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Particle escape from middle-aged SNRs and related gamma-ray signatures

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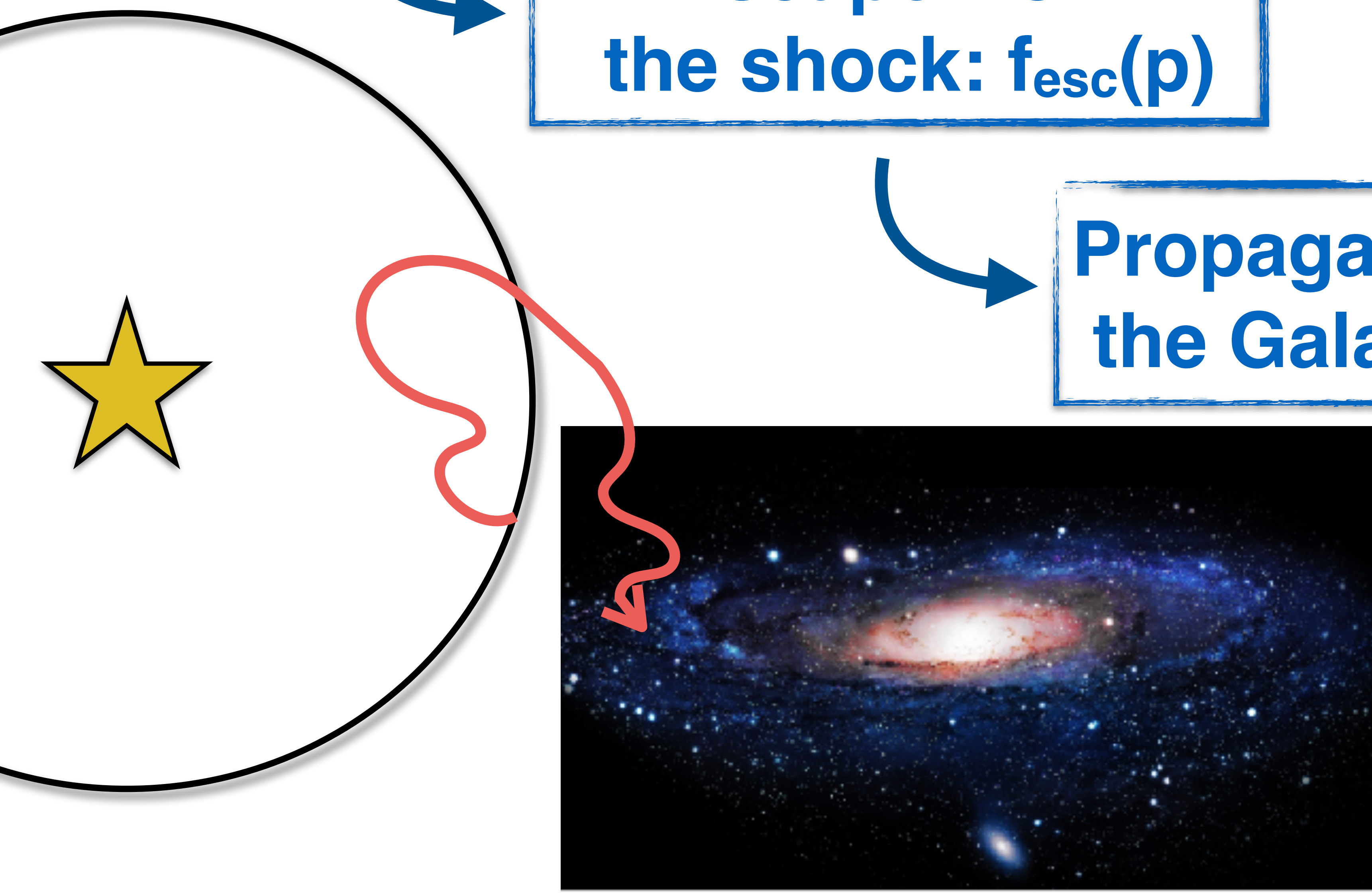
How do accelerated particles become CRs?

Acceleration at the shock: $f_0(p)$

$$f_0(p) \neq f_{\rm esc}(p) \neq f_{\rm prop}(p)$$

Escape from the shock: $f_{esc}(p)$

Propagation inside the Galaxy: $f_{prop}(p)$



How do accelerated particles become CRs?

Acceleration at the shock: $f_0(p)$

Escape from the shock: $f_{esc}(p)$

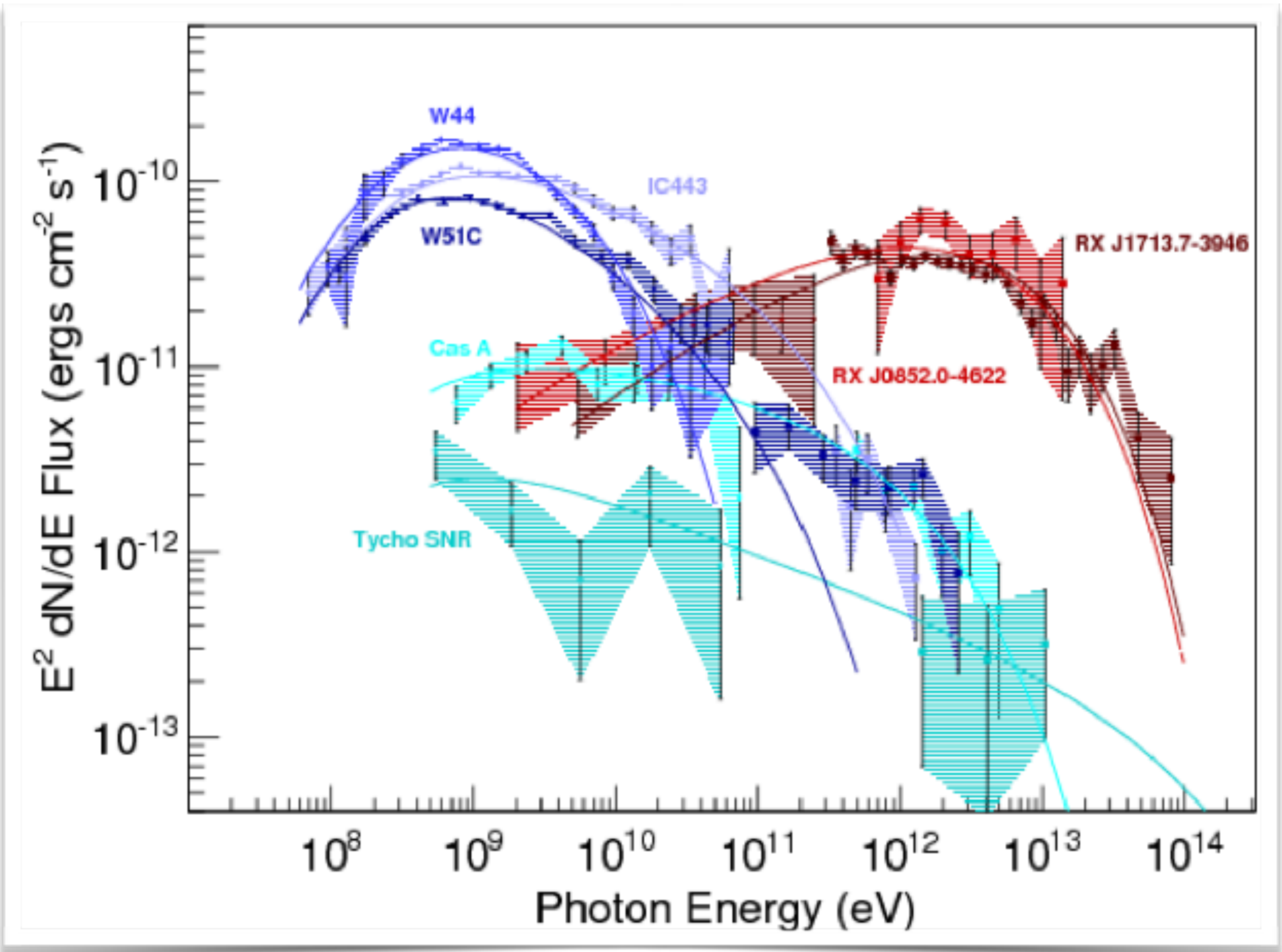
- Ptuskin & Zirakashvili, A&A 429 (2005) 755
- Gabici, Aharonian & Casanova, MNRAS (2009)
- Ohira, Murase & Yamakazi, A&A (2010) 513
- Bell & Shure, MNRAS 437 (2014) 2802
- Cardillo, Amato & Blasi, APh 69 (2015) 1

- Connect the **CR spectrum** observed on Earth with the spectrum of particles released at the sources;
- Understand the current observations of **SNR spectra** ⟶ unveil the presence of **PeV particle accelerators**.

A **phenomenological** model to investigate the particle **escape** through spectral and morphological features of evolved SNRs in the HE and VHE domain.

Celli et al., MNRAS 490 (2019)

SNRs in the HE and VHE domain



Drury et al., A&A 287 (1994) 959

Tsuguya & Fumio, J. Phys. G 20 (1994) 477

Funk et al., ARNPS 65 (2015) 245F

Middle-aged SNRs (20000 yrs)

- hadronic emission
- steep spectra
- E_{max} < 1 TeV

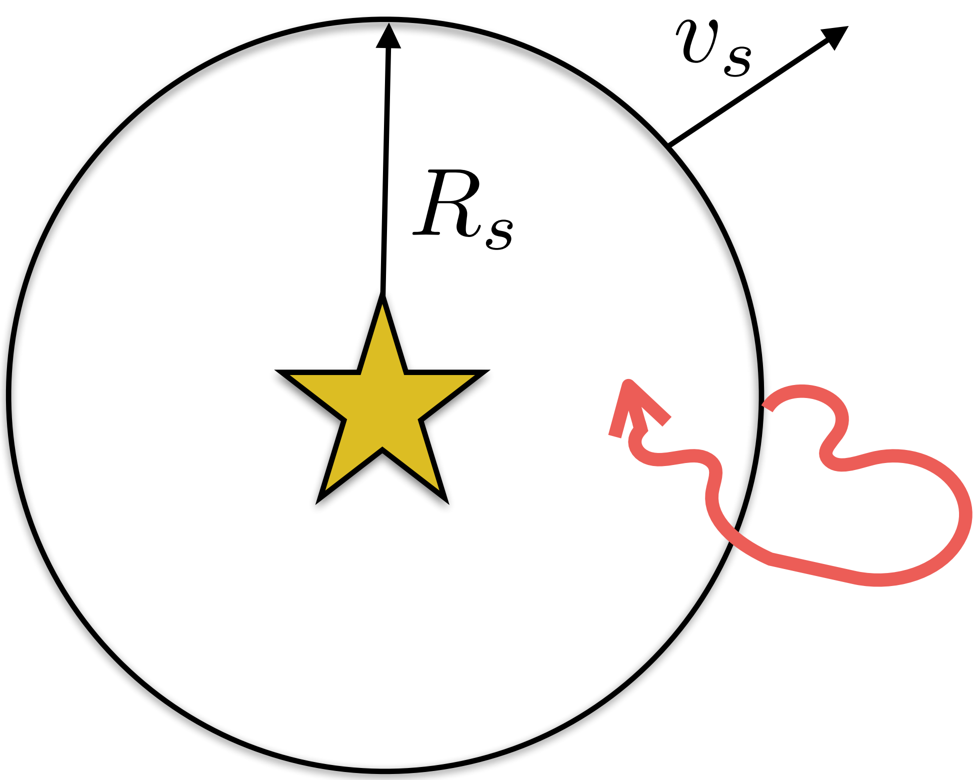
Young SNRs (2000 yrs)

- hadronic/leptonic ?
- hard spectra
- E_{max} = 10 - 100 TeV

Very young SNRs (300 yrs)

- hadronic ?
- steep spectra $E^{-2.3}$
- E_{max} = 10 - 100 TeV

Maximum energy in SNRs



$$t_{\rm acc} = t_{\rm age}$$

acceleration limited by remnant age

$$\frac{D(p_{\rm max})}{v_s^2(t)} = t$$

$$\frac{p_{\rm max}}{B_0\,\mathcal{F}(t)} = v_s^2(t)\,t$$

$$\left(\frac{\delta B(\mathbf{x},t)}{B_0}\right)^2 = \int \mathcal{F}(k,\mathbf{x},t)\,d\ln k$$

$$p_{\rm max,0} \propto \mathcal{F}(t)\,v_s^2(t)\,t$$

ED stage:

$$v_s(t) \simeq \text{const}$$

$$p_{\rm max,0}(t) \propto \mathcal{F}(t)\,t$$

ST stage:

$$v_s(t) \simeq t^{-3/5}$$

$$p_{\rm max,0}(t) \propto \mathcal{F}(t)\,t^{-1/5} \propto t^{-\delta}$$

Maximum energy in SNRs

In the scenario where the maximum momentum confined by the shock is a decreasing function of time, i.e.

$$p_{\mathrm{max},0}(t) = p_{\mathrm{M}} \left(\frac{t}{t_{\mathrm{Sed}}} \right)^{-\delta} \qquad (t \geq t_{\mathrm{Sed}})$$

Ptuskin & Zirakashvili, A&A 429 (2005) 755

$$\longrightarrow \quad t_{\mathrm{esc}}(p) = t_{\mathrm{Sed}} \left(\frac{p}{p_{\mathrm{M}}} \right)^{-1/\delta}$$

$\delta > 0$
high-energy particles escape earlier

where, for a shock expanding into a uniform medium,

$$t_{\mathrm{Sed}} \simeq 1.6 \times 10^3 \,\mathrm{yr} \left(\frac{E_{\mathrm{SN}}}{10^{51}\,\mathrm{erg}} \right)^{-1/2} \left(\frac{M_{\mathrm{ej}}}{10\, M_\odot} \right)^{5/6} \left(\frac{\rho_0}{1\, m_{\mathrm{p}}/\mathrm{cm}^3} \right)^{-1/3}$$

Maximum energy in SNRs

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$$p_{\rm max,0}(t) = p_{\rm M} \left(\frac{t}{t_{\rm Sed}} \right)^{-\delta} \qquad (t \geq t_{\rm Sed})$$

Ptuskin & Zirakashvili, A&A 429 (2005) 755

- Magnetic field not amplified
  $p_{\rm max,0}(t) \propto t^{-1/5}$

- Magnetic field amplification driven by resonant waves
  $p_{\rm max,0}(t) \propto t^{-7/5}$

- Magnetic field amplification driven by non-resonant waves
  $p_{\rm max,0}(t) \propto t^{-2}$

A model for particle propagation

Solution of the transport equation for accelerated **protons**

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = \frac{p}{3} \frac{\partial f}{\partial p} \nabla \cdot \mathbf{v} + \nabla \cdot [D \nabla f]$$

ANALYTICAL DESCRIPTION

Particles confined inside the SNR

$$\frac{\partial f_{\rm conf}}{\partial t} + \mathbf{v} \cdot \nabla f_{\rm conf} = \frac{p}{3} \frac{\partial f_{\rm conf}}{\partial p} \nabla \cdot \mathbf{v}$$

$$p \leq p_{\max,0}(t)$$

Escaped particles

$$\frac{\partial f_{\rm esc}}{\partial t} = \nabla \cdot [D \nabla f_{\rm esc}]$$

$$p > p_{\max,0}(t)$$

Matching condition: $f_{\rm esc}(t_{\rm esc}) = f_{\rm conf}(t_{\rm esc})$

A model for particle propagation

Solution of the transport equation for accelerated **protons**

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Escaped particles

$$\frac{\partial f_{\rm esc}}{\partial t} = \nabla \cdot [D \nabla f_{\rm esc}]$$

Assumption 1: spherical symmetry **f=f(t,r,p)**;

Assumption 2: stationary homogeneous diffusion coefficient is assumed inside and outside the remnant

$$D_{\rm in}(p) = D_{\rm out}(p) \equiv \chi D_{\rm Gal}(p) = \chi 10^{28} \left(\frac{pc}{10\,{\rm GeV}} \right)^{1/3} {\rm cm^2\,s^{-1}}$$

