



A frequentist analysis of three right-handed neutrinos with GAMBIT

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TeVPA, 2 Dec 2019

[M. Chrzaszcz, M. Drewes, T.G., J. Harz, S. Krishnamurthy, C. Weniger, arXiv:1908.02302]

NeutrinoBit



Outline

- 1 Right-handed neutrinos
- 2 Scanning strategy
- 3 Likelihoods and observables
- 4 Results
- 5 Summary and Outlook



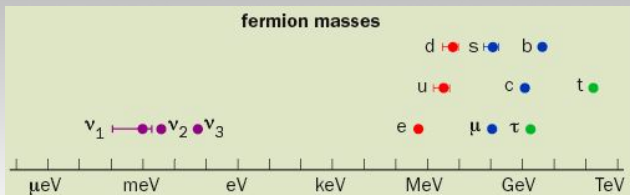
Right-handed neutrinos



Right-handed neutrinos

- Right-handed neutrinos $N_{1,2,3}$
- N_j are SM singlets $N_j \in \{1, 1, 0\}$
- Yukawa couplings \rightarrow Dirac mass terms

$$\mathcal{L} \supset Y_\nu^{ij} L_i N_j \phi = M_D^{ij} \nu_i N_j$$



- $Y_\nu/Y_t \lesssim 10^{12-15}$

Seesaw mechanism

- Majorana mass term for N_j

$$\begin{aligned} \mathcal{L} \supset Y_\nu^{ij} L_i N_j \phi + M^{ij} N_i N_j \\ = M_D^{ij} \nu_i N_j + M_M^{ij} N_i N_j \end{aligned}$$

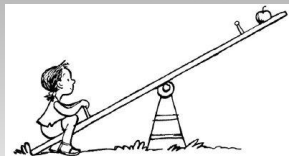
$$M_\nu = \begin{pmatrix} \delta m_\nu^{1-loop} & M_D \\ M_D^T & M_M \end{pmatrix}$$

- “Naturally” **light neutrino masses**

$$m_\nu \sim M_D^T M_M^{-1} M_D, \quad m_N \sim M_M$$

- Neutrino **mixing** matrix

$$\mathcal{U}_\nu = \begin{pmatrix} V_\nu & \Theta \\ \Theta^T & V_N \end{pmatrix} \approx \begin{pmatrix} 1 - \frac{1}{2}\theta\theta^\dagger & \theta \\ -\theta^\dagger & 1 - \frac{1}{2}\theta^\dagger\theta \end{pmatrix} \begin{pmatrix} U_\nu & 0 \\ 0 & U_N \end{pmatrix}$$





The Casas-Ibarra parametrization

- Θ parametrizes the active - sterile neutrino **mixing**
- CI parametrization

[J. A. Casas & A. Ibarra, *Nuc. Phys. B*618, (1-2), 2001]

$$\Theta = iU_\nu \sqrt{m_\nu^{diag}} \mathcal{R} \sqrt{\tilde{M}^{diag}}^{-1}$$

$$\tilde{M}_{IJ} \simeq \tilde{M}_{IJ}^{diag} = M_I \delta_{IJ} \left(1 - \frac{M_I^2}{v^2} l(M_I) \right)$$

- Rotation matrix $\mathcal{R} = \mathcal{R}^{23} \mathcal{R}^{13} \mathcal{R}^{12}$

$$\mathcal{R}_{ii}^{ij} = \mathcal{R}_{jj}^{ij} = \cos \omega_{ij}$$

$$\mathcal{R}_{ij}^{ij} = -\mathcal{R}_{ji}^{ij} = \sin \omega_{ij}$$



Symmetry protected limit

- Approximate $B - L$ symmetry

$$F_{\alpha I} = \Theta_{\alpha I} M_I / v$$

$$M_M = \begin{pmatrix} M(1 - \mu) & 0 & 0 \\ 0 & M(1 + \mu) & 0 \\ 0 & 0 & M' \end{pmatrix},$$
$$F = \begin{pmatrix} F_e(1 + \epsilon_e) & iF_e(1 - \epsilon_e) & F_e \epsilon'_e \\ F_\mu(1 + \epsilon_\mu) & iF_\mu(1 - \epsilon_\mu) & F_\mu \epsilon'_\mu \\ F_\tau(1 + \epsilon_\tau) & iF_\tau(1 - \epsilon_\tau) & F_\tau \epsilon'_\tau \end{pmatrix},$$

- Two-degenerate RHNs \rightarrow pseudo-Dirac fermion

$$\mu, \epsilon_\alpha, \epsilon'_\alpha \ll 1$$

$$M_1 \sim M_2, \quad \Theta_{\alpha 1} \sim i\Theta_{\alpha 2}$$

- Oscillation data does not constraint $|U_{\alpha I}|^2 \equiv |\Theta_{\alpha I}|^2$
- Upper limit purely from other experimental constraints



Model parameters

- Light (left-handed) neutrino masses

$$m_{\nu_i} \quad i \in \{1, 2, 3\} \quad \rightarrow \quad m_{\nu_0}, \Delta m_{12}^2, \Delta m_{3l}^2$$

