Anatomy of a six-parameter fit to the $b \to s\ell^+\ell^-$ anomalies

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Based on work with B. Capdevila and G. Valencia
arXiv:1811.10793

A Global Fit to the Anomalies

Base our analysis on fits described in Descotes-Genon et al [arXiv:1510.04239] and updated in Capdevila et al [arXiv:1704.05340]

Effective description

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i O_i$$

Operators considered

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, \qquad \mathcal{O}_{7'} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu},$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), \qquad \mathcal{O}_{9'} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell),$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell), \qquad \mathcal{O}_{10'} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$$

Best fit (BF) parameters obtained by minimising

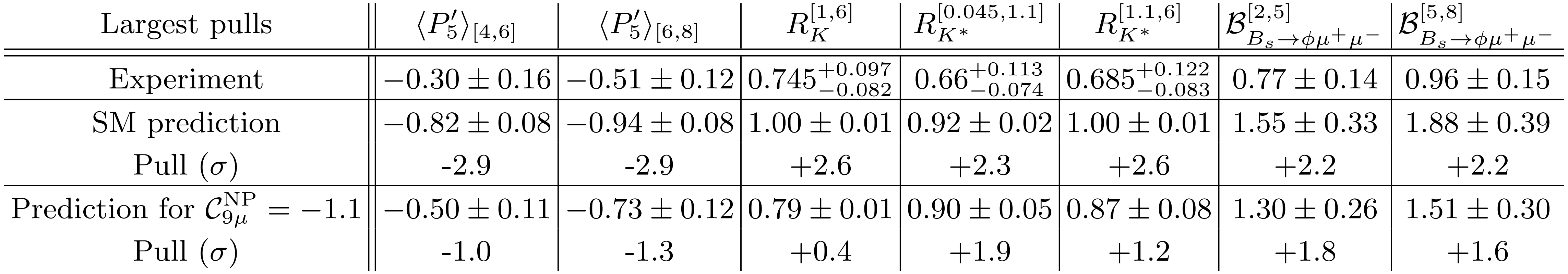
$$\chi^2(\mathcal{C}_k) = \sum_{i,j=1}^{N_{\text{obs}}} \left[O_i^{\text{exp}} - O_i^{\text{th}}(\mathcal{C}_k)\right] (C_{\text{exp}} + C_{\text{th}})^{-1}_{ij} \left[O_j^{\text{exp}} - O_j^{\text{th}}(\mathcal{C}_k)\right]$$

over 175 observables, finding preference of BF point over SM by approximately 5 σ

Presenting Results

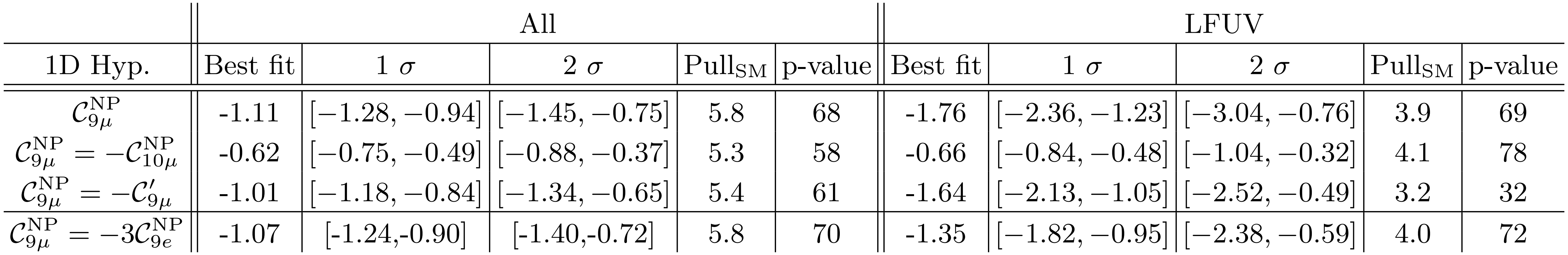
Tabulated results from [arXiv:1704.05340]

0-d (single point)



Largest pulls	$\langle P_5' \rangle_{[4,6]}$	$\langle P_5' \rangle_{[6,8]}$	$R_K^{[1,6]}$	$R_{K^*}^{[0.045,1.1]}$	$R_{K^*}^{[1.1,6]}$	$\mathcal{B}_{B_s \to \phi\mu^+\mu^-}^{[2,5]}$	$\mathcal{B}_{B_s \to \phi\mu^+\mu^-}^{[5,8]}$
Experiment	-0.30 ± 0.16	-0.51 ± 0.12	$0.745^{+0.097}_{-0.082}$	$0.66^{+0.113}_{-0.074}$	$0.685^{+0.122}_{-0.083}$	0.77 ± 0.14	0.96 ± 0.15
SM prediction	-0.82 ± 0.08	-0.94 ± 0.08	1.00 ± 0.01	0.92 ± 0.02	1.00 ± 0.01	1.55 ± 0.33	1.88 ± 0.39
Pull (σ)	-2.9	-2.9	+2.6	+2.3	+2.6	+2.2	+2.2
Prediction for $\mathcal{C}_{9\mu}^{\rm NP} = -1.1$	-0.50 ± 0.11	-0.73 ± 0.12	0.79 ± 0.01	0.90 ± 0.05	0.87 ± 0.08	1.30 ± 0.26	1.51 ± 0.30
Pull (σ)	-1.0	-1.3	+0.4	+1.9	+1.2	+1.8	+1.6

