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Strong first-order phase transitions in the Next-to-Minimal Supersymmetric Standard Model

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Based on: PA, C. Balazs, A. Fowlie, G. Pozzo, G. White, Y. Zhang,
JHEP 1912 (2019) XXX, arXiv:1908.11847

MONASH
University

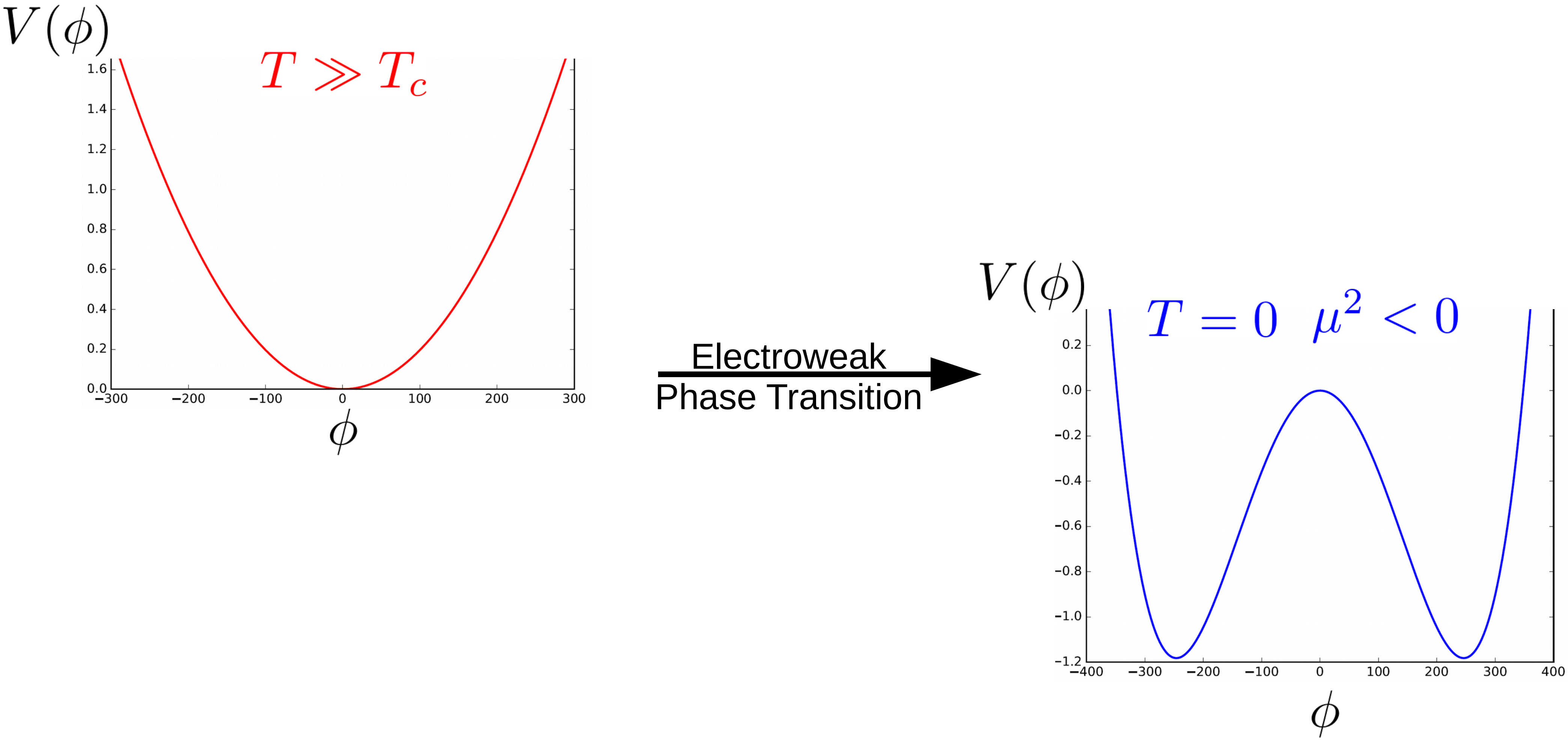
What is a
First Order Phase Transition?

Electroweak Phase Transitions

Finite temperature potential: $\mathrm{V} = \mathrm{V}(\phi, T) = \mu^2|\phi|^2 + \lambda|\phi|^4 + V_T$

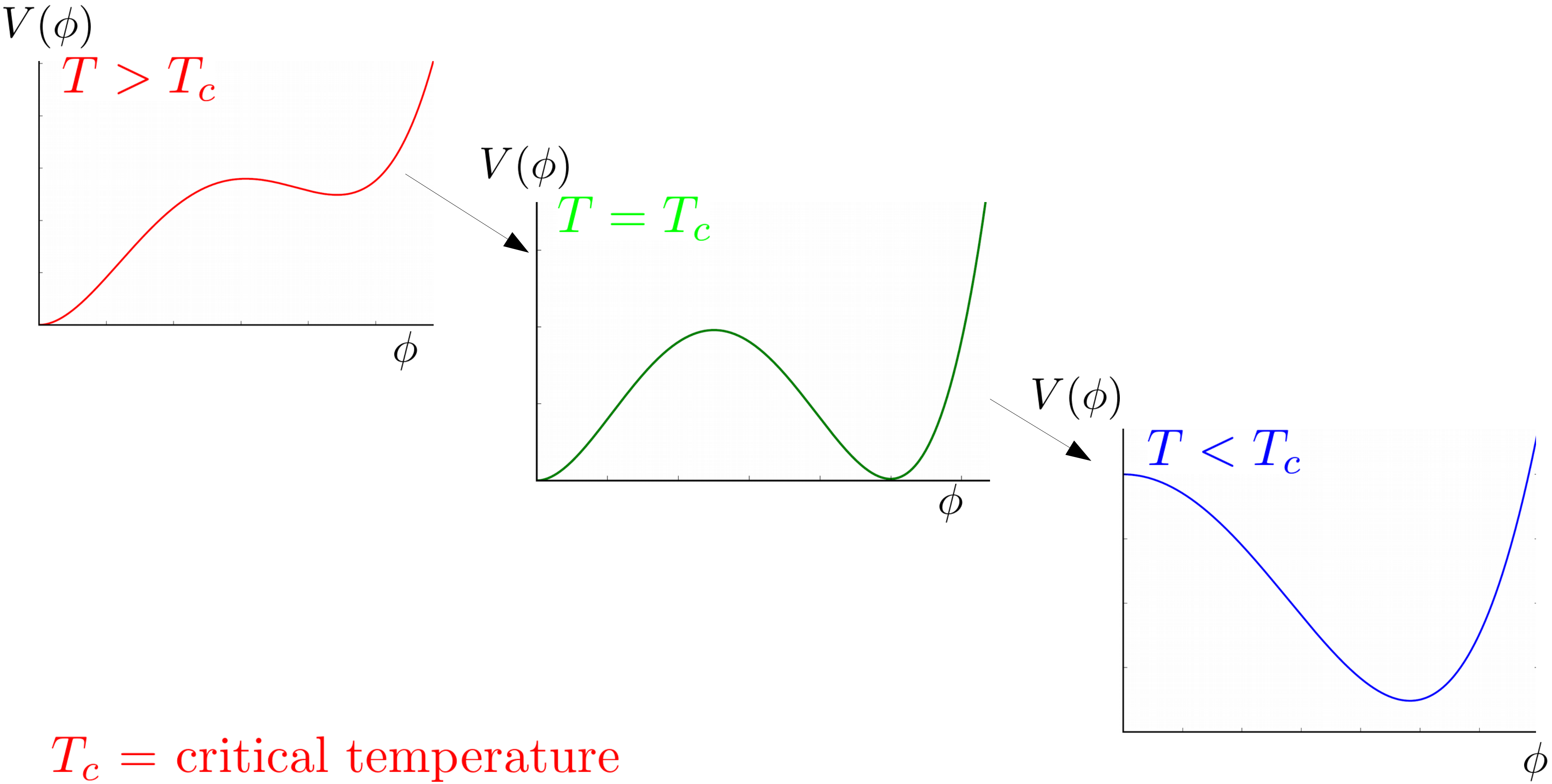
Zero temperature $\mu^2 < 0 \Rightarrow$ Mexican hat shaped potential

High temperature expansion $\Rightarrow V_T \sim T^2|\phi|^2$



Electroweak Phase Transition

First Order Phase Transitions



$$T_c = \text{critical temperature}$$
$$\equiv \text{Temperature where } V \text{ at minima are degenate}$$

Why are
Strong first order phase transitions interesting?

