Precise prediction for the W-boson mass in U(1) extensions of the standard model

(based on arXiv:2305.11931)

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Theory & Experiment #1
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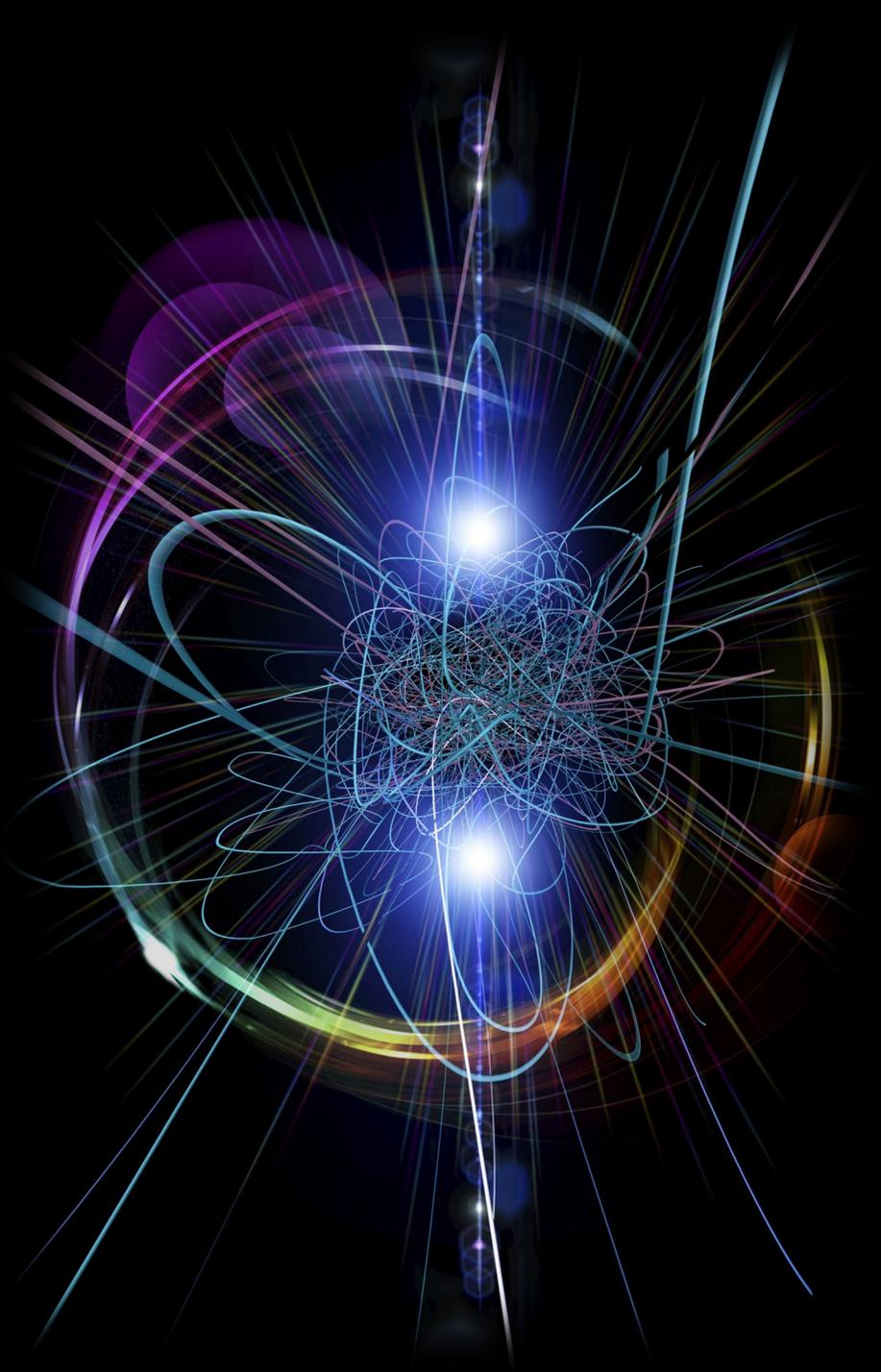
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Outline

