Neutrino masses, oscillations, and 0νββ-decay
Fedor Šimkovic

Even though my friends and I are so small,
we can teach you the biggest things of all.

ν_e

ν_τ

ν_μ

Standard Model
(an astonishing successful theory, based on few principles)

ν **is a special particle in SM:**

- **It is the only fermion that does not carry electric charge**
  (like γ, g, H^0)

- **There are only left-handed ν's (ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$)**

- **ν-mass can not be generated with any renormalizable coupling with the Higgs fields through SSB**

ν**'s oscillations experiments**
$\Rightarrow$ **tiny neutrino masses (!)**
$\Rightarrow$ **Beyond SM physics (!)**

7/26/2023

, etc

After 93/67 years we know

Fundamental ν properties

No answer yet

- 3 families of light (V-A) neutrinos:
  ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

ν_e
ν_τ
ν_μ

- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- **Sterile neutrinos?**
- Statistical properties of ν? Fermionic or partly bosonic?

ETTORE MAJORANA

Currently main issue

Nature, Mass hierarchy, **CP-properties, sterile ν**

$= \nu = $

The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

Majorana fermions

Ettore Majorana
*Teoria simmetrica dell'elettrone e del positrone
(A symmetric theory of electrons and positrons).*
Il Nuovo Cimento, 14: 171–184, 1937.) 171

ν is its own antiparticle

Bruno Pontecorvo
Inverse beta processes and nonconservation of lepton charge
Zhur. Eksptl'. i Teoret. Fiz. 34, 247 (1958)

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]

$\nu \leftrightarrow$ anti-ν oscillation

Steve Weinberg
ν-mass generation
via d=5 eff. oper.
related to unknown
high energy scale (GUT?)

thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \\ 0 & -s_{12} & c_{12} \end{pmatrix}$$

$$\tilde{R}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

3 neutrino masses, 2 mass squared differences

$$\delta m^2 = m_2^2 - m_1^2, \quad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

$$U = R_{23}\tilde{R}_{13}R_{12}$$

3 mixing angles CP-phase

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U^*_{\alpha j}|\nu_j\rangle$$

$$(\alpha = e,\ \mu, \tau)$$

Global neutrino oscillations analysis (PRD 101, 116013 (2020))

	best $-$ fit	1σ	2σ	3σ
	Normal hierarchy (NH)			
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.90
$\Delta m^2/10^{-3}$ eV^2	2.485	2.453-2.514	2.419-2.547	2.2389-2.578
$\sin^2\theta_{12}/10^{-1}$	3.05	2.92-3.19	2.78-3.32	2.65-3.47
$\sin^2\theta_{13}/10^{-2}$	2.22	2.14-2.28	2.07-2.34	2.01-2.41
$\sin^2\theta_{23}/10^{-1}$	5.45	4.98-5.65	4.54-5.81	4.36-5.95
δ/π	1.28	1.10-1.66	0.95-1.90	0-0.07 $\oplus$ 0.81-2.00
	Inverted hierarchy (IH)			
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.91
$-\Delta m^2/10^{-3}$ eV^2	2.465	2.434-2.495	2.404-2.526	2.374-2.556
$\sin^2\theta_{12}/10^{-1}$	3.03	2.90-3.17	2.77-3.31	2.64-3.45
$\sin^2\theta_{13}/10^{-2}$	2.23	2.17-2.30	2.10-2.37	2.03-2.43
$\sin^2\theta_{23}/10^{-1}$	5.51	5.17-5.67	4.60-5.82	4.39-5.96
			$\oplus$ 5.31-6.10	
δ/π	1.52	1.37-1.65	1.23-1.78	1.09-1.90

Source

Detector

$\nu_\alpha \to \nu_\beta$

$distance - L$

Massive neutrinos and neutrino oscillations

S. M. Bilenky

Joint Institute of Nuclear Research, Dubna, Union of Soviet Socialist Republics

S. T. Petcov

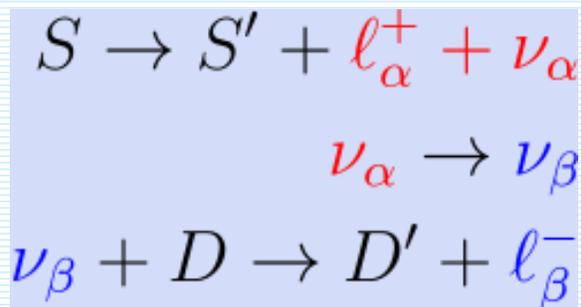*Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, People's Republic of Bulgaria*

The theory of neutrino mixing and neutrino oscillations, as well as the properties of massive neutrinos (Dirac and Majorana), are reviewed. More specifically, the following topics are discussed in detail: (i) the possible types of neutrino mass terms; (ii) oscillations of neutrinos (iii) the implications of *CP* invariance for the mixing and oscillations of neutrinos in vacuum; (iv) possible varieties of massive neutrinos (Dirac, Majorana, pseudo-Dirac); (v) the physical differences between massive Dirac and massive Majorana neutrinos and the possibilities of distinguishing experimentally between them; (vi) the electromagnetic properties of massive neutrinos. Some of the proposed mechanisms of neutrino mass generation in gauge theories of the electroweak interaction and in grand unified theories are also discussed. The lepton number nonconserving processes $\mu \to e\gamma$ and $\mu \to 3e$ in theories with massive neutrinos are considered. The basic elements of the theory of neutrinoless double-β decay are discussed as well. Finally, the existing data on neutrino masses, oscillations of neutrinos, and neutrinoless double-β decay are briefly reviewed. The main emphasis in the review is on the general model-independent results of the theory. Detailed derivations of these are presented.

$$S \to S' + \ell_\alpha^+ + \nu_\alpha$$

$$\nu_\alpha \to \nu_\beta$$

$$\nu_\beta + D \to D' + \ell_\beta^-$$

$$\Gamma_{osc} = \int \frac{d\Phi_\nu(E_\nu)}{dE_\nu} \frac{\mathcal{P}_{\alpha\beta}(E_\nu, L)}{4\pi L^2} \sigma(E_\nu) \, dE_\nu$$

Process is governed by the oscillation probability

$$S + D \to \ell_\alpha^+ + \ell_\beta^- + S' + D'$$

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$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j} e^{-i \, m_j^2 \, L/(2E_\nu)} \right|^2$$

$$\langle f|S^{(2)}|i\rangle = -i \int d^4x_1 J_S^\mu(P_S', P_S) e^{i(P_\alpha + P_S' - P_S)\cdot x_1} \times$$

$$\int d^4x_2 J_D^\mu(P_D', P_D) e^{i(P_\beta + P_D' - P_D)\cdot x_2} \sum_{k=1}^{3} U_{\alpha k}^* U_{\beta k} \times$$

$$\overline{v}(P_\alpha; \lambda_\alpha)\gamma_\mu(1-\gamma_5)\, D(x_2 - x_1, m_k)\, (1-\gamma_5)\gamma_\nu u(P_\beta; \lambda_\beta)$$

The neutrino emission and detection are identified with the charged-current vertices of a single second-order **Feynman diagram** for the underlying process, enclosing neutrino propagation between these two points.

$$D(x; m) = \theta(x_0)D^-(x; m) + \theta(-x_0)D^+(x; m),$$

$$D^\pm(x; m) = \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\mp(-\mathbf{q}\cdot\vec{\gamma} + \omega\gamma^0) + m}{2\omega} e^{\pm i(-\mathbf{q}\cdot\mathbf{x} + \omega x_0)}$$

Integration over time variables results in **energy conservation** and **energy denominator**

$$2\pi i\, \frac{\delta(E_\beta + E_D' - E_D + E_\alpha + E_S' - E_S)}{\omega + E_\alpha + E_S' - E_S + i\varepsilon}.$$

7/26/2023

$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\not{p} + m_k}{2\omega(\omega + E_\alpha + E'_S - E_S + i\varepsilon)} e^{i\mathbf{q}\cdot(\mathbf{x}_2 - \mathbf{x}_1)}$$

$$\simeq \frac{1}{4\pi} \frac{e^{ip_k|\mathbf{x}_2 - \mathbf{x}_1|}}{|\mathbf{x}_2 - \mathbf{x}_1|} \left(\not{Q}_k + m_k\right) \simeq e^{i\mathbf{p}_k\cdot\mathbf{x}_D} \, e^{-i\mathbf{p}_k\cdot\mathbf{x}_S} \frac{e^{ip_k L}}{L} \left(\not{Q}_k + m_k\right)$$

$$Q_k \equiv (E_\nu, \mathbf{p}_k), \quad \mathbf{p}_k = p_k \left(\mathbf{x}_2 - \mathbf{x}_1\right)/|\mathbf{x}_2 - \mathbf{x}_1|, \quad p_k = \sqrt{E_\nu^2 - m_k^2}$$

$$E_\nu = E_S - E'_S - E_\alpha \ (source) = E_\beta + E'_D - E_D \ (detector)$$

Energy conservation

Momentum conservation at source

Momentum conservation at detector

Energy conservation

$$\langle f|S^{(2)}|i\rangle = (2\pi)^7 \delta(E_f - E_i) \sum_k U_{\alpha k} U^*_{\beta k} \frac{e^{ip_k L}}{4\pi L} \times$$

$$T_k^{\alpha\beta} \; \delta^3_{V_S}(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S) \; \delta^3_{V_D}(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_k)$$

with

$$E_f - E_i = E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S$$

$$T_k^{\alpha\beta} = J_S^\mu(P'_S, P_S) J_D^\nu(P'_D, P_D) \, \overline{v}(P_\alpha; \lambda_\alpha)\gamma_\mu(1 - \gamma_5) \, \slashed{Q}_k \, \gamma_\nu u(P_\beta; \lambda_\beta)$$

Lepton current

Master formula

$$d\Gamma^{\alpha\beta}(L) = \sum_{km} U_{\alpha k} U^*_{\beta k} U_{\alpha m} U^*_{\beta m} \frac{e^{i(p_k - p_m)L}}{4\pi L^2} \times \mathcal{F}^{\alpha\beta}_{km}$$

$$\delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m)$$

$$\frac{(2\pi)^7}{4E_S E_D} \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S) \times$$

$$\frac{1}{\hat{J}_S \hat{J}_D} \frac{d\mathbf{p}_\alpha}{2E_\alpha (2\pi)^3} \frac{d\mathbf{p}_\beta}{2E_\beta (2\pi)^3} \frac{d\mathbf{p}'_S}{2E'_S (2\pi)^3} \frac{d\mathbf{p}'_D}{2E'_D (2\pi)^3}$$

$$with$$

$$\mathcal{F}^{\alpha\beta}_{km} = 4\pi \sum_{\text{spin}} \frac{1}{2} \left(T^{\alpha\beta}_k \left(T^{\alpha\beta}_m\right)^* + T^{\alpha\beta}_m \left(T^{\alpha\beta}_k\right)^* \right)$$

$$\langle \Phi^{S,D}(\mathbf{P}_i) | \Phi^{S,D}(\mathbf{P}_k) \rangle = (2\pi)^3 \, 2E_k \, \delta^3_{V_{S,D}} (\mathbf{P}_i - \mathbf{P}_k)$$
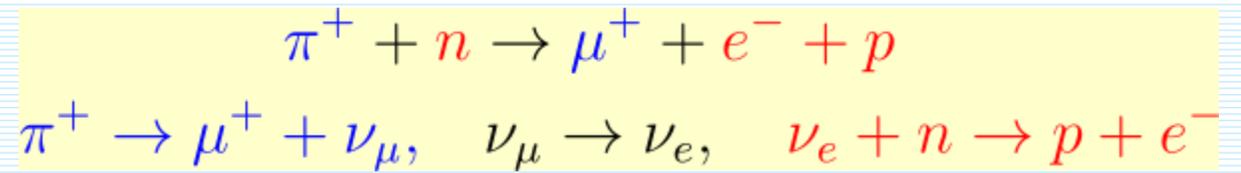
Two normalization volumes:
  i) source;
  ii) Detector.

$$\delta^3_V(\mathbf{Q}_n - \mathbf{P}) \, \delta^3_V(\mathbf{Q}_m - \mathbf{P}) \simeq$$
$$\frac{V}{(2\pi)^3} \frac{1}{2} \left(\delta^3_V(\mathbf{Q}_n - \mathbf{P}) + \delta^3_V(\mathbf{Q}_m - \mathbf{P}) \right)$$

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$$\pi^+ + n \to \mu^+ + e^- + p$$

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

Standard QM approach

$$\Gamma_{osc}^{\pi^+ n} = \int \frac{d\Phi_\nu(E_\nu')}{dE_\nu'} \frac{\mathcal{P}_{\nu_\mu \nu_e}(E_\nu')}{4\pi L^2} \sigma(E_\nu') \, dE_\nu'$$

$$= \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu \nu_e}(E_\nu)}{4\pi L^2} \left(g_V^2 + 3g_A^2\right) \, p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

New QFT approach (no decoherence, no factorization of two processes)

$$\Gamma_{QFT}^{\pi^+ n} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{\mathcal{P}_{\mu e}^{QFT}(E_\nu)}{4\pi L^2} \left(g_V^2 + 3g_A^2\right) \, p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}^{QFT}(E_\nu) = \frac{1}{2} \sum_{km} U_{\beta k} U_{\alpha k}^* \, U_{\beta k}^* U_{\alpha k} \, e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E_\nu^2}\right)$$

Nuovo Cim. 14, 322 (1937)

Second order process with real intermediate neutrinos

$$S + D \to \ell_\alpha^+ + \ell_\beta^+ + S' + D'$$

$$S \to S' + \ell_\alpha^+ + \nu_\alpha, \quad \nu_\alpha \to \overline{\nu}_\beta, \quad \overline{\nu}_\beta + D \to D' + \ell_\beta^+$$

Amplitude proportional to ν−mass

$$T_k^{\alpha\beta} = J_S^\mu(P_S', P_S) J_D^\nu(P_D', P_D) \times$$
$$\overline{v}(P_\alpha; \lambda_\alpha)\gamma_\mu(1 - \gamma_5)m_k\gamma_\nu u(P_\beta; \lambda_\beta)$$

Oscillation probability

$$\mathcal{P}_{\alpha\overline{\beta}}^{\mathrm{QFT}}(E_\nu, L) \equiv |\langle \nu_\beta | \overline{\nu}_\alpha \rangle|^2$$

$$= \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j}^* \frac{m_j}{E_\nu} e^{-im_j^2 L/(2E_\nu)} \right|^2$$

