CoEPP
ARC Centre of Excellence for
Particle Physics at the Terascale

Thermal leptogenesis from a low-scale seesaw

Tomasz Dutka (CoEPP, University of Melbourne)
In collaboration with M. Dolan and R. Volkas

Focus on two problems for the SM

1) Origin of neutrino mass:

From low-energy neutrino experiments and cosmology we know that neutrinos are not massless and significantly lighter than any other fermion in the SM.

Mass generation mechanism? Can their lightness be theoretically motivated?

2) Origin of the Baryon Asymmetry of the Universe (BAU):

We know of a matter-antimatter asymmetry (no evidence of antimatter).

$$\eta_s \equiv \frac{n_B}{s} = (8.718 \pm 0.004) \times 10^{-11} \quad \text{Aghanim et al (Planck) 2018}$$

Challenge for both cosmology and particle physics to explain this observed value.

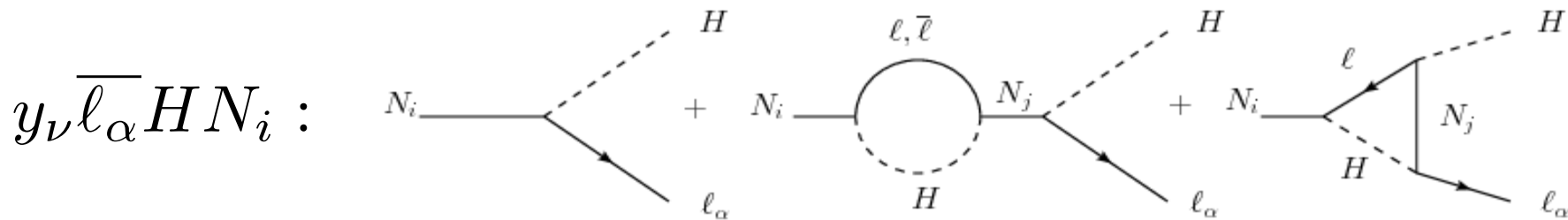Neither are in agreement with SM predictions.

Baryogenesis

Reasonable to assume this small asymmetry arises from a *dynamic* early universe process as opposed to an initial condition - baryogenesis.

Conditions necessary for dynamic generation of a baryon asymmetry written down by Sakharov: Sakharov '67

1)   Baryon Number Violation

2)   C and CP violation

3)   Out of Equilibrium

Many types of baryogenesis models: two popular variants are electroweak baryogenesis and leptogenesis.

Thermal Leptogenesis

$$y_\nu \overline{\ell_\alpha} H N_i :$$

Fukugida, Yanagida;
Liu, Segre;
Flanz et al;
Covi, Roulet , Vissani;
Pilaftsis;

Introduce heavy Majorana neutrinos.

1)   Baryon Number Violation

Lepton number violation (because Majorana),
L asymmetry partially converted to B asymmetry from
known non-perturbative SM effects (as long as asymmetry
generated above phase transition).

2)   C and CP violation

As yet unmeasured.

3)   Out of Equilibrium

Departure from thermal equilibrium due to expanding
universe.

4

Connection to neutrino mass

Mass matrix in the neutrino sector now looks like

$$m_\nu = \frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

Minkowski;
Yanagida; Glashow;
Gell-Mann,
Ramond Slansky;
Mohapatra,
Senjanovic...

Upon diagonalisation (after EWSB), if lepton number violation is large, leads to a light and heavy mass state – seesaw.

$$m_{\nu 1} \sim \frac{m_D^2}{M_R} \quad \text{and} \quad m_{\nu 2} \sim M_R \quad \text{for} \quad M_R \gg m_D$$

Large Majorana mass assumed for leptogenesis leads to one very light (active) state and one very heavy (sterile) state.

$$m_{\nu_1}$$

$$m_{\nu_2}$$
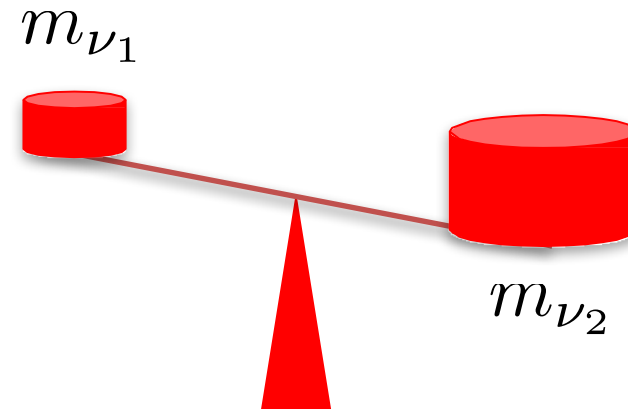
Reasons for considering low-scale variants

Extensive study of high scale thermal leptogenesis reveal certain theoretical issues, for example (backup slides for more info):

- Davidson-Ibarra bound: Requiring successful leptogenesis leads to lower-bound constraint on sterile mass of $\sim 10^9 \text{ GeV}$.

- Vissani bound: Naturalness arguments lead to an upper-bound constraint on the sterile mass of $\sim 10^7 \text{ GeV}$.

- No overlap between these two regions – alternative scenarios or new physics required?

Clarke, Foot, Volkas: 1505.05744

6
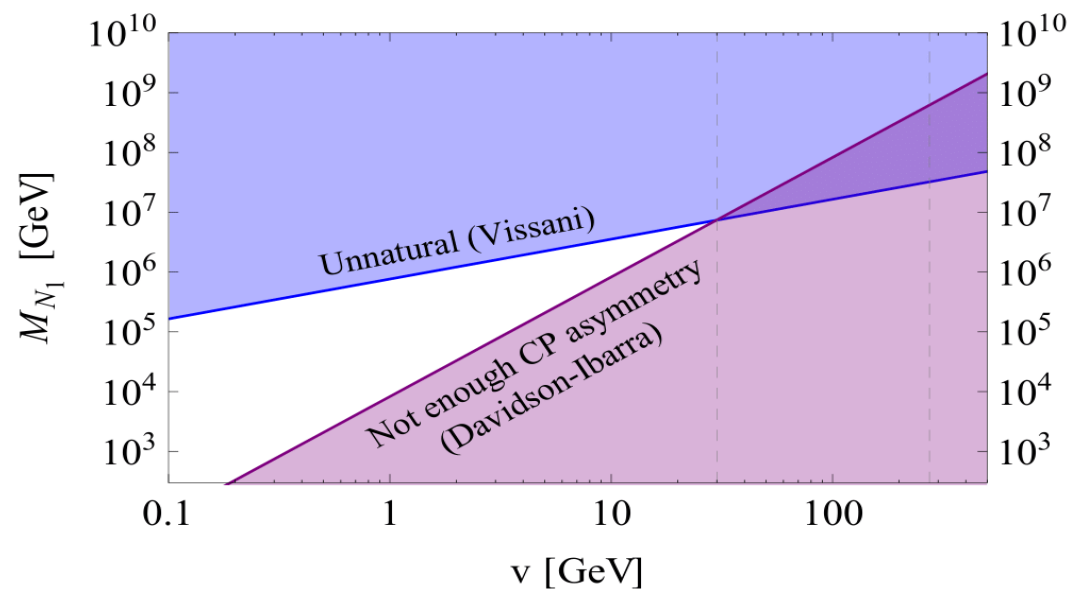
Reasons for considering low-scale variants

Additionally,

CPV phases required for the heavy sterile neutrinos will most likely be experimentally inaccessible.

Importantly, at high temperatures the charged lepton Yukawa interactions are out of equilibrium and flavour effects can be ignored. As a consequence the final lepton asymmetry does not depend on low-energy phases.

Pascoli, Petcov, Riotto: hep-ph/0609125
hep-ph/0611338
Anisimov, Blanchet, Bari: 0707.3024

Abada, Davidson, Ibarra, Josse-Michaux, Losada , Riotto: hep-ph/0605281

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}
$$

Majorana phases not included here for simplicity!

Low-energy phases

If the lepton asymmetry is generated at the low scale, CPV parameters measurable at low-energy neutrino experiments will contribute to the overall lepton asymmetry generated.

Low-scale leptogenesis can be **sensitive to measurable parameters**.

Currently hints that the Dirac-phase of the PMNS matrix is not CP conserving (could even be maximally violating).

nu-fit.org 4.1 (2019)
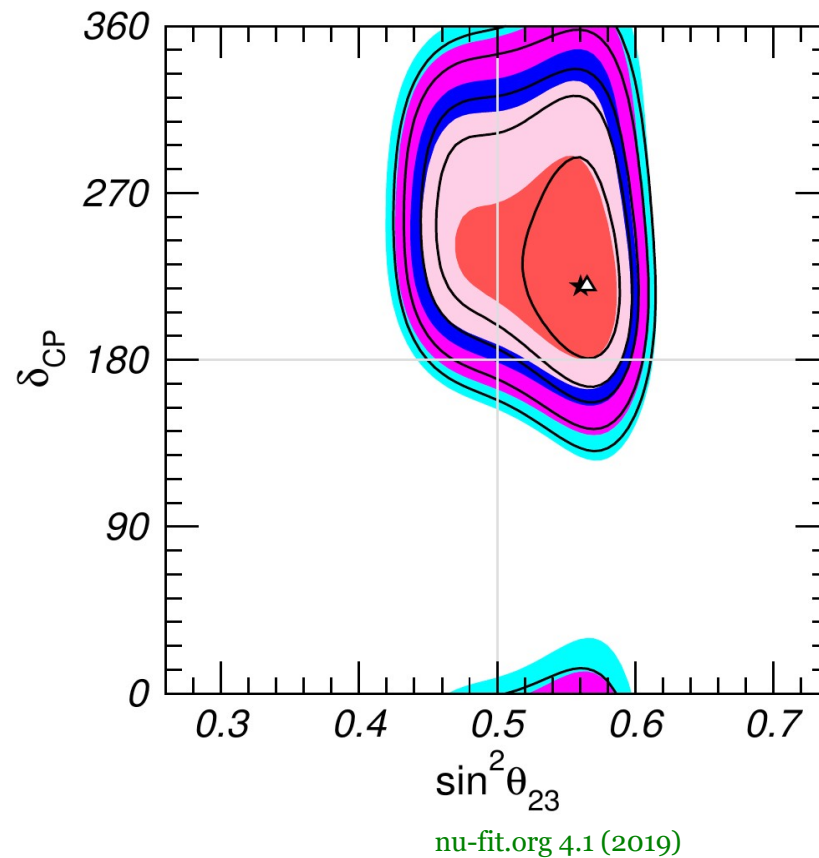
Low-energy phases

1) **Baryon Number Violation** ✔

Lepton number violation (because Majorana)
L asymmetry partially converted to B asymmetry
from non perturbative SM effects.

2) **C and CP violation** ?

