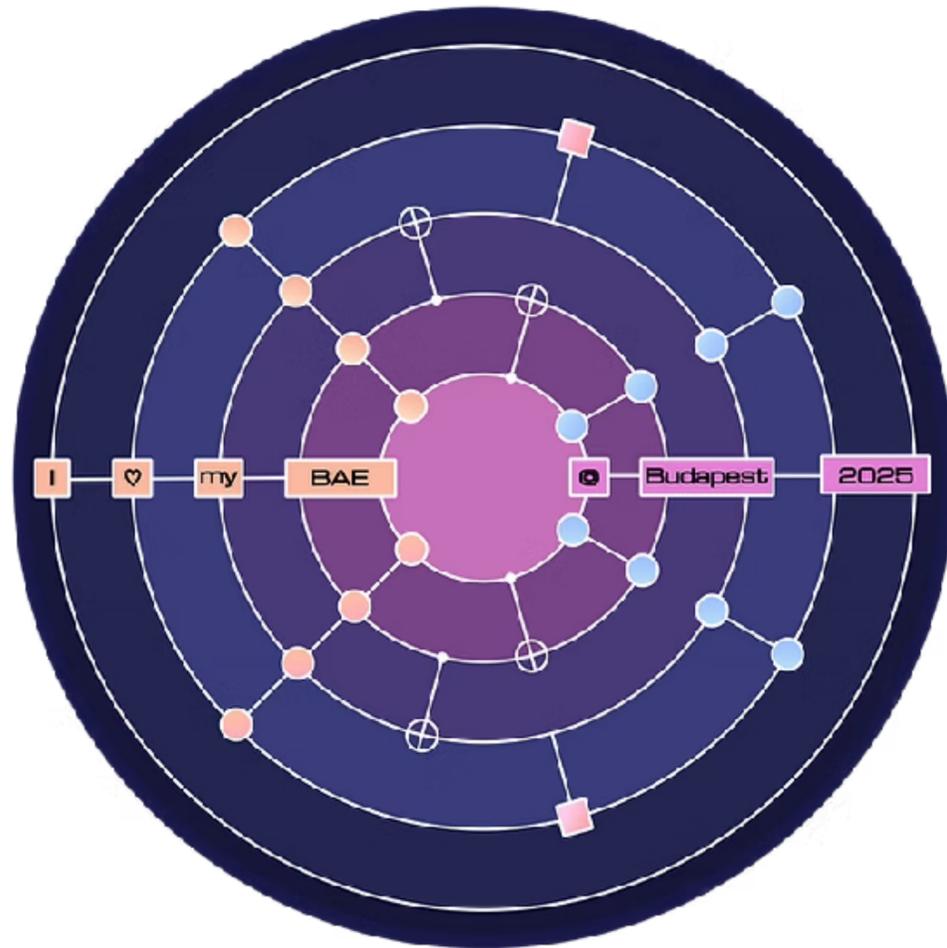# Exact quantum state preparation

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Student Workshop on Integrability
Eötvös Loránd University, Budapest

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Introduction

Huge international investment ($) in building quantum computers

Hope: advantage over classical computers for certain problems, such as in many-body physics

Ex: spin-1/2 chain of length L

$|\psi\rangle \in (\mathbb{C}^2)^{\otimes L}$ dimension 2^L grows *exponentially* in L 🙁

                    # qubits L grows *linearly* in L 🙂   exponential advantage?

How to exactly prepare a given state $|\psi\rangle$ (say, an eigenstate of a Hamiltonian) on a quantum computer?

Beautiful ideas & techniques!

