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CP asymmetries of $\overline{B} \to X_s/X_d\gamma$ in models with three Higgs doublets and the constraints from the electric dipole moment(EDM)

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with Andrew Akeroyd, Stefano Moretti, Tetsuo Shindou [arXiv:2009.05779]
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Overview

1. Motivation
2. Higgs fields in 3HDM (Three-Higgs-Doublet-Model)
3. Mixing matrix and Yukawa couplings for Charged Higgs
4. BR($\bar{B} \to X_s\gamma$) constraint for charged Higgs
5. CP-asymmetry observables ($\mathcal{A}^{\rm tot}_{X_s\gamma}$, $\Delta\mathcal{A}_{X_s\gamma}$, and $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$)
6. Electric-dipole moment (EDM) constraint for charged Higgs

Motivation of charged Higgs and 3HDM (3-Higgs-Doublets-Model)

- Existence of charged Higgs boson?

	SPIN 0	SPIN 1/2	SPIN 1
Charge 0	H	ν_e, ν_μ, ν_τ	γ, Z, g
Charge ± 1	$H^\pm$?	$e^\pm, \mu^\pm, \tau^\pm, u, d, c, s, t, b$	$W^\pm$

Reason for 3HDM:

- Not much literature attention as 2HDM.
- Rich scalar structure
- Dark Matter(Inert doublet)...
- Extra sources of CP-violation in charged scalar sector (vs. generic 2HDM).

Charged Higgs in 3HDM (Weinberg)

- Three active isospin fields $\Phi_i (i = 1, 2, 3)$ are introduced, and each contain a vacuum expectation value with sum rule

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ (v_i + \phi_i^{0,real} + i\phi_i^{0,imag})/\sqrt{2} \end{pmatrix}, \sum_i v_i^2 = v_{sm}^2 = (246 GeV)^2$$

- A unitary 3×3 matrix U is introduced in order to specify charged Higgs mass eigenstates (Left) from charged fields (Right) rotation: [Y. Grossman 1994]

$$\begin{pmatrix} G_1^+ \\ H_2^+ \\ H_3^+ \end{pmatrix} = U \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \phi_3^+ \end{pmatrix}.$$

3HDM Scalar potential under $Z_2 \times Z_2$ symmetry

$$
\begin{aligned}
V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + m_{33}^2\Phi_3^\dagger\Phi_3 \\
&- [m_{12}^2\Phi_1^\dagger\Phi_2 + m_{13}^2\Phi_1^\dagger\Phi_3 + m_{23}^2\Phi_2^\dagger\Phi_3 + \text{h.c.}] \\
&+ \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \frac{1}{2}\lambda_3(\Phi_3^\dagger\Phi_3)^2 \\
&+ \lambda_{12}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_{13}(\Phi_1^\dagger\Phi_1)(\Phi_3^\dagger\Phi_3) + \lambda_{23}(\Phi_2^\dagger\Phi_2)(\Phi_3^\dagger\Phi_3) \\
&+ \lambda_{12}'(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \lambda_{13}'(\Phi_1^\dagger\Phi_3)(\Phi_3^\dagger\Phi_1) + \lambda_{23}'(\Phi_2^\dagger\Phi_3)(\Phi_3^\dagger\Phi_2) \\
&+ \frac{1}{2}[\lambda_{12}''(\Phi_1^\dagger\Phi_2)^2 + \lambda_{13}''(\Phi_1^\dagger\Phi_3)^2 + \lambda_{23}''(\Phi_2^\dagger\Phi_3)^2 + \text{h.c.}],
\end{aligned}
$$

- 6 complex parameters: 3 soft-breaking masses $m_{12}^2, m_{13}^2, m_{23}^2$, 3 quartic couplings $\lambda_{12}'', \lambda_{13}'', \lambda_{23}''$.
- 2 of them could be eliminated under real VEV conditions.
- In addition with three imposed conditions, CP violation in neutral sector could be turned off and leave the $\text{Im}(\lambda_{12}'')$ to be the CP-source in charge sector.