Replacement:

$$U_{\alpha k} \to U_{\alpha k}^*$$
$$U_{\beta m}^* \to U_{\beta m}$$

Particular process:

$$\pi^+ + p \to \mu^+ + e^+ + n$$

Production rate

$$\Gamma_{QFT}^{\pi^+ p} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}} \right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu \overline{\nu}_e}^{\mathrm{QFT}}(E_\nu, L)}{4\pi L^2} \left(g_V^2 + 3g_A^2 \right) p_e E_e$$
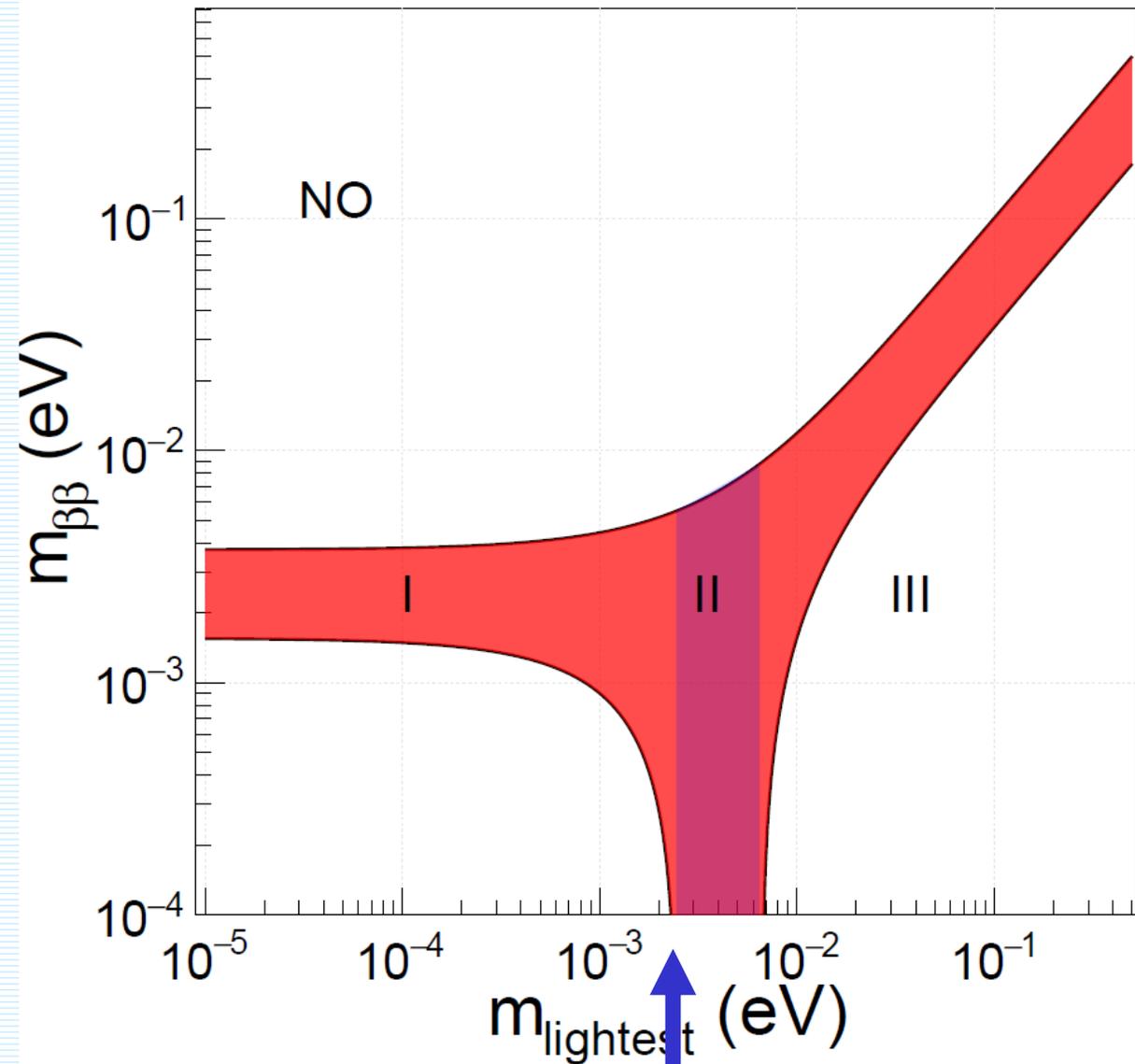
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For L=0

$$\mathcal{P}_{\alpha\bar{\beta}}(E_\nu, L = 0) = \frac{m_{\beta\beta}^2}{E_\nu^2}$$

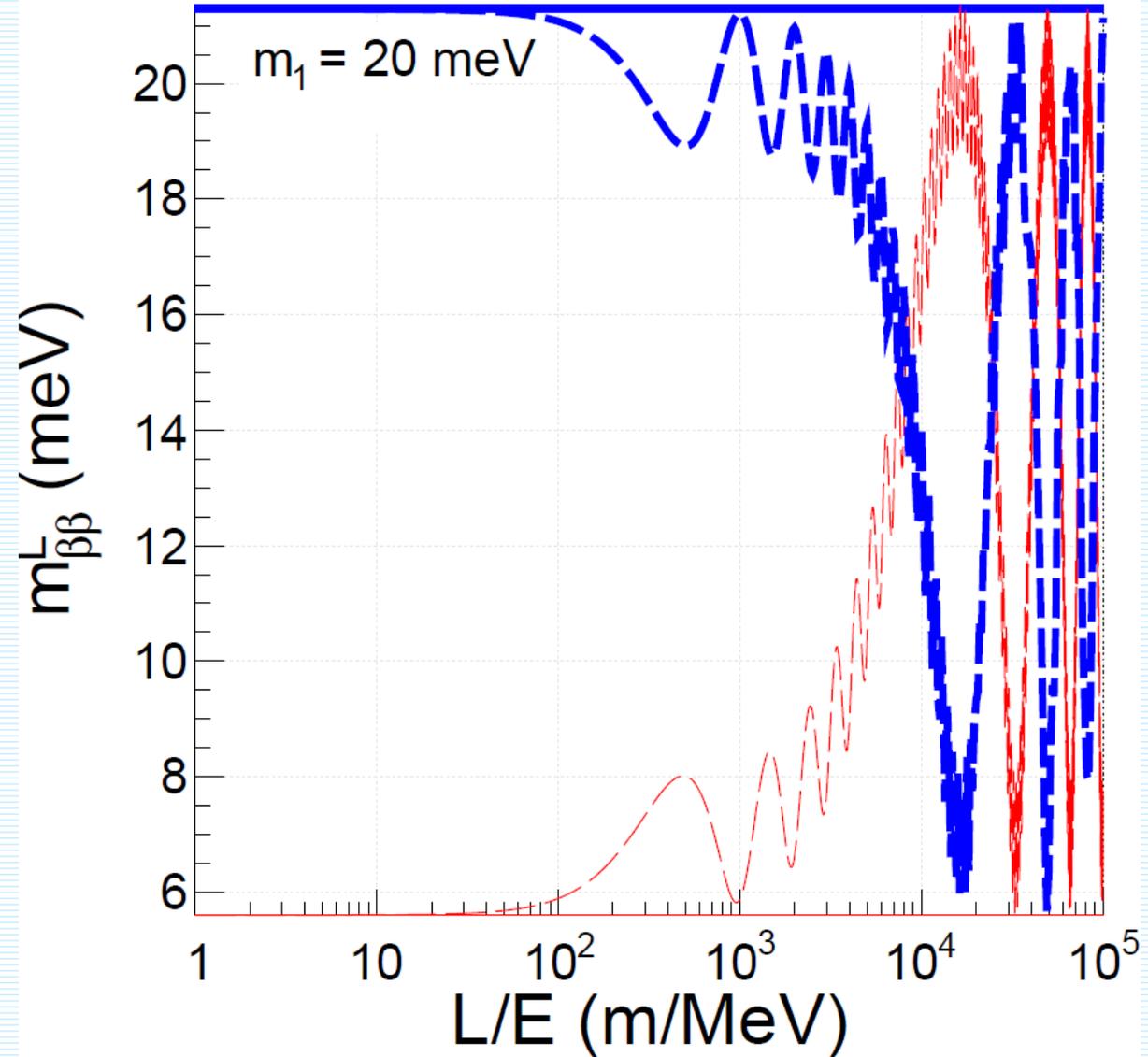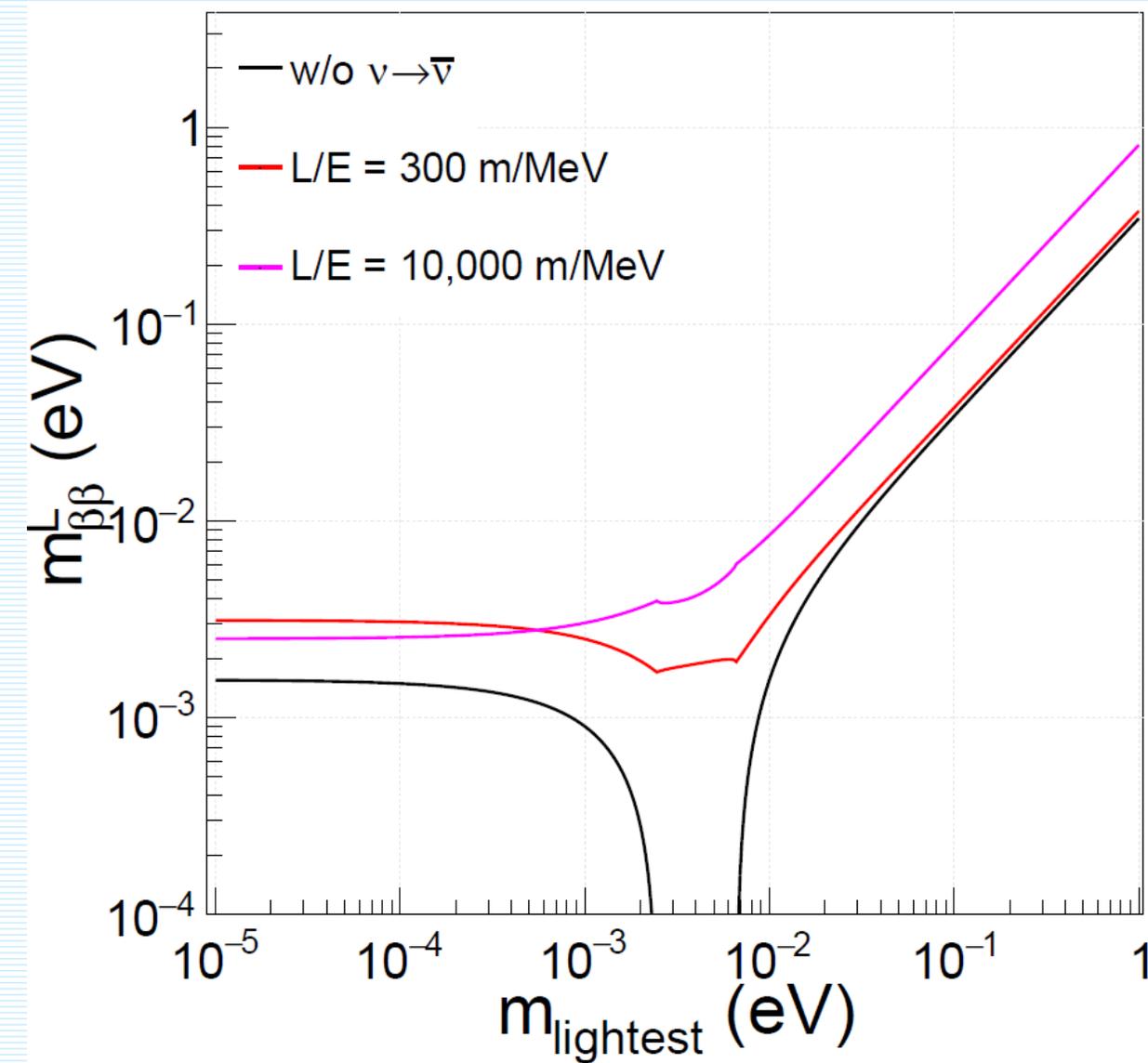$$= \frac{1}{E_\nu^2} \left| \sum_{j=1}^{3} U_{\alpha j} U_{\beta j} m_j \right|^2$$

$$m_{\beta\beta} = |\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3|$$

$$\rho_1 = c_{12}^2 c_{13}^2 m_1, \quad \rho_2 = s_{12}^2 c_{13}^2 m_2, \quad \rho_3 = s_{13}^2 m_3$$

$$\min_{\phi_1, \phi_2} m_{\beta\beta} = \begin{cases} |\rho_2 - \rho_3| - \rho_1, & \text{if } \rho_1 < |\rho_2 - \rho_3| & : \text{region I}, \\ 0, & \text{if } |\rho_2 - \rho_3| \le \rho_1 \le \rho_2 + \rho_3 & : \text{region II}, \\ \rho_1 - (\rho_2 + \rho_3), & \text{if } \rho_2 + \rho_3 < \rho_1 & : \text{region III}. \end{cases}$$

14

Dependence of m^L_{ee} on $m_{lightest}$ and L/E

Left panel axes: $m^L_{\beta\beta}$ (eV) vs $m_{lightest}$ (eV). Legend: w/o $\nu \to \bar{\nu}$; L/E = 300 m/MeV; L/E = 10,000 m/MeV.

Right panel axes: $m^L_{\beta\beta}$ (meV) vs L/E (m/MeV). m_1 = 20 meV

7/26/2023

Symmetry 14, 1383 (2022)

15

Majorana ν-mass => Lepton number violation

The **absence of the RH ν fields** in the SM is the simplest, most economical scenario. The ν-masses and mixing are generated by the **L-number violating Majorana** mass term coming from **dimension-5** effective **Weinberg** operator:

Majorana Neutrinos
- **LN violating**
- $\nu = \nu^c$
- $\nu = \nu_L + (\nu_L)^c$

$$L = \frac{\lambda(LH)(LH)}{\Lambda} + h.c.$$

(LH) is a SM singlet, Λ - mass scale, λ -dimensionless coupling. After SSB

$$L = \frac{\lambda v^2}{2\Lambda}(\nu_L \nu_L + h.c.)$$

The Majorana ν-mass term violates total lepton number

Make $\nu_L \to e^{i\phi}\nu_L$, L changes by $e^{2i\phi}$
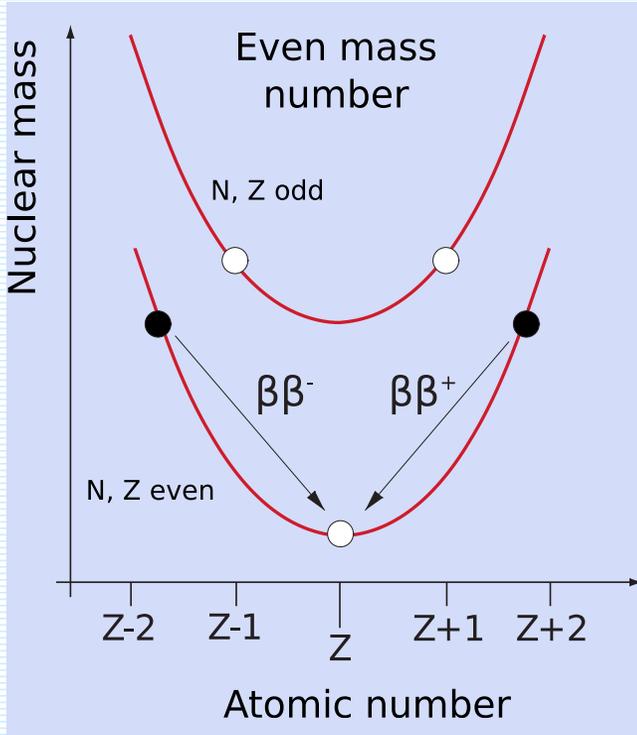
In the SM, The term like $\nu_L \nu_L$ is not allowed by **SU(2)xU(1)** => there is no natural Majorana mass term for the LH ν. However, dim-5 L-number non-conserving operator is allowed leading to a **Majorana mass**

$$m_M = \frac{\lambda v^2}{\Lambda}$$

This is a seesaw formula, in the sense that small ν-mass can be understood when Λ is large. To get **meV mass**, we need

$$\Lambda = 10^{16} GeV \text{ (GUT scale)}$$

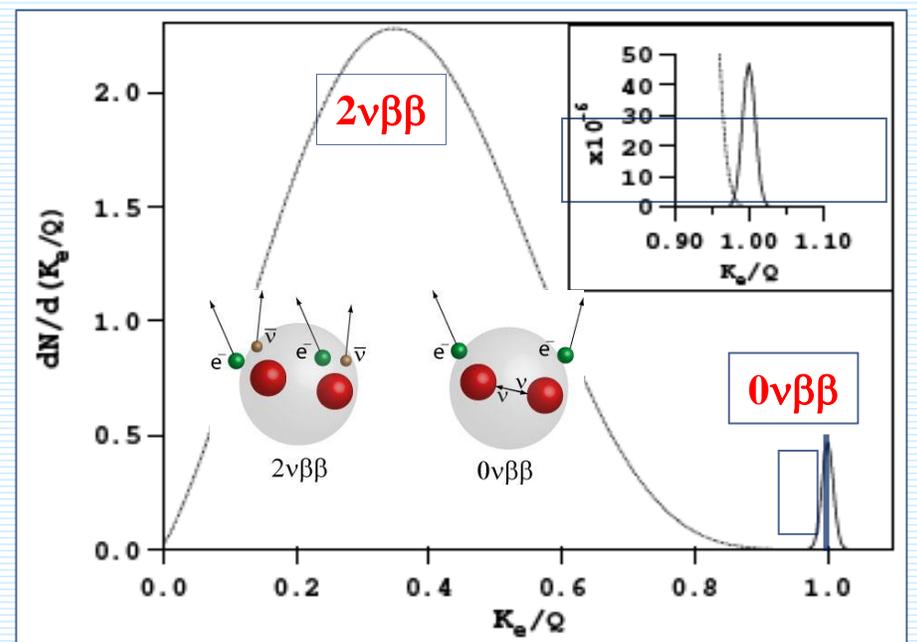**Nuclear double-β decay**
(even-even nuclei, pairing int.)