Outline
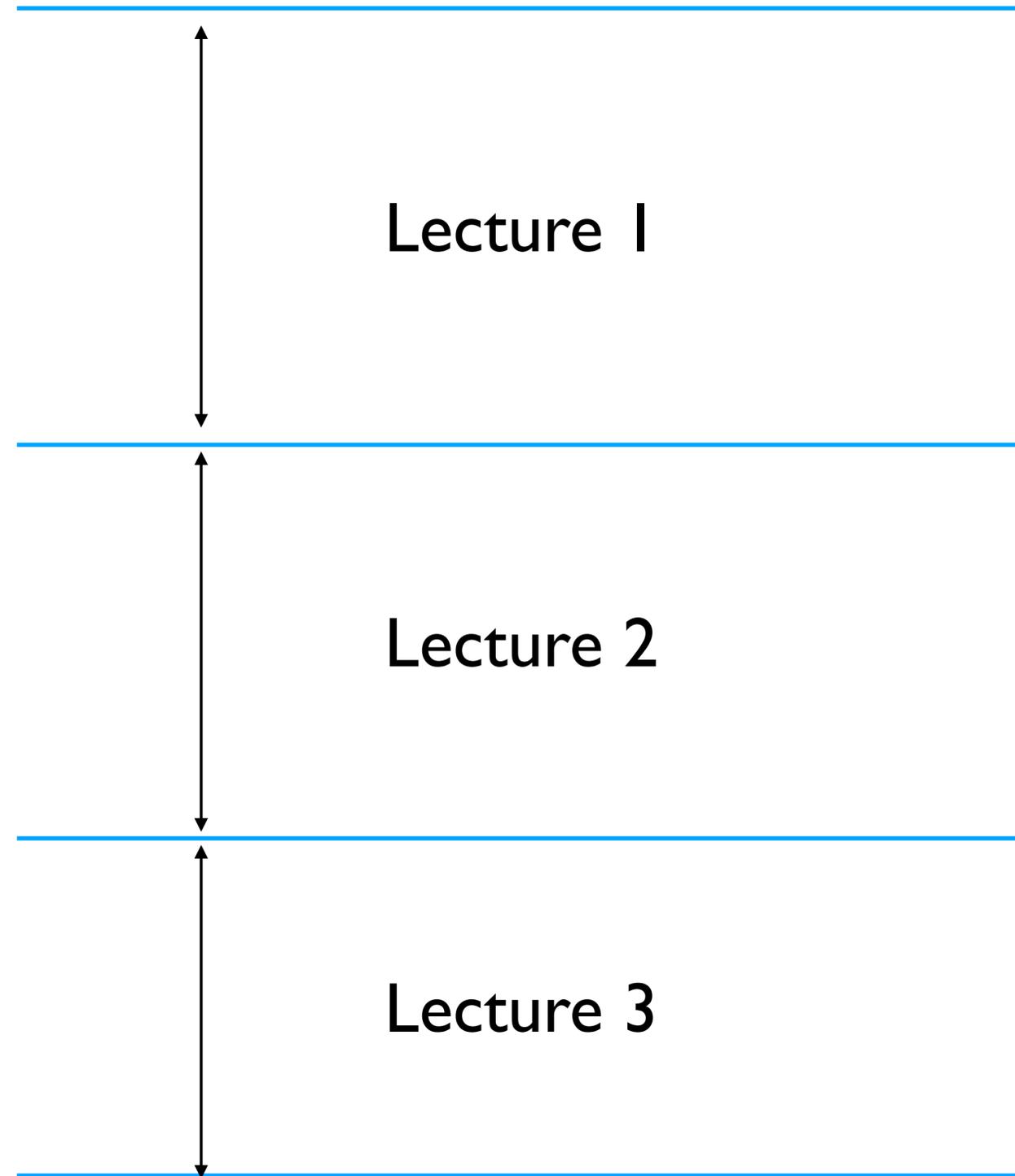
Quantum computing basics

GHZ states & dynamic circuits

Matrix Product States (MPS)

Fusion measurements

AKLT states

Dicke states

Bethe states

Lecture 1

Lecture 2

Lecture 3

Quantum computing basics

"Never underestimate the joy people derive from hearing something they already know."