- Heavy (right-handed) neutrino masses

$$M_I \quad I \in \{1, 2, 3\} \quad \rightarrow \quad M_1, \Delta M_{21}, M_3$$

- Active neutrino mixing parameters

$$\{\theta_{12}, \theta_{13}, \theta_{23}, \alpha_1, \alpha_2, \delta_{CP}\}$$

- Active-sterile neutrino mixing angles

$$\Re(\omega_{ij}), \Im(\omega_{ij})$$



Scanning strategy

GAMBIT: The Global And Modular BSM Inference Tool

gambit.hepforge.org

EPJC **77** (2017) 784

arXiv:1705.07908

- Extensive model database – not just SUSY
- Extensive observable/data libraries
- Many statistical and scanning options (Bayesian & frequentist)
- *Fast* LHC likelihood calculator
- Massively parallel
- Fully open-source
- Fast definition of new datasets and theories
- Plug and play scanning, physics and likelihood packages



Members of:

ATLAS, Belle-II, CLiC, CMS, CTA, *Fermi*-LAT, DARWIN, IceCube, LHCb, SHiP, XENON

Authors of:

DarkSUSY, DDCalc, Diver, FlexibleSUSY, gamlike, GM2Calc, IsaTols, nulike, PolyChord, Rivet, SoftSUSY, SuperISO, SUSY-AI, WIMPSim



Recent collaborators:

Peter Athron, Csaba Balázs, Ankit Beniwal, Sanjay Bloor, Torsten Bringmann, Andy Buckley, José Eliel Camargo-Molina, Marcin Chrzęszcz, Jonathan Cornell, Matthias Danninger, Joakim Edsjö, Ben Farmer, Andrew Fowlie, Tomás E. Gonzalo, Will Handley, Sebastian Hoof, Selim Hotinli, Felix Kahlhoefer, Anders Kvellestad, Julia Harz, Paul Jackson, Farvah Mahmoudi, Greg Martinez, Are Raklev, Janina Renk, Chris Rogan, Roberto Ruiz de Austri, Pat Scott, Patrick Stöcker, Aaron Vincent, Christoph Weniger, Martin White, Yang Zhang



Scanning strategy

- Likelihood is mostly flat
- Saturate couplings upper limits
 - Differential model ΔM_{21}
 - Coupling slide

$$s \log |U_{\alpha I}|^2 + m \log M_I$$

- Split the M_I range

[0.06, 0.3162], [0.3162, 2], [2, 60], [60, 500]

- Diver 1.0.4, [19200, 10^{-10} , λ_j DE]

[GAMBIT Scanner WG, Eur.Phys.J. C77(2017) 761]

- Postprocess, combine and symmetrize

Parameter	Value/Range	Prior
<i>Active neutrino parameters</i>		
θ_{12} [rad]	[0.547684, 0.628144]	flat
θ_{23} [rad]	[0.670206, 0.925025]	flat
θ_{13} [rad]	[0.139452, 0.155509]	flat
m_{ν_0} [eV]	$[10^{-7}, 0.23]$	log
Δm_{21}^2 [10^{-5} eV ²]	[6, 9]	flat
Δm_{3l}^2 [10^{-3} eV ²]	$[\pm 2, \pm 3]$	flat
α_1, α_2 [rad]	$[0, 2\pi]$	flat
<i>Sterile neutrino parameters</i>		
δ [rad]	$[0, 2\pi]$	flat
Re ω_{ij} [rad]	$[0, 2\pi]$	flat
Im ω_{ij}	$[-15, 15]$	flat
M_I [GeV]	[0.06, 500]	log
R_{order}	[1, 6]	flat
<i>Nuisance parameters</i>		
m_H [GeV]	[124.1, 127.3]	flat



Likelihoods and observables



Likelihoods and observables

- Active neutrino parameters

[I.Esteban, M.C.Gonzalez-Garcia, M.Maltoni,

I.Martinez-Soler, T.Schwetz in JHEP 1701 (2017)

087]

	NH	IH
$\sin^2 \theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.307^{+0.013}_{-0.012}$
$\sin^2 \theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.554^{+0.023}_{-0.033}$
$\sin^2 \theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.02227^{+0.00074}_{-0.00074}$
δ_{CP}	234^{+43}_{-31}	278^{+26}_{-29}
$\frac{\Delta m_{21}^2}{10^{-5} \text{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$7.40^{+0.21}_{-0.20}$
$\frac{\Delta m_{3l}^2}{10^{-5} \text{eV}^2}$	$2.494^{+0.033}_{-0.031}$	$-2.465^{+0.032}_{-0.031}$

- Gaussian likelihoods
- Planck limit $\sum m_\nu < 0.23 \text{ eV}$

- Modified Fermi constant

$$G_\mu^2 = G_F^2 (1 - (\theta\theta^\dagger)_{\mu\mu} - (\theta\theta^\dagger)_{ee})$$

- EWPO parameters [PDG]

$$m_W = 80.385(15)$$

$$s_{eff}^2 = 0.23155 \pm 0.00005$$

- EWPO decays [PDG]

$$\Gamma_{inv} = 0.499 \pm 0.016$$

$$\Gamma_{W \rightarrow e\bar{\nu}_e} = 0.223 \pm 0.006$$

$$\Gamma_{W \rightarrow \mu\bar{\nu}_\mu} = 0.222 \pm 0.005$$

$$\Gamma_{W \rightarrow \tau\bar{\nu}_\tau} = 0.237 \pm 0.006$$



Likelihoods and observables

- Direct searches for RHN in meson, tau and gauge boson decays
- Beam dump and peak search experiments
- M_i vs $|\Theta_{\alpha i}|^2$ exclusion limits
- Poisson likelihoods

