SPEED OF SOUND IN A DYNAMICAL CHIRAL QUARK MODEL

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OUTLINE

- Problem statement – Dense Matter asymptotics
- Quasiparticle models
- Speed of sound
- Solution - Dynamical quark model
- Mean field v/s Dynamical models
- Speed of sound (revisited)
- Quark number susceptibility
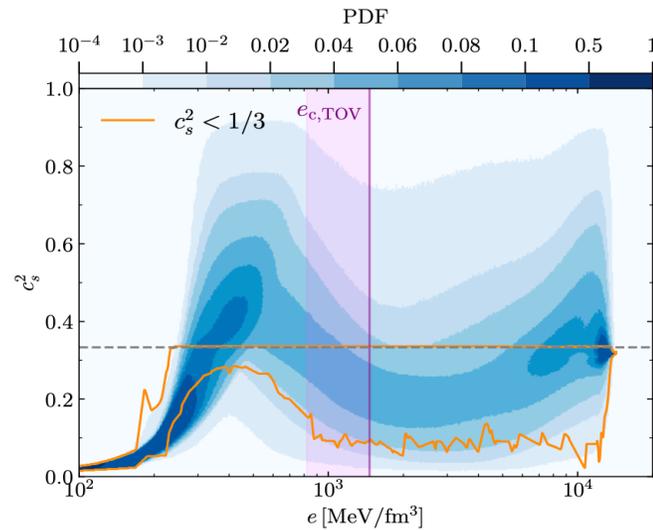- Summary
- Outlook

SPEED OF SOUND ASYMPTOTICS

Figure 1. PDF of the sound speed squared as function of the energy density. The purple region marks the 95%-interval of maximum central energy densities, so that the vertical purple line represents an estimate for the largest possible energy density in a neutron star. The orange contour marks the region containing EOSs with $c_s^2 < 1/3$.

On the Sound Speed in Neutron Stars

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ABSTRACT

Determining the sound speed c_s in compact stars is an important open question with numerous implications on the behaviour of matter at large densities and hence on gravitational-wave emission from neutron stars. To this scope, we construct more than 10^7 equations of state (EOSs) with continuous sound speed and build more than 10^8 nonrotating stellar models consistent not only with nuclear theory and perturbative QCD, but also with astronomical observations. In this way, we find that EOSs with sub-conformal sound speeds, i.e., with $c_s^2 < 1/3$ within the stars, are possible in principle but very unlikely in practice, being only 0.03% of our sample. Hence,

$$\frac{\delta\Omega}{\delta m^*} = 0 = \frac{\delta\Omega}{\delta\mu^*}$$

WALECKA

NJL

$$\mu^* = \mu - g_\omega\,\omega$$
$$m^* = m - g_\sigma\,\sigma$$

$$\sigma = -\left(\frac{g_\sigma}{m_\sigma^2}\right)\frac{\partial P}{\partial m^*}$$

