

Topological avatars of the Standard Model

Archil Kobakhidze



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Monash U, 19 Feb 2019

What is the next energy scale to be probed?

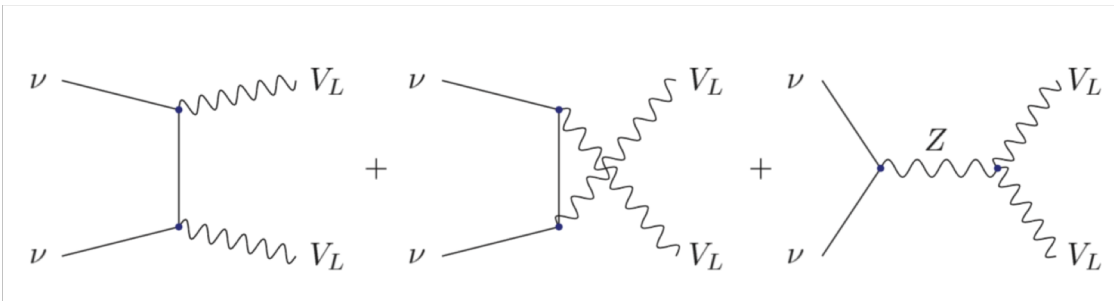
- **Neutrino masses** - robust evidence from particle physics (neutrino oscillation) experiments

Add neutrino mass to the SM Lagrangian (EW gauge invariance is still OK, but nonlinear):

$$\mathcal{L}_\nu = -\frac{1}{2} m_\nu \nu_L^T C \nu_L + h.c. \equiv -\frac{1}{2} m_\nu [L^T \epsilon \Sigma] C [\Sigma^T \epsilon L]$$

$$L = (\nu, \ell), \quad \Sigma = \exp\{i\sigma^a \pi^a(x)\}(0, 1)$$

Consider in this theory neutrino scattering off longitudinal EW bosons:



Perturbative unitarity implies:

$$\Lambda \lesssim \frac{4}{\alpha_2} \cdot \frac{M_W^2}{m_\nu} \sim 10^{11} \text{ GeV}$$

Maltoni, Niczyporuk, and Willenbrock, 01'

What is the next energy scale to be probed?

- **Dark Matter** – robust, but only observed in gravitational interactions

Assuming non-relativistic DM is produced thermally via weak-strength scatterings with SM particles, we arrive at the ‘WIMP miracle’:

$$\Omega_X h^2 \sim \frac{3 \cdot 10^{-27} \text{ cm}^3/\text{sec}}{\langle \sigma v_{\text{rel}} \rangle}$$

Cross section is constrained from perturbative unitarity:

$$\sigma_J \leq \pi(2J + 1)/p_i^2 \approx 16\pi(2J + 1)/(m_X^2 v_{\text{rel}}) \implies m_X^2 \leq 16\pi/(\sigma_{J=0} v_{\text{rel}}), [v_{\text{rel}} \approx 1/4]$$

$$\Lambda \sim m_X \lesssim 100 \text{ TeV}$$

Griest and Kamionkowski, 90'

What is the next energy scale to be probed?

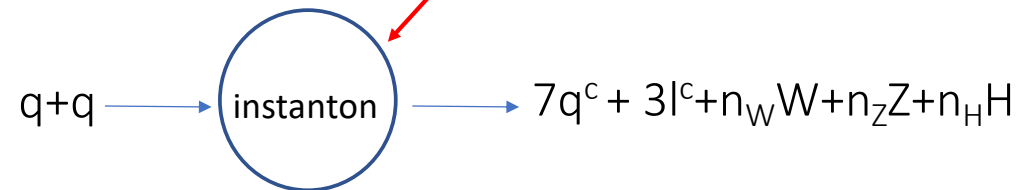
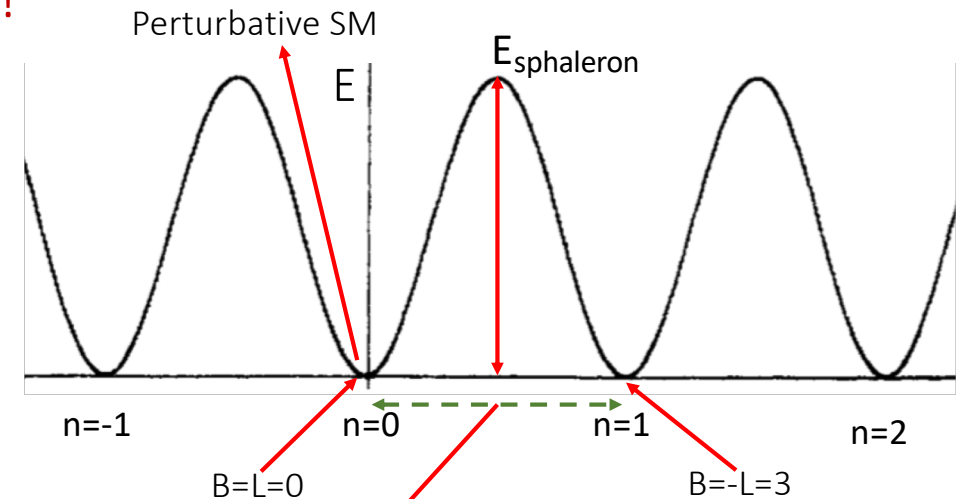
- EW vacuum has topologically non-trivial structure [SU(2) sector].
- Transition between vacua change B and L by 3 units: $\Delta B = -\Delta L = 3\Delta n$ (quantum anomaly); $\Delta(B-L) = 0$.

