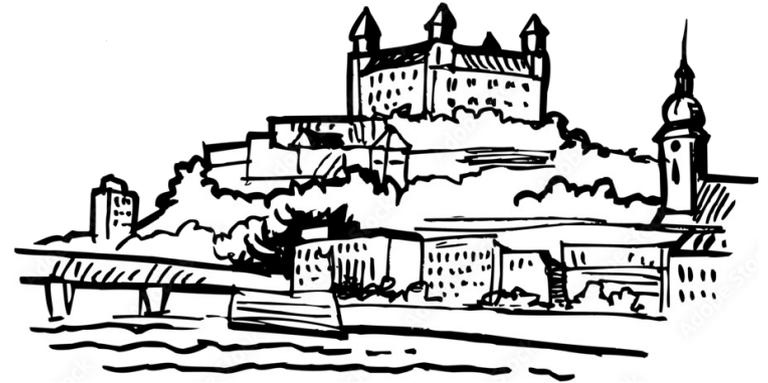
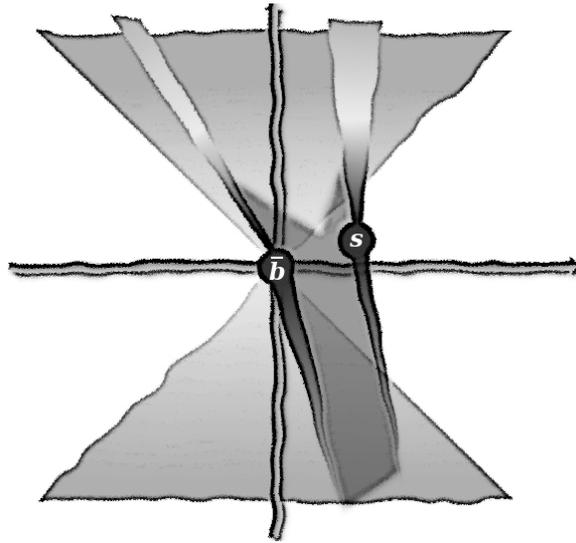


# Hadrons in Covariant Confined Quark Model



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**BRATISLAVA, SLOVAKIA**

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**Theory and Experiment in High Energy Physics**

# Overview

- Motivation
- Main characteristics of the model
- Example calculation  $B \rightarrow D_{(s)}^{(*)} + \pi(\rho)$
- Summary and outlook

# Motivation

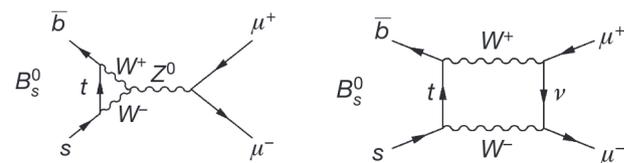
## ● Domain with large experimental activity:

### ➔ *Heavy quark factories*

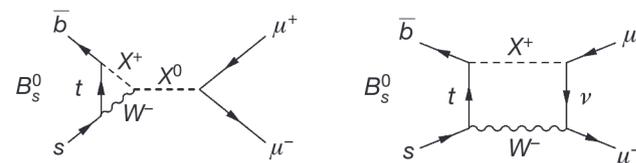
- Ongoing experiments: LHCb, BaBar, Belle (II), BESII
- Near future: SuperKEKB (has beams already)

### ➔ *Unceasing appearance of new and unique data*

- Rare heavy meson decays
- Multiquark state claims
- Precision measurements: decay widths, branching fractions, EW parameters



Standard Model



beyond Standard Model

Nature 522 (2015) 68-72

## ● Theory catching up

### ➔ *Theoretical description – a challenge*

- ChPT, Dyson-Schwinger equations, QCD sum rules, Lattice QCD, HQET despite impressive progress still limited application domain  $\Rightarrow$  *quark models!*

### ➔ *Covariant confined quark model*

- Successful tool for a unified description of the multiquark states (mesons, baryons, tetraquarks, ...)
- Effective quantum field approach to hadronic interactions (hadrons interacting with constituent quarks, gluons are absent)
- Limited number of free parameters: constituent quarks masses, infrared cutoff and parameters describing hadron effective size.