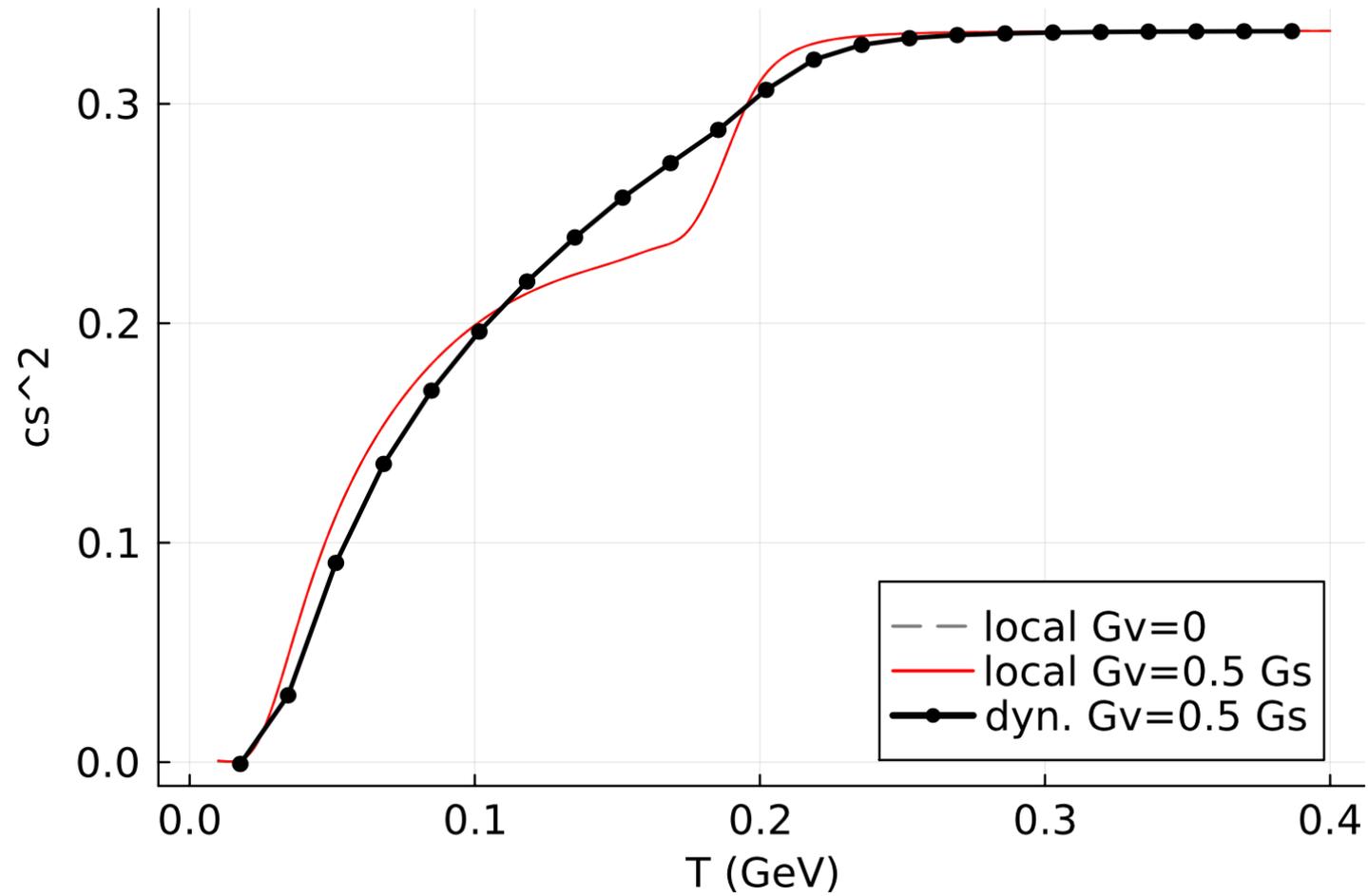$$\omega = -\left(\frac{g_\omega}{m_\omega^2}\right)\frac{\partial P}{\partial \mu^*}$$

$$\mu^* = \mu - 4N_f N_c G_v \int \frac{d^3p}{(2\pi)^3}\left(n(E_p) - \bar{n}(E_p)\right)$$