Matter anti-matter asymmetry

Baryon asymmetry of the universe:

$$Y_B := \frac{n_B - n_B}{s} \approx \frac{n_B}{s} = 8.65 \pm 0.09 \times 10^{11}$$ [Planck 2015]

To generate an asymmetry we need:

1) Baryon number violation
2) Charge and Charge-Parity violation
3) Departure from thermal equilibrium

Sakharov conditions

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$\Big\}$ Sakharov conditions

First order phase transition → Departure from thermal equilibrium

$$\Gamma \sim \exp[-E_{\rm Sph}(T_c)/T_c]$$

$$E_{\rm Sph}(T) \approx \frac{4\pi}{g}\langle\phi(T_c)\rangle$$

$$\Rightarrow \frac{\langle\phi(T_c)\rangle}{T_c} \equiv \text{“strength”}$$

For sufficient sphaleron suppression inside bubble:

$$\frac{\langle\phi(T_c)\rangle}{T_c} \gtrsim 1$$

Gravitational Waves

Bubbles of different phase colliding are violent events

→ Gravitational waves

FOPTs give gravitational waves through three effects:

$$\Omega_{\mathrm{GW}}h^2 \approx \underset{\substack{\text{Bubble}\\ \text{wall}\\ \text{collisions}}}{\Omega_{\mathrm{col}}h^2} + \underbrace{\underset{\substack{\text{Sound}\\ \text{waves}}}{\Omega_{\mathrm{sw}}h^2} + \underset{\text{turbulance}}{\Omega_{\mathrm{turb}}h^2}}_{\text{Dominant contributions}}$$

Again we want *strong* FOPT:

$$\frac{\langle\phi(T_c)\rangle}{T_c} \gtrsim 1$$

So now it is simple?

- ✔ Find EWSB minimum at T=0 and high T with a single minimum at origin

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  ➔ BSM physics needed for EWBG

- Cosmological history can be much more complicated with multiple intermediate phase transitions

- Need more sophisticated method than bisection to trace phases and calculate critical temperatures

Next-to-Minimal Supersymmetric Standard Model (NMSSM)

$$\begin{aligned}\mathcal{W}_{\rm NMSSM} \quad = \quad & (Y_u)_{ij}\,\hat{Q}_i \cdot \hat{H}_u\,\hat{u}^c_j + (Y_d)_{ij}\,\hat{Q}_i \cdot \hat{H}_d\,\hat{d}^c_j + (Y_e)_{ij}\,\hat{L}_i \cdot \hat{H}_d\,\hat{e}^c_j \\ & +\lambda\,\hat{S}\,\hat{H}_u \cdot \hat{H}_d + \frac{1}{3}\kappa\,\hat{S}^3,\end{aligned}$$

- Arguably the simplest supersymmetry extension without fine tuning (solves "mu-problem" of the MSSM)
- The MSSM has no strong FOPT unless $\mathrm{m}_{\tilde{t}} \lesssim m_t$

$$\begin{aligned}V^{\rm tree}_{\rm NMSSM} \quad = \quad & |\lambda S|^2\left(|H_u|^2 + |H_d|^2\right) + \left|\lambda H_u \cdot H_d + \kappa S^2\right|^2 \\ & +\frac{1}{8}(g^2 + g'^2)(|H_u|^2 - |H_d|^2)^2 + \frac{1}{2}g^2|H_u^\dagger H_d|^2 \\ & +m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2 + m^2_S|S|^2 + [\lambda A_\lambda S H_u \cdot H_d + \frac{1}{3}\kappa A_\kappa S^3 + {\rm h.c.}]\end{aligned}$$

$$V_{\rm NMSSM} = V^{\rm tree}_{\rm NMSSM} + V^{\text{1-loop CW}} + V_T + V_{\rm Daisy}$$

Numerical Calculations

- **PhaseTracer** development version for phases and critical temperatures
- **FlexibleSUSY-2.1.0** with **SARAH-4.12.3**: EFT calculation of Higgs potential, masses and effective couplings
- **Lilith-1.1.4_DB-17.05** and **HiggsBounds—5.3.2beta** for checking Higgs signals and limits
- **MultiNest-3.10** for efficent sampling with, $\chi^2 = \chi^2_{\text{Higgs}} + \chi^2_{\text{SFOPT}} + \chi^2_{\text{LEP}}$

Parameter	Range	Metric
λ	$0,\ \pi/2$	flat
$\lvert\kappa\rvert$	$0,\ \pi/2$	flat
$\lvert A_\lambda\rvert$	$0,\ 10\,\text{TeV}$	hybrid
$\lvert A_\kappa\rvert$	$0,\ 10\,\text{TeV}$	hybrid
$\lvert A_t\rvert$	$0,\ 10\,\text{TeV}$	hybrid
m_{SUSY}	$1,\ 10\,\text{TeV}$	log
$\lvert v_S\rvert$	$0,\ 10\,\text{TeV}$	hybrid
$\tan\beta$	$1,\ 60$	log

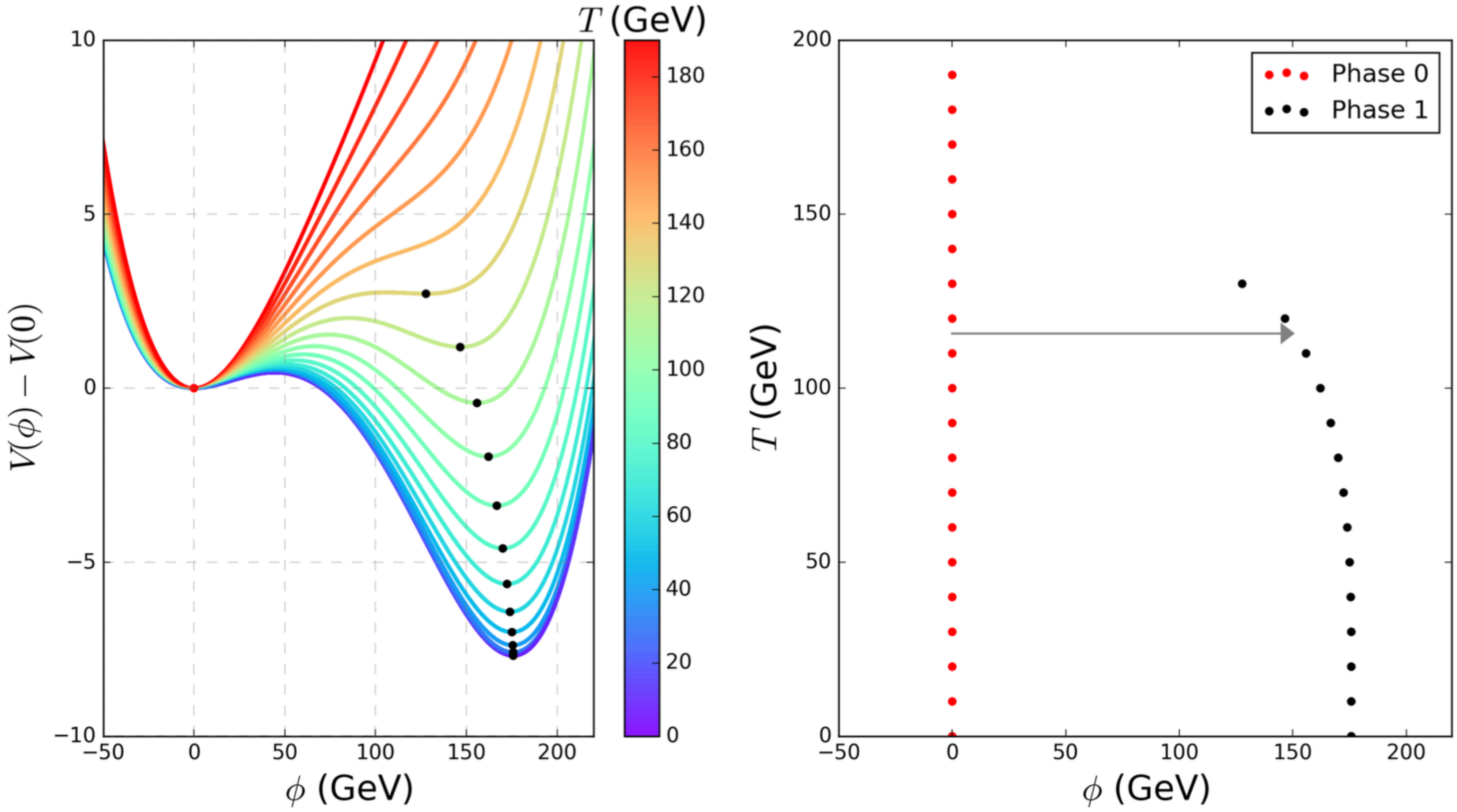
We collected more than 3,000,000 valid points …