Charged Higgs mixing matrix U in 3HDM

- The matrix U can be written explicitly as a function of four parameters $\tan\beta$, $\tan\gamma$, θ, and δ, where

$$\tan\beta = v_2/v_1, \qquad \tan\gamma = \sqrt{v_1^2 + v_2^2}/v_3 \, .$$

- v_1, v_2, and v_3 are the vacuum expectation values of the three Higgs doublets.
- θ is the mixing angle between H_2^+ and H_3^+
- δ is the CP-violating phase source.
- The explicit form of U given as :

$$= \begin{pmatrix} s_\gamma c_\beta & s_\gamma s_\beta & c_\gamma \\ -c_\theta s_\beta e^{-i\delta} - s_\theta c_\gamma c_\beta & c_\theta c_\beta e^{-i\delta} - s_\theta c_\gamma s_\beta & s_\theta s_\gamma \\ s_\theta s_\beta e^{-i\delta} - c_\theta c_\gamma c_\beta & -s_\theta c_\beta e^{-i\delta} - c_\theta c_\gamma s_\beta & c_\theta s_\gamma \end{pmatrix}$$

Here s, c denote the sine or cosine of the respective parameter.

Yukawa Couplings of charged Higgs in 3HDM

- Charged Higgs Yukawa interactions are written by:

$$\mathcal{L}_{H_i^\pm} = -\sum_{i=2}^{3} H_i^+ \{ \frac{\sqrt{2}V_{ud}}{v_{sm}} \bar{u}(m_d X_i P_R + m_u Y_i P_L) d + \frac{\sqrt{2}m_l}{v_{sm}} Z_i \bar{\nu}_L l_R \} + H.c.$$

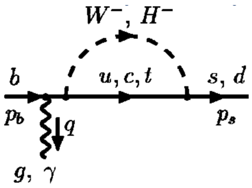
- Yukawa couplings for H_i^+ (with $i = 2, 3$) can be written as:

$$X_i = \frac{U^\dagger_{di}}{U^\dagger_{d1}}, \qquad Y_i = -\frac{U^\dagger_{ui}}{U^\dagger_{u1}}, \qquad Z_i = \frac{U^\dagger_{\ell i}}{U^\dagger_{\ell 1}}.$$

- Five independent versions of Yukawa interactions of 3HDM with NFC based on charged assignment of two softly-broken discrete Z_2 symmetries.

	u	d	ℓ
3HDM(Type I)	2	2	2
3HDM(Type II)	2	1	1
3HDM(Lepton-specific)	2	2	1
3HDM(Flipped)	2	1	2
3HDM(Democratic)	2	1	3

BR($\bar{B} \to X_s\gamma$) constraint for $H^\pm$



- BR($\bar{B} \to X_s\gamma$) limits $|Y_i^2|$, $X_i Y_i^*$ and $M_{H_i^\pm}$, $i = 2, 3$.
- Parameter space difference between 2HDM ($\tan\beta = \frac{v_2}{v_1}$) and 3HDM ($\tan\beta$, $\tan\gamma$, θ, δ)
- Evaluate NLO BR($\bar{B} \to X_s\gamma$) through 2HDM [F. Borzumati and C. Greub, arXiv:hep-ph/9802391]and extrapolate to 3HDM account from two charged Higgs boson states and corresponding Yukawa couplings.

CP-asymmetry observables ($\mathcal{A}^{\rm tot}_{X_s\gamma}$, $\Delta\mathcal{A}_{X_s\gamma}$, and $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$)

- $\bar{B} \to X_s\gamma$ alone does not give any evidence on new Physics since SM and experiment are quite good agreement even in NNLO prediction.
- Direct CP-asymmetry $\mathcal{A}^{\rm tot}_{X_{s(d)}\gamma}$: $\frac{\Gamma(\overline{B}\to X_{s(d)}\gamma)-\Gamma(B\to X_{s(d)}\gamma)}{\Gamma(\overline{B}\to X_{s(d)}\gamma)+\Gamma(B\to X_{s(d)}\gamma)}$
- Difference of CP-asymmetry $\Delta\mathcal{A}_{X_s\gamma}$: $\mathcal{A}^{\pm}_{X_s\gamma} - \mathcal{A}^{0}_{X_s\gamma}$
- Untagged CP-asymmetry $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$: $\frac{\mathcal{A}_{X_s\gamma}+R_{ds}\mathcal{A}_{X_d\gamma}}{1+R_{ds}}$.$R_{ds} \approx |V_{td}/V_{ts}|^2$
- B^0 for neutral B mesons ($\mathcal{A}^{0}_{X_{s(d)}\gamma}$) and $B, \bar{B}$ for charged B mesons(B^+, B^-) ($\mathcal{A}^{\pm}_{X_{s(d)}\gamma}$).
- Main contribution from (current-current operator) C_2 , (magnetic penguin operator) $C_7(b \to s\gamma)$ and $C_8(b \to sg)$ constrain $|Y_i^2|$, $X_i Y_i^*$ and $M_{H_i^{\pm}}$, $i = 2, 3$.