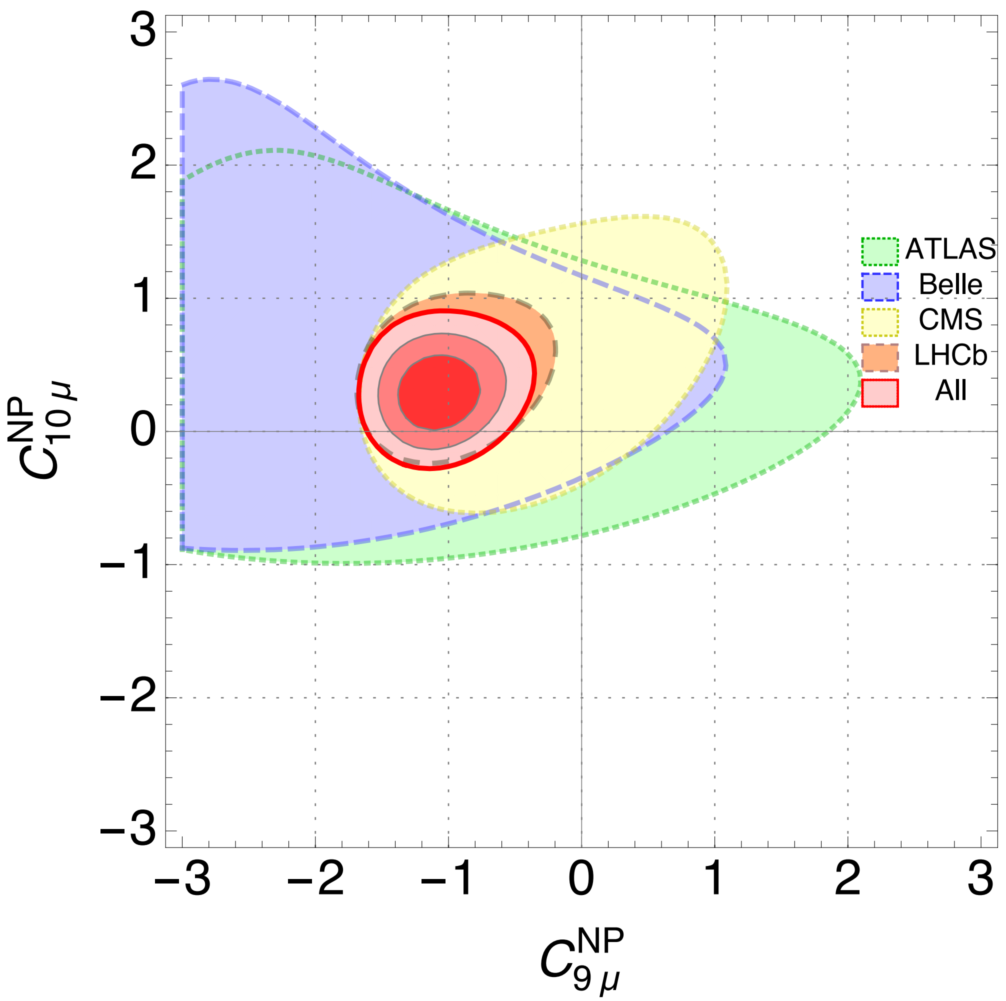
1-d

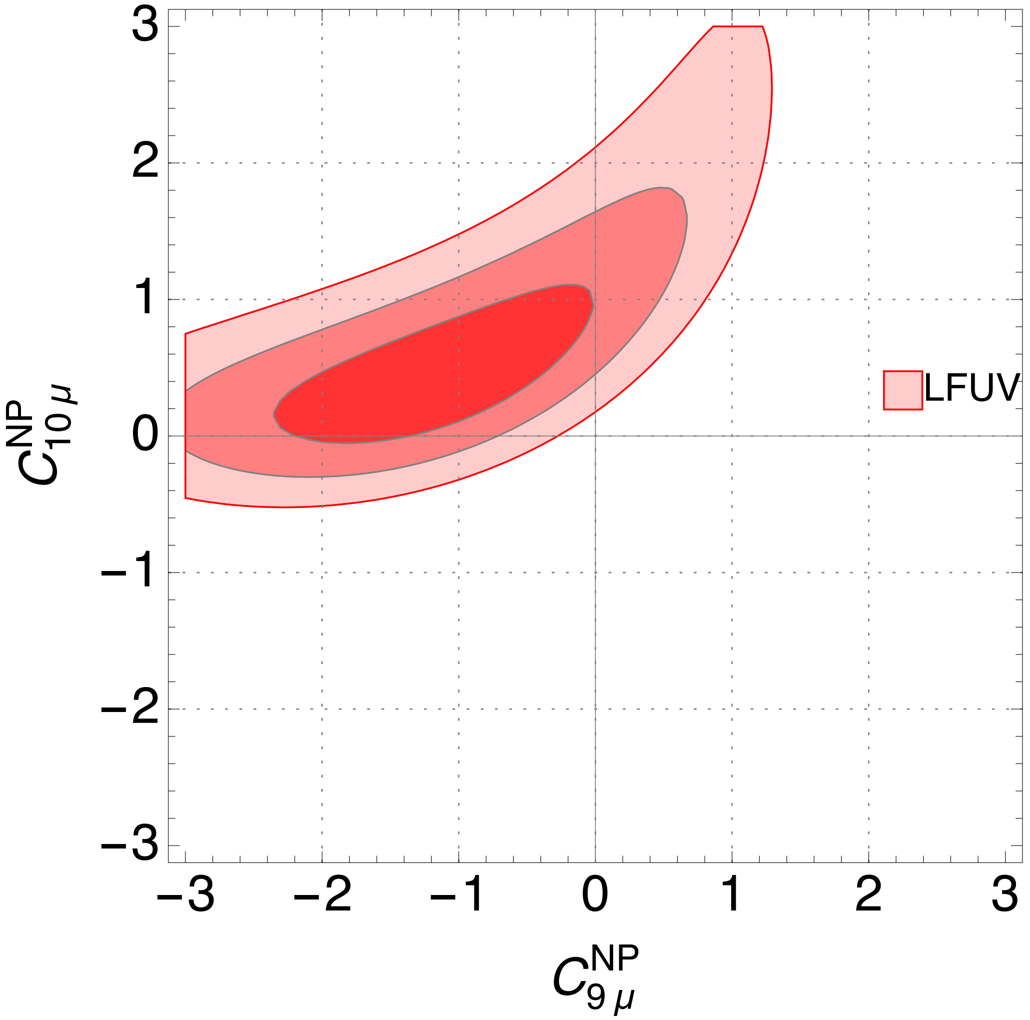


	All					LFUV				
1D Hyp.	Best fit	1 σ	2 σ	$\text{Pull}_{\rm SM}$	p-value	Best fit	1 σ	2 σ	$\text{Pull}_{\rm SM}$	p-value
$\mathcal{C}_{9\mu}^{\rm NP}$	-1.11	$[-1.28, -0.94]$	$[-1.45, -0.75]$	5.8	68	-1.76	$[-2.36, -1.23]$	$[-3.04, -0.76]$	3.9	69
$\mathcal{C}_{9\mu}^{\rm NP} = -\mathcal{C}_{10\mu}^{\rm NP}$	-0.62	$[-0.75, -0.49]$	$[-0.88, -0.37]$	5.3	58	-0.66	$[-0.84, -0.48]$	$[-1.04, -0.32]$	4.1	78
$\mathcal{C}_{9\mu}^{\rm NP} = -\mathcal{C}_{9\mu}'$	-1.01	$[-1.18, -0.84]$	$[-1.34, -0.65]$	5.4	61	-1.64	$[-2.13, -1.05]$	$[-2.52, -0.49]$	3.2	32
$\mathcal{C}_{9\mu}^{\rm NP} = -3\mathcal{C}_{9e}^{\rm NP}$	-1.07	[-1.24,-0.90]	[-1.40,-0.72]	5.8	70	-1.35	$[-1.82, -0.95]$	$[-2.38, -0.59]$	4.0	72

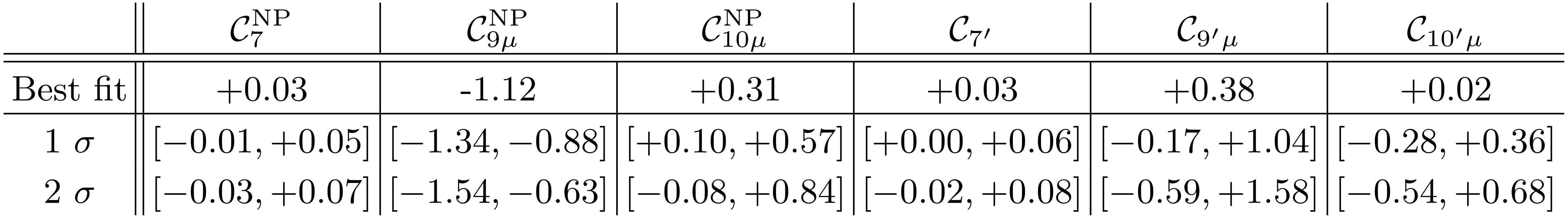
Presenting Results

2d fits from [arXiv:1704.05340]





Six parameter fit



	$\mathcal{C}_7^{\rm NP}$	$\mathcal{C}_{9\mu}^{\rm NP}$	$\mathcal{C}_{10\mu}^{\rm NP}$	$\mathcal{C}_{7'}$	$\mathcal{C}_{9'\mu}$	$\mathcal{C}_{10'\mu}$
Best fit	+0.03	-1.12	+0.31	+0.03	+0.38	+0.02
$1\ \sigma$	$[-0.01, +0.05]$	$[-1.34, -0.88]$	$[+0.10, +0.57]$	$[+0.00, +0.06]$	$[-0.17, +1.04]$	$[-0.28, +0.36]$
$2\ \sigma$	$[-0.03, +0.07]$	$[-1.54, -0.63]$	$[-0.08, +0.84]$	$[-0.02, +0.08]$	$[-0.59, +1.58]$	$[-0.54, +0.68]$

6d fit from [arXiv:1704.05340]