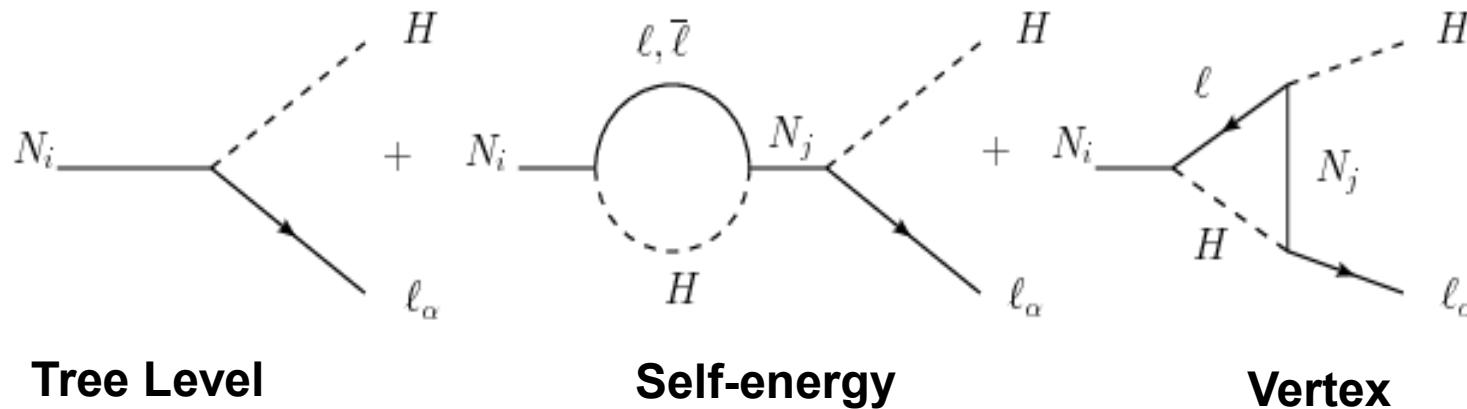Dirac or Majorana phase within the PMNS
matrix?

3) **Out of Equilibrium** ✔

Departure from thermal equilibrium due to
expanding universe

Resonance: an attractive feature

Tree Level   **Self-energy**   **Vertex**

For a quasi-degenerate spectrum of Majorana neutrinos, self-energy effects dominate the leptonic CP-asymmetry.

Resonantly enhanced when $M_{N_i} - M_{N_j} \simeq \Gamma_{N_{i,j}}$.

Can allow for a significant reduction in the masses of the heavy sterile neutrinos required – moving past the Davidson-Ibarra bound.

Can allow for mass scales as low as the EW scale at the cost of a tuned mass spectrum. Can such a spectrum naturally arise in a theory?

10

Low scale alternative

If instead of introducing a single singlet fermion we introduce two

$$m_\nu = \frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} & \overline{S_L} \end{pmatrix} \begin{pmatrix} 0 & m_D & m_L \\ m_D^T & \mu_N & M_N \\ m_L^T & M_N^T & \mu_S \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ S_L^c \end{pmatrix}$$

seesaw looks very different:

$$\mu_S, \ \mu_N \, m_L \to 0, m_{\nu_1} \to 0 \text{ and } m_{\nu_2} \to m_{\nu_3} = \sqrt{M_N^2 + m_D^2}$$

Can explain light neutrinos now not by a large LNV term but by *small* LNV effects (technically natural).

$$m_L \to 0 :$$ Known as **Inverse Seesaw.**   Mohapatra, Valle: PRD 34 (1986) 1642

$$\mu_N, \mu_S \to 0 :$$ Known as **Linear Seesaw.**   Akhmedov, Lindner, Schnapka, Valle: hep-ph/9509255

11

Low-scale alternative

With non-zero (but small) LNV parameters, the spectrum of neutrino masses are given by

$$m_{\nu_1} \simeq m_D M_N^{T-1} \mu M_N^{-1} m_D^T$$
$$-(m_D(m_L M_N^{-1})^T + (m_L M_N^{-1})m_D^T)$$

$$m_{\nu_{2,3}} \simeq M_N \pm \frac{1}{2}\mu$$

Quasi-degenerate heavy sterile masses!

Attractive link between resonant leptogenesis and small neutrino masses.

Further reading if interested:
Ilakovac, Pilaftsis: hep-ph/9403398,   Deppisch, Valle: hep-ph/0406040,   Deppisch, Kosmas, Valle: hep-ph/0512360,
Gu, Sarkar: 1007.2323,   Dib, Moreno, Neill: 1409.1868   Garayoa, Gonzaez-Garcia, Rius: hep-ph/0611311
Wang, Han: 1508.00706   Das, Nomura, Okada, Roy: 1704.02078

Leptogenesis implications

Considered two scenarios:

TPD, R. Volkas and M. Dolan: 1802.08373; JCAP06(2018)012

Scenario 1: Limiting case of **Pure Inverse,** scanned over $\mu_S < M_N$.

Scenario 2: Full case including both **Inverse and Linear** contributions $\mu_S,\ m_L < M_N$.

Under the hood:

- Dirac phase set to maximally violating. Only CPV parameter set to be non-zero as a simplifying assumption.

- Overall mass of sterile fermions set to $M_N = 1\,\mathrm{TeV}$ to consider a low-scale scenario.

- Dirac mass matrix m_D generated using a Casas-Ibarra parametrisation to ensure light neutrino data satisfied, e.g. mass differences, mixings etc. Different parametrisation in each scenario – see backup slides.

- Degenerate and hierarchical spectrum of heavy sterile neutrinos considered for both scenarios – see backup for more details.

- For computational convenience we set all LNV matrices to be diagonal. Results insensitive to this choice unless very special matrix textures are chosen. To be discussed later.

Note: Pure Linear scenario absent (predicts exactly degenerate sterile spectrum.

Phenomenological Constraints

Seesaw models predict a deviation in unitarity of the PMNS matrix which can be observed in low-energy experiments. Constrained from a combination of electroweak precision constraints.
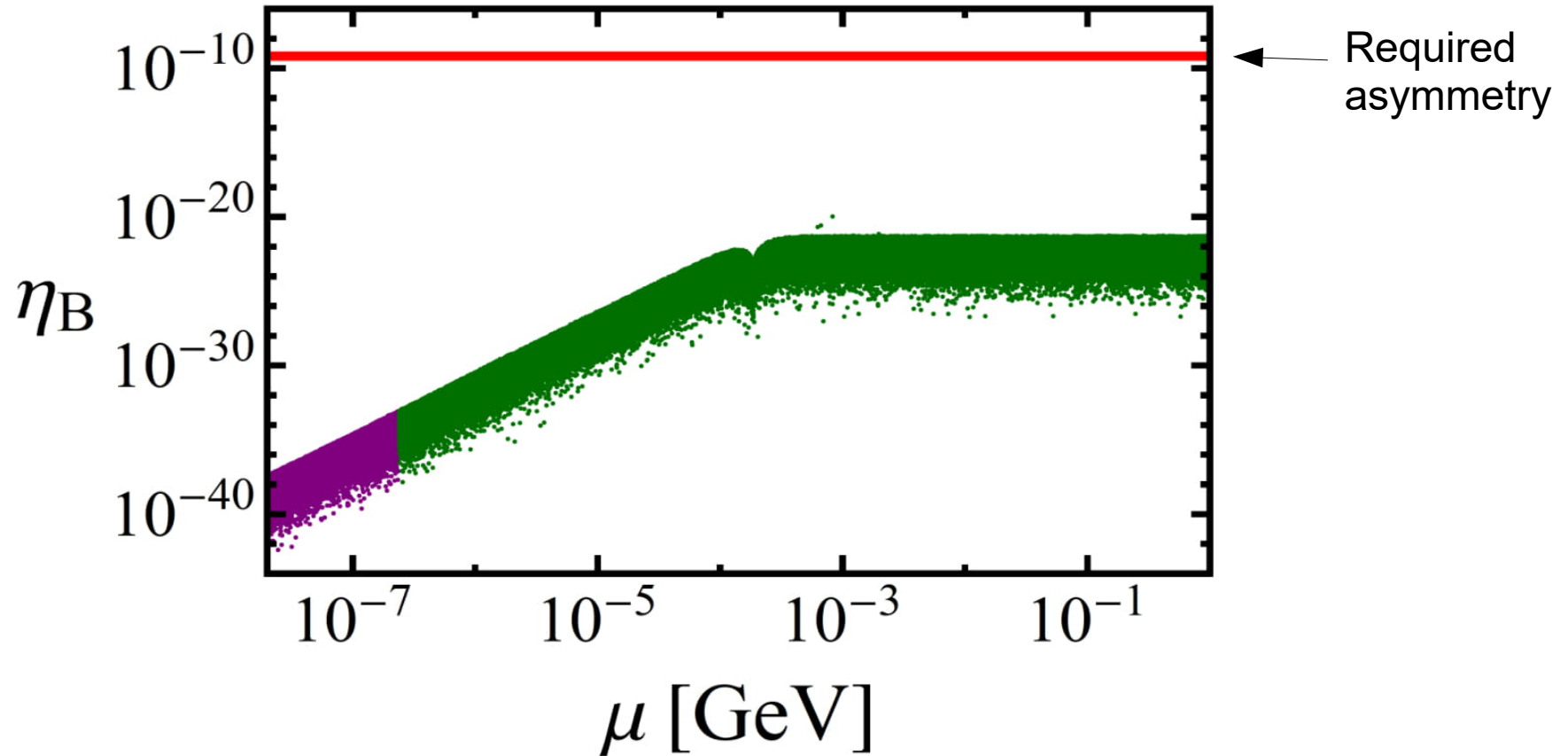
$$U_{3\times3} \simeq (1 - \eta)\, U_{PMNS}$$
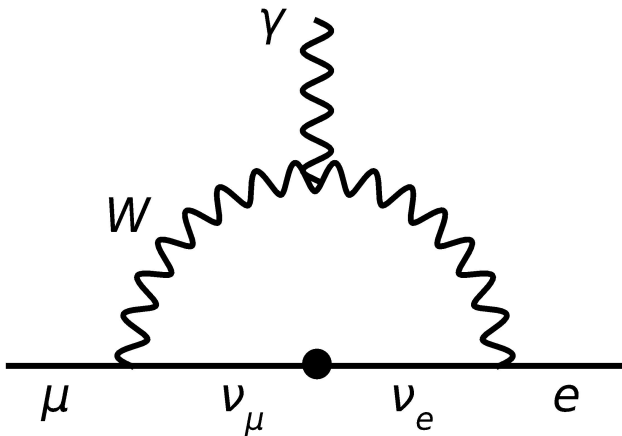
Constraints from future lepton flavour violation experiments. The future upper limit on the most constraining LFV processes are estimated to be

$$Br(\mu \to e\gamma) \lesssim 5 \times 10^{-14}$$

coming from (MEGII, COMET and Mu2e).

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_N \\ 0 & M_N^T & \mu \end{pmatrix}$$

Scenario 1: Pure Inverse

In this scenario cannot generate required asymmetry. Washout effects cause a strong suppression in final asymmetry generated.