- W-boson mass

- Muon decay in the standard model

- Muon decay in U(1) extensions of the standard model

- Comparison of the full and a truncated version of the 1-loop correction to the W-mass

W-boson mass

Theory and experiment

Prediction & measurement of M_W

- Theory (SM):
$M_W^{\text{theo}} = 80353 \pm 4(\text{P.T.}) \pm 8(\text{param})$ MeV with 2022 PDG inputs

- Theory has ~ 10 MeV combined uncertainty

(main sources are uncertainties of m_{t} and $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$)

Prediction & measurement of M_W

- Experiment (2022 PDG world. avg.):

$$M_W^{\text{exp.}} = 80377 \pm 12\text{MeV}$$

Improved ATLAS result (2023 march):

$$M_W^{\text{ATLAS}} = 80360 \pm 16\text{MeV}$$

- Lot of attention from experimentalists lately:

CDFII measurement (2022 april):

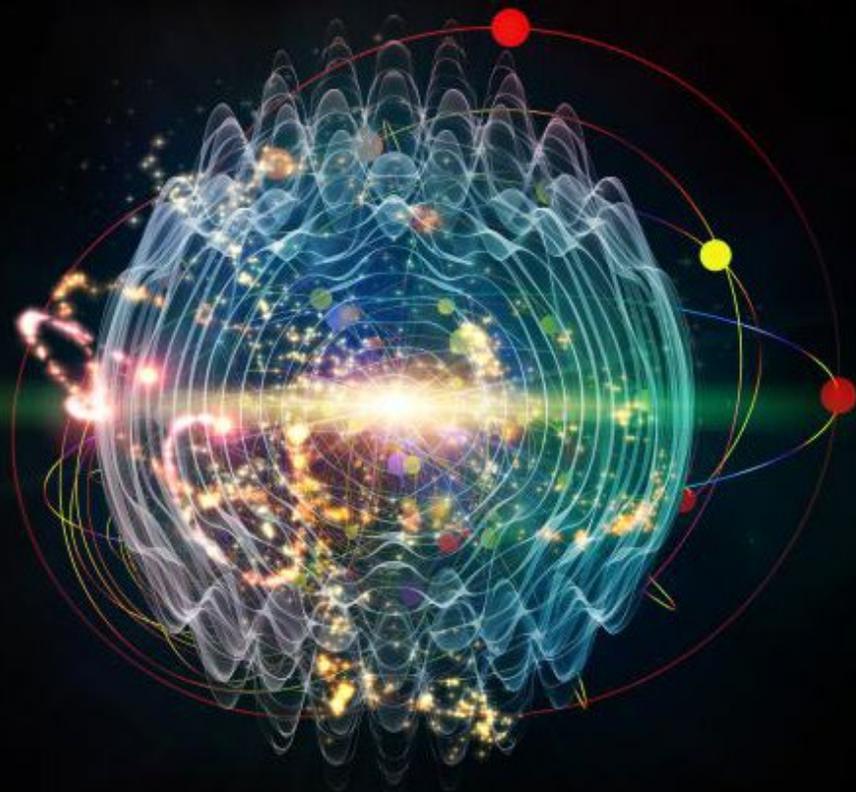$$M_W^{\text{CDF}} = 80433.5 \pm 9.4\text{MeV},$$

BSM physics and M_W

- Exclusion bands for BSM models:
  $$|M_W^{\text{exp.}} - M_W^{\text{theo}}| < 2\sigma$$

- Both the theory and experimental uncertainties are ~ 10 MeV

- A U(1) extension even has tree level correction to the W-mass

- We need to know these corrections precisely

Muon decay

in the standard model

Brief overview of the progress in the SM

Prediction for M_W using fiducial parameters

$G_F = 1.166378 \times 10^{-5}~\text{GeV}^{-2}$
from the lifetime of the muon

$\alpha^{-1} = 137.036$
from electron g-2 /
Rydberg constant +
atomic recoil

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\,\alpha}{2\,M_W^2\,s_W^2}(1 + \Delta r) \quad \text{with } c_W = \frac{M_W}{M_Z}$$

$\Delta r = 0.0366$
radiative corrections:
- first full 1-loop Sirlin 1980
- now full 2-loop + leading 3-loop corrections are available