A model for particle propagation

Assumption 3: at every time, a constant fraction $\xi_{\rm CR}$ of the shock ram pressure is converted into CR pressure, such that the acceleration spectrum reads as

$$f_0(t,p) = \frac{3\xi_{\rm CR}\rho_{\rm up} v_s^2(t)}{4\pi c (m_p c)^{4-\alpha} \Lambda(p_{\max}(t))} p^{-\alpha} \theta\left[p_{\max}(t) - p\right]$$

acceleration efficiency constant in time

normalization factor such that $P_{\rm CR} = \xi_{\rm CR}\rho_{\rm up} v_s^2(t)$

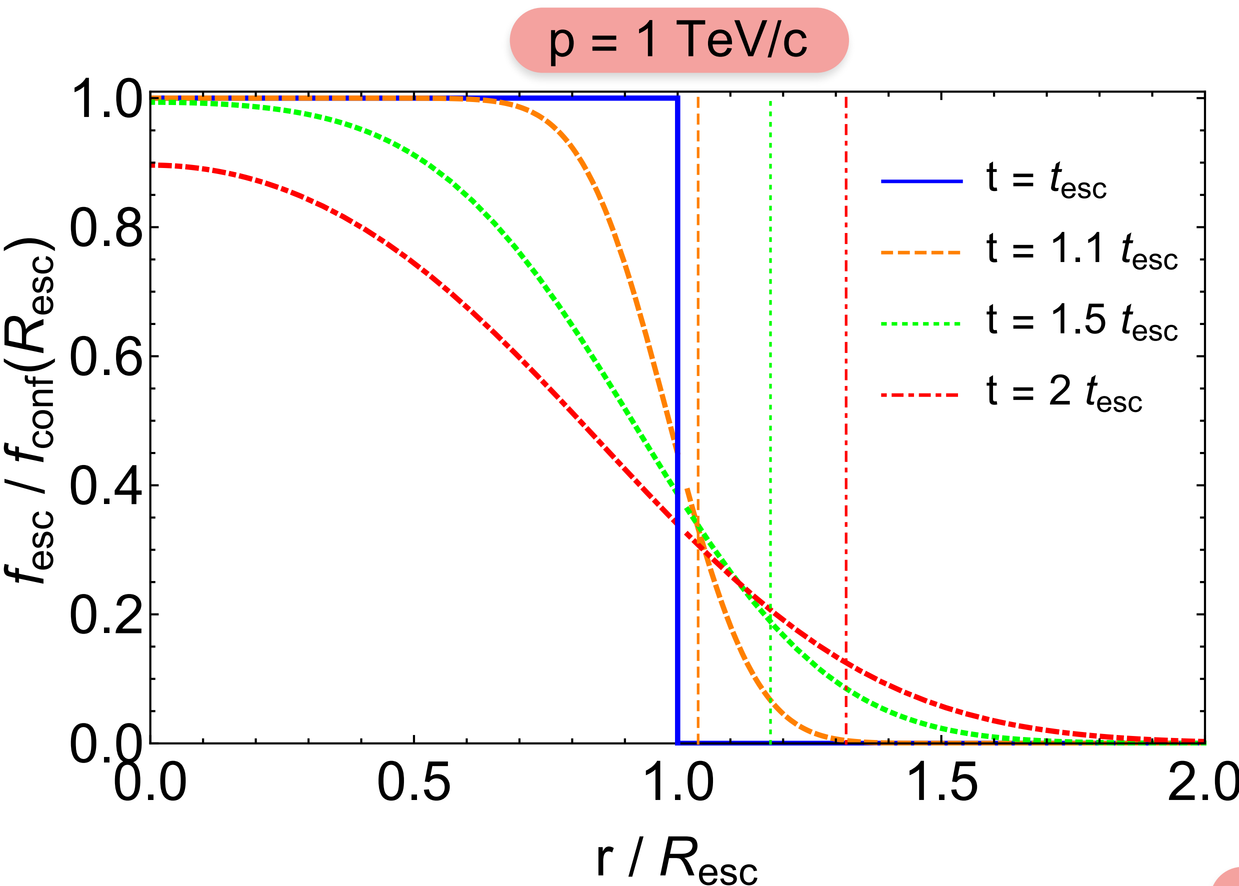
acceleration spectrum ($\alpha \sim 4$ from DSA)

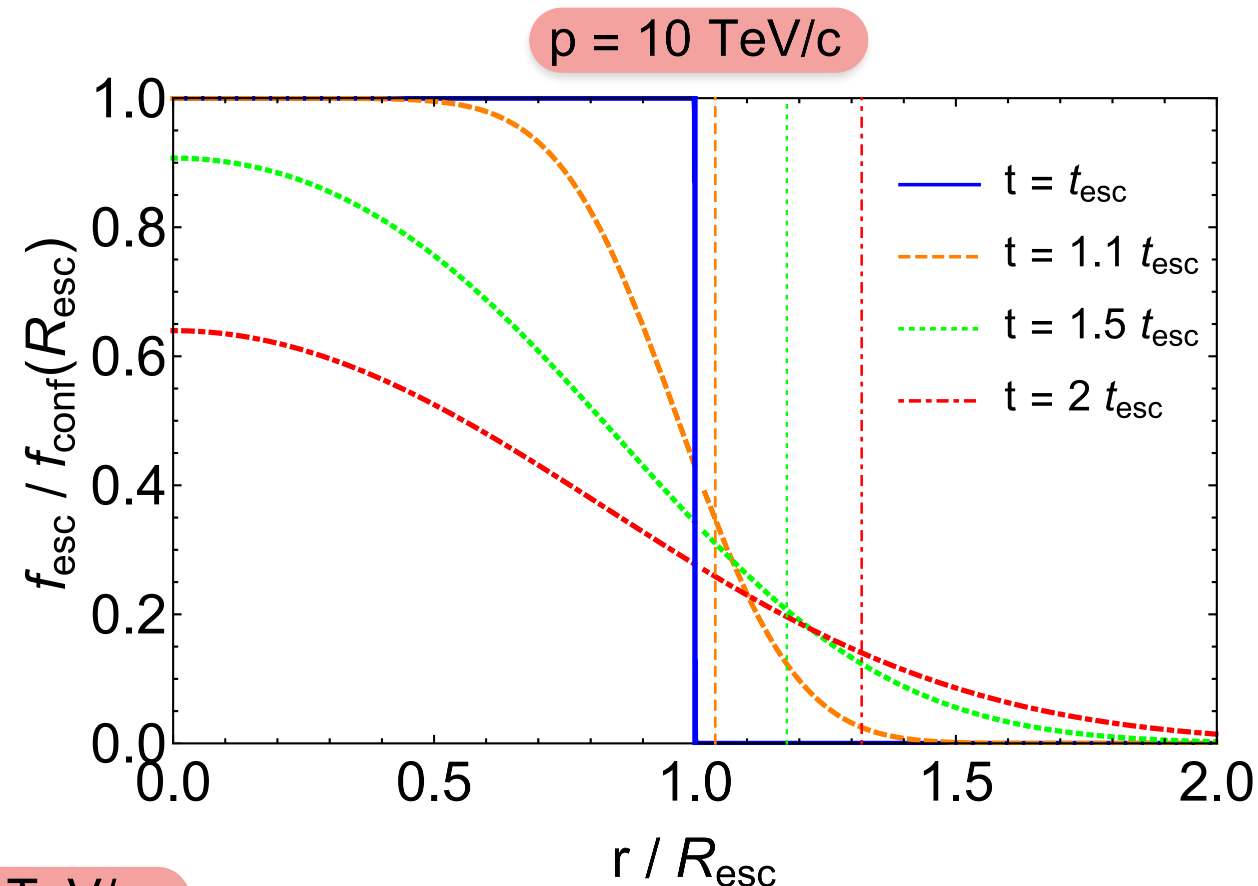
Ptuskin & Zirakashvili, A&A 429 (2005) 755

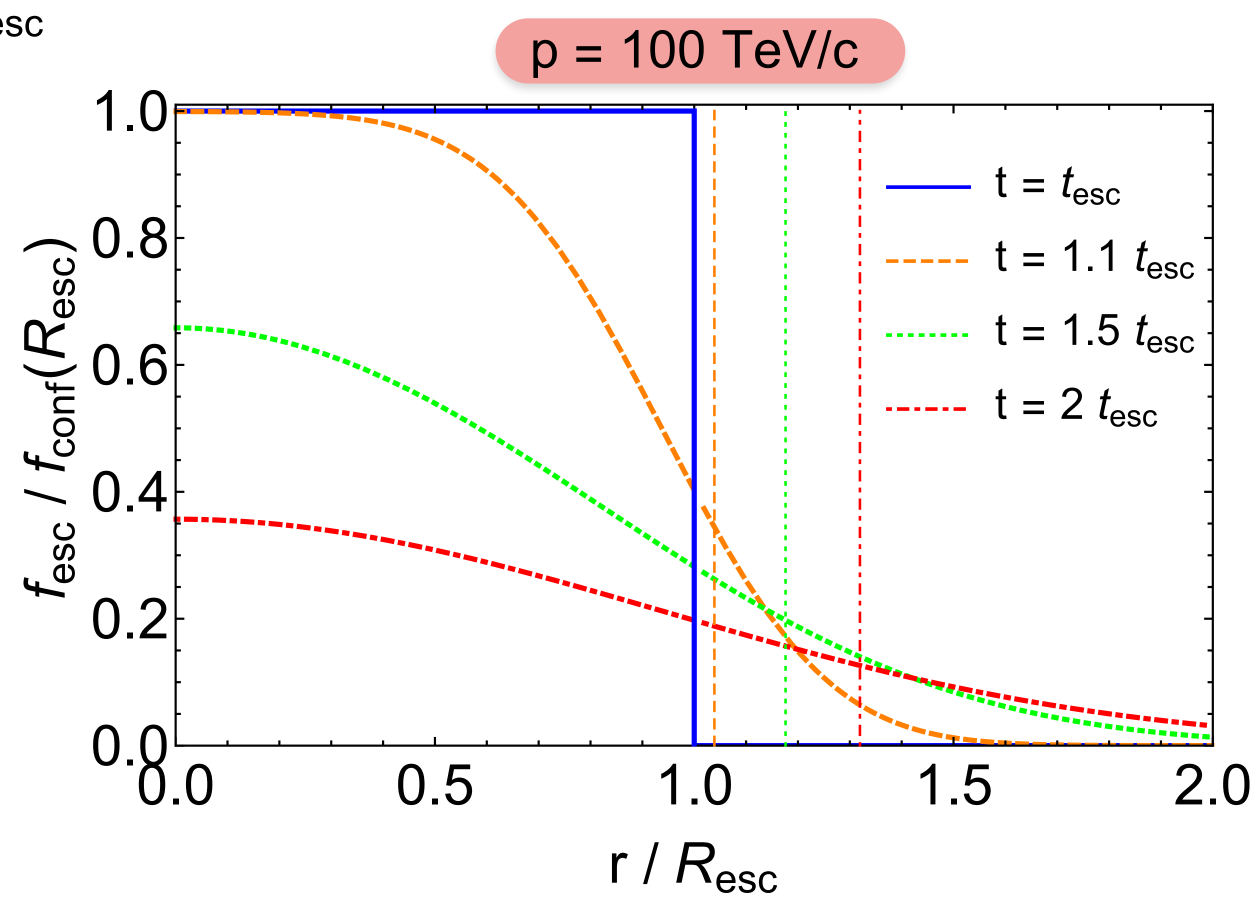
Assumption 4: the shock is evolving through the ST phase

$$R_s(t) \propto t^{2/5} \qquad v_s(t) \propto t^{-3/5}$$

Density of non-confined particles





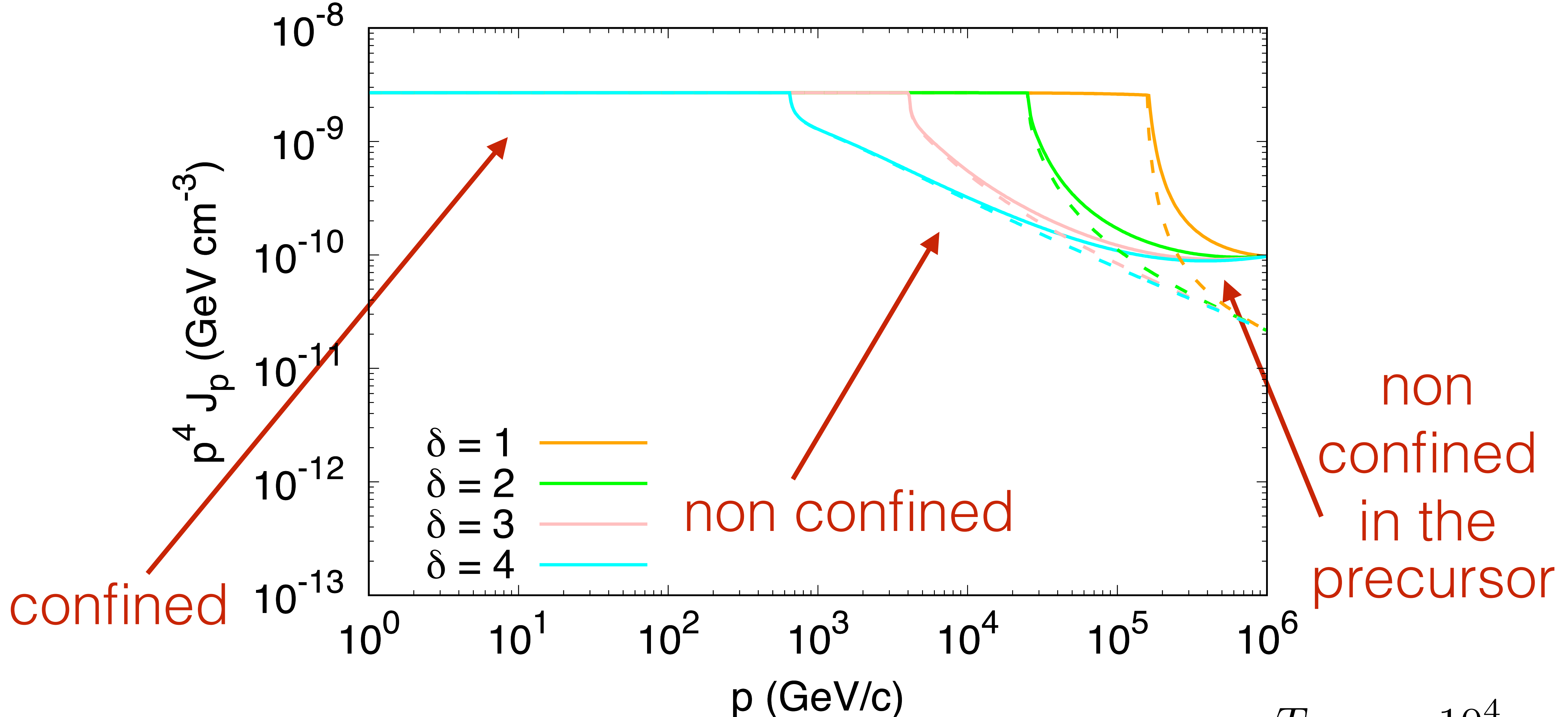


$\alpha = 4$

$\chi = 0.01$

The spectrum of protons inside the SNR

$$J_p^{\rm in}(t,p) = \frac{4\pi}{V_{\rm SNR}} \int_0^{R_{\rm sh}(t)} \left[f_{\rm esc}(t,r,p) + f_{\rm p,esc}(t,r,p) + f_{\rm conf}(t,r,p)\right] r^2 dr$$



