

Lie Symmetries and Bi-Hamiltonian structures of the Pais-Uhlenbeck Oscillator

The Lie symmetries of the Pais-Uhlenbeck Oscillator (PUO) are identified. They are then used to generate the Bi-Hamiltonian structure of this system. We then study how we might leverage this Bi-Hamiltonian structure to mitigate the pathologies associated with theories where, as in the case of the PUO, the lagrangian admits time derivatives of order two or higher. Theories of this nature are usually thought to admit classical Hamiltonians that are unbounded from below, and either unbounded energy eigenvalue spectra or negative norm "ghost" states at the quantum level. For this reason they are often generally referred to as "ghost ridden". However the appearance of a Bi-Hamiltonian structure can be shown to allow for the generation of positive definite Hamiltonians with appropriate symplectic structure in the case of the free theory. In addition, we leverage the Bi-Hamiltonian structure to construct families of transforms between the higher derivative Pais-Uhlenbeck oscillator and a two dimensional system where the dynamics and symplectic structure are preserved. We will also discuss the implications of including interactions, which provides an important caveat to this argument. The results presented here are reported in [1,2].

References

- [1] Fring A, Felski A, and Turner B. Equivalent representations and alternative formulations of higher time-derivative models. in preperation.
- [2] Fring A, Taira T, and Turner B. Quantisations of exactly solvable ghostly models. 2025. arXiv: 2503.21447 [quant-ph]. Submitted for publication in J. Phys. A: Math. Theor.

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