Phys. Rev. 48, 512 (1935)

Two-neutrino double-β decay – LN conserved
$$(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$$
Goepert-Mayer – 1935. 1st observation in 1987

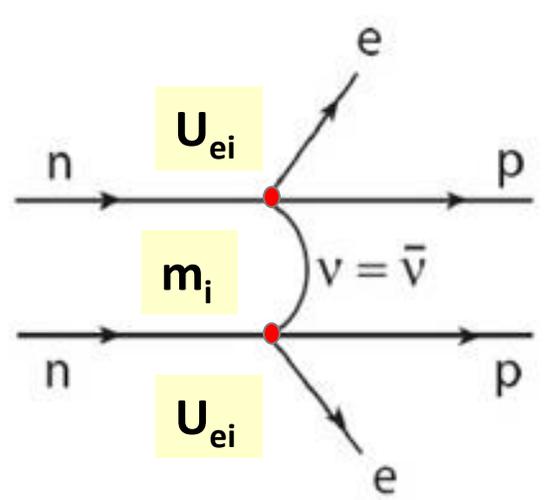**Nuovo Cim. 14, 322 (1937)**   **Phys. Rev. 56, 1184 (1939)**

Neutrinoless double-β decay – LN violated
$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$ **(Furry 1937)**
Not observed yet. Requires massive Majorana ν's

0νββ-decay
(LNV at ≈GUT scale, exchange of three light ν)
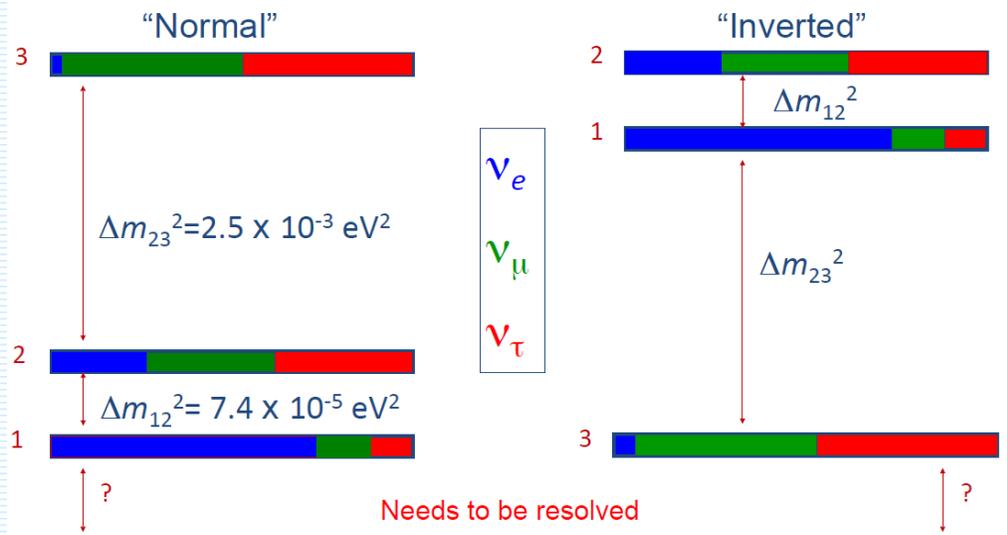
$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

?

⟵ **Phase space factor well understood**

↑ *NME must be evaluated using tools of nuclear theory*

Effective Majorana mass can be evaluated. It depends on **m_1, m_2, m_3, θ_{12}, θ_{13}, α_1, α_2 (3 unknown parameters: m_1/m_3, α_1, α_2 and ν-mass hierarchy)**

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$

"Normal"

"Inverted"

$\Delta m_{23}^2 = 2.5 \times 10^{-3}$ eV²

Δm_{12}^2

Δm_{23}^2

ν_e
ν_μ
ν_τ

$\Delta m_{12}^2 = 7.4 \times 10^{-5}$ eV²

? Needs to be resolved ?

$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Effective Majorana ν-mass $m_{\beta\beta}$ *(prediction due ν-oscillations)*

Constraint from cosmology

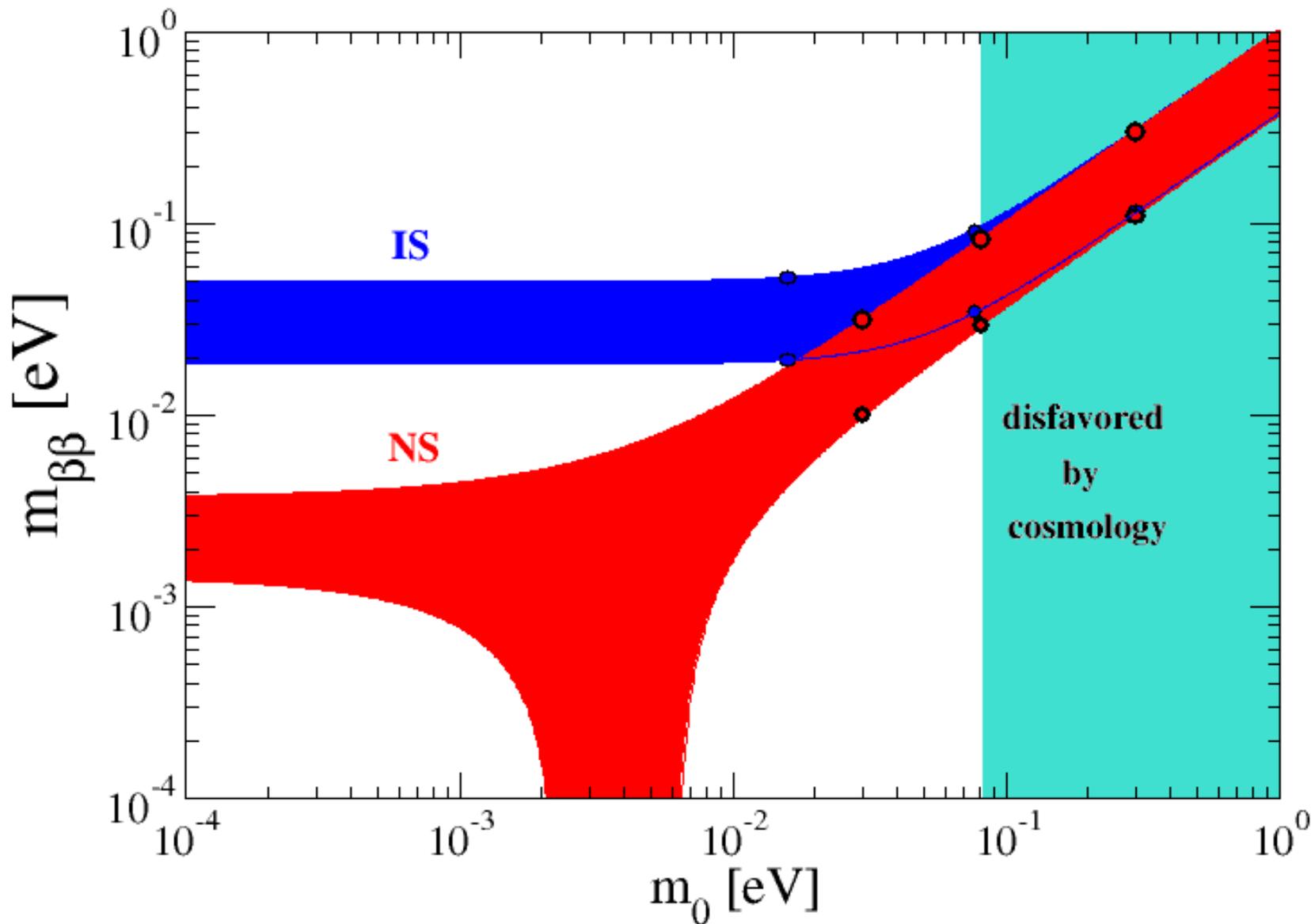$\Sigma = m_1 + m_2 + m_3$

  < 0.90 eV

  < 0.26 eV (Planck coll.)

  < 0.12 eV

Contrary, the constraint from $0\nu\beta\beta$-decay (KLZ)

  $m_{\beta\beta} < 0.036\text{-}0.156$ eV implies

   $\Sigma < 0.12$ eV

IS

NS

disfavored by cosmology

$m_{\beta\beta}$ [eV]

m_0 [eV]

$0\nu\beta\beta$ decay isotopes and experiments

【Current CANDLES detector】

Candidates	$Q_{\beta\beta}$(MeV)	N.A. (%)
$^{48}Ca \rightarrow ^{48}Ti$	4.268	0.187
$^{76}Ge \rightarrow ^{76}Se$	2.039	7.8
$^{82}Se \rightarrow ^{82}Kr$	2.998	8.8
$^{96}Zr \rightarrow ^{96}Mo$	3.356	2.8
$^{100}Mo \rightarrow ^{100}Ru$	3.034	9.7
$^{110}Pd \rightarrow ^{110}Cd$	2.017	11.7
$^{116}Cd \rightarrow ^{116}Sn$	2.813	7.5
$^{124}Sn \rightarrow ^{124}Te$	2.293	5.8
$^{130}Te \rightarrow ^{130}Xe$	2.528	34.1
$^{136}Xe \rightarrow ^{136}Ba$	2.458	8.9
$^{150}Nd \rightarrow ^{150}Sm$	3.371	5.6

CANDLE
CaF scintillating crystal

SuperNEMO
Se source foil

GERDA, MAJORANA
Ge crystal

CUPID-0
ZnSe scintillating crystal

m= 31 g   m= 505 g   m= 281 g

Aurora
CdWO$_4$ crystal

CUORE
TeO$_2$ crystal

Amore
CaMoO$_4$ crystal

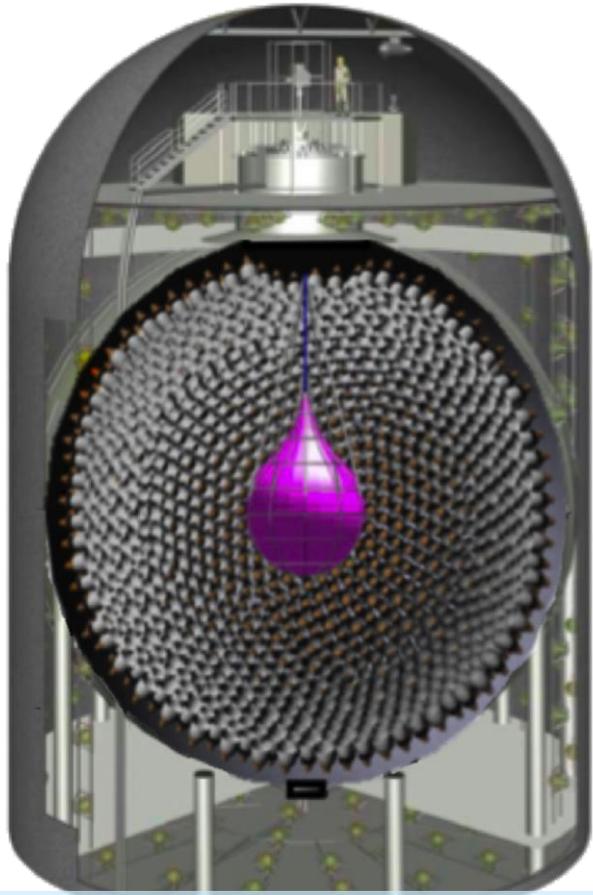**EXO, KamLAND-Zen**
Liquid Xe

A monoenergetic peak at the Q-value is searched for.
Need a large amount of decay isotope and low radioactive environment

Experiment	Isotope	Exposure [kg yr]	$T^{0\nu}_{1/2}[10^{25}$ yr]	$m_{\beta\beta}$ [meV]
Gerda	^{76}Ge	127.2	18	79-180
Majorana	^{76}Ge	26	2.7	200-433
CUPID-0	^{82}Se	5.29	0.47	276-570
NEMO3	^{100}Mo	34.3	0.15	620-1000
CUPID-Mo	^{100}Mo	2.71	0.18	280-490
Amore	^{100}Mo	111	0.095	1200-2100
CUORE	^{130}Te	1038.4	2.2	90-305
EXO-200	^{136}Xe	234.1	3.5	93-286
KamLAND-Zen	^{136}Xe	970	23	36-156

KamLAND-Zen 400 and KamLAND-Zen 800 combined results
Limit: $T^{0\nu}_{1/2} > 2.3 \times 10^{26}$ year (90%C.L.), $m_{\beta\beta} < 36$–156 meV
(g_A=1.27, NME = 1.11-4.77 are assumed)
Currently, the most strict $0\nu\beta\beta$ limit

Large volume liquid scintillator detector
LS: 30.5 m³, ^{136}Xe: 677 kg

7/26/2023

LEGEND-200

- **Builds on the past successes of the MAJORANA DEMONSTRATOR and GERDA**
- **Low-risk approach to meeting background and sensitivity goals**
- **LEGEND-200: start data taking in 2022**

LEGEND-1000 **is a next-generation Experiment aiming for unambiguous discovery of $0\nu\beta\beta$ with 10^{28} years of sensitivity targeting 10 years of exposure**

- **in Conceptual Design phase**
- **20x reduction to LEGEND-200 background goal**
- **Next-generation R&D efforts including Germanium Machine learning in progress**

^{76}Ge (91% enr.)
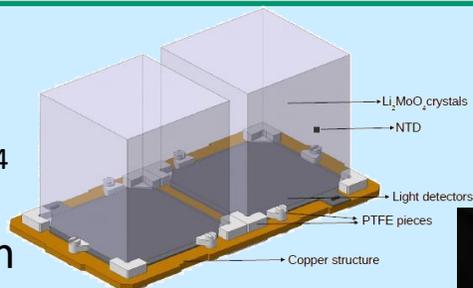
LEGEND 1000

Inverted Mass Ordering

LEGEND 200

MJD/GERDA

10 $m_{\beta\beta}^{min}$ range
— Background free
— 0.025 counts/FWHM-t-y
--- 0.1 counts/FWHM-t-y
-·- 1.0 count/FWHM-t-y
···· 10 counts/FWHM-t-y

CUORE + CUPID-Mo
→ CUPID: 1 ton-class ^{100}Mo 0νββ experiment

CUPID-Mo

CUPID-0

CUORE

→ CUPID → CUPID Reach / CUPID-1T

- Single module: $Li_2^{100}MoO_4$
  45×45x45 mm − ~280 g
- 57 towers of 14 floors with
  2 crystals each - **1596 crystals**
- **~240 kg of ^{100}Mo** with
  >95% enrichment
- **~1.6×10^{27} ^{100}Mo atoms**
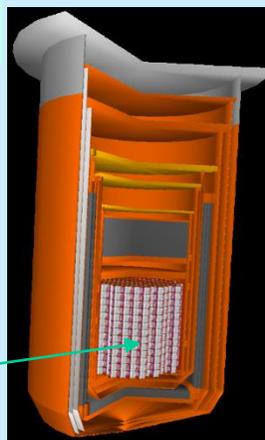- **Bolometric Ge light detectors**
  as in CUPID-Mo, CUPID-0

arXiv:1907.09376

J. Ouellet, TAUP 2021

prototype tower

Li$_2$MoO$_4$crystals
NTD
Light detectors
PTFE pieces
Copper structure

CUPID is built on successful CUPID-Mo + CUORE

Li$_2$MoO$_4$ scintillating bolometer technology, with demonstration of energy resolution, crystal radiopurity and α rejection

Ton-scale bolometric experiment is possible Electronics and data analysis tools **Reuse CUORE infrastructure**

CUPID sensitivity

Data driven background model
- Information from CUPID-Mo, CUPID-0
- CUORE background model (same infrastructure!)