CP-asymmetry measurements recent and future

	BELLE	BABAR	World Average
$\mathcal{A}^{\rm tot}_{X_s\gamma}$	$(1.44 \pm 1.28 \pm 0.11)\%$	$(1.73 \pm 1.93 \pm 1.02)\%$	$1.5\% \pm 1.1\%$
$\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$	$(2.2 \pm 3.9 \pm 0.9)\%$	$(5.7 \pm 6.0 \pm 1.8)\%$	$1.0\% \pm 3.1\%$
$\Delta\mathcal{A}_{X_s\gamma}$	$(3.69 \pm 2.65 \pm 0.76)\%$	$(5.0 \pm 3.9 \pm 1.5)\%$	$4.1\% \pm 2.3\%$

BELLE II	SM Prediction	Leptonic tag	Hadronic tag	Sum of exclusives
$\mathcal{A}_{X_s\gamma}$	$-1.9\% < \mathcal{A}_{X_s\gamma} < 3.3\%$	x	x	0.19%
$\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$	0	0.48%	0.70%	x
$\Delta\mathcal{A}_{X_s\gamma}$	0	x	1.3%	0.3%

- Both SM $\Delta\mathcal{A}_{X_s\gamma}$ and $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$ are 0 due to real Wilson coefficients and CKM unitarity respectively.
- Due to SM $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$ is essentially zero, 2.5% with an error 0.5% could constitute a 5σ signal of physics beyond SM.

$\bar{B} \to X_s\gamma$ and three CP-asymmetry observables

- 6 parameters (two charged Higgs masses($M_{H_2^\pm}, M_{H_3^\pm}$) and mixing parameters($\tan\beta, \tan\gamma, \theta, \delta$)) to constrain $|Y_i^2|$, $X_i Y_i^*$.
- We take the 3σ measured allowed range : $2.87 \leq \bar{B} \to X_s\gamma \leq 3.77$ $(\times 10^{-4})$ with central value $3.32(\times 10^{-4})$ to set the limit. And take 3σ world average CP-asymmetry observables for allowed parameter space.
- Type II, Flipped and Democratic 3HDM parameter space are taken since the Yukawa coupling structure are similar.

$$X_i = \frac{U^\dagger_{di}}{U^\dagger_{d1}}, \qquad Y_i = -\frac{U^\dagger_{ui}}{U^\dagger_{u1}}, \qquad Z_i = \frac{U^\dagger_{\ell i}}{U^\dagger_{\ell 1}}.$$

	u	d	ℓ
3HDM(Type I)	2	2	2
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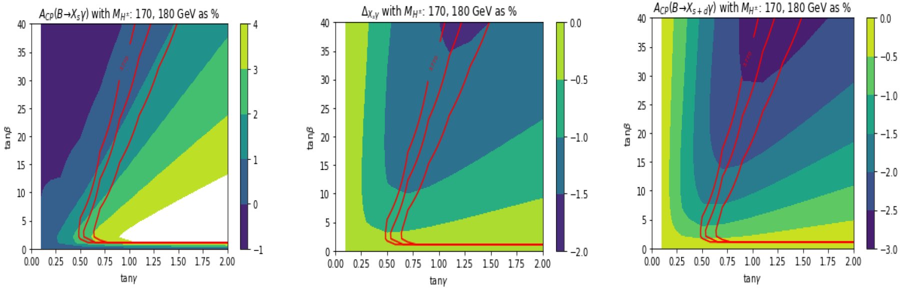
$\mathcal{A}^{\rm tot}_{X_s\gamma}$, $\Delta\mathcal{A}_{X_s\gamma}$, and $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$ under $[\tan\gamma, \tan\beta]$