PIENU	0.06 - 0.129 GeV	Θ_{ei}
PS191	0.02 - 0.45 GeV	$\Theta_{ei}, \Theta_{\mu i}$
E949	0.175 - 0.3 GeV	$\Theta_{\mu i}$
CHARM	0.01 - 2.8 GeV	$\Theta_{ei}, \Theta_{\mu i}, \Theta_{\tau i}$
NuTeV	0.25 - 2 GeV	$\Theta_{\mu i}$
DELPHI (S)	3 - 50 GeV	$\Theta_{ei}, \Theta_{\mu i}, \Theta_{\tau i}$
DELPHI (L)	0.5 - 4.2 GeV	$\Theta_{ei}, \Theta_{\mu i}, \Theta_{\tau i}$
ATLAS	50 - 500 GeV	$\Theta_{ei}, \Theta_{\mu i}$
CMS	1 - 10^3 GeV	$\Theta_{ei}, \Theta_{\mu i}$

- Big Bang Nucleosynthesis

$$\begin{array}{ll}
 N_I \rightarrow \pi^0 \nu_\alpha & N_I \rightarrow H^+ l_\alpha^- \\
 N_I \rightarrow \eta \nu_\alpha & N_I \rightarrow \rho^0 \nu_\alpha \\
 N_I \rightarrow \eta' \nu_\alpha & N_I \rightarrow \rho^+ l_\alpha^- \\
 N_I \rightarrow \nu_\alpha \bar{\nu}_\beta \nu_\beta & N_I \rightarrow l_{\alpha \neq \beta}^- l_\beta^+ \nu_\beta \\
 N_I \rightarrow \nu_\alpha l_\beta^+ l_\beta^- & N_I \rightarrow \nu_\alpha u \bar{u} \\
 N_I \rightarrow \nu_\alpha d \bar{d} & N_I \rightarrow l_\alpha u n \bar{d}_m
 \end{array}$$

→ Conservative limit
 $\tau_N < 0.1s$

[O.Ruchayskiy, A.Ivashko, JCAP1210(2012) 014]

- CKM Unitarity

$$\begin{aligned}
 K_L \rightarrow \pi^+ e^- \bar{\nu}_e &\rightarrow |V_{us}^{CKM}| = 0.2163(6) \\
 \tau \rightarrow K \nu &\rightarrow |V_{us}^{CKM}| = 0.2214(22)
 \end{aligned}$$

[FlaviaNet, HFAG, PDG]

$$\rightarrow |V_{us}^{CKM}|^2 + |V_{ud}^{CKM}|^2 = 1$$



Likelihoods and observables

- Lepton flavour violating decays

Process	Branch. Frac.
$\mu^- \rightarrow e^- \gamma$	4.2×10^{-13}
$\tau^- \rightarrow e^- \gamma$	5.4×10^{-8}
$\tau^- \rightarrow \mu^- \gamma$	5.0×10^{-8}
$\mu^- \rightarrow e^- e^- e^+$	1.0×10^{-12}
$\tau^- \rightarrow e^- e^- e^+$	1.4×10^{-8}
$\tau^- \rightarrow \mu^- \mu^- \mu^+$	1.2×10^{-8}
$\tau^- \rightarrow \mu^- e^- e^+$	1.1×10^{-8}
$\tau^- \rightarrow e^- e^- \mu^+$	0.84×10^{-8}
$\tau^- \rightarrow e^- \mu^- \mu^+$	1.6×10^{-8}
$\tau^- \rightarrow \mu^- \mu^- e^+$	0.98×10^{-8}
$\mu - e$ (Ti)	4.3×10^{-12}
$\mu - e$ (Au)	7×10^{-13}
$\mu - e$ (Pb)	4.6×10^{-11}

[MEG, BaBar, Belle, SINDRUM, SINDRUM II, ATLAS, LHCb]

- Upper bounds on $|\Theta_{\alpha I}|^2$
- One-sided gaussian likelihoods

- Lepton Universality

$$R_{\alpha\beta}^X = \frac{\Gamma(X^+ \rightarrow l_\alpha^+ \nu_\alpha)}{\Gamma(X^+ \rightarrow l_\beta^+ \nu_\beta)}, \quad \pi, K, \tau, W$$

$$R_Y = \frac{\Gamma(B^{0/\pm} \rightarrow Y^{0/\pm} l_\alpha^+ l_\alpha^-)}{\Gamma(B^{0/\pm} \rightarrow Y^{0/\pm} l_\beta^+ l_\beta^-)}, \quad K, K^*$$

$\rightarrow R_D$ and R_{D^*} not impacted

- Neutrinoless Double β Decay

$$T_{1/2}^{0\nu} \geq 2.1 \times 10^{25} \text{ yr} \quad [\text{GERDA}]$$

$$T_{1/2}^{0\nu} \geq 1.07 \times 10^{26} \text{ yr} \quad [\text{KamLAND-Zen}]$$