Projected background index: **1×10^{-4} c/(keV kg y)**

Critical background component: random coincidence of 2νββ events (^{100}Mo fastest 2νββ emitter: $T_{1/2}$ = 7.1×10^{18} y)

10 y discovery sensitivity 1.1×10^{27}

$m_{\beta\beta}$ < 12 − 20 meV

Possible follow-up of CUPID

CUPID-reach - Same sensitive mass and cryostat as CUPID Background improvement by factor 5 **2.3 × 10^{27} y** → $m_{\beta\beta}$ < 7.9 − 14 meV
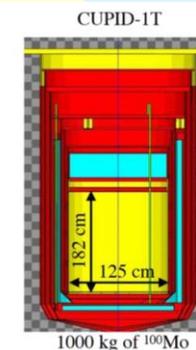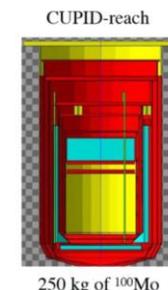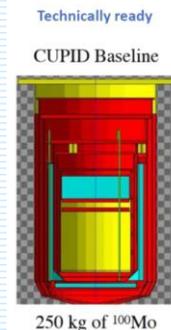
CUPID-1T - 1 ton isotope → new cryostat Background improvement by factor 20 **9.2 × 10^{27} y** → $m_{\beta\beta}$ < 4.0 − 6.9 meV

Surface events

Phased approach

Technically ready

CUPID Baseline

CUPID-reach

CUPID-1T

182 cm

125 cm

250 kg of ^{100}Mo

250 kg of ^{100}Mo

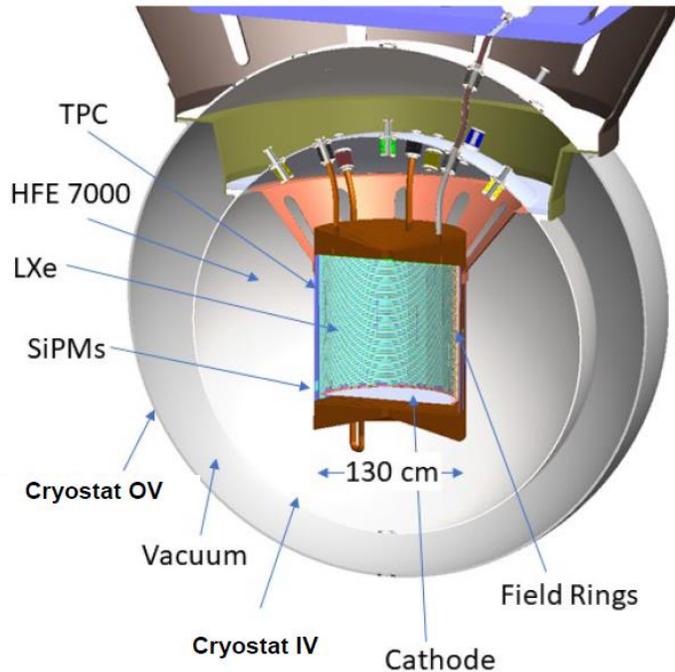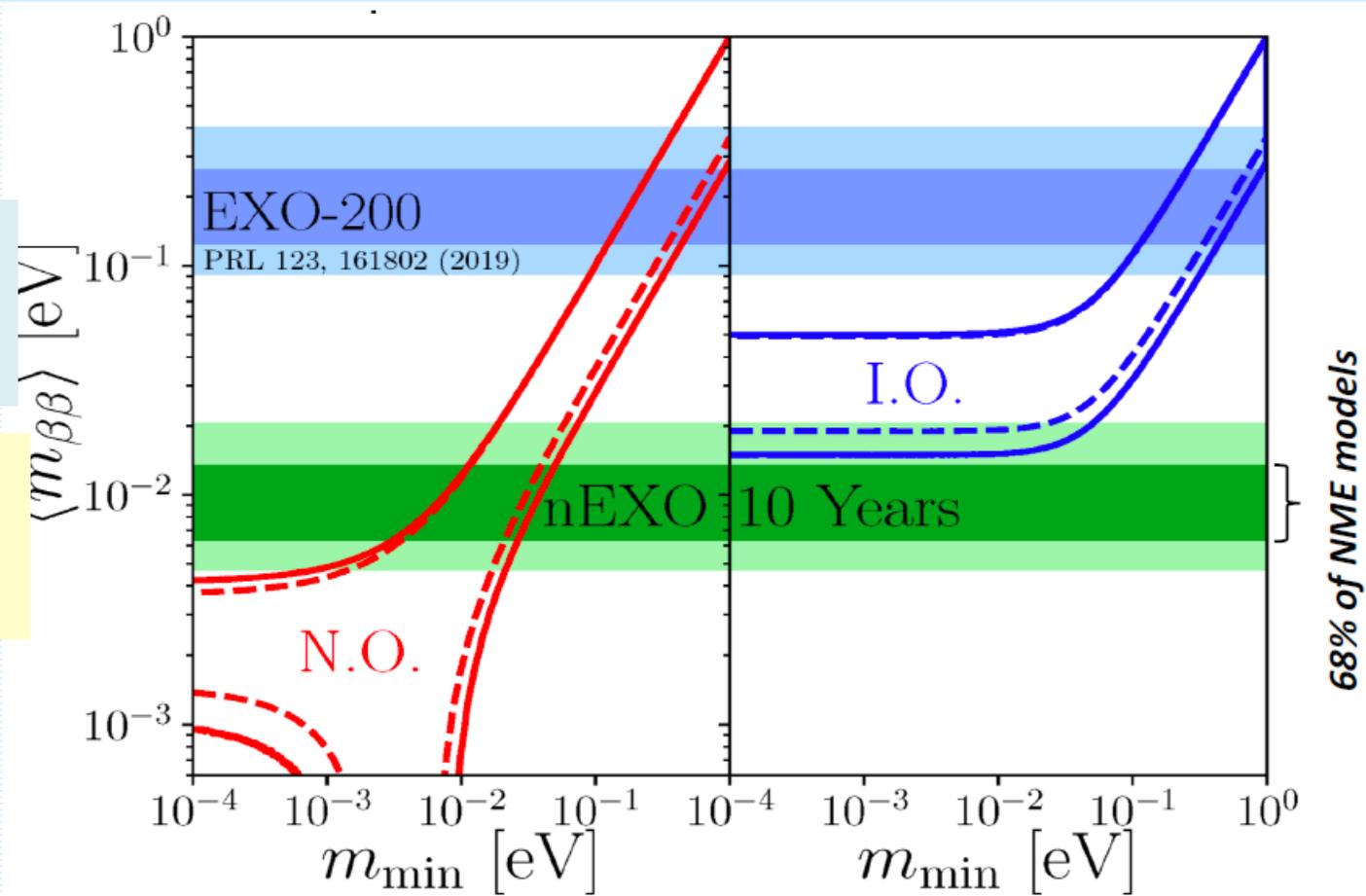1000 kg of ^{100}Mo

nEXO
5 ton-class ^{136}Xe $0\nu\beta\beta$ experiment

EXO-200, 1st 100 kg-class $0\nu\beta\beta$-experiment, excellent background-essential for nEXO design, Sensitivity increased linearly with exposure.

nEXO, discovery $0\nu\beta\beta$ experiment, reaches sensitivity of 10^{28} yr in **6.5 yr** data taking, probes $m_{\beta\beta}$ down to **15 meV**, scalable experiment.

	isotope	$m_{\beta\beta}$ [meV] 90% excl. sensitivity	$m_{\beta\beta}$ [meV] 3 σ discovery potential
Legend	^{76}Se	**8.2**	**11.1**
CUPID	^{100}Mo	**11.1**	**12.0**
nEXO	^{136}Xe	**12.9**	**15.0**

See-saws

A natural theoretical way to understand why 3 ν-masses are very small.

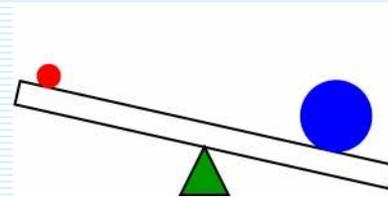| Planck	GUT	TeV LHC	Fermi	keV hot DM
10^{19} GeV	10^{16} GeV	10^3 GeV	10^{-6} GeV	10^{-6} GeV

Type-I Seesaw

Type-II Seesaw

Light ν mass $\approx (m_D/m_{LNV}) \, m_D$

Heavy ν mass $\approx m_{LNV}$

$$M_\nu \approx -v^2 Y_\nu \frac{1}{M_R} Y_\nu^T$$

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$

$$M_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

Type-I Seesaw: a right-handed Majorana neutrinos is added into the SM.

Type-II Seesaw: a few right-handed Majorana neutrinos and one Higgs triplet are both added into the SM.

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \left| \sum_N \left(U_{eN}^2 m_N \right) m_p M'^{0\nu}(m_N, g_A^{\text{eff}}) \right|^2$$

General case

light ν exchange

$$M'^{0\nu}(m_N, g_A^{\text{eff}}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x\, d^3y\, d^3p$$

$$M'^{0\nu}(m_N \to 0, g_A^{\text{eff}}) = \frac{1}{m_p m_e} M_\nu'^{0\nu}(g_A^{\text{eff}})$$

$$\times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+| J^{\mu\dagger}(\mathbf{x})|n\rangle \langle n| J_\mu^\dagger(\mathbf{y})|0_I^+\rangle}{\sqrt{p^2 + m_N^2}(\sqrt{p^2 + m_N^2} + E_n - \frac{E_I - E_F}{2})}$$

$$M'^{0\nu}(m_N \to \infty, g_A^{\text{eff}}) = \frac{1}{m_N^2} M_N'^{0\nu}(g_A^{\text{eff}})$$
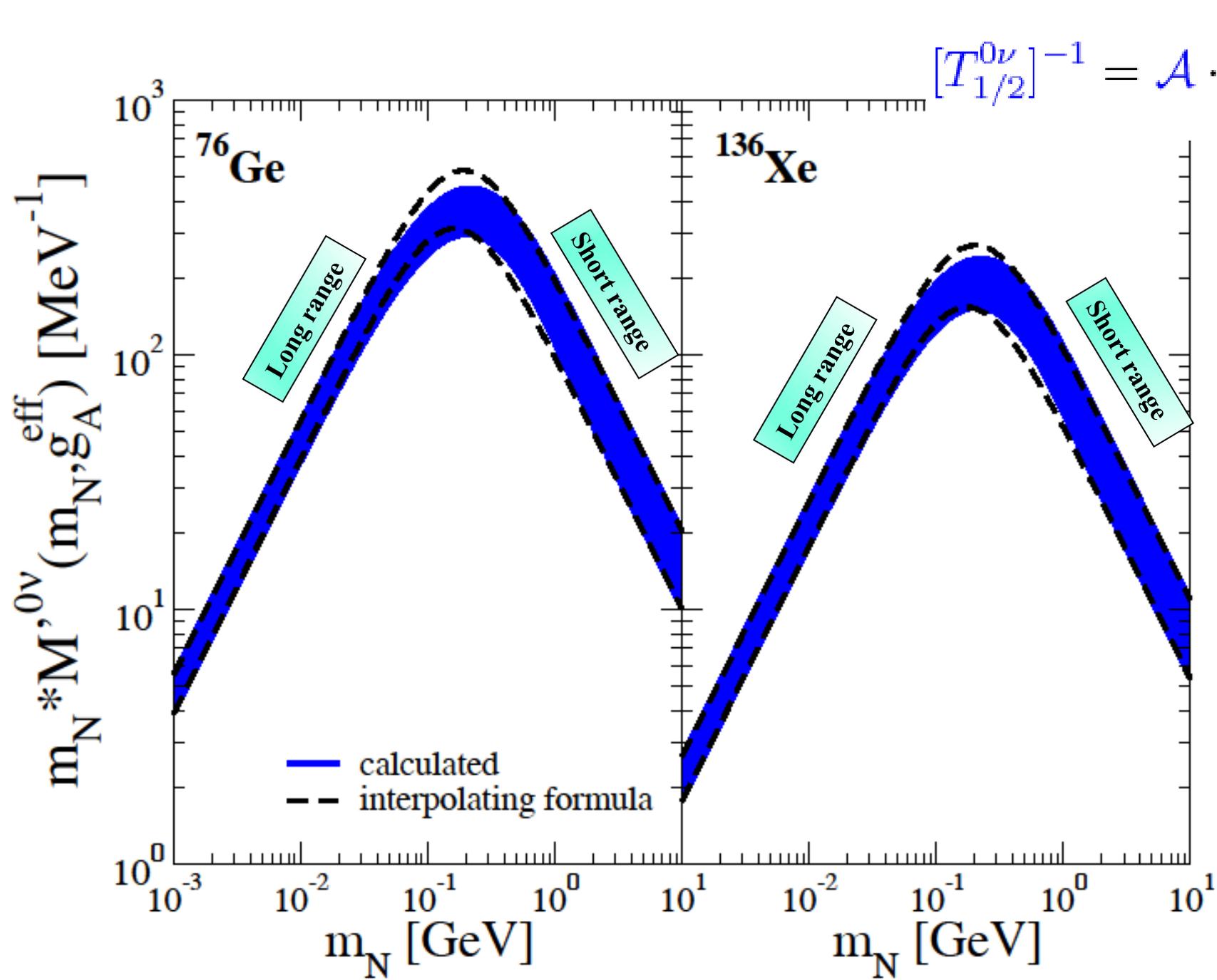
Particular cases

heavy ν exchange

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \times$$

$$\times \begin{cases} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M_\nu'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \ll p_F \\ \left| \langle \frac{1}{m_N} \rangle m_p \right|^2 \left| M_N'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \gg p_F \text{ or Simkovic} \end{cases}$$