CP-asymmetry observables in the plane $[\tan\gamma, \tan\beta]$, with $\theta = -\frac{\pi}{4}$, $\delta = 2.64$.
Left Panel: $\mathcal{A}^{\rm tot}_{X_s\gamma}$. Middle Panel: $\Delta\mathcal{A}_{X_s\gamma}$. Right Panel: $\mathcal{A}_{\rm CP}(B \to X_{s+d}\gamma)$.
Between the red color lines are allowed by $\bar{B} \to X_s\gamma$. 2.5% can be achieved.

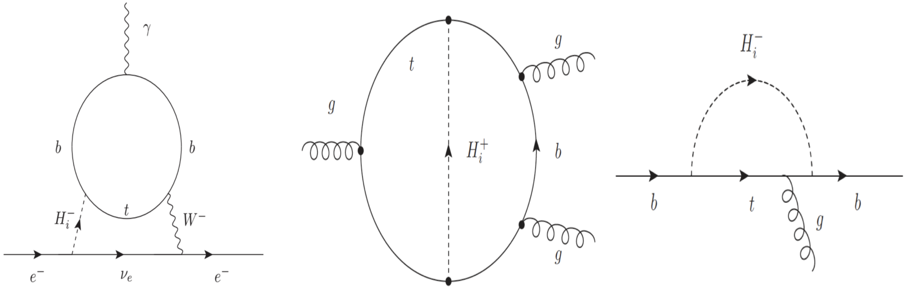
Electric-diopole moment(EDM) constraints for CP-violation

	SM Prediction	Experimental bound
Neutron-EDM(nEDM)	$\sim 10^{-31} - 10^{-32}$ e cm.	$\lvert d_n \rvert < 1.8 \times 10^{-26}$ e cm. [arXiv:2001.11966]
Electron-EDM(eEDM)	$\sim 10^{-38}$ e cm.	$\lvert d_e \rvert < 1.1 \times 10^{-29}$ e cm. [*Nature. 562 (7727): 355–360*]

- nEDM and eEDM in SM with CKM phases [Maxim and Adam 2005, arXiv:hep-ph/0504231].

$$\mathcal{L}_{H_i^\pm} = -\sum_{i=2}^{3} H_i^+ \{ \frac{\sqrt{2} V_{ud}}{v_{sm}} \bar{u}(m_d X_i P_R + m_u Y_i P_L) d + \frac{\sqrt{2} m_l}{v_{sm}} Z_i \bar{\nu}_L l_R \} + H.c.$$

EDMs in the charged scalar sector in 3HDM



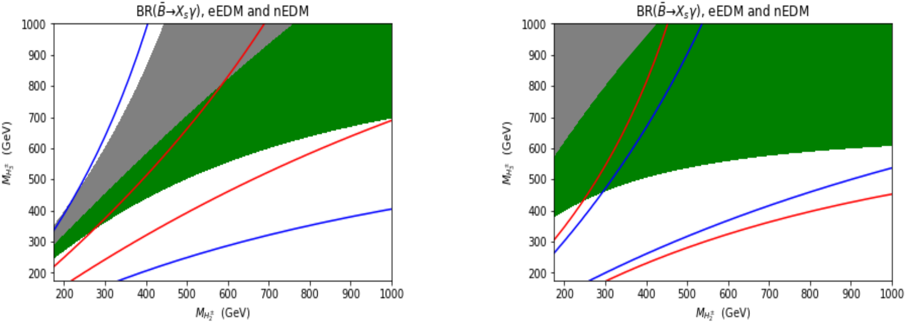
- eEDM via Yukawa couplings. Dominant contribution comes from 2-loop Barr-Zee type diagrams. Left.
- nEDM via the dominant Weinberg operator (three gluon operators) and b-quark Chromo-EDM(effective 5 operators). Middle and Right.

