PROBING ULTRALIGHT DARK MATTER USING GALACTIC KINEMATICS

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Weizmann Institute of Science
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TeVPA 2019
03/12/2019

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DARK MATTER CANDIDATE ZOO

MASS	DM Candidate	Example(s)
YYYeV, ZYYeV	MaCHOs	Black Holes Boson Stars
. . .	Heavy Bound States	Dark Blobs
TeV, GeV	WIMPs	Supersymmetry "Extra" Higgs
MeV, keV	Warm Dark Matter	Sterile Neutrinos Dark Photons SIMPs
eV, meV, μeV	Axions	QCD Axions
. . . zeV, yeV	Axion-Like Particles	String/GUT Axions

$M_\odot$

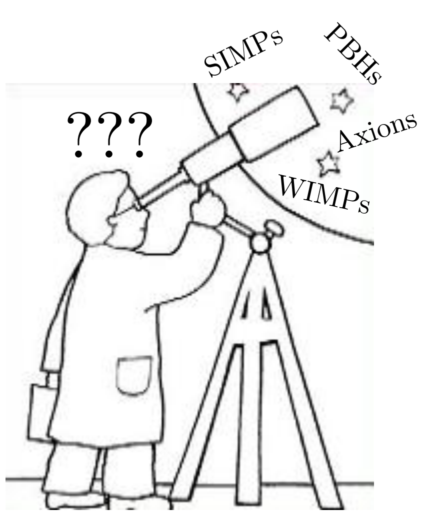
(badly abusing metric prefixes)

(this conference?)

(proton mass)

$10^{-22}\,eV$

> 70 orders of magnitude!



DARK MATTER CANDIDATE ZOO

MASS	DM Candidate	Example(s)
$M_\odot$		
YYYeV	*Heavy objects*	
ZYYeV		
⋮		
TeV	*Very massive particles*	
GeV		
MeV		
keV		
eV	*Wavelike DM*	
meV		
μeV		
⋮		
zeV	*This range: focus of my talk*	
yeV		

(badly abusing metric prefixes)

(this conference?) — TeV

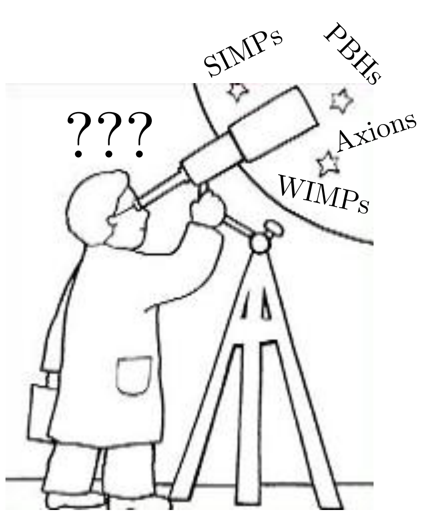
(proton mass) — GeV

$10^{-22}\ eV$

>70 orders of magnitude!

Ultralight Dark Matter (ULDM)

("Fuzzy" DM?)



WHY ULDM?

1. Represents absolute lower limit of mass for DM candidates
2. Can throw away “half” of candidates: ULDM is bosonic
   - Pauli exclusion: not enough quantum states to “fill up” galaxy with fermions

$$\mathcal{N} \sim 10^{94} \times \left(\frac{\rho_{\rm local}}{0.4\,{\rm GeV/cm^3}}\right)\left(\frac{10^{-22}\,{\rm eV}}{m_\phi}\right)^4\left(\frac{10^{-3}\,c}{\sigma}\right)^3$$

3. Large (galactic-scale) effects on de Broglie length scales

$$\lambda_{dB} \sim {\rm kpc} \times \left(\frac{10^{-22}\,{\rm eV}}{m_\phi}\right)\left(\frac{10^{-3}c}{\sigma}\right)$$

4. Simple: described by classical wave equation

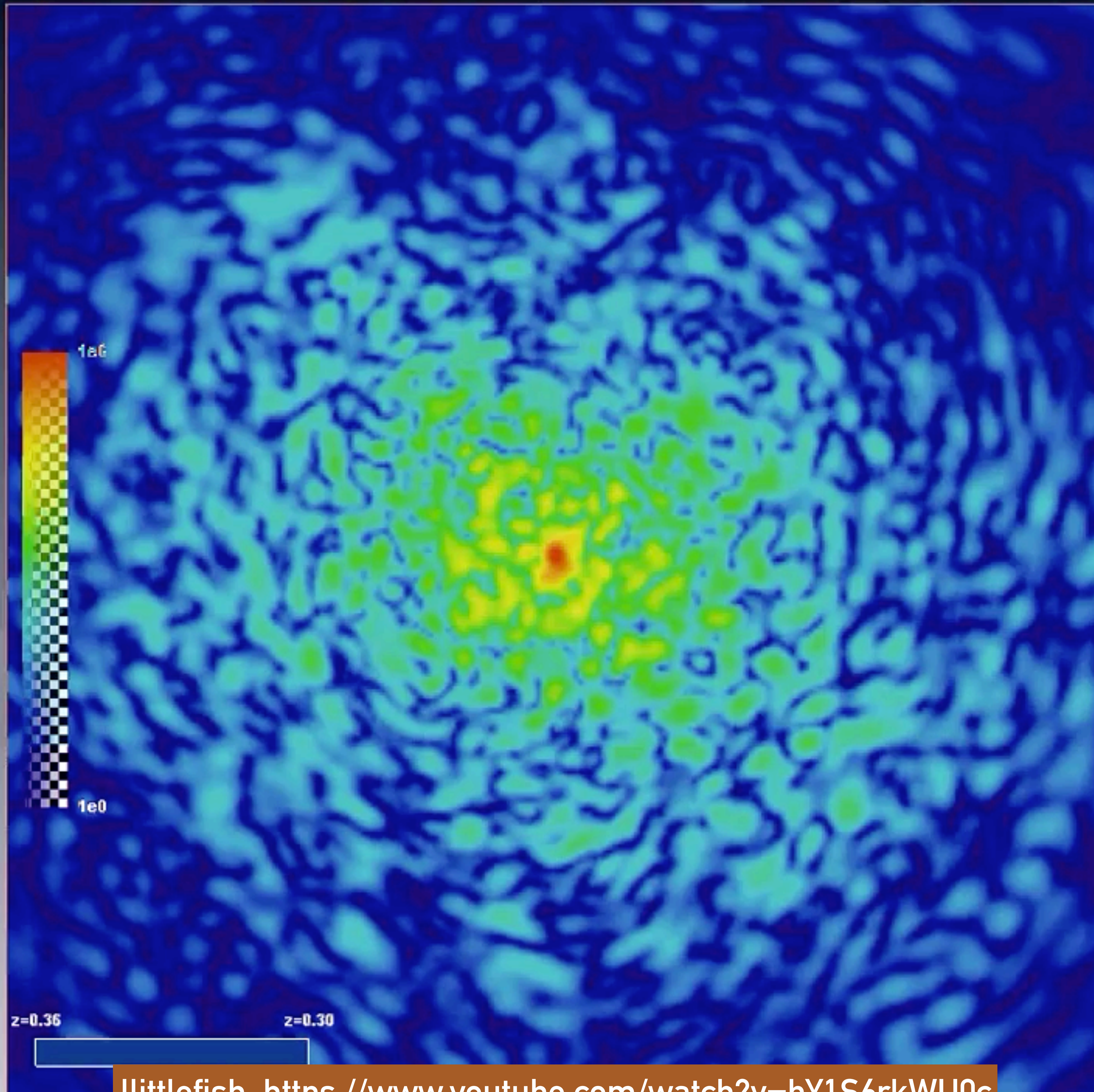
Oscillations Newtonian Gravity

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{\nabla^2}{2m_\phi} + V_g\left(|\psi|^2\right) + \cancel{V_{int}\left(|\psi|^2\right)}\right]\psi$$

Kinetic energy Self-interactions (ignore here)

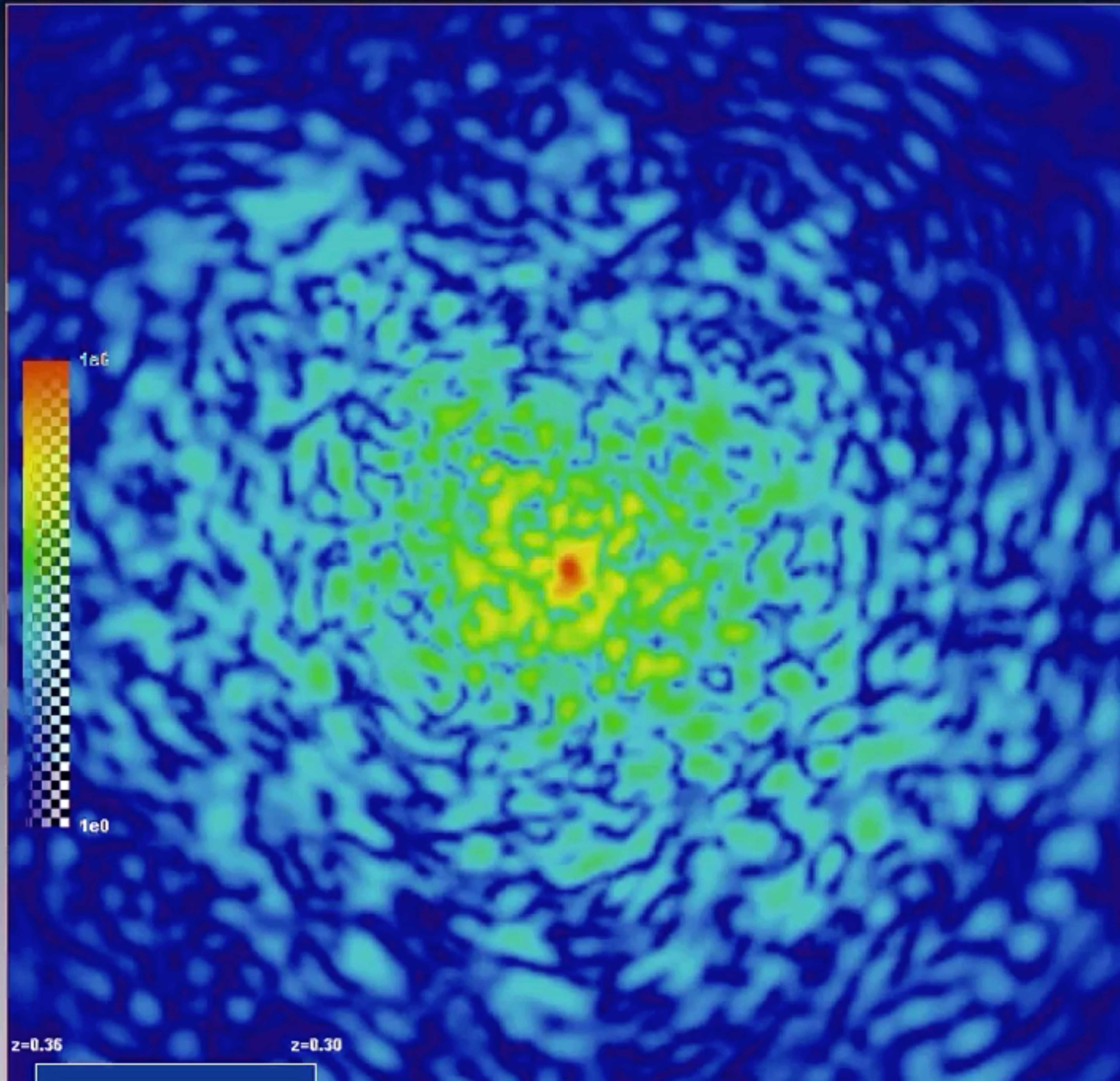
⇒ *Can probe ULDM in a model-independent way, using gravity only*

SIMULATING WAVE-LIKE DARK MATTER



llittlefish: https://www.youtube.com/watch?v=bY1S6rkWU0c

SIMULATING WAVE-LIKE DARK MATTER



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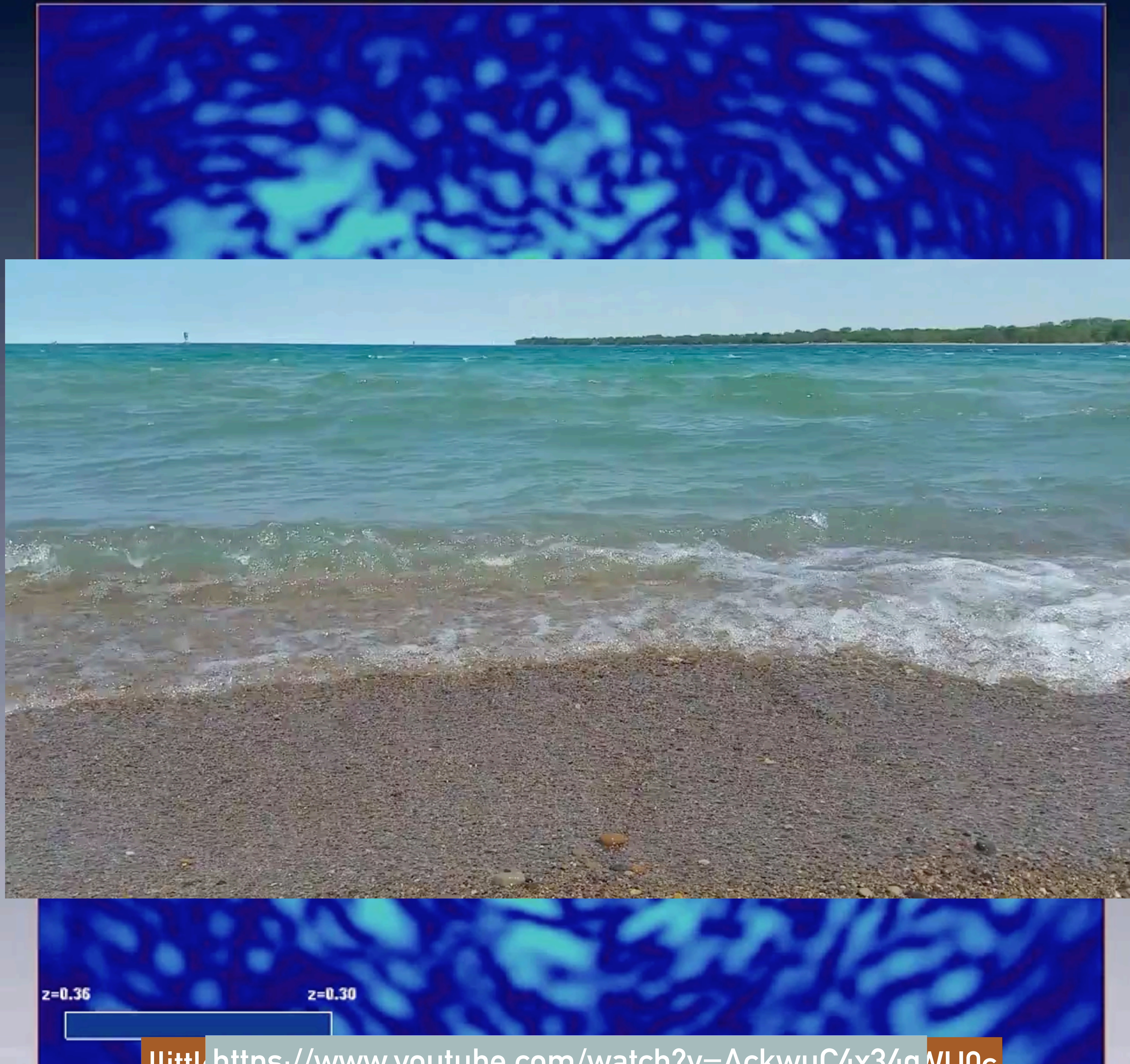
SIMULATING WAVE-LIKE DARK MATTER