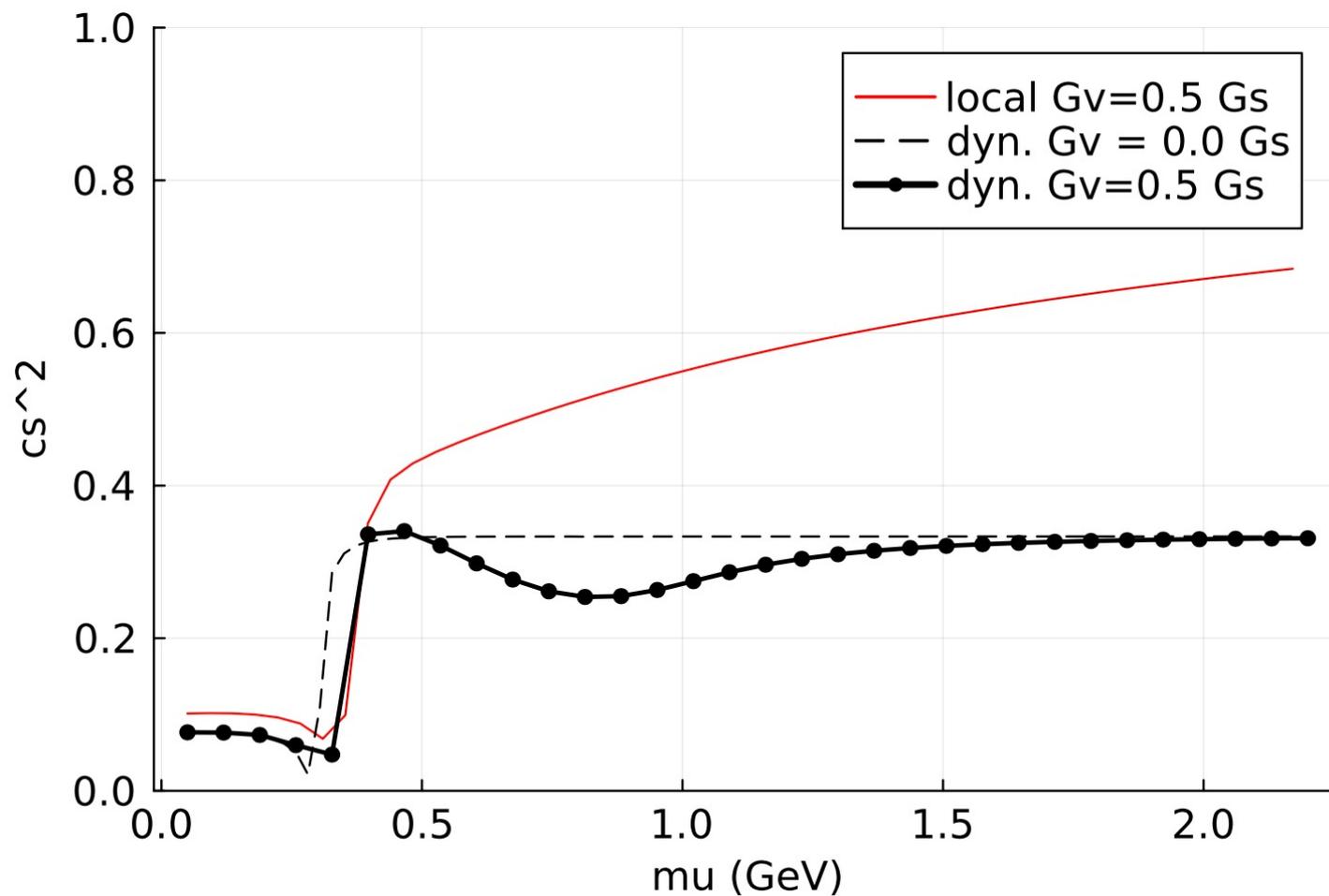$$m^* = m + 4N_f N_c G_v \int \frac{d^3p}{(2\pi)^3}\frac{m^*}{E_p}\left(1 - n(E_p) - \bar{n}(E_p)\right)$$

QUASIPARTICLE MODELS

SPEED OF SOUND – DENSE MATTER DOMAIN

NJL fails conformal limit

$$\mu' = \mu - 2G_V \int \frac{d^3q}{(2\pi)^3} \left(n_F - \bar{n}_F\right)$$

$$\propto \mu'^3$$

$$\mu' \propto \mu^{\frac{1}{3}} \longrightarrow c_S^2 \to 1$$

$$c_s^2 = \frac{n_v}{\mu} \Big/ \frac{dn_v}{d\mu}$$

DYNAMICAL CHIRAL QUARK MODEL

$$\mathscr{L} = \bar{\psi}(x)(i\partial\!\!\!/_x - m)\psi(x) - \frac{1}{2}\int d^4y\, \rho^a(x)V^{ab}(x,y)\rho^b(y)$$