\rightarrow Negligible in $B - L$ limit

$$\rightarrow [T_{1/2}^{0\nu}]^{-1} \propto |\sum_I \Theta_{eI}^2 M_I|^2$$

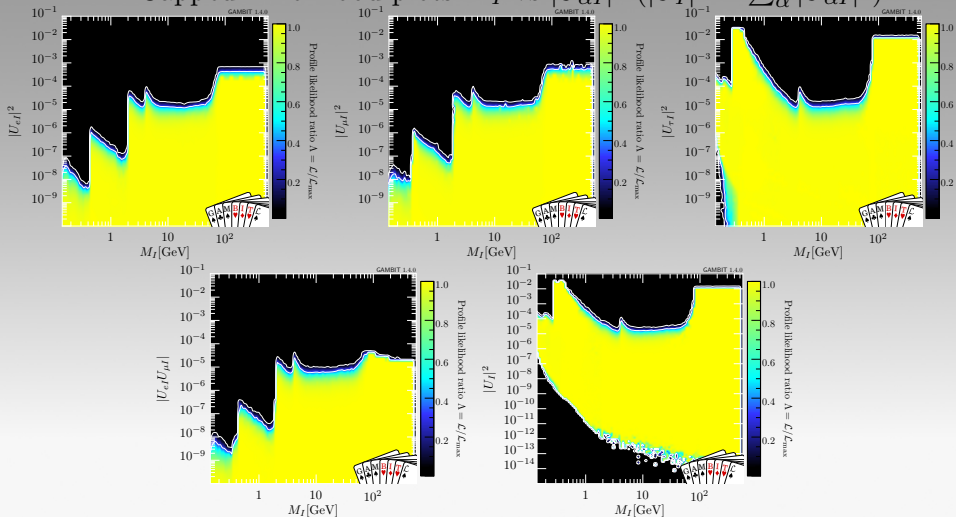


Results

For NH, unless otherwise stated

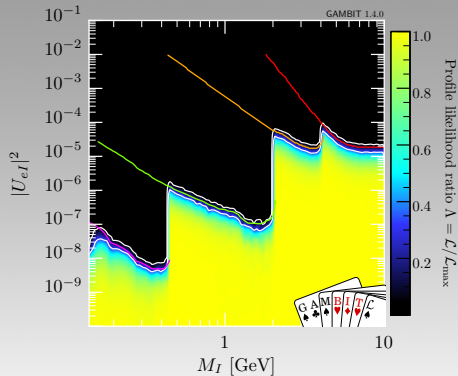
Results

- “Capped” likelihood plots M_I vs $|U_{\alpha I}|^2$ ($|U_I|^2 = \sum_{\alpha} |U_{\alpha I}|^2$)

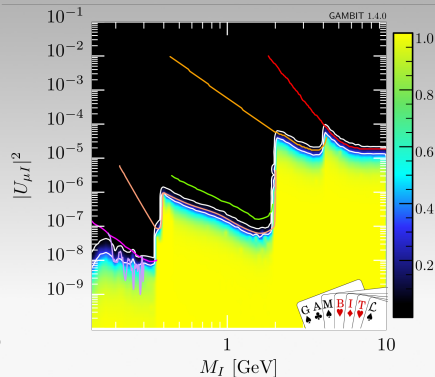


Results

- Upper limits from direct searches for $M_I < 80$ GeV

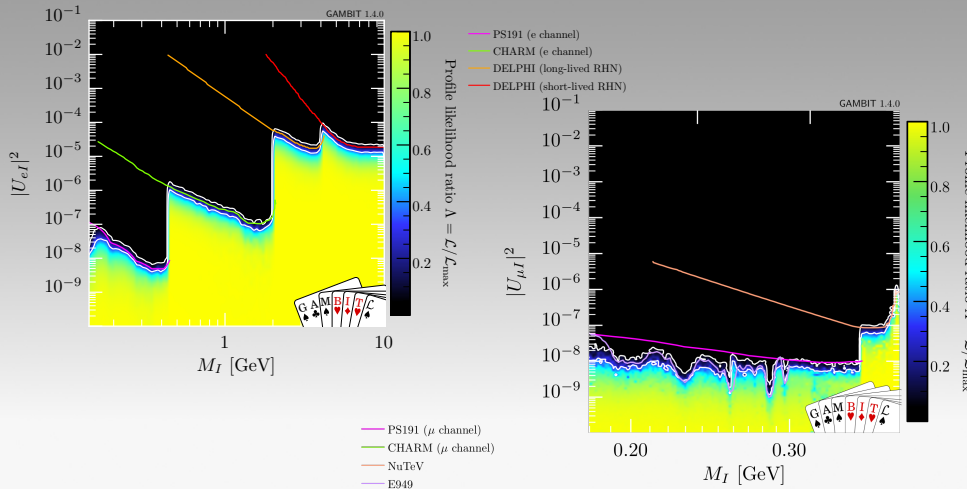


- PS191 (μ channel)
- CHARM (μ channel)
- DELPHI (long-lived RHN)
- DELPHI (short-lived RHN)
- NuTeV
- E949