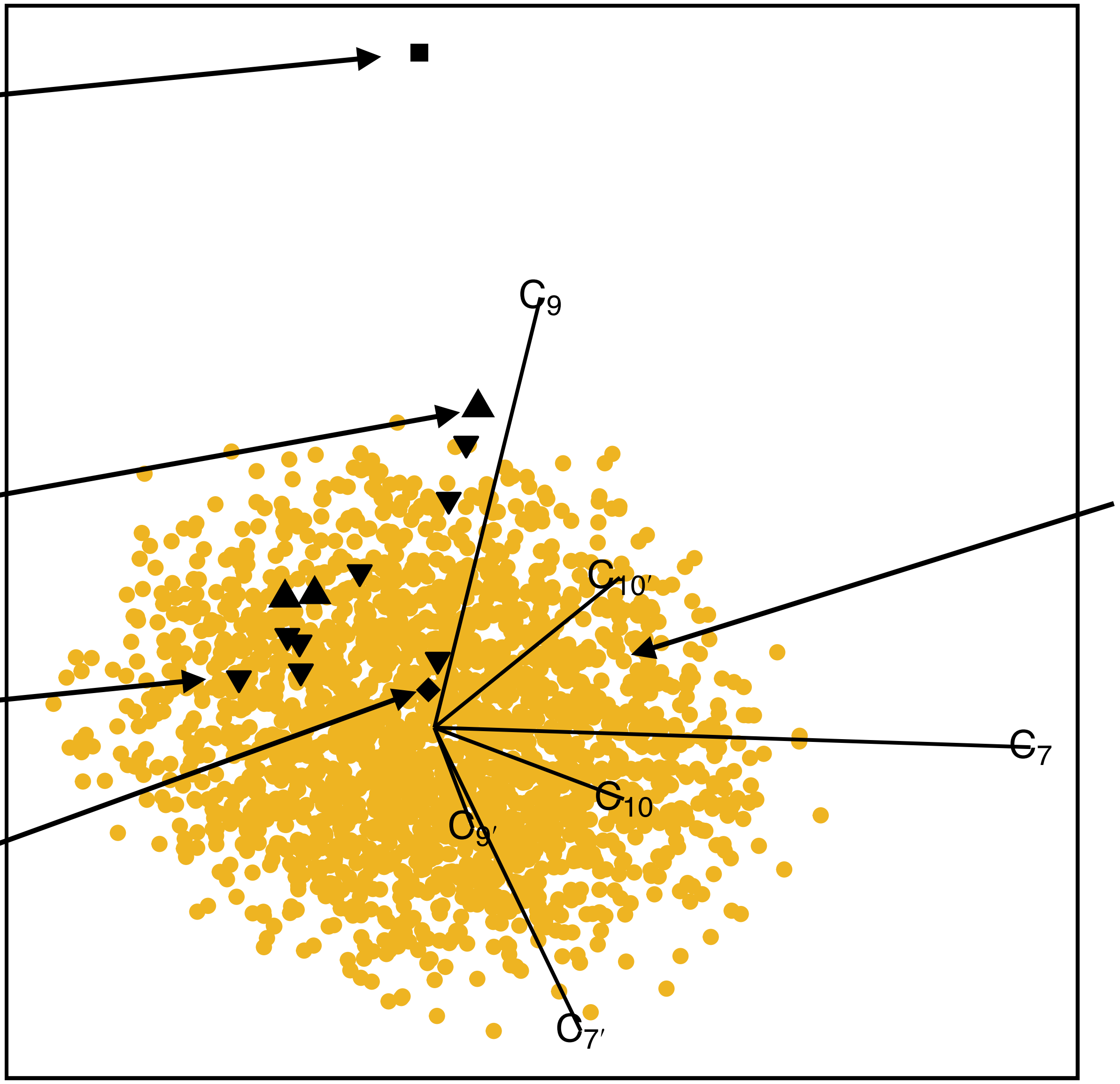
A lot of information is lost

- ? Characterisation beyond just the best fit (BF) point
- ? How does the 6d fit differ from lower dimensional ones
- ? Which observables are important in constraining the parameters
- ? Fit in observable space
- ? Role of correlation
- ? Predictions in the context of the fit rather than for the BF only

Ideas

- Use tools for visualisation of high-dimensional distributions (the grand tour) to compare fits in 6 parameters
- Systematically compare predictions between BF and SM for all observables, also taking into account correlation between observables
- Construct quadratic approximation and use it to select set of points that are representative of the fit
- Use the set of points to discuss:
    - Fit uncertainty in observable space and for predictions
    - Connection between observables and constraints in specific directions in parameter space