# CQM Lagrangian

## ● Non-local Lagrangian (density):

$$L_{\text{int}} = g_H \cdot H(x) \cdot J_H(x)$$

$$J_M(x) = \int dx_1 \int dx_2 F_M(x, x_1, x_2) \cdot \bar{q}_{f_1}^a(x_1) \Gamma_M q_{f_2}^a(x_2)$$

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3 F_B(x, x_1, x_2, x_3) \times \Gamma_1 q_{f_1}^{a_1}(x_1) \left( q_{f_2}^{a_2}(x_2) C \Gamma_2 q_{f_3}^{a_3}(x_3) \right) \cdot \varepsilon^{a_1 a_2 a_3}$$

$$J_T(x) = \int dx_1 \dots \int dx_4 F_T(x, x_1, \dots, x_4) \\ \times \left( q_{f_1}^{a_1}(x_1) C \Gamma_1 q_{f_2}^{a_2}(x_2) \right) \cdot \left( \bar{q}_{f_3}^{a_3}(x_3) \Gamma_2 C \bar{q}_{f_4}^{a_4}(x_4) \right) \cdot \varepsilon^{a_1 a_2 c} \varepsilon^{a_3 a_4 c}$$

## ● Translational invariance and calculational convenience:

$$F_H(x, x_1, \dots, x_n) = \delta \left( x - \sum_{i=1}^n w_i x_i \right) \Phi_H \left( \sum_{i<j} ((x_i - x_j)^2) \right) \quad w_i = m_i / \sum_{j=1}^n m_j$$

$$\bar{\Phi}_H(-k^2) = \exp(k^2 / \Lambda_H^2)$$

## ● CCQM Lagrangian:

- ➔ Full Lorentz invariance
- ➔ Standard QFT diagram techniques

# Electromagnetic interactions

## Non-local strong interaction theory

⇒ inclusion of EM interactions requires dedicated treatment.

[S. Mandelstam, Annals Phys.19, 1 (1962); J. Terning, Phys. Rev. D44, 887 (1991)]

## Minimal substitution for free the free-quark Lagrangian

$$\partial^\mu q_i \rightarrow (\partial^\mu - ieA^\mu)q_i \quad \mathcal{L}_{int}^{em-min}(x) = \sum_q e_q \bar{q}(x) \hat{A}(x) q(x)$$

## Gauge field exponential introduced for the strong-interaction part

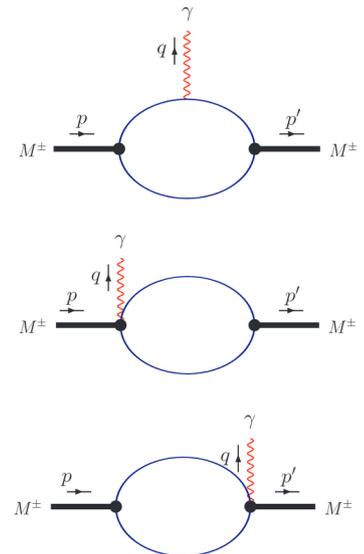
$$q_i(x_i) \rightarrow e^{-ie_{q_i} I(x_i, x, P)} q_i(x_i) \quad I(x_i, x, P) = \int_x^{x_i} dz_\mu A^\mu(z) \quad \frac{\partial}{\partial x^\mu} I(x, y, P) = A_\mu(x)$$

## The full Lagrangian is invariant under simultaneous transformations

$$q_i(x) \rightarrow e^{ie_{q_i} f(x)} q_i(x) \quad \bar{q}_i(x) \rightarrow \bar{q}_i(x) e^{-ie_{q_i} f(x)} \quad A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu f(x)$$