Dynamical effect Gluon exchange

$$\rho^a(x) = \bar{\psi}(x)\gamma^0 T = T^a\psi(x)$$

Natural medium dependence

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2}(n(E_q) - \bar{n}(E_q))$$
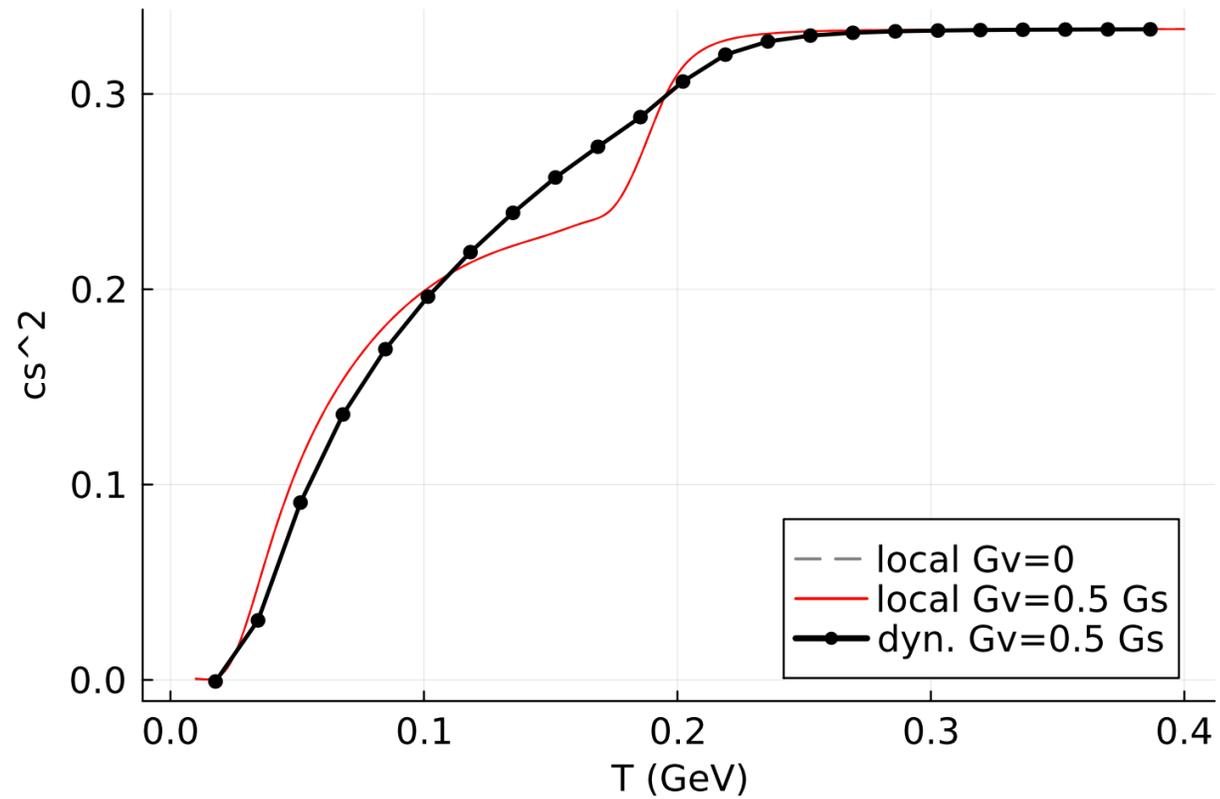
Not non-local NJL

$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{A(q)\,\hat{p}\cdot\hat{q}}{2E_q}(1 - n(E_q) - \bar{n}(E_q))$$

Quasiparticle property modified

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{B}{2E_q}(1 - n(E_q) - \bar{n}(E_q))$$