Visualisation in 6 Dimensions



SM

1-d BF

2-d BF

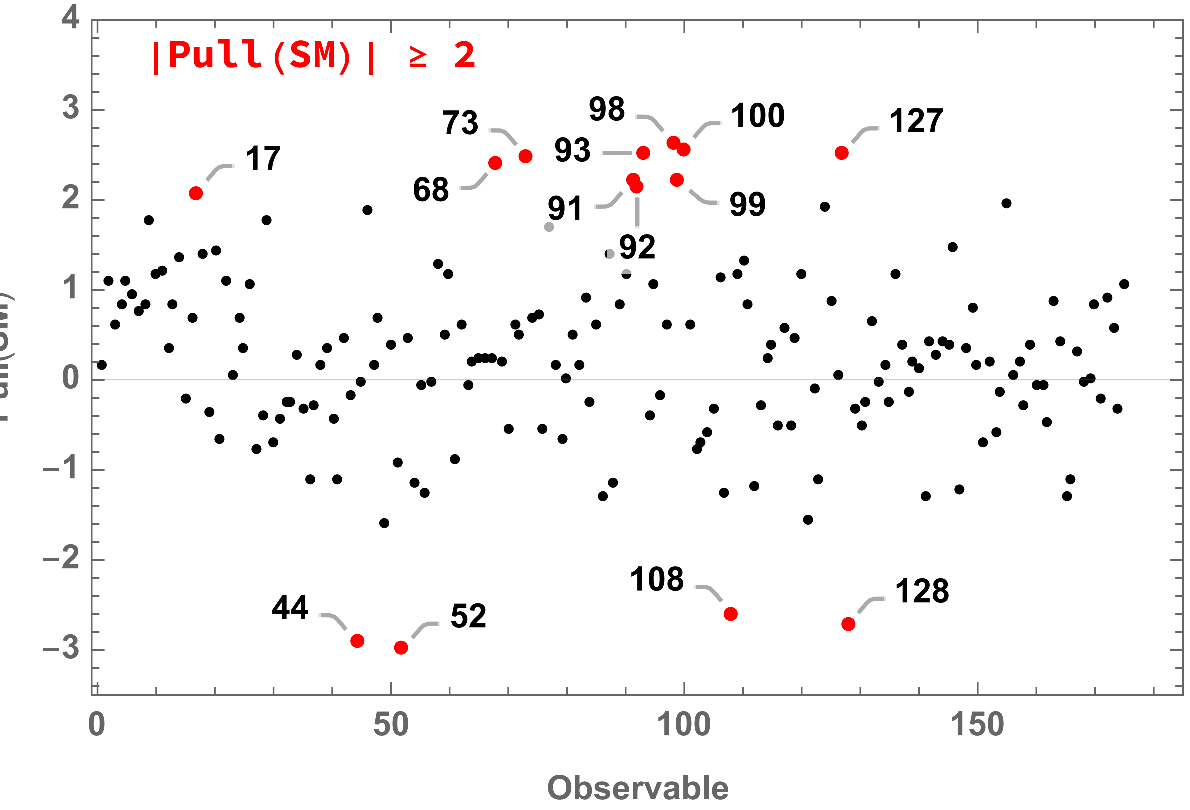
6-d BF

Within 1 σ of the BF

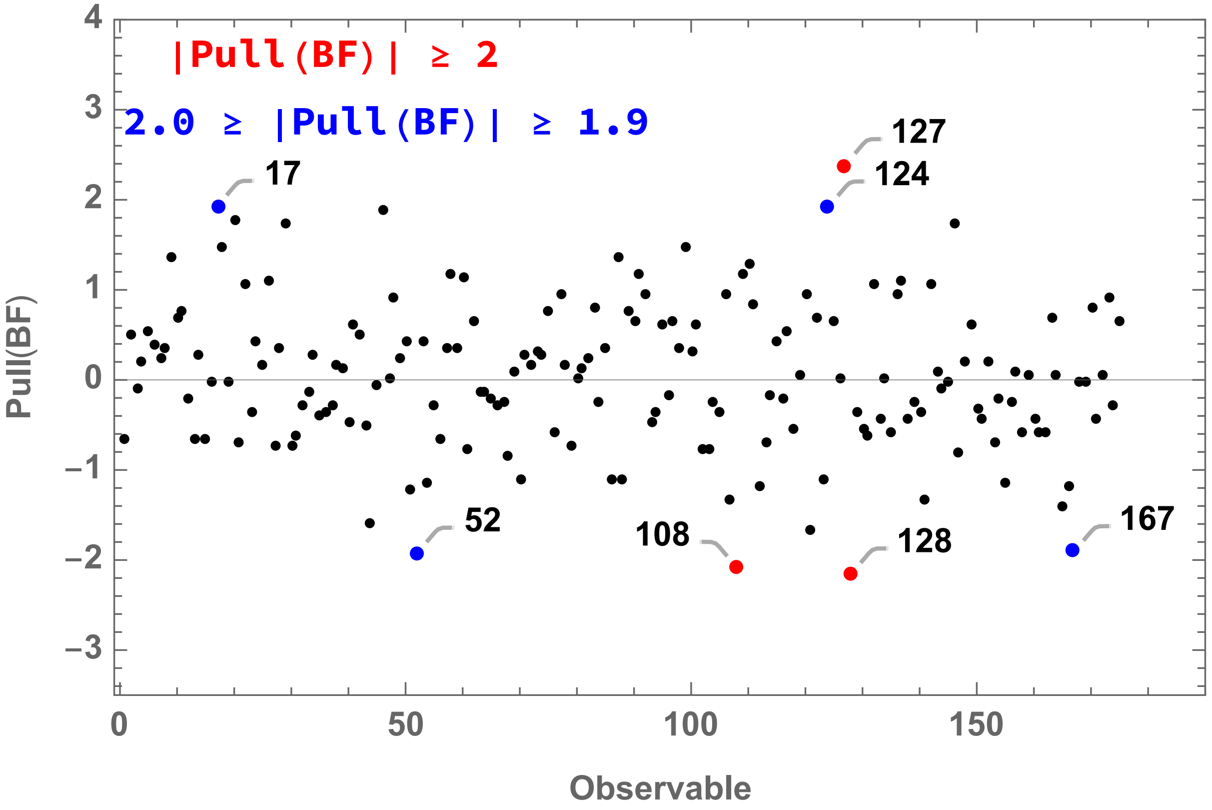
Pull

$$\mathrm{Pull}(p) = \frac{T(p) - \mathcal{O}}{\sqrt{\Delta^2_{exp} + \Delta^2_{T(p)}}}$$

SM



BF



44	$P_5'(B \to K^* \mu\mu)[4-6]$
52	$P_5'(B \to K^* \mu\mu)[6-8]$

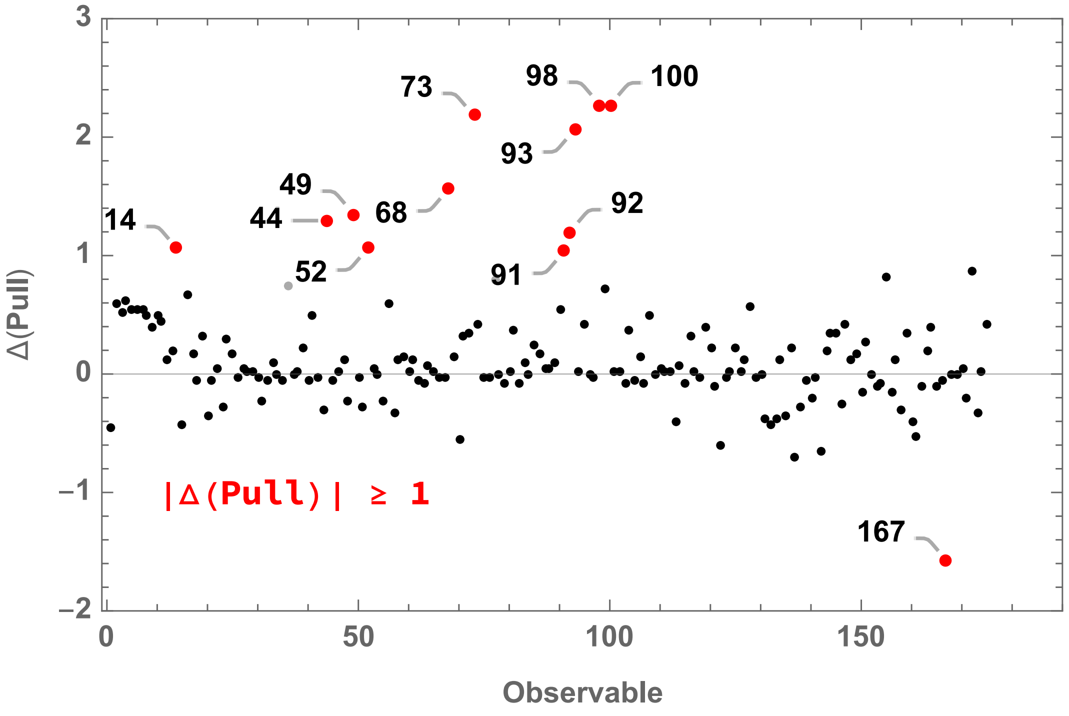
98	$R_K(B^+ \to K^+)[1-6]$
99	$R_{K^*}(B^0 \to K^{0*})[0.045-1.1]$
100	$R_{K^*}(B^0 \to K^{0*})[1.1-6]$

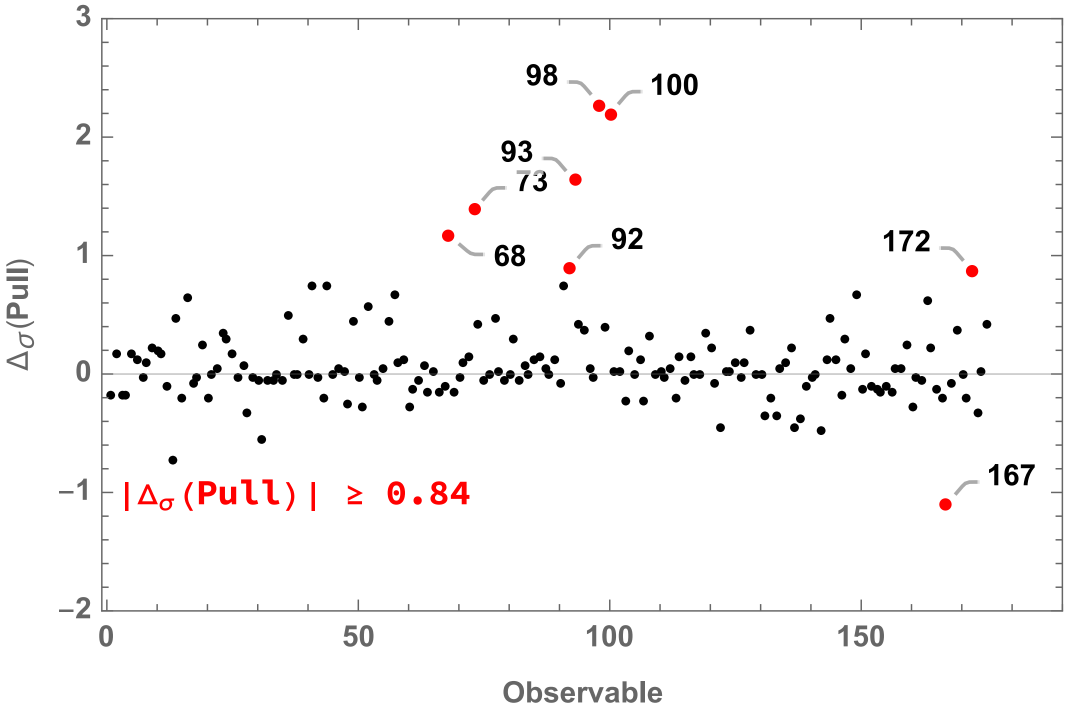
Can take into account correlated uncertainties by defining