Classifying Phase Transitions

We classified phase transitions according to the first transition:

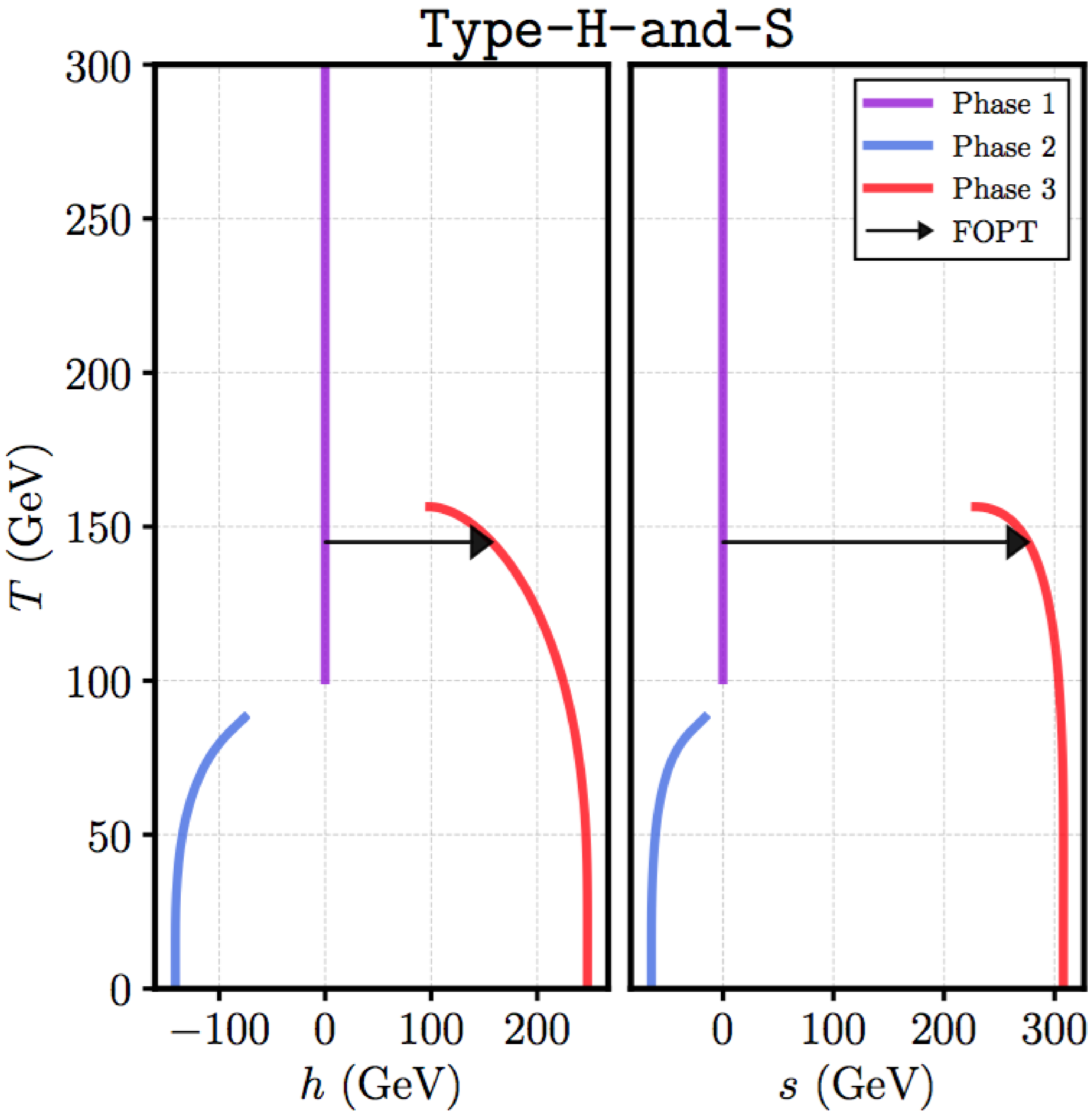
- Type-H-and-S
- Type-only-H
- Type-only-S-maintain
- Type-only-S-flip

To show these phase transition patterns I will be show using plots like the right panel:



Classifying Phase Transitions: Type-H-and-S

First phase transition gives EW and singlet VEVs



First (and only) PT at:
T = 145 GeV:

$$(0,0,0) \to (106,117,276)$$

Note: $\mathrm{h} := \sqrt{h_u^2 + h_d^2}$

$$\text{and } \gamma_{EW} := \frac{\Delta h(T_c)}{T_c}$$

FOPT strength:

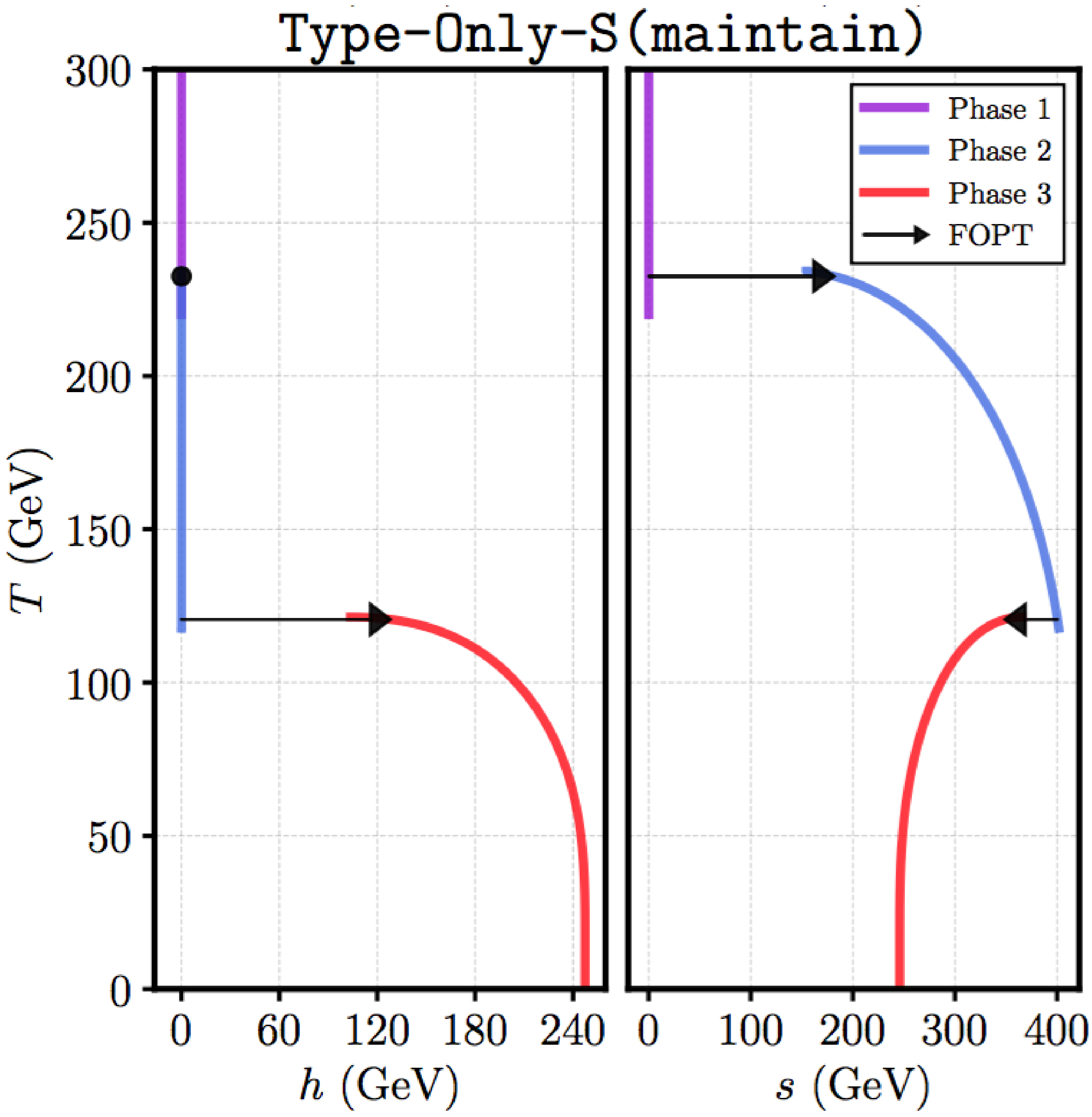
$$\gamma_{EW} = 1.09$$

Nucleation temperature:

$$T_N = 116 \text{ GeV}$$

Classifying Phase Transitions: Type-Only-S-maintain

First transition gives only singlet a VEV, with same sign as $\langle S \rangle$ at $T = 0$



First PT:

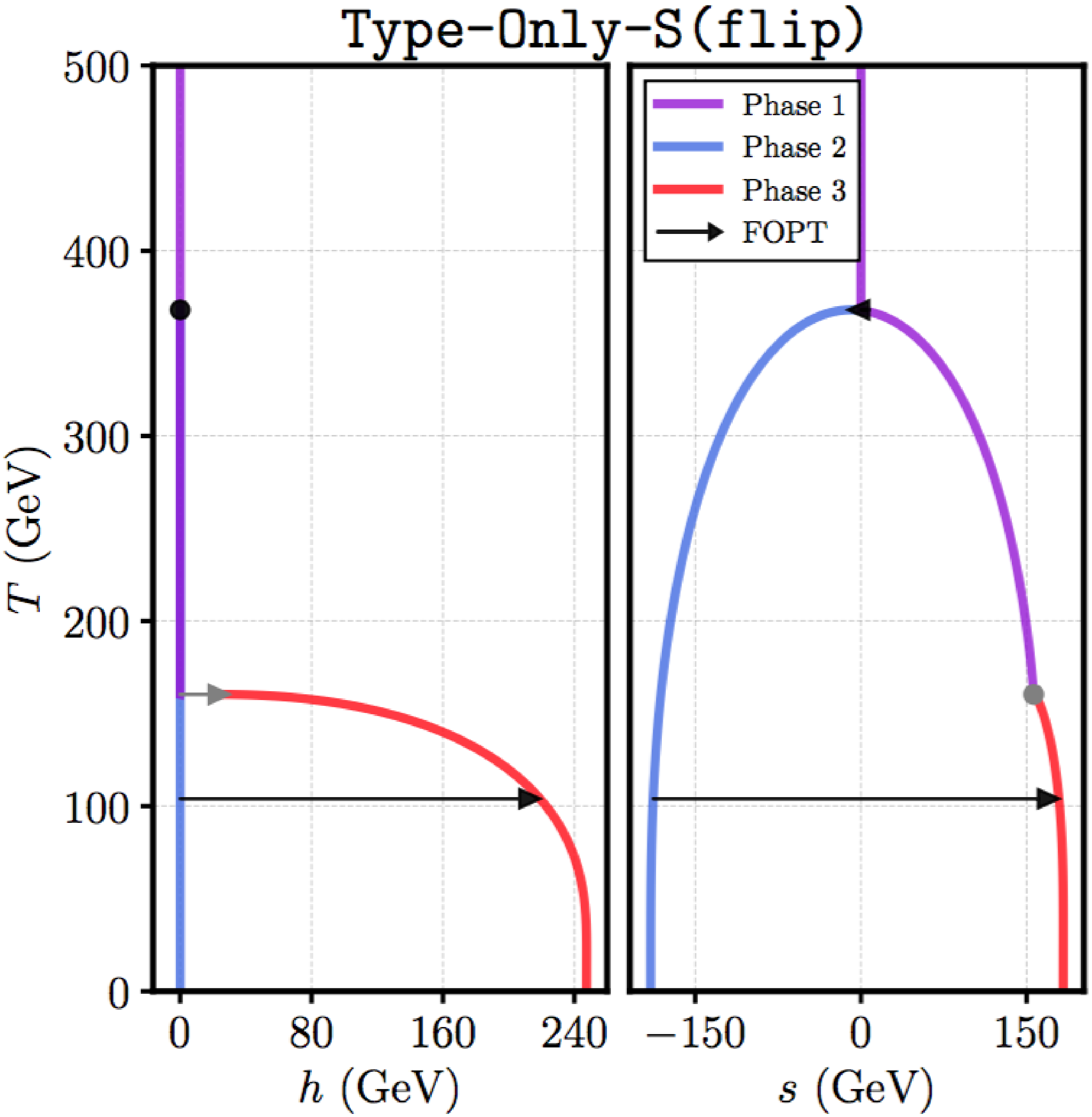
$$T_c = 233\mathrm{GeV}$$
$$\gamma_s = 0.78$$
$$T_N = 230\mathrm{GeV}$$

Second PT:

$$T_c = 121\mathrm{GeV}$$
$$\gamma_{\mathrm{EW}} = 1.1$$
$$T_N = 119\mathrm{GeV}$$

Classifying Phase Transitions: Type-Only-S-flip

First transition gives only singlet a VEV, with opposite sign to $\langle S \rangle$ at $T = 0$



First PT:

$$T_c = 368\text{GeV}$$

$$\gamma_s << 1$$

$$T_N = 367\text{GeV}$$

Second PT:

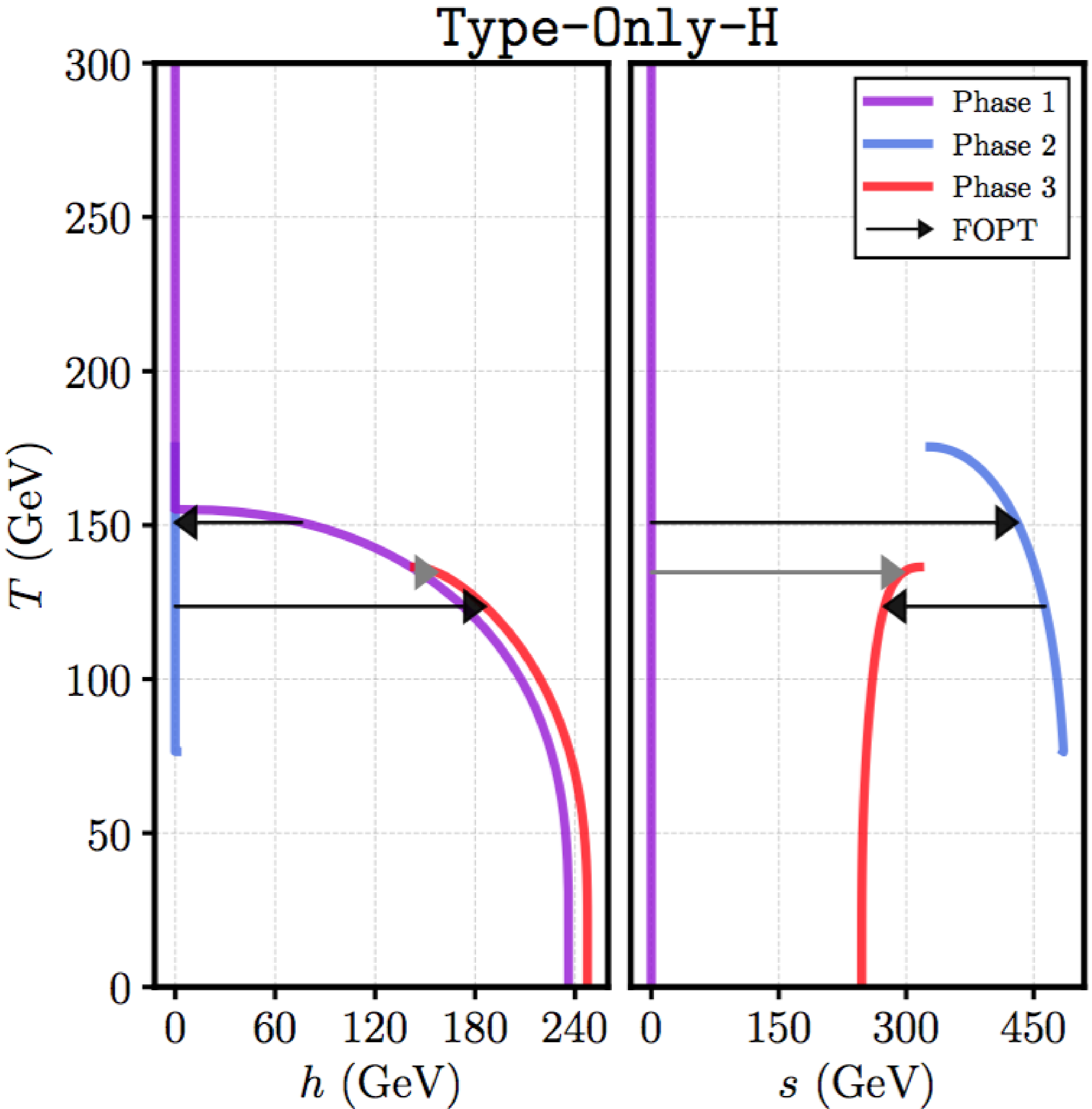
$$T_c = 104\text{GeV}$$

$$\gamma_{\text{EW}} = 2.1$$

No Bubble Nucleation!

Classifying Phase Transitions: Type-Only-H

The first phase transition breaks EW symmetry without a singlet VEV



First PT:

$$T_c = 155\text{GeV}$$

Crossover PT

Second PT:

$$T_c \approx 150\text{GeV}$$

$$\gamma_s \approx 2.7$$

No Bubble Nucleation!

Third PT:

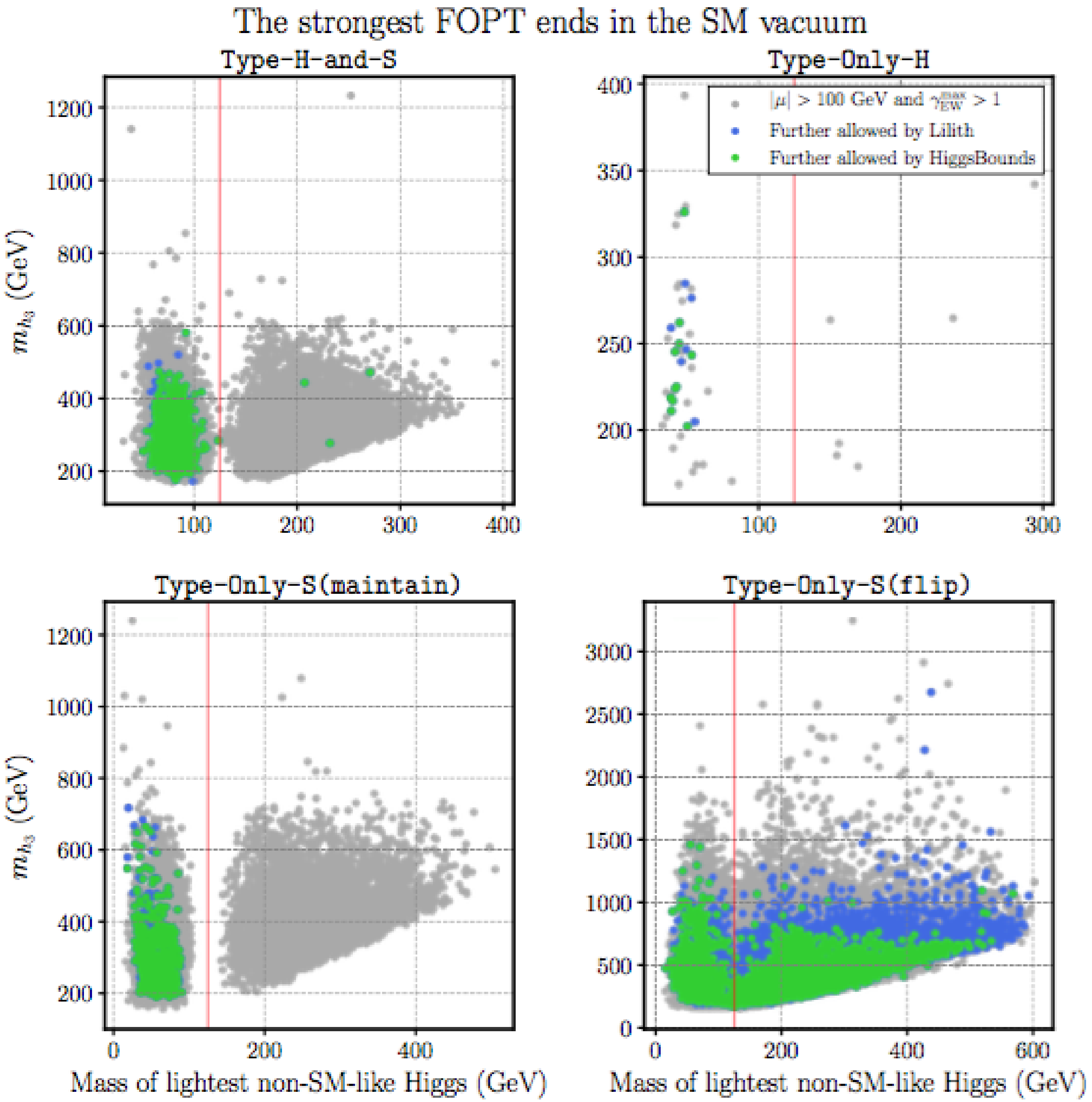
$$T_c = 124\text{GeV}$$

$$\gamma_{\text{EW}} = 1.5$$

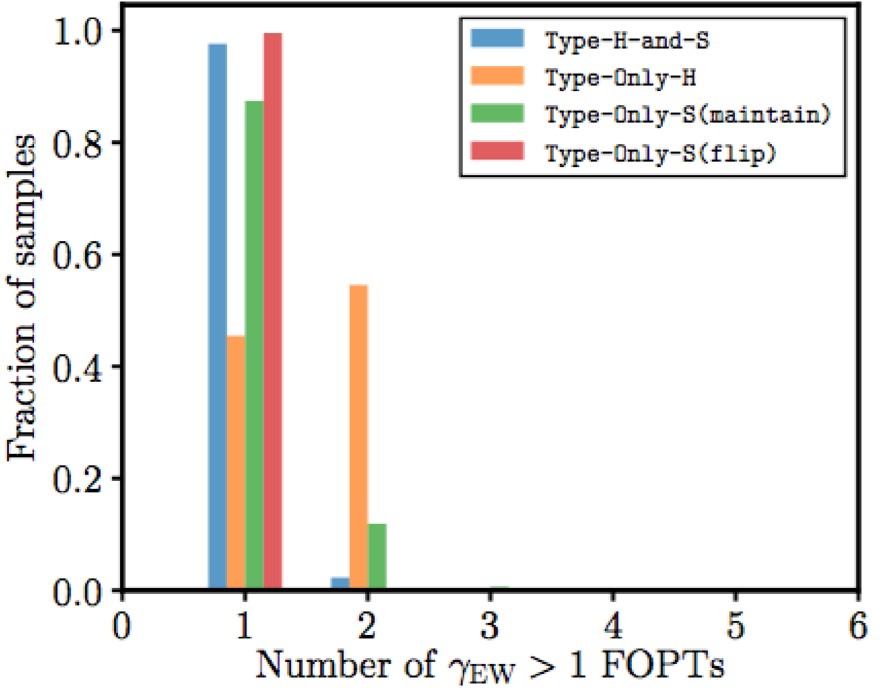
$$T_N = 119\text{GeV}$$

Higgs masses

$$\chi^2_{\text{Higgs}} - \min \chi^2_{\text{Higgs}} \leq 6.18,\ \ \mu_{\text{eff}} \geq 100 \text{ and } \gamma_{\text{EW}} \geq 1$$



The strongest FOPT may not end in the SM vacuum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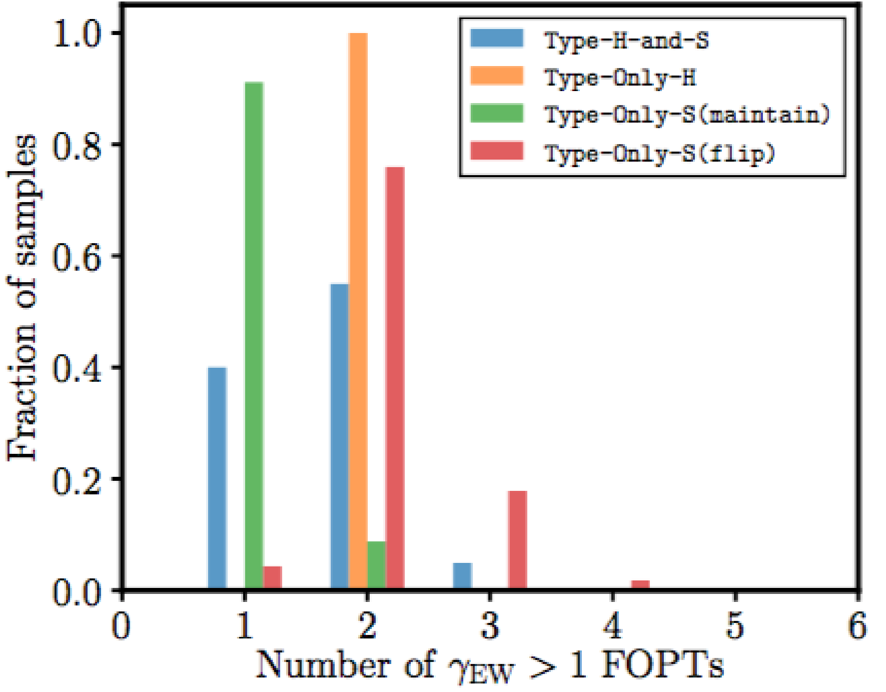

The strongest FOPT does not end in the SM vacuum

→ Multiple strong first order phase transitions may produce very exciting gravitational waves signatures

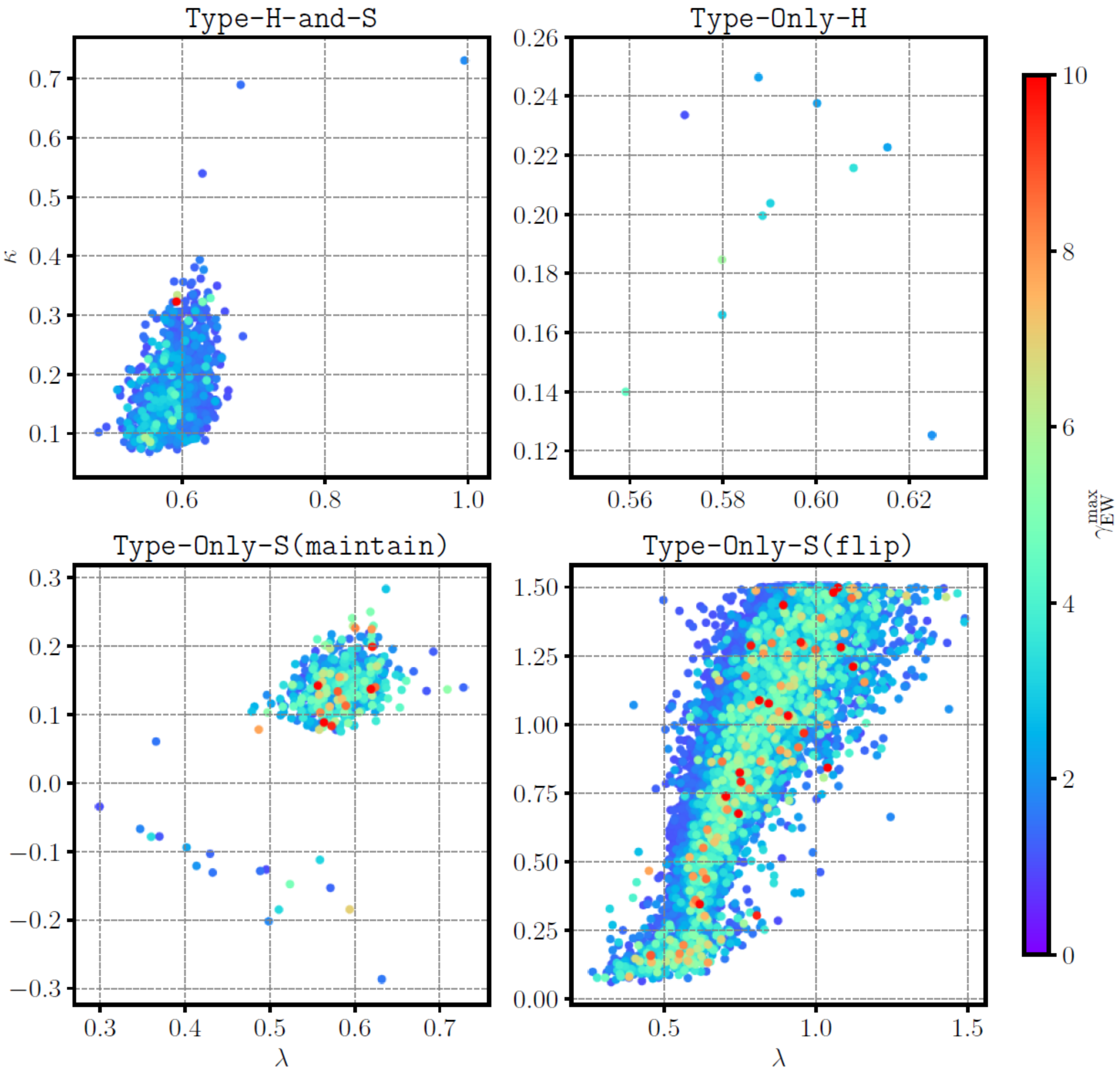
Conclusions

- First order phase transitions are needed for EWBG and could generate gravitational waves
- The NMSSM admits complicated phase transition patterns, including
    - Multiple strong first order phase transitions
    - Transitions breaking, restoring and breaking EW symmetry again
    - Scenarios where the transition does not complete (be wary)
    - Or where one may be fooled by a simple looking transition not present in the cosmological history
- We found that a light singlet Higgs was very common in our samples
- The NMSSM has excellent prospects for viable electropweak baryogenesis scenarios and interesting gravitational wave signals