$M_Z = 91.1876~\text{GeV}$
from collider: LEP
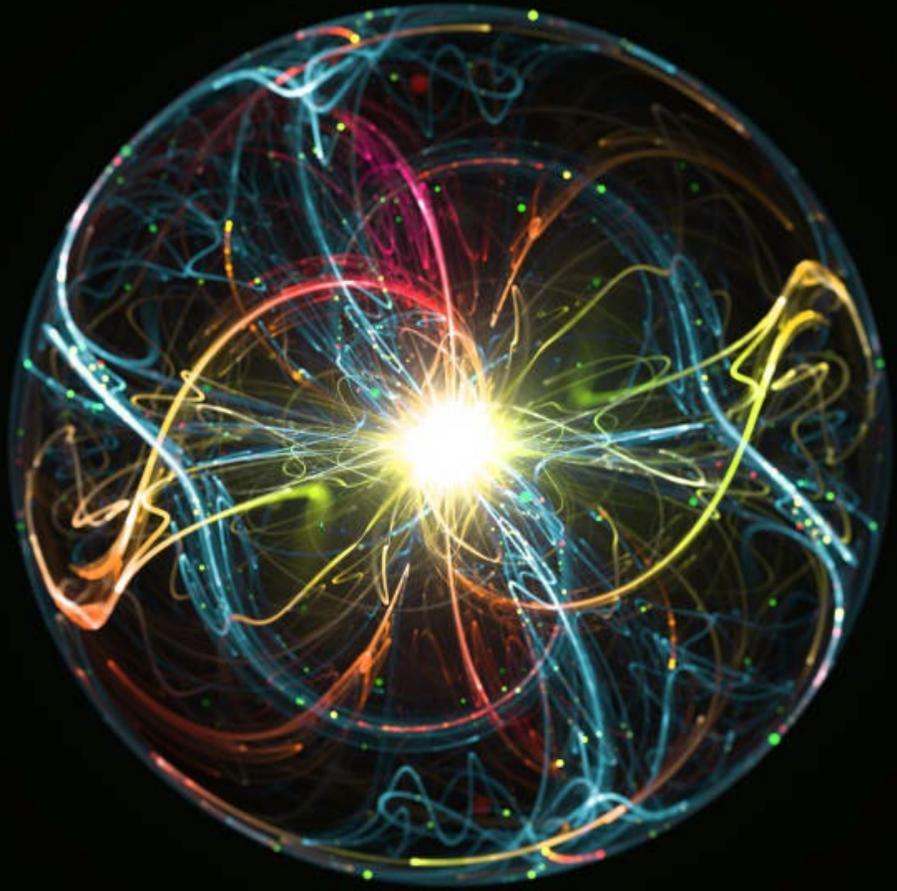
Δr at one-loop in the on-shell scheme:

$$\Delta r = \underbrace{\frac{2\delta e}{e}}_{} + \underbrace{\left(\frac{\mathrm{Re}\Pi_{WW}(M_W^2) - \Pi_{WW}(0)}{M_W^2}\right) + \delta_{\mathrm{BV}}}_{}$$

Renormalization of the electric charge, formula known all order

Diagrammatic corrections to the muon decay graph: W-propagator and box and vertex diagrams

$$\underbrace{+ \frac{c_W^2}{s_W^2}\left(\frac{\mathrm{Re}\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\mathrm{Re}\Pi_{WW}(M_W^2)}{M_W^2}\right)}_{}$$

Renormalization of s_W

Muon decay in U(1) extensions of the SM

What is a U(1) extension and how it affects the W-boson mass

What may be new in a U(1) extension?

