Structure of Local Conservation Laws in Quantum Integrable Systems

In this talk, I explain the structure of the local consrvetaion law in several interacting integrable systems.

Quantum integrable systems are exactly solvable by the Bethe ansatz.

Behind their exact solvability, there is an infinite number of local conserved quantities $\{Q_k\}_{k=2,3,4,...}$

Although the existence of Q_k itself is guaranteed from the quantum inverse scattering method, obtaining their explicit expressions is challenging.

This difficulty lies not only in the expensive computational cost for higher order charges but also in finding a general pattern in the enormous amounts of data that emerge from this calculation.

I will review recent progress on this problem, including the derivation of general explicit expressions for the local conserved quantities in the spin-1/2 XYZ chain [Nozawa, Fukai, Phys. Rev. Lett., 2020] and the one-dimensional Hubbard model [Fukai, Phys. Rev. Lett., 2023].

In addition, I will explain how the complex structure of the local conserved quantities can be simplified using matrix product operator (MPO) representations, as demonstrated for the Heisenberg chain [Yamada, Fukai, SciPost Phys. Core, 2023].

In the case of the XYZ chain, we observe that a weighted-path generalization of the Catalan numbers emerges in the structure of the local conserved quantities—extending the Catalan tree pattern originally found by Grabowski and Mathieu in the Heisenberg chain [Grabowski, Mathieu, J. Math. Phys., 1995].

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