$$D(p) = 10^{27} \left(\frac{pc}{10\ \rm GeV}\right)^{1/3} \rm cm^2 s^{-1}$$

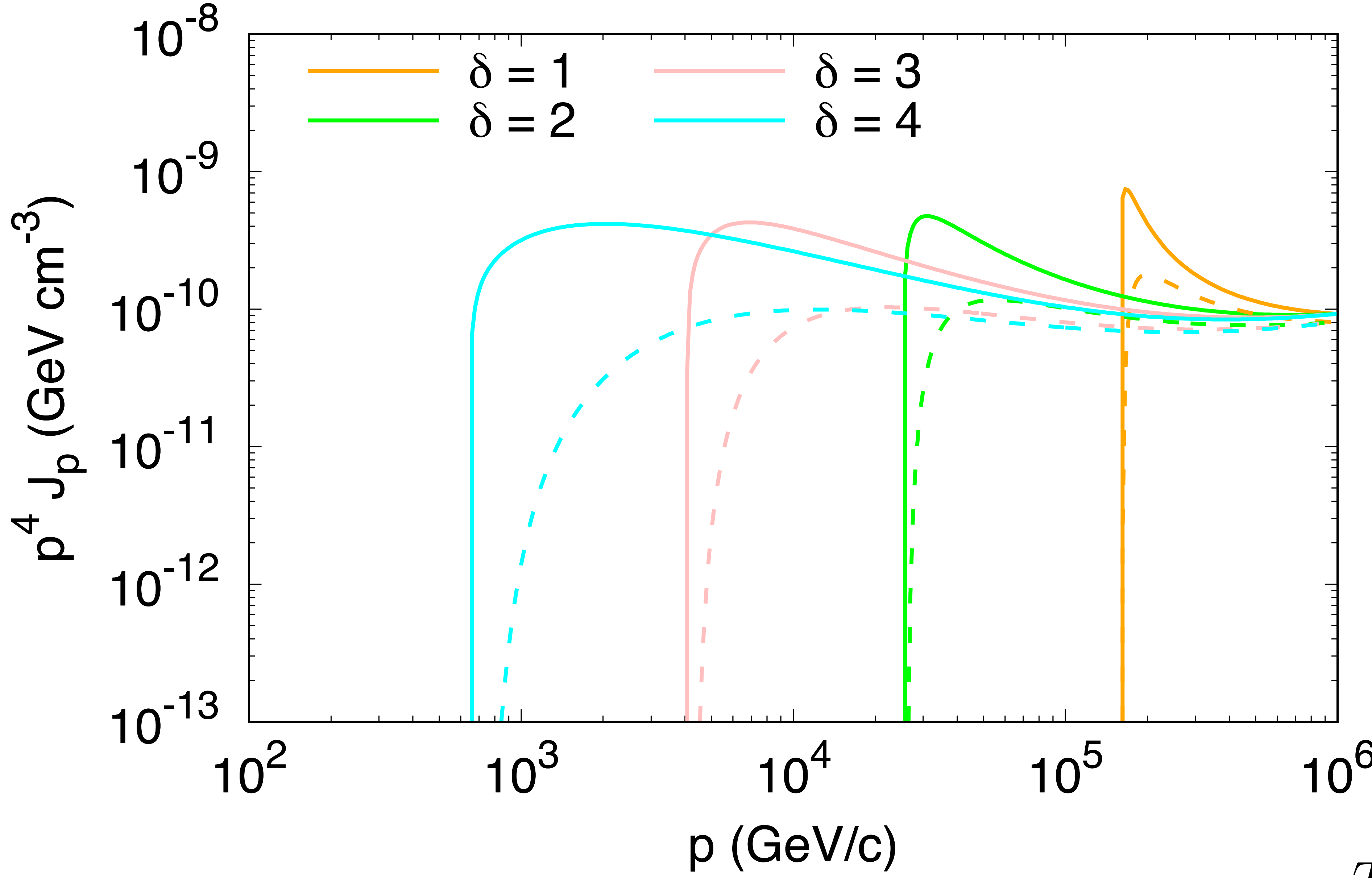
$T_{\rm SNR} = 10^4\,\rm yr$

$\xi_{\rm CR} = 10\%$

$n_{\rm up} = 1\,\rm cm^{-3}$

The spectrum of protons outside the SNR

$$J_p^{\mathrm{out}}(t,p) = \frac{3}{R_2^3 - R_1^3} \int_{R_1}^{R_2} \left[f_{\mathrm{esc}}(t,r,p) + f_{\mathrm{p,esc}}(t,r,p) \right] r^2 dr$$



SOLID:
$R_1 = R_{SNR}$
$R_2 = 2R_{SNR}$

DASHED:
$R_1 = 2R_{SNR}$
$R_2 = 3R_{SNR}$

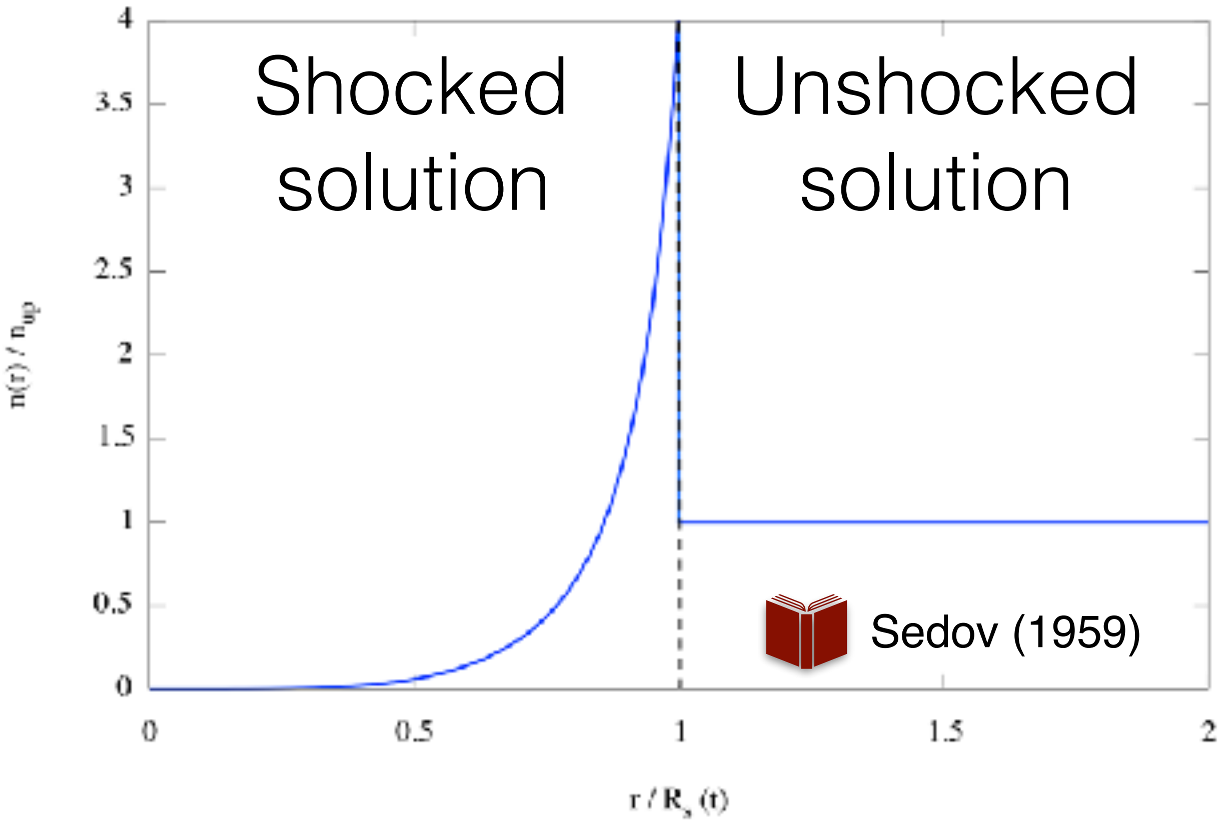
$$D(p) = 10^{27} \left(\frac{pc}{10\ \mathrm{GeV}} \right)^{1/3} \mathrm{cm}^2 \mathrm{s}^{-1}$$

$T_{\mathrm{SNR}} = 10^4\,\mathrm{yr}$

$\xi_{\mathrm{CR}} = 10\%$

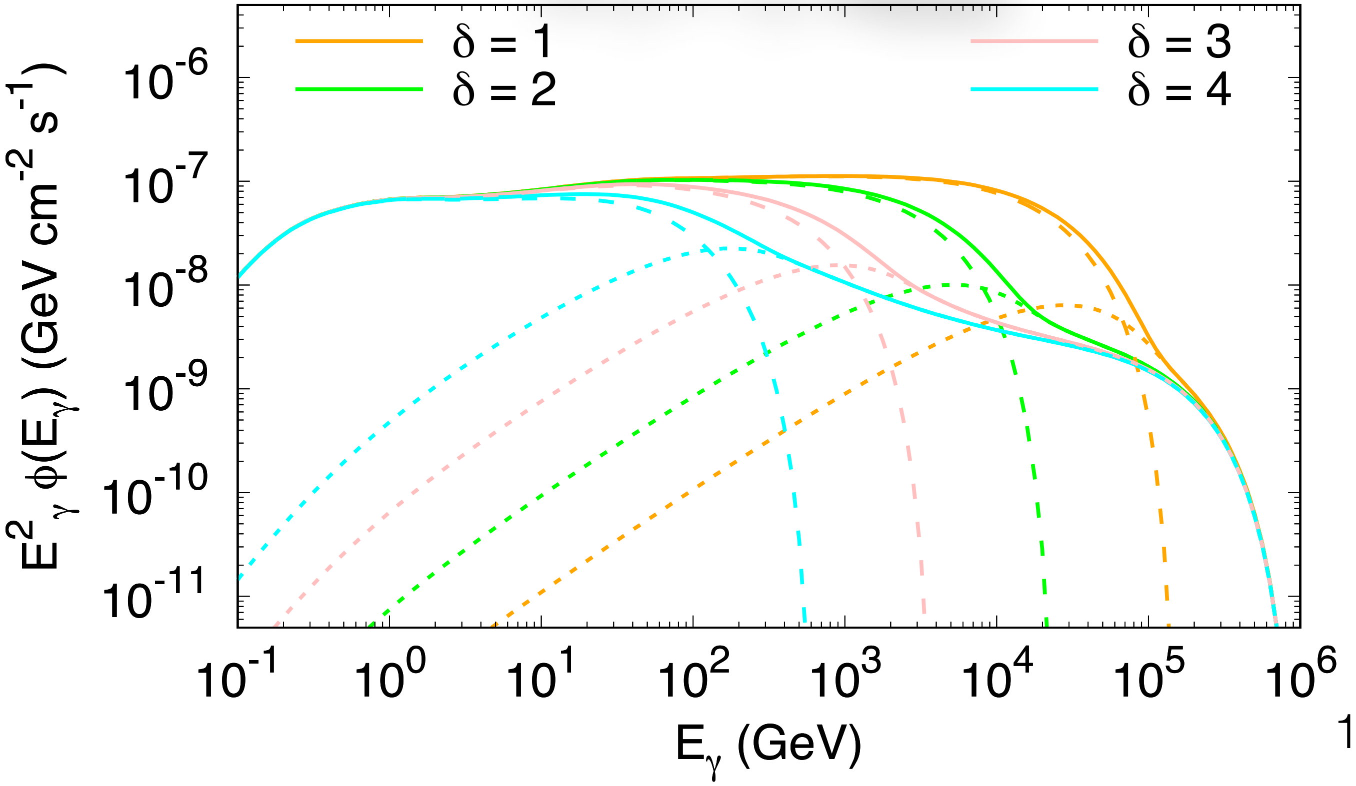
$n_{\mathrm{up}} = 1\,\mathrm{cm}^{-3}$

Volume integrated gamma-ray emission from hadronic (pp) interactions

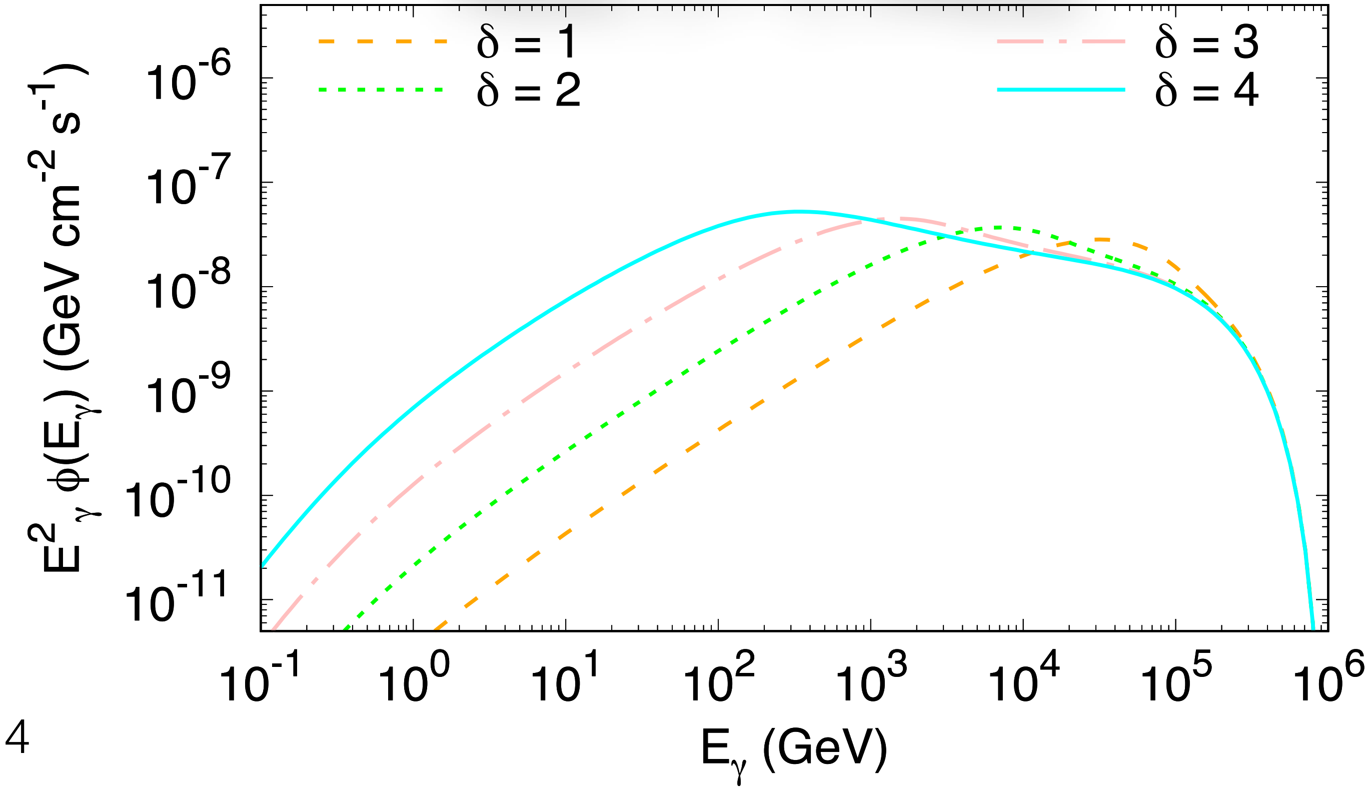


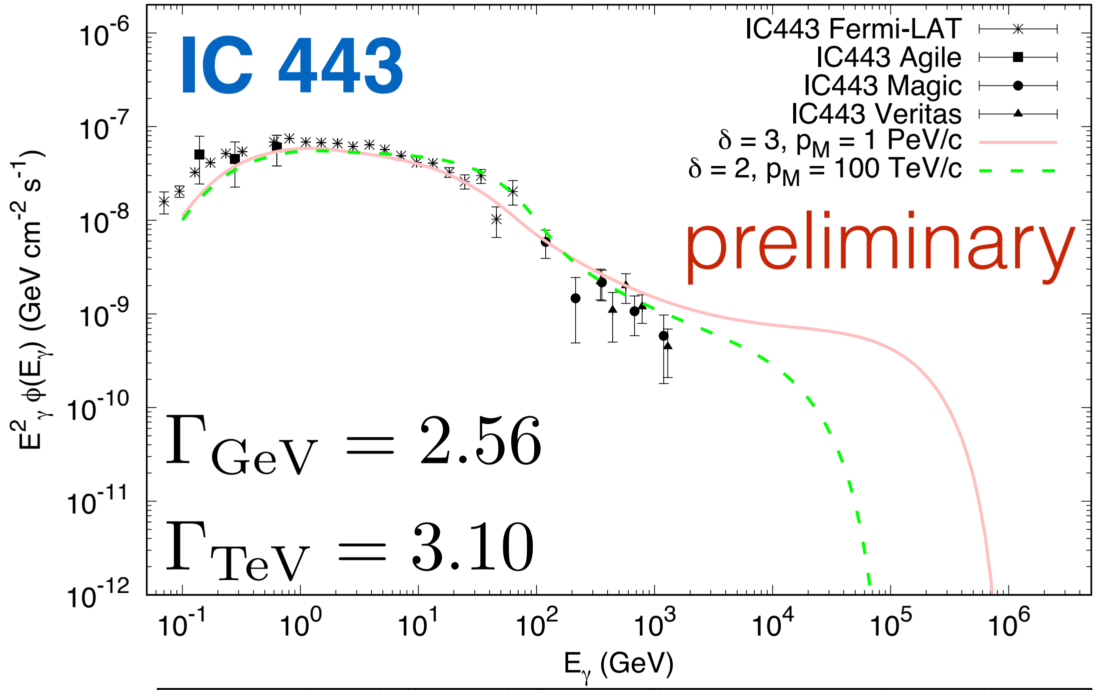
$$f_0(p) \propto p^{-4}$$
$$D(10\ \mathrm{GeV}/c) = 10^{27}\ \mathrm{cm^2 s^{-1}}$$
$$\xi_{\rm CR} = 10\%$$
$$n_{\rm up} = 1\ \mathrm{cm}^{-3}$$
$$T_{\rm SNR} = 10^4\ \mathrm{yr}$$
$$d = 1\ \mathrm{kpc}$$