llittl https://www.youtube.com/watch?v=AckwuC4x34g VU0c

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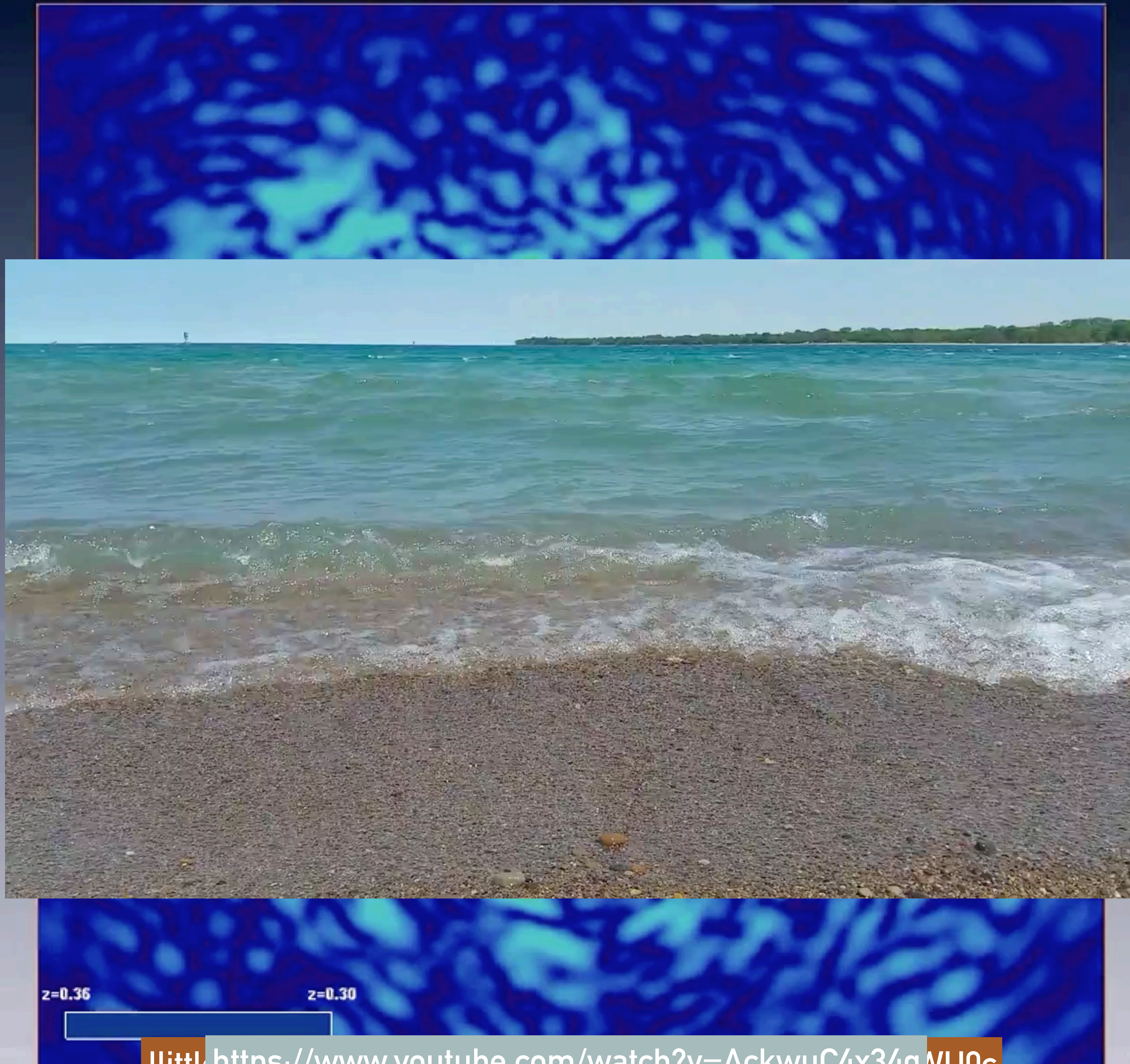

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ULDM CORE FORMATION

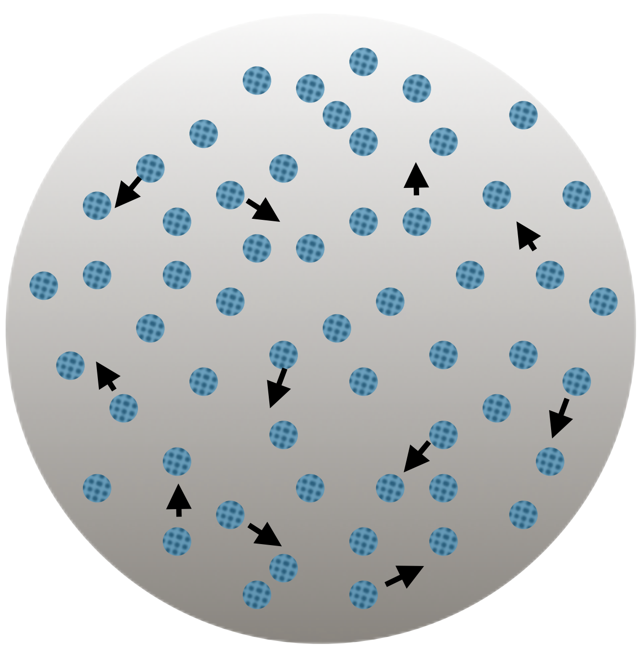
Hui, Ostriker, Tremaine, Witten (1610.08297)

Bar-Or, Fouvry, Tremaine (1809.07673)

- Recent studies: ULDM described by 'quasiparticles' with

$$m_{\rm eff} \approx \frac{\rho}{(m_\phi \sigma)^3} \approx 10^5\, M_\odot \left(\frac{10^{-22}\,{\rm eV}}{m_\phi}\right)^3 \left(\frac{200\,{\rm km/sec}}{\sigma}\right)^3 \qquad \lambda_{\rm dB} \approx \frac{1}{m_\phi \sigma} \approx {\rm kpc} \left(\frac{10^{-22}\,{\rm eV}}{m_\phi}\right)\left(\frac{200\,{\rm km/sec}}{\sigma}\right)$$

- Gravitational relaxation of quasiparticles gives rise to cored structure



Quasiparticle dispersion

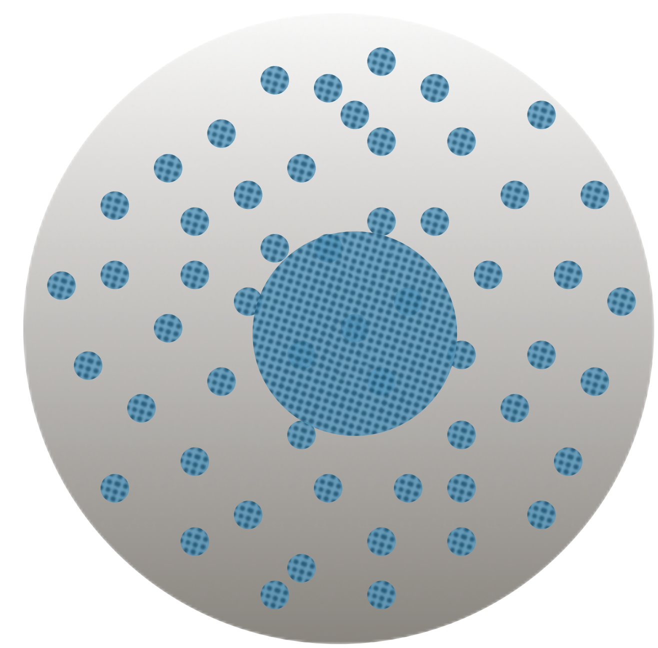
Binney and Tremaine, "Galactic Dynamics"

Velocity change per crossing

$$\Delta v^2 \simeq 8\,N \left(\frac{G\,M}{R_{\rm gal}\, v}\right) \ln N$$

Fractional velocity change

$$\frac{\Delta v^2}{v^2} \simeq \frac{8\,\ln N}{N}$$

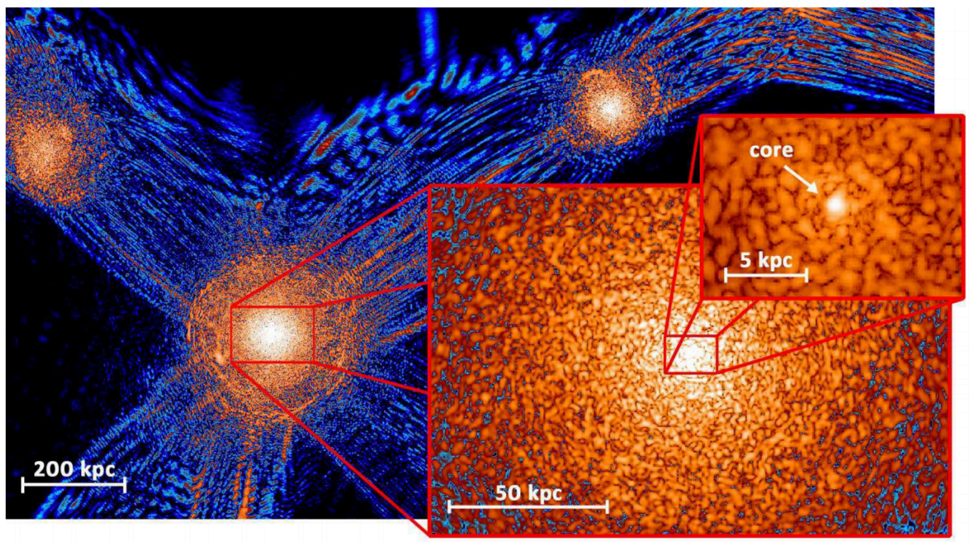


Core formation

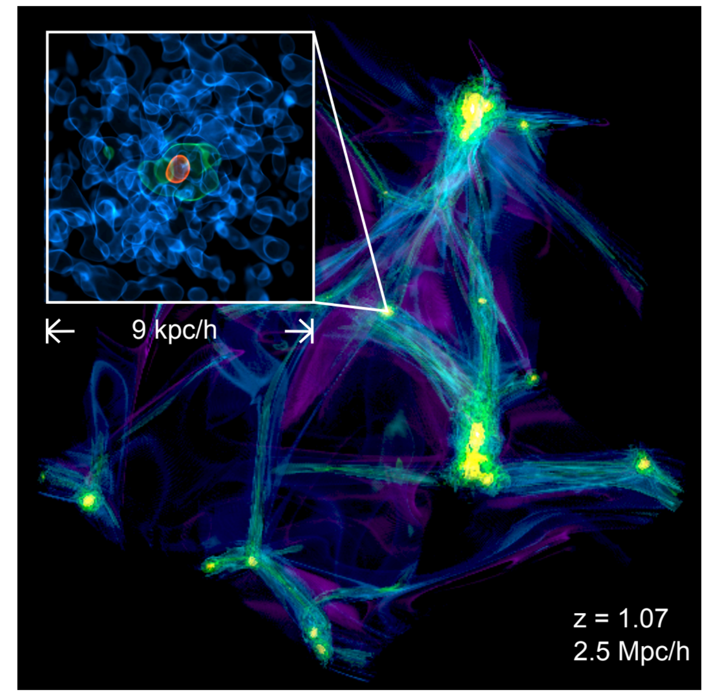
Relaxation to ground state after $t_{\rm relax} \simeq \dfrac{0.1\,N}{\ln N} t_{\rm cross}$

SIMULATIONS CONFIRM THIS PICTURE

- ULDM, $m_\phi \lesssim 10^{-19}\,\mathrm{eV}$



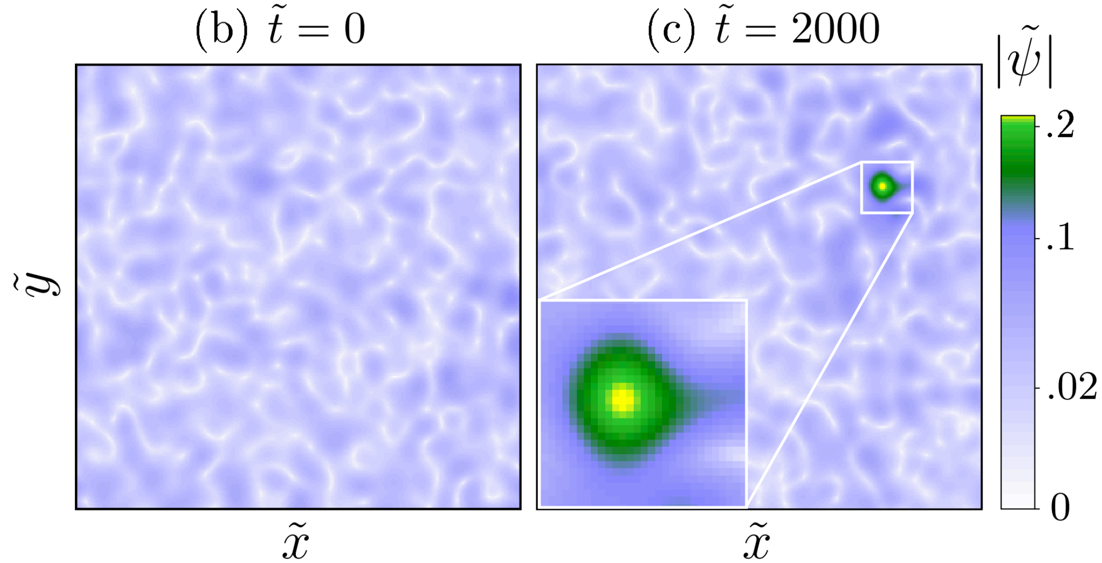
Schive, Chiueh, Broadhurst (1406.6586)



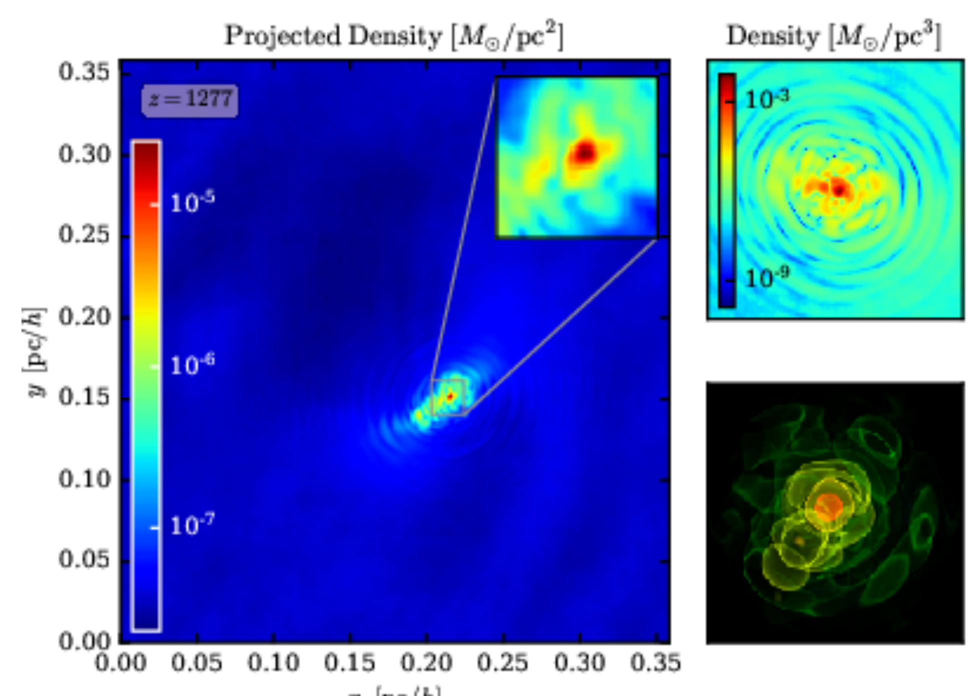
Veltmaat, Niemeyer, Schwabe (1804.09647)

+others…

- QCD axions, $m_\phi \sim 10^{-6}\,\mathrm{eV}$



Levkov, Panin, Tkachev (1804.05857)