$$\mathrm{Pull}_\sigma(p) = \sum_j \sigma_{ij}^{-1/2} (T(p) - O)_j$$

Pull Differences and Correlation

$$\begin{aligned}\Delta(\mathrm{Pull}) &= |\mathrm{Pull(SM)}| - |\mathrm{Pull(BF)}| \\ \Delta_\sigma(\mathrm{Pull})_i &= \left|\sum_j \sigma_{ij}^{-1/2}(T(SM)-\mathcal{O})_j\right| - \left|\sum_j \sigma_{ij}^{-1/2}(T(BF)-\mathcal{O})_j\right|\end{aligned}$$





$$49 \quad \Big| \quad P_2(B \to K^* \mu\mu)[6-8]$$

Including correlation information reduces the Pull difference in particular for angular observables because patterns in residuals are more consistent with the covariance matrix as seen from the SM compared to the BF point

Quadratic Approximation

use Hessian matrix to approximate χ^2 function near the global minimum

$$\chi^2 = \chi_0^2 + \sum_{i,j} H_{ij}\, y_i\, y_j \,,$$

Hessian matrix

$$H_{ij} = \frac{1}{2} \left(\frac{\partial^2 \chi^2}{\partial y_i \, \partial y_j} \right)_0 \,,$$

$$y_i = a_i - a_i^0$$

fit parameter

value at the minimum

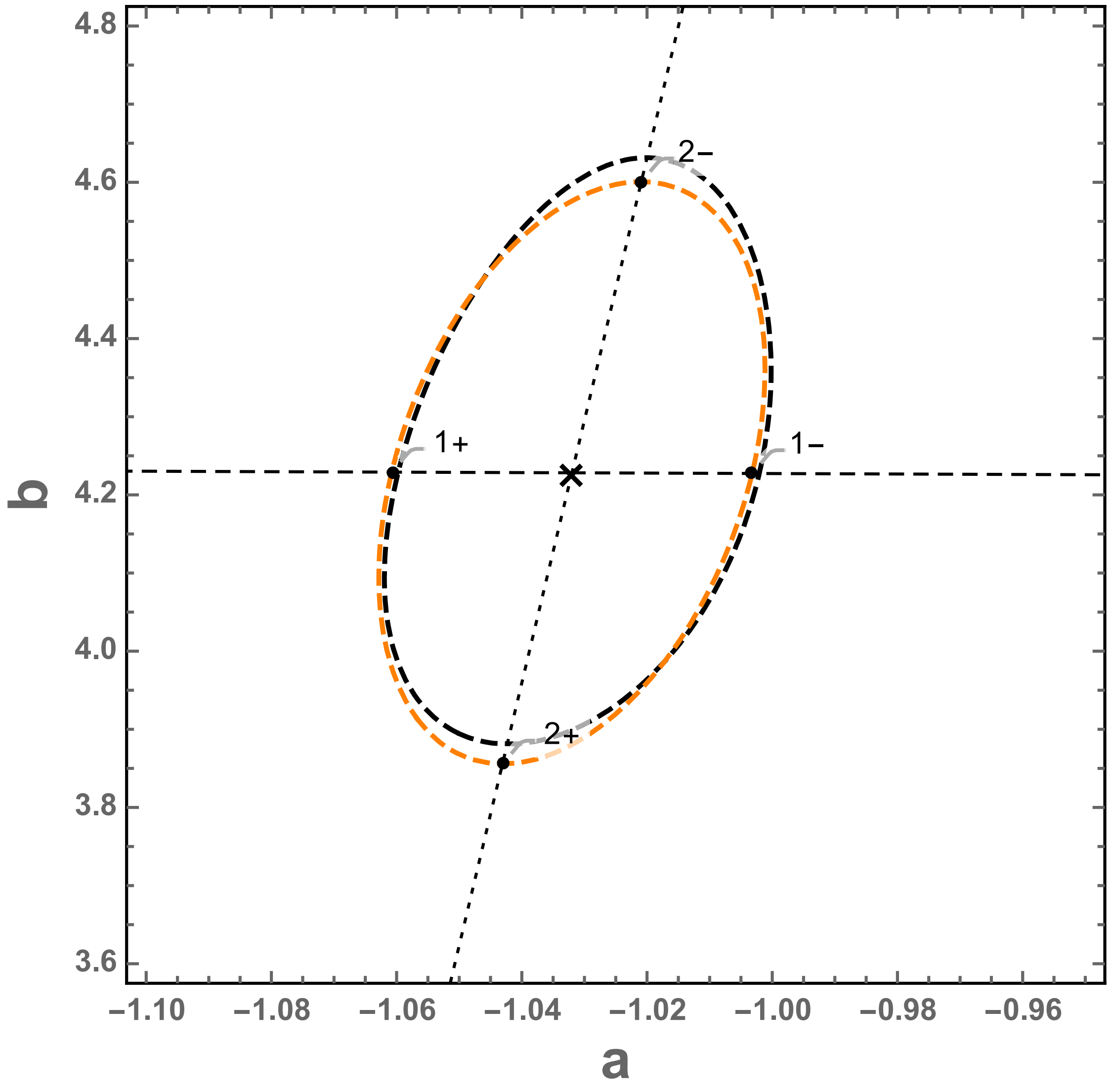
See Pumplin et al [arXiv:0008191]

Features of the Hessian

- Eigenvectors are principal axes of the approximate confidence level ellipsoids
- Eigenvalues encode how tightly each direction is constrained by the data

➡ Use this to define normalised step size in each eigendirection, i.e. we can define step size in terms of $\Delta\chi^2$

➡ Construct set of representative points by moving 1σ away from the BF point along each eigendirection

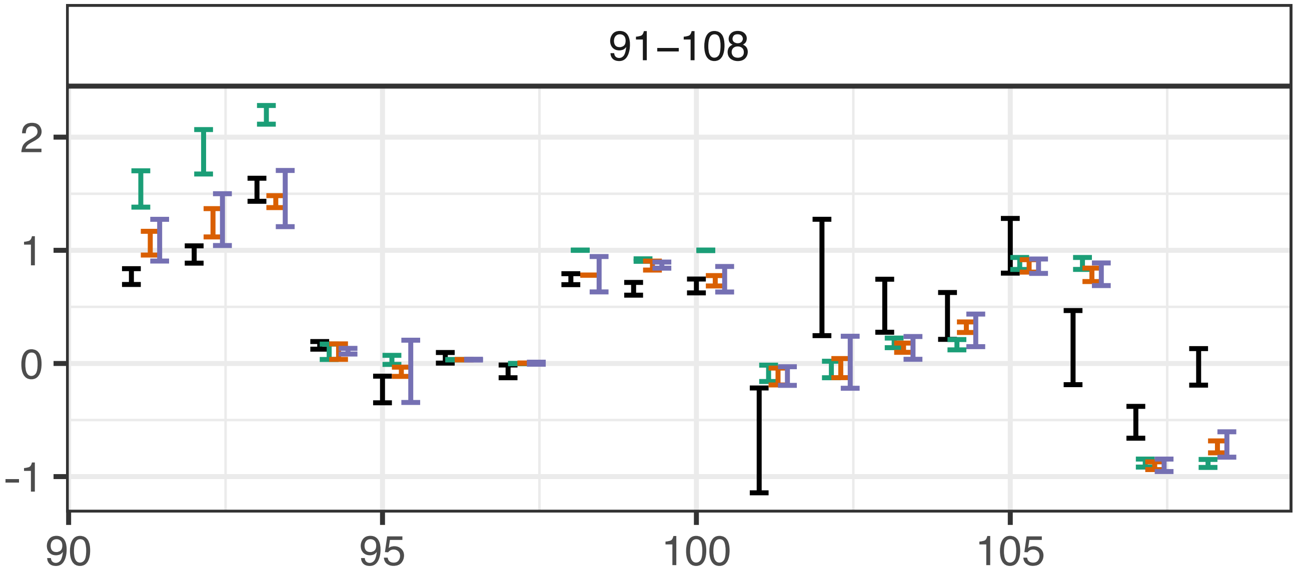


See example in [arXiv:1806.09742]