$$\langle m_\nu \rangle = \sum_N U_{eN}^2 m_N$$

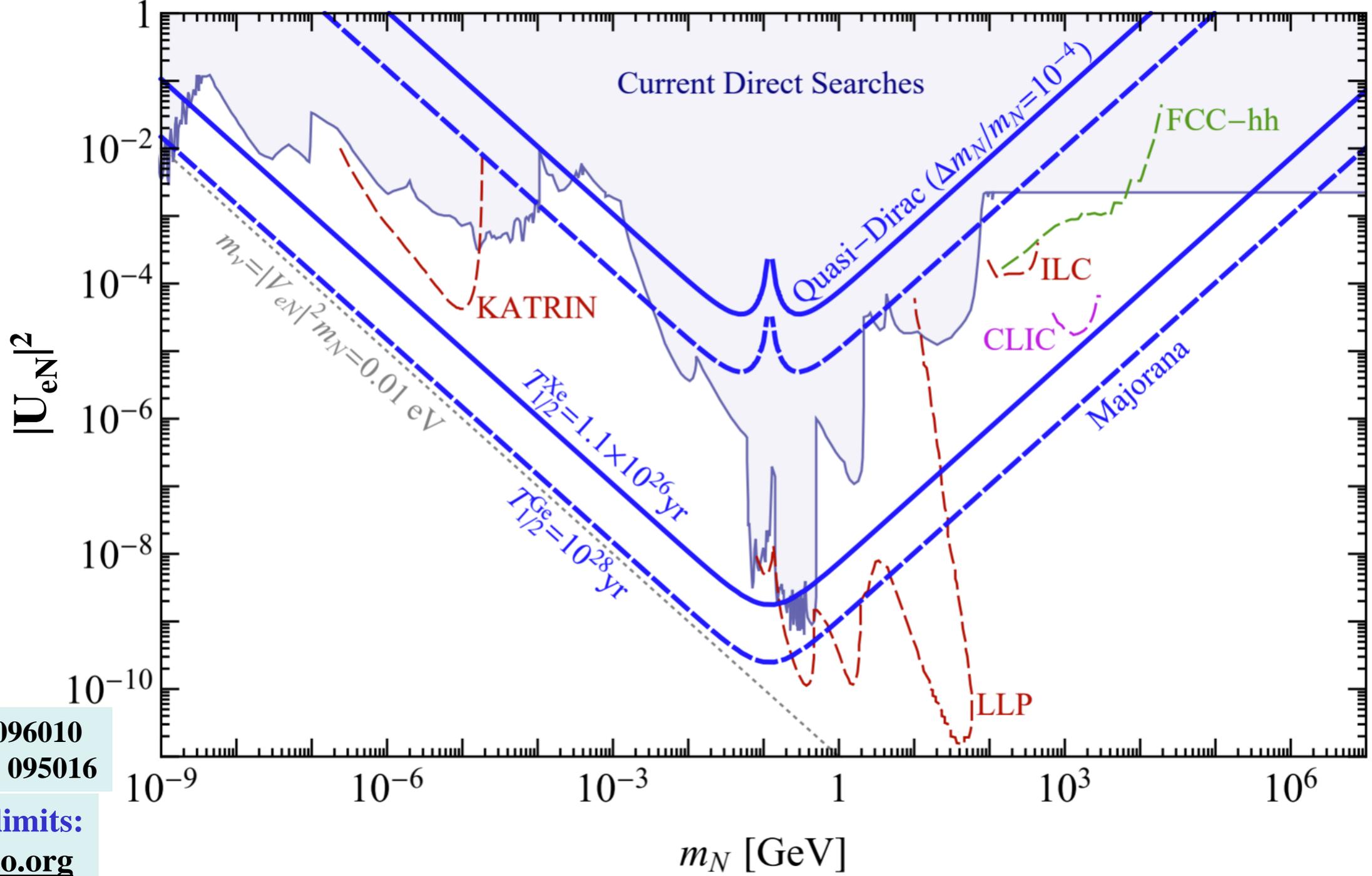$$\left\langle \frac{1}{m_N} \right\rangle = \sum_N \frac{U_{eN}^2}{m_N}$$

$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot \left| m_{\mathrm{P}} \sum_{\mathrm{N}} U_{e\mathrm{N}}^2 \frac{m_{\mathrm{N}}}{\langle p^2 \rangle + m_{\mathrm{N}}^2} \right|^2,$$

$$\langle p^2 \rangle = m_{\mathrm{p}} m_{\mathrm{e}} \frac{M_{\mathrm{N}}'^{0\nu}}{M_{\nu}'^{0\nu}}$$

Interpolating formula

28

Sterile ν (U_{en}, m_N)

Constraints from Direct Searches are less Stringent as those from 0νββ

PRD 90 (2014) 096010
PRD 102 (2020) 095016

Direct search limits:
sterile-neutrino.org

Current Direct Searches

Quasi-Dirac ($\Delta m_N / m_N = 10^{-4}$)

FCC-hh

KATRIN

ILC

CLIC

$m_\nu = |V_{eN}|^2 m_N = 0.01$ eV

$T^{Xe}_{1/2} = 1.1 \times 10^{26}$ yr

$T^{Ge}_{1/2} = 10^{28}$ yr

Majorana

LLP

$|U_{eN}|^2$

m_N [GeV]

$$[T^{0\nu}_{1/2}]^{-1} = \eta^2_{\nu N} \, C_{\nu N} \qquad C_{\nu N} = g^4_A \left| M'^{0\nu}_\nu \right|^2 G^{0\nu}$$

$$\nu_{eL} = \sum_{j=1}^{3} \left(U_{ej} \nu_{jL} + S_{ej}(N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left(T^*_{ej} (\nu_{jL})^C + V^*_{ej} N_{jR} \right)$$

Mixing of 3 light and 3 heavy neutrinos generally parametrized with 15 angles + 15 phases

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

Effective LNV parameter within LRS model (due interpolating formula)

$$\langle p^2 \rangle = m_p m_e \frac{M'^{0\nu}_N}{M'^{0\nu}_\nu}$$

$$\eta^2_{\nu N} = \left| \sum_{j=1}^{3} \left(U^2_{ej} \frac{m_j}{m_e} + S^2_{ej} \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j} \frac{M_j}{m_e} \right) \right|^2$$
$$+ \lambda^2 \left| \sum_{j=1}^{3} \left(T^2_{ej} \frac{m_j}{m_e} + V^2_{ej} \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M^2_j} \frac{M_j}{m_e} \right) \right|^2$$

The dominance of light and heavy ν-mass contributions to 0nbb-decay rate can not be established by observing this process at different nuclei.

7/26/2023

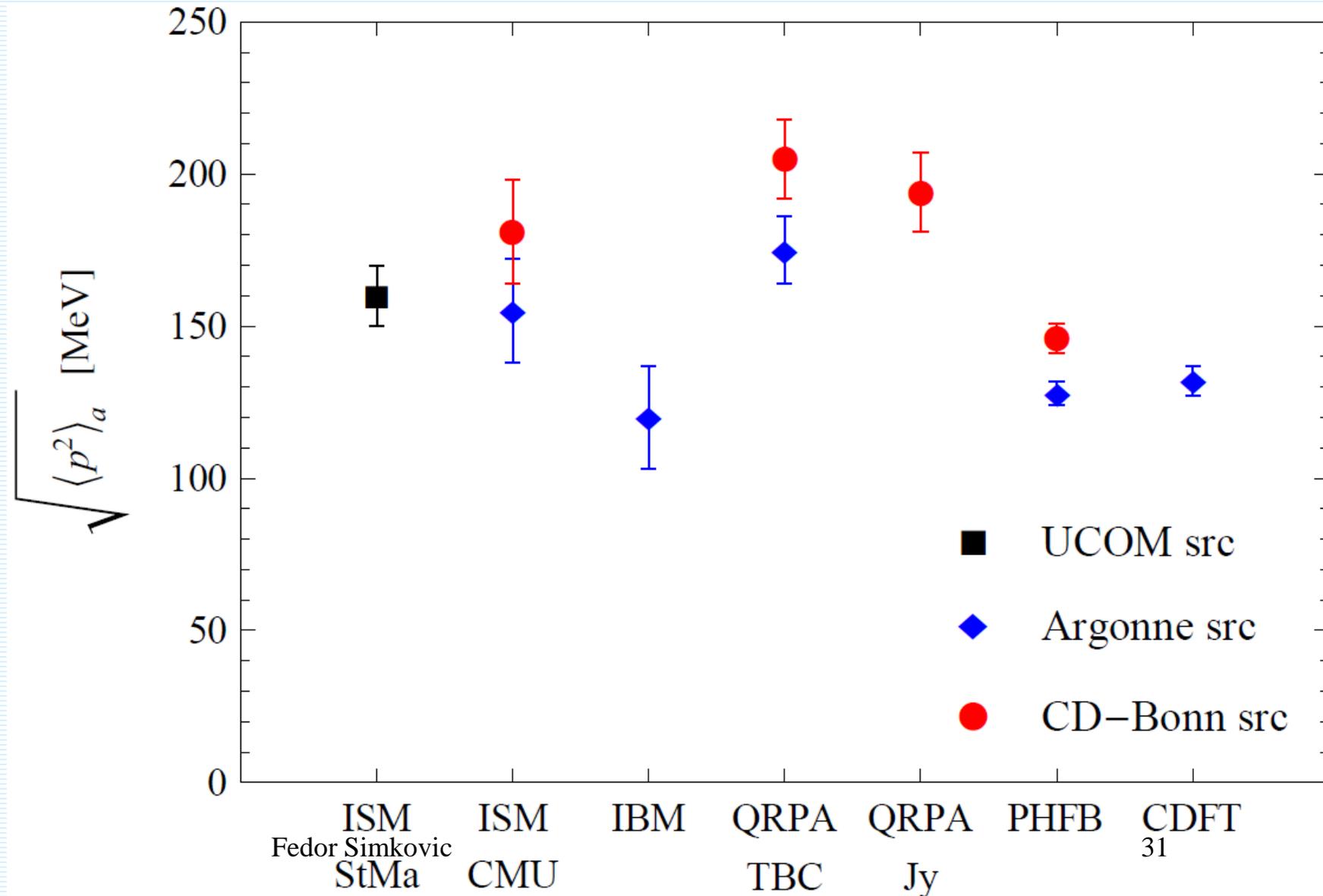**\<p²\> only weakly depends on A**

$$\langle p^2 \rangle = m_{\mathrm{p}} m_{\mathrm{e}} \frac{M_{\mathrm{N}}'^{0\nu}}{M_{\nu}'^{0\nu}}$$

Supported by detailed analysis:
Degeneracies of particle and nuclear physics uncertainties in 0νββ−decay
PRD 92, 093004 (2018)

As a consequence contributions to 0νββ decay rate from light and heavy neutrino mass mechanisms can not be distinquished.

PRD 98, 015003 (2018)

Fedor Simkovic

31

6x6 PMNS see-saw ν-mixing matrix
(the most economical one)

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\,1 \\ -\zeta\,1 & V_0 \end{pmatrix}$$

ζ - see-saw parameter

$U_0 = U_{\text{PMNS}}$ **PRD 98, 015003 (2018)**

$$\eta^2_{\nu N} = \frac{1}{m_e^2}\left(m_{\beta\beta}{}^2 + (M_{\beta\beta}^R)^2\right)$$

The mixing for heavy ν follows from the unitarity of U

$$V_0 = U_{\text{PMNS}}^\dagger =$$

$$\begin{pmatrix} c_{12}\,c_{13}\,e^{-i\alpha_1} & \left(-s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{-i\delta}\right)e^{-i\alpha_1} & \left(s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{-i\delta}\right)e^{-i\alpha_1} \\ s_{12}\,c_{13}\,e^{-i\alpha_2} & \left(c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{-i\delta}\right)e^{-i\alpha_2} & \left(-c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{-i\delta}\right)e^{-i\alpha_2} \\ s_{13}\,e^{i\delta} & c_{13}\,s_{23} & c_{13}\,c_{23} \end{pmatrix}$$

Assumption about heavy neutrino masses M_i (by assuming see-saw)

$$m_i\,M_i \simeq m_D^2$$

Inverse proportional

$$M_{\beta\beta}^R = \lambda\,\frac{\langle p^2\rangle_a}{m_D^2}\left|\sum_{j=1}^3 (U_0^\dagger)_{ej}^2\,m_j\right|$$

$M^R_{\beta\beta}$ depends on "Dirac" CP phase δ unlike "Majorana" CP phases α_1 and α_2

$$m_i \simeq \zeta^2 M_i$$

Proportional

$$M_{\beta\beta}^R = \lambda\,\zeta^2\left|\sum_{j=1}^3 (U_0^\dagger)_{ej}^2\,\frac{\langle p^2\rangle_a}{m_j}\right|$$

Heavy Majorana mass $M^R_{\beta\beta}$ depends on the "Dirac" CP violating phase δ