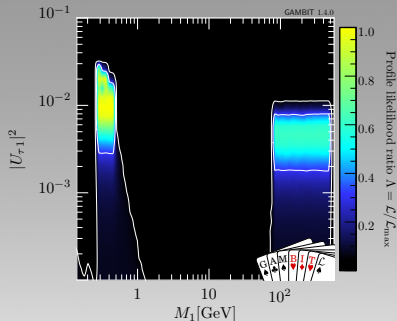
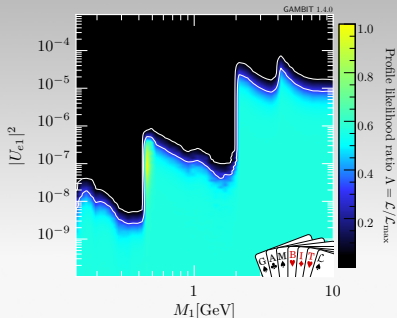
Results

- Upper limits from direct searches for $M_I < 80$ GeV



Results

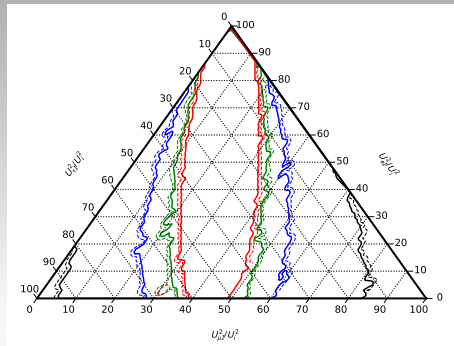
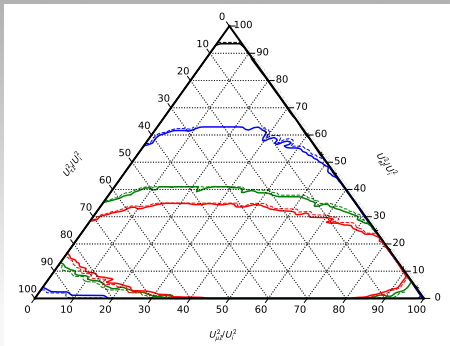
- Full likelihood
- Excess in $\ln L$ at large $|U_\tau|^2$
 - Invisible width Γ_Z
 - Fit to CKM entries
 - Lepton universality R_τ



- Excess in $\ln L$ at large $|U_e|^2$
 - Lepton universality R_K
- Low significance $\lesssim 1\sigma$

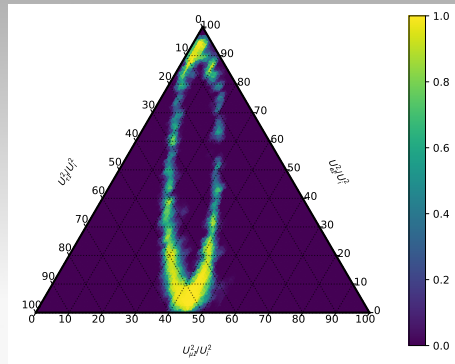
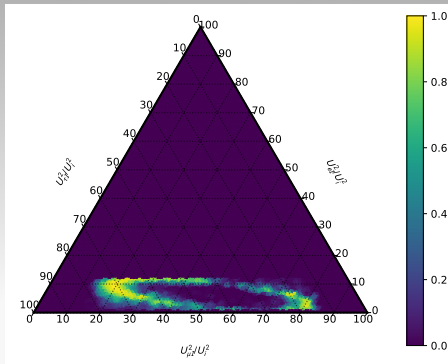
Results

- Flavour mixing pattern ($m_{\nu_0} = (1, 10^{-1}, 10^{-2})$ meV)
- Massless neutrino limit $m_{\nu_0} \rightarrow 0$



Results

- $m_{\nu_0} \rightarrow 0 \rightsquigarrow M_3$ is totally decoupled \rightsquigarrow 2 RHN model
- Symmetric ($B - L$) vs fine-tuned points
 - NO: $(\omega_{12}, \omega_{13}, \omega_{23}) \sim (0, \pi/2, \omega)$
 - IO: $(\omega_{12}, \omega_{13}, \omega_{23}) \sim (\omega, 0, 0)$





Summary and Outlook



Summary and Outlook

- Fully explore the parameter space $M_I \in [0.06, 100]$ GeV
- Saturate direct detection limits
 - Maximal $|U_{\alpha I}|^2 \rightarrow$ “large” Yukawas $F \sim 10^{-2}$
 - Proper statistical combination of likelihoods
- Reach BBN bounds, no seesaw lower limit
- 3RHN model is a better fit than SM ($\sim 2\sigma$)
 - $\Gamma_Z, \text{CKM}, R_K, R_\tau$
- Recover symmetry protected scenario, approximate $B - L$
 - Massless neutrino limit $m_{\nu_0} \rightarrow 0 \rightsquigarrow M_3$ decouples ($\Theta_{\alpha 3} \rightarrow 0$)
 - Pseudo-Dirac pair $M_1 \sim M_2, \Theta_{\alpha 1} \sim i\Theta_{\alpha 2}$
 - Weaker constraints (oscillations, $0\nu\beta\beta, \dots$) \rightsquigarrow higher couplings
 - $B - L$ symmetric subset is equivalent to the 2RHN model