— Enrico Fermi

"qubit" : 2-state system

basis: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

"Quantum computer":

- device with n qubits

"computational basis"

basis: $|x\rangle_n = |x_{n-1}\rangle \otimes \cdots \otimes |x_0\rangle$ $x_j \in \{0,1\}$ $x = \sum_{j=0}^{n-1} x_j 2^j$ integer $0 \leq x < 2^n$

binary bits

Ex: $2 = 010_2$

$$|2\rangle_3 = |0\rangle \otimes |1\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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binary bits

Tensor products are often suppressed!

$$|x\rangle_n = |x_{n-1}\rangle \cdots |x_0\rangle$$

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binary bits

- can initialize each qubit $|0\rangle^{\otimes n}$

- can perform unitary transformations on qubits (decomposed into 1-qubit & 2-qubit unitary "gates")

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binary bits

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Example: $\quad$ NOT $\quad X = \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{aligned} X|0\rangle &= |1\rangle \\ X|1\rangle &= |0\rangle \end{aligned} \qquad X|x\rangle = |x \oplus 1\rangle \qquad x \in \{0,1\}$

$\oplus$: addition mod 2

"qubit" : 2-state system

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  "computational basis"

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  binary bits

- can initialize each qubit $|0\rangle^{\otimes n}$

- can perform unitary transformations on qubits (decomposed into 1-qubit & 2-qubit unitary "gates")

  Example: Hadamard $\quad H = \dfrac{1}{\sqrt{2}}\left(X + Z\right) = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad\qquad H|0\rangle = \dfrac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

  $$H|1\rangle = \dfrac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

"qubit" : 2-state system

basis: $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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"computational basis"

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binary bits

- can initialize each qubit $|0\rangle^{\otimes n}$

- can perform unitary transformations on qubits (decomposed into 1-qubit & 2-qubit unitary "gates")

"target" changes to $X|y\rangle$ if $x = 1$

Example: CNOT $\qquad$ $C^X |x\rangle |y\rangle = |x\rangle |x \oplus y\rangle$

$x, y \in \{0,1\}$

"control" does not change

$\oplus$: addition mod 2

"qubit" : 2-state system

basis: $\quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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  binary bits

- can initialize each qubit $|0\rangle^{\otimes n}$

- can perform unitary transformations on qubits (decomposed into 1-qubit & 2-qubit unitary "gates")

- can perform projective measurements of $\displaystyle\sum_{0 \le x < 2^n} x |x\rangle \langle x| \qquad |\Psi\rangle_n = \sum_{0 \le x < 2^n} \alpha_x |x\rangle \mapsto |x\rangle$

  probability $\quad |\alpha_x|^2$

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"quantum circuit"

probability $\quad |\alpha_x|^2$

"Circuit diagrams":

- represent qubits by horizontal "wires" :

$$|x\rangle_n = |x_{n-1}\rangle \otimes \cdots \otimes |x_0\rangle$$

$|x_0\rangle$ ——————

$\vdots$

$|x_{n-1}\rangle$ ——————

$= \quad |x\rangle_n \quad \overset{n}{\diagup}$ ——————

- represent gates symbolically:

Examples: $\quad X$

X

C^X

X

"control"

$=$

"Circuit diagrams":

- represent qubits by horizontal "wires" :

$$|x\rangle_n = |x_{n-1}\rangle \otimes \cdots \otimes |x_0\rangle$$

$|x_0\rangle$ ———————

$\vdots$

$|x_{n-1}\rangle$ ———————

$=$ $|x\rangle_n$ $\overset{n}{\diagup}$ ———————

- represent gates symbolically:

- "time" flows to the right:

$$V U |\Psi\rangle_n$$

$|\Psi\rangle_n$ $\overset{n}{\diagup}$ ———$\boxed{U}$———$\boxed{V}$———

time →

Currently: $n \sim 10^2$ IBM, Google, …   "noisy" - make many errors!   Noisy Intermediate-Scale Quantum era

Dream: $n \sim 10^4$ fault-tolerant

In the meantime, can test algorithms ("quantum circuits") using noiseless *simulators*

Here: IBM Qiskit simulators $n \sim 30$

Exercise: show

=

change order + new gate

Exercise: show

=

i.e.