- SM gauge group + an extra U(1) adds a new interaction
- May add new scalar field(s), can stabilize the EW vacuum
- May add right-handed (sterile) neutrinos: neutrino mass generation via see-saw, dark matter

New parameters: 5 (gauge + scalar)

Gauge sector:

- M'_Z: mass of the new gauge boson Z'

- s_Z : new gauge mixing angle, rotation of gauge eigenstates to mass eigenstates:

$$\begin{pmatrix} B_\mu \\ W^3_\mu \\ B'_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W & 0 \\ s_W & c_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_Z & -s_Z \\ 0 & s_Z & c_Z \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

Scalar sector:

- $\tan\beta = \frac{w}{v}$: ratio of new VEV to BEH VEV

- M_S: mass of the new scalar boson

- s_S : new scalar mixing angle to mass eigenstates

$$\begin{pmatrix} \phi^0 \\ \chi \end{pmatrix} = \begin{pmatrix} c_S & -s_S \\ s_S & c_S \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}$$

Gauge boson masses

Concise relation:

$$\frac{M_W^2}{c_W^2} = c_Z^2 M_Z^2 + s_Z^2 M_{Z'}^2$$

Express predictions with Lagrangian couplings or **pheno parameters** e.g.:

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2} = 1 - s_Z^2 \left(1 - \frac{M_{Z'}^2}{M_Z^2} \right)$$

Renormalization in on-shell scheme

- Split bare parameters into $g^{(0)} \rightarrow g + \delta g$

- The Weinberg angle changes at tree level:

$$M_W^2 \frac{\delta c_W^2}{c_W^2} = \delta M_W^2 - c_W^2 \left(c_Z^2 \, \delta M_Z^2 + s_Z^2 \, \delta M_{Z'}^2 - 2 s_Z \left(M_Z^2 - M_{Z'}^2 \right) \delta s_Z \right)$$
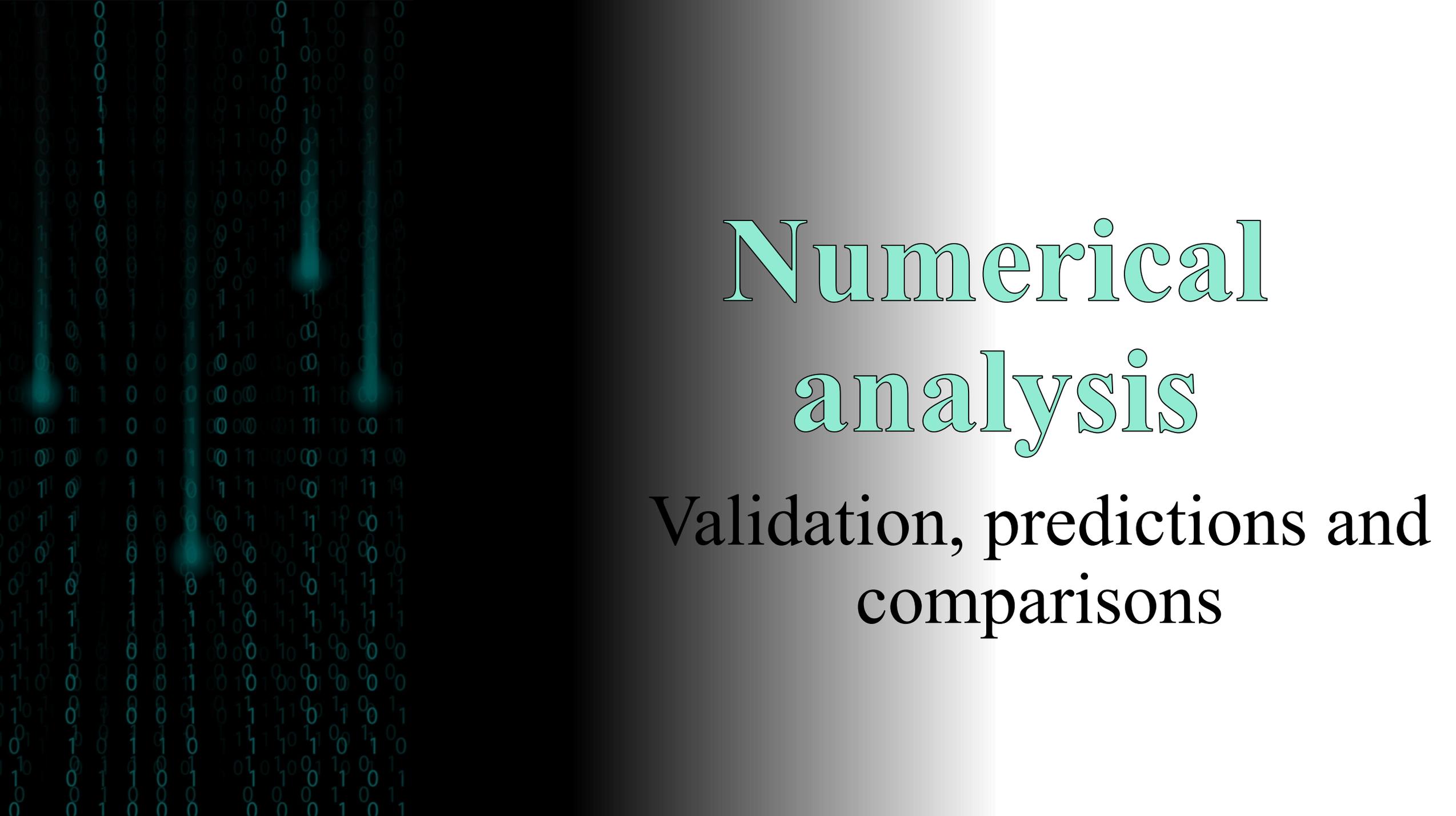
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Δr receives completely new corrections:

