-1)^F = N$

independent on β, L (E=0 only contribute,E>0 cancel due to SUSY), so calculate semiclassically at small L

calculated in Hamiltonian on $\mathbb{T}^3$ with twist

Witten 1982, 2000

- twist removes zero modes of all fields, gap, only discrete set of E=0 states
- exactly N zero energy classical states: A=0 and x_3 center-symmetry transforms thereof

  center in x_1, x_2 acts trivially

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path integral expression for Witten index is

$[DD]$ denotes aux. field… localization?

$$\mathcal{N} = \sum_{\nu \in \mathbb{Z}} \int [DA_\mu][D\lambda][D\bar{\lambda}][DD]\; e^{-S_{SYM} - i\theta\nu}\Big|_{n_{12}=-k\,, n_{34}=0}$$

(sum over integer topological charges)

at small $\mathbb{T}^4$, weak coupling: sum over zero action saddle points (each contributes 1 due to susy)

—> more formal argument using SUSY localization…worthy of pursuit… !

there are exactly N^2 gauge nonequivalent zero action configurations

González-Arroyo 1998

can be described abstractly, but also explicitly as A=0 and x_3, x_4 center-symmetry transforms thereof

(essentially because Euclidean path integral can be taken to have either x_3, x_4 as time)

thus, path integral has us take $\mathcal{N}^{-1} = N^2$, restoring all agreement with SUSY and $\mathbb{R}^4$

The nagging point is: based on our understanding of Hamiltonian <—> path integral there should be agreement between the two

N vs N^2 in "Witten index":

due to center-symmetry transforms on A=0 saddles in x_3 vs in both x_3 and x_4 in path integral

what principle in Euclidean path integral says one should omit one of x_4/x_3 center symmetry images of A=0 saddle?

likewise, N vs N^2 in gaugino condensate:

due to center-symmetry images of Q=k/N saddles in x_3 vs in both x_3 and x_4 in path integral

perhaps easier to answer for Witten index!

- path integral calculation of Witten index with twist was never done!

deep stuff? localization, complexification, middle dimensional cycles,… ???

as we saw, understanding this has implications for semiclassics on twisted $\mathbb{T}^4$

summary of the main points of my talk:

1. **semiclassical objects contributing to gaugino condensate on the torus are related to center vortices and monopoles, argued responsible for chiral symmetry breaking and confinement**

(just stated; didn't describe any of this…other talks?)

2. **3.**

calculated $\langle \lambda^{2k} \rangle$ on small $\mathbb{T}^4$, gcd(N,k)=1; agrees with $\mathbb{R}^4$

$$\langle (\mathrm{tr}\,\lambda^2)^k \rangle \Big|_{\text{also } \mathbb{R}^4} = \mathcal{N}^{-1}\, N^2 \left(16\pi^2 \Lambda^3\right)^k \quad \text{for } \mathcal{N}^{-1} = N^2$$

→ rationale…

→ nagging point

two backups

Pf($\widehat{m}$) and not on μ. It would be interesting to try to verify this by direct

[13] The idea of the proof is to first reduce to $\mu = 1$ by replacing $\widehat{m}$ by $\widehat{m}/\mu$ and s by s/μ. Then one shows that one can by an $SL(4,\mathbf{Z})$ transformation set $\widehat{m}_{12} = 1$, after which by an $SL(4,\mathbf{Z})$ transformation one can set $\widehat{m}_{ij} = 0$ for $i = 1,2$ and $j = 3,4$ and (therefore) $\widehat{m}_{34} = \mathrm{Pf}(\widehat{m})$.

2000 “Supersymmetric index in 4d gauge theories”

study of path integrals, but we will not do that in the present paper. Section 4 of the paper is devoted to microscopic calculations verifying the predictions that we have presented up to this point, but these calculations will be done from a Hamiltonian point of view.

(formal) localization argument:

$$g^2 S_{SYM} = \delta^\alpha \mathcal{O}_\alpha + \delta_{\dot\alpha} \mathcal{O}^{\dot\alpha}, \text{ where } \mathcal{O}_\alpha = \frac{1}{8}\int_{\mathbb{T}^4} d^4x\, \Delta_\alpha, \;\; \mathcal{O}^{\dot\alpha} = \frac{1}{8}\int_{\mathbb{T}^4} d^4x\, \Delta^{\dot\alpha}$$

$$\Delta_\alpha \equiv \sigma_{\mu\nu\,\alpha}{}^{\beta}\; \lambda^a_\beta\; F^a_{\mu\nu} + \lambda^a_\alpha D^a, \;\; \Delta^{\dot\alpha} \equiv \bar\sigma_{\mu\nu}{}^{\dot\alpha}{}_{\dot\beta}\; \bar\lambda^{\dot\beta\, a}\; F^a_{\mu\nu} + \bar\lambda^{\dot\alpha\, a} D^a$$

$$\mathcal{N} = \sum_{\nu\in\mathbb{Z}} \int [DA_\mu][D\lambda][D\bar\lambda][DD]\; e^{-S_{SYM} - i\theta\nu}\Big|_{n_{12}=-k\,, n_{34}=0}$$

$$\frac{d\mathcal{N}}{dg^{-2}} = -\int_{\mathbb{T}^4 \text{ with } n_{12}\neq 0\, (\text{mod} N)} \mathcal{D}A\, \mathcal{D}\lambda\, \mathcal{D}\bar\lambda\, \mathcal{D}D\; \left[\delta^\alpha(\mathcal{O}_\alpha e^{-S_{SYM}}) + \delta_{\dot\alpha}(\mathcal{O}^{\dot\alpha} e^{-S_{SYM}})\right] = 0.$$

hence, calculate at $g^2 \to 0$: integral localizes to sum over zero action saddles + 1-loop

(usually, localization leads to an integral over moduli; here: a discrete set of points… should be simpler?)