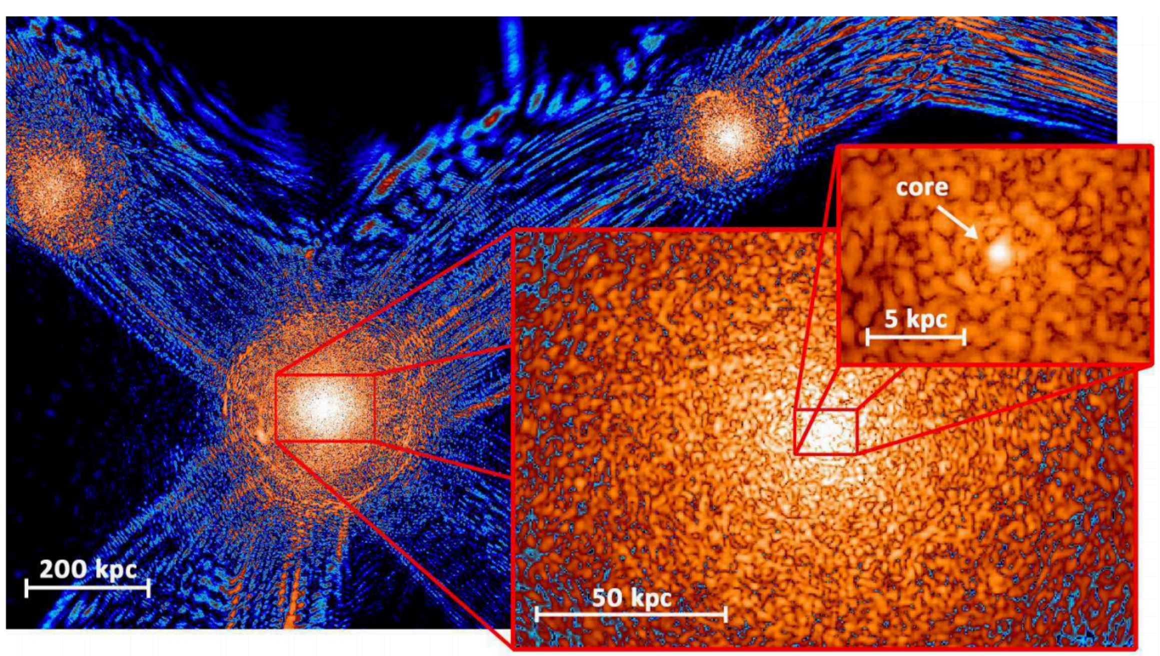
Eggemeier and Niemeyer (1906.01348)

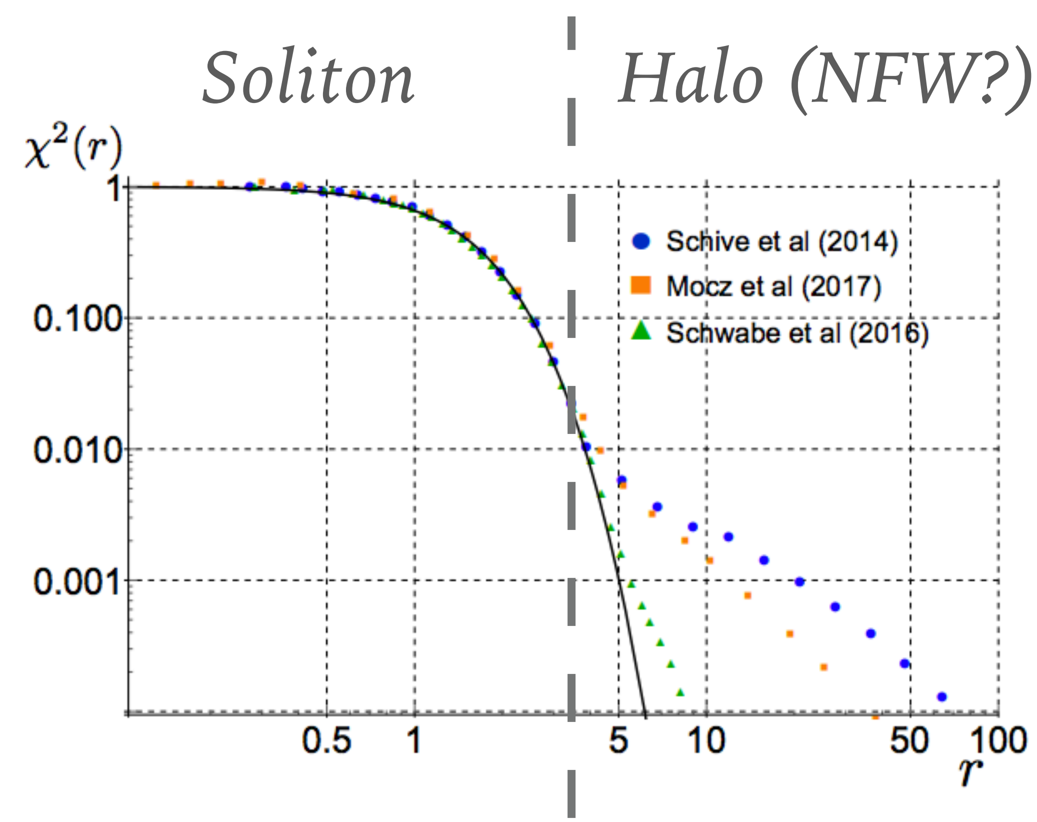
ULDM CORES

- ULDM core = soliton = self-gravitating ground state of axion field

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{\nabla^2}{2m_\phi} + V_g\left(|\psi|^2\right)\right]\psi \qquad \Rightarrow \quad R_\star \approx \frac{M_P^2}{m_\phi^2 M_\star}$$

Balance these forces *Stable under perturbations and decay*





SOLVING FOR SOLITONS

$$\psi(r,t) = \left(\frac{m_\phi M_P}{\sqrt{4\pi}}\right) e^{-i\mu m_\phi t}\chi(x) \qquad r = x/m_\phi \qquad V_g = m_\phi \Phi$$

- Solitons exist along a continuous family, one free parameter

$$\dot{\psi} = \left[-\frac{\nabla^2}{2m_\phi} + V_g\right]\psi$$

$$\nabla^2 V_g = 4\pi G m_\phi^2 |\psi|^2$$

$$\nabla_x^2 \chi = 2\left(\Phi - \mu\right)\chi$$

$$\nabla_x^2 \Phi = \chi^2$$

Simple equations of motion

- Spherically symmetric: easy to solve using shooting method

- Baryonic potentials introduce (disc-like) background potential

$$\nabla_x^2 \chi = 2\left(\Phi + \Phi_b - \mu\right)\chi$$

$$\nabla_x^2 \Phi = \chi^2$$

Difficult equations of motion!

- Our work: develop simple algorithm to solve for soliton in presence of azimuthally-symmetric potential (i.e. include baryons!)

Bar, Blum, JE, Sato (1903.03402)

CLUES FROM SIMULATION: CORE-HALO RELATION

- Curious connection between central soliton and host halo:

Schive, Chiueh, Broadhurst (1406.6586)

Schive et al. (1407.7762)

Veltmaat, Niemeyer, Schwabe (1804.09647)

$$M_{sol} \approx 10^9 M_\odot \left(\frac{10^{-22}\, eV}{m_\phi}\right)\left(\frac{M_{halo}}{10^{12} M_\odot}\right)^{1/3}$$

- Can be recast as

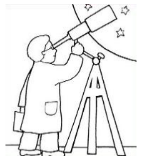
$$\left(\frac{E}{M}\right)_{sol} \sim \left(\frac{E}{M}\right)_{halo}$$

Bar, Blas, Blum, Sibiryakov (1805.00122)

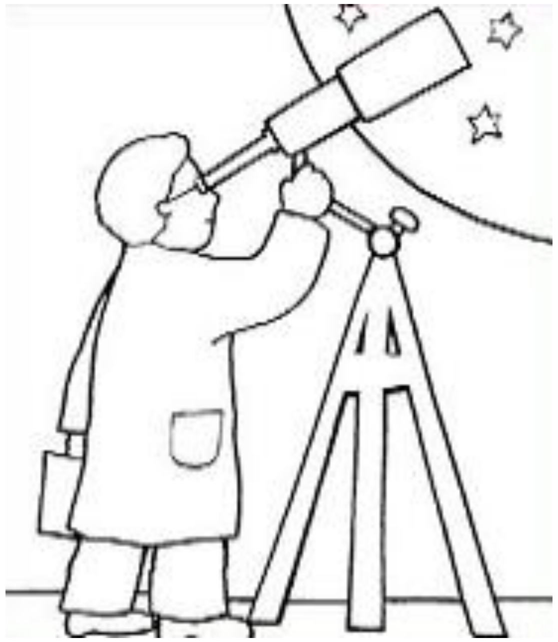
- Which implies (!)

$$(v_{core}^{peak})^2 \sim (v_{NFW}^{peak})^2$$

Potential new observable in galactic kinematic data!

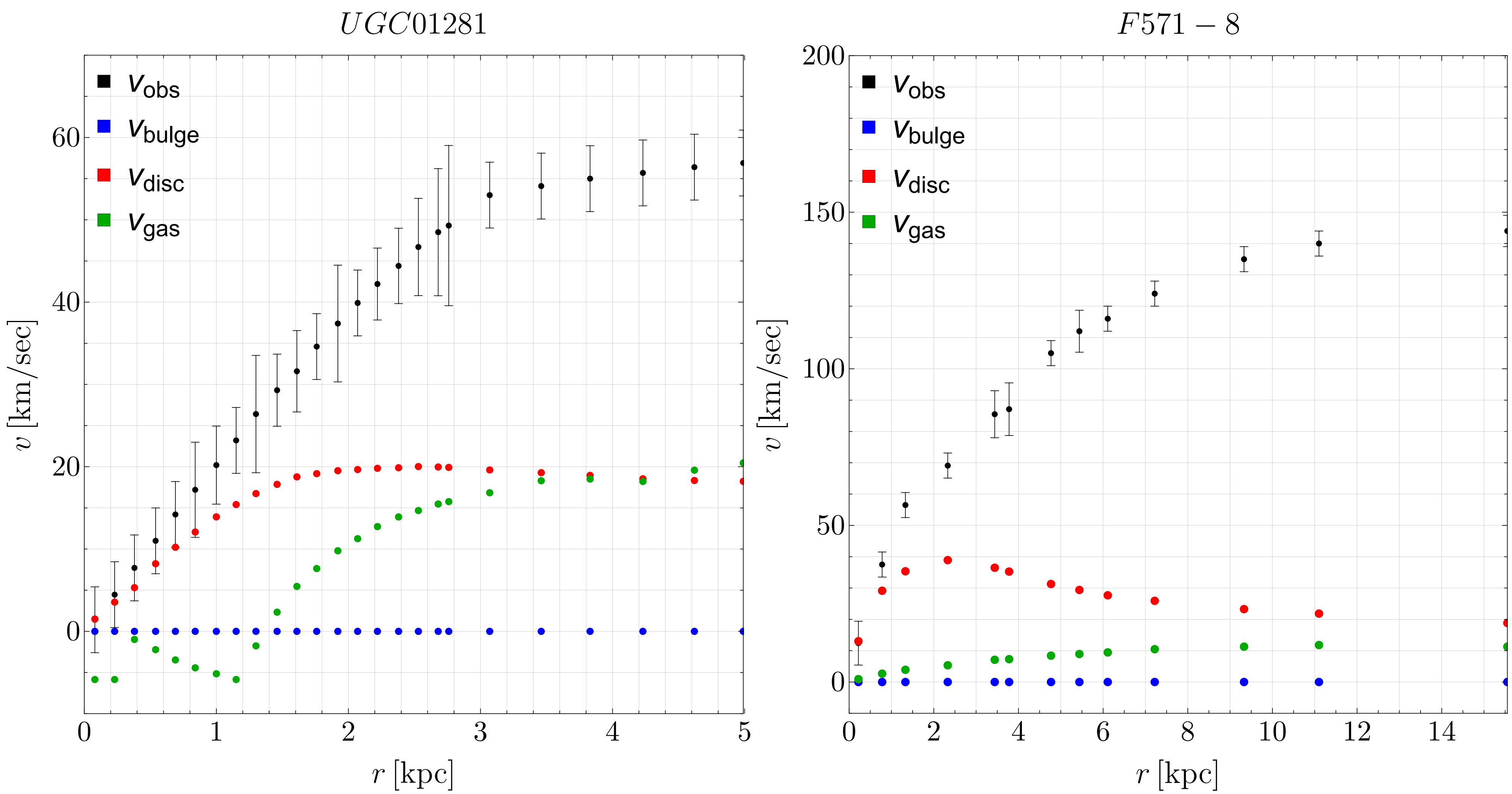


HOW DO WE TEST IT?



GALACTIC KINEMATIC DATA

- Significant data sets available: e.g. SPARC database
  Lelli, McGaugh, Schombert (1606.09251)
- Over 175 galaxies: disc, bulge, and (sometimes) gas modelling



ULDM Tests using SPARC:

- Spherical symmetry
  Bar, Blas, Blum, Sibiryakov (1805.00122)
- Azimuthal symmetry
  Bar, Blum, JE, Sato (1903.03402)

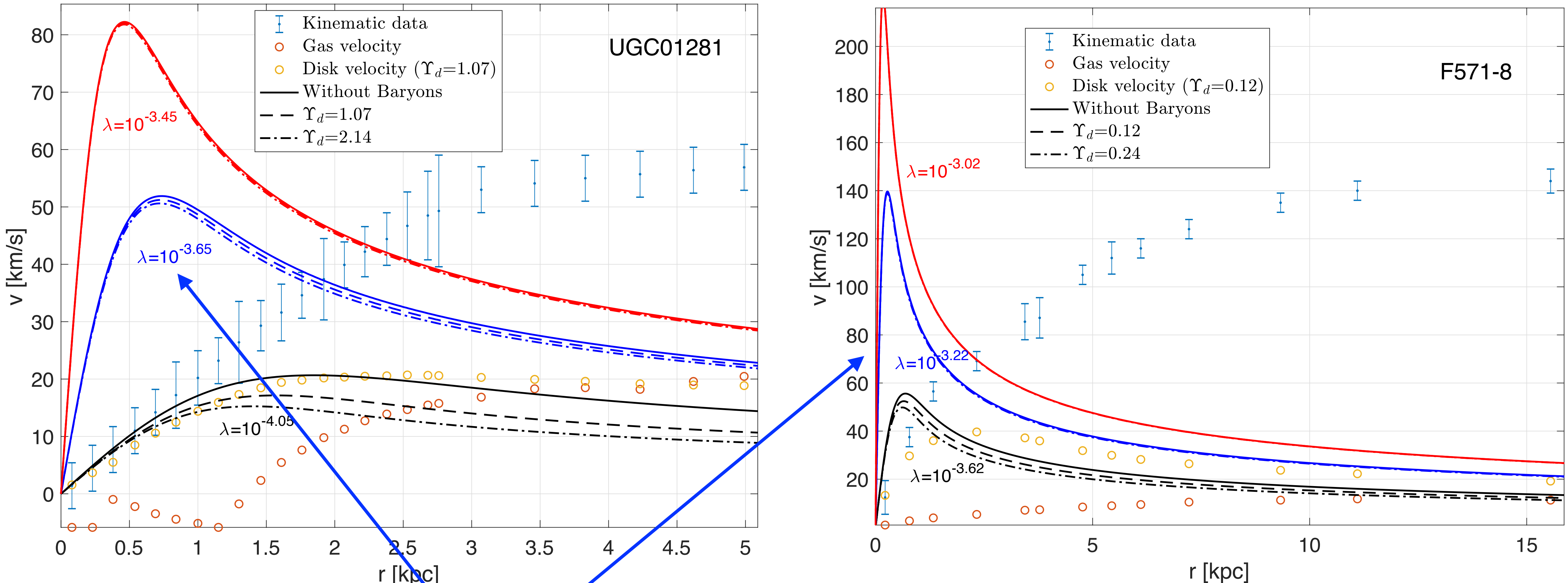
- Ideal tests: Clean; DM-dominated; mass range matches simulations
- Test ULDM prediction!