$|x_0\rangle$

$|x_1\rangle$

$|x_2\rangle$

=

$|x_0\rangle$

$|x_1\rangle$

$|x_2\rangle$

for all $\quad x_i \in \{0, 1\}$

$$C^X |x\rangle |y\rangle = |x\rangle |x \oplus y\rangle$$

Exercise: show

i.e.

for all $\quad x_i \in \{0, 1\}$

$$C^X |x\rangle |y\rangle = |x\rangle |x \oplus y\rangle$$

LHS: 1 $|x_2\rangle |x_1\rangle |x_0\rangle$

          2    $|x_2\rangle |x_0 \oplus x_1\rangle |x_0\rangle$

          3    $|x_0 \oplus x_1 \oplus x_2\rangle |x_0 \oplus x_1\rangle |x_0\rangle$

RHS: 1 $|x_2\rangle |x_1\rangle |x_0\rangle$

          2    $|x_0 \oplus x_2\rangle |x_1\rangle |x_0\rangle$

          3    $|x_0 \oplus x_1 \oplus x_2\rangle |x_1\rangle |x_0\rangle$

          4    $|x_0 \oplus x_1 \oplus x_2\rangle |x_0 \oplus x_1\rangle |x_0\rangle$ = LHS   ✓

Bell states

$$|B_{x,y}\rangle = \frac{1}{\sqrt{2}}(|y\rangle|0\rangle + (-1)^x|y \oplus 1\rangle|1\rangle) \qquad x, y \in \{0, 1\} \qquad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Ex: $\quad |B_{0,0}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$

The 4 Bell states form a basis of 2-qubit states

Exercise: show

$$|B_{x,y}\rangle =$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

1 $\quad |y\rangle|x\rangle$

2 $\quad |y\rangle H|x\rangle = |y\rangle \frac{1}{\sqrt{2}}(|0\rangle + (-1)^x|1\rangle) \;=\; \frac{1}{\sqrt{2}}(|y\rangle|0\rangle + (-1)^x|y\rangle|1\rangle)$

3 $\quad \frac{1}{\sqrt{2}}(|y\rangle|0\rangle + (-1)^x X|y\rangle|1\rangle) \;=\; \frac{1}{\sqrt{2}}(|y\rangle|0\rangle + (-1)^x|y \oplus 1\rangle|1\rangle) = |B_{x,y}\rangle \qquad \checkmark$

Bell measurement

Bell measurement of Bell state:

$$H^2 = 1$$

$$\left(C^X\right)^2 = 1$$

Bell measurement

Bell measurement of Bell state:

$$|B_{x,y}\rangle \left\{ \right. \quad = \quad |x\rangle \quad |y\rangle \quad = \quad |x\rangle \quad |y\rangle \quad |x\rangle \quad |y\rangle$$

Bell measurement

Bell measurement of Bell state:

$$|B_{x,y}\rangle \Bigg\{$$

Bell measurement of general 2-qubit state:

$$|\Psi\rangle = \sum_{x,y=0}^{1} \alpha_{x,y}|B_{x,y}\rangle \Bigg\{$$

with probability $\quad |\alpha_{x,y}|^2$

Bell measurement

Bell measurement of Bell state:

$$|B_{x,y}\rangle \left\{ \right.$$

$=$

$=$

Bell measurement of general 2-qubit state:

$$|\Psi\rangle = \sum_{x,y=0}^{1} \alpha_{x,y}|B_{x,y}\rangle \left\{ \right.$$

$$\left. \right\} |B_{x,y}\rangle \quad \text{with probability} \quad |\alpha_{x,y}|^2$$

$$|\Psi\rangle \rightarrow |B_{x,y}\rangle\langle B_{x,y}|\Psi\rangle$$

GHZ states & dynamic circuits

GHZ (Greenberger-Horne-Zeilinger) states

$$|\mathrm{GHZ}_n\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes n} + |1\rangle^{\otimes n})$$

For n=2:

$$|\mathrm{GHZ}_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \qquad \text{Bell state} \quad |B_{0,0}\rangle$$

For n=4:

$$|\mathrm{GHZ}_4\rangle =$$

1    $|0\rangle^{\otimes 4}$

2    $|0\rangle^{\otimes 3}H|0\rangle = |0\rangle^{\otimes 3}\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes 4} + |0\rangle^{\otimes 3}|1\rangle)$

3    $\frac{1}{\sqrt{2}}(|0\rangle^{\otimes 4} + |0\rangle^{\otimes 2}|1\rangle|1\rangle)$