Fit Uncertainty

Use set of representative points to evaluate fit uncertainty by selecting extreme values

Measured
SM prediction
BF prediction
Fit uncertainty



ID	Observable	Exp
91	$10^7 \times Br(B_s \to \Phi\mu\mu)[2-5]$	LHCb [26]
92	$10^7 \times Br(B_s \to \Phi\mu\mu)[5-8]$	LHCb [26]
93	$10^7 \times Br(B_s \to \Phi\mu\mu)[15-18.8]$	LHCb [26]
94	$F_L(B \to K^*ee)[0.0020-1.120]$	LHCb [27]
95	$P_1(B \to K^*ee)[0.0020-1.120]$	LHCb [27]
96	$P_2(B \to K^*ee)[0.0020-1.120]$	LHCb [27]
97	$P_3(B \to K^*ee)[0.0020-1.120]$	LHCb [27]
98	$R_K(B^+ \to K^+)[1-6]$	LHCb [28]
99	$R_{K^*}(B^0 \to K^{0*})[0.045-1.1]$	LHCb [29]
100	$R_{K^*}(B^0 \to K^{0*})[1.1-6]$	LHCb [29]
101	$P_4'(B \to K^*ee)[0.1-4]$	Belle [30]
102	$P_4'(B \to K^*\mu\mu)[0.1-4]$	Belle [30]
103	$P_5'(B \to K^*ee)[0.1-4]$	Belle [30]
104	$P_5'(B \to K^*\mu\mu)[0.1-4]$	Belle [30]
105	$P_4'(B \to K^*ee)[4-8]$	Belle [30]
106	$P_4'(B \to K^*\mu\mu)[4-8]$	Belle [30]
107	$P_5'(B \to K^*ee)[4-8]$	Belle [30]
108	$P_5'(B \to K^*\mu\mu)[4-8]$	Belle [30]

Relating Observables and Parameter Directions

Use set of representative points to evaluate variation in theory predictions in the eigendirections of the parameter space

Change in predicted value when moving 1 σ in eigendirection i, normalised to the error

$$\delta_i \quad = \quad \frac{(T_i - T_{BF})}{\sqrt{\Delta^2_{exp} + \Delta^2_{BF}}}$$

Taking into account correlated errors

$$\delta_{\sigma,i} \quad = \quad \sum_l \sigma_{il}^{-1/2} (T_{pt} - T_{BF})_l$$

❓Which observables change most in each eigendirection
❓Which directions result in the larges variation in predictions
❓How important are correlation effects

Ranking Observables

Without correlation

Mostly C_7

1+		1-	
ID	δ^2	ID	δ^2
171	4.07	171	5.03
170	0.58	170	0.74
41	0.56	41	0.52
90	0.34	90	0.46
49	0.31	49	0.39

Constraints clearly dominated by observable 171

$10^4 \times Br(B \to X_s \gamma)$

Mostly C_9

moving towards the SM

5+		5-	
ID	δ^2	ID	δ^2
57	0.93	49	0.64
49	0.72	68	0.58
52	0.56	155	0.49
44	0.56	41	0.43
171	0.35	93	0.42

$P_2(B \to K^* \mu\mu)[6-8]$

$10^7 \times Br(B^0 \to K^{0*} \mu\mu)[15-19]$

$10^7 \times Br(B \to K^* \mu\mu)[16-19]$

Constraints much more balanced → combination of observables is important

Ranking Observables

With correlation

Mostly C_7

1+		1-	
ID	δ^2	ID	δ^2
171	4.07	171	5.03
170	0.49	170	0.64
41	0.30	49	0.35
49	0.24	41	0.27
169	0.13	169	0.17

Similar picture as before

Mostly C_9

moving towards the SM

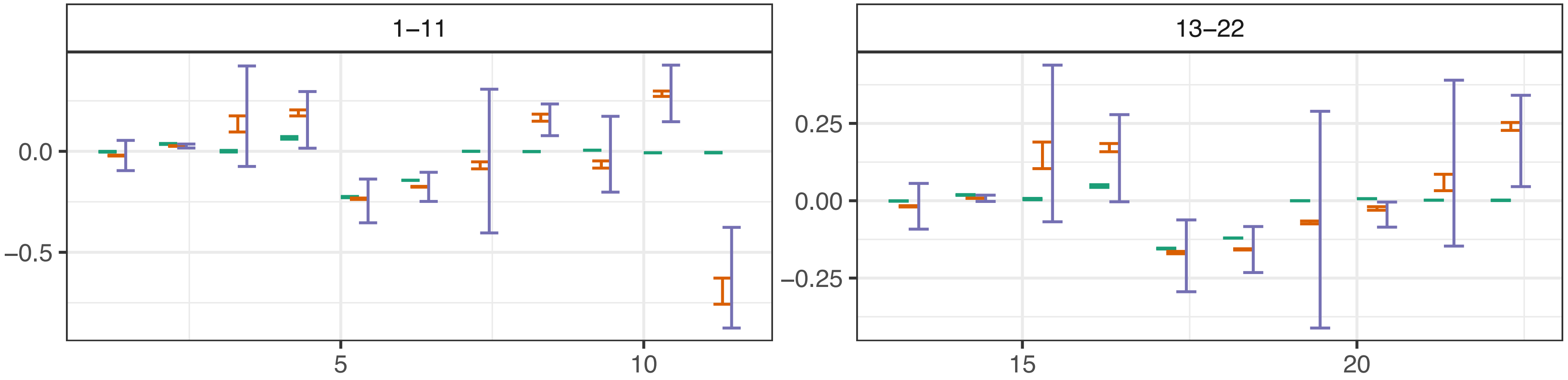
5+		5-		
ID	δ^2	ID	δ^2	
49	1.20	49	1.14	$P_2(B \to K^* \mu\mu)[6-8]$
57	1.15	41	0.47	$P_2(B \to K^* \mu\mu)[4-6]$
52	0.66	171	0.37	
44	0.48	44	0.27	$P_5'(B \to K^* \mu\mu)[4-6]$
56	0.42	57	0.26	

Quite different, BR observables drop out, angular observables become more important

Summary

- Visualisation and systematic comparison provide more complete picture of the fit, also in observable space
- Correlations should be considered when judging the impact of each observable in a global fit, e.g. angular observables appear less relevant when including correlation
- When applicable the Hessian approximation allows discussion of fit uncertainties, relating parameter directions to observables