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_N \\ 0 & M_N^T & \mu \end{pmatrix}$$
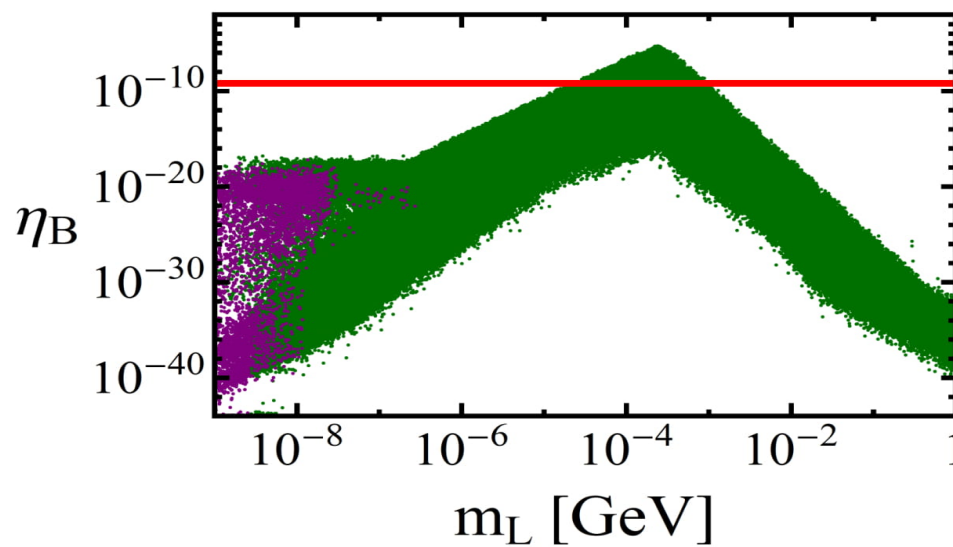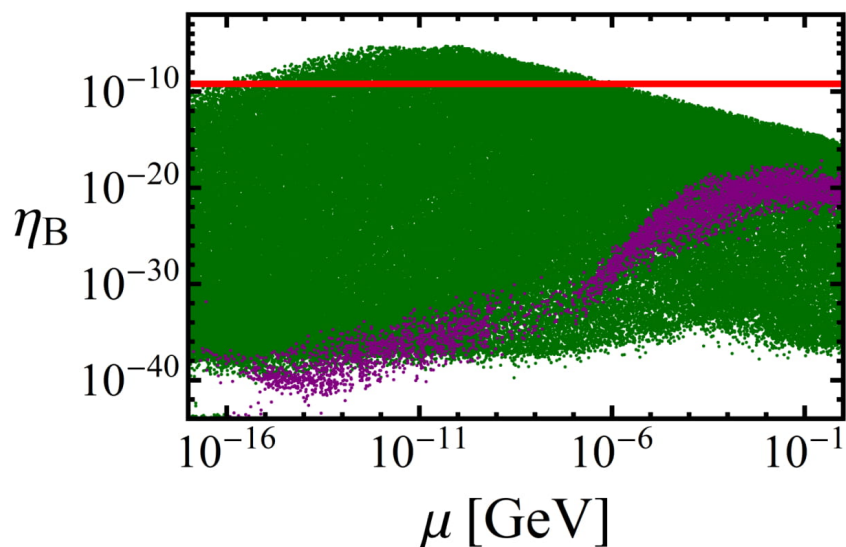
Scenario 1: Pure Inverse
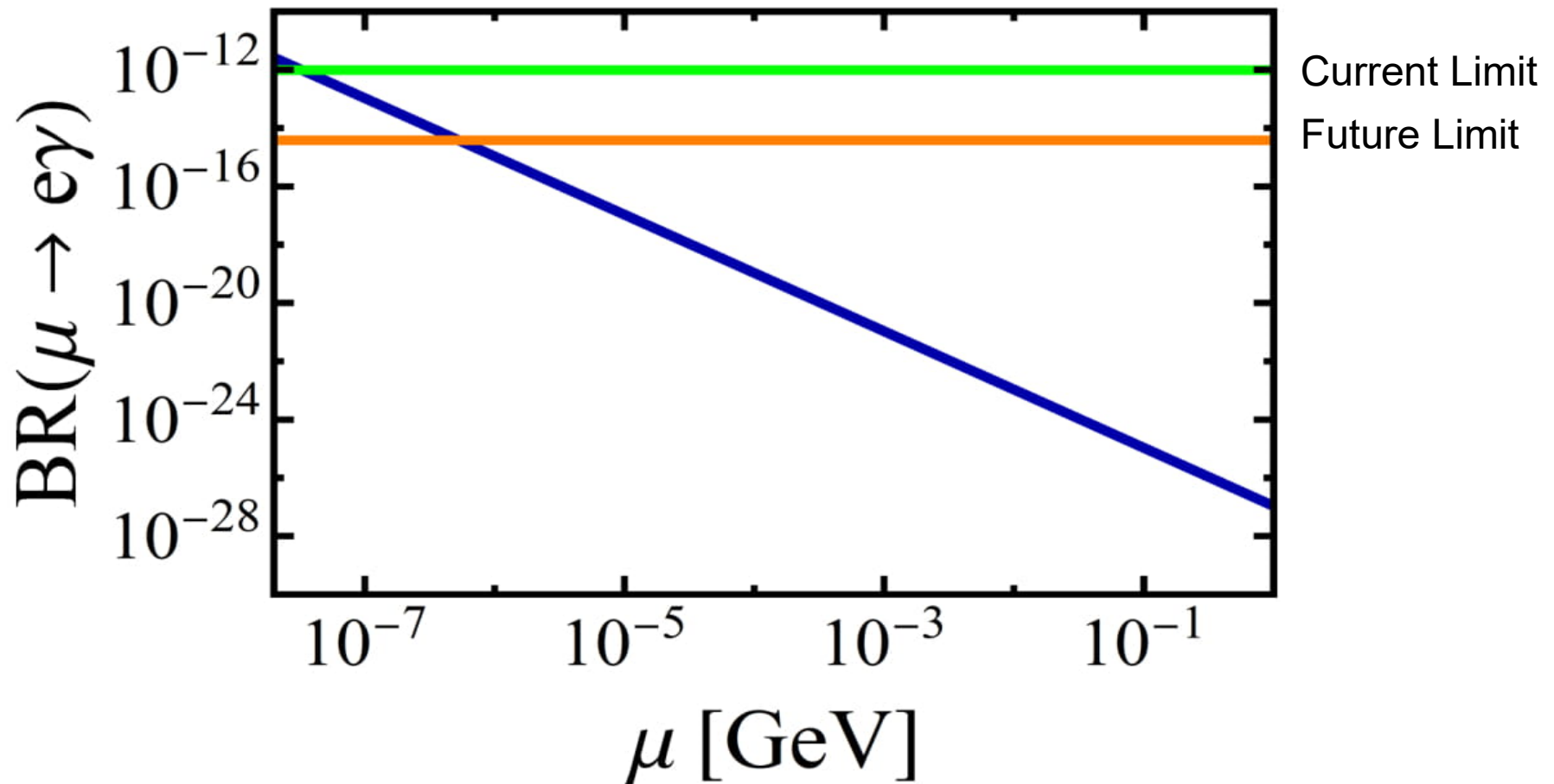
Future cLFV experiments will begin to be sensitive to regions of the inverse seesaw parameter space.

$$\begin{pmatrix} 0 & m_D & \textcolor{red}{m_L} \\ m_D^T & 0 & M_N \\ \textcolor{red}{m_L^T} & M_N^T & \textcolor{red}{\mu} \end{pmatrix}$$

Scenario 2: Inverse + Linear

Degenerate Spectrum

Dirac phase alone capable of generating sufficient asymmetry.

Correct asymmetry possible for

$$10^{-16} \lesssim \mu/\mathrm{GeV} \lesssim 10^{-6}$$

assuming the Dirac phase is maximally CPV. Such small values are motivated by small neutrino mass and is not an ad-hoc assumption.

17

$$\begin{pmatrix} 0 & m_D & m_L \\ m_D^T & 0 & M_N \\ m_L^T & M_N^T & \mu \end{pmatrix}$$

Scenario 2: Inverse + Linear

Unlike high-scale type-I, results are relatively insensitive to a degenerate or hierarchical spectrum.
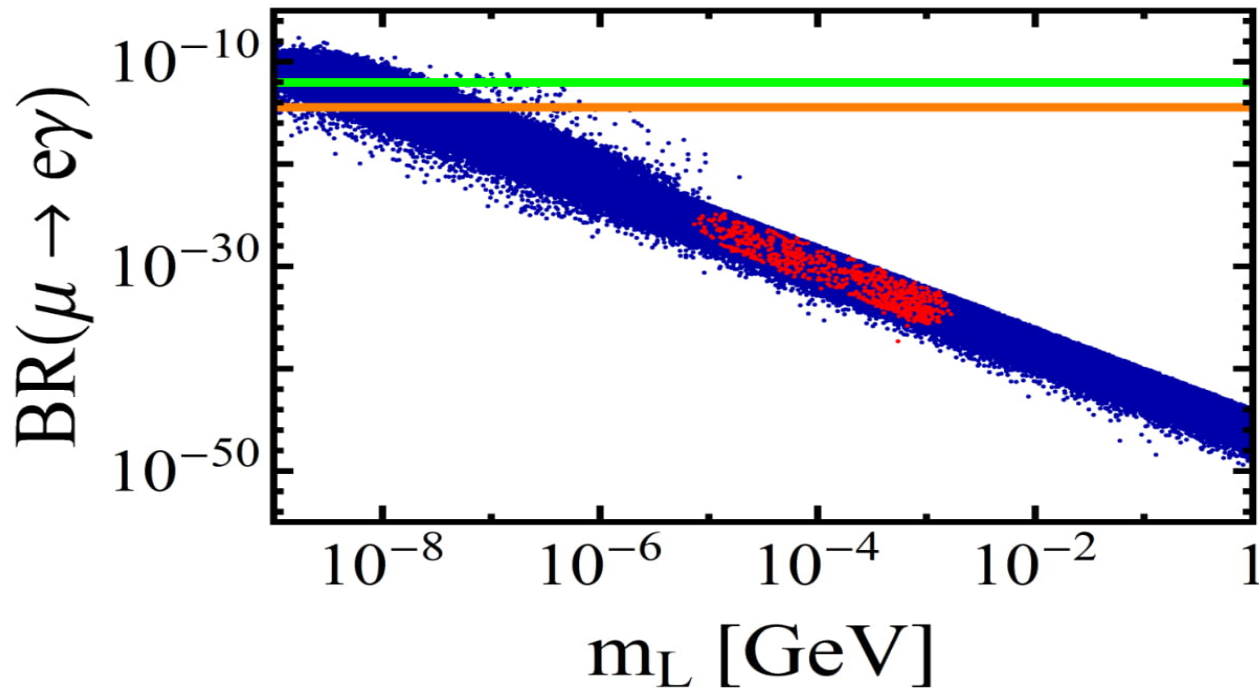
Successful Dirac-phase leptogenesis candidate however contribution to cLFV observables too small for future measurements.