4    $\frac{1}{\sqrt{2}}(|0\rangle^{\otimes 4} + |0\rangle|1\rangle|1\rangle|1\rangle)$

5    $\frac{1}{\sqrt{2}}(|0\rangle^{\otimes 4} + |1\rangle^{\otimes 4}) \qquad \checkmark$

For general n:

time

"width" = #qubits = n

"size" = #2-qubit gates = n-1

"depth" = # time steps = n

Noisy Intermediate-Scale Quantum era :   focus on circuits whose size & depth does not grow too rapidly with width

time

For general n:

"width" = #qubits = n

"size" = #2-qubit gates = n-1

"depth" = # time steps = n

Noisy Intermediate-Scale Quantum era :   focus on circuits whose size & depth does not grow too rapidly with width

Can we prepare GHZ states in *constant* depth (independent of n) ?

Yes - use "dynamic circuits"!   mid-circuit measurements/feed forward operations   - not unitary!

Conditional gate

$x \in \{0, 1\}$

X is (or is not) applied, depending on the result of the measurement

$$X^x$$

"Principle of deferred measurement"

=

Merging conditional gates

=

$\oplus$: addition mod 2

Constant-depth preparation of GHZ state

Claim:

Constant-depth preparation of GHZ state

Claim:

Proof:

Changing the order of CNOT gates

Constant-depth preparation of GHZ state

Claim:

Proof:

Omit CNOT gates conditioned on $|0\rangle$

Constant-depth preparation of GHZ state

Claim:

Proof:

Omit CNOT gates conditioned on $|0\rangle$

Constant-depth preparation of GHZ state

Claim:

These qubits are involved only at the very end,
so we can use them beforehand and then reset

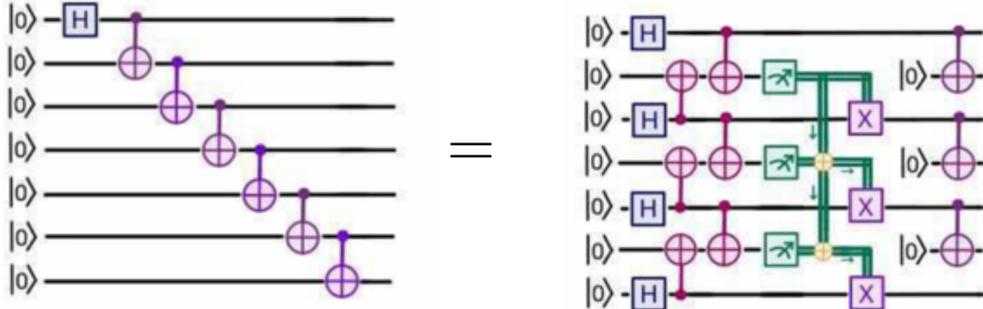$$|+\rangle = H|0\rangle$$

Proof:

Constant-depth preparation of GHZ state

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Constant-depth preparation of GHZ state