$0 \leq r \leq R_{\rm SNR}$



$R_{\rm SNR} \leq r \leq 2R_{\rm SNR}$



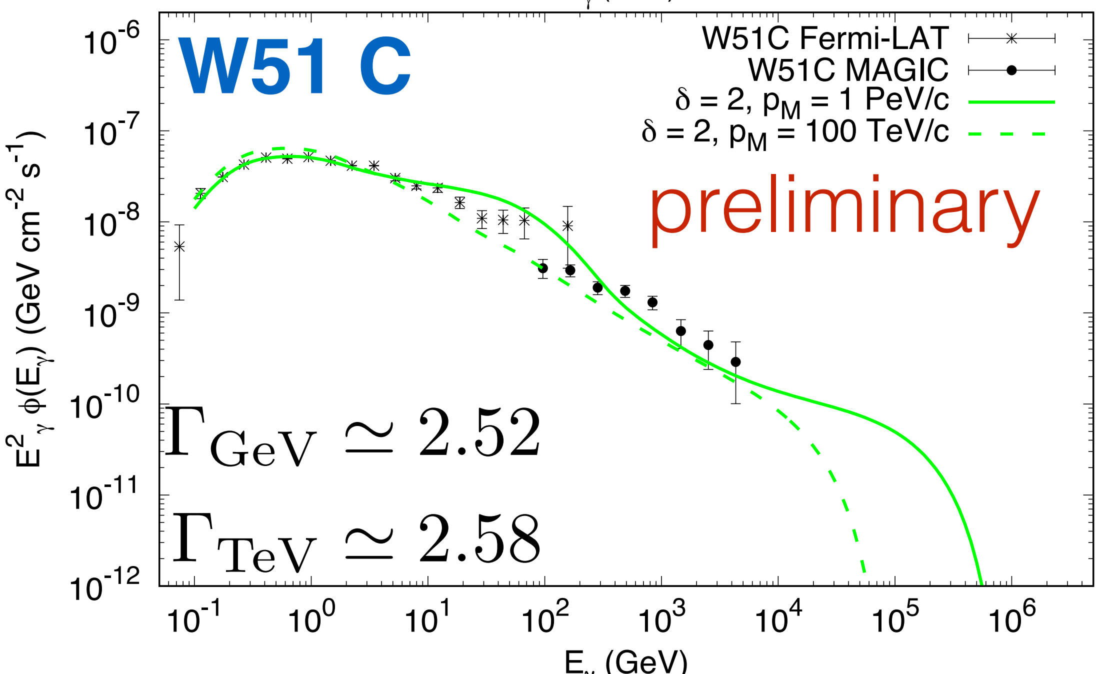


$$f_0(p) \propto p^{-4}$$

$$T_{\rm SNR} = 1.5 \times 10^4\,{\rm yr}, n_{\rm up} = 10\ {\rm cm}^{-3}$$

$$d = 1.5\ {\rm kpc}, \xi_{\rm CR} = 2\%$$

$$D(10\ {\rm GeV}/c) = 10^{27}\ {\rm cm}^2{\rm s}^{-1}$$

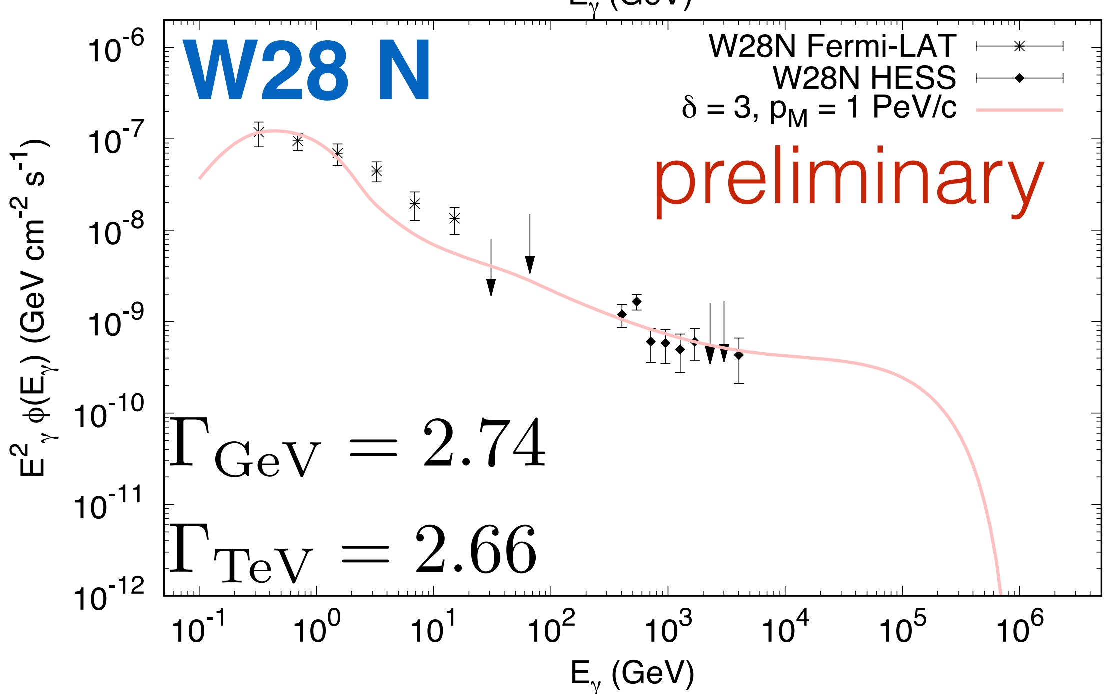


$$f_0(p) \propto p^{-(4+1/3)}$$

$$T_{\rm SNR} = 3 \times 10^4\,{\rm yr}, n_{\rm up} = 10\ {\rm cm}^{-3}$$

$$d = 5.4\ {\rm kpc}, \xi_{\rm CR} = 12\% - 15\%$$

$$D(10\ {\rm GeV}/c) = 3 \times 10^{26}\ {\rm cm}^2{\rm s}^{-1}$$



$$f_0(p) \propto p^{-4}$$

$$T_{\rm SNR} = 4 \times 10^4\,{\rm yr}, n_{\rm up} = 10\ {\rm cm}^{-3}$$

$$d = 2.0\ {\rm kpc}, \xi_{\rm CR} = 15\%$$

$$D(10\ {\rm GeV}/c) = 3 \times 10^{27}\ {\rm cm}^2{\rm s}^{-1}$$

$$D/D_{\rm Gal} \leq 0.3$$

Suppression of diffusion coefficient required:

- local turbulence?
- CR-induced turbulence (streaming instability)?

Malkov et al., ApJ 768 (2013) 63

Nava et al., MNRAS 461 (2016) 3552N

D'Angelo et al., MNRAS 474 (2018) 1944D

$$\delta \geq 2$$

How does turbulence evolve with time?

Needs to include damping effects (MHD cascade, ion-neutral friction).

$$\xi_{\rm CR} \simeq 2 - 20\%$$

Standard assumption in the SNR paradigm for the origin of GCRs.

Conclusions

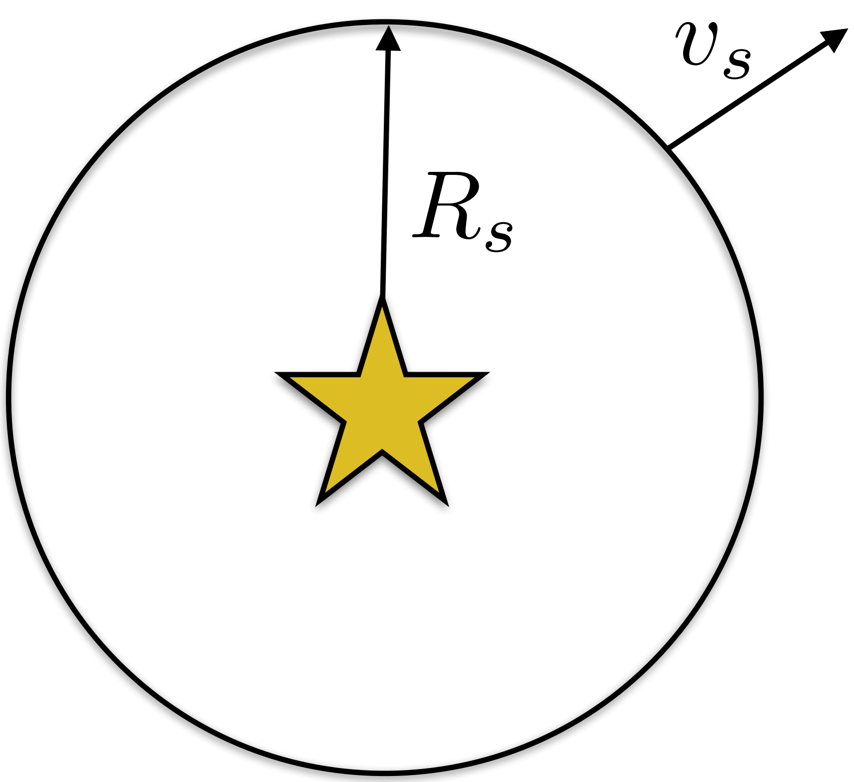
- Particle **escape** is a poorly understood mechanism, strongly embedded in the process of particle acceleration
  ⟶ it depends on the time evolution of magnetic turbulence;

- Reasonable arrangement of parameters can explain the **steep spectra** observed in the **HE and VHE** emission of several middle-aged SNRs (IC 443, W 51C, W 28)
  ⟶ constraints on escape from SNR population studies?

- Results obtained can be used in the future as a strategy to search for **PeVatrons**: TeV halos around SNRs observable with CTA?