ULDM WITH BARYONS $m_\phi = 10^{-22}\,eV$

Bar, Blas, Blum, Sibiryakov (1805.00122)

Bar, Blum, JE, Sato (1903.03402)

Blue = Core-Halo Relation



+dozens of similar results for other SPARC galaxies

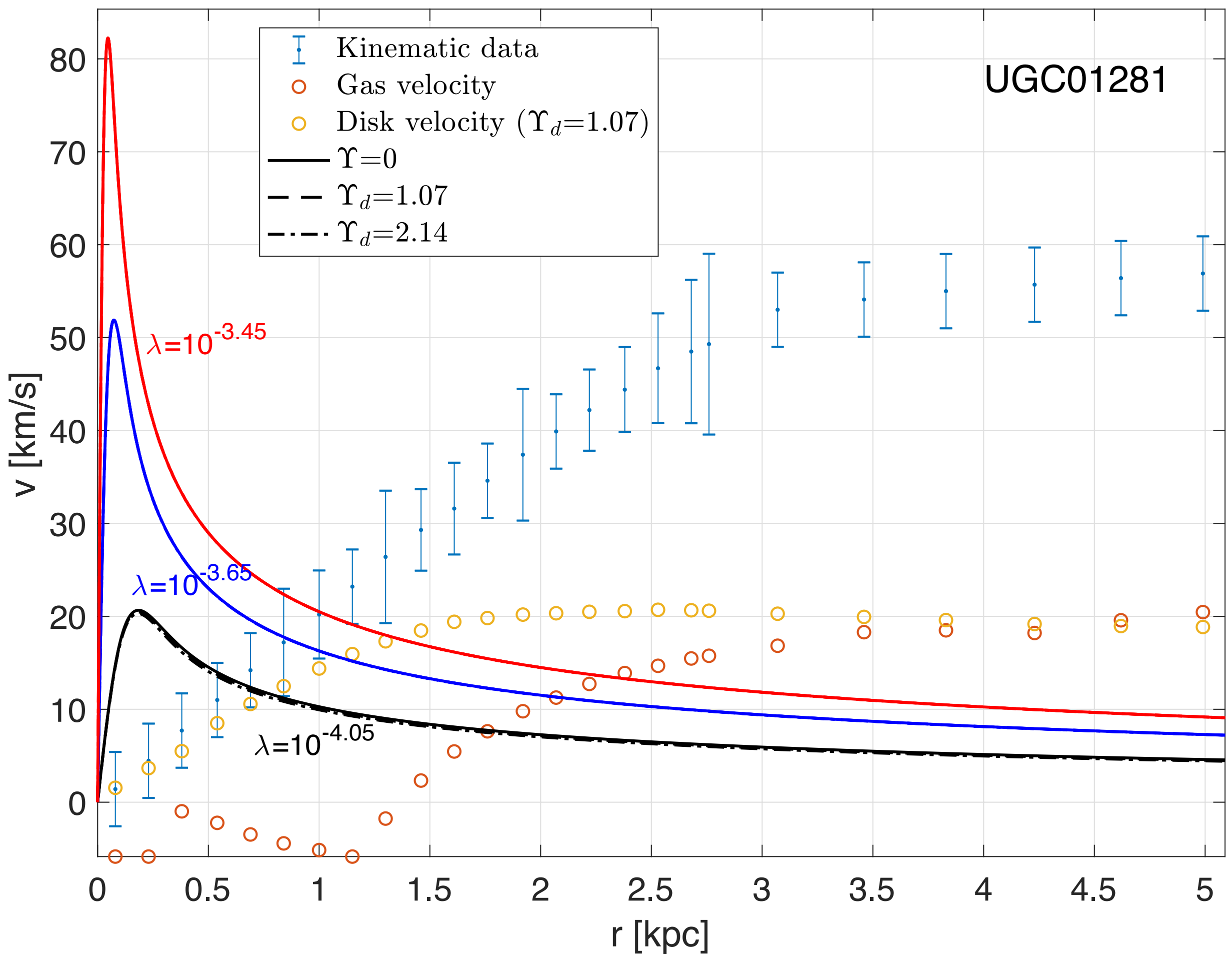
- ➤ **<u>ULDM prediction</u>** of peak velocity in core **<u>fails badly</u>** in numerous galaxies
- ➤ If simulations are correct, 10^{-22} eV $\lesssim m_\phi \lesssim 10^{-21}$ eV in tension with data
- ➤ Complimentary to constraint from Lyman-α, $m_\phi \lesssim 10^{-21}$ eV

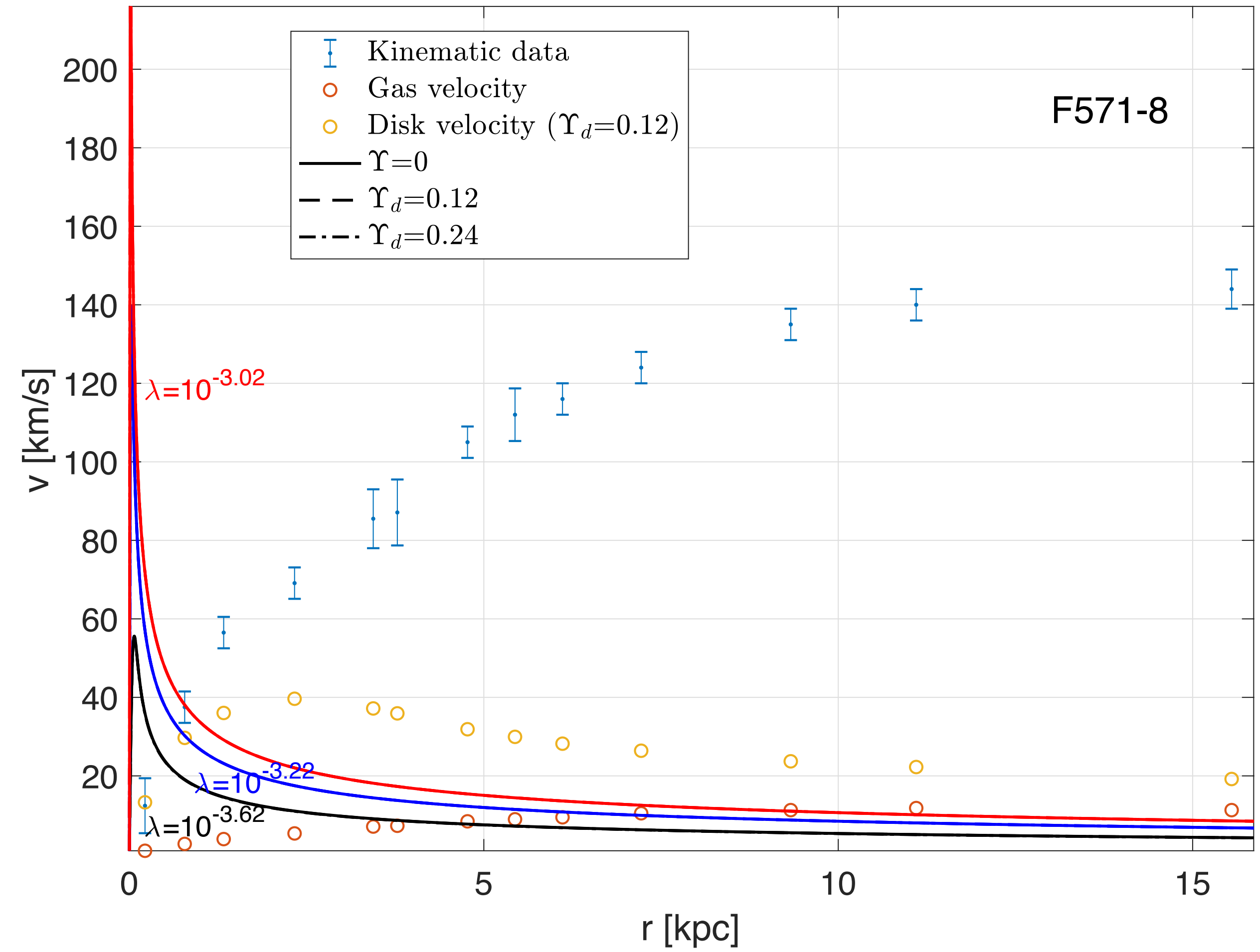
ULDM WITH BARYONS $m_\phi = 10^{-21}\,eV$

Bar, Blas, Blum, Sibiryakov (1805.00122)

Bar, Blum, JE, Sato (1903.03402)

Blue = Core-Halo Relation





- At higher masses, ULDM prediction of peak velocity in core marginally consistent
- Data at the level to constrain roughly $m_\phi \simeq 10^{-21}$ eV
  - To do better requires <10 pc observations. Difficult!
  - Aside: Recent simulation with baryons suggests further core growth
    $\rightarrow$ If true, increases discrepancy with data *Veltmaat, Schwabe, and Niemeyer (1911.09614)*

ANOTHER OBSERVABLE: ULDM FLUCTUATIONS OUTSIDE THE CORE

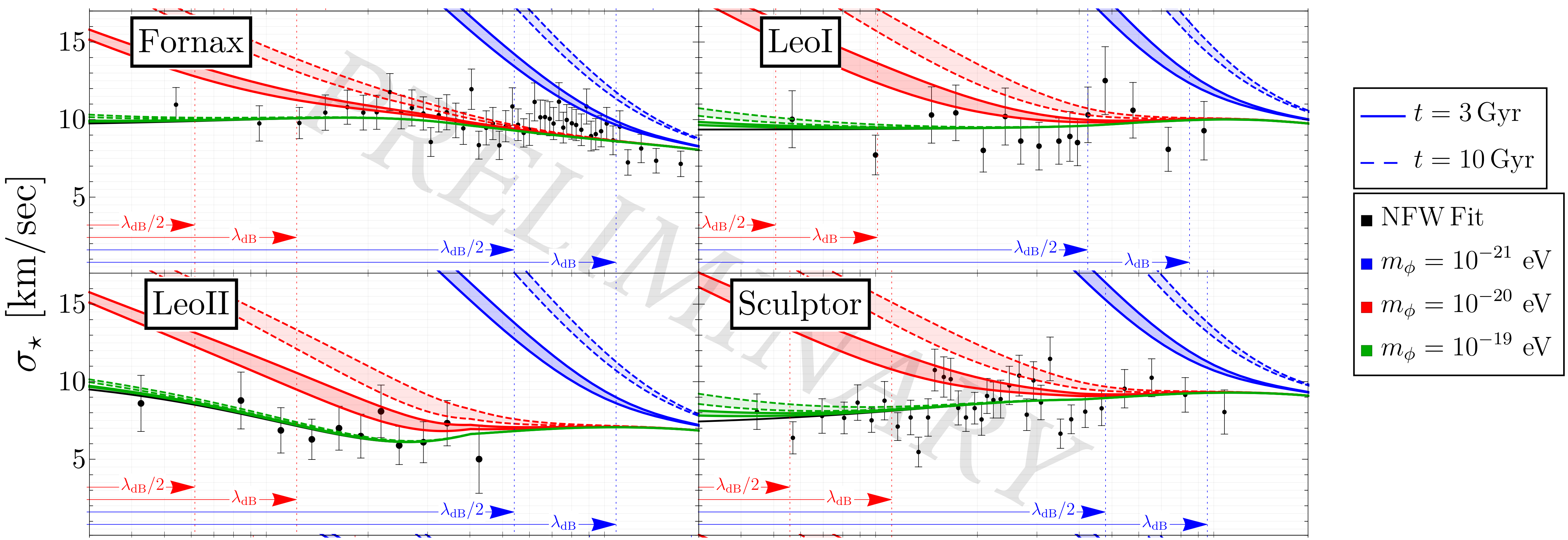
Blum, JE, Kim (To Appear)

- Two-body encounters with ULDM quasiparticles ‘heat’ populations of stars

$$\frac{d\sigma_\star^2}{dt} \simeq \frac{\sigma^2}{T_{\rm heat}}\left(1+\frac{2\,\sigma_\star^2}{\sigma^2}\right)^{-\frac{3}{2}} \quad \text{with } T_{\rm heat} \simeq 0.14\,\text{Gyr}\left(\frac{m_\phi}{10^{-21}\,\text{eV}}\right)^3\left(\frac{0.01\frac{M_\odot}{\text{pc}^3}}{\rho}\right)^2\left(\frac{\sigma}{10\,\frac{\text{km}}{\text{sec}}}\right)^6$$

Bar-Or, Fouvry, Tremaine (1809.07673)

- Can be probed in Milky Way Dwarf Spheroidal galaxies!

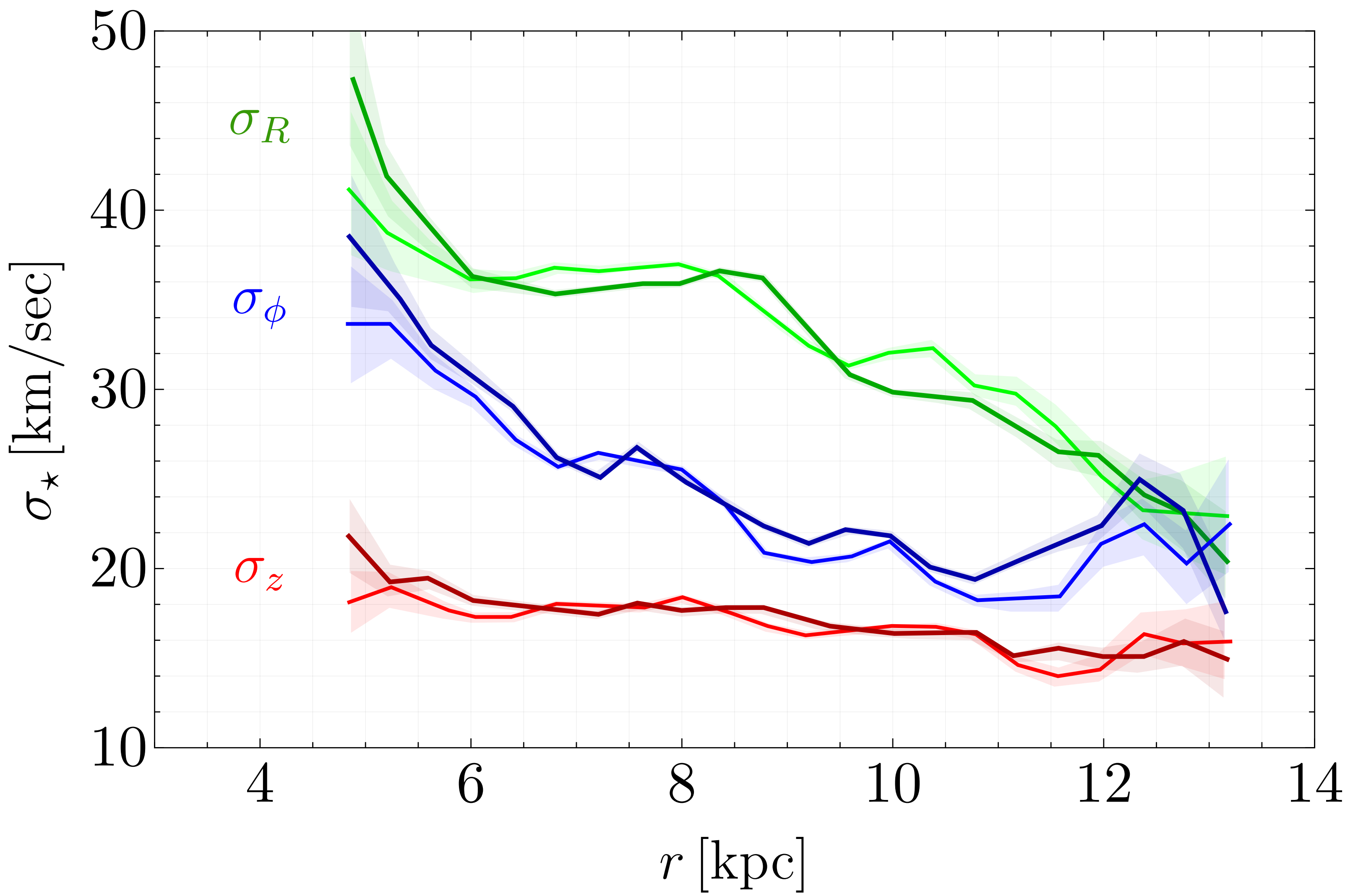


- Modifies Jeans analysis; possible tension; depends sensitively on details near $\lambda_{\rm dB}$

PROBING ULDM WITH GAIA?

Blum, JE, Kim (To Appear)

- GAIA measures stellar velocity dispersion in the Milky Way (among other things)



- Can we probe ULDM and quasiparticle 'heating' of stellar populations with GAIA?
- Maybe not: Heating effect suppressed due to disc rotation
- Stay tuned to find out!

CONCLUSIONS

- Ultralight dark matter exhibits wave-like dynamics on galactic scales
- Simulations predict solitonic cores with particular mass and density
  - Resulting peak in circular velocity *not observed* in large sample of galaxies with high-precision kinematic data
  - Tension for 10^{-22} eV $\lesssim m_\phi \lesssim 10^{-21}$ eV (depends on simulations)
- ULDM quasiparticles 'heat' stellar populations; possible observational reach to 10^{-21} eV $\lesssim m_\phi \lesssim 10^{-20}$ eV
- All such constraints depend on gravity only, and are thus extremely model-independent
- We can close the 70+ order-of-magnitude space in DM models!

Thanks!