## The exponential is expanded into powers of the coupling (example for meson)

$$\mathcal{L}_{int}^{em-nonloc}(x) = g_M M(x) \int dx_1 \int dx_2 \int dz E_M^\mu(x; x_1, x_2, z) A_\mu(z) \bar{q}_1(x_1) \Gamma_M q_2(x_2)$$



# Parameters and compositeness condition

## Free parameters:

- ➔ Constituent quark masses [4], hadron-size related parameters [N] and universal cut-off [1]:  $N+5$
- ➔ Fitting basic observables (leptonic decay constants, EM decay widths)  $\Rightarrow$  num. values:  $m_{u,d} = 0.241$ ,  $m_s = 0.428$ ,  $m_c = 1.67$ ,  $m_b = 5.05$ ,  $\lambda_{\text{cut-off}} = 0.181$ ,  $\Lambda_\pi = 0.87$ ,  $\Lambda_K = \dots$  in GeV

## Compositeness condition:

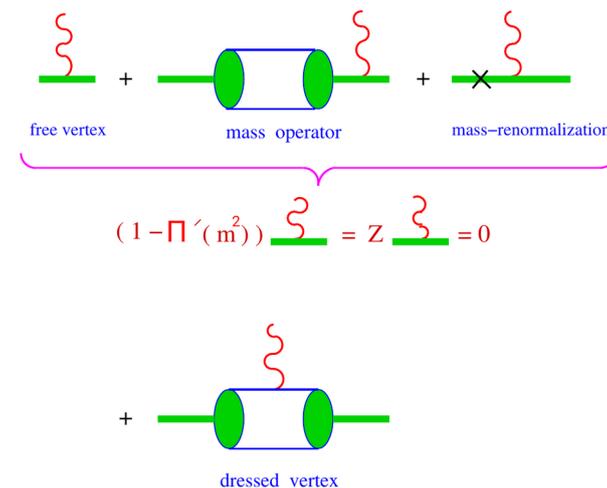
- ➔  $L_{\text{int}}$ : quarks and hadrons elementary  $\leftrightarrow$  *nature*: hadrons compound of quarks
- ➔ Compositeness condition: renormalization constant  $Z_H^{1/2}$ : the matrix element between a physical state and the corresponding bare state.

- ➔  $Z_H^{1/2} = \langle H_{\text{bare}} | H_{\text{dressed}} \rangle = 0 \Rightarrow$  Physical state does not contain bare state and is properly described as a bound state.

[ A. Salam, Nuovo Cim. 25, 224 (1962), S. Weinberg, Phys. Rev. 130, 776 (1963) ]

- ➔ Couplings removed as free parameters  
( $\Pi_H$  – meson mass operator)

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2} \tilde{\Pi}'_H(m_H^2) = 0$$



# Calculation methods

## ● Feynman diagram:

➔ General form: 
$$\Pi(p_1, \dots, p_j) = \int [d^4 k]^\ell \prod_{i_1=1}^m \Phi_{i_1+n}(-K_{i_1+n}^2) \prod_{i_3=1}^n S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3})$$

- $j$  external momenta
- $n$  quark propagators
- $l$  loop integrations
- $m$  vertices

$$K_{i_1+n}^2 = \sum_{i_2} (\tilde{k}_{i_1+n}^{(i_2)} + \tilde{p}_{i_1+n}^{(i_2)})^2$$

- $\tilde{k}_i$ : linear combination of loop momenta  $k_i$
- $\tilde{p}_i$ : linear combination of external momenta  $p_i$

## ● Calculation steps:

➔ Schwinger representation of quark propagators:

$$\tilde{S}_q(k) = (m + \hat{k}) \int_0^\infty d\alpha e^{[-\alpha(m^2 - k^2)]}$$

➔ Loop momenta integration:

$$\int d^4 k P(k) e^{2kr} = \int d^4 k P \left( \frac{1}{2} \frac{\partial}{\partial r} \right) e^{2kr} = P \left( \frac{1}{2} \frac{\partial}{\partial r} \right) \int d^4 k e^{2kr}$$