- EW instantons are classical solution of Euclidean e.o.m., with action, e.g., for $\Delta n = 1$,

$$S_{\text{inst.}} = \frac{2\pi}{\alpha_2}$$

(multiple of W, Z, H particles in a coherent state)

- describe vacuum-to-vacuum transitions)



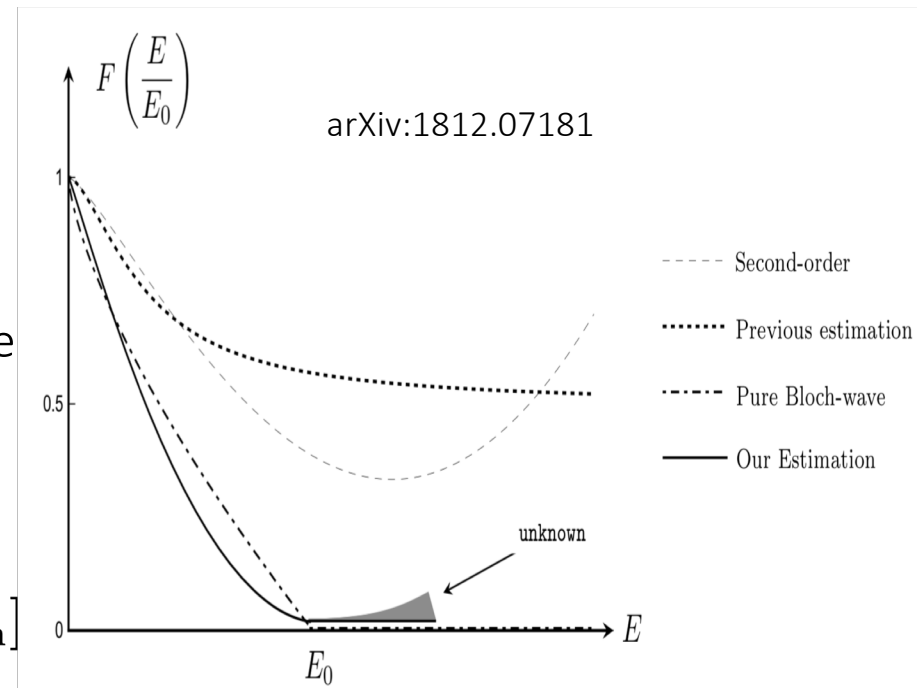
$$\mathcal{A}_{\text{inst.}} \sim \exp\{-S_{\text{inst.}}\} \approx 10^{-80}$$

$$\Lambda \sim E_{\text{sphaleron}} \sim \frac{4M_W}{\alpha_2} \approx 10 \text{ TeV}$$

Sphalerons at LHC?

- Jump over barrier is described by production and subsequent decay of a sphaleron.
- Sphaleron is an unstable particle-like classical solution with a typical size $\sim 1/M_W$ and mass ~ 10 TeV.
- Spectacular B+L – violating processes with multiple of W,Z and H (background-free!)
- Cross section:

$$\sigma \propto \exp\{-2S_{\text{ints.}} F(E/E_0)\}, \quad [E_0 \equiv E_{\text{sphaleron}}]$$



Ringwald; McLerran, Vainstein and Voloshin 90'
Bezrukov and Levkov, 03'
Tye and Wong, 15'

Electroweak monopoles

[Arunasalam, Collison, AK, 18']

- Standard (and incorrect) argument against electroweak monopoles:

$$H^\dagger H \equiv \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 \stackrel{r \rightarrow \infty}{=} \rho_0^2$$

Map of S^2 (boundary at spatial infinity) onto the vacuum manifold S^3 . The map is trivial, hence topological ('t Hooft-Polyakov) monopoles do not exist.