18

Resonance from flavour symmetries

TPD, R. Volkas and M. Dolan: 1812.11964; Phys. Rev. D 99, 123508 (2019)

Previous work found that inverse and linear scenarios alone were unable to generate sufficient asymmetry.

BSM models with non-minimal seesaws can often predict a specific seesaw depending on particle content.

What additional physics beyond the simple inclusion of sterile fermions could exist in order to make such scenarios work?

In the past a 'minimal lepton flavour violation' (MLFV) flavour symmetry hypothesis has been proposed as a potential motivated theory containing near-degenerate mass RHNs.

Branco, Buras, Jager, Uhlig, Weiler: hep-ph/0609067,
Cirigliano, De Simone, Isidori, Masina, Riotto: 0711.0778
Deppisch, Pilaftsis: 1012.1834
Dev, Millington, Pilaftsis, Teresi: 1504.07640

We applied a similar MLFV hypothesis to the case of the inverse and linear seesaws and its implications on leptogenesis.

Resonance from flavour symmetries

The basic idea is the assumption that the Yukawa sector is the only flavour-breaking sector within the Lagrangian. Therefore the bare Majorana masses must be flavour diagonal.

$$SU(3)_{\ell_L} \times SU(3)_{e_R} \times SO(3)_{\nu_R}$$

The breaking of the flavour symmetry by the Yukawa sector introduces radiative corrections to the diagonal Majorana neutrino mass lifting the mass degeneracy. A very convenient feature for resonant leptogenesis.

$$M_R = m_R \mathbb{1} + a(y_D^\dagger y_D + y_D^T y_D^*)$$

An import consequence of the ansatz is that the mass degeneracy is broken by terms proportional to the Yukawa couplings and is not a free parameter.

It was found that MLFV leptogenesis in type-I scenario leads to a suppression in asymmetry generation due to the structure of the radiative corrections.
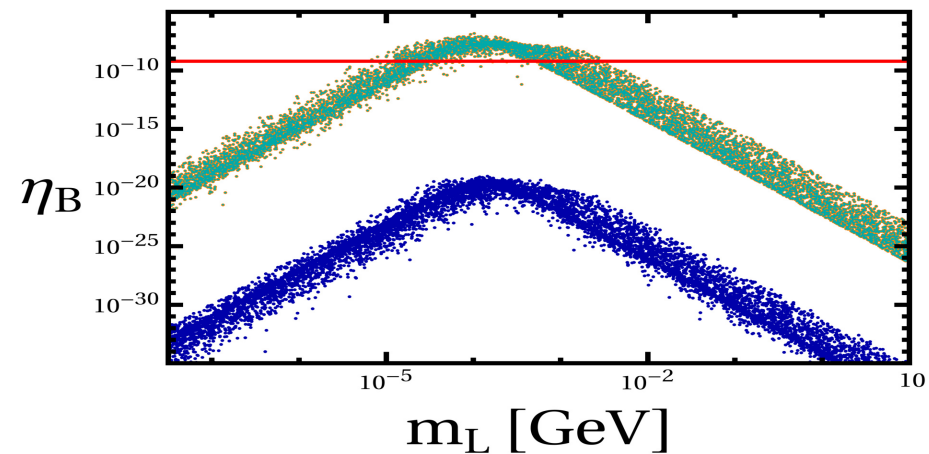
20

Scenario 3: MLFV Inverse Seesaw

Able to generate sufficient asymmetry in both scenarios when the radiative effects on the

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mu_N + \delta\mu_N & M_N \\ 0 & M_N^T & \mu_S \end{pmatrix}$$

$$\begin{pmatrix} 0 & m_D & m_L \\ m_D^T & \delta\mu_N & M_N \\ m_L^T & M_N^T & \delta\mu_S \end{pmatrix}$$

21
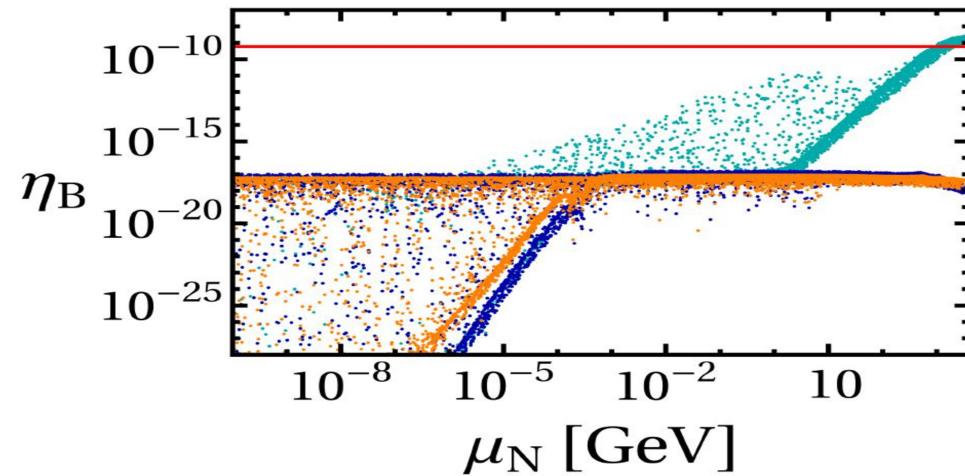
Concluding Remarks

- Although successful, possible theoretical issues within high-scale leptogenesis models may suggest the SM + type-I is not the complete picture.

- The inverse and linear seesaws provide a possible alternative scenario of neutrino mass and we have studied the leptogenesis implications of these models.

- The future measurement of the Dirac phase of the PMNS matrix can give insights into the matter-antimatter asymmetry problem. If it is measured to be large, will necessarily contribute to CPV in low-scale leptogenesis scenarios. Can even potentially be the dominant source. This is unlike the CKM phase.

- Low scale models of neutrino mass and their implications for leptogenesis provide an exiting avenue of research due to their potential testability. The measurement of δ_{CP} can prove crucial to our understanding of baryogenesis.

- -

- If the Yukawa sector is the only source of flavour violation a degenerate spectrum of sterile masses is predicted. Radiative effects can lead to enhanced asymmetry generation.

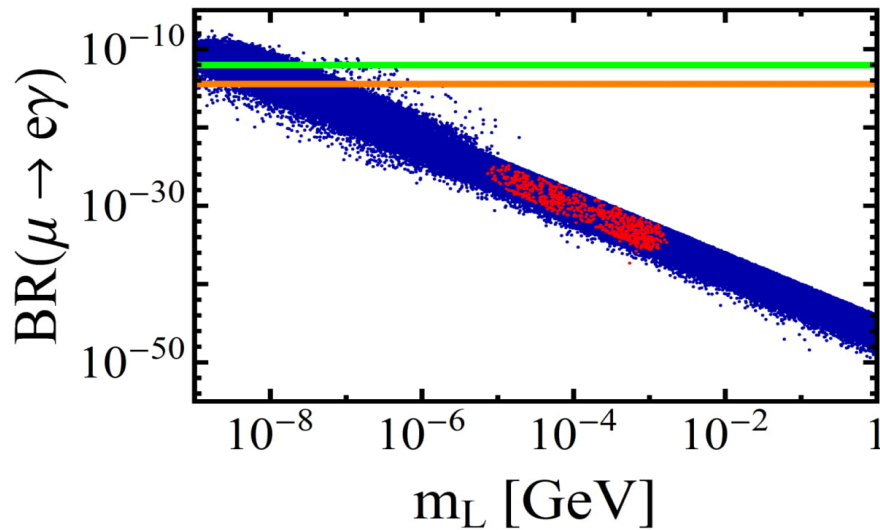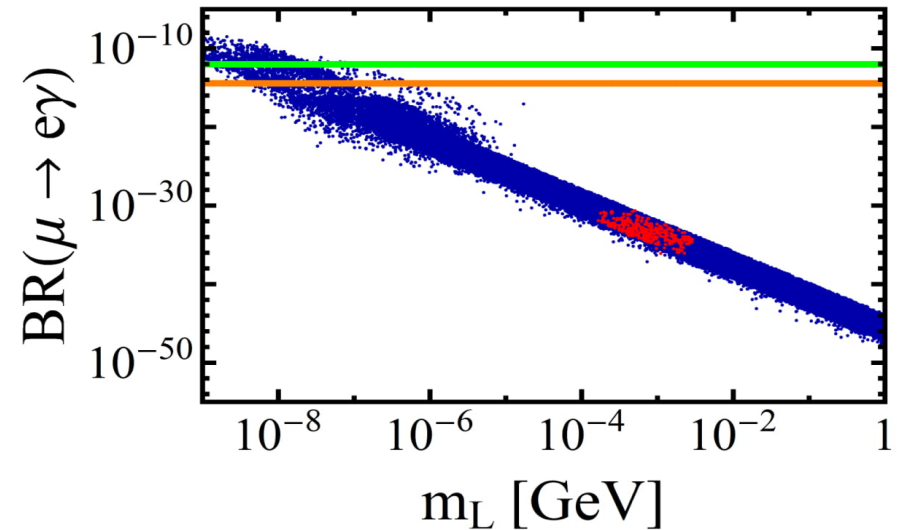- Matter-antimatter asymmetry could arise due to flavour symmetries.