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Constant-depth preparation of GHZ state
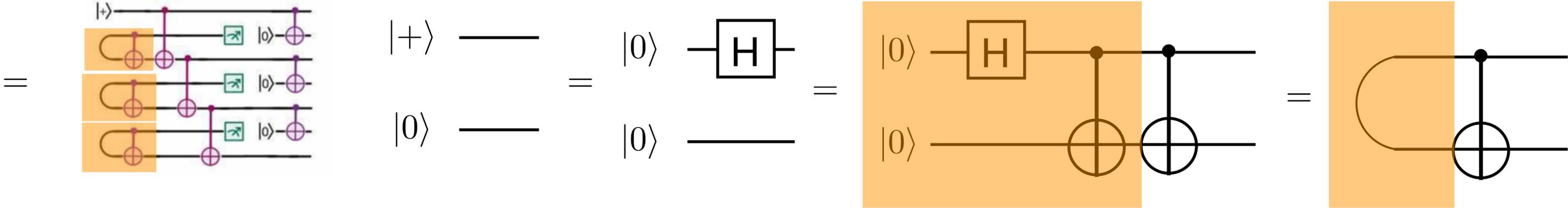
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Constant-depth preparation of GHZ state
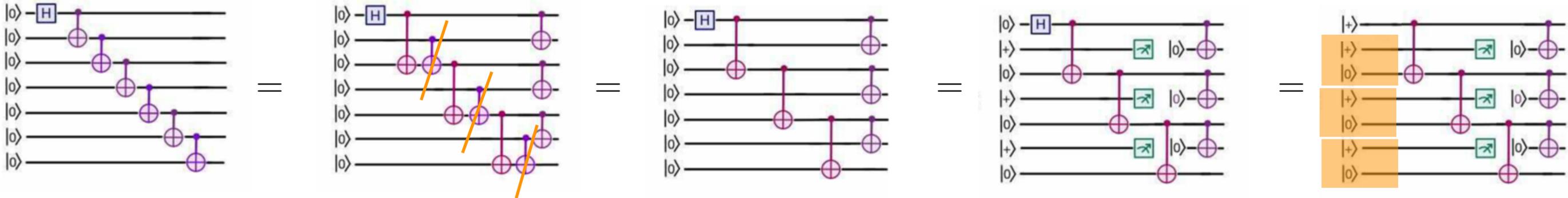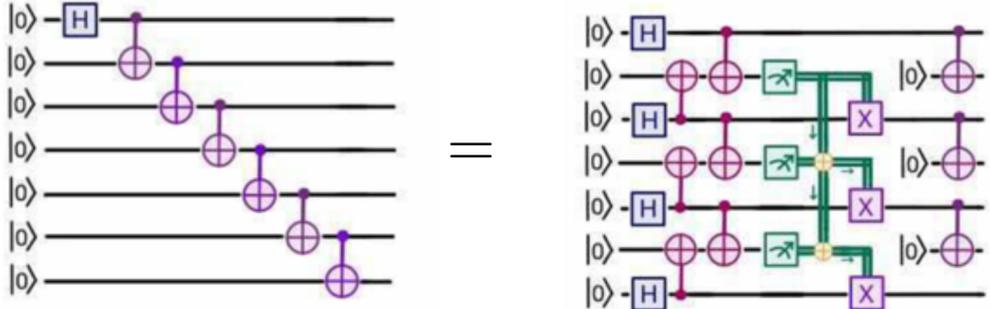
Claim:

Proof:

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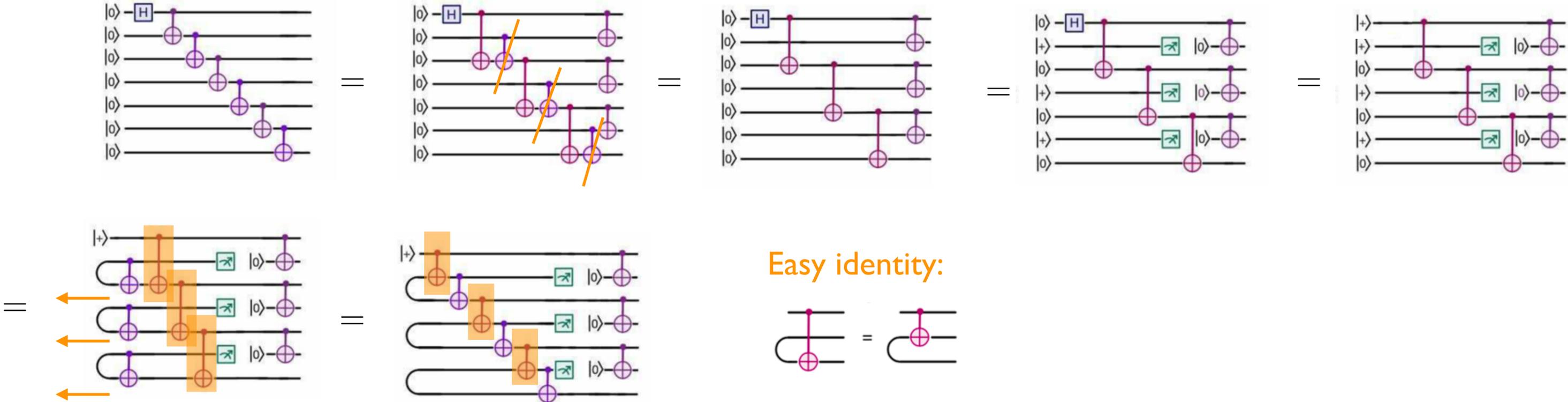Represent Bell state by a "cup"