➔ Further simplifications:

$$\int_0^\infty d^n \alpha P \left( \frac{1}{2} \frac{\partial}{\partial r} \right) e^{-\frac{r^2}{a}} = \int_0^\infty d^n \alpha e^{-\frac{r^2}{a}} P \left( \frac{1}{2} \frac{\partial}{\partial r} - \frac{r}{a} \right), \quad r = r(\alpha_i), \quad a = a(\Lambda_H, \alpha_i)$$

# Infrared confinement

## Introduction of infrared cut-off

➔ Unity in form of  $\delta$ -function introduced  $\Rightarrow$  single cut-off parameter

$$1 = \int_0^\infty dt \delta\left(t - \sum_{i=1}^n \alpha_i\right)$$

$$\Pi = \int_0^\infty d^n \alpha F(\alpha_1, \dots, \alpha_n) = \int_0^\infty dt t^{n-1} \int_0^1 d^n \alpha \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, \dots, t\alpha_n)$$

➔ Universal value  $\lambda_{\text{cut-off}} = 0.181$  established for all processes.

➔  $\Pi$  becomes a smooth function, thresholds in the quark loop diagrams and corresponding branch points are removed

➔ Numerical integration

## Analogy: Confined propagators

$$\frac{1}{m^2 - k^2} = \int_0^\infty d\alpha e^{[-\alpha(m^2 - k^2)]} \rightarrow \int_0^{1/\lambda^2} d\alpha e^{[-\alpha(m^2 - k^2)]} = \frac{1 - e^{-(m^2 - k^2)/\lambda}}{m^2 - k^2}$$

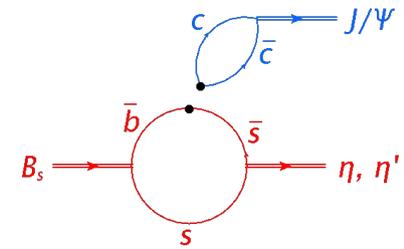
Entire function without poles, indicating absence of a single quark as asymptotic state.

# Form factors and weak decays

## ● Main ingredients:

- ➔ Effective theory (Wilson coefficients) used to describe quark flavor transition
- ➔ Factorization: convolution of form factor and term proportional to decay constant

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_t \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$



$$\langle P'_{[\bar{q}_3, q_2]}(p_2) | \bar{q}_2 O^\mu q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \rangle = \mathbf{F}_+(q^2) P^\mu + \mathbf{F}_-(q^2) q^\mu$$

$$\langle P'_{[\bar{q}_3, q_2]}(p_2) | \bar{q}_2 (\sigma^{\mu\nu} q_\nu) q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \rangle = \frac{i}{m_1 + m_2} (q^2 P^\mu - q \cdot P q^\mu) \mathbf{F}_T(q^2)$$

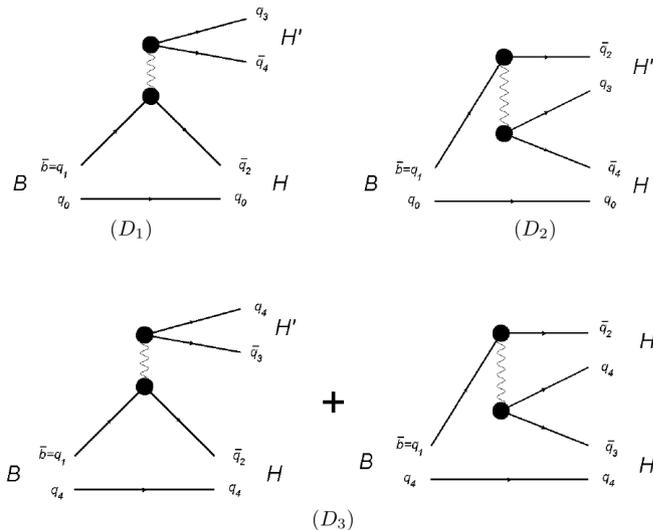
$$\langle V_{[\bar{q}_3, q_2]}(p_2, \epsilon_2) | \bar{q}_2 O^\mu q_1 | P_{[\bar{q}_3, q_1]}(p_1) \rangle = \frac{\epsilon_\nu^\dagger}{m_1 + m_2} \left[ -g^{\mu\nu} P \cdot q \mathbf{A}_0(q^2) + P^\mu P^\nu \mathbf{A}_+(q^2) \right. \\ \left. + q^\mu P^\nu \mathbf{A}_-(q^2) + i\epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta \mathbf{V}(q^2) \right]$$