Backup Slides

Inverse + Linear

Hierarchical Spectrum
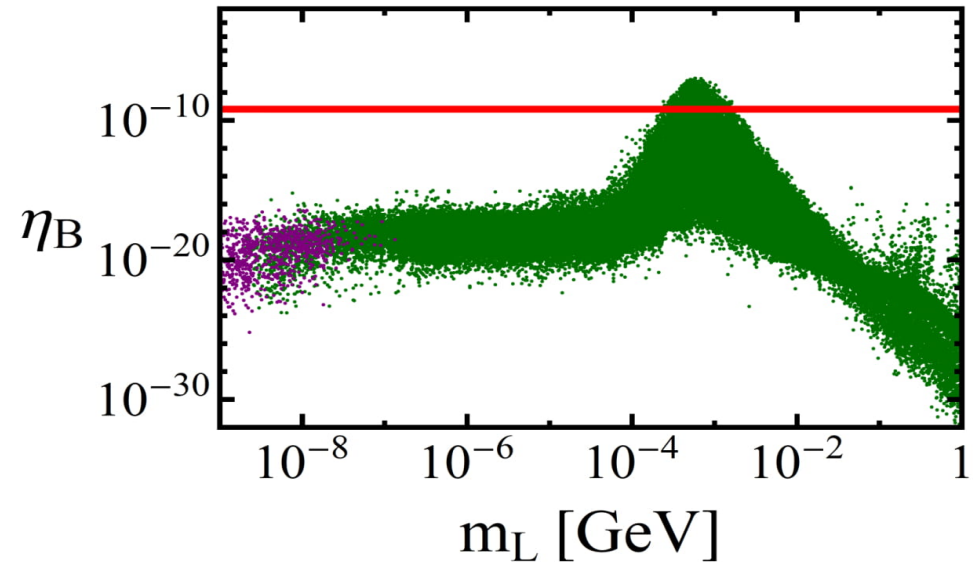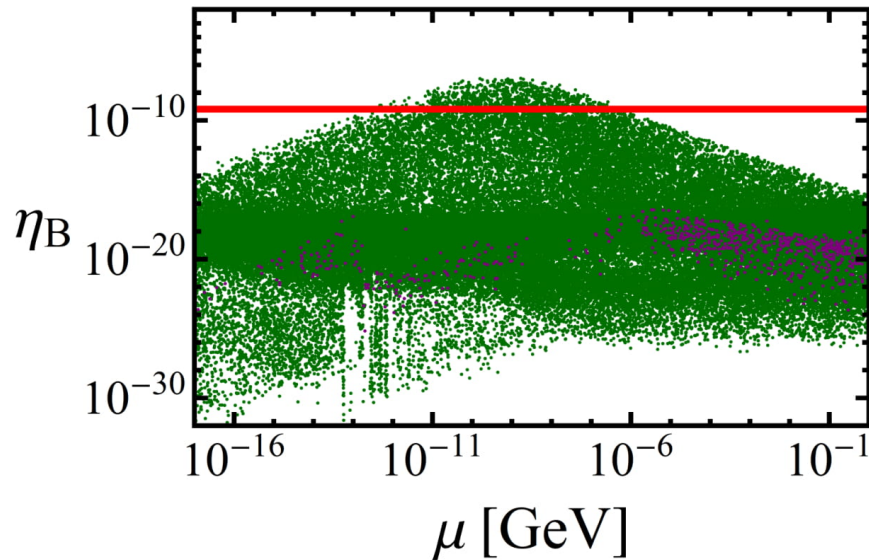
Taking the heavy steriles to be hierarchical (contribution to asymmetry only from the lightest) produces similar predictions on the LNV couplings and masses albeit slightly more constrained.

$$10^{-12} \lesssim \mu/\mathrm{GeV} \lesssim 10^{-6}$$

$$10^{-4} \lesssim m_L/\mathrm{GeV} \lesssim 10^{-3}$$

Inverse + Linear

Degenerate Spectrum   **Hierarchical Spectrum**

Similar to scenario 1 only small values of the LNV coupling will provide signals of cLFV. For larger values of the coupling even smaller branching fractions predicted due to linear relationship .

If signals seen in future MEGII, Mu2e and COMET of cLFV can rule out contribution of Dirac Phase to leptogenesis for these models (in the absence of extra symmetries which may introduce additional resonances – flavour symmetries?).

25

Phenomenological constraints

As seesaw models predict extra heavy states, the PMNS rotation matrix can no longer be exactly unitary. In high scale models deviation from unitarity negligible, potentially significant in low scale models.

Rodejohann '09;

$$U_{3\times3} \simeq (1 - \eta) \, U_{PMNS}$$

In inverse/linear seesaw schemes: $\eta = \dfrac{1}{2} \left[m_D M_N^{-1} \right] \left[m_D M_N^{-1} \right]^T$

$$|\eta_{ij}| < \begin{pmatrix} 2.0 \times 10^{-3} & 3.5 \times 10^{-5} & 8.0 \times 10^{-3} \\ 3.5 \times 10^{-5} & 8.0 \times 10^{-4} & 5.1 \times 10^{-3} \\ 8.0 \times 10^{-3} & 5.1 \times 10^{-3} & 2.7 \times 10^{-3} \end{pmatrix}$$

Constrained from combination of observables including universality tests of weak interactions, rare leptonic decays, invisible width of Z-boson and neutrino oscillation data.

Low-scale tests of neutrino mass models

Lepton Flavour Violation

Very difficult to constrain seesaw models at colliders due to low productions cross sections, if seesaw is embedded in some larger theory e.g GUTS, SUSY discovery may be viable in other signals.

The future upper limit on the most constraining LFV processes are estimated as coming from (MEGII, COMET and Mu2e)

$$Br(\mu \to e\gamma) \lesssim 5 \times 10^{-14}$$

$$Br(\mu^- N \to e^- N) \lesssim 3.84 \times 10^{-15}$$

for $\mu \to e\gamma$ and $\mu \to e$ conversion in nuclei respectively.
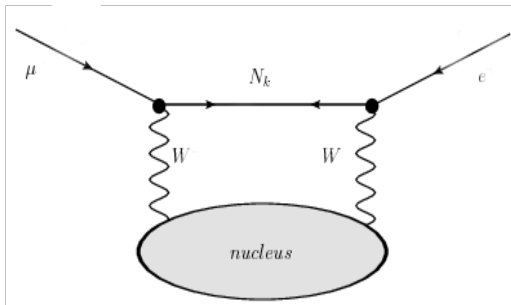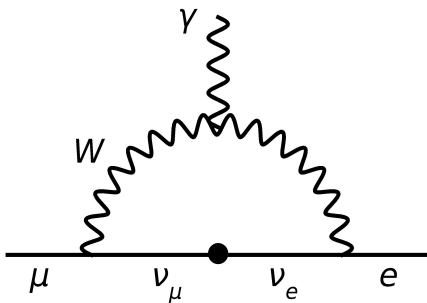
High-scale models predict very small contributions to such processes, but can be relevant for low-scale models such as the Inverse, Linear etc.
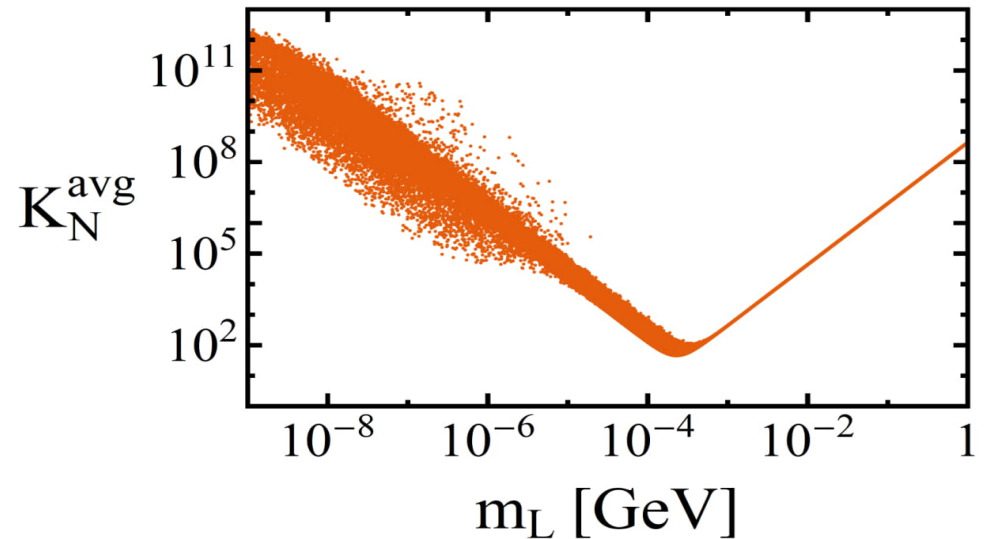
Similarly these processes can be model dependent, e.g. embedding seesaw model into larger theory, SUSY, L-R, GUT can lead to other exotics which contribute to such processes.

27

Above phase transition neutrinos and charged leptons massless but heavy sterile fermions still massive.

$$M_\nu \to M'_\nu = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} y'_D & \frac{v}{\sqrt{2}} y'_L \\ \frac{v}{\sqrt{2}} \left(y'_D\right)^T & M_R - \frac{1}{2}\mu & 0 \\ \frac{v}{\sqrt{2}} \left(y'_L\right)^T & 0 & M_R + \frac{1}{2}\mu \end{pmatrix}$$

$$y'_D = \frac{i}{v} \left[\left(1 + \frac{\mu}{4M_R}\right) m_D - m_L \right]$$

$$y'_L = \frac{1}{v} \left[\left(1 - \frac{\mu}{4M_R}\right) m_D + m_L \right].$$

Inverse Casas-Ibarra:

$$m_D = U_{PMNS} m_d^{1/2} R \mu^{-1/2} M_R^T$$

$$R = \begin{pmatrix} c_y c_z & -s_x c_z s_y - c_x s_z & s_x s_z - c_x s_y c_z \\ c_y s_z & c_x c_z - s_x s_y s_z & c_z(-s_x) - c_x s_y s_z \\ s_y & s_x c_y & c_x c_y \end{pmatrix}$$
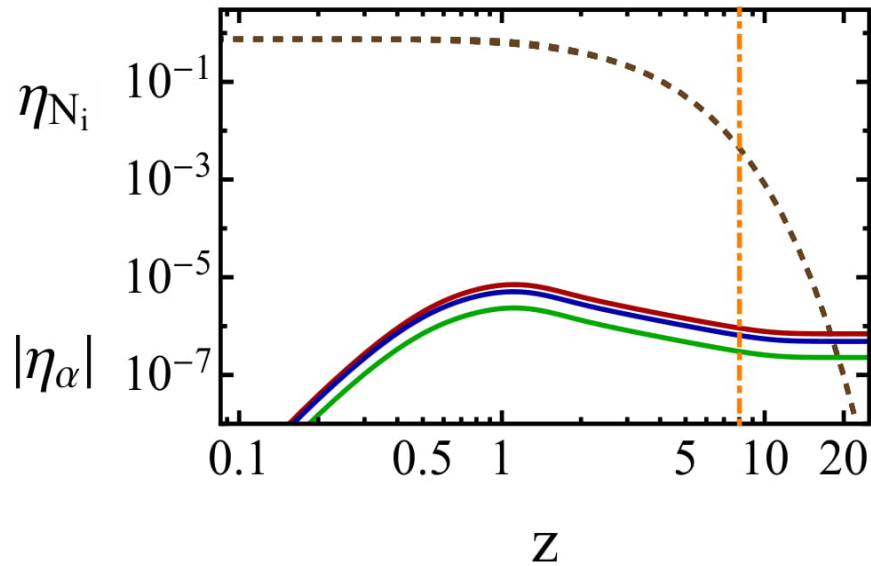
Linear Casas-Ibarra:

$$m_D = U_P m_d^{1/2} R m_d^{1/2} U^T m_L^{T-1} M_R^T.$$
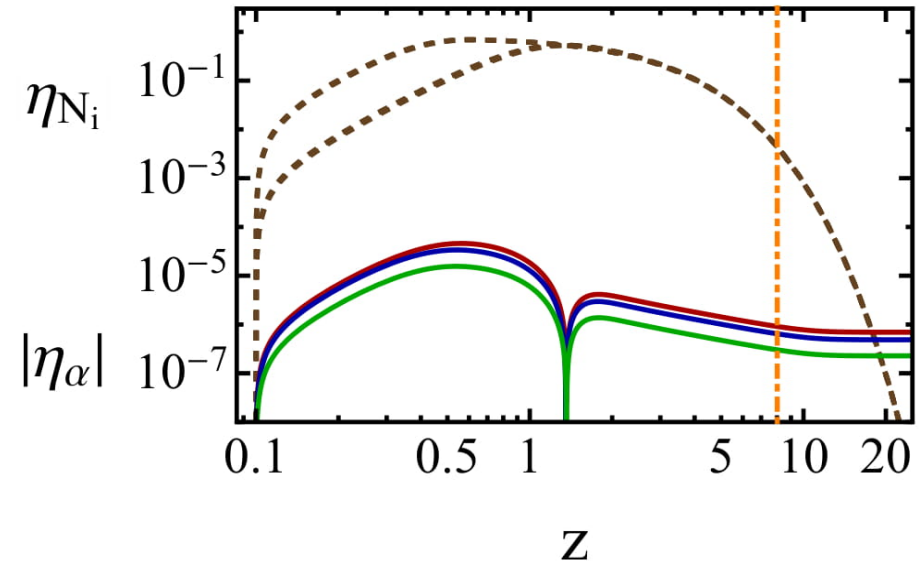
Inverse and Linear Casas-Ibarra:

$$U_{PMNS}^* m_d^{1/2} R m_d^{1/2} U_{PMNS}^\dagger = \frac{1}{2} m_D M_R^{-1} \mu M_R^{T-1} m_D^T - m_L M_R^{-1} m_D^T$$

$$R = \begin{pmatrix} \frac{1}{2} & r_1 & r_2 \\ -r_1 & \frac{1}{2} & r_3 \\ -r_2 & -r_3 & \frac{1}{2} \end{pmatrix}$$

Final asymmetry independent of initial conditions for this strong washout regime.
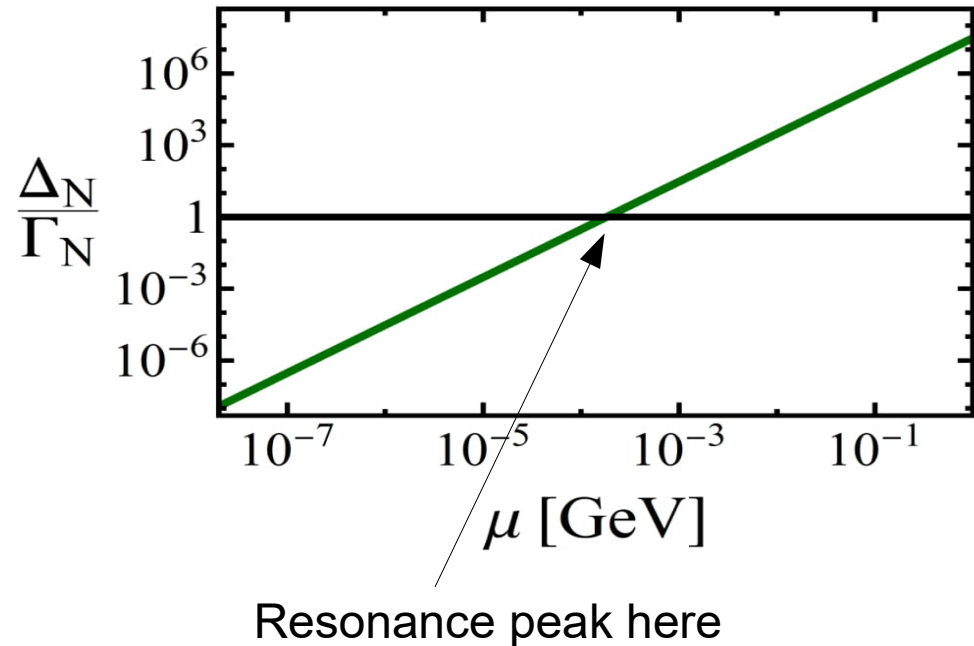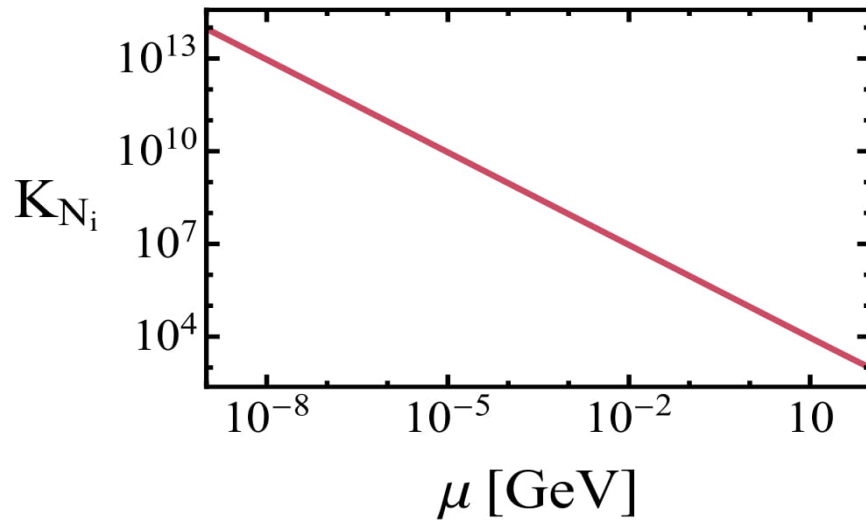

$$\eta_{N_i}(z_{\text{in}}) = 0 \qquad\qquad \eta_{N_i}(z_{\text{in}}) = \eta_{N_i}^{\text{eq}}$$

$$\eta_B = -\frac{28}{1377}\Sigma_\alpha \eta_\alpha = 7.229 \times 10^{-10}$$

In both cases

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_N \\ 0 & M_N^T & \mu \end{pmatrix}$$

Scenario 1: Pure Inverse

K_{N_i} vs $\mu \, [\text{GeV}]$

$\frac{\Delta_N}{\Gamma_N}$ vs $\mu \, [\text{GeV}]$

Resonance peak here

Decreasing μ can cause a resonance peak by tuning the mass splitting but also causes increase in washout by orders of magnitude. Resonant leptogenesis models can cope with strong washout regimes only if the washout is not too strong.

Leptogenesis not possible if the only source of mass splitting arises from μ .

Hierarchical limit for low-scale Inverse/Linear
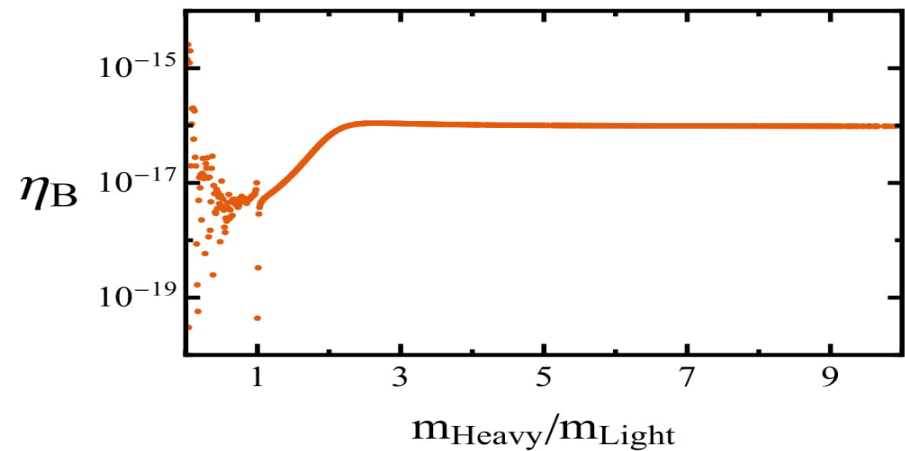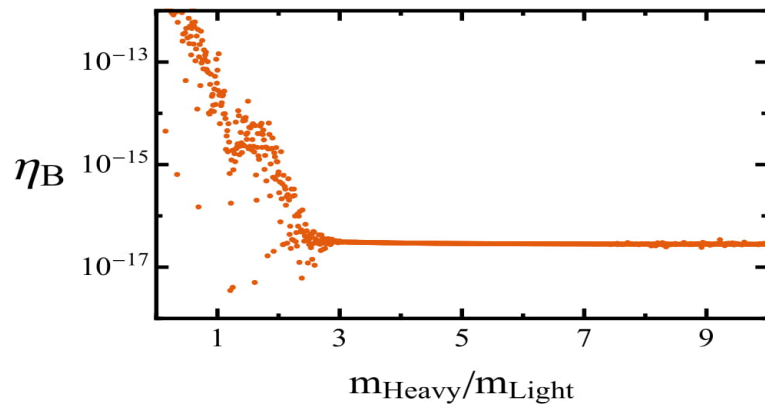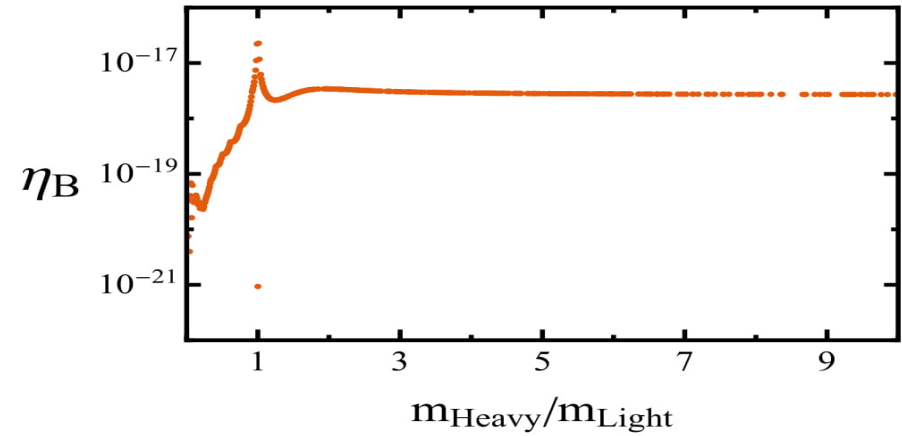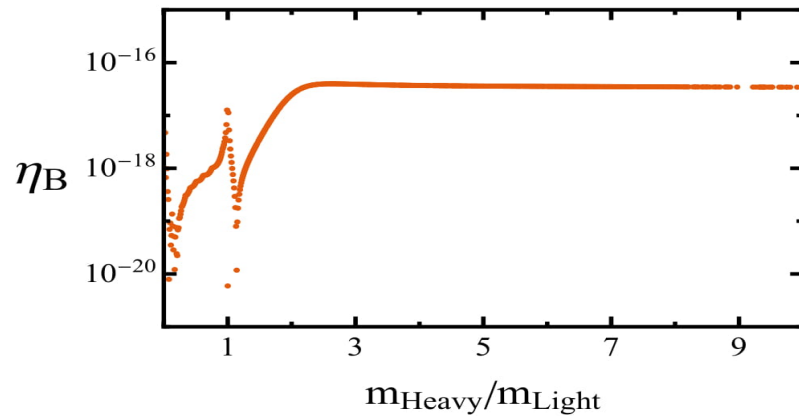
Increasing mass of heavy states does not change the final asymmetry →
hierarchical limit.