$$\Delta r = \left(\text{formally } \Delta r^{\text{SM}} \text{ with BSM loops}\right) -$$

$$-s_Z^2 \frac{c_W^2}{s_W^2} \frac{c_W^2}{M_W^2} \left(\text{Re}\Pi_{ZZ}(M_Z^2) - \text{Re}\Pi_{Z'Z'}(M_{Z'}^2) + 2\left(M_Z^2 - M_{Z'}^2\right)\frac{\delta s_Z}{s_Z}\right)$$

Numerical analysis

Validation, predictions and comparisons

Checks

- The ε poles cancel in Δr in R_ξ-gauge with general z-charge assignment

- For several benchmark points Δr is independent of the gauge parameters ξ_i, $i = W, A, Z, Z'$

Checks

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- Compare Δr in two cases:
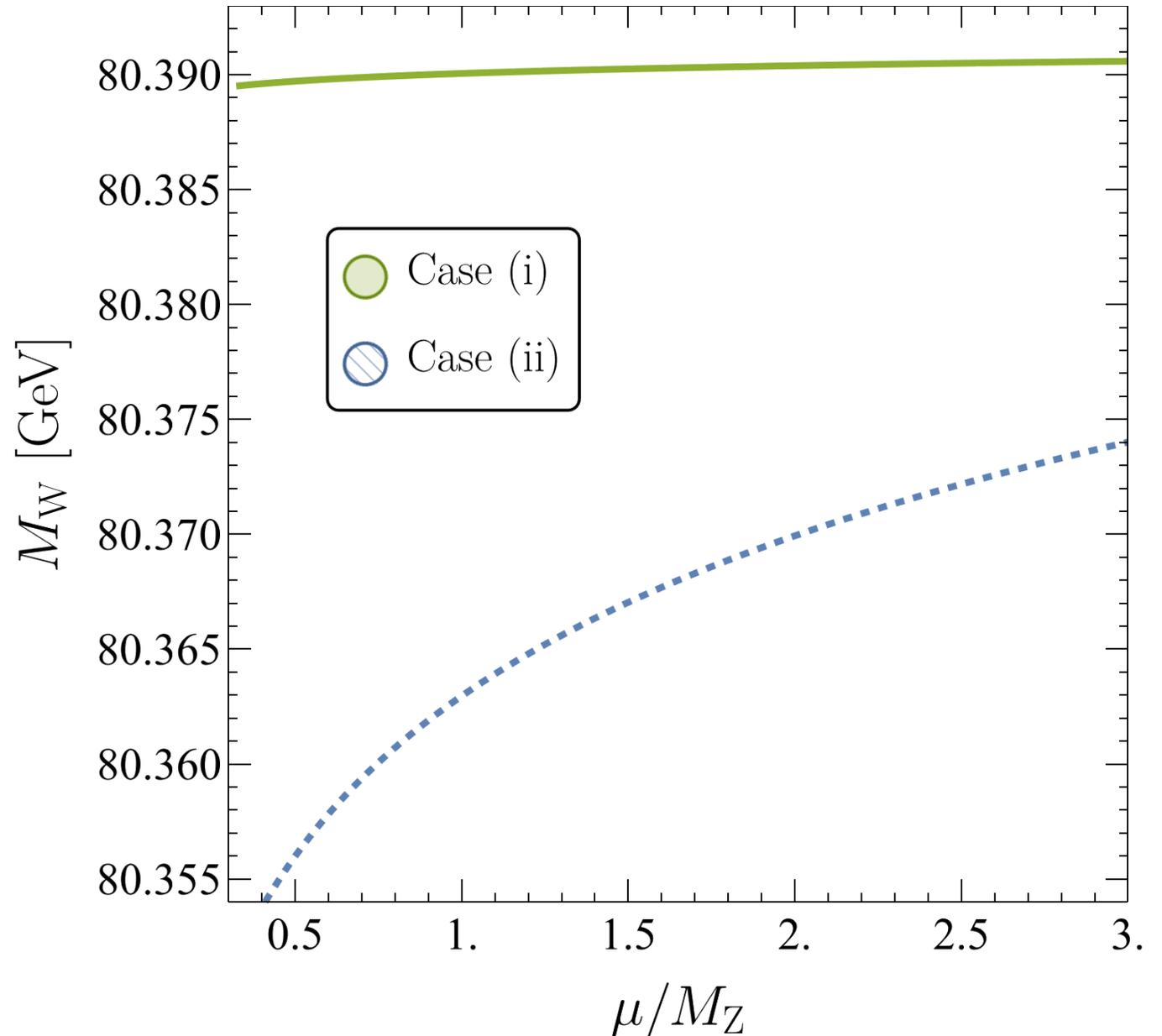
Case I. :
$$\Delta r = \left(\text{formally } \Delta r^{\text{SM}} \text{ with BSM loops}\right) -$$
$$-s_Z^2 \frac{c_W^2}{s_W^2} \frac{c_W^2}{M_W^2} \left(\text{Re}\Pi_{ZZ}(M_Z^2) - \text{Re}\Pi_{Z'Z'}(M_{Z'}^2) + 2\left(M_Z^2 - M_{Z'}^2\right)\frac{\delta s_Z}{s_Z} \right)$$