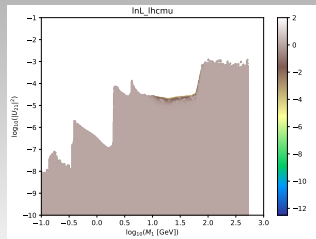
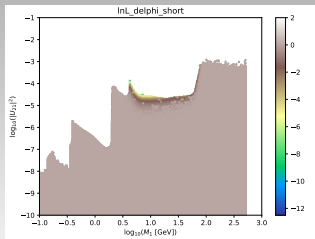
Backup

Why a global fit

- Proper use of **nuisance parameters**

$$\Delta m_{12}^2, \Delta m_{3l}^2, \theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}, m_H$$

- Combination of different **likelihoods**



- Statistical interpretation of the results
- Smart scanning strategies



Likelihoods and observables

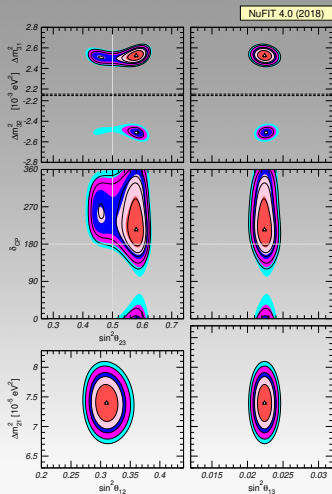
- Active neutrino parameters

[E. Fernandez-Martinez, J. Hernandez-Garcia,

J. Lopez-Pavon, JHEP 1608 (2016) 033]

	NH	IH
$\sin^2 \theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.306^{+0.012}_{-0.012}$
$\sin^2 \theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.587^{+0.020}_{-0.024}$
$\sin^2 \theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.02179^{+0.00076}_{-0.00076}$
δ_{CP}	261^{+51}_{-59}	277^{+40}_{-46}
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.50^{+0.19}_{-0.17}$
$\frac{\Delta m_{3l}^2}{10^{-5} \text{ eV}^2}$	$2.524^{+0.039}_{-0.040}$	$-2.514^{+0.038}_{-0.041}$

- 2D gaussian likelihoods
- Planck limit $\sum m_\nu < 0.23 \text{ eV}$





Likelihoods and observables

- Modified **Fermi constant**

$$G_{\mu}^2 = G_F^2 (1 - (\theta\theta^\dagger)_{\mu\mu} - (\theta\theta^\dagger)_{ee})$$

- EWPO parameters

$$\sin^2 2\theta_w = [\sin^2 2\theta_w]_{SM} \sqrt{1 - (\theta\theta^\dagger)_{\mu\mu} - (\theta\theta^\dagger)_{ee}}$$

$$\frac{m_W^2}{[m_W^2]_{SM}} = \frac{[s_w^2]_{SM}}{s_w^2} \sqrt{1 - (\theta\theta^\dagger)_{\mu\mu} - (\theta\theta^\dagger)_{ee}},$$

- EWPO decays

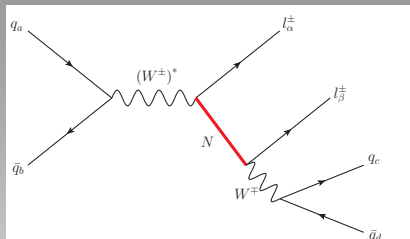
$$\Gamma_{\text{inv}} = \frac{G_{\mu} m_Z^3}{12\sqrt{2}\pi} \sum_{ij} \frac{(V_{\nu} V_{\nu}^\dagger)_{ij}}{\sqrt{1 - (\theta\theta^\dagger)_{\mu\mu} - (\theta\theta^\dagger)_{ee}}}$$

$$\Gamma_{W \rightarrow l_{\alpha} \bar{\nu}} = \frac{G_{\mu} m_W^3}{6\sqrt{2}\pi} \frac{(1 - \frac{1}{2}(\theta\theta^\dagger)_{\alpha\alpha})(1 - x_{\alpha})^2(1 + x_{\alpha})}{\sqrt{1 - (\theta\theta^\dagger)_{\mu\mu} - (\theta\theta^\dagger)_{ee}}}$$

Obs.	Value
m_W	80.385(15)
s_{eff}^2	0.23155 ± 0.00005
Γ_{inv}	0.499 ± 0.016
$\Gamma_{W \rightarrow e \bar{\nu}_e}$	0.223 ± 0.006
$\Gamma_{W \rightarrow \mu \bar{\nu}_{\mu}}$	0.222 ± 0.005
$\Gamma_{W \rightarrow \tau \bar{\nu}_{\tau}}$	0.237 ± 0.006

Likelihoods and observables

- Direct searches for RHN in **meson, tau and gauge boson** decays
- **Beam dump and peak search** experiments
- M_i vs $|\Theta_{\alpha i}|^2$ exclusion limits
- Poisson likelihoods



