The standard model on the lattice

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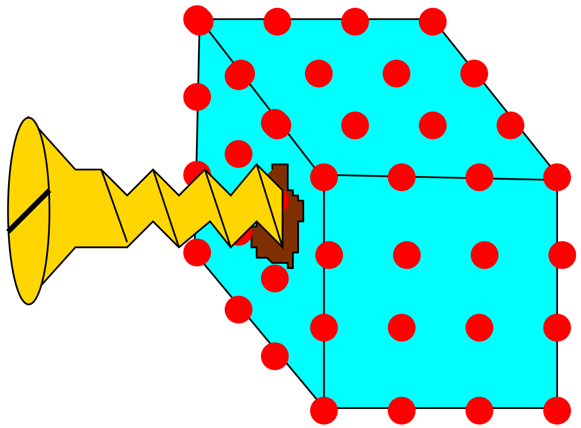
BNL



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Motivation

- neutrinos exist and violate parity
- lattice provides a definition of a field theory

Want exact local $SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetries

Remaining issue

- non-asymptotically free Higgs and $U(1)$ field

Approach based on Smit-Swift model

- P.V.D. Swift, Pys. Lett. B **145**, 256 (1984).
- J. Smit, Acta Phys. Polon. B17, 531l (1986)

Main additions

- twisted interplay of weak and strong groups
  - requires inclusion of entire generation
- pseudo-reality of SU2
  - anti-particles of left handed doublets
  - are right handed doublets

- Witten forbids single chiral multiplet with $SU(2)$
  - must work with even number of doublets
  - involve both quarks and leptons
- $SU(3)$ invariance forces use of entire generation
- Higgs required for masses and doubler removal
- anomaly gives proton decay

Need to understand anomalies: “instantons”

- strong anomaly gives η' mass
- weak processes give proton decay
  - $p \leftrightarrow e^+$ and $n \leftrightarrow \overline{\nu}$ mixing

Typical paths non-differentiable

- zero modes not robust
- replaced by real chiral eigenvalues

Fields

Consider a full generation as a unit

- 8 4-component fermion fields on lattice sites
- $u^r, u^g, u^b, d^r, d^g, d^b, \nu, e^-$
  - three colors for up and down quarks $\{r, g, b\}$
  - include right-handed neutrino
- complex Higgs doublet $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$ on sites

Gauge fields on lattice bonds

- SU(3) for the strong interactions: U_{su3}
- SU(2) matrix for the weak interactions: U_{su2}
- U(1) matrix for hypercharge: U_Y

Standard plaquette action for the gauge fields

- three independent gauge couplings

In our single generation

Two vectorlike strong $SU(3)$ triplets

- $$u = \begin{pmatrix} u^r \\ u^g \\ u^b \end{pmatrix} \qquad d = \begin{pmatrix} d^r \\ d^g \\ d^b \end{pmatrix}$$

Four left handed weak $SU(2)$ doublets

- $$r = \begin{pmatrix} u^r \\ d^r \end{pmatrix}_L \; g = \begin{pmatrix} u^g \\ d^g \end{pmatrix}_L \; b = \begin{pmatrix} u^b \\ d^b \end{pmatrix}_L \; l = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

Local gauge symmetries

Strong gauge transformation works on triplets

- $\psi_{ud} \to g_{su3}\psi_{ud}$

Weak group acts on left handed doublets

- $\psi_{rgbl} \to \left(g_{su2}\frac{1-\gamma_5}{2} + \frac{1+\gamma_5}{2}\right)\psi_{rgbl}$

Gauge matrices transform as usual

- $$\begin{array}{l} U^{ij}_{su3} \to g^i_{su3}\; U^{ij}_{su3}\; {g^\dagger_{su3}}^j \\ U^{ij}_{su2} \to g^i_{su2}\; U^{ij}_{su2}\; {g^\dagger_{su2}}^j \end{array}$$

Weak SU(2) group also acts on Higgs fields

- $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \to g_{su2} H$

SU(2) is pseudo-real

- $g^* = \tau_2 g \tau_2$
  - $H' \equiv \tau_2 H^* \tau_2 = \begin{pmatrix} -H_2^* \\ H_1^* \end{pmatrix}$
- transforms equivalently to H
  - $H' \to g_{su2} H'$

- $u^r, u^g, u^b, d^r, d^g, d^b, \nu, e^-$

Hypercharge $\psi \to U_Y \psi$

- $Y_L = (1/3, 1/3, 1/3, 1/3, 1/3, 1/3, -1, -1) = 2Q \pm 1$
- $Y_R = (4/3, 4/3, 4/3, -2/3, -2/3, -2/3, 0, -2) = 2Q$
- $Y_H = 1,\ Y_{H'} = -1$
- gauge fields neutral under hypercharge

$SU(3)$, $SU(2)$ and $U(1)$ groups all commute!