Cancellation in charged Higgs contribute to EDMs

$$\mathcal{L}_{H_i^\pm} = -\sum_{i=2}^{3} H_i^+\{\frac{\sqrt{2}V_{ud}}{v_{sm}}\bar{u}(m_d X_i P_R + m_u Y_i P_L)d + \frac{\sqrt{2}m_l}{v_{sm}} Z_i \bar{\nu}_L l_R\} + H.c.$$

- After the effective field framework framework and dimensional analysis work as A2HDM[arXiv:1308.6283], results are extrapolated and calculated to constrain the $\mathrm{Im}(X_i Y_i^*)$ (nEDM) and $\mathrm{Im}(Y_i^* Z_i)$ (eEDM).
- Taking $M_{H_2^\pm}, M_{H_3^\pm}$, $\tan\beta, \tan\gamma, \theta, \delta$, we took the experiment bound as the up-limit to finalise the results.
- The effect of CP-violation via charged Higgs exchange can be suppressed by making two physical- charged Higgs degenerate in mass.

BR($\bar{B} \to X_s\gamma$) and two EDMs(nEDM and eEDM) under $[M_{H_2^\pm}, M_{H_3^\pm}]$



2 σ BR($\bar{B} \to X_s\gamma$)(Grey and Green area), eEDM(between blue lines), nEDM(between red lines) with $\theta = -\pi/2.1$, $\tan\beta = 20$, $\delta = \pi/2$.
Left Panel: $\tan\gamma = 1$. Right Panel: $\tan\gamma = 2$.
Tunnel effect generated from charged Higgs mass degeneracy (GIM-like) due to unitarity of charged Higgs mixing matrix U.

Summary

- We studied charged Higgs sector in 3HDM (contains 3 active doublets)
- We studied NLO BR($\bar{B} \to X_s\gamma$) in 3HDM.
- CP-asymmetry observables ($\mathcal{A}^{\rm tot}_{X_s\gamma}$, $\Delta\mathcal{A}_{X_s\gamma}$, and $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$) can give a signal at BELLE II in 3HDM.
- $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$ is of particular interest as SM prediction is essentially zero.
- In the case of isolation of neutral sector, the contribution of EDMs (nEDM and eEDM) for charged sector are calculated based on the mixing parameters.
- In particular, the suppression of EDM constraint from degeneracy of two charged Higgs masses ($M_{H_2^\pm}, M_{H_3^\pm}$) are realised.

Thanks for Listening

Backup slides

Branching ratio of $\bar{B} \to X_s\gamma$

- short distance pertubative $b \to s\gamma$ ($|\bar{D}|$)
- short distance perturbative $b \to s\gamma g$ (A) (gluon Bremsstrahlung process)
- Long distance non perturbative corrections to scale $\frac{1}{m_b^2}$ and $\frac{1}{m_c^2}$.

$$
\begin{aligned}
\Gamma(\bar{B} \to X_s\gamma) &= \frac{G_F^2}{32\pi^4}|V_{ts}^* V_{tb}|^2 \alpha_{em} m_b^5 \\
&\times \left\{ |\bar{D}|^2 + A + \frac{\delta_\gamma^{NP}}{m_b^2}|C_7^{0,\text{eff}}(\mu_b)|^2 \right. \\
&+ \left. \frac{\delta_c^{NP}}{m_c^2}\mathrm{Re}\left[[C_7^{0,\text{eff}}(\mu_b)]^* \times \left(C_2^{0,\text{eff}}(\mu_b) - \frac{1}{6} C_1^{0,\text{eff}}(\mu_b) \right) \right] \right\}. \\
\mathrm{BR}(\bar{B} \to X_s\gamma) &= \frac{\Gamma(\bar{B} \to X_s\gamma)}{\Gamma_{SL}} \mathrm{BR}_{SL}
\end{aligned}
$$

- Γ_{SL} is the semileptonic decay width and BR_{SL} is the measured semileptonic decay branching ratio

Formulas for charged Higgs in eEDM

$$
\begin{aligned}
d_e(M_{H_2^\pm}, M_{H_3^\pm})_{BZ} &= -m_e \frac{24 G_F^2 M_W^2}{(4\pi)^4} |V_{tb}|^2 \\
&\times \Big[\mathrm{Im}(-Y_2^* Z_2) \left(q_t F_t(z_{H_2^\pm}, z_W) + q_b F_b(z_{H_2^\pm}, z_W) \right) \\
&+ \mathrm{Im}(-Y_3^* Z_3) \left(q_t F_t(z_{H_3^\pm}, z_W) + q_b F_b(z_{H_3^\pm}, z_W) \right) \Big]
\end{aligned}
$$

where $q_t = 2/3$ and $q_b = -1/3$ are quark electric charges, $z_a = M_a^2/m_t^2$.