$$\langle V_{[\bar{q}_3, q_2]}(p_2, \epsilon_2) | \bar{q}_2 [\sigma^{\mu\nu} q_\nu (1 + \gamma^5)] q_1 | P_{[\bar{q}_3, q_1]}(p_1) \rangle = \epsilon_\nu^\dagger \left[ -\left( g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) P \cdot q \mathbf{a}_0(q^2) \right. \\ \left. + \left( P^\mu P^\nu - q^\mu P^\nu \frac{P \cdot q}{q^2} \right) \mathbf{a}_+(q^2) + i\epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta \mathbf{g}(q^2) \right]$$

# Example decay $B \rightarrow D_{(s)}^{(*)} + \pi(\rho)$

## Large set of processes with rich properties

(various diagram topologies and spin structures).



Spin structure	Diagram type		
	$D_1$	$D_2$	$D_3$
(A) $\underline{PS} \rightarrow \underline{PS} + PS$	$B^0 \rightarrow D^- + \pi^+$ $B^0 \rightarrow \pi^- + D^+$ $B^0 \rightarrow \pi^- + D_s^+$ $B^+ \rightarrow \pi^0 + D_s^+$	$B^0 \rightarrow \pi^0 + \bar{D}^0$	$B^+ \rightarrow \bar{D}^0 + \pi^+$
(B) $\underline{PS} \rightarrow \underline{PS} + V$	$B^0 \rightarrow D^- + \rho^+$ $B^0 \rightarrow \pi^- + D_s^{*+}$ $B^+ \rightarrow \pi^0 + D_s^{*+}$ $B^+ \rightarrow \pi^0 + D_s^{*+}$	$B^0 \rightarrow \pi^0 + \bar{D}^{*0}$	$B^+ \rightarrow \bar{D}^0 + \rho^+$
(C) $\underline{PS} \rightarrow \underline{V} + PS$	$B^0 \rightarrow D^{*-} + \pi^+$ $B^0 \rightarrow \rho^- + D_s^+$ $B^+ \rightarrow \rho^0 + D_s^+$	$B^0 \rightarrow \rho^0 + \bar{D}^0$	$B^+ \rightarrow \bar{D}^{*0} + \pi^+$
(D) $\underline{PS} \rightarrow \underline{V} + V$	$B^0 \rightarrow D^{*-} + \rho^+$ $B^0 \rightarrow \rho^- + D_s^{*+}$ $B^+ \rightarrow \rho^0 + D_s^{*+}$	$B^0 \rightarrow \rho^0 + \bar{D}^{*0}$	$B^+ \rightarrow \bar{D}^{*0} + \rho^+$