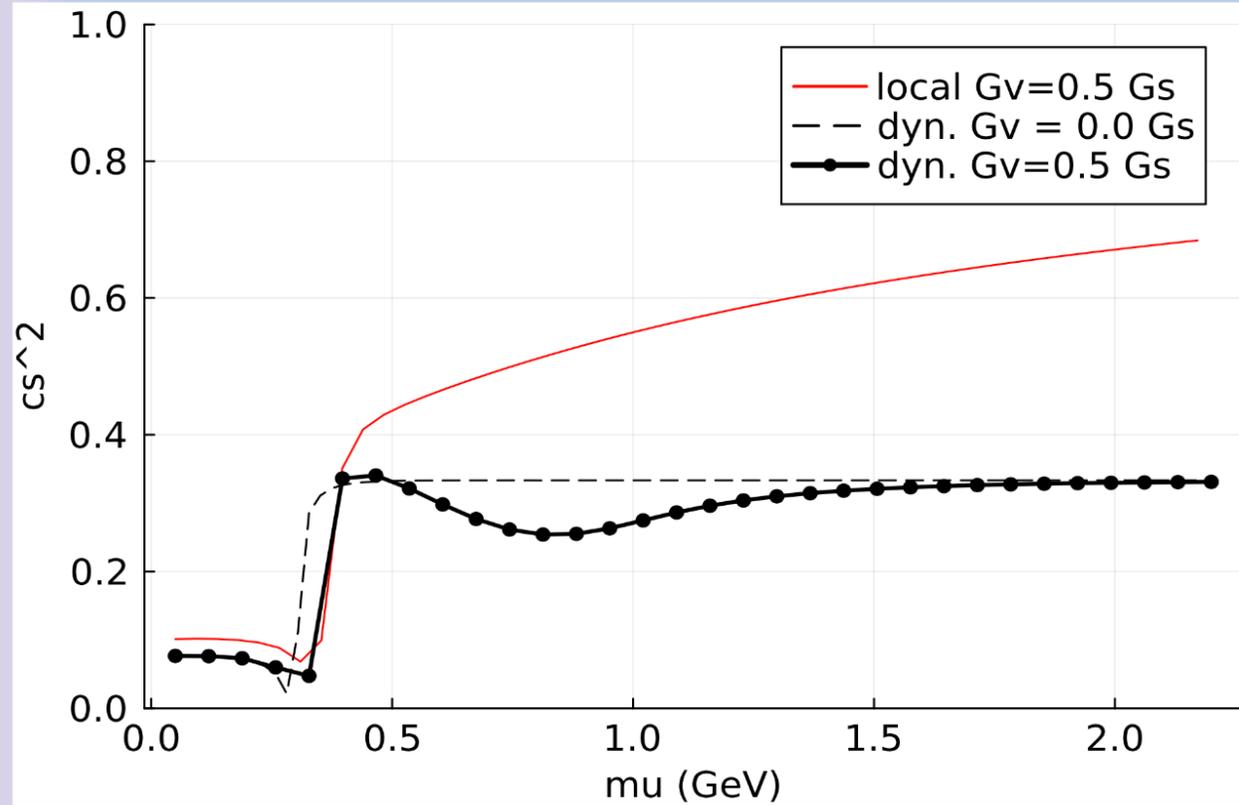$$E_q = \sqrt{A_q^2\, q^2 + B_q^2}$$

SPEED OF SOUND - FINITE TEMPERATURE DOMAIN

NJL obeys conformal limit

Dynamical model obeys conformal limit

$$c_s^2 = \frac{n_v}{\mu} \bigg/ \frac{dn_v}{d\mu}$$

$$\mu'(p) = \mu + \int \frac{d^3q}{(2\pi)^3} \frac{1}{2} V(\vec{p} - \vec{q}) \, (n_F - \bar{n}_F)$$

If V->0 as p-> Inf: $\qquad \mu' \to \mu \qquad \longrightarrow \qquad c_s^2 \to \frac{1}{3}$

e.g. $\quad V(p, q) \approx G_0 \, e^{-p^2} e^{-q^2}$

$$\chi_2 = \frac{\partial n_v^{QP}}{\partial \mu} = \frac{\partial n_v^{QP}}{\partial \mu'} \times \frac{\partial \mu'}{\partial \mu}$$

$$\chi_2 = \frac{\chi_2^{(0)}}{1 + 2G_v \chi_2^{(0)}}$$

$$\chi_2^{(0)} = \frac{\partial n_v^{QP}}{\partial \mu'}$$

QUARK NUMBER SUSCETIBILITY (QNS)

We showed that the enhancement of χ_q obtained in the lattice simulations may be attributed to the vanishing or an abrupt decrease of the interaction between q–q̄ and hence the correlations in the vector channel after the chiral restoration, which has been

Quark-number susceptibility and fluctuations in the vector channel at high temperatures ☆