Backup slides

The hydrodynamical evolution of an SNR



I. Ejecta-dominated stage

$$M_{\rm ej} \gg \frac{4}{3}\pi\rho R_s^3(t)$$

⟶ free expansion

II. Sedov-Taylor stage

$$M_{\rm ej} \sim \frac{4}{3}\pi\rho R_s^3(t)$$

⟶ energy conservation

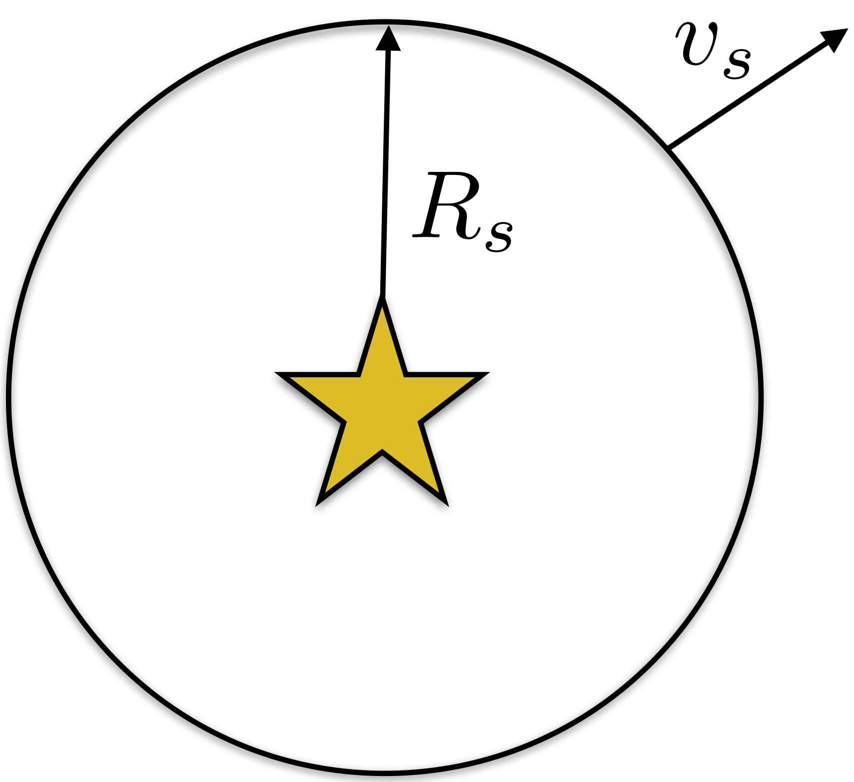
III. Radiative stage

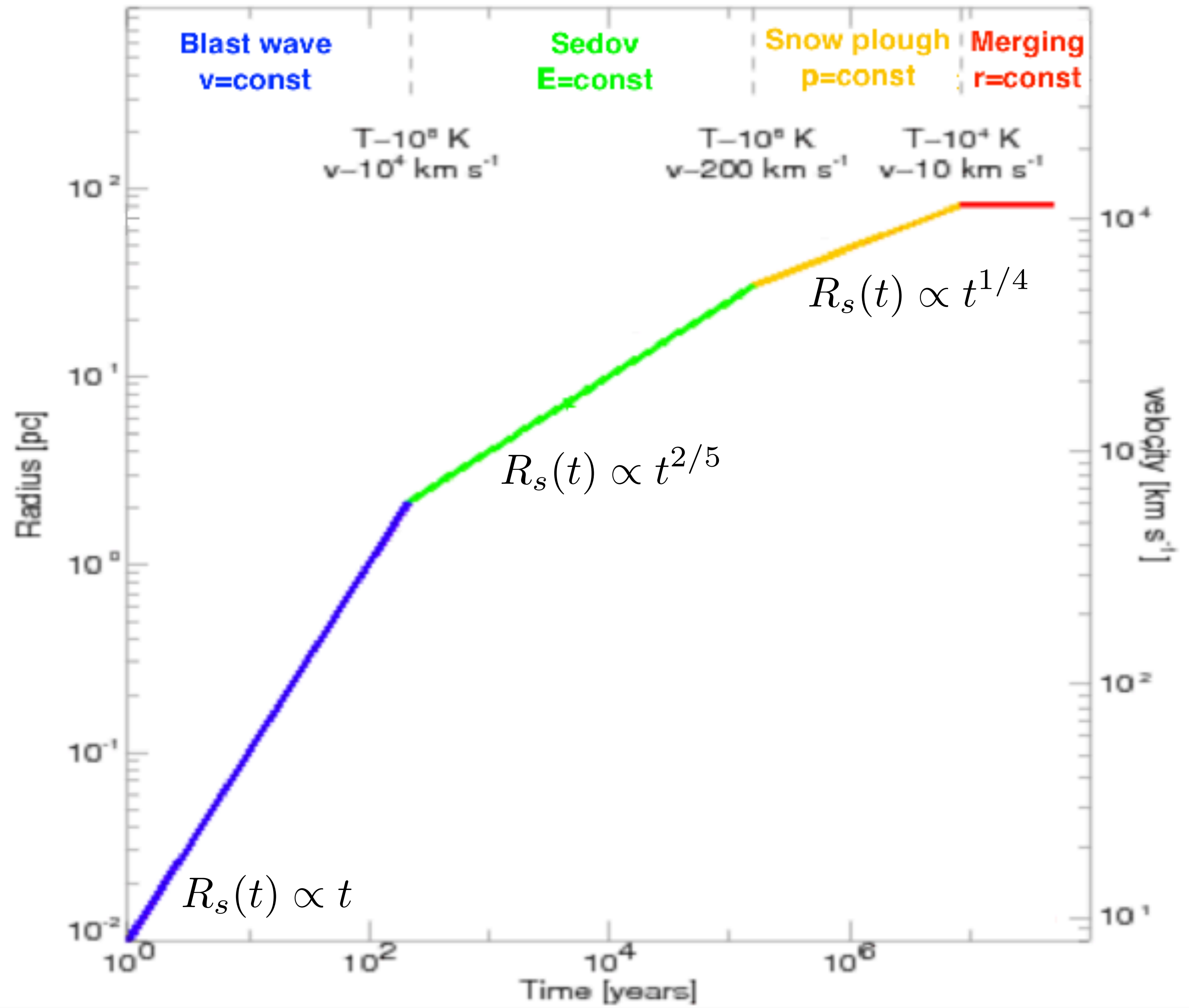
⟶ momentum conservation

IV. Merging phase

⟶ pressure comparable to ISM

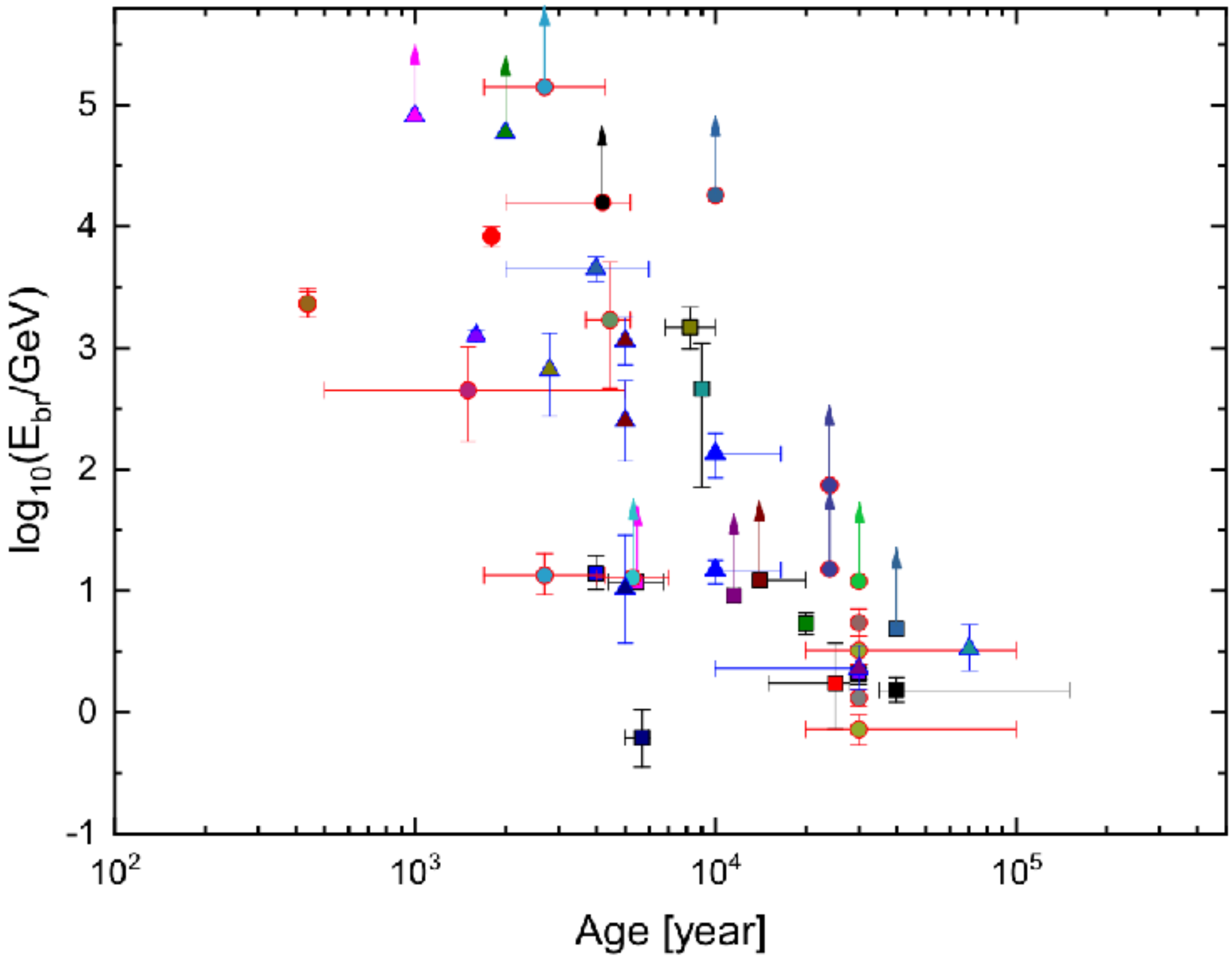
The hydrodynamical evolution of an SNR





$$t_{\rm Sed} \simeq 1.6 \times 10^3 \,{\rm yr} \left(\frac{E_{\rm SN}}{10^{51}\,{\rm erg}} \right)^{-1/2} \left(\frac{M_{\rm ej}}{10\, M_\odot} \right)^{5/6} \left(\frac{\rho_0}{1\, m_{\rm p}/{\rm cm}^3} \right)^{-1/3}$$

A population study of evolved SNRs

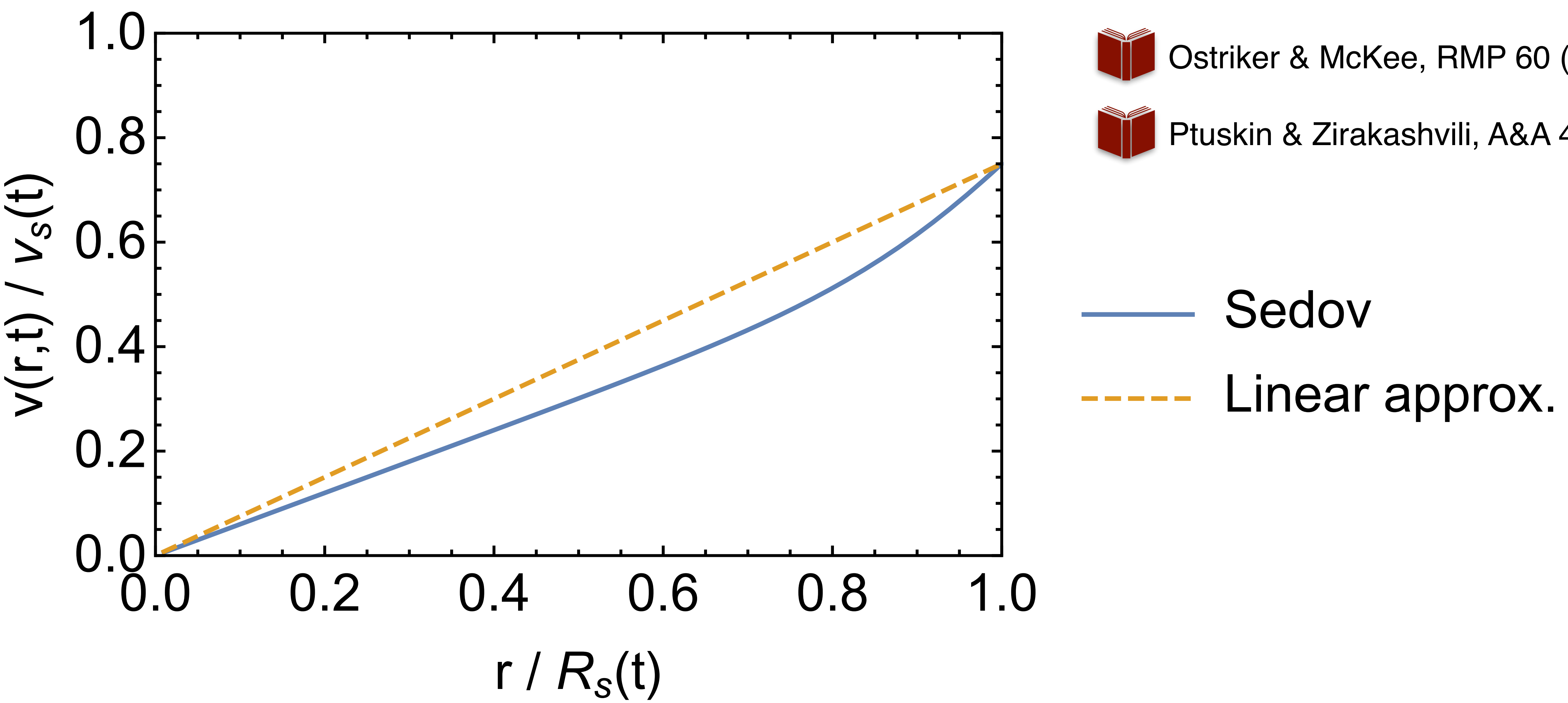


Zeng et al., ApJ 874 (2019) 50Z

The velocity profile in the downstream

The **velocity field** in the downstream plasma, adopted for solution of the confined particle equation, follows from the ST solution in a homogeneous medium

⟶ linear approximation: $$v(r,t) = \left(1 - \frac{1}{\sigma}\right)\frac{r}{R_s(t)} v_s(t)$$

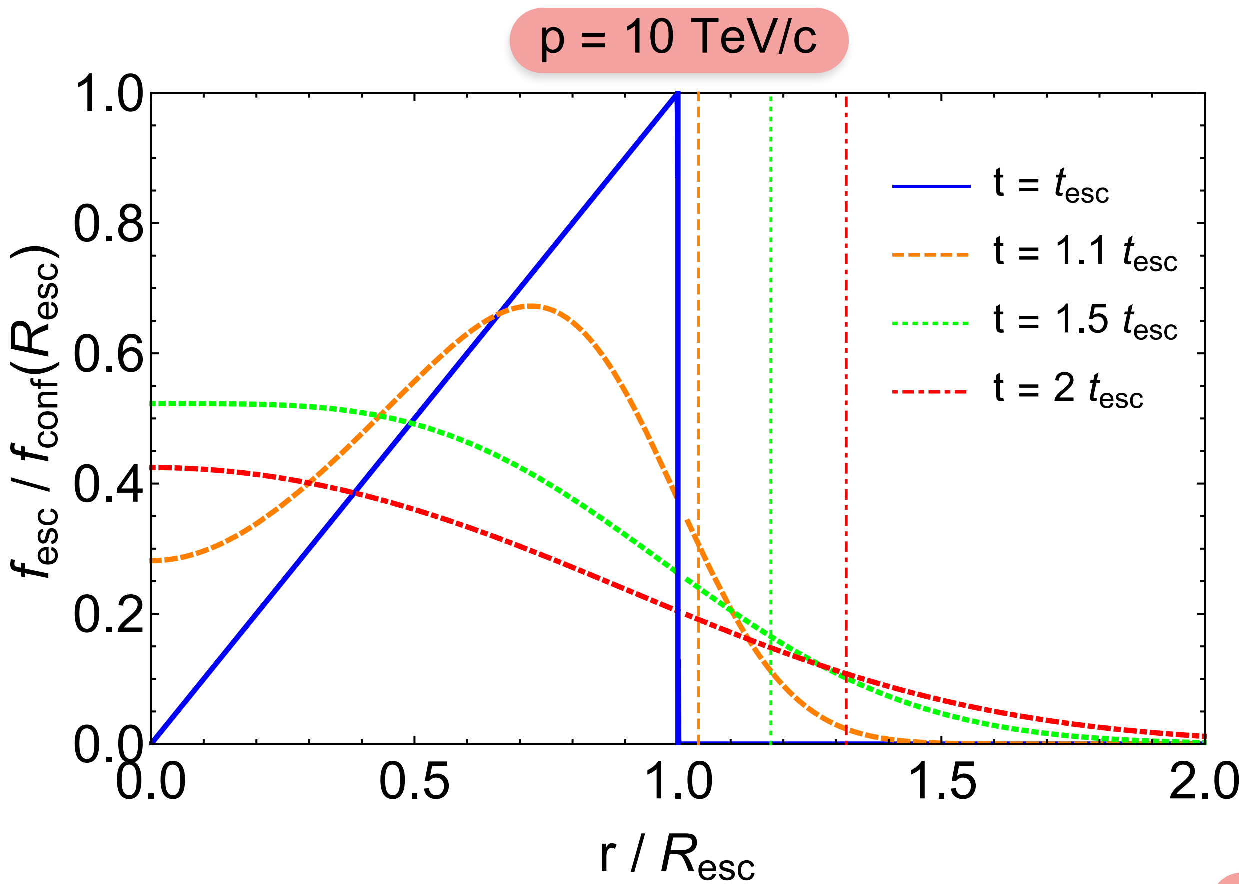


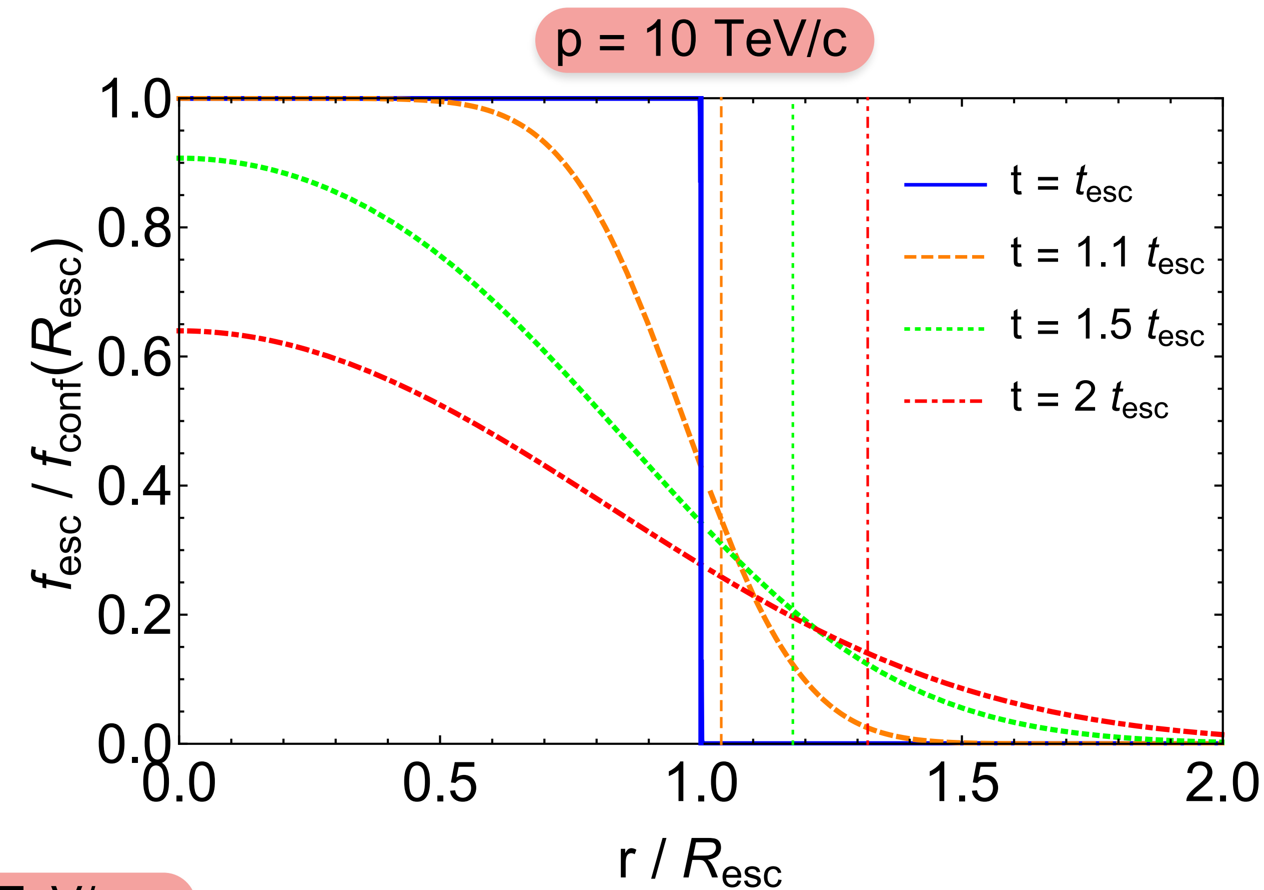
Ostriker & McKee, RMP 60 (1988) 1

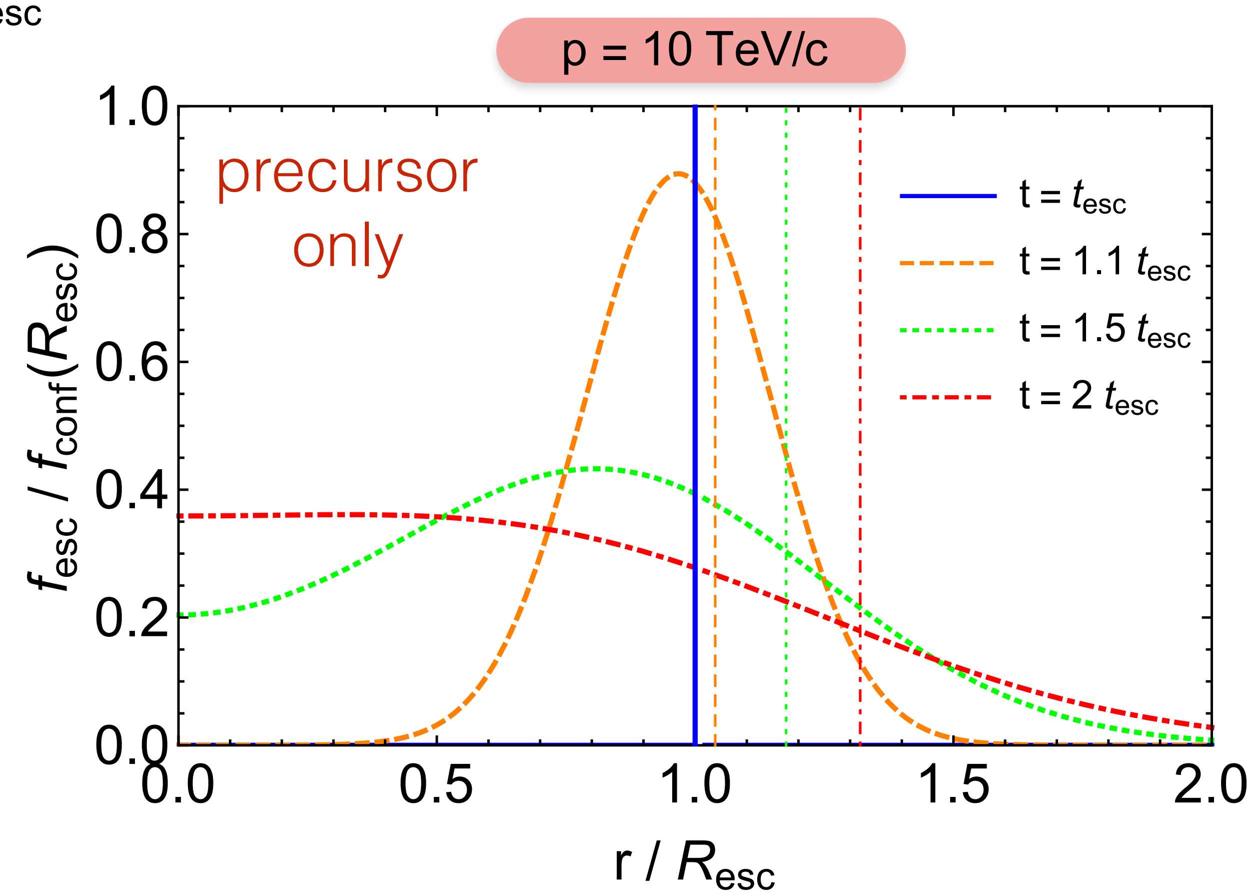
Ptuskin & Zirakashvili, A&A 429 (2005) 755