Contribution from exchange of heavy neutrino to 0νββ-decay rate might be large

Inverse proportional

$$m_i M_i \simeq m_D^2$$

$$V_0 = U_{PMNS}^\dagger$$

$$M_i = m_D^2/m_i \quad m_D \simeq 5 \text{ MeV}$$

$$\lambda = 7.7 \times 10^{-4}$$

$$M_{\beta\beta}^{\text{R}} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \, m_j \right|$$
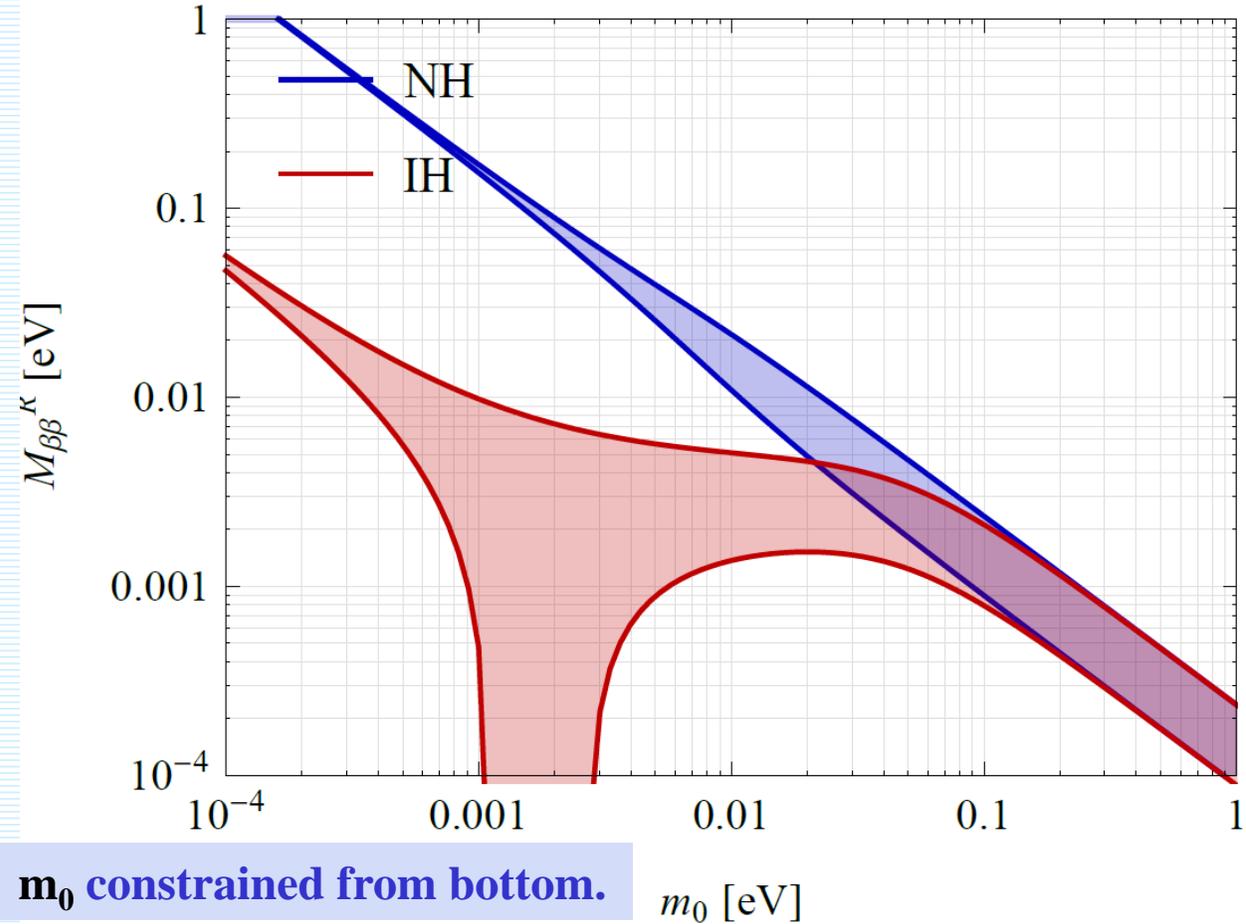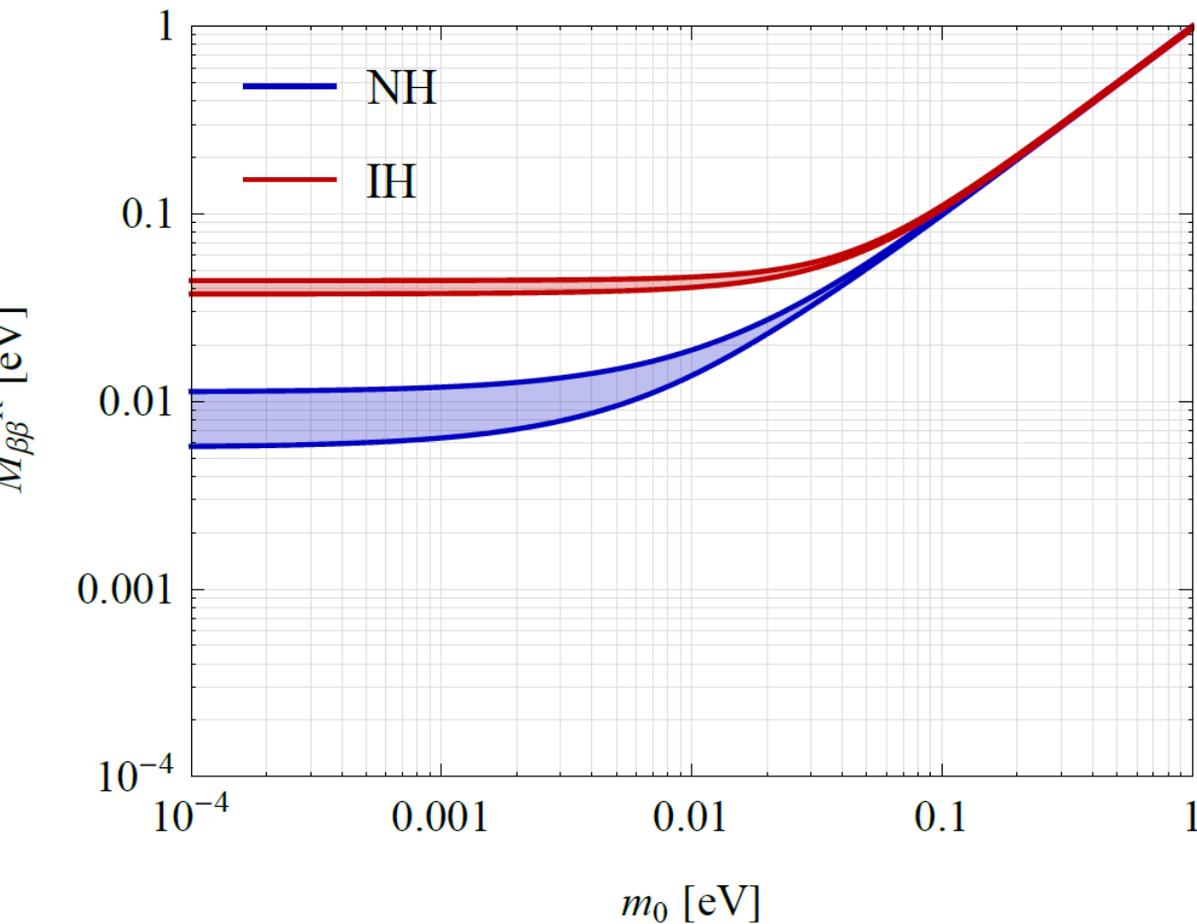
Proportional

$$m_i \simeq \zeta^2 M_i$$

$$V_0 = U_{PMNS}^\dagger$$

$$\zeta = m_i/M_i \quad \zeta^2 \simeq 5 \times 10^{-17}$$

$$\lambda = 7.7 \times 10^{-4}$$

$$M_{\beta\beta}^{\text{R}} = \lambda \zeta^2 \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \, \frac{\langle p^2 \rangle_a}{m_j} \right|$$

m₀ constrained from bottom.

0νββ governed by exotic mechanisms

Light ν-mass mechanism can be strongly suppressed: $m_{\beta\beta} < 1$ meV
- It is not possible to discover 0νββ with **10-100 ton-class** experiment
- It should be a **subject of theory** to justify it
- There might be a dominance of other 0νββ mechanisms

$m_{\beta\beta}$ mech. strongly suppressed
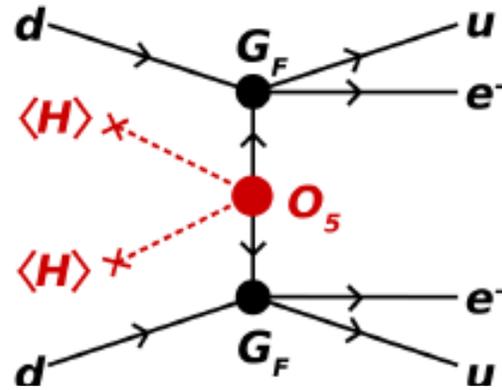
Any 0νββ mech. generates a small correction to ν-mass

Schechter, Valle: PRD 1982

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \boxed{\frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + O(\frac{1}{\Lambda^3})}$$

short range: d=9 (d=11)

Amplitude for (A,Z)→(A,Z+2)+2e⁻ can be divided into:

long range: d=7

+

mass mechanism: d=5

$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_i H)(L_j H)$$

Weinberg, 1979

$$\mathcal{O}_2 \propto LLLe^c H$$
$$\mathcal{O}_3 \propto LLQd^c H$$
$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^c H$$
$$\mathcal{O}_8 \propto L\bar{e}^c \bar{u}^c d^c H$$

**Babu, Leung: 2001
de Gouvea, Jenkins: 2007**

$$\mathcal{O}_5 \propto LLQd^c HHH^\dagger$$
$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^c HH^\dagger H$$
$$\mathcal{O}_7 \propto LQ\bar{e}^c \bar{Q}HHH^\dagger$$
$$\mathcal{O}_9 \propto LLLe^c Le^c$$
$$\mathcal{O}_{10} \propto LLLe^c Qd^c$$
$$\mathcal{O}_{11} \propto LLQd^c Qd^c$$
$$\ldots \ldots$$

Valle

Quark Condensate Seesaw Mechanism for Neutrino Mass

This operator contributes to the Majorana-neutrino mass matrix due to chiral symmetry breaking via the light-quark condensate.

The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}_{\alpha\beta}^{u,d}}{\Lambda^3} \, \overline{L_\alpha^C} \, L_\beta \, H \left\{ (\overline{Q}\, u_R), \, (\overline{d_R}\, Q) \right\}$$
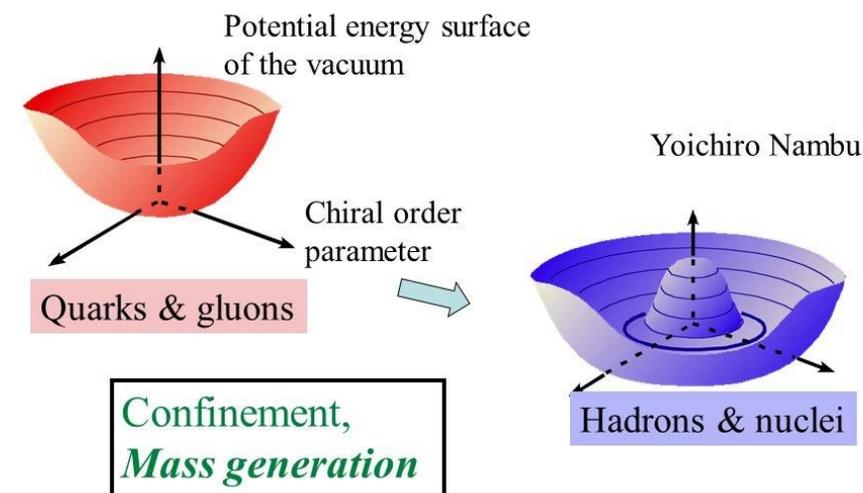
After the EWSB and ChSB one arrives at the Majorana mass matrix of active neutrinos

$$m_{\alpha\beta}^{\nu} = g_{\alpha\beta} \, v \, \frac{\langle \bar{q}q \rangle}{\Lambda^3}$$

$$= g_{\alpha\beta} \, v \left(\frac{\omega}{\Lambda} \right)^3$$

$$g_{\alpha\beta} = g_{\alpha\beta}^u + g_{\alpha\beta}^d, \quad v/\sqrt{2} = \langle H^0 \rangle$$

$$\omega = -\langle \bar{q}q \rangle^{1/3}, \quad \langle \bar{q}q \rangle^{1/3} \approx -283 \,\mathrm{MeV}$$

Spontaneous breaking of **chiral (χ) symmetry**

Potential energy surface of the vacuum

Chiral order parameter

Quarks & gluons

Confinement, *Mass generation*
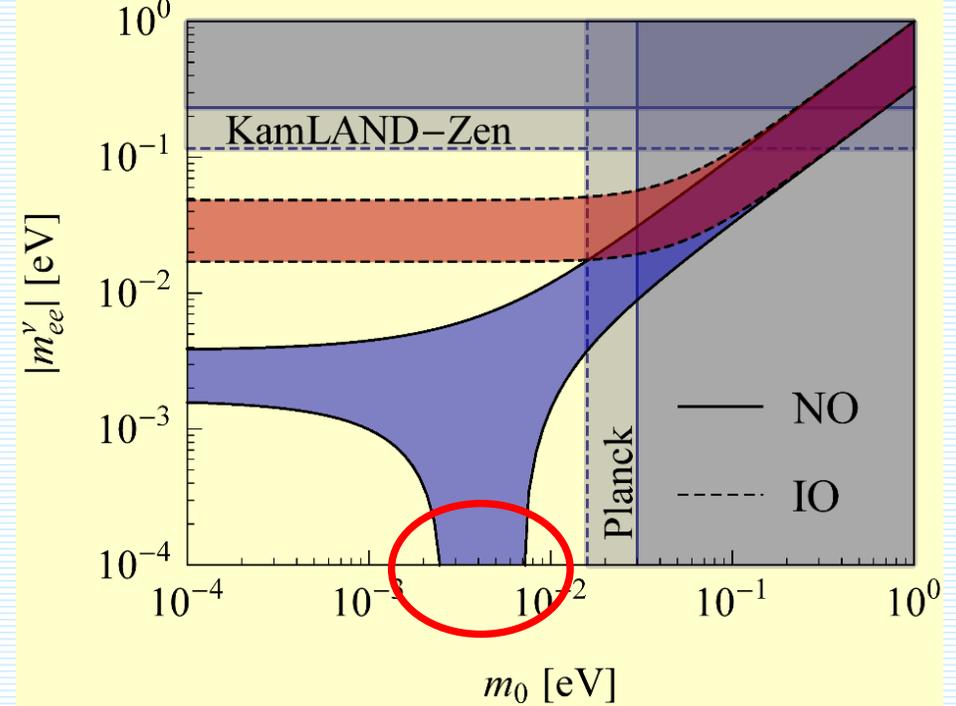
Yoichiro Nambu

Hadrons & nuclei

$\Lambda \sim$ **a few TeV** we get the neutrino mass in the **sub-eV** ballpark

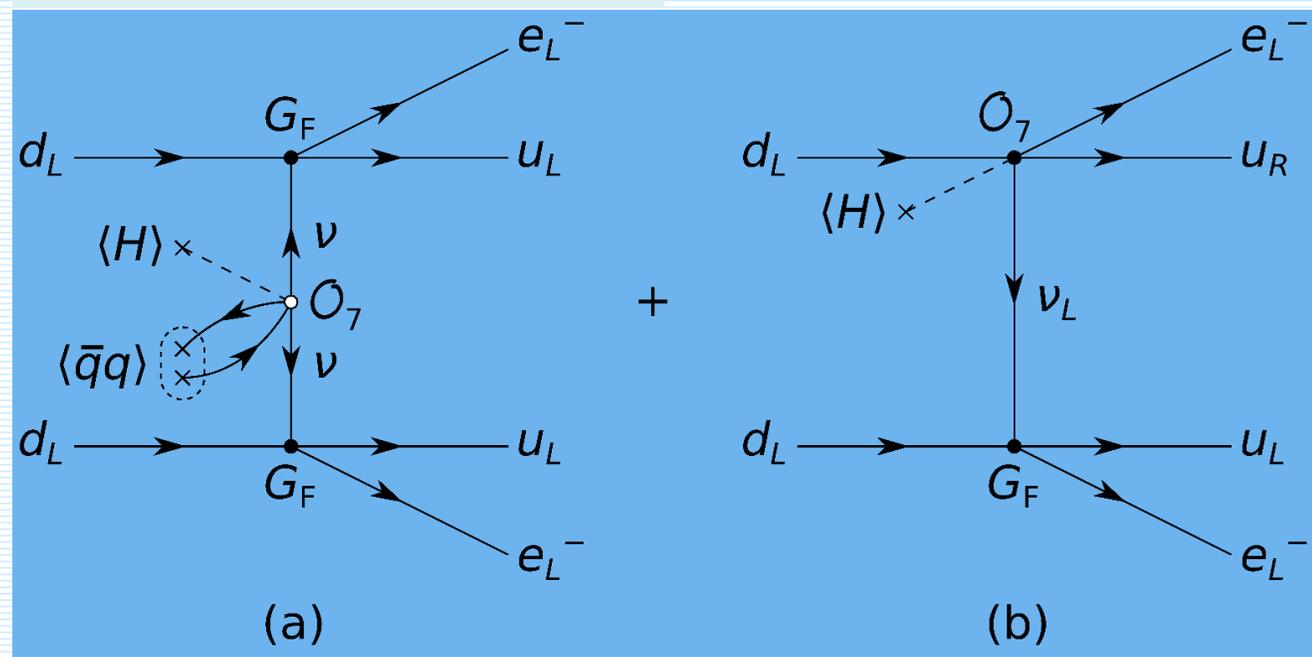# The genuine QCSS scenario
(predicts NH and ν-mass spectrum)

$$\mathcal{L}_7 = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \frac{v}{\Lambda^3} \overline{\nu^C_{\alpha L}} \nu_{\beta L} (g^u_{\alpha\beta} \overline{u_L} u_R + g^d_{\alpha\beta} \overline{d_R} d_L) + \text{H.c.}$$

$$m^\nu_{\alpha\beta} = -\frac{g_{\alpha\beta}}{\sqrt{2}} v \frac{\langle \bar{q}q \rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}} v \left(\frac{\omega}{\Lambda}\right)^3$$

(a) PRL 112, 142503 (2014).   **(b)** PLB 453, 194 (1999).