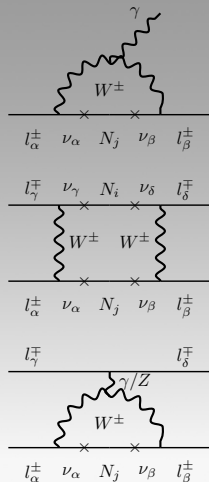
PIENU	0.06 - 0.129 GeV	Θ_{ei}
PS191	0.02 - 0.45 GeV	$\Theta_{ei}, \Theta_{\mu i}$
E949	0.175 - 0.3 GeV	$\Theta_{\mu i}$
CHARM	0.01 - 2.8 GeV	$\Theta_{ei}, \Theta_{\mu i}, \Theta_{\tau i}$
NuTeV	0.25 - 2 GeV	$\Theta_{\mu i}$
DELPHI (S)	3 - 50 GeV	$\Theta_{ei}, \Theta_{\mu i}, \Theta_{\tau i}$
DELPHI (L)	0.5 - 4.2 GeV	$\Theta_{ei}, \Theta_{\mu i}, \Theta_{\tau i}$
ATLAS	50 - 500 GeV	$\Theta_{ei}, \Theta_{\mu i}$
CMS	1 - 10^3 GeV	$\Theta_{ei}, \Theta_{\mu i}$

- [M. Aoki et al, Phys. Rev. D, 84(5), 2011]
 [G. Bernardi et al, Phys. Lett. B, 203(3), 1988]
 [A. V. Artamonov et al, Phys. Rev. D 91, 2015]
 [CHARM, Phys. Lett. B166(4), 1986]
 [FNAL-E815, Phys. Rev. Lett. 83, 1999]
 [DELPHI, Z. Phys. C, 74(1), 1997]
 [DELPHI, Z. Phys. C, 74(1), 1997]
 [ATLAS, JHEP 07:162, 2015]
 [CMS, arXiv:1802.02965v1]

Likelihoods and observables

- Lepton flavour violating decays

Process	Branch. Frac.	Reference
$\mu^- \rightarrow e^- \gamma$	4.2×10^{-13}	[MEG]
$\tau^- \rightarrow e^- \gamma$	5.4×10^{-8}	[BaBar, Belle]
$\tau^- \rightarrow \mu^- \gamma$	5.0×10^{-8}	[BaBar, Belle]
$\mu^- \rightarrow e^- e^- e^+$	1.0×10^{-12}	[SINDRUM]
$\tau^- \rightarrow e^- e^- e^+$	1.4×10^{-8}	[BaBar, Belle]
$\tau^- \rightarrow \mu^- \mu^- \mu^+$	1.2×10^{-8}	[ATLAS, BaBar, Belle, LHCb]
$\tau^- \rightarrow \mu^- e^- e^+$	1.1×10^{-8}	[BaBar, Belle]
$\tau^- \rightarrow e^- e^- \mu^+$	0.84×10^{-8}	[BaBar, Belle]
$\tau^- \rightarrow e^- \mu^- \mu^+$	1.6×10^{-8}	[BaBar, Belle]
$\tau^- \rightarrow \mu^- \mu^- e^+$	0.98×10^{-8}	[BaBar, Belle]
$\mu - e$ (Ti)	4.3×10^{-12}	[SINDRUM II]
$\mu - e$ (Au)	7×10^{-13}	[SINDRUM II]
$\mu - e$ (Pb)	4.6×10^{-11}	[SINDRUM II]



- Upper bounds on $|\Theta_{\alpha I}|^2$
- One-sided gaussian likelihoods



Likelihoods and observables

- **Big Bang Nucleosynthesis** \rightarrow lower bound on $|U_I|^2$

$$N_I \rightarrow \pi^0 \nu_\alpha, N_I \rightarrow H^+ l_\alpha^-, N_I \rightarrow \eta \nu_\alpha, N_I \rightarrow \eta' \nu_\alpha, N_I \rightarrow \rho^+ l_\alpha^-,$$

$$N_I \rightarrow \rho^0 \nu_\alpha, N_I \rightarrow \sum_{\alpha, \beta} \nu_\alpha \bar{\nu}_\beta \nu_\beta, N_I \rightarrow l_{\alpha \neq \beta}^- l_\beta^+ \nu_\beta, N_I \rightarrow \nu_\alpha l_\beta^+ l_\beta^-,$$

$$N_I \rightarrow \nu_\alpha u \bar{u}, N_I \rightarrow \nu_\alpha d \bar{d}, N_I \rightarrow l_\alpha u_n \bar{d}_m$$

\rightarrow Conservative limit on the lifetime

$$\tau_N \propto M_I^{-5} < 0.1s$$

- **Neutrinoless Double β Decay**

$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \left| m_p \sum_I \frac{\Theta_{eI}^2 M_I}{\langle p^2 \rangle + M_I^2} \right|^2, \quad \begin{aligned} T_{1/2}^{0\nu} &\geq 2.1 \times 10^{25} \text{ yr, GERDA (Ge)} \\ T_{1/2}^{0\nu} &\geq 1.07 \times 10^{26} \text{ yr, KamLAND-Zen (Xe)} \end{aligned}$$

\rightarrow Loses effectiveness in $B - L$ limit



Likelihoods and observables

- Lepton Universality

$$R_{\alpha\beta}^X = \frac{\Gamma(X^+ \rightarrow l_\alpha^+ \nu_\alpha)}{\Gamma(X^+ \rightarrow l_\beta^+ \nu_\beta)}, \quad X = \pi, K, \tau, W$$

$$R_Y = \frac{\Gamma(B^{0/\pm} \rightarrow Y^{0/\pm} l_\alpha^+ l_\alpha^-)}{\Gamma(B^{0/\pm} \rightarrow Y^{0/\pm} l_\beta^+ l_\beta^-)}, \quad Y = K, K^*$$