$\alpha = 4 + 1/3$

$\alpha = 4$



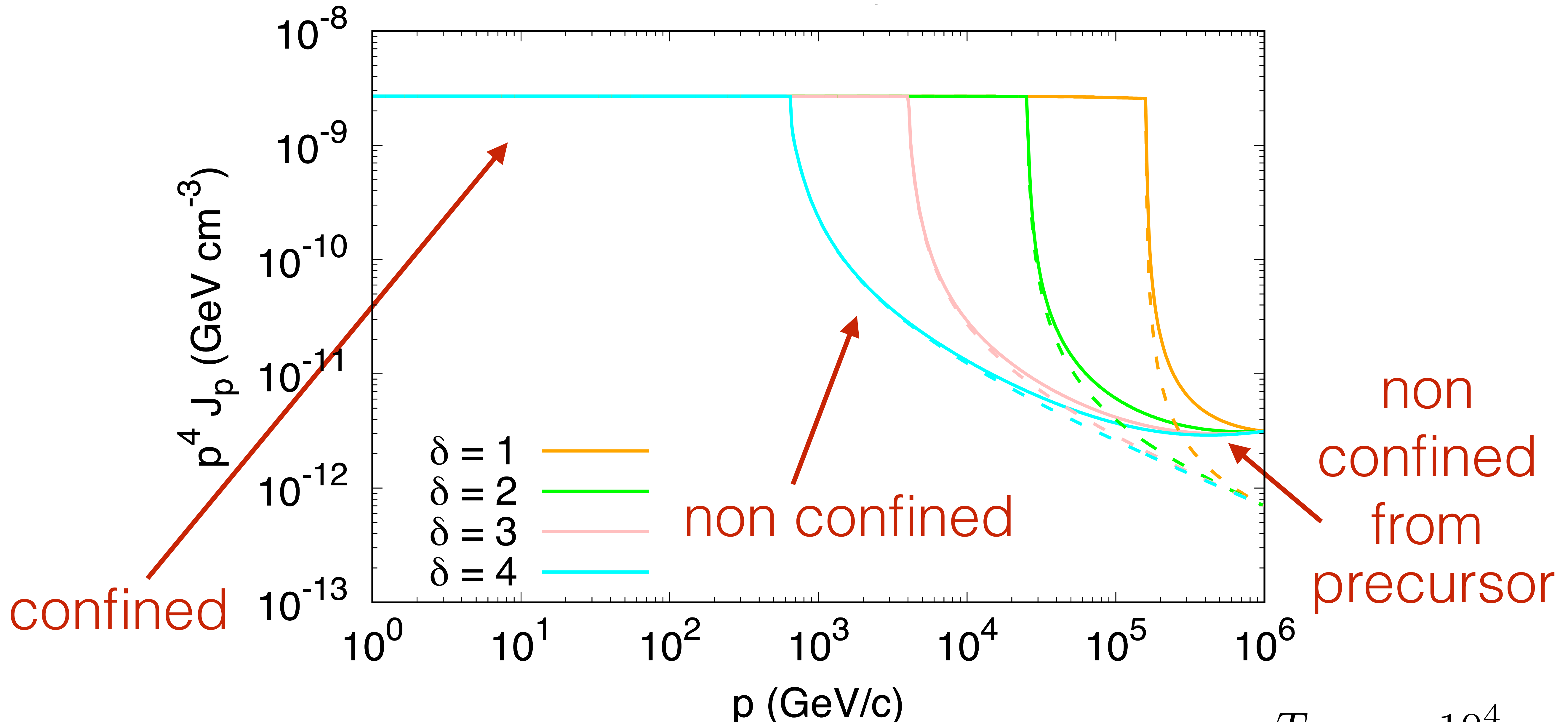




$\chi = 0.01$

The spectrum of protons inside the SNR

$$J_p^{\rm in}(t,p) = \frac{4\pi}{V_{\rm SNR}} \int_0^{R_{\rm sh}(t)} \left[f_{\rm esc}(t,r,p) + f_{\rm p,esc}(t,r,p) + f_{\rm conf}(t,r,p)\right] r^2 dr$$



$$D(p) = 10^{28} \left(\frac{pc}{10\ {\rm GeV}}\right)^{1/3} {\rm cm}^2 {\rm s}^{-1}$$

$T_{\rm SNR} = 10^4\,{\rm yr}$

$\xi_{\rm CR} = 10\%$

$n_{\rm up} = 1\,{\rm cm}^{-3}$

A model for particle propagation

1. Confined particle density ⟶ Method of characteristics

$$f_{\rm conf}(t,r,p) = f_0\left(t_0(t,r), p\left(\frac{R_s(t)}{R_s(t_0)}\right)^{3/4}\right)$$

adiabatic losses

2. Escaped particle density ⟶ Laplace transformation

$$\frac{f_{\rm esc}(r,t,p)}{f_{\rm conf}(t_{\rm esc},p)} = \frac{1}{2}\left[{\rm Erf}\left[\frac{R_+}{R_d}\right] + {\rm Erf}\left[\frac{R_-}{R_d}\right] + \frac{R_d}{\sqrt{\pi} r}\left(e^{-\left(\frac{R_+}{R_d}\right)^2} - e^{-\left(\frac{R_-}{R_d}\right)^2}\right)\right]\theta[t - t_{\rm esc}(p)]$$

$$R_+ = (R_{esc} + r) \qquad R_- = (R_{esc} - r) \qquad R_d(t,p) = 2D(p)\sqrt{t - t_{\rm esc}(p)}$$

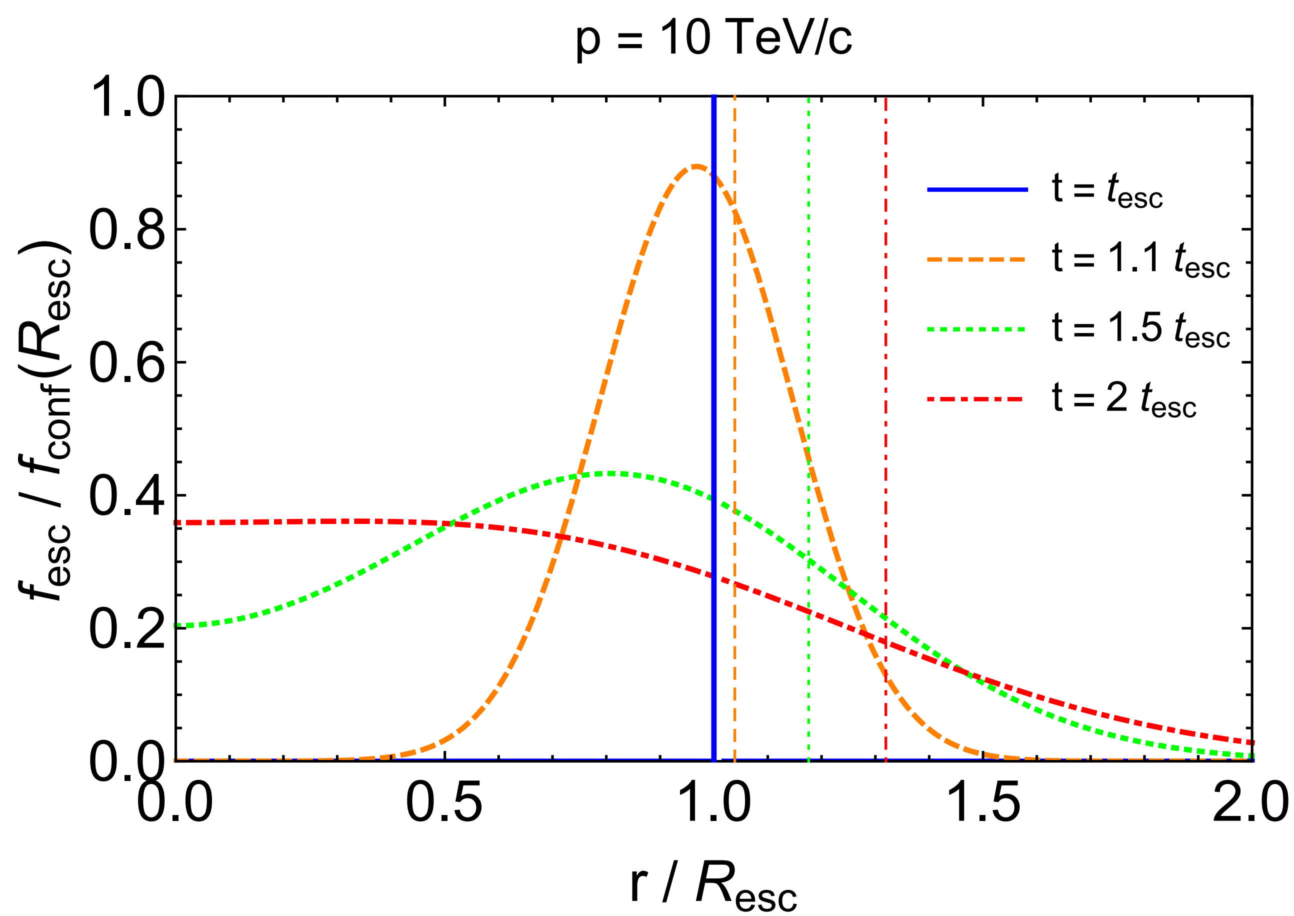
$$\alpha = 4$$

The precursor contribution to the density of non-confined particles

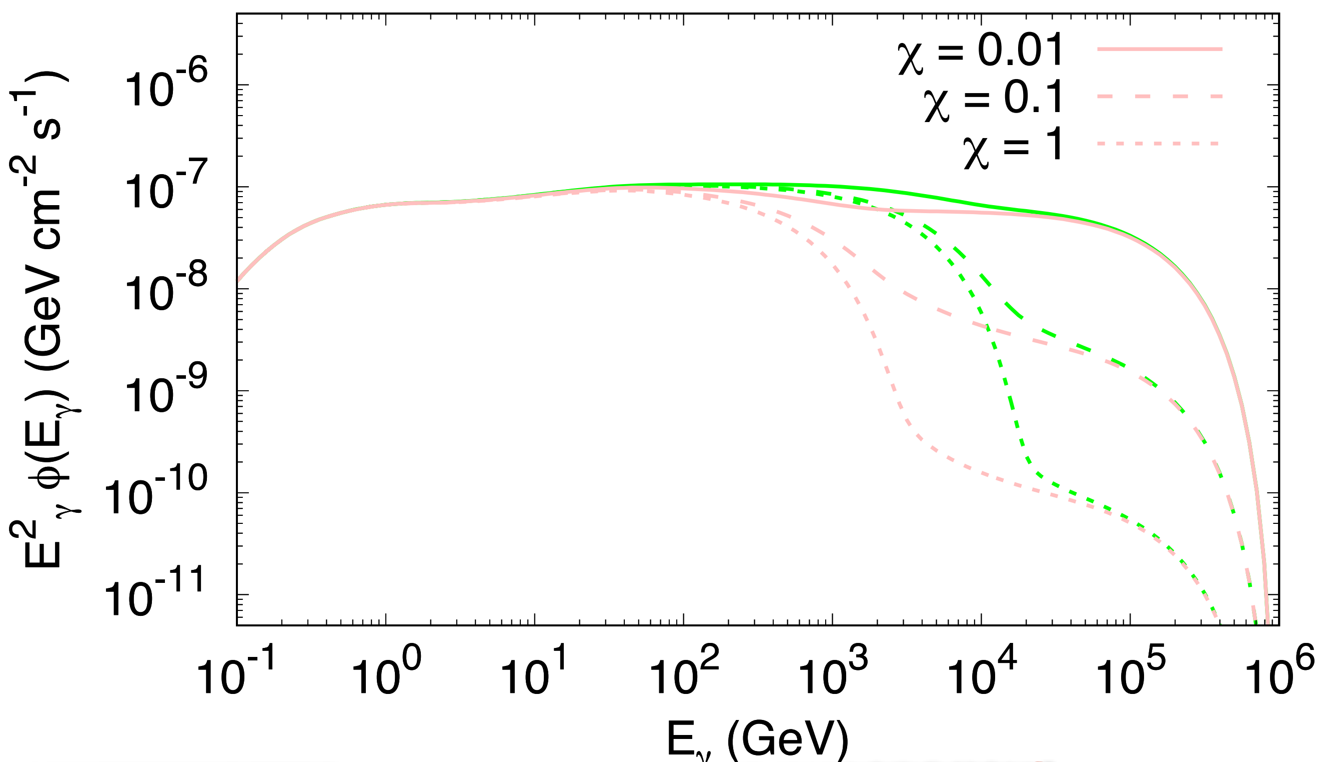
$$f_{\rm p}(t,r,p) = f_0(t,p)\exp\left[-\frac{u_{\rm sh}(t)}{D_{\rm p}(p)}(r-R_{\rm sh})\right]$$