Neutrino spectrum (NH) !!!
2 meV < m$_1$ < 7 meV
9 meV < m$_2$ < 11 meV
50 meV < m$_2$ < 51 meV

Prediction for m$_\beta$
9 meV < m$_b$ < 12 meV

Prediction for cosmology (Σ)
62 meV < m$_1$ + m$_2$ + m$_3$ < 69 meV

Six Quasi-Dirac neutrinos and 0νββ-decay

Symmetry 12, 1310 (2020).

M_D - 3x3 complex matrix (18 real numb.)
$M_{L,R}$ - 3x3 symmetric matrix (12 real numb.)
(42 parameters)

Dirac-Majorana mass term

$$\mathcal{L}_m = \frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \mathcal{M} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

$|M_{L,R}| \ll |M_D|$

6 eigenvalues:
3 Dirac masses $m_{1,2,3}$, **3 mass splitting** $\epsilon_{1,2,3}$

$$\mathcal{U}^T \tilde{\mathcal{M}} \mathcal{U} = \mathcal{M}$$

$$m_i^{\pm} = \pm m_i + \epsilon_i$$

Diagonalization: 6x6 unitary mixing matrix
(15 mixing angles plus 15 phases)

$$\mathcal{U} = \mathcal{X} \cdot \mathcal{A} \cdot \mathcal{S}$$

$$\mathcal{A} \equiv \begin{pmatrix} U^T & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{X} = \begin{pmatrix} 1 & X^{\dagger} \\ -X & 1 \end{pmatrix} + O(X^2)$$

$$\mathcal{S} \equiv \begin{pmatrix} 1 & 0 \\ 0 & V^{\dagger} \end{pmatrix}$$

Product of 3 unitary matrices.
A and S mix exclusively active and sterile neutrino flavors, each given by 3 angles and 3 phases.

X given by 9 angles and 9 phases, small parameters.

Simplified Quasi-Dirac neutrino mixing scheme
(6x6 generalization of the PMNS matrix)

$$\mathcal{U}_{\mathrm{QD}} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

$$m_i^{\pm} = \pm m_i + \epsilon \quad (\epsilon > 0)$$

3 Dirac masses and 1 universal Majorana mass splitting ϵ

Oscillation probabilities among active neutrinos

$$
\begin{aligned}
P_{\alpha\beta} = \quad & \delta_{\alpha\beta} - \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 \sin^2 \frac{m_i \epsilon}{E} L - \sum_{i>j=1}^{3} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \left(\sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{4E} L \right. \\
& + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{4E} L \bigg) \\
& + \frac{1}{2} \sum_{i>j=1}^{3} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \left(\sin \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{2E} L \right. \\
& + \sin \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{2E} L \bigg)
\end{aligned}
$$

**The survival probability of
electron antineutrinos**

$$P_{\bar{\nu}_e \to \bar{\nu}_e}(\epsilon \neq 0) = P_{\bar{\nu}_e \to \bar{\nu}_e}(\epsilon = 0) - \frac{\epsilon^2 L^2}{E^2}\left[c_{13}^4 c_{12}^4 m_1^2 + c_{13}^4 s_{12}^4 m_2^2 + s_{13}^4 m_3^2\right]$$

$$- \frac{\epsilon^2 L^2}{4E^2}\left[4\, c_{13}^4 s_{12}^2 c_{12}^2 \Sigma m_{21}^2 \cos\frac{\Delta m_{21}^2 L}{2E} + 4 s_{13}^2 c_{13}^2 c_{12}^2 \Sigma m_{31}^2 \cos\frac{\Delta m_{31}^2 L}{2E}\right.$$

$$\left. + 4\, s_{13}^2 c_{13}^2 s_{12}^2 \Sigma m_{32}^2 \cos\frac{\Delta m_{32}^2 L}{2E}\right] + \mathcal{O}(\epsilon^4),$$

Tritium β-decay

$$m_\beta = \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 c_{13}^2 s_{12}^2 + m_3^2 s_{13}^2 + \epsilon^2}$$

$$= m_\beta^{(0)}\left(1 + \frac{1}{2}\left(\epsilon/m_\beta^{(0)}\right)^2 + \dots\right)$$

Cosmology

$$\frac{1}{2}\sum_{i=1}^{6}\left|\tilde{\mathcal{M}}_{ii}\right| = \sum_{i=1}^{3} m_i$$

**Restriction from
Daya-Bay data (3σ):**

**Survival probabilities
with non-zero ε are
the same 3ν cases.**

0νββ-decay

$$m_{\beta\beta} = \left[M_L\right]_{ee}$$

$$= \epsilon\left|c_{12}^2 c_{13}^2 + e^{2i\alpha_{21}} c_{13}^2 s_{12}^2 + e^{2i\alpha_{31}} s_{13}^2\right|$$

Ior Simkovic

$$m_{\beta\beta} \lesssim 30 \text{ meV} \quad \text{for} \quad \text{NO}$$
$$\lesssim 1 \text{ meV} \quad \text{for} \quad \text{IO}$$

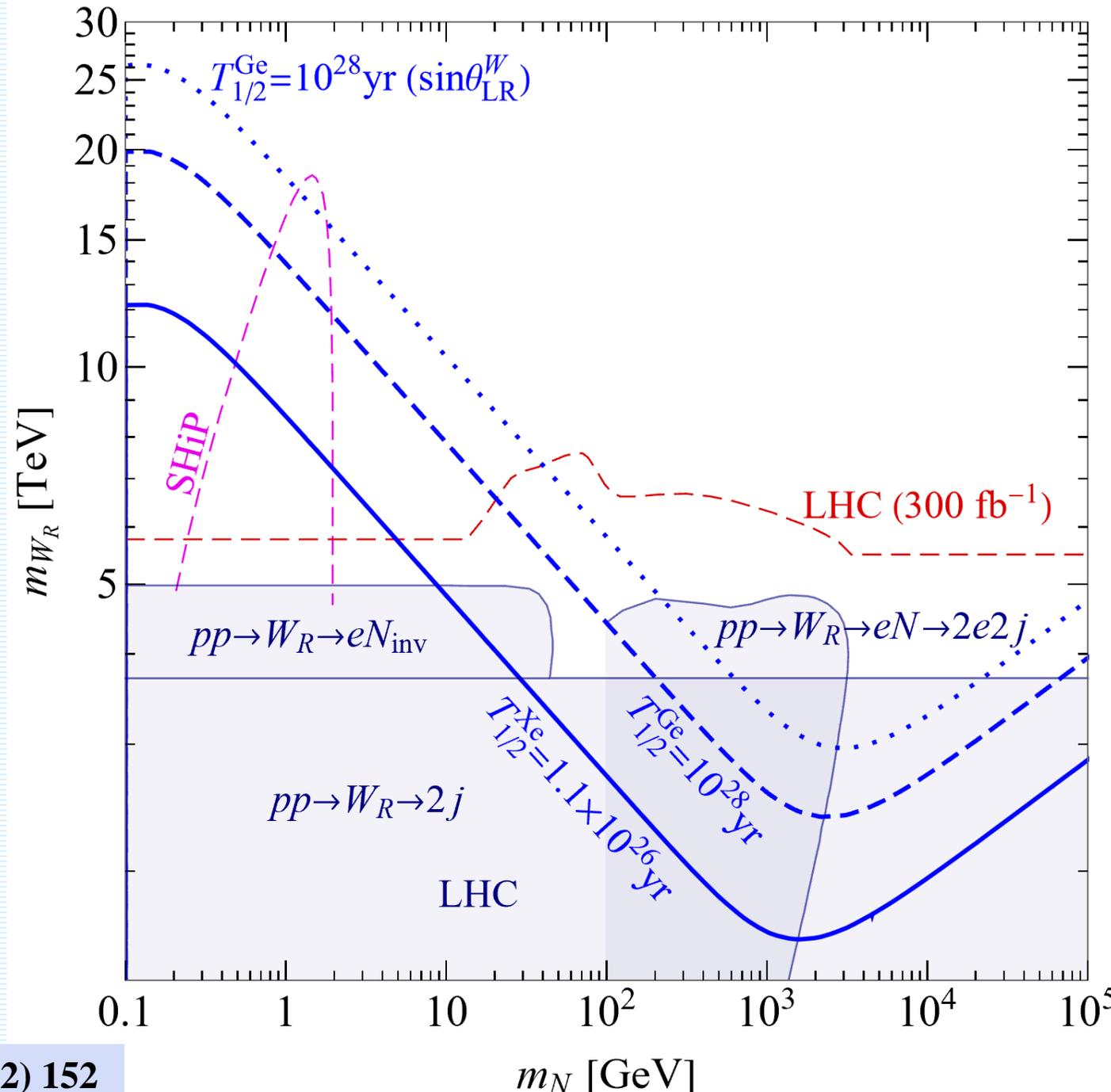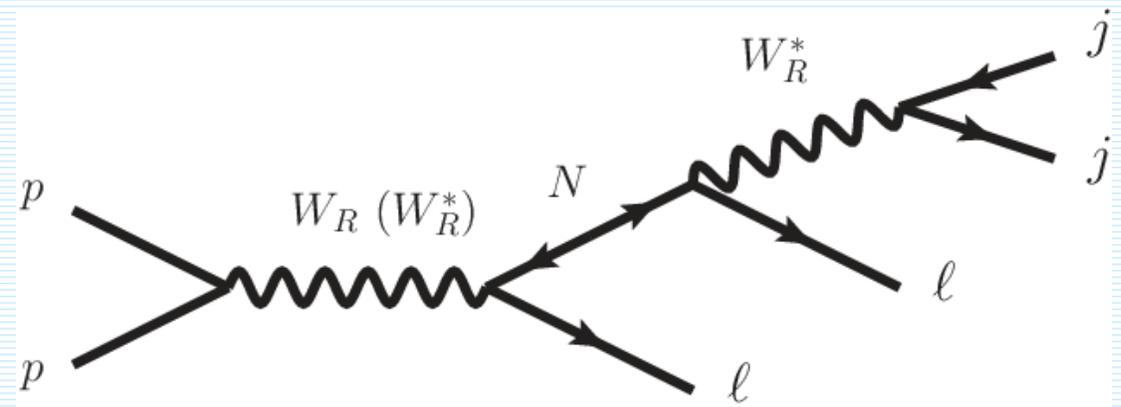# Quasi-Dirac neutrino oscillations at different distances

Direct probe of Majorana nature of heavy neutrino (or HNL) at LHC

Keung-Senjanovic processes:

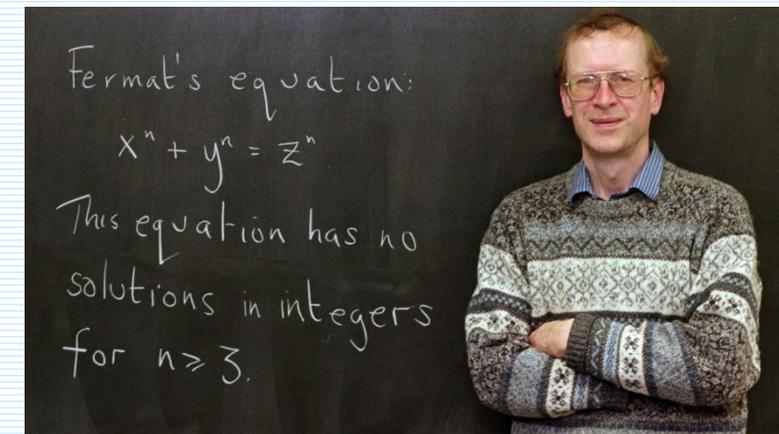$pp \rightarrow W_R \rightarrow eN \rightarrow 2e2j$
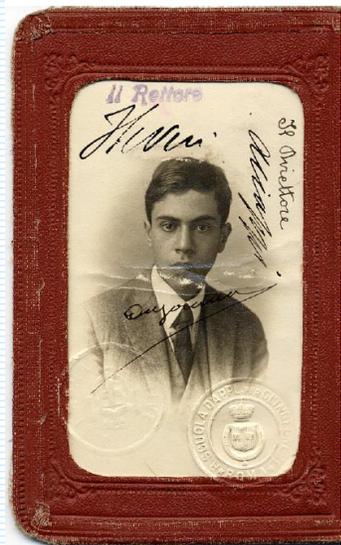or
$pp \rightarrow W_R \rightarrow \mu N \rightarrow 2\mu2j$

JHEP **03** (2022) 152

PIERRE DE FERMAT 1601-1665 RF
LA POSTE 2001

$x^n + y^n = z^n$
$x^n + y^n = z^n$

4,50 F
0,69 €

n'a pas de solution pour des entiers n > 2

ITVF LAVERGNE

Around 1637, Pierre de Fermat wrote in the margin of a book that the more general equation

$$a^n + b^n = c^n$$

had no solutions in positive integers if n is an integer greater than 2.

Fermat's equation:
$x^n + y^n = z^n$
This equation has no solutions in integers for n ≥ 3.

After 358 years

The proof was published by Andrew Wiles in 1995.

Some long-standing tasks of humanity …

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

NICOLA ZANICHELLI EDITORE

© Erasmo Recami & M.M.
Ogni riproduzione totale o parziale è assolutamente vietata

1937

After 85 years

n-ton-class 0νββ exp. with discovery potential
KamLAND-Zen 800
SNO+
LEGEND
nEXO
NEXT
CUPID
etc

After ? years

If $m_{\beta\beta} < 1$ meV, what technology is needed for observation of 0νββ?

43

0νββ decay NMEs

Before: 2004 (factor 10 uncertainty)
few groups, 2 nuclear structure methods:
Nuclear Shell Model, QRPA

Present: 2022 (factor 2-3)
Recognized priority task in nuclear physics
many groups, many nuclear structure methods:
Nuclear Shell Model, QRPA, PHFB, IBM, EDF

Attempts (light nuclear systems):
Ab initio calculations by different approaches – No Core Shell Model, Green's Function Monte Carlo, Coupled Cluster Method, Lattice QCD etc

Calculated values
| Nuclear matrix |^2
^{76}Ge

N_{CALC}

F_N (yr^{-1})

PRC 70, 033012 (2004)

Additional problems:
+ Effective weak coupling constant
$g^{eff}_A \approx 0.4 - 1.27$ (factor ≈ 3)
+ contact term (factor ≈ 2)

Evaluation of the 0νββ-decay NMEs calculation - Approximations needed

Nuclear Shell Model (**Madrid-Strasbourg, Michigan, Tokyo**): **Relatively small model space (1 shell), all correlations included, solved by direct diagonalization**

QRPA (**Tuebingen-Bratislava-Calltech, Jyvaskyla, Chapel Hill, Lanzhou**): **Several major shells, only simple correlations included**

Interacting Boson Method (**Yale-Concepcion**): **Small space, important proton-neutron Pairing correlations missing**

Projected Hartree-Fock-Bogoliubov Method (**Lucknow**): **Several major shells, missing GT proton-neutron residual interaction.**

Energy Density Functional theory (**Madrid, Beijing**): **>10 shells, important proton-neutron pairing missing**

An initio approaches:

The nuclear w. f. of (A,Z), (A,Z+1)*, (A,Z+2) Many-body methods of choice:

The 0νββ nuclear transition operators (F, GT, and tensor type):

+ **Transiton operators involving complete set of states of intermediate nucleus**

+ **Transition operators in closure approximation – just two-body operators**

+ **Chiral effective field theory two-body transition operators with contact term**

+

! Isospin, and spin-isospin symmetries ($M_{Fcl} \approx 0$, M_{GTcl} strongly suppressed):

Initial 0^+ g.s.: (T, T)
Final 0^+ g.s.: (T-2, T-2) $\Rightarrow$ $\Delta T = 2$ (!)