Constant-depth preparation of GHZ state

Claim:

Proof:

$$|+\rangle = H|0\rangle$$
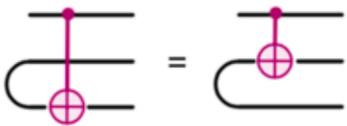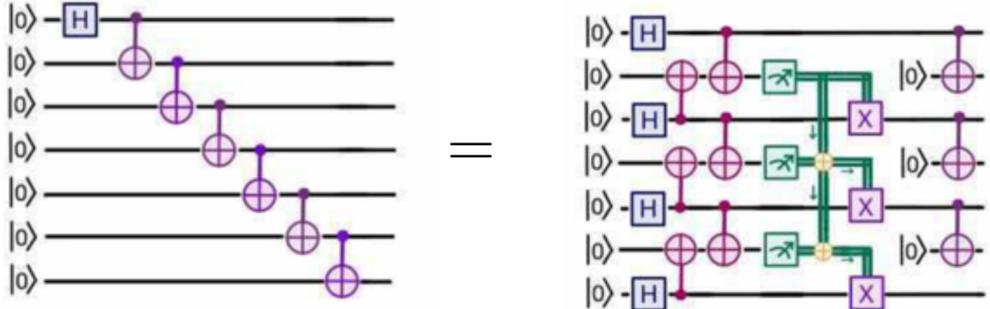

Easy identity:

Constant-depth preparation of GHZ state
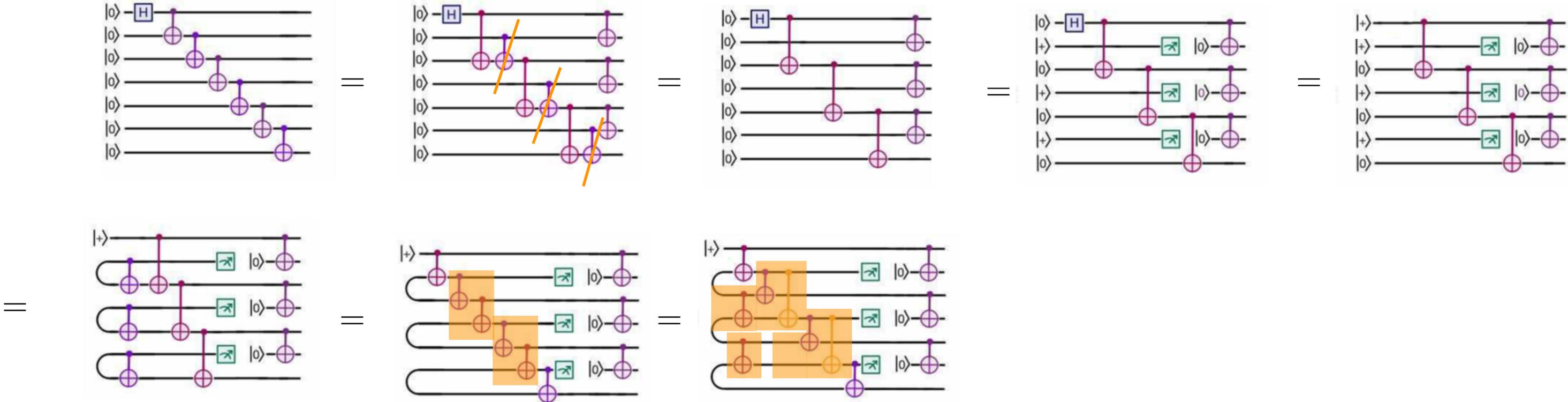
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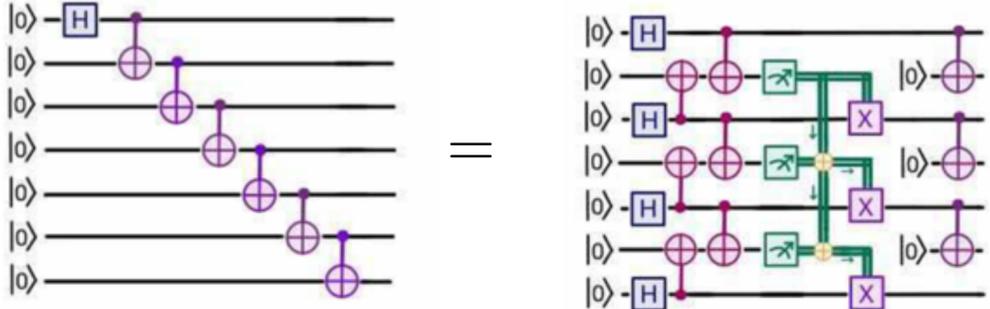