Teiji Kunihiro

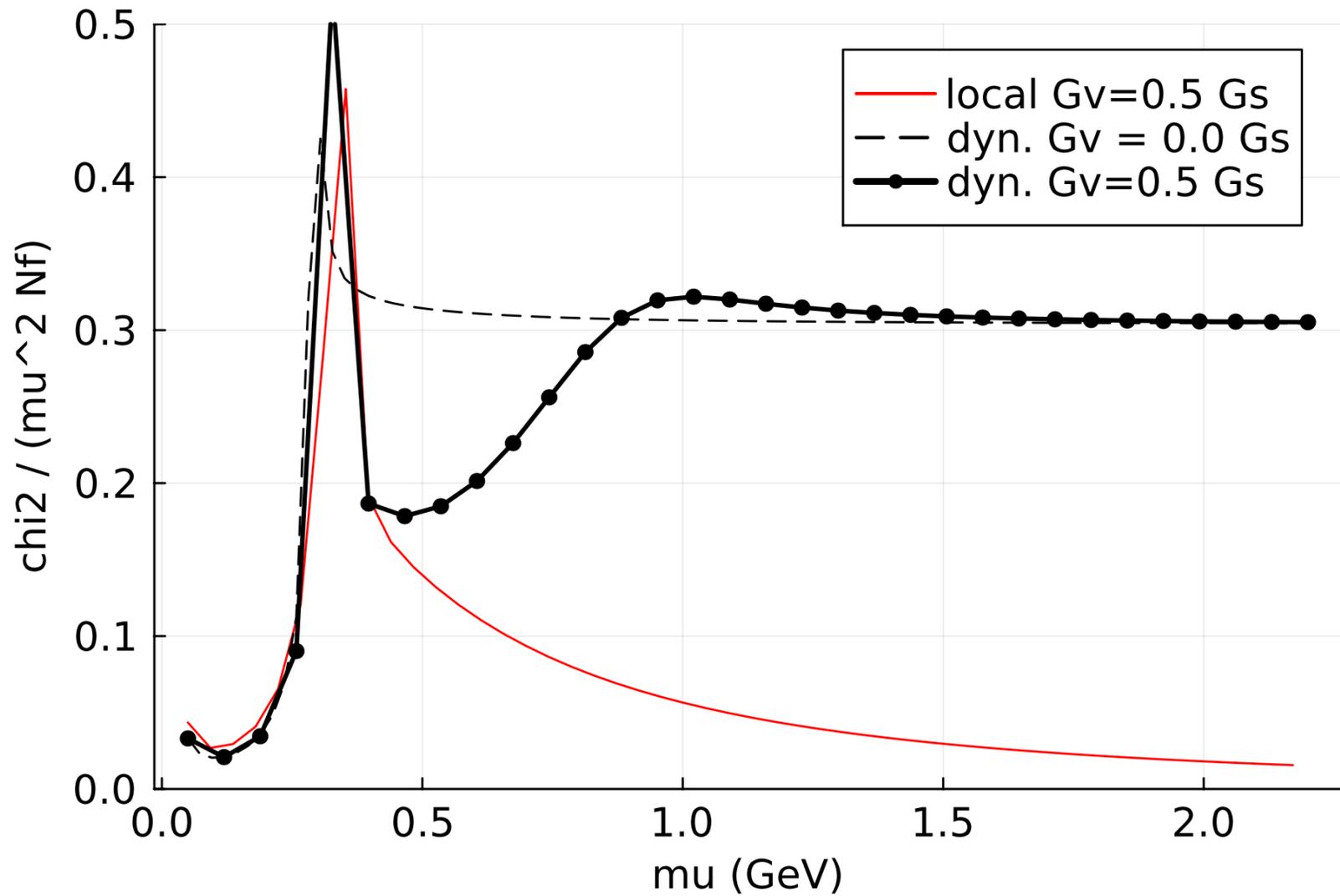*Faculty of Science and Technology, Ryukoku University, Seta, Otsu-city 520-21, Japan*

The quark-number susceptibility χ_q is examined as an observable which may help to reveal the physical picture of the high-temperature phase of QCD. It is emphasized that χ_q is intimately related with the fluctuations in the vector channel of the system. It is shown that the results of the recent lattice simulations of χ_q can be understood in terms of a possible change of the interactions between quark and anti-quarks in the vector channel, and imply that the fluctuations in the vector channel is greatly suppressed in the high-temperature phase in contrast with those in the scalar and pseudo-scalar ones.

QNS- FINITE TEMPERATURE DOMAIN

QNS- DENSE MATTER DOMAIN

local Gv=0.5 Gs
dyn. Gv = 0.0 Gs
dyn. Gv=0.5 Gs

SUMMARY

- NJL fails to reach conformal limit in dense matter
- Speed of sound highly sensitive to dynamical effect
- Momentum-dependence modifies quasiparticle dispersion relation, generating natural medium dependence -> conformal limit
- A dense matter theoretical model must ensure correct dressing of chemical potential to ensure correct asymptotics

OUTLOOK

- So far we only fix UV: conformal limit / perturbative QCD.
- IR: confinement and chiral symmetry <-> formally related to the gluon exchange.
- Can we find a density functional that gives comparable results to a dynamical model?