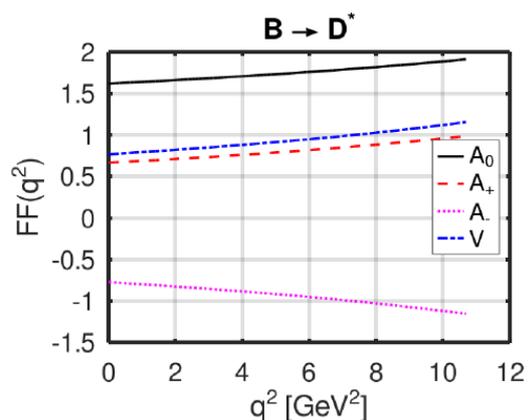
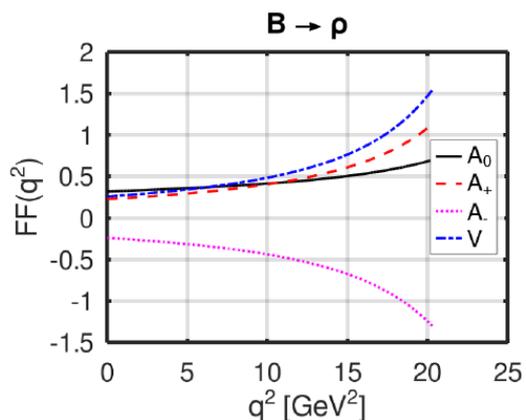
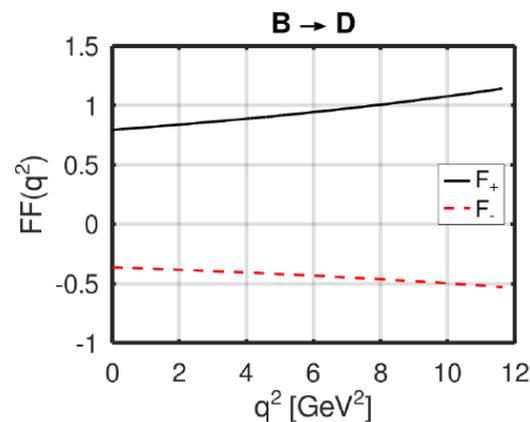
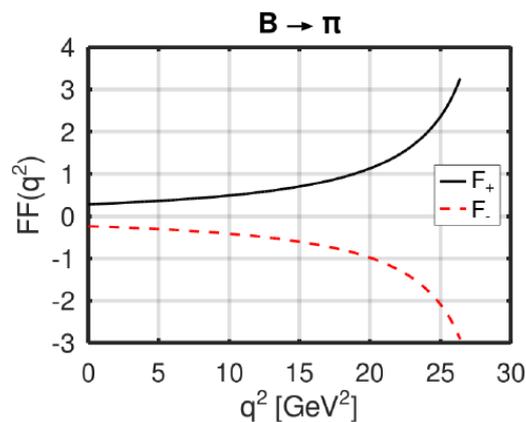
## We test

- Our model on new (and old) data.
- Our understanding of the weak sector (5 CKM elements enter).
- Factorization assumption validity.

## We complete the CCQM predictions with up-to-now non-addressed B decays.

# Example decay $B \rightarrow D_{(s)}^{(*)} + \pi(\rho)$

● **An overall overshooting** of the experimental data is seen and this was observed also by several other authors for similar processes [25-28]. Some of them consider this observation to be the sign of new physics and label it as „novel puzzle“.



[25] T. Huber, S. Kränkl, and X.-Q. Li  
JHEP, 09:112, (2016).

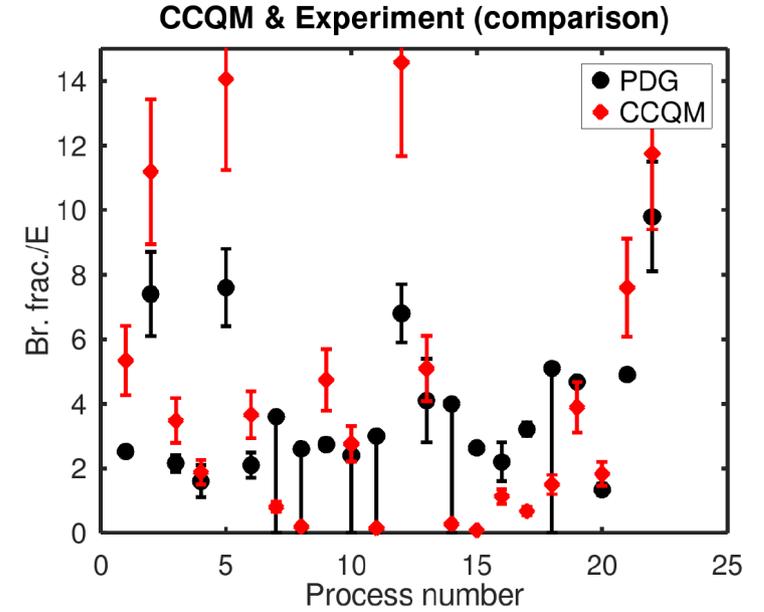
[26] M. Bordone, et al.  
Eur. Phys. J. C, 80(10):951, (2020).