→ R_D and R_{D^*} are not impacted

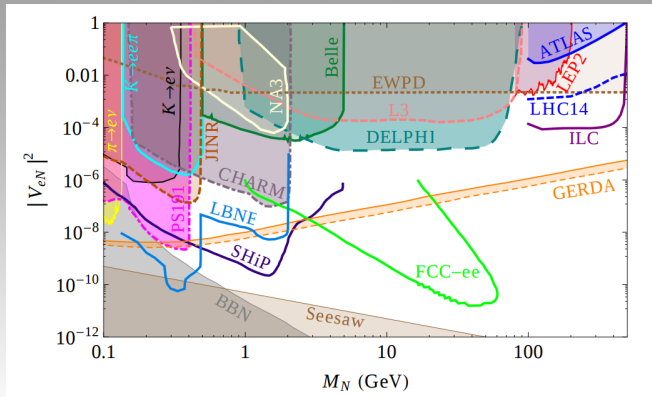
- CKM Unitarity $|V_{us}^{CKM}|^2 + |V_{ud}^{CKM}|^2 = 1$

$$|(V_{CKM}^{exp})_{us,ud}^i|^2 = |(V_{CKM})_{us,ud}|^2 [1 + f^i(\Theta)],$$

e.g. $K_L \rightarrow \pi^+ e^- \bar{\nu}_e : 1 + f^1(\Theta) = \frac{G_F^2}{G_\mu^2} [1 - (\theta\theta^\dagger)_{ee}]$

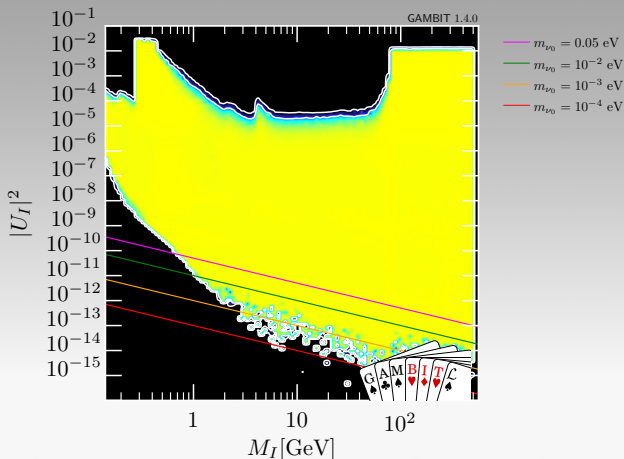
Results

- Upper limits from direct searches + EWPO



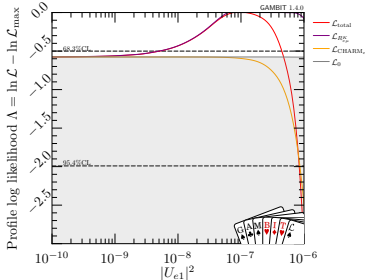
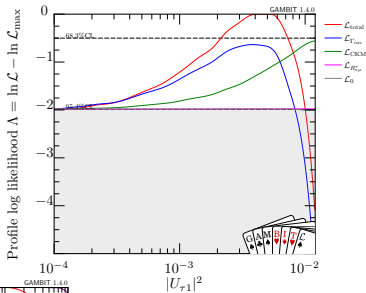
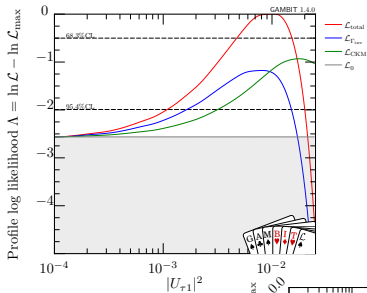
Results

- Lower limits: seesaw and BBN limits ($U_I^2 \gtrsim m_{\nu_0}/M_I$)





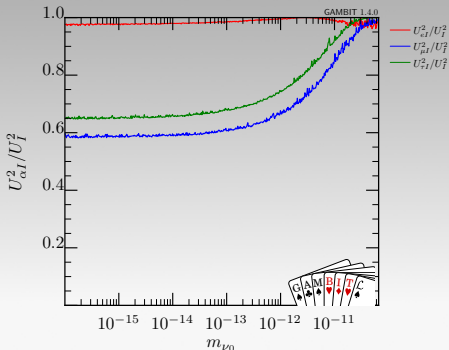
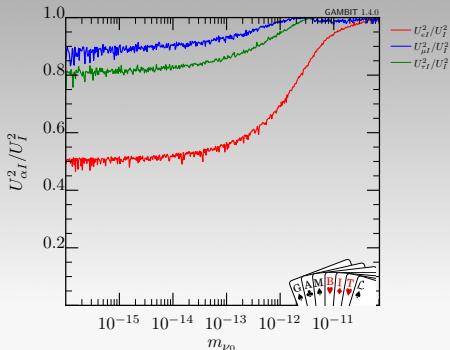
Results





Results

- Flavour mixing pattern ($m_{\nu_0} = (1, 10^{-1}, 10^{-2})$ meV)
- Massless neutrino limit $m_{\nu_0} \rightarrow 0$



Summary and Outlook