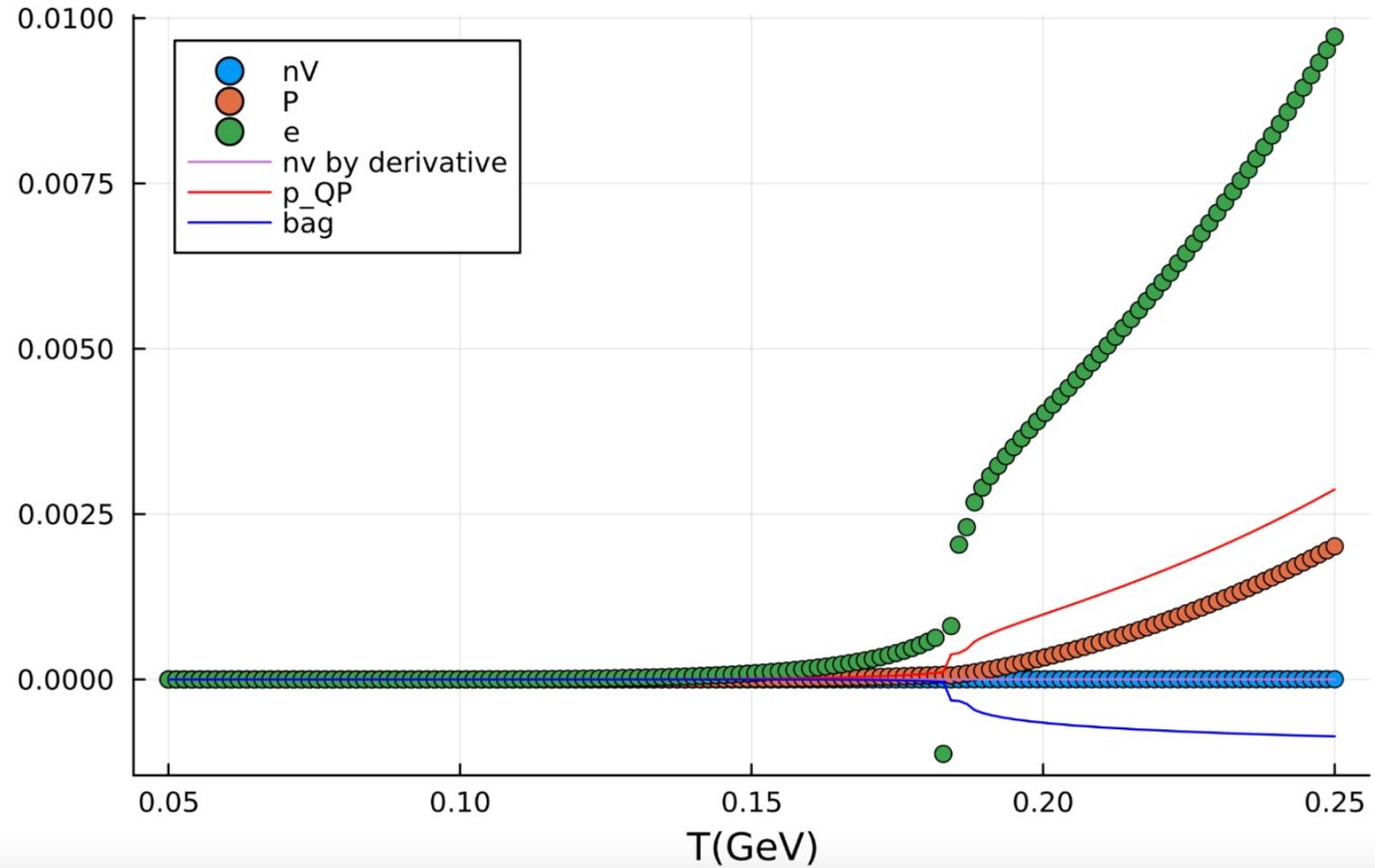$$\mu' = \mu + \frac{\partial U}{\partial \bar{n}_V}$$

- Dynamical effect in diquark gap and superconductivity.

BACKUP SLIDES

WALECKA – FINITE TEMPERATURE DOMAIN

Bag < QP

obs @ fixed mu=0.001

WALECKA – DENSE MATTER DOMAIN

Bag > QP