Case II. :
$$\Delta r = \left(\text{formally } \Delta r^{\text{SM}} \text{ with BSM loops}\right)$$

Checks

- The ε poles cancel in Δr in R_ξ-gauge with general z-charge assignment

- For several benchmark points Δr is independent of the gauge parameters ξ_i, with $i = W, A, Z, Z'$

- Weak dependence on the renormalization scale μ at fixed benchmark points

- The new mixing s_Z has to be small (or excluded)
- $M_{Z'} < M_Z \; \rightarrow \;$ lighter W-boson
- $M_{Z'} > M_Z \; \rightarrow \;$ heavier W-boson
- Much weaker dependence on M_S, s_S than $M_{Z'}, s_Z$
- For a heavy $M_Z \ll M_{Z'}$:
  $w \; \ll \; M_{Z'}$ is unphysical (new gauge coupling is nonperturbative)
- Case (ii) works well if $M_{Z'} \ll M_Z$
- Case (i) is needed if $M_Z \ll M_{Z'}$!

Remarks

Benchmarks: $M_W - M_{W,\text{SM}}$ [MeV]

SMALL $M_{Z'} = 50$ MeV
and $s_S = 0.1$
Irrelevant

s_Z	$5 \cdot 10^{-4}$			
$\tan\beta$ ╲ M_S	0.5 TeV		5 TeV	
	(i)	(ii)	(i)	(ii)
0.1	-1	-1	-2	-2
1	-1	-1	-2	-2
10	-1	-1	-2	-2

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LARGE $M_{Z'} = 5$ TeV
and $s_S = 0.1$
Potentially relevant

s_Z $\diagdown$	$5 \cdot 10^{-4}$				$7 \cdot 10^{-4}$			
M_S	0.5 TeV		5 TeV		0.5 TeV		5 TeV	
$\tan\beta$	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
10	37	10	35	13	75	29	73	36
20	39	34	35	34	81	76	74	79
30	40	38	35	37	83	85	75	85

- Precise predictions in BSM models are important

- Full Δr at 1-loop in U(1) extensions is computed

- Neglected terms in the incomplete Δr may become important for heavy $M_{Z'}$

- Fig. shows region where : $\left| M_W^{\text{exp.}} - M_W \right| < 2\sigma$

Conclusions

Backup slides

Gauge boson masses

- M_Z and $M_{Z'}$ have cumbersome expressions

- Any simplification?