Thanks to Zuckerman STEM Leadership Program for financial support

DIRECTOR'S CUT (INCLUDES DELETED SCENES)

NON-SPHERICAL POTENTIALS

- Baryonic disc potentials introduce non-sphericity
- Shooting method inadequate to solve 2-D equations of motion
- Our work: successive over-relaxation method to solve 2-D EoM

Input: Φ_0, χ_0 Φ_b $\xrightarrow[\textit{solve}]{\textit{Iteratively}}$

$$\nabla^2\Phi(R,z) = \chi(R,z)^2$$

update Φ

update χ

$$\frac{\partial\chi}{\partial\tau} = \nabla^2\chi - 2\left(\Phi + \Phi_b\right)\chi$$

Converges

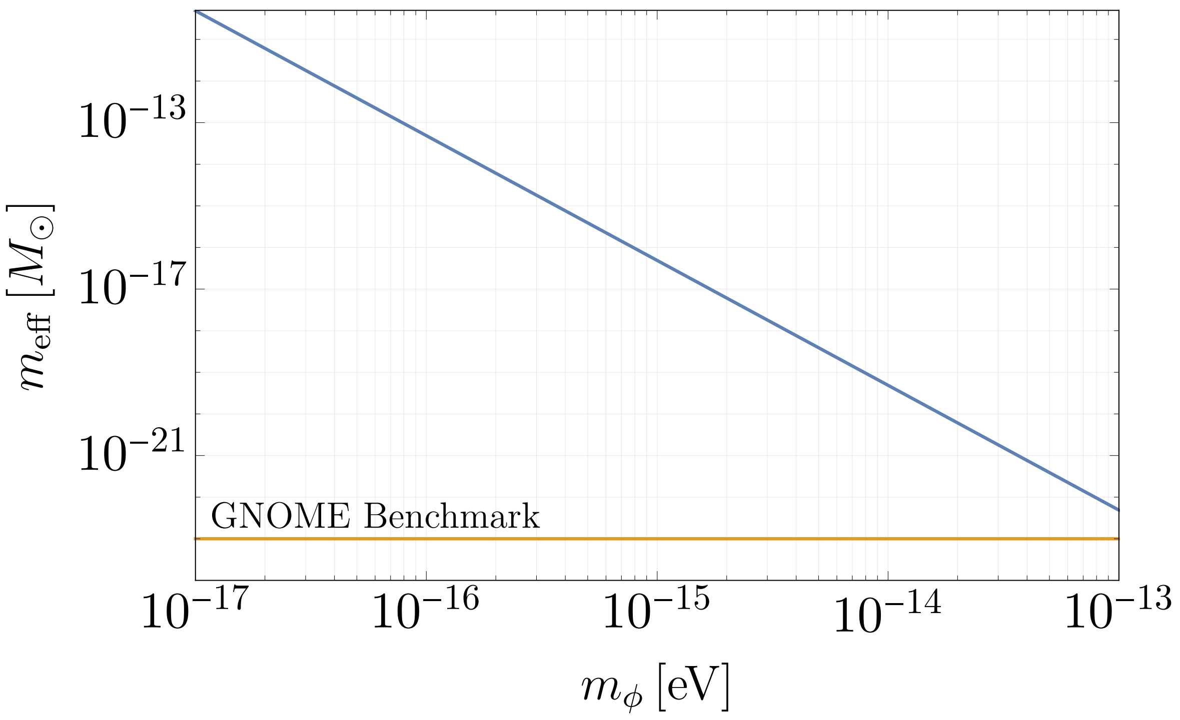
$$\Phi_{sol}, \chi_{sol}$$

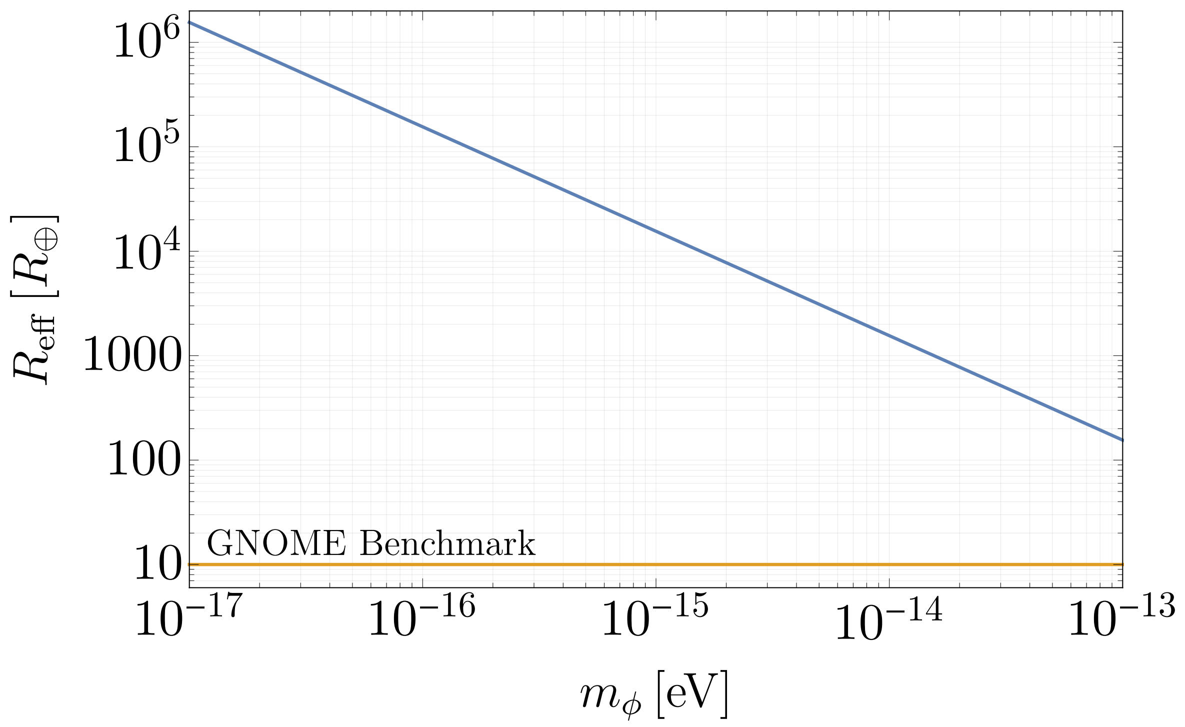
$$\lim_{\tau\to\infty} \chi_{sol}(\tau) \propto e^{-2\mu\tau}\chi_{sol}(0)$$

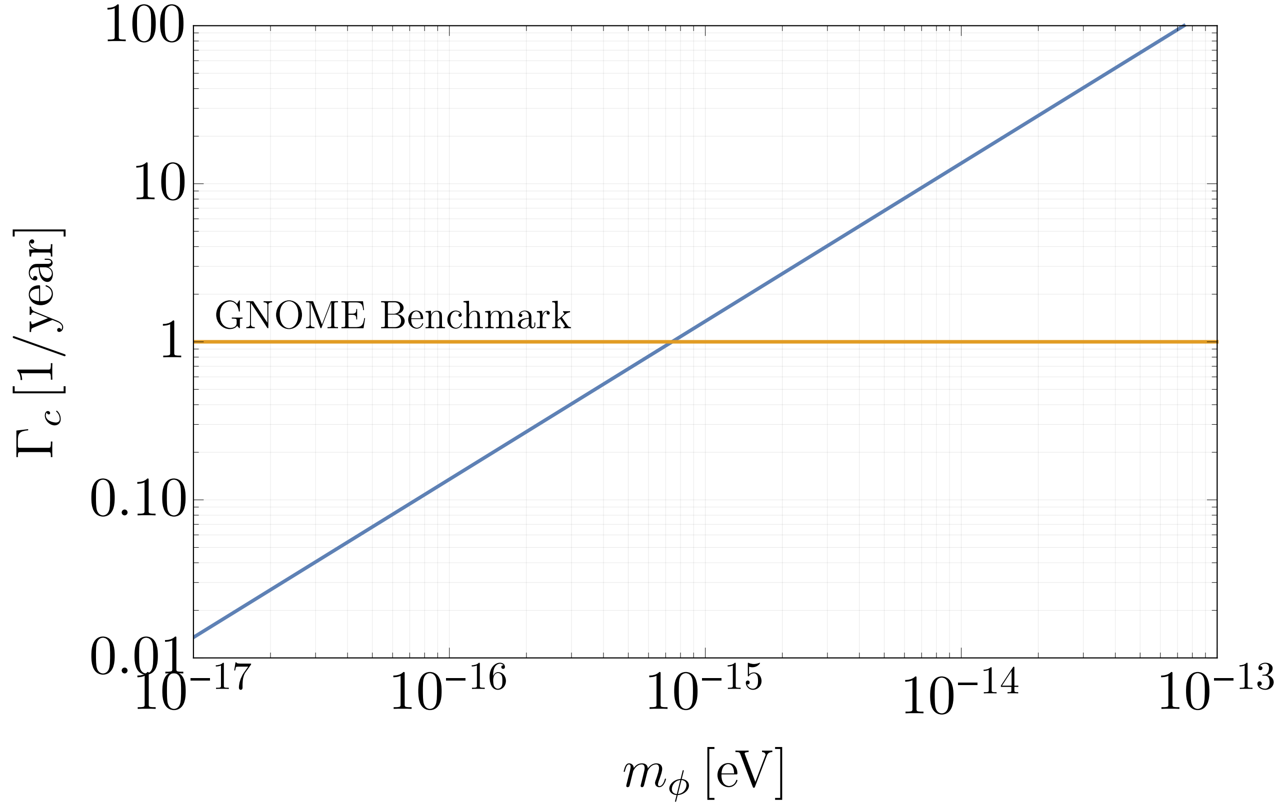
SPARC w/ spherical symmetry Bar, Blas, Blum, Sibiryakov (1805.00122)

SPARC w/ azimuthal symmetry Bar, Blum, JE, Sato (1903.03402)

QUASIPARTICLE ENCOUNTERS

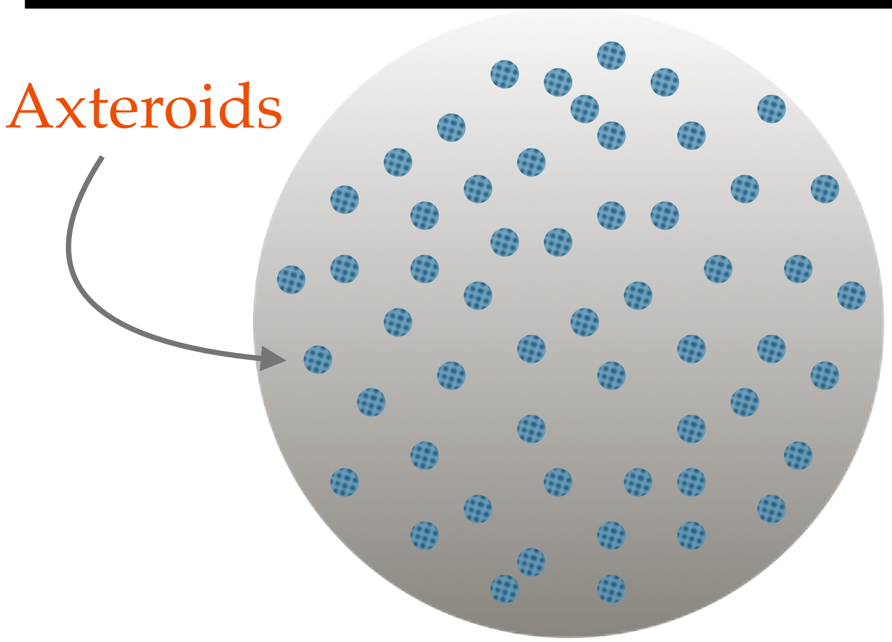






AXION STAR SCALES

"QCD" axion	"FDM" axion ("Fuzzy Dark Matter")
$m \sim 10^{-5}$ eV and $f \sim 10^{11}$ GeV	$m \sim 10^{-22}$ eV and $f \sim 10^{16}$ GeV
Natural scale	Natural scale
$M \sim 10^{-11} M_\odot$	$M \sim 10^{10} M_\odot$
$R \sim 100$ km	$R \sim 100$ pc

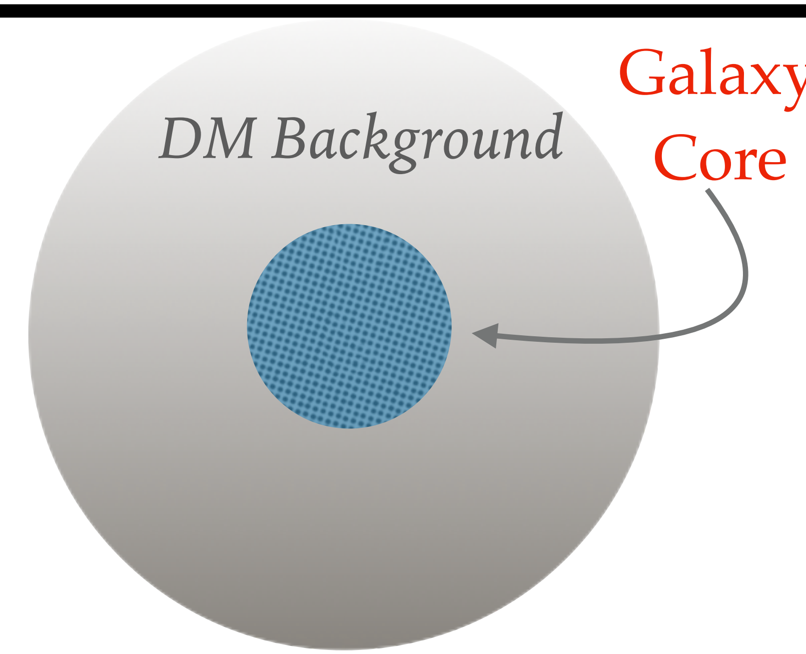


Kolb and Tkachev (PRL 1993)

Vaquero, Redondo, Stadler (1809.09241)

Simulations give very different models for halo substructure

Could have $O(1)$ DM fraction in clumps



Schive, Chiueh, Broadhurst (1406.6586)

ASIDE: OTHER BOUND STATES

(attractive ϕ^4)

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{\nabla^2}{2m} \cancel{+ V_g\left(|\psi|^2\right)} + V_{int}\left(|\psi|^2\right)\right]\psi$$

Chavanis (1103.2050), with Delfini (1103.2054)

"Transition axion star"

Unstable to perturbations

$(1 - \cos\phi/f)$

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{\nabla^2}{2m} \cancel{+ V_g\left(|\psi|^2\right)} + V_{int}\left(|\psi|^2\right)\right]\psi$$

Braaten, Mohapatra, Zhang (1512.00108)

"Dense axion star"

Unstable to decay

JE, Suranyi, Wijewardhana (1512.01709), + Ma (1705.05385)

(repulsive ϕ^4)

$$i\frac{\partial\psi}{\partial t} = \left[\cancel{-\frac{\nabla^2}{2m}} + V_g\left(|\psi|^2\right) + V_{int}\left(|\psi|^2\right)\right]\psi$$

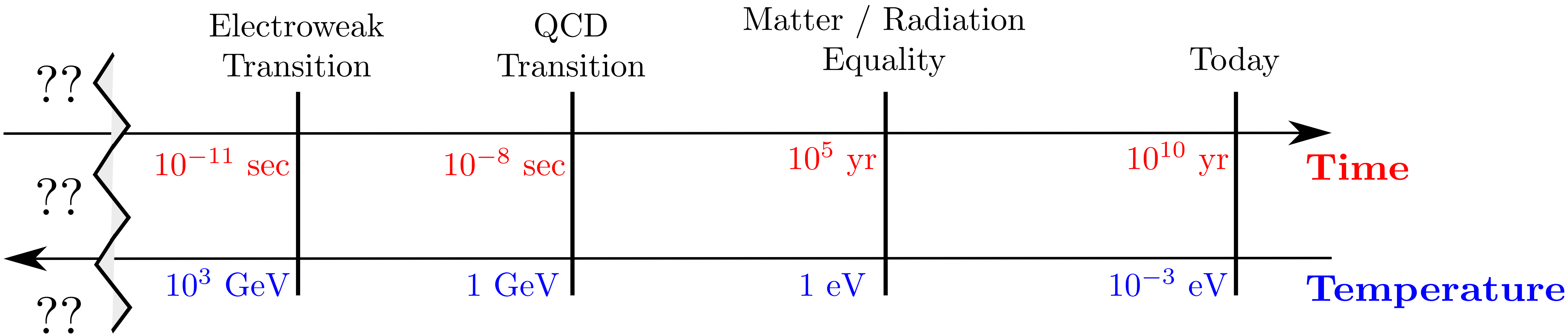
Colpi, Shapiro, Wasserman (PRL 1986)

Bohmer and Harko (0705.4158); now many others

"Repulsive boson star"?