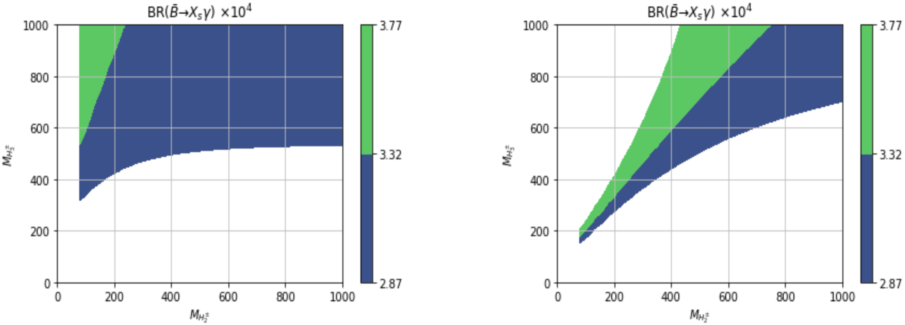
$$
\begin{aligned}
F_q(z_{H_i^\pm}, z_W) &= \frac{T_q(z_{H_i^\pm}) - T_q(z_W)}{z_{H_i^\pm} - z_W}, \\
T_t(x) &= \frac{1-3x}{x^2}\frac{\pi^2}{6} + \left(\frac{1}{x} - \frac{5}{2}\right)\log x - \frac{1}{x} - \left(2 - \frac{1}{x}\right)\left(1 - \frac{1}{x}\right)\mathrm{Li}_2(1-x), \\
T_b(x) &= \frac{2x-1}{x^2}\frac{\pi^2}{6} + \left(\frac{3}{2} - \frac{1}{x}\right)\log x + \frac{1}{x} - \frac{1}{x}\left(2 - \frac{1}{x}\right)\mathrm{Li}_2(1-x).
\end{aligned}
$$

CP-asymmetry observables $\mathcal{A}_{X_{s(d)}\gamma}$ and $\Delta\mathcal{A}_{X_s\gamma}$

$$\begin{aligned}
\mathcal{A}_{X_{s(d)}\gamma} &\approx \pi\Bigg\{\left[\left(\frac{40}{81}-\frac{40}{9}\frac{\Lambda_c}{m_b}\right)\frac{\alpha_s}{\pi}+\frac{\tilde{\Lambda}^c_{17}}{m_b}\right]\text{Im}\,\frac{C_2}{C_{7\gamma}} \\
&- \left(\frac{4\alpha_s}{9\pi}-4\pi\alpha_s e_{\text{spec}}\frac{\tilde{\Lambda}_{78}}{m_b}\right)\text{Im}\,\frac{C_{8g}}{C_{7\gamma}} \\
&- \left(\frac{\tilde{\Lambda}^u_{17}-\tilde{\Lambda}^c_{17}}{m_b}+\frac{40}{9}\frac{\Lambda_c}{m_b}\frac{\alpha_s}{\pi}\right)\text{Im}\left(\epsilon_{s(d)}\frac{C_2}{C_{7\gamma}}\right)\Bigg\} \\
\Delta\mathcal{A}_{X_s\gamma} &\approx 4\pi^2\alpha_s\frac{\tilde{\Lambda}_{78}}{m_b}\text{Im}\,\frac{C_{8g}}{C_{7\gamma}}\,.
\end{aligned}$$