- weak group doesn't change colors
- strong group doesn't break weak chirality
- hypercharge constant on each multiplet

For each doublet (H, ψ_L) and (H', ψ_L) SU(2) singlets

- $(H, \psi_L) \equiv H_1^* \psi_1 + H_2^* \psi_2$

- $(H', \psi_L) \equiv -H_2 \psi_1 + H_1 \psi_2$

- two SU(2) invariant states per doublet

- physical left handed particles "composite"

Physical left particles are "composite"

- divide out "vacuum expectation" $v = |H|$

$$
\begin{aligned}
e_L &= (H, l)/v & Q = (Y_l - Y_H)/2 = -1 \\
\nu_L &= (H', l)/v & Q = (Y_l + Y_H)/2 = 0 \\
u_{rgb\,L} &= (H, rgb)/v & Q = (Y_{rgb} - Y_H)/2 = 2/3 \\
d_{rgb\,L} &= (H', rgb)/v & Q = (Y_{rgb} + Y_H)/2 = -1/3
\end{aligned}
$$

Equivalent to perturbative "unitary" gauge

Masses from the Higgs mechanism

Use above $SU(2)$ invariant combinations

- $\chi_L = \frac{1}{v}\begin{pmatrix} H, \psi_L \\ H', \psi_L \end{pmatrix}$

To construct on site gauge singlet mass terms

- $\overline{\psi}_R M \chi_L + h.c.$

Use Higgs mechanism to also remove doublers

- Wilson term using these "physical" fields
- $\overline{\psi}_{Ri+e_\mu}(1+\gamma_\mu)\chi_{Li}/2 + \overline{\psi}_{Ri}(1-\gamma_\mu)\chi_{Li+e_\mu}/2 + h.c.$
  - mimics ∂^2
  - formally "irrelevant" operator
- doublers moved to cutoff scale
- requires additive mass tuning

Weak bosons on bonds ij

- $(H'_i, U_{su2ij} H_j)$ $\qquad Q = 1$ $\qquad W^+$
- $(H_i, U_{su2ij} H'_j)$ $\qquad Q = -1$ $\qquad W^-$

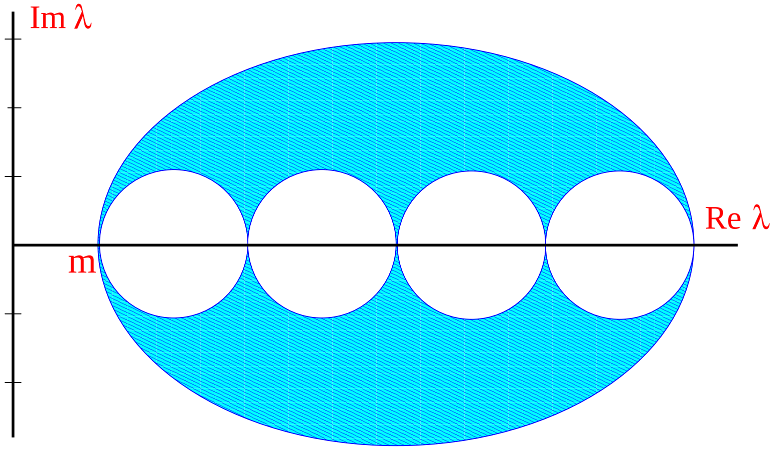
Z and Higgs hopping ∂H mix

- $(H'_i, U_{su2ij} H'_j)$ $\qquad Q = 0$
- $(H_i, U_{su2ij} H_j)$ $\qquad Q = 0$

Anomalies and real eigenvalues

Gamma 5 hermeticity: $\gamma_5 D \gamma_5 = D^\dagger$

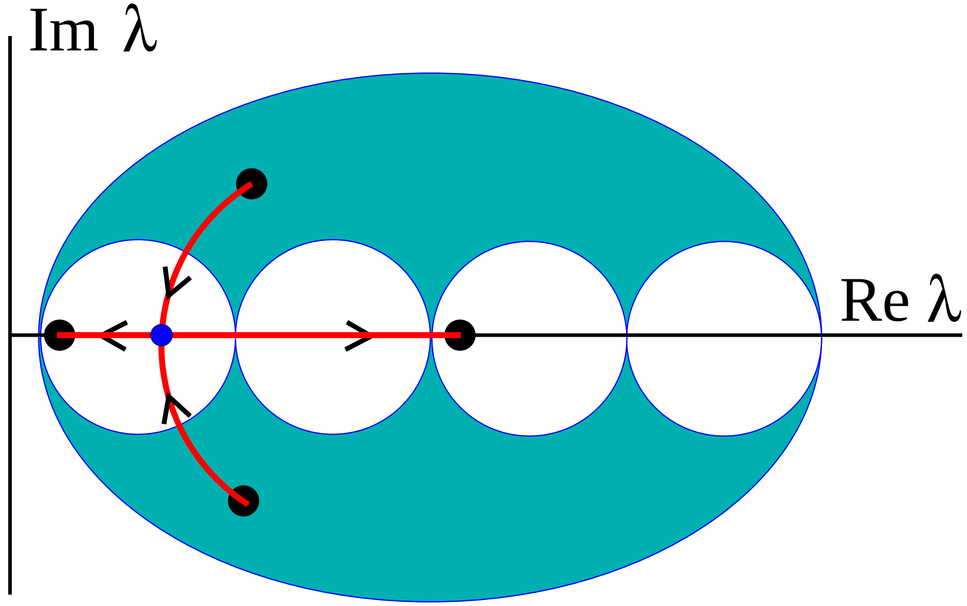
- all eigenvalues of D are in complex pairs or real
  - eigenvalue spectrum for free Wilson fermions