Also stable! See however Jiji Fan (1603.06580)

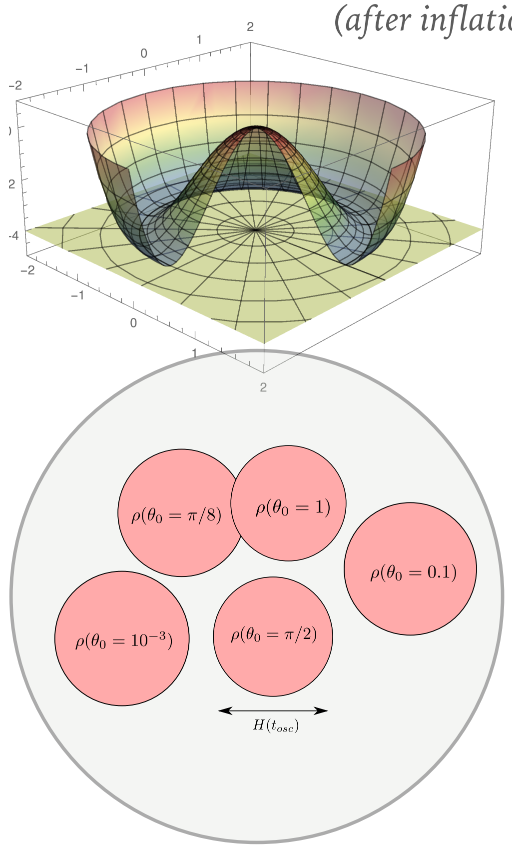
A BRIEF HISTORY



High energy scale f:

Broken global symmetry

(after inflation)



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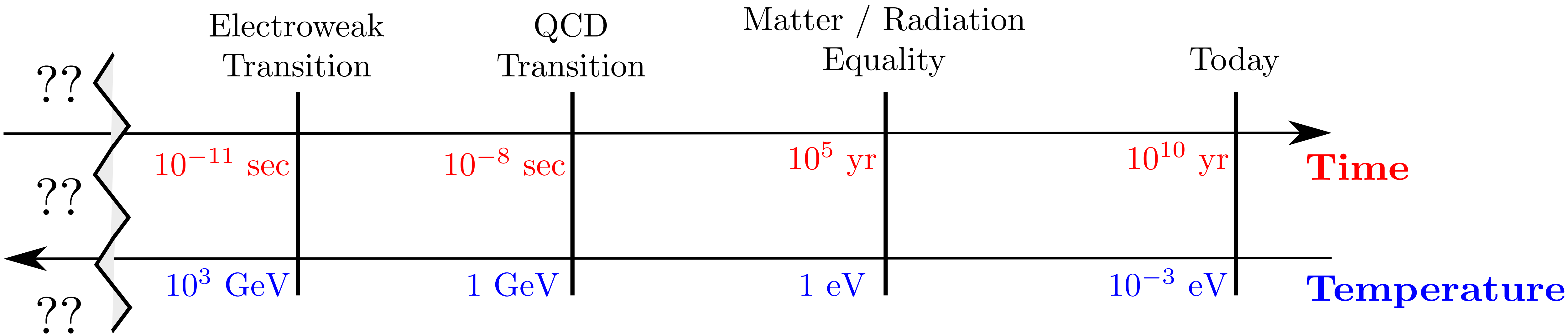

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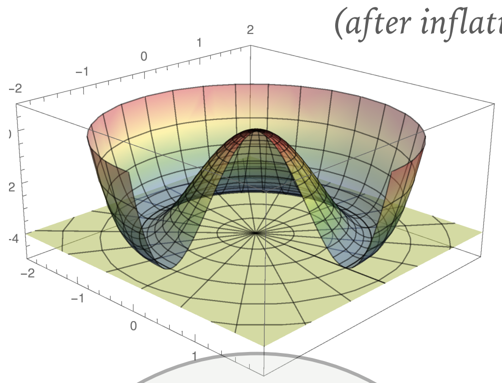
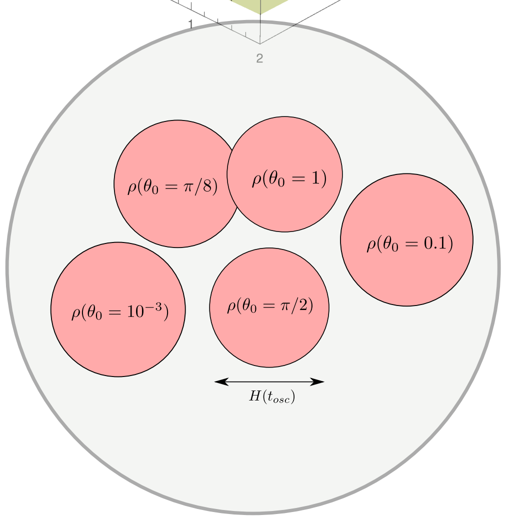
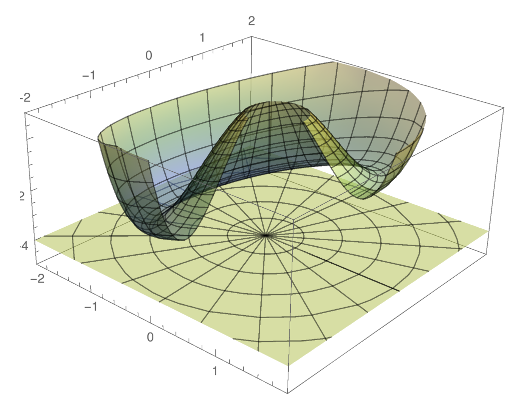




Lower energy scale: $\Lambda = \sqrt{m_\phi f}$

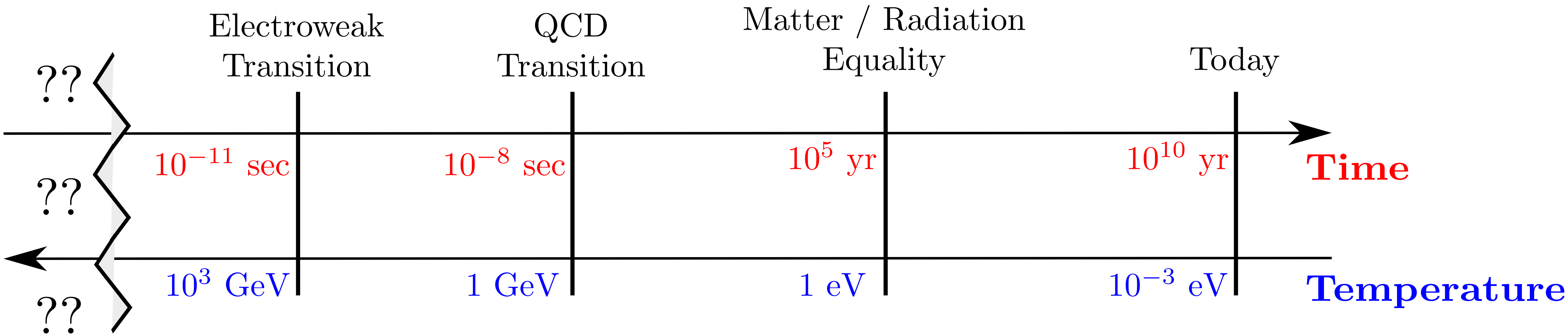
Tilt potential, obtain mass



Axion

Miniclusters

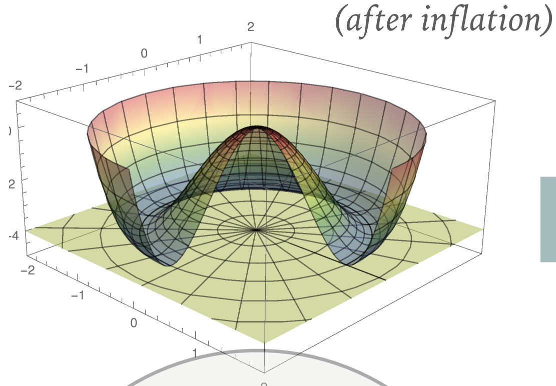
A BRIEF HISTORY



High energy scale f:

Broken global symmetry

(after inflation)



Lower energy scale: $\Lambda = \sqrt{m_\phi f}$

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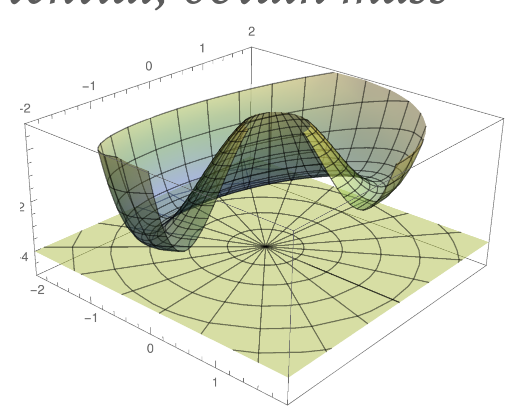


Coherently Oscillating

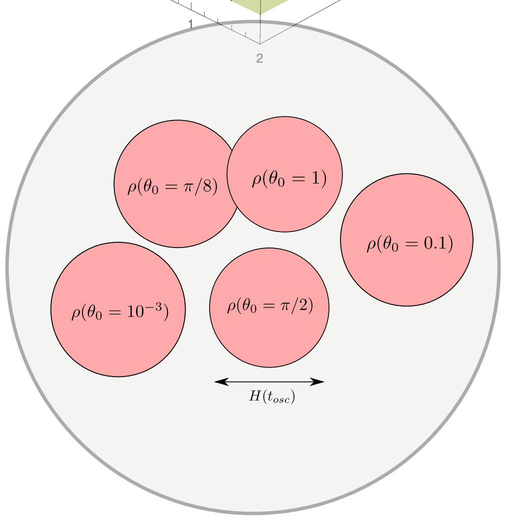
Scalar Field

$\omega \simeq m_\phi$

Relaxation?

Cooling?

Fragmentation?

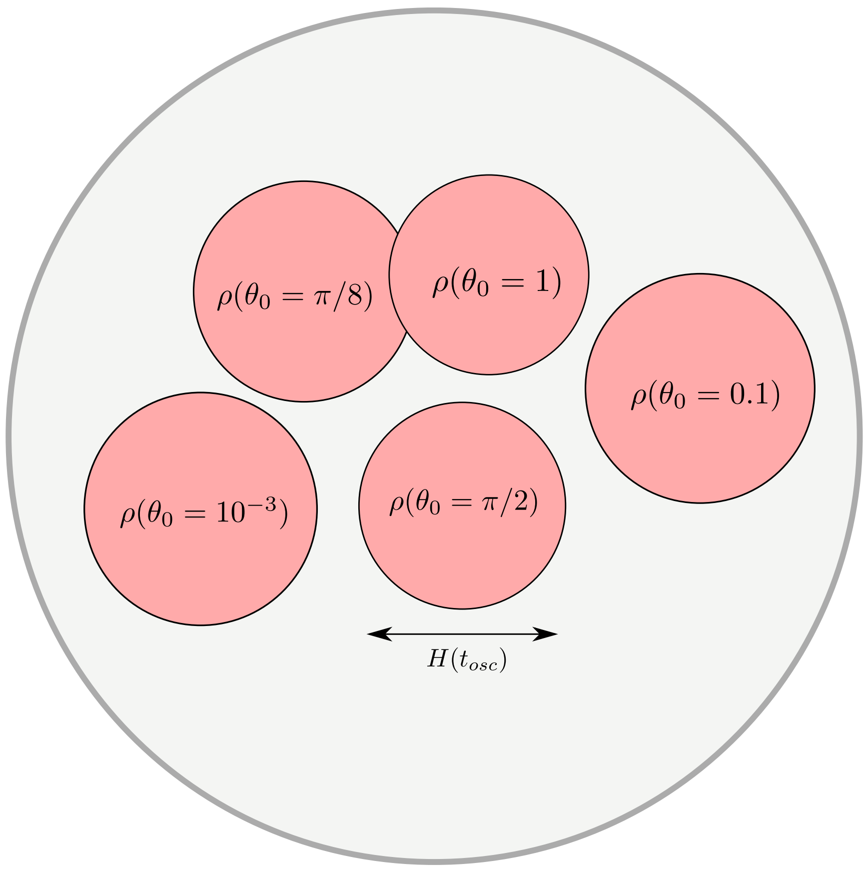


Axion

Miniclusters

FORMATION IN EARLY UNIVERSE (QCD)

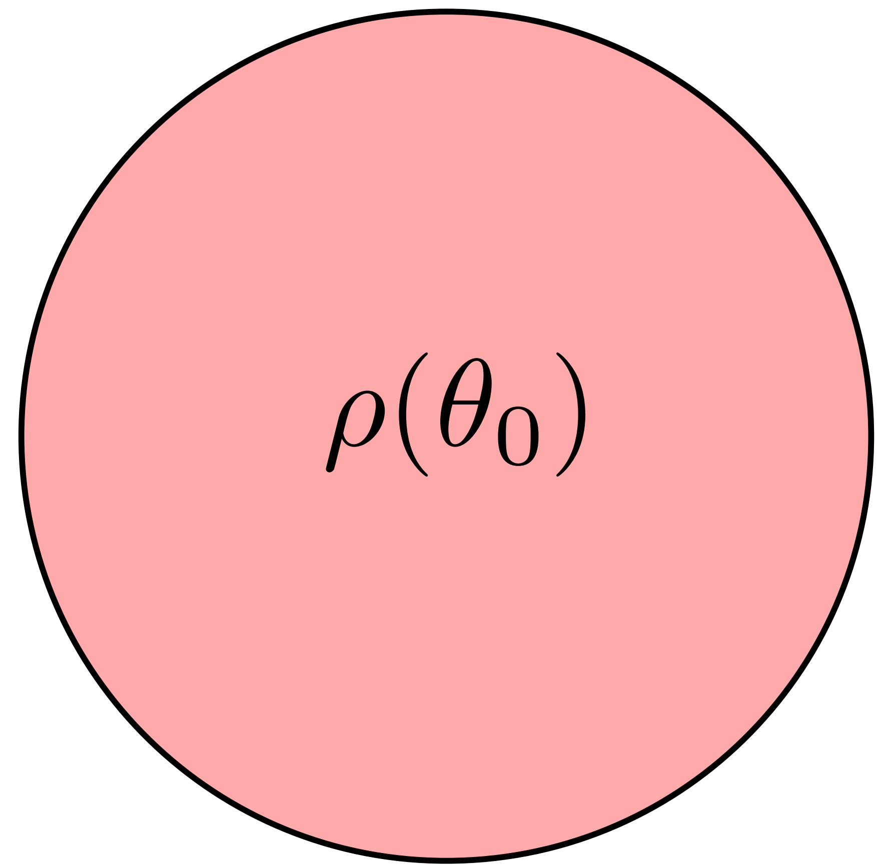
Inflation before PQ breaking



$$\Omega_a \sim \left(\frac{f}{10^{12}\,\mathrm{GeV}}\right)^{7/6} \langle {\theta_0}^2 \rangle$$

Axion Miniclusters

Inflation after PQ breaking



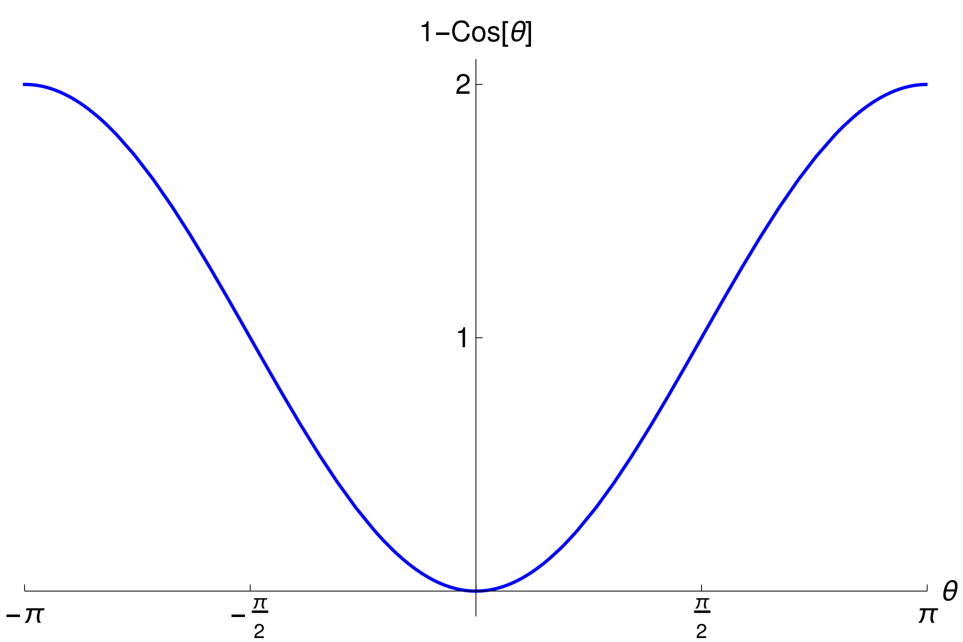
$$\Omega_a \sim \left(\frac{f}{10^{12}\,\mathrm{GeV}}\right)^{7/6} {\theta_0}^2$$