Fit predictions for future observables

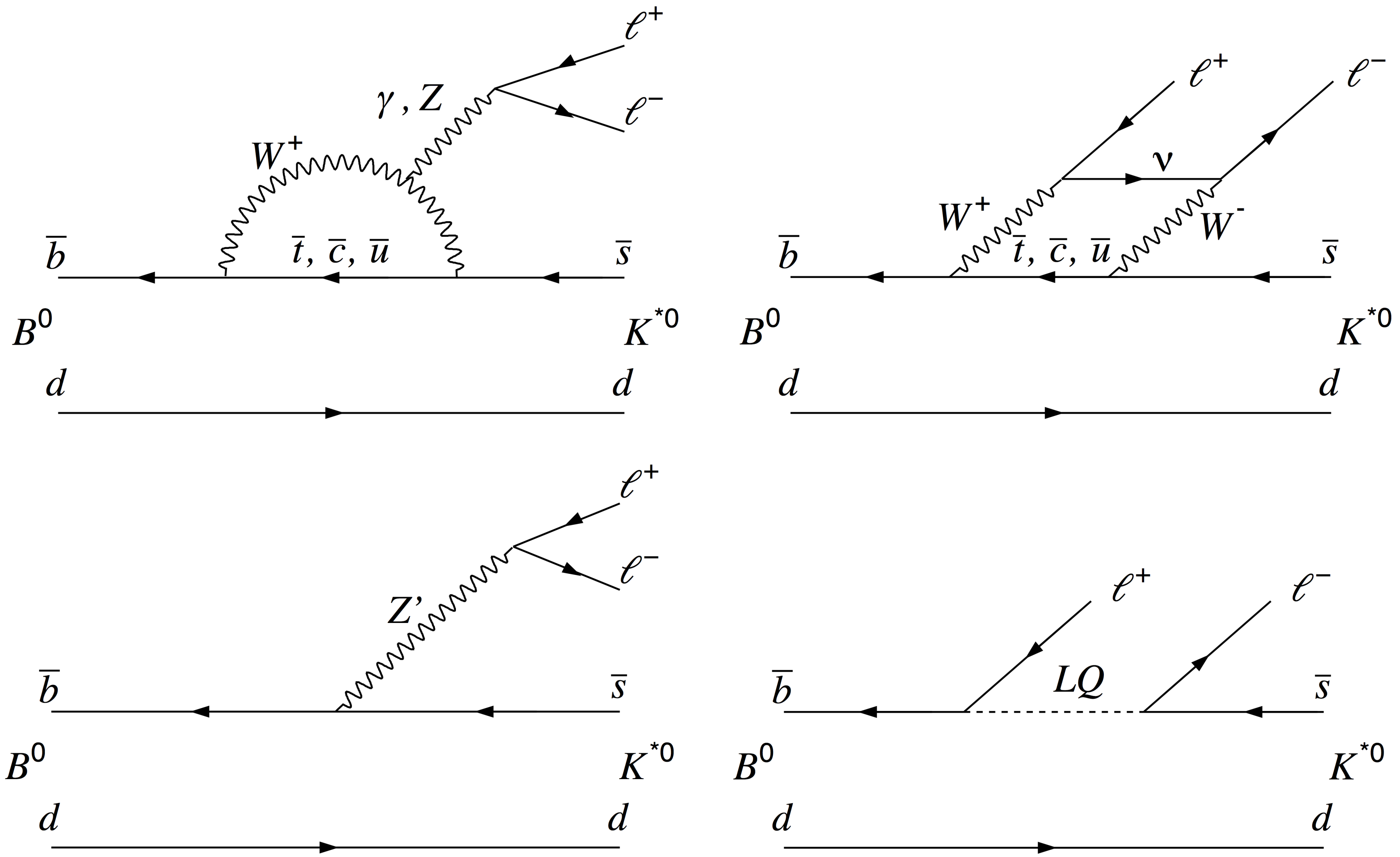


BACKUP

$B^0 \to K^{*0}\ell^+\ell^-$

SM

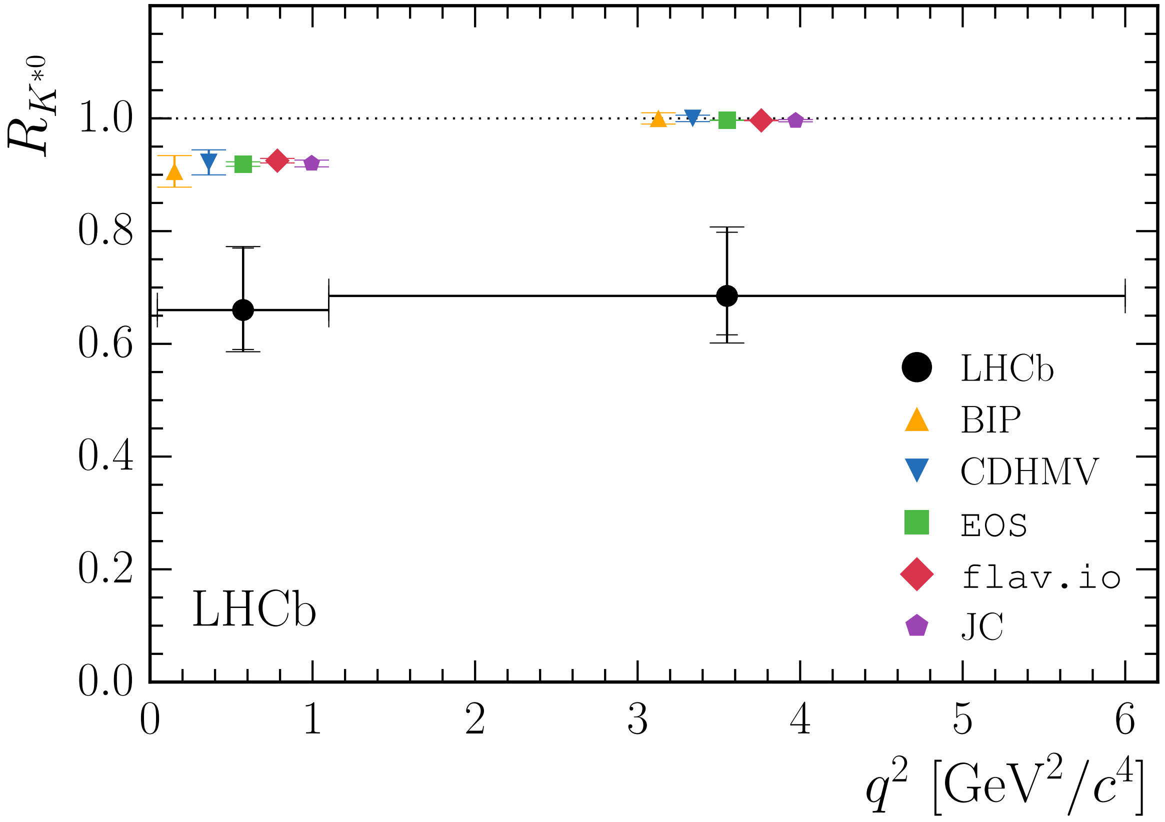
NP



Taken from arXiv:1705.05802

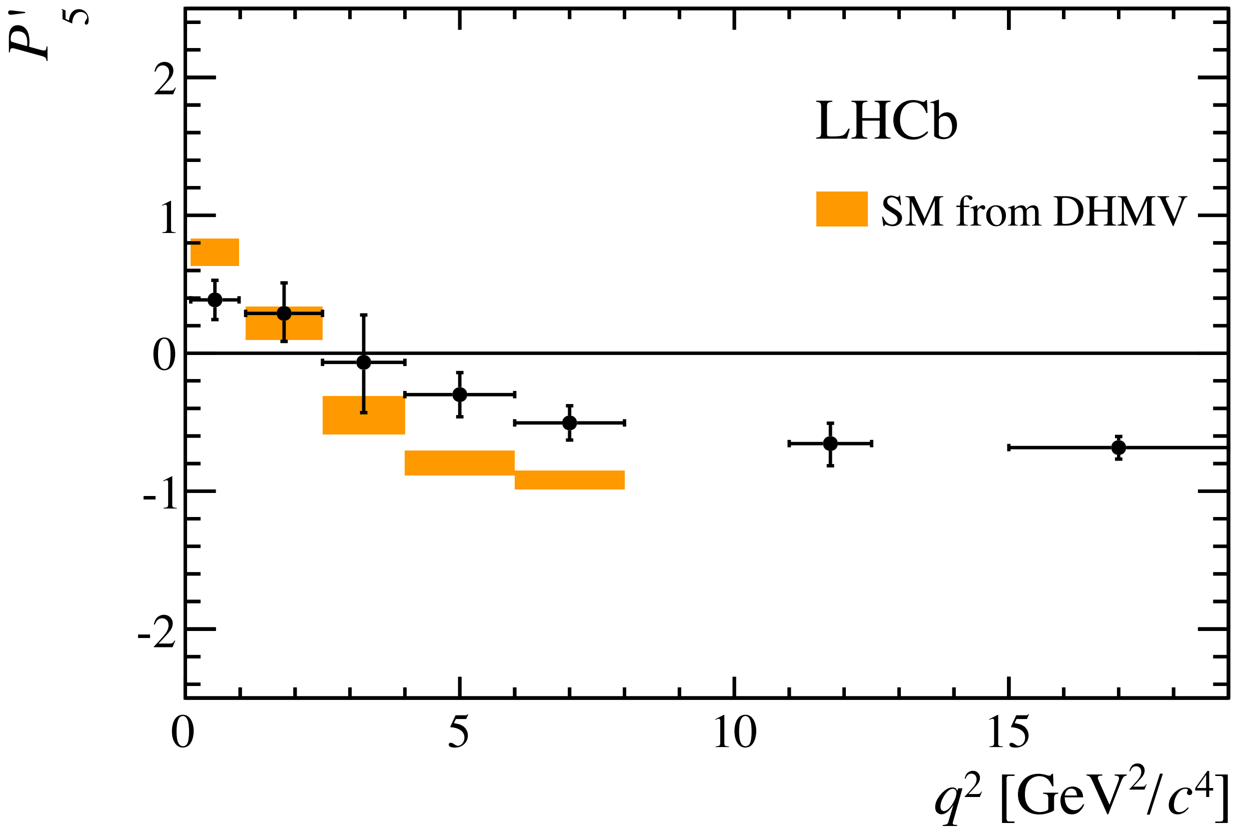
Anomalies observed

Testing lepton universality



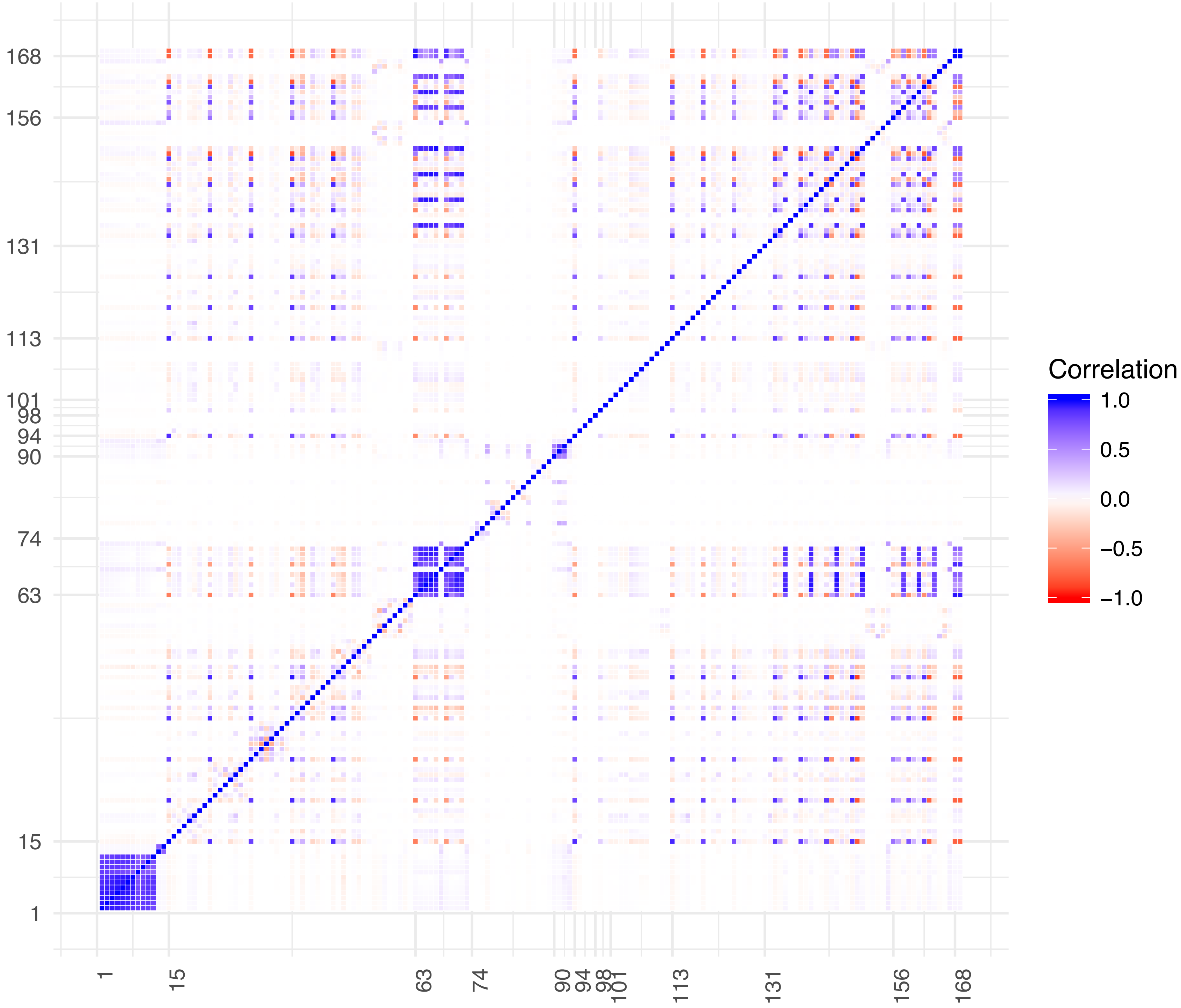
arXiv:1705.05802

Angular observables



arXiv:1512.04442

Correlation Map



Representative Points

EV	C_7	C_9	C_{10}	$C_{7'}$	$C_{9'}$	$C_{10'}$	$\Delta\chi^2_{\text{exact}}$	$\Delta\chi^2_{\text{quad}}$	$\sum\delta^2_\sigma$
1^+	0.0504	-1.06	0.341	0.0147	0.375	-0.0412	6.8	7.1	7.1
1^-	-0.0418	-1.07	0.342	0.0225	0.374	-0.0403	8.1	7.1	8.5
2^+	0.000137	-1.07	0.341	-0.0313	0.374	-0.0398	6.5	7.1	7.3
2^-	0.00852	-1.06	0.342	0.0686	0.375	-0.0416	7.	7.1	7.1
3^+	0.011	-1.16	0.234	0.0267	0.248	0.253	4.9	7.1	7.2
3^-	-0.00239	-0.972	0.449	0.0106	0.501	-0.335	8.9	7.1	7.2
4^+	0.0118	-1.27	0.722	0.0185	0.289	-0.00429	7.7	7.1	6.6
4^-	-0.00315	-0.859	-0.0387	0.0188	0.46	-0.0772	6.3	7.1	7.7
5^+	0.0173	-1.54	0.13	0.0195	0.511	-0.21	4.9	7.1	9.6
5^-	-0.00866	-0.59	0.553	0.0178	0.238	0.129	9.1	7.1	6.7
6^+	0.00484	-1.03	0.595	0.0131	1.72	0.64	7.4	7.1	10.0
6^-	0.00382	-1.1	0.0881	0.0241	-0.971	-0.722	8.6	7.1	11.5