[27] S. Iguro and T. Kitahara  
Phys. Rev. D, 102(7):071701, (2020).

[28] F.-M. Cai, W.-J. Deng, X.-Q. Li, and Y.-D. Yang  
JHEP,10:235, (2021).

# Example decay $B \rightarrow D_{(s)}^{(*)} + \pi(\rho)$

Process	Diagram	$\mathcal{B}_{\text{CCQM}}/E$	$\mathcal{B}_{\text{PDG}}/E$	E
1 $B^0 \rightarrow D^- + \pi^+$	$D_1$	$5.34 \pm 1.07$	$2.52 \pm 0.13$	$10^{-3}$
2 $B^0 \rightarrow \pi^- + D^+$	$D_1$	$11.19 \pm 2.24$	$7.4 \pm 1.3$	$10^{-7}$
3 $B^0 \rightarrow \pi^- + D_s^+$	$D_1$	$3.48 \pm 0.70$	$2.16 \pm 0.26$	$10^{-5}$
4 $B^+ \rightarrow \pi^0 + D_s^+$	$D_1$	$1.88 \pm 0.38$	$1.6 \pm 0.5$	$10^{-5}$
5 $B^0 \rightarrow D^- + \rho^+$	$D_1$	$14.06 \pm 2.81$	$7.6 \pm 1.2$	$10^{-3}$
6 $B^0 \rightarrow \pi^- + D_s^{*+}$	$D_1$	$3.66 \pm 0.73$	$2.1 \pm 0.4$	$10^{-5}$
7 $B^+ \rightarrow \pi^0 + D_s^{*+}$	$D_1$	$0.804 \pm 0.16$	$< 3.6$	$10^{-6}$
8 $B^+ \rightarrow \pi^0 + D_s^{*+}$	$D_1$	$0.197 \pm 0.04$	$< 2.6$	$10^{-4}$
9 $B^0 \rightarrow D^{*-} + \pi^+$	$D_1$	$4.74 \pm 0.95$	$2.74 \pm 0.13$	$10^{-3}$
10 $B^0 \rightarrow \rho^- + D_s^+$	$D_1$	$2.76 \pm 0.55$	$< 2.4$	$10^{-5}$
11 $B^+ \rightarrow \rho^0 + D_s^+$	$D_1$	$0.149 \pm 0.03$	$< 3.0$	$10^{-4}$
12 $B^0 \rightarrow D^{*-} + \rho^+$	$D_1$	$14.58 \pm 2.92$	$6.8 \pm 0.9$	$10^{-3}$
13 $B^0 \rightarrow \rho^- + D_s^{*+}$	$D_1$	$5.09 \pm 1.02$	$4.1 \pm 1.3$	$10^{-5}$
14 $B^+ \rightarrow \rho^0 + D_s^{*+}$	$D_1$	$0.275 \pm 0.06$	$< 4.0$	$10^{-4}$
15 $B^0 \rightarrow \pi^0 + \bar{D}^0$	$D_2$	$0.085 \pm 0.02$	$2.63 \pm 0.14$	$10^{-4}$
16 $B^0 \rightarrow \pi^0 + \bar{D}^{*0}$	$D_2$	$1.13 \pm 0.23$	$2.2 \pm 0.6$	$10^{-4}$
17 $B^0 \rightarrow \rho^0 + \bar{D}^0$	$D_2$	$0.675 \pm 0.14$	$3.21 \pm 0.21$	$10^{-4}$
18 $B^0 \rightarrow \rho^0 + \bar{D}^{*0}$	$D_2$	$1.50 \pm 0.30$	$< 5.1$	$10^{-4}$
19 $B^+ \rightarrow \bar{D}^0 + \pi^+$	$D_3$	$3.89 \pm 0.78$	$4.68 \pm 0.13$	$10^{-3}$
20 $B^+ \rightarrow \bar{D}^0 + \rho^+$	$D_3$	$1.83 \pm 0.37$	$1.34 \pm 0.18$	$10^{-2}$
21 $B^+ \rightarrow \bar{D}^{*0} + \pi^+$	$D_3$	$7.60 \pm 1.52$	$4.9 \pm 0.17$	$10^{-3}$
22 $B^+ \rightarrow \bar{D}^{*0} + \rho^+$	$D_3$	$11.75 \pm 2.35$	$9.8 \pm 1.7$	$10^{-3}$