Real eigenvalues from colliding complex complex modes

- small eigenvalues have doubler counterparts
- opposite chirality

MC, Lattice 2002, Boston



On the set of real eigenvalues $[D, \gamma_5] = 0$

- γ_5 and D can be simultaneously diagonalized
- real eigenvalues can be sorted by chirality

"Topology" excess of small eigenvalues of one winding

- on the lattice, chiral real modes robust
- for smooth fields this becomes the index theorem

The 't Hooft process

Small eigenvalues of D suppress partition function

- $Z = \int (dA)(d\overline{\psi} d\psi)\ e^{-S_g + \overline{\psi} D \psi} = \int (dA)\ e^{-S_g(A)}\ \prod \lambda_i$

Are zero modes irrelevant?

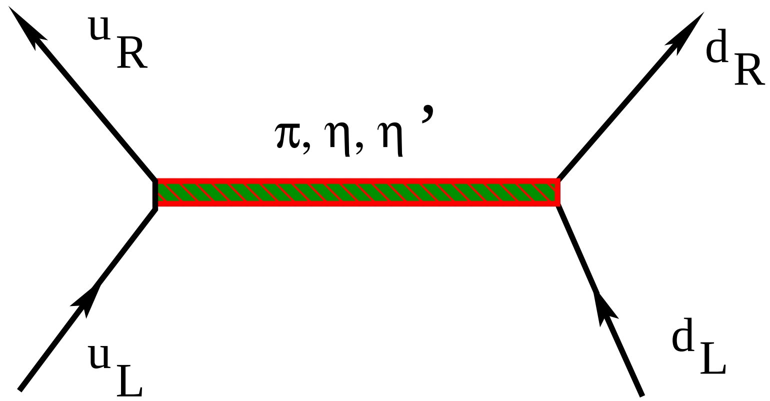
't Hooft: No, observables can enhance them!

Introduce sources η and $\overline{\eta}$

- $Z(\eta,\overline{\eta}) = \int (dA)\ (d\overline{\psi} d\psi)\ e^{-S_g + \overline{\psi} D\psi + \overline{\psi}\eta + \overline{\eta}\psi}$
- Differentiation (Grassman) gives Green's functions
- complete the square
- $Z = \int (dA)\ e^{-S_g + \overline{\eta} D^{-1} \eta/4} \prod \lambda_i.$
- D^{-1} factor can cancel supression

Strong interactions

- chiral eigenmode couples left and right fermions
- $\langle \psi_R D^{-1} \psi_L \rangle \neq 0$
- applies to all strong triplets



gives η' mass

Weak interactions

For each doublet its conjugate is right handed

- $\psi^c = \tau_2 \gamma_2 \psi^*$ $\qquad$ $\overline{\nu}$ right handed

Pair each doublet with a second conjugate

- $\overline{\psi}_i^c D^{-1} \psi_j \qquad i, j \in \{r, g, b, l\}$

- removes zero mode supppression

Antisymmetrize to restore strong gauge invariance

- $\epsilon_{ijkl}\ \langle \overline{\psi}^c_i D^{-1} \psi_j\ \overline{\psi}^c_k D^{-1} \psi_l \rangle\ \neq 0$

Vertex changes baryon number and lepton number by 1

- preserves B-L
- Hamiltonian: modes crossing from Dirac sea

Fermion number changes by 2

- consistent with $SU(2)$ since pseudo-real
- consistent with $SU(3)$ since $\overline{3} \in 3 \otimes 3$ and two flavors

"Effective" neutron anti-neutrino and proton positron mixing

- $\begin{pmatrix} n \\ p \end{pmatrix} \Longleftrightarrow \begin{pmatrix} \overline{\nu} \\ e^+ \end{pmatrix}$

$p \rightarrow e^+ + \pi$ allowed

- very small $O(e^{-1/\alpha})$

Summary

One generation fits nicely onto a Wilson lattice

$SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetries exact

Must include full generation

Baryon and lepton number violation, B-L preserved

Same parameters as in continuum discussion

- fermion masses, gauge couplings, Higgs potential