Isocurvature Only

AXION POTENTIAL

$$V(\phi) = m^2 f^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$$= \frac{m^2}{2}\phi^2 - \frac{m^2}{4!\,f^2}\phi^4 + \ldots$$



NR Limit:

$$\phi = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi \underbrace{+c.c.}\right]$$

ϕ *(If is real)*

$$V_{eff} = V(\phi) - \frac{m^2}{2}\phi^2$$

$$= m^2 f^2 \left[1 - J_0\left(\sqrt{\frac{\psi^*\psi}{2\,m\,f^2}}\right)\right] - \frac{m}{2}|\psi|^2 + \mathcal{O}(e^{\pm imt})$$

$$= -\frac{1}{8f^2}|\psi|^4 + \ldots$$

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V_g\left(|\psi|^2\right) + V_{eff}\left(|\psi|^2\right)\right]\psi$$

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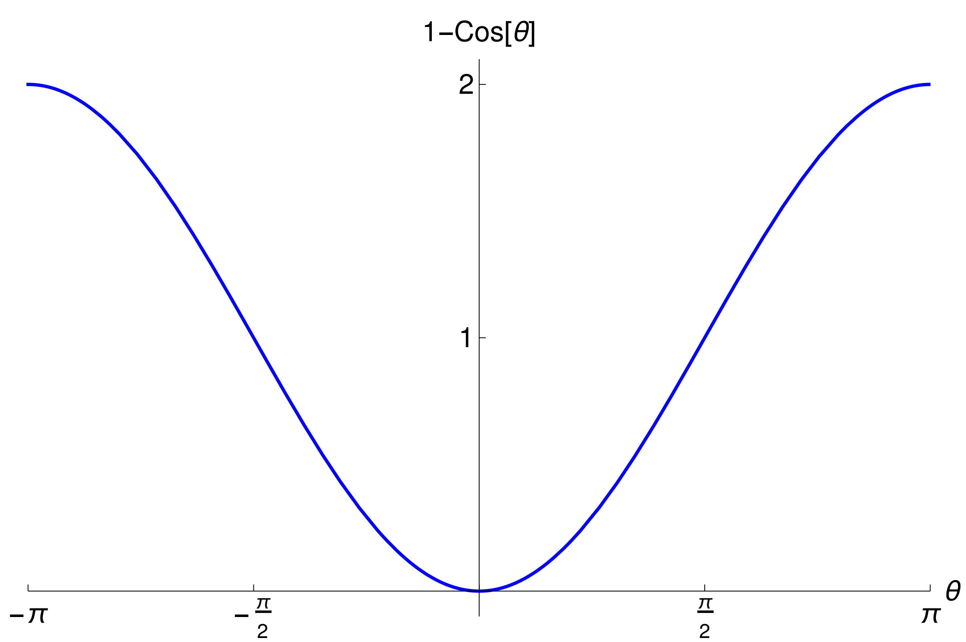

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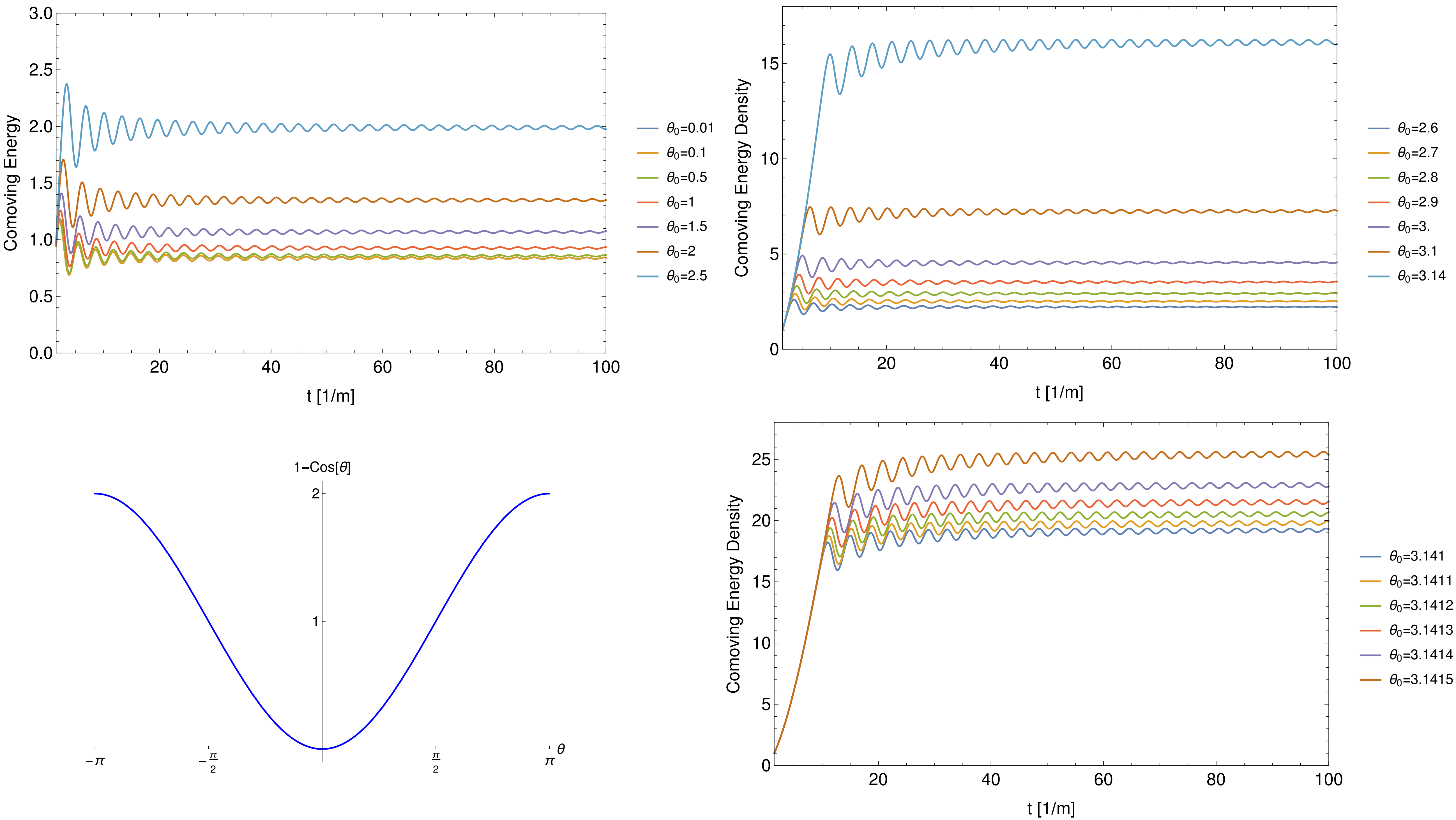
EARLY UNIVERSE OVERDENSITIES



ORIGINAL AXION MINICLUSTER SIMULATIONS

VOLUME 71, NUMBER 19 PHYSICAL REVIEW LETTERS 8 NOVEMBER 1993

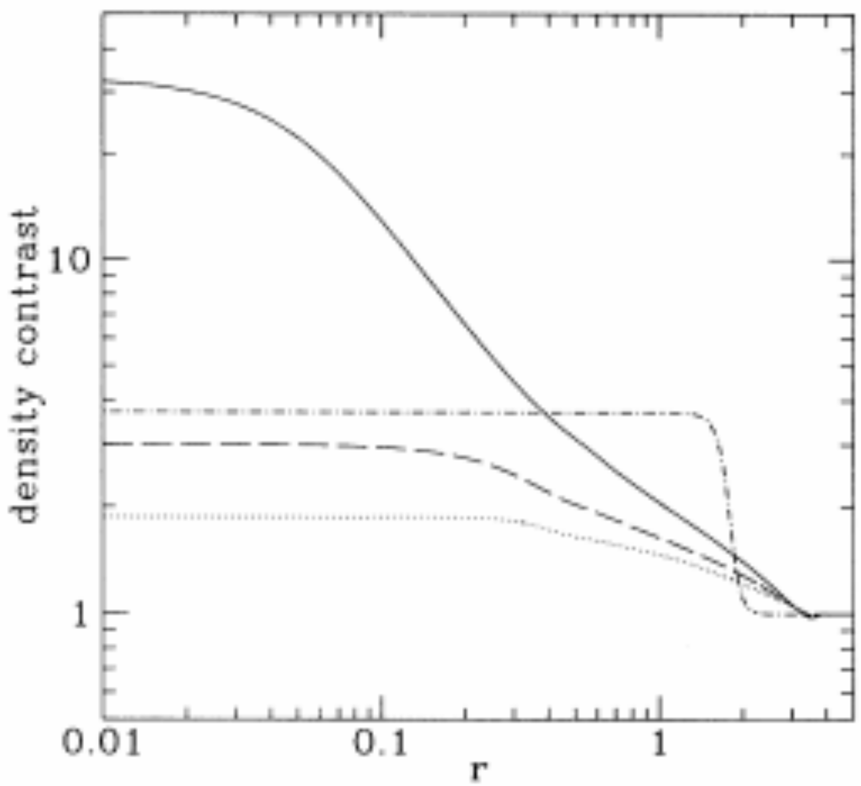


FIG. 2. Energy density profiles at $\bar{\eta}=4$ for identical initial fluctuations evolved with different Lagrangians. Solid line: axion case; dashed line: $V(\theta)\propto\theta^2/2$; dotted line: $V(\theta)\propto\theta^2/2+\theta^4/4$; dash-dotted line: axion potential with field gradients switched off.

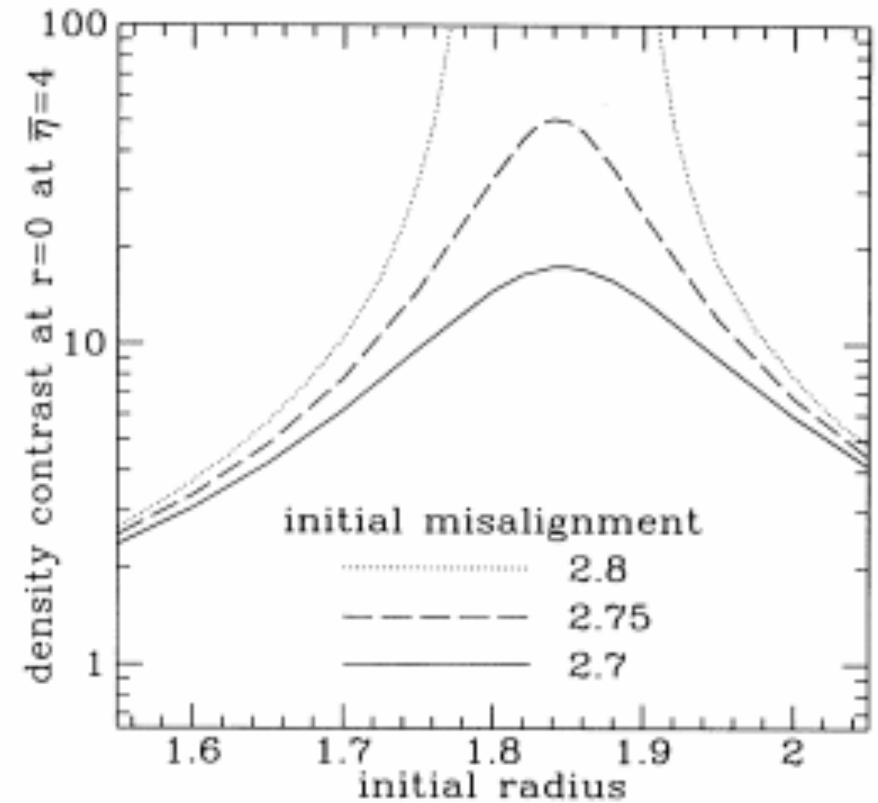


FIG. 3. Dependence of density contrast in the center of a fluctuation at $\bar{\eta}=4$ upon the initial radius of a fluctuation for several values of the initial misalignment angle inside the fluctuation.

[Kolb and Tkachev, PRL 1993]

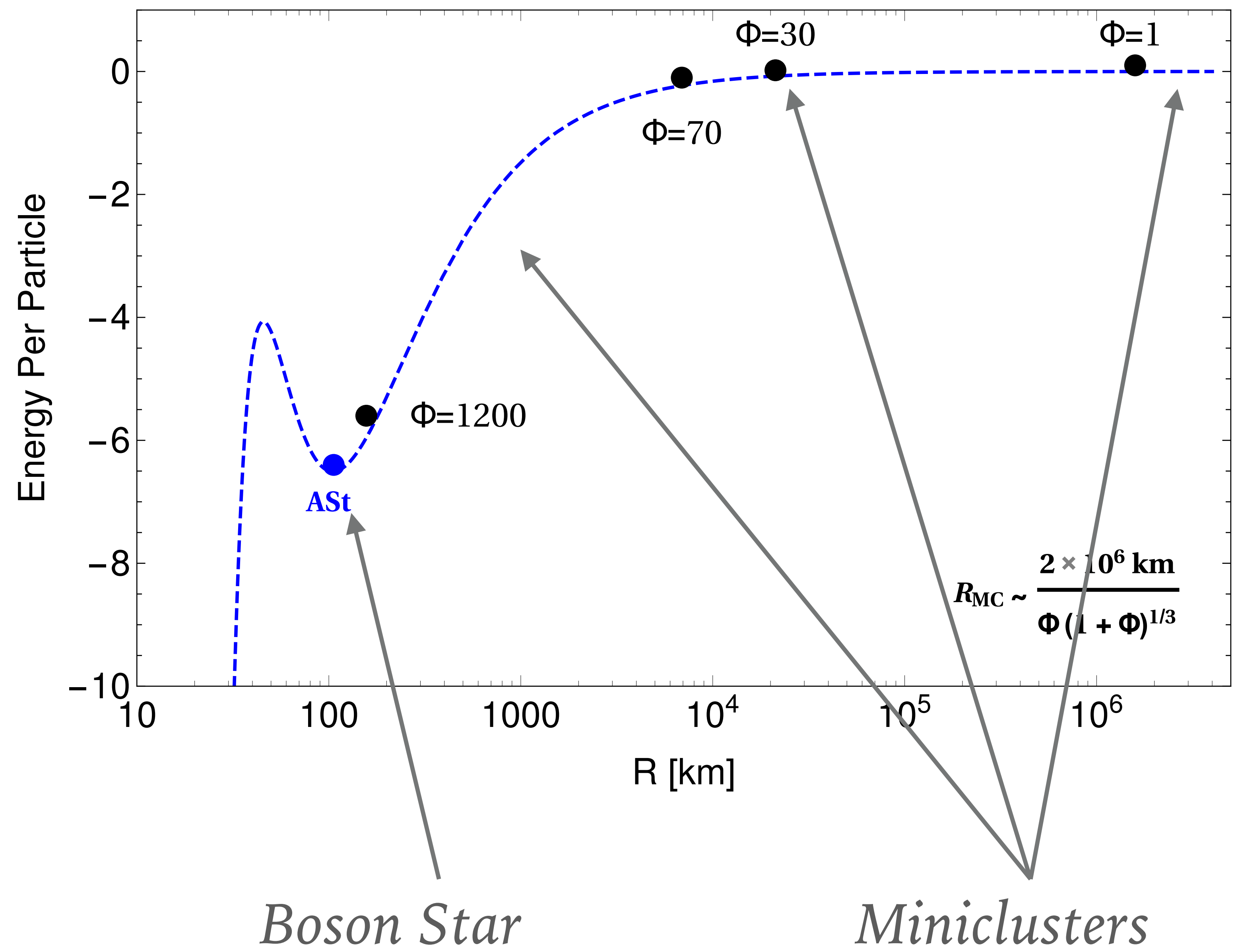
RELAXATION?