Decay mode	our results	[25]	[28]
$B^0 \rightarrow D^- \pi^+$	$5.34 \pm 1.07$	$3.93^{+0.43}_{-0.42}$	$4.74^{+0.61}_{-0.69}$
$B^0 \rightarrow D^{*-} \pi^+$	$4.74 \pm 0.95$	$3.45^{+0.53}_{-0.50}$	$4.26^{+0.75}_{-0.80}$
$B^0 \rightarrow D^- \rho^+$	$14.06 \pm 2.81$	$10.42^{+1.24}_{-1.20}$	$12.28^{+1.40}_{-1.63}$
$B^0 \rightarrow D^{*-} \rho^+$	$14.58 \pm 2.92$	$9.24^{+0.72}_{-0.71}$	$11.61^{+1.88}_{-2.01}$

# Summary and outlook

## Summary

### ➔ *Predictions of the CCQM for various processes*

- Light and heavy hadrons
- Mesons, baryons, tetraquarks
- Strong and weak decays

### ➔ *Validity and suitability of the CQM consists in*

- Nice agreement with experimental data
- Wide application range
- High activity in this physics area

## Successful and cited model: being noticed (by large Collaborations) and cited (219).

ATLAS Coll. (2022), JHEP 08 (2022) 087  
[<https://inspirehep.net/literature/2044968>]

LHCb Coll. (2022), Phys. Rev. Lett. 128, 041801  
[<https://inspirehep.net/literature/1908214>]

LHCb Coll. (2022), Phys. Rev. D 105, 012010  
[<https://inspirehep.net/literature/1908217>]

[5] M.A. Ivanov, J.G. Korner and P. Santorelli, *Exclusive semileptonic and nonleptonic decays of the  $B_c$  meson*, *Phys. Rev. D* **73** (2006) 054024 [[hep-ph/0602050](#)] [[INSPIRE](#)].

[6] S. Dubnicka, A.Z. Dubnickova, A. Issadykov, M.A. Ivanov and A. Liptaj, *Study of  $B_c$  decays into charmonia and  $D$  mesons*, *Phys. Rev. D* **96** (2017) 076017 [[arXiv:1708.09607](#)] [[INSPIRE](#)].

[20] S. Dubnička, A. Z. Dubničková, M. A. Ivanov, A. Liptaj, P. Santorelli, and C. T. Tran, *Study of  $B_s \rightarrow \ell^+ \ell^- \gamma$  decays in covariant quark model*, *Phys. Rev. D* **99**, 014042 (2019).

[20] S. Dubnička, A. Z. Dubničková, M. A. Ivanov, A. Liptaj, P. Santorelli, and C. T. Tran, *Study of  $B_s \rightarrow \ell^+ \ell^- \gamma$  decays in covariant quark model*, *Phys. Rev. D* **99**, 014042 (2019).

## Outlook

### ➔ Hadron physics at heavy quark factories - rich source for applications