Leptonic CPV

Hints of a CPV phase in the lepton sector from neutrino oscillation experiments seen in active neutrino mixing.

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}
$$

atmospheric, accelerator   **accelerator, reactor**   **solar, reactor**

Currently this CPV phase known as the Dirac phase, is unconstrained but **CP conservation disfavoured at 2 sigma**. Expected to be measured in the coming decade.

There are hints that this phase might be maximally violating, could it be relevant in the context of leptogenesis?

Aside: There are potentially two extra phases appearing in this mixing matrix due to the potential Majorana nature of the neutrino sector which do not appear in the quark sector. Much more difficult to measure these as oscillation experiments not sensitive to them. Neutrinoless double beta decay signals may be able to place limits on them.

$$\theta_{12} = 33.6^{\text{o}} \pm 0.8^{\text{o}}$$
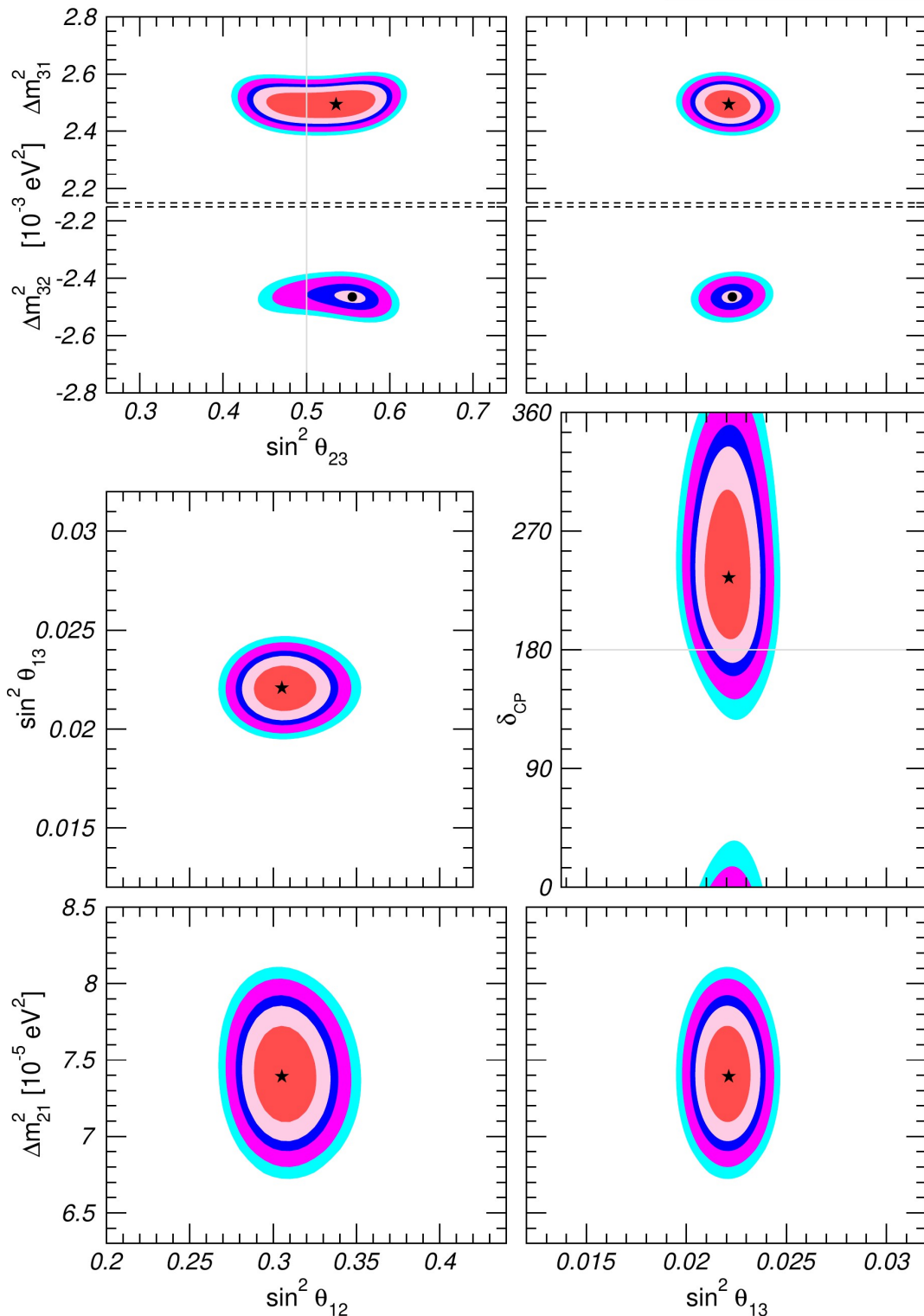
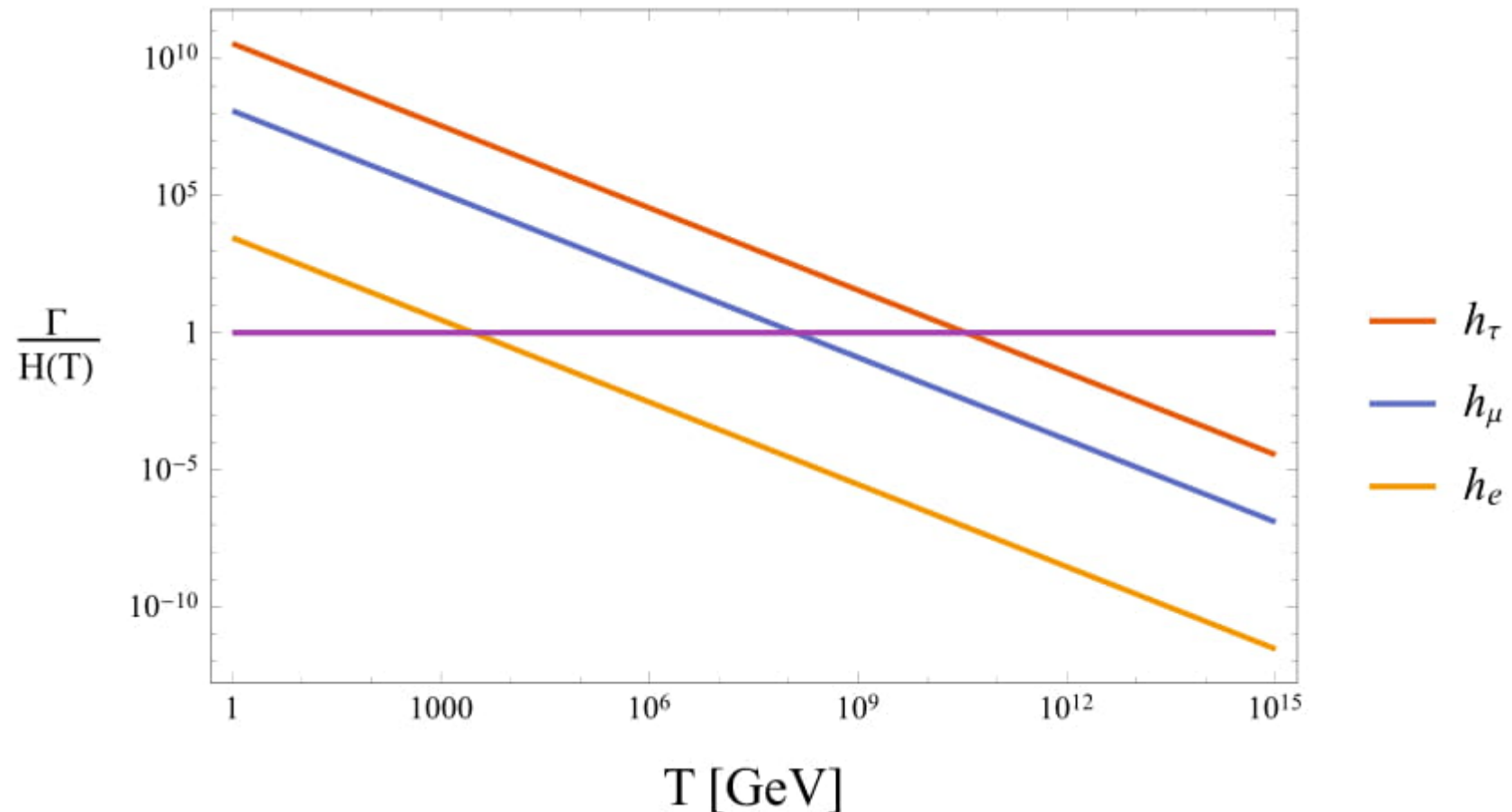$$\Delta m_{21}^2 = +(7.5 \pm 0.2) \times 10^{-5} \text{eV}^2$$

$$\theta_{23} = (38 - 50)^{\text{o}}(3\sigma)$$

$$|\Delta m_{32}^2| = (2.5 \pm 0.4) \times 10^{-3} \text{eV}^2$$

$$\theta_{13} = 8.4^{\text{o}} \pm 0.2^{\text{o}}$$

$$\delta_{CP} = [0, 2\pi]$$

34

Lepton Flavour effects in Leptogenesis

Temperature Regimes:

1) $10^{12}\,\mathrm{GeV} \lesssim \mathrm{T} \lesssim 10^{13}\,\mathrm{GeV}$ - h_τ Yukawa interaction enters Thermal Equilibrium

2) $10^{9}\,\mathrm{GeV} \lesssim \mathrm{T} \lesssim 10^{12}\,\mathrm{GeV}$ - h_μ Yukawa interaction enters Thermal Equilibrium

3) $T \lesssim 10^{9}\,\mathrm{GeV}$ -   Flavour coherence completely broken

Lepton Flavour effects in Leptogenesis

In models of high scale leptogenesis $T > 10^{12}\mathrm{GeV}$ processes involving the three lepton flavours are out of equilibrium in the thermal bath. Can simply count the number of leptons created or destroyed independently of its lepton flavour – 'no flavour' approximation.

As the temperature drops however, SM charged Yukawa interactions enter into thermal equilibrium w.r.t the Hubble expansion causing the thermal bath to distinguish lepton flavours apart and the no flavour approximation is no longer valid. Requires a proper treatment involving tracking the asymmetries in lepton flavour space for the relevant flavours in equilibrium.

More interestingly:
- Outside the 'no flavour' approximation extra dependence on the 3 low energy PMNS phases. Particularly δ_{CP} which is soon to be measured in neutrino oscillations, will act as a source of CP violation during the RHN decay.
- When the 'no flavour' approximation is valid these low energy phases *cannot* contribute to the lepton asymmetry and therefore the baryon asymmetry due to the unitarity of the mixing matrix.