- Use relations for the sin and cos of the Goldstone mixing angle:

$$M_{Z'}(c_Z - \kappa\, s_z) = M_Z c_z \tau$$
$$M_Z(s_Z + \kappa\, c_z) = M_{Z'} s_z \tau$$

Gauge boson masses

Rotation angles:

- $M_A = 0 \text{ GeV}: e = g_L s_W = g_y c_W$

- Express new angle with effective couplings:
$$\tan(2\theta_Z) = -\frac{2\kappa}{1 - \kappa^2 - \tau^2}$$

- κ and τ are functions of the 2 new Lagrangian couplings

Tree level masses:

$$M_W = \frac{1}{2} g_L\, v$$

$$M_Z = \frac{M_W}{c_W}\sqrt{R(c_Z, s_Z)} \text{ and } M_{Z'} = \frac{M_W}{c_W}\sqrt{R(s_Z, -c_Z)}$$

$$R(x, y) = (x - \kappa\, y)^2 + (\tau\, y)^2$$

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Concise relation:

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- **Express predictions with** Lagrangian couplings or effective couplings or **pheno parameters** e.g.:

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How to obtain δs_Z I.

- Relate unrotated and rotated fields:

$$B_\mu^{(0)} = c_W^{(0)} A_\mu^{(0)} - s_W^{(0)} (c_Z^{(0)} Z_\mu^{(0)} - s_Z^{(0)} Z_\mu'^{(0)})$$

$$B_\mu'^{(0)} = s_Z^{(0)} Z_\mu^{(0)} + c_Z^{(0)} Z_\mu'^{(0)}$$

- Also true for renormalized fields:

$$B_\mu = c_W A_\mu - s_W (c_Z Z_\mu - s_Z Z_\mu')$$

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Gauge sector:

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$$B_\mu = c_W A_\mu - s_W \left(c_Z \, Z_\mu - s_Z \, Z_\mu' \right)$$

$$B_\mu' = s_Z Z_\mu + c_Z Z_\mu'$$

- Unrotated fields are renormalized such that

$$B_\mu^{(0)} = \sqrt{Z_B} B_\mu \ \text{ and } \ B_\mu'^{(0)} = \sqrt{Z_{B'}} B_\mu'$$

- Rotated fields may mix:

$$\begin{pmatrix} A_\mu^{(0)} \\ Z_\mu^{(0)} \\ Z_\mu'^{(0)} \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{AA}} & \frac{1}{2} Z_{AZ} & \frac{1}{2} Z_{AZ'} \\ \frac{1}{2} Z_{ZA} & \sqrt{Z_{ZZ}} & \frac{1}{2} Z_{ZZ'} \\ \frac{1}{2} Z_{Z'A} & \frac{1}{2} Z_{Z'Z} & \sqrt{Z_{Z'Z'}} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z_\mu' \end{pmatrix}$$

How to obtain δs_Z II.

- Express bare fields with renormalized ones and collect coefficients:

$$\sqrt{Z_B}\, c_W = c_W^{(0)} \sqrt{Z_{AA}} - \frac{1}{2} s_W^{(0)} \left(c_Z^{(0)} Z_{ZA} - s_Z^{(0)} Z_{Z'A} \right)$$

$$\sqrt{Z_{B'}}\, s_Z = s_Z^{(0)} \sqrt{Z_{ZZ}} + \frac{1}{2} c_Z^{(0)} Z_{Z'Z}$$

$$\sqrt{Z_{B'}}\, c_Z = \frac{1}{2} s_Z^{(0)} Z_{ZZ'} + c_Z^{(0)} \sqrt{Z_{Z'Z'}}$$

- First equation is used to derive δe

 (U(1) Ward identity $\sqrt{Z_B}\, Z_{g_y} = 1$)

- 2nd and 3rd ones are divided to cancel $\sqrt{Z_{B'}}$ and express δs_Z