- However, ϕ_i can be singular (gauge d.o.f.). In that case the vacuum manifold may not be S^3 .

- Consider an ansatz:

Cho and Maison, 96'

$$H = \frac{1}{\sqrt{2}} \rho(r) \zeta, \quad \zeta = i \begin{pmatrix} \sin(\theta/2) e^{-i\phi} \\ -\cos(\theta/2) \end{pmatrix},$$
$$\mathbf{A}_\mu = -\frac{1}{g_2} A(r) \partial_\mu t \hat{r} + \frac{1}{g_2} (f(r) - 1) \hat{r} \times \partial_\mu \hat{r},$$
$$B_\mu = -\frac{1}{g_1} B(r) \partial_\mu t - \frac{1}{g_1} (1 - \cos \theta) \partial_\mu \phi.$$

Singular at $\theta = \pi/2$

Electroweak monopoles

- Denote the components of doublet Higgs as: $z_1 \equiv \phi_1 + i\phi_2, z_2 \equiv \phi_3 + i\phi_4$
- Are defined up to hypercharge gauge transformations: $(z_1, z_2)^T \equiv (\lambda z_1, \lambda z_2)^T, \lambda \in U(1)_Y$. Hence could be viewed as coordinates on a complex plane C^2 (modulo singularities).
- Remove singularities by using the gauge freedom and defining two monopole solutions on two different patches of space:

$$H_N = i \frac{\rho(r)}{\sqrt{2}} \begin{pmatrix} \sin(\theta/2) e^{-i\phi} \\ -\cos(\theta/2) \end{pmatrix}, \quad B_\phi^N = -\frac{1}{g'} \frac{1 - \cos \theta}{r \sin \theta} \quad \text{for } 0 \leq \theta \leq \pi/2, \text{ and}$$

$$H_S = i \frac{\rho(r)}{\sqrt{2}} \begin{pmatrix} \sin(\theta/2) \\ -\cos(\theta/2) e^{i\phi} \end{pmatrix}, \quad B_\phi^S = \frac{1}{g'} \frac{1 + \cos \theta}{r \sin \theta} \quad \text{for } \pi/2 \leq \theta \leq \pi.$$

- At the equator ($\theta = \pi/2$) the transition function $e^{i\phi}$ is a holomorphic function $\Rightarrow (z_1, z_2)$ actually span a projective complex plane CP^1 .
- Hence, monopole solution is topologically nontrivial: $\pi_2(CP^1) = \pi_2(S^2) = \mathbb{Z}$

Electroweak monopoles

- Considering, two monopole solutions on the whole space (with opposite magnetic charges), one gets monopole-antimonopole bound state, which actually is a sphaleron!
- Monopole – particle scattering is known unsuppressed (Rubakov 81'; Callan 82'). By crossing symmetry the process of production of monopole-antimonopole pair in two-particle collision must not be suppressed either. Monopole-antimonopole pair then can form sphaleron:

$$q + q \rightarrow M + M^c \rightarrow 7q^c + 3l^c + n_W W + n_Z Z + n_H H$$

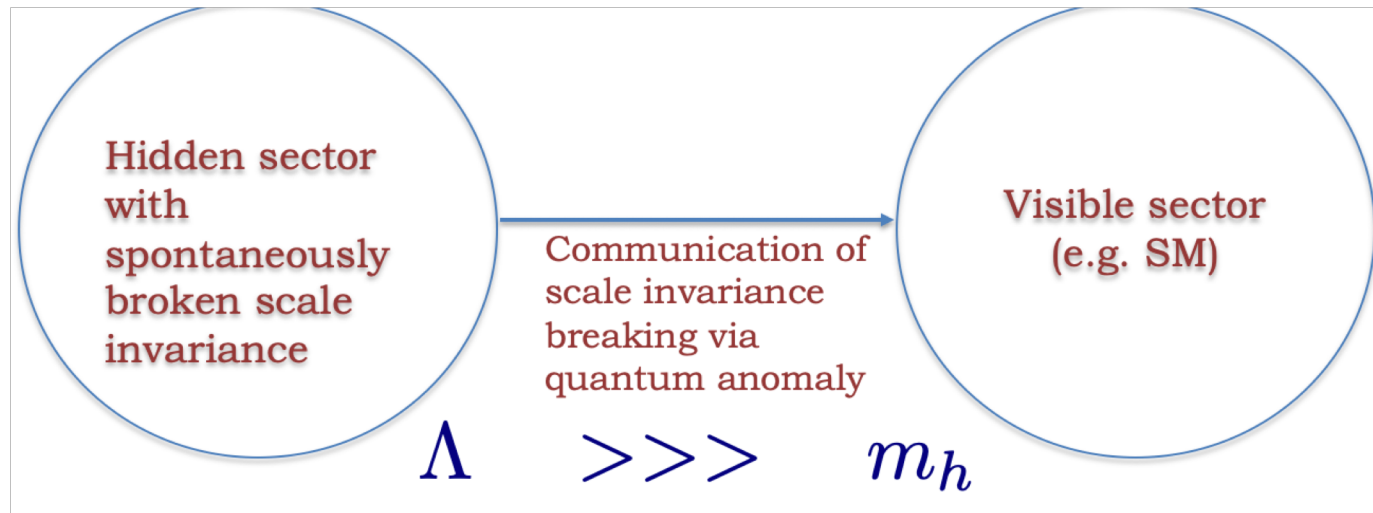
- EW monopoles inevitably introduce new CP violating phase (Witten effect):