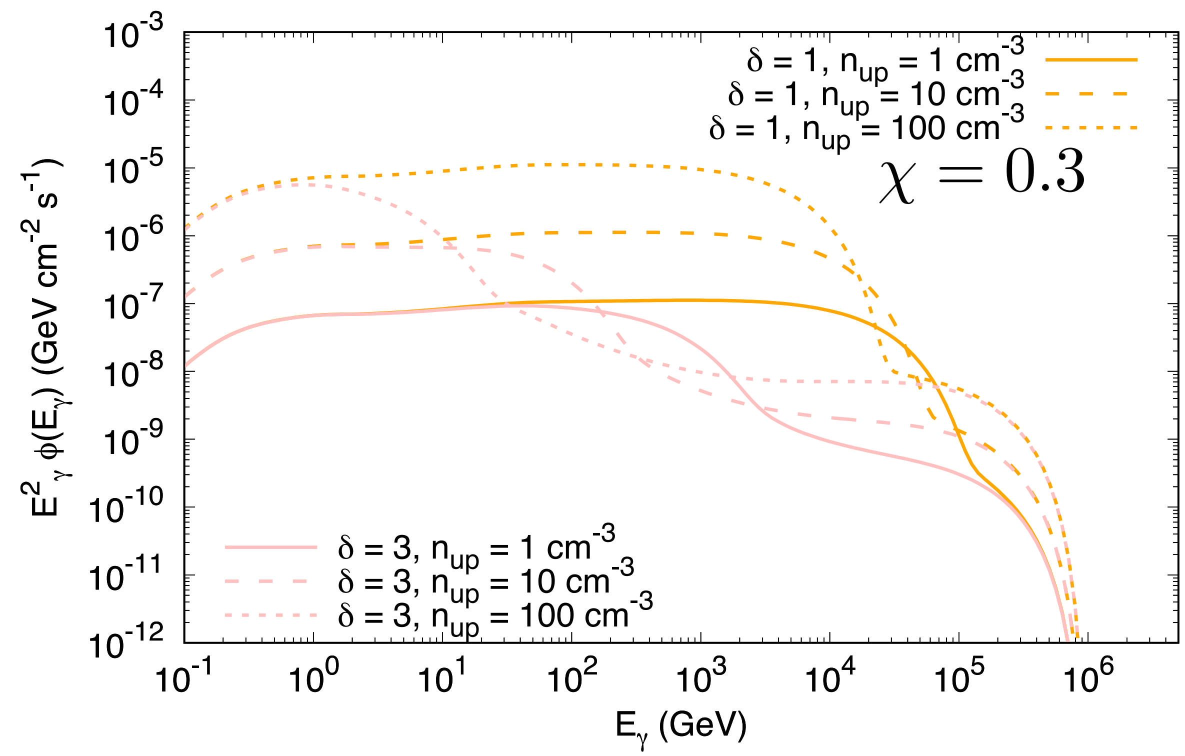
$$f_{\rm p,conf}(t,r,p) \simeq f_0(t,p)\frac{D_{\rm p}(p)}{u_{\rm sh}(t)}\delta(r-R_{\rm sh})$$

$$\longrightarrow \frac{f_{\rm p,esc}(r,t,p)}{f_0(p,t_{\rm esc})} = \frac{1}{\sqrt{\pi}}\frac{R_{\rm esc}}{R_d}\frac{D_p(p)}{v_s(t_{\rm esc})r}\left[e^{-(\frac{R_-}{R_d})^2} - e^{-(\frac{R_+}{R_d})^2}\right]\theta[t-t_{\rm esc}(p)]$$

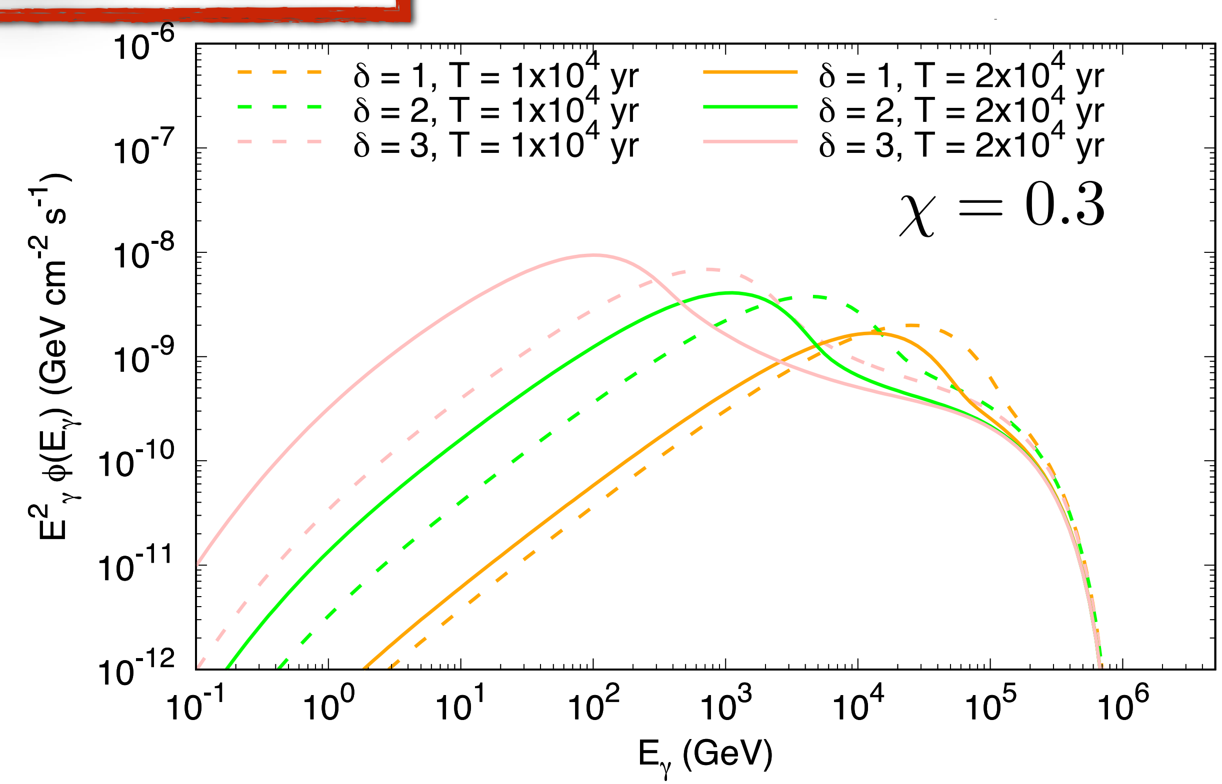


Volume integrated gamma-ray emission from hadronic interactions



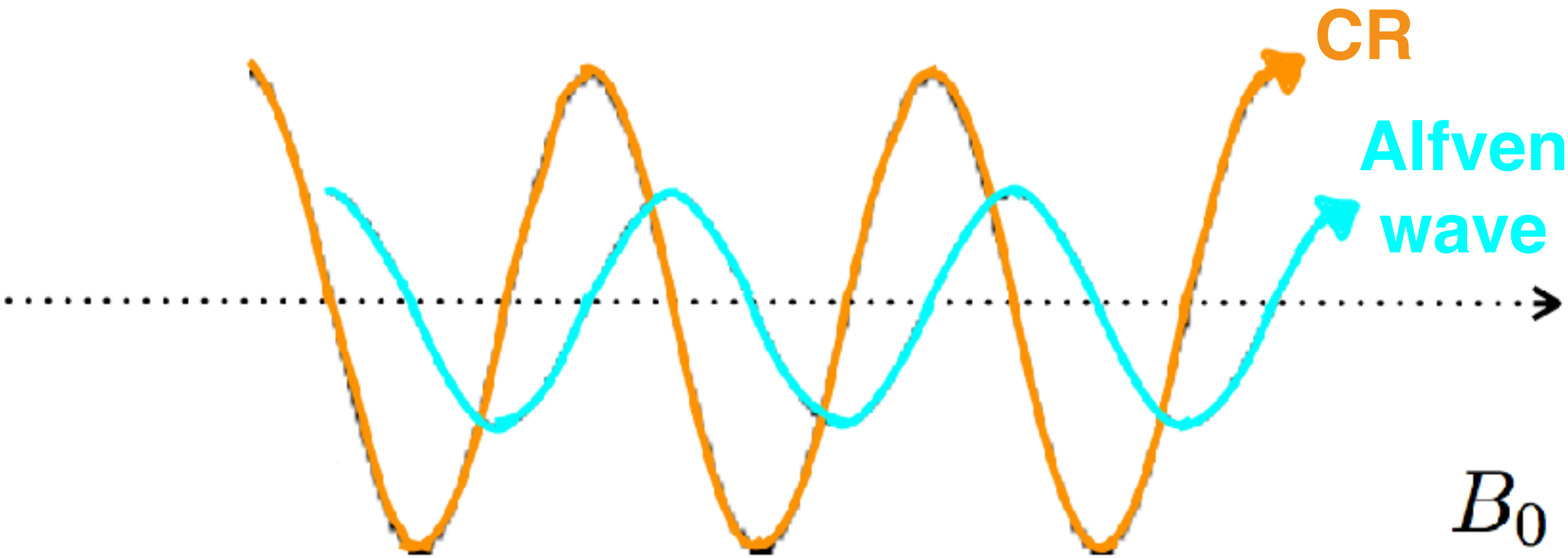


$$D(p) \equiv \chi D_{\rm Gal} = \chi 10^{28} \left(\frac{pc}{10\ {\rm GeV}}\right)^{1/3}\ {\rm cm^2 s^{-1}}$$



$$\xi_{\rm CR} = 10\%$$
$$n_{\rm up} = 1\ {\rm cm^{-3}}$$
$$T_{\rm SNR} = 10^4\ {\rm yr}$$
$$d = 1\,{\rm kpc}$$

Self-amplification of the magnetic field: the streaming instability

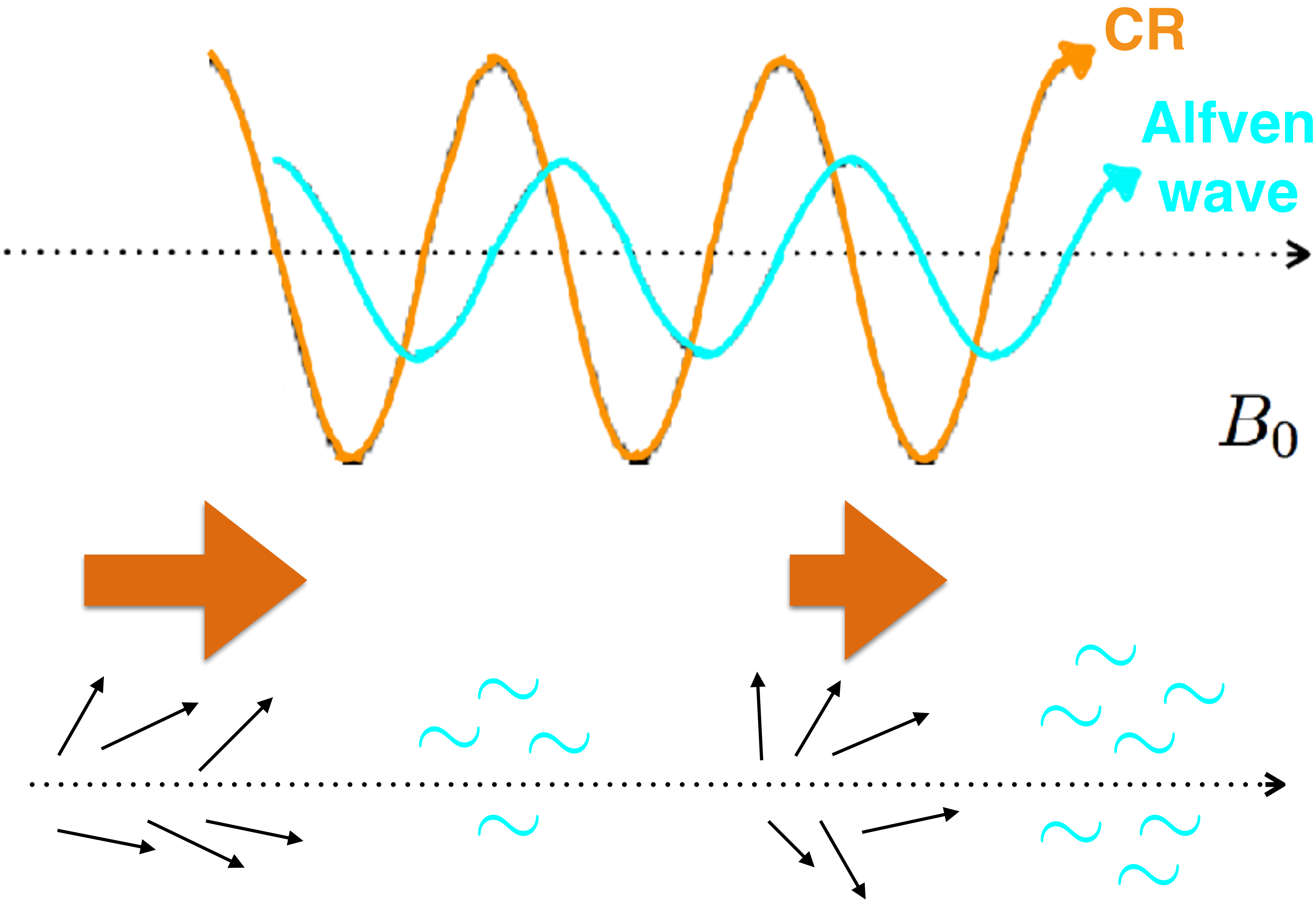


Kulsrud & Pearce, 156 (1969) 445

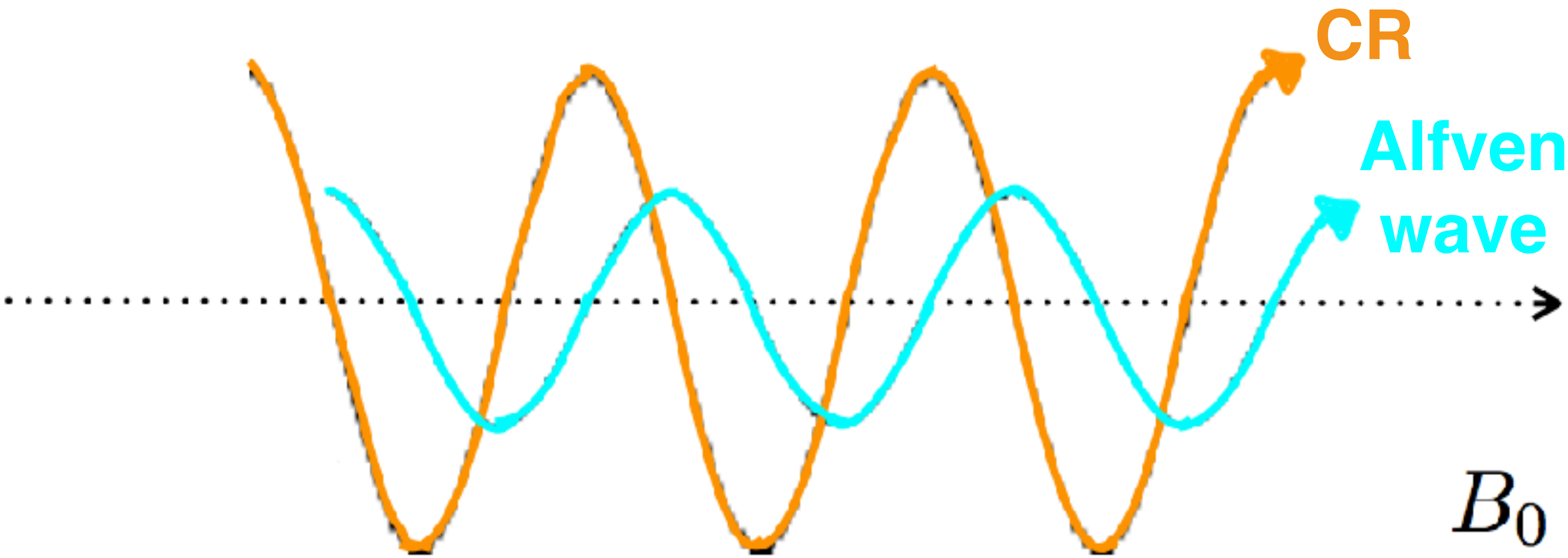
Skilling, ApJ 170 (1971) 265

$$r_L \sim \frac{1}{k}$$ → Power transfer from CRs to resonant magnetic waves

Self-amplification of the magnetic field: the streaming instability



Self-amplification of the magnetic field: the streaming instability



$$\left(\frac{\delta B(\mathbf{x},t)}{B_0}\right)^2 = \int \mathcal{F}(k,\mathbf{x},t)\, d\ln k$$

instability growth rate

$$\Gamma_{\rm CR}(k) = \frac{16}{3}\pi^2 \frac{v_A}{B_0^2 \mathcal{F}(k)} \left[p^4 v(p) \nabla f\right]_{p=p_{\rm res}}$$

Self-generated turbulence

$$\Gamma_{\rm CR}(k) = \frac{16\pi^2}{3}\frac{v_A}{B_0^2 \mathcal{F}(k)}\left[p^4 v(p)\frac{\partial f}{\partial r}\right]_{p=p_{\rm res}}$$