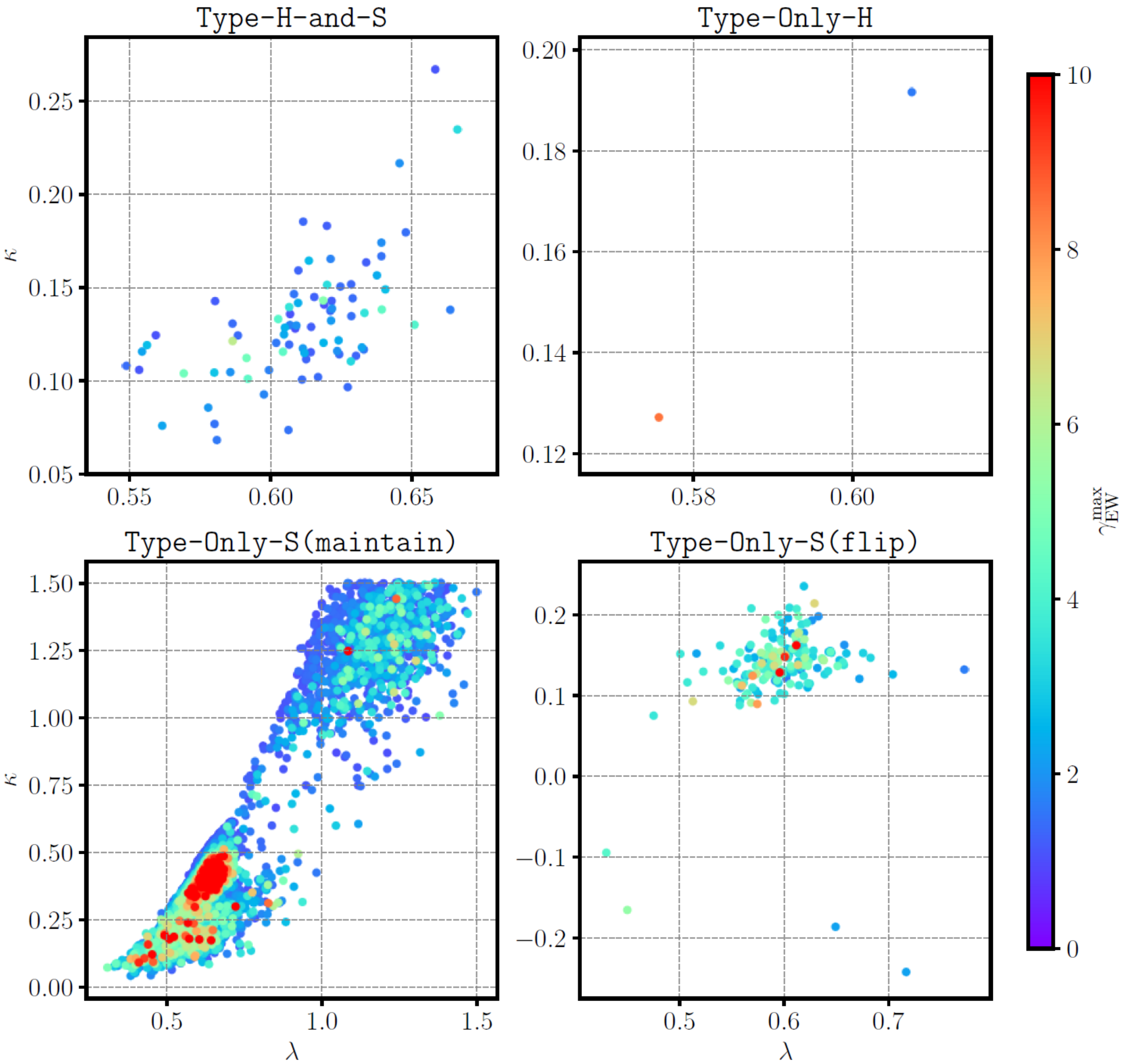
The End

Thanks for listening!

Strongest PT ends in deepest T=0 minimum



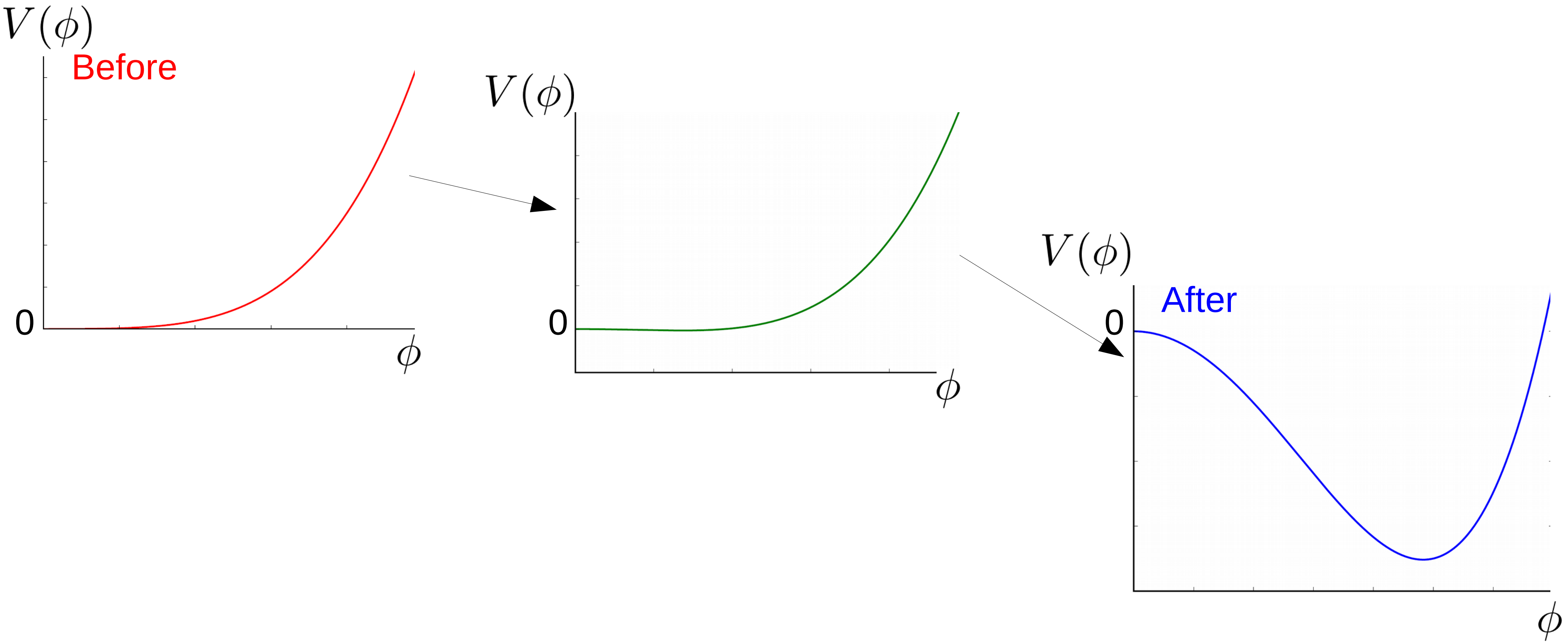
Strongest PT *does not* end in deepest T=0 minimum