Till now, this issue addressed only within the QRPA PRC 98, 064325 (2018)

0νββ-decay NME status 2022

All models missing essential physics

Impossible to assign rigorous uncertainties

Differencies:
- **Many-body approxim.**
- **Size of the m.s.**
- **Residual interactions**

unquenched g_A

Assuming that the 0νββ process is mediated by a light-Majorana-neutrino exchange, a systematic analysis in chiral effective field theory shows that already at leading order **a contact operator** is required **to ensure renormalizability** of the amplitude for **nn -> pp + ee process.** Without the strong $^1S_0$ short range interaction (which appears universally in all nuclear potentials) there would be no need of contact term.

$$M^{0\nu} = \langle \Psi_f | O^{0\nu} | \Psi_i \rangle$$

$$V_\alpha^{0\nu}(r_{ab}) = \frac{1}{g_A^2} \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}_{ab}} V_\alpha^{0\nu}(\mathbf{q}^2)$$

Nuclear matrix element

$$O_F^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_F^{0\nu}(r_{ab}) \tau_a^+ \tau_b^+ ,$$

ν-potential (< E$_{aver.}$> = 0).

$$V_\alpha^{0\nu}(\mathbf{q}^2) = \frac{1}{\mathbf{q}^2} v_\alpha(\mathbf{q}^2)$$

$$O_{GT}^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_{GT}^{0\nu}(r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+ ,$$

Standard type contrib.

regularized with a dipole form-factors

$$v_F(\mathbf{q}^2) = -g_V^2(\mathbf{q}^2),$$

$$O_T^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_T^{0\nu}(r_{ab}) S_{ab} \tau_a^+ \tau_b^+ .$$

$$v_{GT}(\mathbf{q}^2) = g_A^2(\mathbf{q}^2) + \frac{2}{3}\frac{\mathbf{q}^2}{2m_N} g_A(\mathbf{q}^2) g_P(\mathbf{q}^2)$$

$$+ \frac{1}{3}\frac{\mathbf{q}^4}{4m_N^2} g_P^2(\mathbf{q}^2) + \frac{2}{3}\frac{\mathbf{q}^2}{4m_N^2} g_M^2(\mathbf{q}^2),$$

Contact term contrib.

$$O_S^{0\nu} = (4\pi R_A) \sum_{a \neq b} V_S^{0\nu}(r_{ab}) \tau_a^+ \tau_b^+$$

$$V_S^{0\nu}(r_{ab}) = 2\frac{g_\nu^{NN}}{g_A^2} \delta_R^{(3)}(\mathbf{r}_{ab}),$$

PPNP 112, 1037 (2020)

$$v_T(\mathbf{q}^2) = -\frac{2}{3}\frac{\mathbf{q}^2}{2m_N} g_A(\mathbf{q}^2) g_P(\mathbf{q}^2) - \frac{1}{3}\frac{\mathbf{q}^4}{4m_N^2} g_P^2(\mathbf{q}^2)$$

$$+ \frac{1}{3}\frac{\mathbf{q}^2}{4m_N^2} g_M^2(\mathbf{q}^2),$$

Magnitude of the contact term 0νββ-decay NME

Nucleus	QRPA $M_S^{0\nu}/M_L^{0\nu}$ (%)	NSM $M_S^{0\nu}/M_L^{0\nu}$ (%)
^{48}Ca		$23-62$
^{76}Ge	$32-73$	$15-42$
^{82}Se	$30-70$	$15-41$
^{96}Zr	$29-69$	
^{100}Mo	$49-108$	
^{116}Cd	$26-61$	
^{124}Sn	$36-81$	$17-46$
^{128}Te	$35-77$	$17-46$
^{130}Te	$34-77$	$17-47$
^{136}Xe	$30-70$	$17-47$

$$[t_{1/2}^{0\nu}]^{-1} = G_{0\nu}\, g_A^4\, |M_L^{0\nu} + M_S^{0\nu}|^2 \frac{m_{\beta\beta}^2}{m_e^2}$$

Long-range
$$M_L^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

Short-range
$$M_S^{0\nu} = \frac{2R}{\pi g_A^2} \langle 0_f^+ | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr) h_S(q^2)\, q^2 dq |0_i^+\rangle$$

$$h_S(q^2) = -2 g_\nu^{NN} e^{-q^2/(2\Lambda^2)}$$

**Regularized with Gaussian
(maybe better with dipole form-f. like weak magn. contr.)**

Open urgent questions:
- **A correspondence** of the standard and the chiral field theory formalisms. Is contact term involved in the standard mechanism (completeness …).
- What is the **magnitude** of the contact term NME? **Can it be large?** Justification with other phenomenology needed – pion and heavy-ion DCX, etc.

Supporting nuclear physics experiments
(Measurements still not conclusive for $0\nu\beta\beta$ NME)

- ✓ β-decay, EC and $2\nu\beta\beta$ decay
- ✓ μ-capture
- ✓ (π^+, π^-), single charge exchange
- ✓ $(^3\text{He,t}), (\text{d},^2\text{He})$, transfer reactions
- ✓ γ-ray spectroscopy, $\gamma\gamma$-decay
- ✓ A promising experimental tool:
  Heavy-Ion Double Charge-Exchange

Multipole decomposition of light and heavy $0\nu\beta\beta$-decay NMEs normalized to unity

Higher multiploles are populated mostly due large ν-momenta transfer

Fedor Simkovic

Understanding of the 2νββ-decay NMEs is of crucial importance for correct evaluation of the 0νββ-decay NMEs

There is no reliable calculation of the *2νββ-decay* NMEs yet

$$M_{GT}^{2\nu} = \sum_m \frac{<0_f^+||\tau^+\sigma||1_m^+><1_m^+||\tau^+\sigma||0_i^+>}{E_m - E_i + \Delta}$$

HSD, higher states dominance

1⁺

^{100}Tc

SSD, 1⁺ ground level dominance

J.Abad et.al. Ann.Fis.A80,9(1984)

0⁺

^{100}Mo

0⁺ ^{100}Ru

The spread of 2νββ and 0νββ NMEs is large and small, respectively.
Reasons:
- **2νββ NMEs governed by contribution from single (1⁺) multipole unlike 0νββ NMEs by contributions from all multipoles (Jᵖ) of int. nucl.**
- **Neutrino potential vs 2νββ potential**

7/26/2

Both 2νββ and 0νββ operators connect the same states. Both change two neutrons into two protons. Explaining 2νββ-decay is necessary but not sufficient

50

NEMO-3 ^{100}Mo, Esum > 1.4 MeV — HSD χ^2/ndf= 55.86

NEMO-3 ^{100}Mo, Esum > 1.4 MeV — SSD χ^2/ndf= 1.45

Looking for SSD/HSD effect

SSD favored
$\Rightarrow$
Strong trans. through low lying states of (A,Z+1);
$M_F^{2\nu} \approx 0$

NEMO3 Collaboration, Eur. Phys. J. C79, 440 (2019)

HSD, higher levels contribute to the decay

1^+
^{100}Tc

SSD, 1^+ level dominates in the decay
(Abad et al., 1984, Ann. Fis. A 80, 9)

0^+
^{100}Mo

7/26/2023   Fedor Šimkovic

CUPID-0 Coll., PRL 123, 262501 (2019)

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

$$\left[T_{1/2}^{2\nu\beta\beta} \right]^{-1} \simeq \left(g_A^{\text{eff}} \right)^4 \left| M_{GT-3}^{2\nu} \right|^2 \frac{1}{\left| \xi_{13}^{2\nu} \right|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu} \right)$$

$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)} \qquad \xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4\, m_e^3}{(E_n - (E_i + E_f)/2)^3} \qquad \xi_{15}^{2\nu} = \frac{M_{GT-5}^{2\nu}}{M_{GT-1}^{2\nu}}$$

CD-Bonn (solid), Argonne (dashed)

Left panel: $(d\Gamma^{2\nu}/dE_{e_1})/\Gamma^{2\nu}$ [MeV^{-1}] versus $E_{e_1} - m_e$ [MeV]

- $\xi_{31} = 0.536,\ \xi_{51} = 0.197$ (QRPA)
- $\xi_{31} = 0.368,\ \xi_{51} = 0.135$ (SSD)
- $\xi_{31} = 0.,\ \xi_{51} = 0.$ (HSD)

residual: (QRPA) − (HSD), (SSD) − (HSD)

Right panel: $(d\Gamma^{2\nu}/d(E_{e_1} + E_{e_2}))/\Gamma^{2\nu}$ [MeV^{-1}] versus $E_{e_1} + E_{e_2} - 2m_e$ [MeV]

residual: (QRPA) − (HSD), (SSD) − (HSD)

Ab initio β-decay study: g_A is unquenched
(light nuclear systems)

Discrepancy between experimental and theoretical β-decay rates resolved from first principles

P. Gysbers[1,2], G. Hagen[3,4*], J. D. Holt[1], G. R. Jansen[3,5], T. D. Morris[3,4,6], P. Navráti[...]
S. Quaglioni[7], A. Schwenk[8,9,10], S. R. Stroberg[1,11,12] and K. A. Wendt[7]

Once meson-exchange currents and 3-body forces are considered there is no need for any "quenching"

Measurement of GT strength via μ-capture
$$\mu^-_b + (A,Z) \rightarrow (A,Z-1) + \nu_\mu$$

Contradicting results:
- **Strong quenching ($g_A \approx 0.6$)**

PRC 100, 014619 (2019)

- **Weak quenching ($g_A \approx 1.1$)**

PRC 74, 024326 (2006)

PRC 79, 054323 (2009)

J-PARC 3-50 GeV p, ν, μ

I. Hashim H. Ejiri , MXG16, PR C 97 2018

Momentum transfer q~80 MeV

⇒ **Small basis nuclear structure calculations (NSM, IBM) are disfavored.** ⇒

Fedor S

Muon capture rates evaluated within QRPA
In agreement with soft quenching ($g_A \approx 1.1$)

Multipole decomposition of B^μ_{GT} and $M^{0\nu}_{GT}$

PRC 102, 034301 (2020).

7/26/2023

Experiment Monument at PSI
will study contributions from all multipoles

Heavy-ion DCE as surrogate processes of ββ-decay

Double GT Giant resonances (exhausts a major part of sum-rule strength)

ββ-decay

E_x in grand-daughter nucleus

- ✓ **Induced by strong interaction**
- ✓ **Sequential nucleon transfer mechanism 4th order: Kinematical matching**
- ✓ **Meson exchange mechanism 1st or 2nd order**
- ✓ **Possibility to go in both directions**
- ✓ **Low cross section**

Tiny amount of DGT strength for low lying states

NUMEN erc NURE

RIKEN

RCNP

Future: INFN-LNS

Sum rule almost exhausted by DGT Giant Mode, still not observed

$(^{18}O, ^{16}O)$

| ^{76}Se | ^{77}Se | ^{78}Se |

$(^{18}O, ^{18}F)$

$(\pi^+, \pi^-), (^{20}Ne, ^{20}O), ββ$ decay

$(^{20}Ne, ^{18}O)$

^{76}As

$(^{18}O, ^{20}Ne)$

^{75}As

$(^{18}O, ^{18}Ne)$

$(^{20}Ne, ^{20}F)$

^{77}As

^{74}Ge | ^{75}Ge | ^{76}Ge

$(^{20}Ne, ^{22}Ne)$

The ^{130}Te(^{20}Ne,^{20}O)^{130}Xe DCE reaction

- g.s. → g.s. transition maybe isolated
- Absolute cross section measured

Resolution ~ 500 keV FWHM

No spurious counts at
-10 < E_x < -2 MeV

State (MeV)	Counts	Absolute coss section (nb)	Cross section 95% limit (nb)
g.s. (0$^+$) + 2$^+$ (536 keV)	5	13	[3--18]

Analysis of cross-section sensitivity < 0.1 nb in the Region Of Interest

2νββ is sensitive to New Physics as well

All 100 kg- and ton-class 0νββ experiments can also study a diverse range of exotic phenomena, e.g. through spectral distortion in 2νββ. Future searches will probe the 2νββ with high statistics about 10^5-10^6 events.

Common subjects:

Majoron(s) emission (partly)bosonic neutrinos, Lorentz invariance violation

Recent subjects:

Lepton-number conserving right-handed currents (PRL 125 (2020) 17, 171801)

Neutrino self-interactions (PRD 102 (2020) 5, 051701)

Sterile neutrino and light fermion searches through energy end point (PRD 103 (2021) 5, 055019; PLB 815 (2021) 136127)

Legend: 2νββ, 0νM2, 0νM3, 0νM7, 2ν-LIV, 2ν-Boson

$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q - \varepsilon_1 - \varepsilon_2)^n \left[p_1 \varepsilon_1 F(\varepsilon_1) \right] \left[p_2 \varepsilon_2 F(\varepsilon_2) \right]$$

Spectral index n

$$\mathcal{L} = \frac{G_F \cos\theta_C}{\sqrt{2}} \left[(1 + \delta_{\mathrm{SM}}) j_L^\mu J_{L\mu} + V_{eN} j_L^{N\mu} J_{L\mu} + \epsilon_{LR} j_R^{N\mu} J_{L\mu} + \epsilon_{RR} j_R^{N\mu} J_{R\mu} \right] + \mathrm{h.c.}$$

Exotic 2νββ Sterile Neutrino (or HNL)

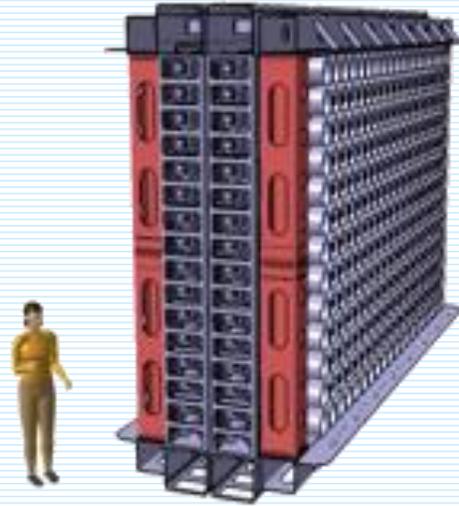**Consider either mixing of sterile ν with active ν, or right-handed currents**

Phys.Rev.D 103 (2021) 055019

in

GPT-4

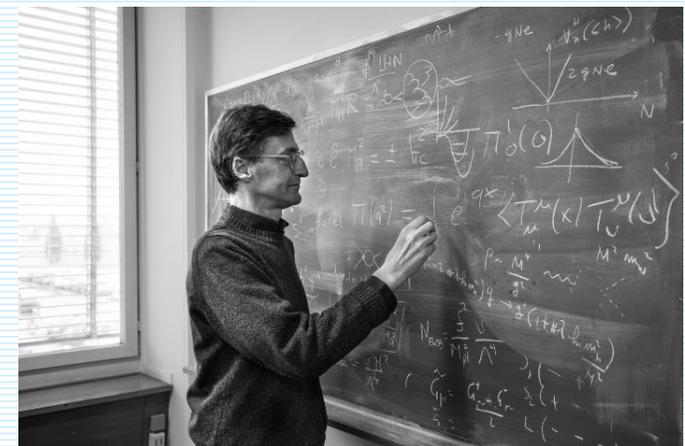**AI can not solve ν-physics problems!**

**ν-physics problems can be solved
only by ν-experiments (and ν-theory),
i.e., by skilled experimentalists and theorists**

Experience
Theory

Experimentalist

Fedor Simkovic

63

Thank you for your attention!