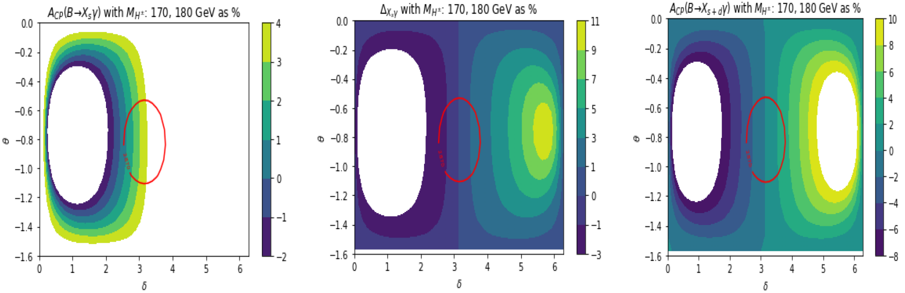
- $\tilde{\Lambda}^u_{17}, \tilde{\Lambda}^c_{17}, \tilde{\Lambda}_{78}$ are long distance hadronic parameters. $\Lambda_c = 0.38$ GeV.
- $e_{spec} = -\frac{1}{3}(\mathcal{A}^0_{X_{s(d)}\gamma})$ or $\frac{2}{3}(\mathcal{A}^{\pm}_{X_{s(d)}\gamma})$.
- $\epsilon_s = (V_{ub}V^*_{us})/(V_{tb}V^*_{ts}), \epsilon_d = (V_{ub}V^*_{ud})/(V_{tb}V^*_{td})$.

BR($\bar{B} \to X_s\gamma$) under $[M_{H_2^\pm}, M_{H_3^\pm}]$



3 σ BR($\bar{B} \to X_s\gamma$) in the plane $[m_{H_2^\pm}, m_{H_3^\pm}]$, with $\theta = -\pi/2.1$, $\tan\beta = 10$, $\tan\gamma = 1$. Left Panel: $\delta = 0$. Right Panel: $\delta = \pi/2$. δ has large effect on BR

$\mathcal{A}^{\rm tot}_{X_s\gamma}$, $\Delta\mathcal{A}_{X_s\gamma}$, and $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$ under $[\delta, \theta]$



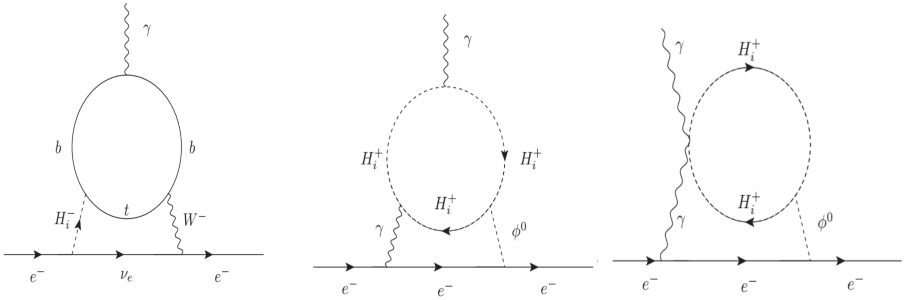
CP-asymmetry observables in the plane $[\delta, \theta]$, with $\tan\beta = 35$, $\tan\gamma = 1.32$. Left Panel: $\mathcal{A}^{\rm tot}_{X_s\gamma}$. Middle Panel: $\Delta\mathcal{A}_{X_s\gamma}$. Right Panel: $\mathcal{A}_{\rm CP}(\bar{B} \to X_{s+d}\gamma)$. Inside the red color ellipse is allowed by $\bar{B} \to X_s\gamma$. 2.5% can be achieved.

Formulas for charged Higgs in nEDM

$$
\begin{aligned}
|d_n(C_W)/e| &= \left[1.0\,{}^{+1.0}_{-0.5}\right] \times 20\,\text{MeV}\; C_W(\mu_h),\, \mu_h \approx 1\text{GeV} \\
C_W(\mu_h) &= \eta_{c-h}^{\kappa_W}\eta_{b-c}^{\kappa_W}\left(\eta_{t-b}^{\kappa_W} C_W(\mu_{tH}) + \eta_{t-b}^{\kappa_C}\frac{g_s^3(\mu_b)}{8\pi^2 m_b}\frac{d_b^C(\mu_{tH})}{2}\right) \\
\frac{d_b^C(\mu_{tH})}{2} &= -\frac{G_F}{\sqrt{2}}\frac{1}{16\pi^2}|V_{tb}|^2 m_b(\mu_{tH})[\text{Im}(-X_2Y_2^*)x_{tH_2}\left(\frac{\log(x_{tH_2})}{(x_{tH_2}-1)^3}\right. \\
&+ \left.\frac{(x_{tH_2}-3)}{2(x_{tH_2}-1)^2}\right) + \text{Im}(-X_3Y_3^*)x_{tH_3}\left(\frac{\log(x_{tH_3})}{(x_{tH_3}-1)^3} + \frac{(x_{tH_3}-3)}{2(x_{tH_3}-1)^2}\right)]
\end{aligned}
$$