Davidson-Ibarra Bound

Consider the conventional type-I seesaw

$$m_\nu \simeq -v^2 Y_\nu M_R^{-1} Y_\nu^T$$

However m_ν has upper bound from cosmology and Tritium decay experiments

$$m_\nu \leq 0.1 \text{ eV}$$

Therefore the parameters on the right hand side are coupled; lowering the Majorana mass scale of the RHN lowers the Yukawa coupling strength

A smaller coupling leads to a smaller generated asymmetry and turns out can express amount of asymmetry generated as a function of RHN masses (in hierarchical case)

$$|\epsilon_1| \lesssim \frac{3}{8\pi} \frac{M_1}{v^2} (m_3 - m_1)$$

Leads to **lower bound** on lightest RHN mass, any mass lower wont generate enough asymmetry
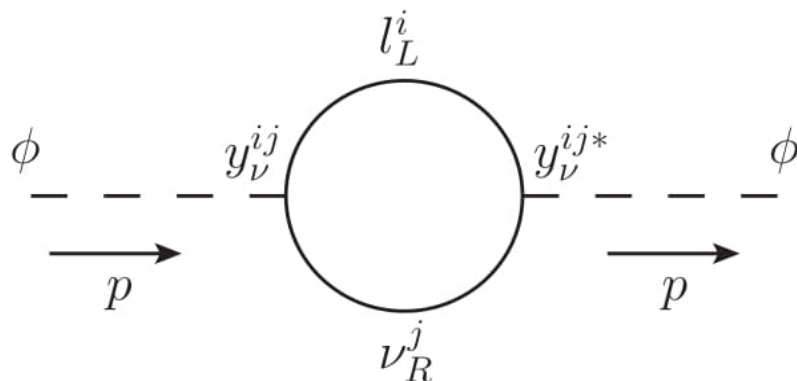
$$M_{N_1} \gtrsim 10^{8-9} \text{ GeV}$$

Hierarchical? $\rightarrow 200 M_1 \lesssim M_2 < M_3$

37

Vissani Bound

If a physical scale much larger than the electroweak scale exists this *can* lead to large corrections to the electroweak parameter μ. Large corrections may imply a fine-tuning (naturalness) problem.

The seesaw does introduce a large physical scale (the Majorana mass scale) whose coupling gets larger as the physical scale increases. Can compute the correction to the electroweak parameter from seesaw

$$\delta\mu^2 \simeq \frac{1}{4\pi^2}\frac{1}{v^2}m_\nu m_N^3$$

Leads to **upper bound** on lightest RHN mass

$$M_N \lesssim 3 \times 10^7 \text{ GeV}$$
$$\text{for } \delta\mu^2 \lesssim \text{TeV}^2$$
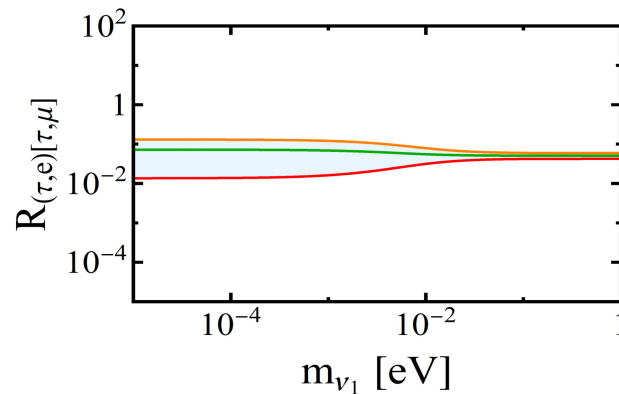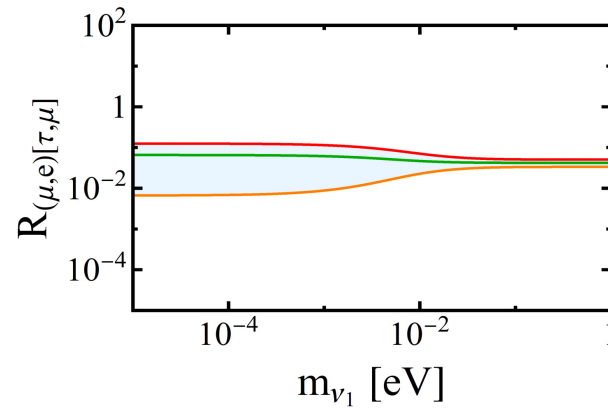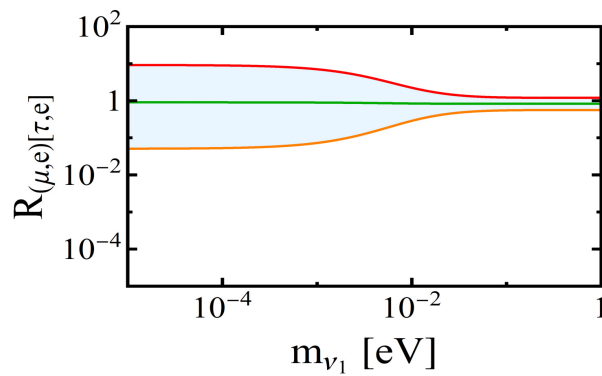

Contribution to $\delta\mu^2$ 1502.01352

Amount of fine-tuning allowed is subjective and can vary; if more fine-tuning allowed can increase the upper bound

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \textcolor{red}{\mu_N + \delta\mu_N} & M_N \\ 0 & M_N^T & \textcolor{red}{\mu_S} \end{pmatrix}$$

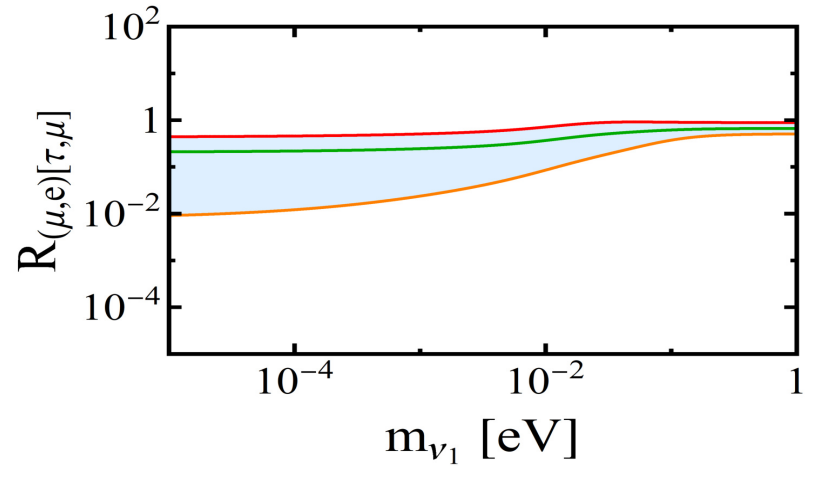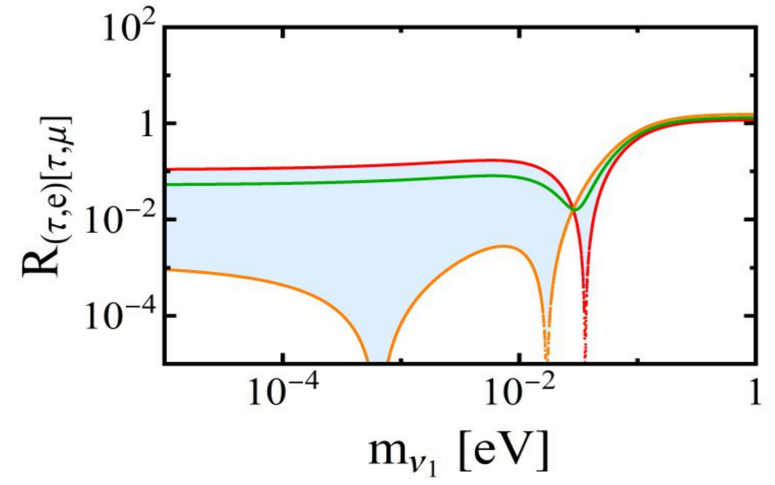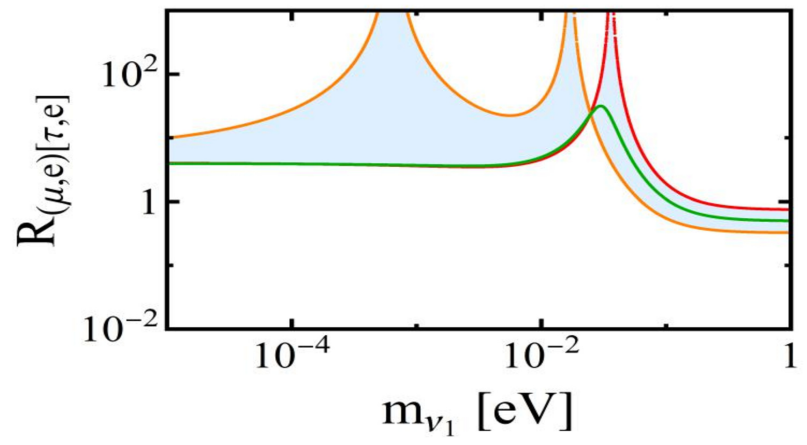# Scenario 3: MLFV Inverse Seesaw

Similar cLFV constraints to previously.

Tests of MLFV inverse seesaw

$$\begin{pmatrix} 0 & m_D & m_L \\ m_D^T & \delta\mu_N & M_N \\ m_L^T & M_N^T & \delta\mu_S \end{pmatrix}$$

Scenario 4: MLFV Linear Seesaw

Minimal Lepton Flavour Violation

MLFV is an extension of the MFV hypothesis in the quark sector.

MFV/MLFV hypothesis recognises that in the massless limit (absent Yukawa couplings) the SM Lagrangian is invariant under a product of flavour symmetries.

$$SU(3)_{q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{\ell_L} \times SU(3)_{e_R}$$

It is an ansatz introduced and based off of two asumptions:

- The broken flavour symmetry above is a 'good' symmetry that the UV sector respects. The *only* source of flavour symmetry breaking within the theory arise from unspecified UV dynamics (spurions) which at the low scale appear as Yukawa couplings.

- The SM is an **effective theory** for which all renormalisable and non-renormalisable operators must respect the gauge *and* flavour symmetries.

Unflavoured

$$\epsilon_{N_i} \propto \mathrm{Im}(h^\dagger h)_{ij}^2$$

Flavoured

$$\epsilon_{N_i}^{\alpha} \propto \sum_{j \neq i} \mathrm{Im}(h_{\alpha i} h_{\alpha j}) \mathrm{Re}((h^\dagger h)_{ij})$$

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R + \delta M_R \end{pmatrix} \rightarrow \begin{pmatrix} 0 & m_D \mathcal{O} \\ \mathcal{O}^T m_D^T & \mathcal{O}^T (M_R + \delta M_R) \mathcal{O} \end{pmatrix}$$

$$\delta M_R \propto \mathrm{Re}(m_D^\dagger M_D)$$