$$\mathcal{L}_\theta = \theta_2 F_{\mu\nu}^a \tilde{F}^{a\mu\nu} + \theta_1 B_{\mu\nu} \tilde{B}^{\mu\nu} \implies \mathcal{L}_\theta = \theta_{ew} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}, \quad \theta_{ew} = \theta_2 - \theta_1$$

- Contribute to EDM of known particles
- Successful electroweak baryogenesis scenario [Arunasalam and AK, 17']

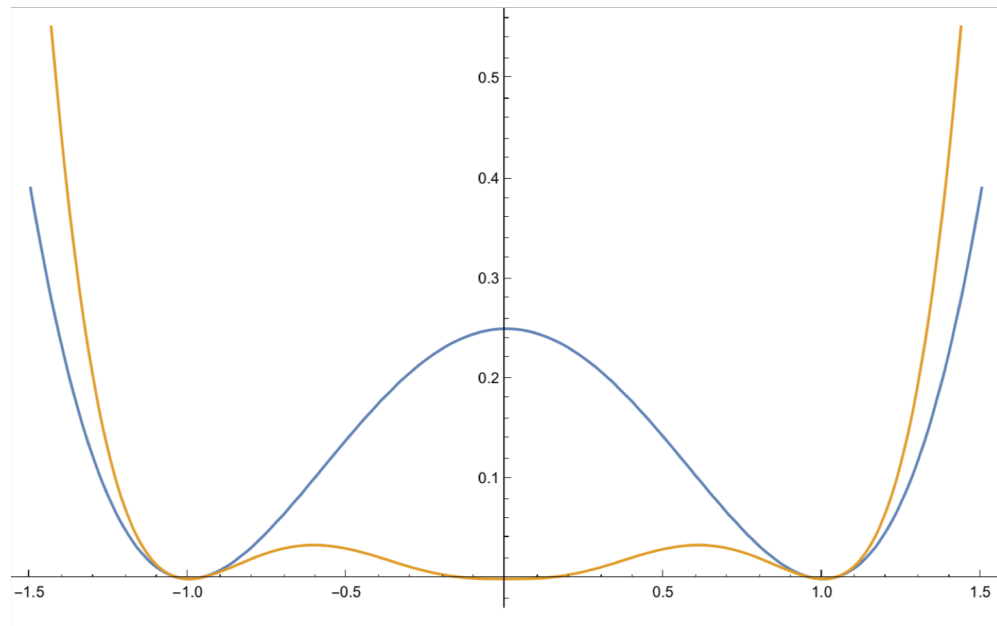
Scale invariant paradigm

- We have not found yet anticipated new physics at the LHC => old problems remain unsolved
- Scale invariant paradigm for solving the electroweak scale stability (aka hierarchy, aka naturalness) problem [Wetterich 84'; Bardeen 95'; Meissner and Nicolai 07'; Foot, AK, Volkas 07']



Scale invariant paradigm: Sydney version

- The minimal model is just SM with very feebly coupled dilaton of mass 10^{-8} eV (can be a dark matter).
- Almost indistinguishable from the SM in the perturbative sector, but...



- Can the model be probed through non-perturbative effects?

Conclusion

- The robust prediction for a new physics scale within SM is ~ 10 TeV.
- This scale is associated with a non-perturbative aspects of electroweak theory and potentially provides (less explored) portal to the BSM physics.
- Several theoretical/computational issues must be solved.
- Reminder: LHC remains a discovery machine...we must be prepared to face different manifestations of the 'deity' known as the Standard Model.