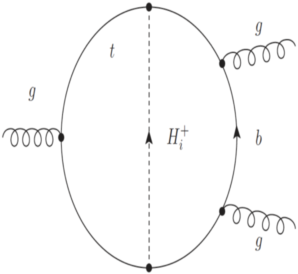
- where $x_{tH_i} = m_t^2/M_{H_i^\pm}^2$ and $C_W(\mu_{tH}) = 0$.
- $\kappa_i = \gamma_i/(2\beta_0)$, where $\gamma_W = N_C + 2n_f$ and $\gamma_C = 10C_F - 4N_C$.

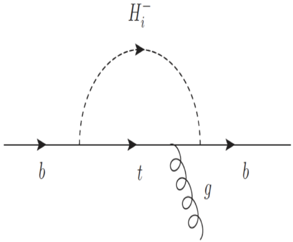
Barr-Zee diagram for eEDM



- $H_i^+ f_u \bar{f}_d$ is the dominant contribution.
- $\Phi^0 e^+ e^-$ and $\Phi^0 H_i^+ H_i^-$ are not included as neutral sector contain no CP-violation phase.
- $\Phi^0 H_i^+ H_j^-$ may have CP-violation however the coupling does not appear in such diagram as photon couples to charged Higgs is diagonal.

3-gluon contribution and Chromo-EDM for nEDM





- Weinberg operator on the left.
- b-quark Chromo-EDM on the right.

Cancellation in charged Higgs contribute to EDMs continued

- Such sort of GIM-like mechanism is due to the unitarity of the charged Higgs mixing matrix U.
- nEDM constrains $\mathrm{Im}(X_i Y_i^*)$ and eEDM constrains $\mathrm{Im}(Y_i^* Z_i)$.
- eEDM contribution is zero for Type I and Flipped 3HDM as $Y_i = Z_i$.
- nEDM contribution is zero for Type I and Lepton-specific 3HDM as $X_i = Y_i$.

$$X_i = \frac{U^\dagger_{di}}{U^\dagger_{d1}}, \qquad Y_i = -\frac{U^\dagger_{ui}}{U^\dagger_{u1}}, \qquad Z_i = \frac{U^\dagger_{\ell i}}{U^\dagger_{\ell 1}}.$$

	u	d	ℓ
3HDM(Type I)	2	2	2
3HDM(Type II)	2	1	1
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3HDM(Flipped)	2	1	2
3HDM(Democratic)	2	1	3

Cancellation in charged Higgs contribute to EDMs continued

By taking the Democratic-type model for nEDM as an example,

$$
\begin{aligned}
X_i Y_i^* &= -\frac{U_{1i}^\dagger U_{i2}}{U_{11}^\dagger U_{12}}, \\
\sum_{i=2}^{3} \mathrm{Im}(X_i Y_i^*) f(M_{H_i^+}) &= -\frac{1}{U_{11}^\dagger U_{12}} [\mathrm{Im}(U_{12}^\dagger U_{22}) f(M_{H_2^+}) \\
&+ \mathrm{Im}(U_{13}^\dagger U_{32}) f(M_{H_3^+})].
\end{aligned}
$$

- where $f(M_{H_i^+})$ represents the dependence of the diagram on the charged Higgs boson mass.
- In the case of $M_{H_2^\pm} = M_{H_3^\pm} \equiv m$, such result will be $= -\frac{1}{U_{11}^\dagger U_{12}} \mathrm{Im}(\delta_{12}) f(m) = 0$ as $\mathrm{Im}(X_2 Y_2^*)$ = - Im $(X_3 Y_3^*)$