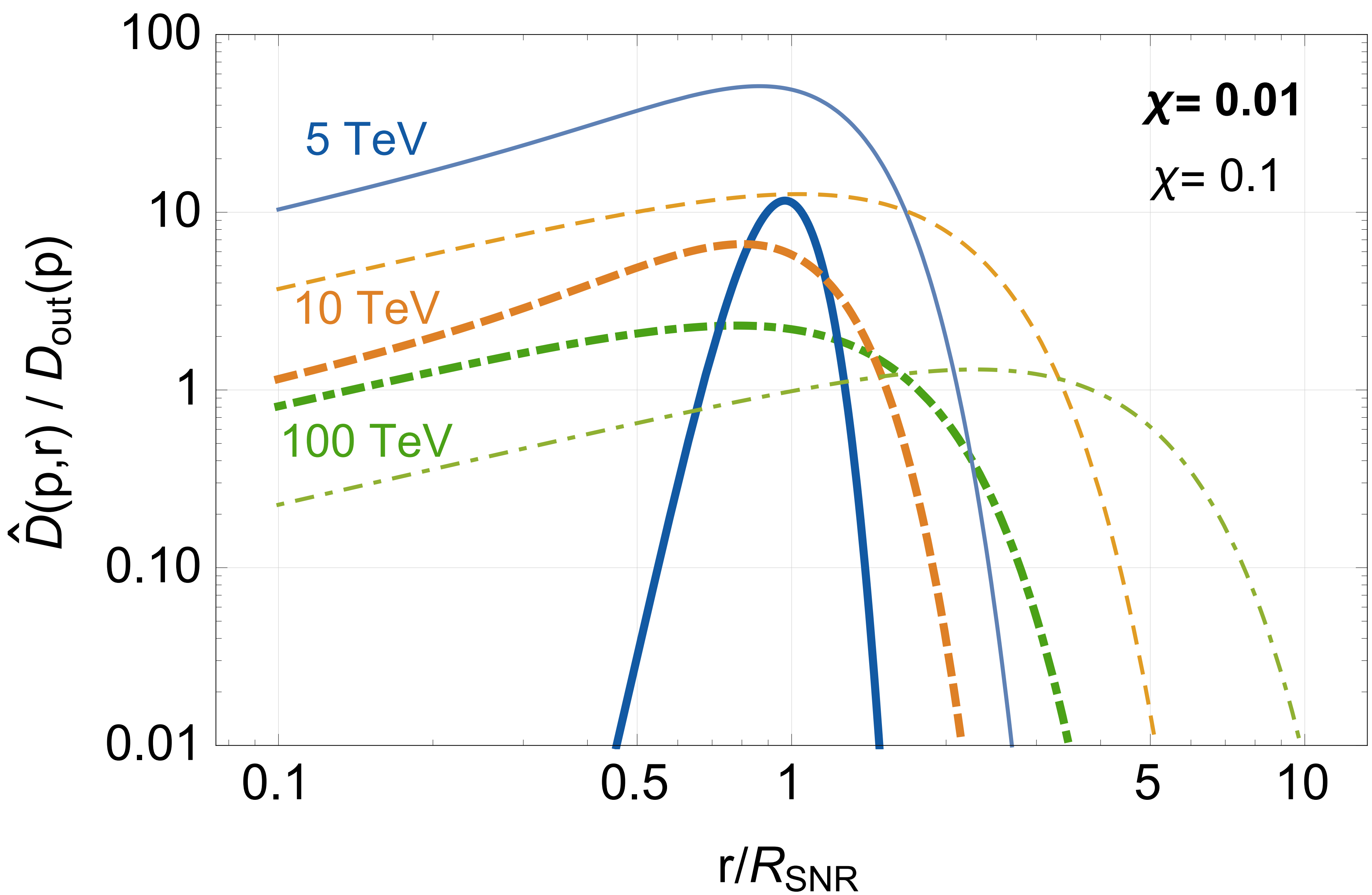
growth rate by resonant streaming instability

Skilling, ApJ 170 (1971) 265

$$\Gamma_{\rm NLD}(k) = (2c_k)^{-3/2}\, k v_A \sqrt{\mathcal{F}(k)}$$

non-linear damping rate

Ptuskin & Zirakashvini, A&A 403 (2003) 1



$\xi_{\rm CR} = 10\%$

$n_{\rm up} = 1\ {\rm cm}^{-3}$

$T_{\rm SNR} = 10^4\,{\rm yr}$

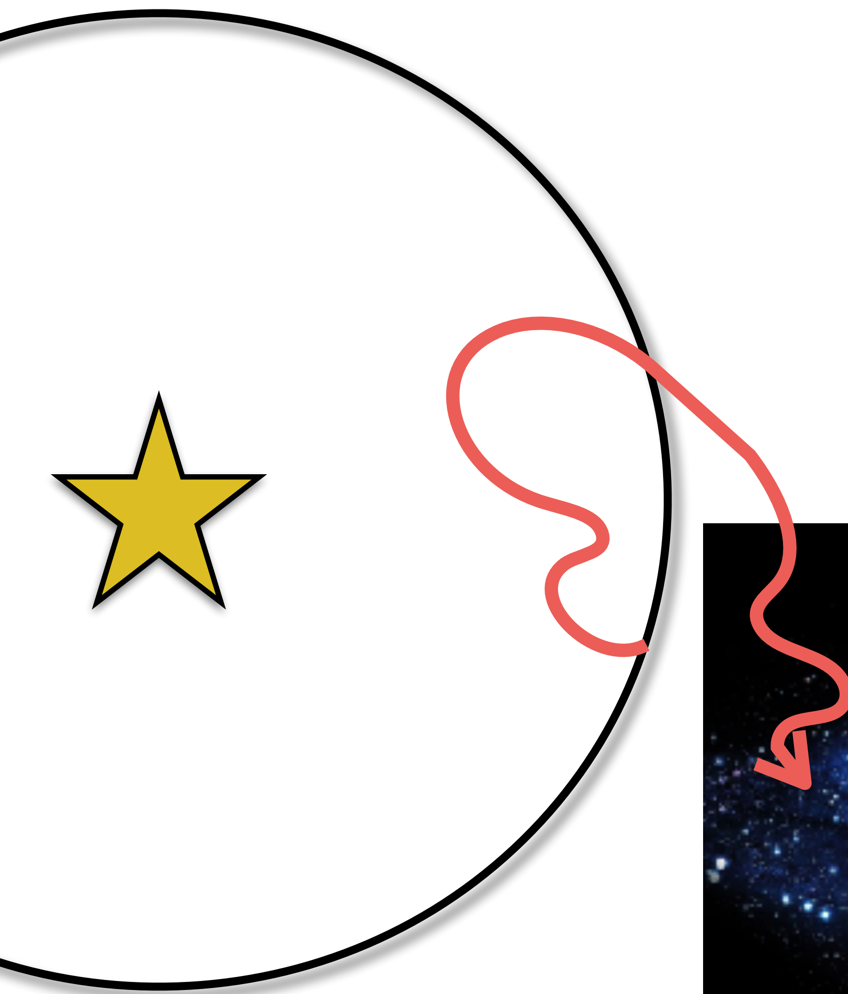
The CR spectrum injected into the Galaxy

$$f_{\rm inj}(p) = 4\pi \int_0^{R_{\rm esc}(p)} r^2 f_{\rm conf}\left(t_{\rm esc}(p), r, p\right) dr$$

$$\longrightarrow f_{\rm inj}(p) \propto v_{\rm esc}^2(p) R_{\rm esc}^3(p) \frac{p^{-\alpha}}{\Lambda(p)}$$

$$\longrightarrow f_{\rm inj}(p) \propto \frac{p^{-\alpha}}{\Lambda(p)}$$

during the ST phase



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during the ST phase

- Ultra-relativistic limit ($p \gg m_p c$):

$$f_{\rm inj}(p) \propto \begin{cases} p^{-\alpha} & \alpha > 4 \\ p^{-4} & \alpha < 4 \end{cases}$$

Bell & Shure, MNRAS 437 (2014) 2802

Cardillo, Amato & Blasi, APh 69 (2015) 1

The CR spectrum injected into the Galaxy

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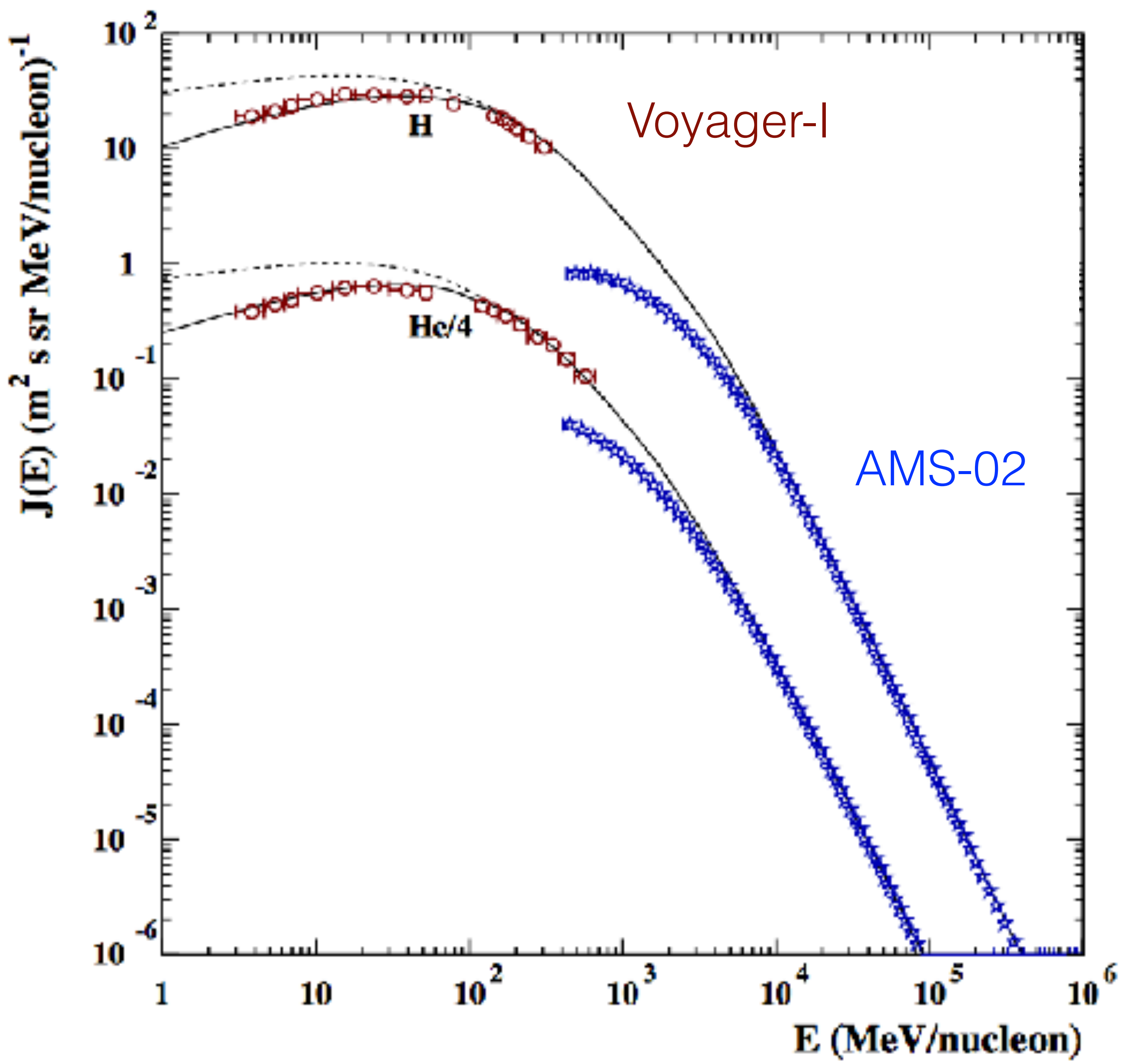
during the ST phase

- Ultra-relativistic limit ($p \gg m_p c$): $f_{\rm inj}(p) \propto \begin{cases} p^{-\alpha} & \alpha > 4 \\ p^{-4} & \alpha < 4 \end{cases}$
- Non-relativistic limit ($p \ll m_p c$): $f_{\rm inj}(p) \propto \begin{cases} p^{-\alpha} & \alpha > 5 \\ p^{-5} & \alpha < 5 \end{cases}$

The CR spectrum injected into the Galaxy



- Non-relativistic limit ($p \ll m_p c$):

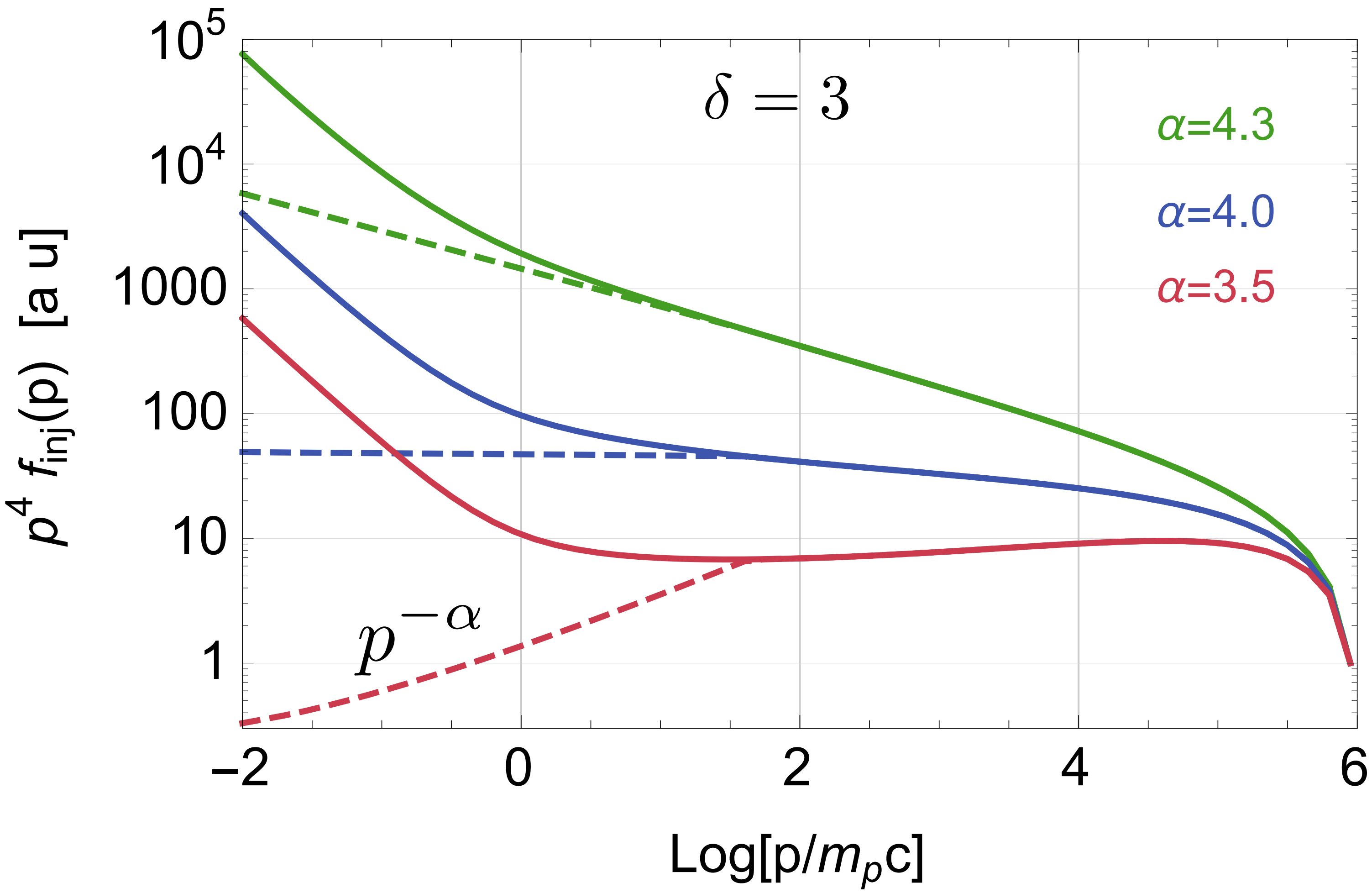
$$f_{\rm inj}(p) \propto \begin{cases} p^{-\alpha} & \alpha > 5 \\ p^{-5} & \alpha < 5 \end{cases}$$

The CR spectrum injected into the Galaxy

What if acceleration suddenly stops when the remnant enters the radiative phase of its evolution?

$$T \leq 10^6\,\mathrm{K},\; v_\mathrm{s} \simeq 200\,\mathrm{Km/s},\; t_\mathrm{rad} \simeq 47\,\mathrm{kyr}$$

$$p_{\max,0}(t_\mathrm{rad}) \simeq 40\,\mathrm{GeV/c}$$



→ particles with $p < p_{\max,0}(t_\mathrm{rad})$ do not suffer further adiabatic losses and are soon released in the ISM