- Initial overdensity

$$\rho_{mc,osc} \equiv \delta_0\, \rho_{bg,osc}$$

- Grows with scale factor

$$\rho_{mc} \propto \delta_0^4\, \rho_{bg}$$



BOSON STARS

Gross-Pitaevskii+Poisson (GPP)

- Nonrelativistic, classical field:

Poisson Gravity

$$\nabla^2 V_g = 4\pi G m_\phi^2 \, |\psi|^2$$

(Attractive)

Normalization

$$m_\phi \int d^3r\, |\psi|^2 = M_\star$$

Leading time dependence

$$\dot\psi \sim (\mu - m_\phi)\psi \ll m_\phi \psi$$

$$i\frac{\partial\psi}{\partial t} = \left[-\frac{\nabla^2}{2m_\phi} + V_g\left(|\psi|^2\right) + V_{int}\left(|\psi|^2\right)\right]\psi$$

Kinetic energy

(Repulsive)

Self-interactions

For axion potential,

$$V(\phi) = m_\phi^2 f^2\left[1 - \cos\left(\frac{\phi}{f}\right)\right] = \frac{m_\phi^2}{2}\phi^2 - \frac{1}{4!}\left(\frac{m_\phi}{f}\right)^2\phi^4 + \frac{1}{6!f^2}\left(\frac{m_\phi}{f}\right)^2\phi^6 - \dots$$

$$E(\psi) = \int d^3r\left[\frac{|\nabla\psi|^2}{2m_\phi} + \frac{1}{2}V_g|\psi|^2 - \frac{\lambda}{16m_\phi^2}|\psi|^4 + \frac{g}{m_\phi^5}|\psi|^6 - \dots\right]$$

$$\frac{E}{M_\star} \sim \underbrace{\frac{a}{R_\star^2} - \frac{b\,M_\star}{R_\star}}_{\text{Large radius}} - \underbrace{\frac{c\,M_\star}{R_\star^3} + \frac{d\,M_\star^2}{R_\star^6}}_{\text{Small radius}} - \dots$$

STABLE BOUND STATES

Dilute Chavanis (1103.2050), with Delfini (1103.2054)

Dense Braaten, Mohapatra, Zhang (1512.00108)

- Finding energy minima:

JE, Leembruggen, Suranyi, Wijewardhana (1608.06911)

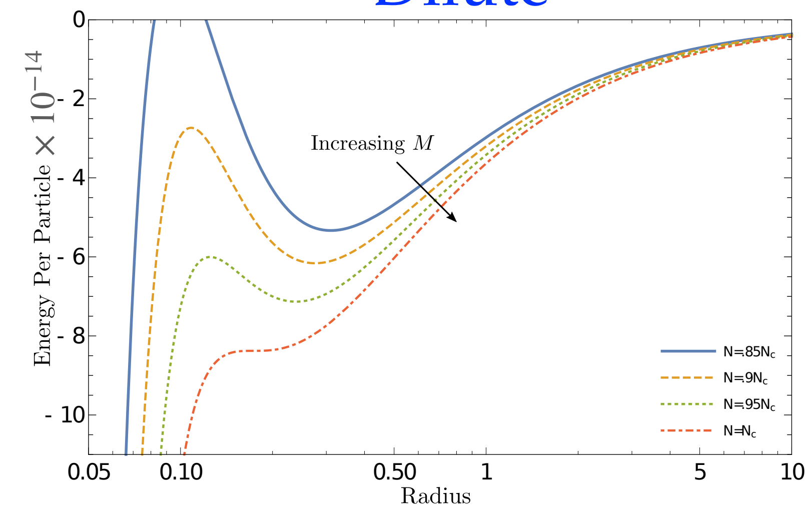
$$M_{max} \approx 10 \frac{M_P f}{m_\phi}$$

$$M \approx \frac{M_P^2}{m_\phi^2 R}$$

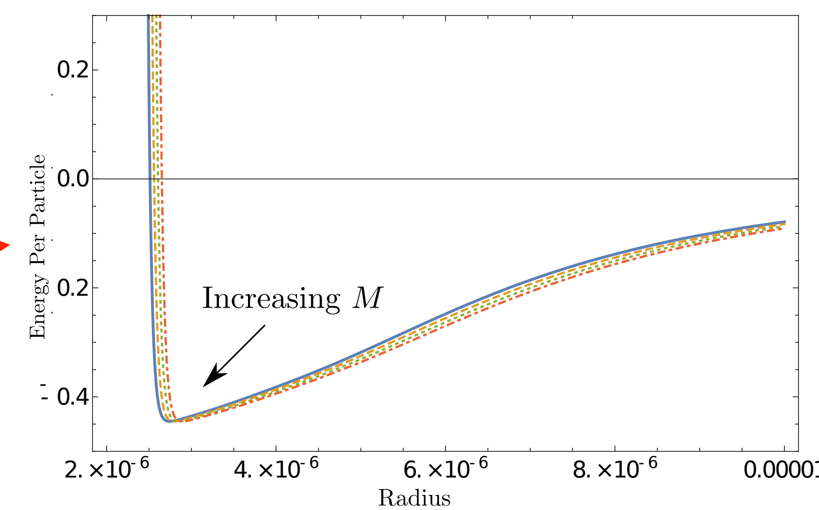
$$\frac{E}{M} \sim \overbrace{\frac{a}{R^2} - \frac{b\,M}{R}}^{} - \underbrace{\frac{c\,M}{R^3} + \frac{d\,M^2}{R^6}}_{} - \ldots$$

$$M \approx m_\phi^2 f^2 R^3$$

Dilute



Dense



- Example: QCD axion

$M_{max} \sim \mathcal{O}(10^{-11}) M_\odot$

$R_{dilute} \sim \mathcal{O}(100)\ km$ $\qquad \rho_{dilute} \sim m_\phi^2 f^2 \left(\frac{f^2}{M_P^2}\right) \lesssim \rho_{water}$

$R_{dense} \sim \mathcal{O}(10)\ cm$ $\qquad \rho_{dense} \sim m_\phi^2 f^2 \sim \Lambda_{QCD}^4$

- Nuggets of stable, nuclear-density dark matter??
  - Dense states decay fast to relativistic scalars

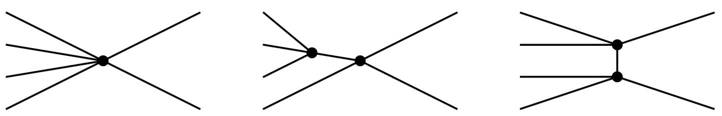
Radius scale: $R \sim \frac{M_P}{m_\phi f}$

Mass scale: $M \sim \frac{M_P f}{m_\phi}$

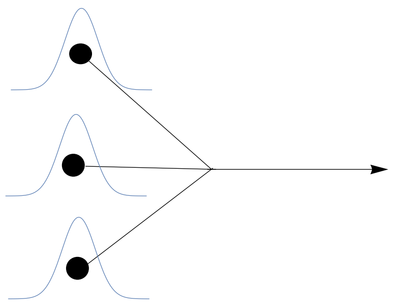
JE, Suranyi, Wijewardhana (1512.01709), with Ma (1705.05385)

DENSE: NOT SO STABLE AFTER ALL

- Recall that axions are ***real*** scalars: no number conservation
  - Decay to photons: $a \to \gamma\gamma$, small rate if $m \lesssim eV$
  - Can decay through self-interactions:
    - On-shell: $4\,a \to 2\,a$
      Braaten, Mohapatra, Zhang (1609.05182)



    - Off-shell decays in bound state: $3\,a_{bound} \to a_{free}$
      JE, *Suranyi, Wijewardhana (1512.01709), with Ma (1705.05385)*



- 3—>1 dominant, large when $|E - m| \lesssim 0.998$

$$\frac{dM}{dt} \propto e^{-1/\sqrt{1-(E/m)^2}}$$

Known from oscillon literature,

(bound state of no gravity)

Fodor, Forgacs, Horvath, Mezei (0903.0953)

Hertzberg (1003.3459)

Mukaida, Takimoto, Yamada (1612.07750)

…ALSO NOT SO NON-RELATIVISTIC

- Non-relativistic limit assumes field energy $E = \mathcal{O}(m)$
- Relativistic corrections are legion:
  a. Special Relativity: $E_K = \sqrt{p^2 + m^2} - m \approx \dfrac{p^2}{2m} - \dfrac{p^4}{8\,m^3} + \ldots$
  b. Scalar field contains higher harmonic modes:

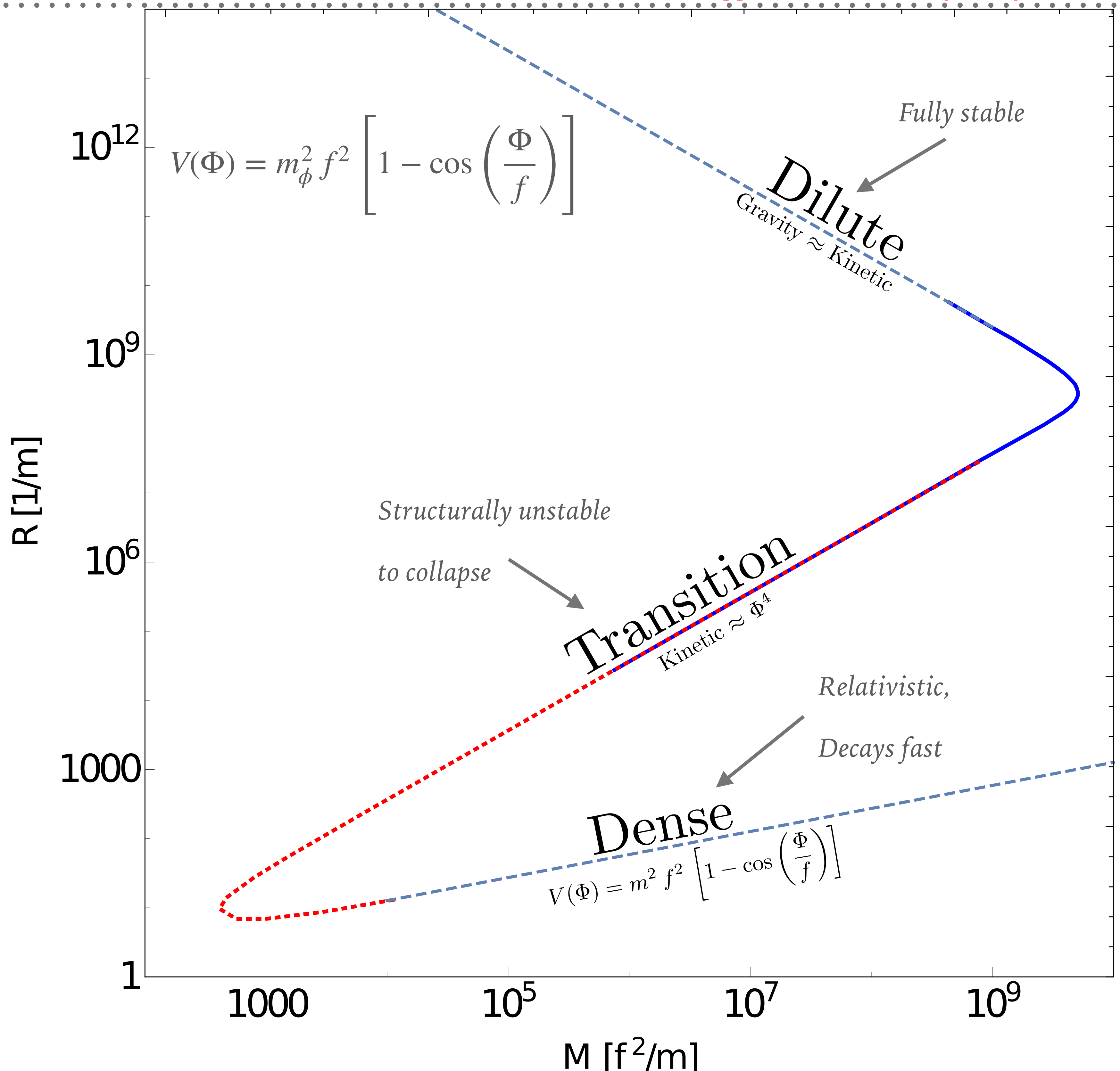
$$\phi = \left[\phi_1\, e^{-iEt} + \phi_3\, e^{-3\,iEt} + \ldots\right] + h.c.$$

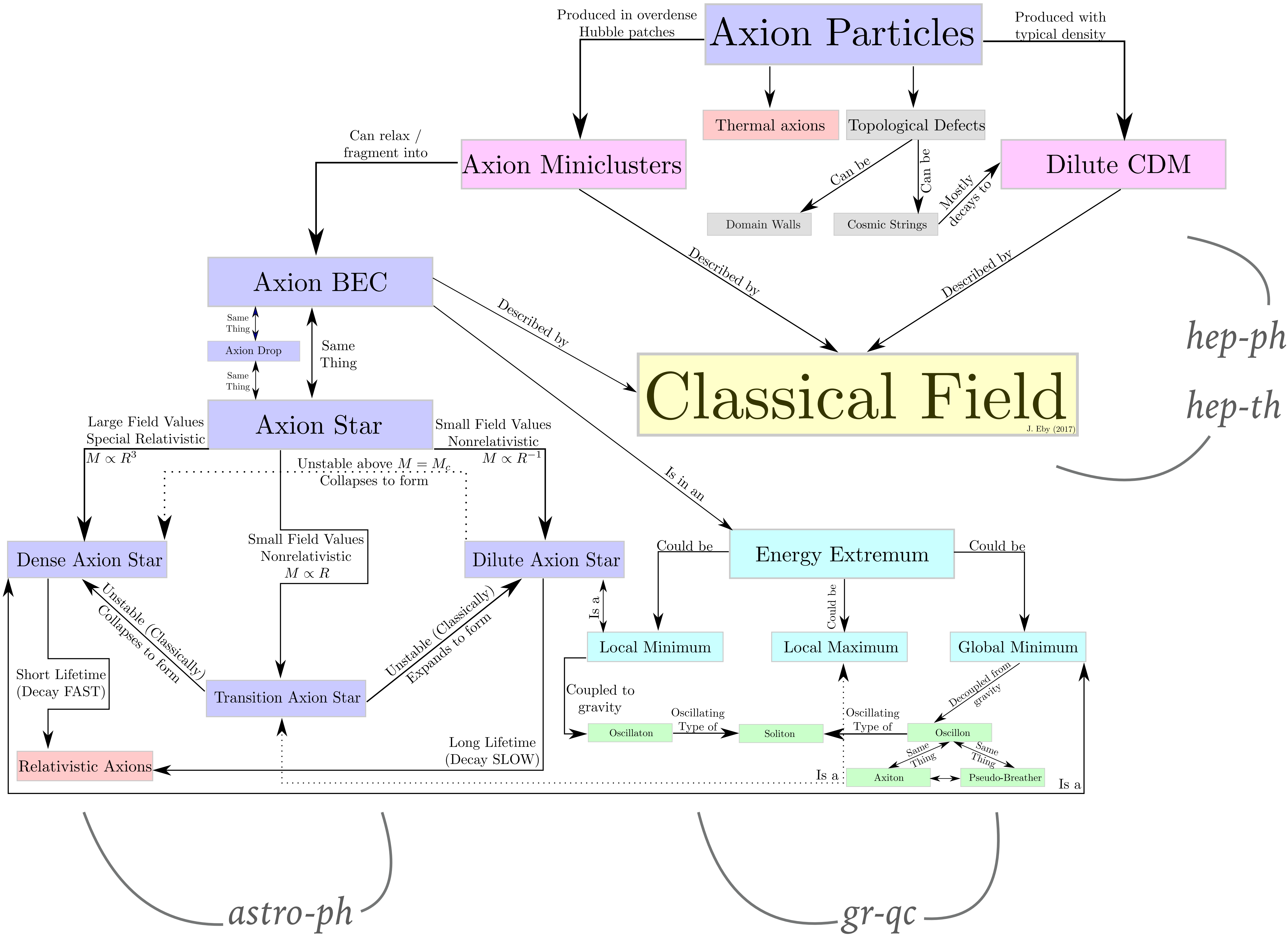
  c. General Relativity??
     ! (thankfully, gravity is small in crossover to dense region!)

'FULL' MASS SPECTRUM

JE, Suranyi, Wijewardhana (1712.04941)

JE, Leembruggen, Street, Suranyi, Wijewardhana (1905.00981)





J. Eby, Weizmann Institute