Electroweak Phase Transitions

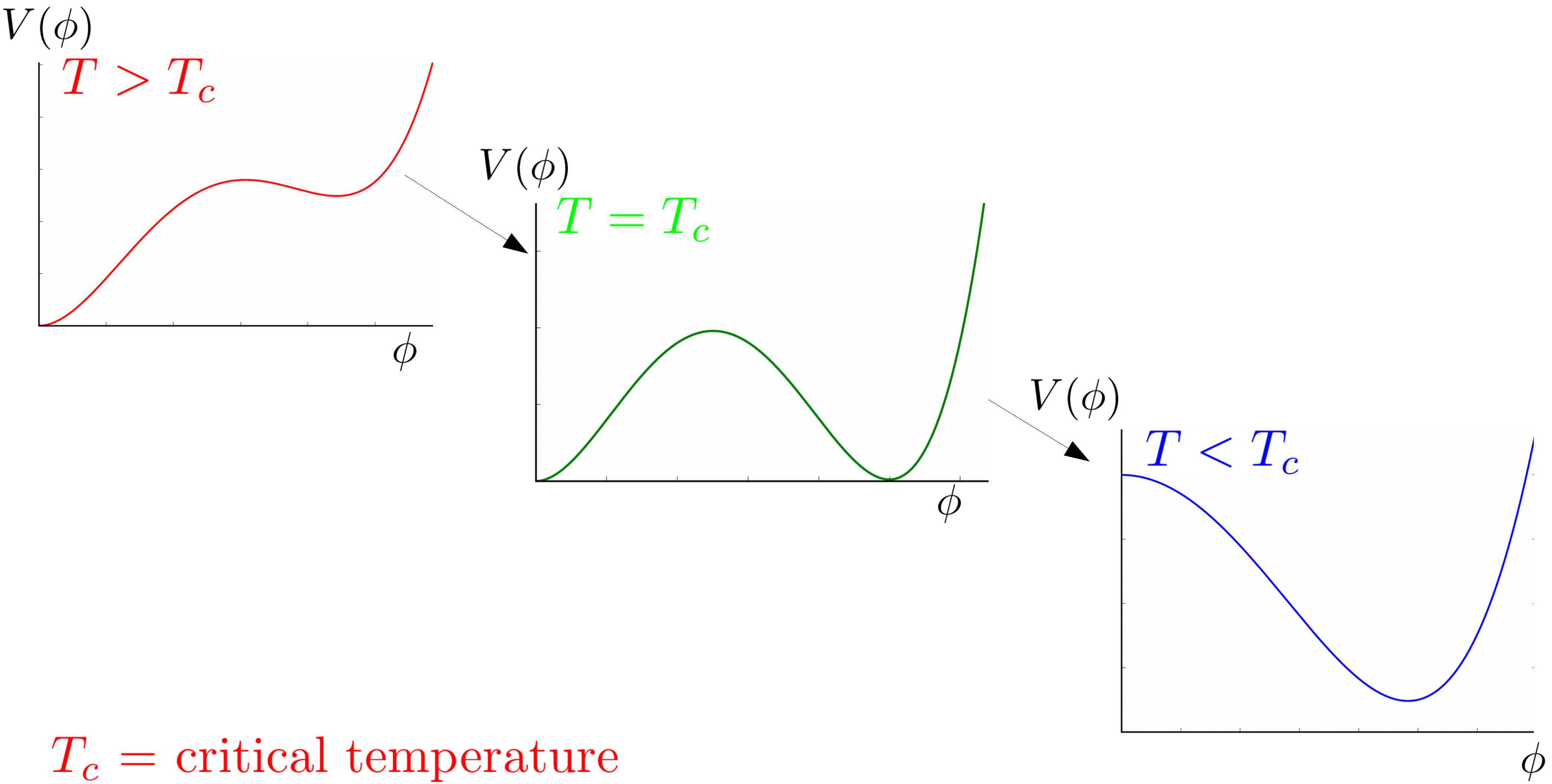
Finite temperature potential: $\mathrm{V} = \mathrm{V}(\phi, T)$

The phase transition can proceed in different ways, e.g.



First Order Phase Transitions

Or as a first order phase transition (FOPT):



$$\begin{aligned} T_c &= \text{critical temperature} \\ &\equiv \text{Temperature where } V \text{ at minima are degenate} \end{aligned}$$

Matching conditions to THDMS

$$V_{\text{THDMS}}^{\text{tree}} = \frac{1}{2}\lambda_1|H_d|^4 + \frac{1}{2}\lambda_2|H_u|^4 + (\lambda_3+\lambda_4)|H_u|^2|H_d|^2 - \lambda_4|H_u^\dagger H_d|^2$$
$$+ \lambda_5|S|^2|H_d|^2 + \lambda_6|S|^2|H_u|^2 + (\lambda_7 S^{*2} H_d\cdot H_u + \text{h.c.}) + \lambda_8|S|^4$$
$$+ m_1^2|H_d|^2 + m_2^2|H_u|^2 + m_3^2|S|^2 - (m_4 S H_d\cdot H_u + \text{h.c.}) - \frac{1}{3}(m_5 S^3 + \text{h.c.})$$

$$\lambda_1 = \frac{1}{4}\left(g'^2+g^2\right), \quad \lambda_2 = \frac{1}{4}\left(g'^2+g^2\right) + \Delta\lambda_2, \quad \lambda_3 = \frac{1}{4}\left(g^2-g'^2\right),$$
$$\lambda_4 = \frac{1}{2}\left(2|\lambda|^2-g^2\right), \quad \lambda_5=\lambda_6=|\lambda|^2, \quad \lambda_7=-\lambda\kappa^*, \quad \lambda_8=|\kappa|^2,$$
$$m_1^2 = m_{H_d}^2, \quad m_2^2 = m_{H_u}^2, \quad m_3^2 = m_S^2, \quad m_4 = A_\lambda\lambda, \quad m_5 = -A_\kappa\kappa.$$

$$\Delta\lambda_2 = \frac{3y_t^4A_t^2}{8\pi^2M_{\text{SUSY}}^2}\left(1-\frac{A_t^2}{12M_{\text{SUSY}}^2}\right)$$

Higgs Potential corrections

We calculate the potential in the THDMS EFT after the matching and running procedure

$$V = V^{\text{tree}}_{\text{THDMS}} + \Delta V^{\text{CW}} + \Delta V_T + V_{\text{Daisy}}$$

$$\Delta V^{\text{CW}} = \frac{1}{64\pi^2}\left(\sum_h n_h m_h^4(\xi)\left[\ln\left(\frac{m_h^2(\xi)}{Q^2}\right) - 3/2\right] + \sum_V n_V m_V^4\left[\ln\left(\frac{m_V^2}{Q^2}\right) - 5/6\right]\right.$$
$$\left. - \sum_V \tfrac{1}{3} n_V (\xi m_V^2)^2\left[\ln\left(\frac{\xi m_V^2}{Q^2}\right) - 3/2\right] - \sum_f n_f m_f^4\left[\ln\left(\frac{m_f^2}{Q^2}\right) - 3/2\right]\right).$$

$$\Delta V_T = \frac{T^4}{2\pi^2}\left[\sum_h n_h\left(\frac{m_h^2(\xi)}{T^2}\right) + \sum_V n_V\left(\frac{m_V^2}{T^2}\right) - \sum_V \tfrac{1}{3} n_V\left(\frac{\xi m_V^2}{T^2}\right) + \sum_f n_f\left(\frac{m_f^2}{T^2}\right)\right],$$

$$V_{\text{daisy}} = -\frac{T}{12\pi}\left(\sum_h n_h\left[(\bar{m}_h^2)^{\frac{3}{2}} - (m_h^2)^{\frac{3}{2}}\right] + \sum_V \tfrac{1}{3} n_V\left[(\bar{m}_V^2)^{\frac{3}{2}} - (m_V^2)^{\frac{3}{2}}\right]\right),$$