Changing the order of CNOT gates

Constant-depth preparation of GHZ state

Claim:

Proof:

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"Principle of deferred measurement"

Constant-depth preparation of GHZ state

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Proof:

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Use definitions

Constant-depth preparation of GHZ state

Claim:

Proof:

$$|+\rangle = H|0\rangle$$

Merging conditioned gates

Constant-depth preparation of GHZ state

Claim:

Proof:

$$|+\rangle = H|0\rangle$$

Matrix Product States

Matrix Product States (MPS)

n-qubit state: $\quad |\psi\rangle = \sum_{\vec{m}} \boxed{a_{\vec{m}}} |\vec{m}\rangle \qquad\qquad |\vec{m}\rangle = |m_n \ldots m_2 m_1\rangle \qquad m_i = 0, 1 \qquad i = 1, \ldots, n$

qubits $\qquad$ "physical space"

$a_{\vec{m}} = \langle L | A_n^{m_n} \ldots A_2^{m_2} A_1^{m_1} | R \rangle$

$A_i^{m_i} \qquad \chi \times \chi \qquad$ matrices

$\chi \qquad$ "bond dimension"

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$A_i^{m_i} \qquad \chi \times \chi \quad$ matrices $\qquad$ basis $\quad |\underline{j}\rangle \qquad j = 0, 1, \dots, \chi - 1 \qquad$ qudit $\qquad$ "bond space"

$\chi \quad$ "bond dimension"

Matrix Product States (MPS)

n-qubit state: $\quad |\psi\rangle = \sum_{\vec{m}} \boxed{a_{\vec{m}}} |\vec{m}\rangle \qquad\qquad |\vec{m}\rangle = |m_n \dots m_2 m_1\rangle \qquad m_i = 0, 1 \qquad i = 1, \dots, n$

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$\chi \quad$ "bond dimension" $\qquad\qquad |L\rangle, |R\rangle \quad$ boundary states $\quad$ "auxiliary", "ancilla", "memory", "virtual"

Matrix Product States (MPS)

n-qubit state: $\quad |\psi\rangle = \sum_{\vec{m}} \boxed{a_{\vec{m}}} |\vec{m}\rangle \qquad |\vec{m}\rangle = |m_n \dots m_2 m_1\rangle \qquad m_i = 0, 1 \qquad i = 1, \dots, n$

qubits   "physical space"

$a_{\vec{m}} = \langle L| A_n^{m_n} \dots A_2^{m_2} A_1^{m_1} |R\rangle$

$$\langle L| \boxed{A} - \boxed{A} - \boxed{A} - \boxed{A} |R\rangle$$

$A_i^{m_i} \qquad \chi \times \chi \quad \text{matrices} \qquad \text{basis} \quad |\underline{j}\rangle \qquad j = 0, 1, \dots, \chi - 1$ qudit   "bond space"

$\chi \quad$ "bond dimension" $\qquad\qquad |L\rangle, |R\rangle \quad$ boundary states $\quad$ "auxiliary", "ancilla", "memory", "virtual"

Matrix Product States (MPS)

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$$a_{\vec{m}} = \langle L| A_n^{m_n} \ldots A_2^{m_2} A_1^{m_1} |R\rangle$$

qubits "physical space"

$A_i^{m_i} \qquad \chi \times \chi \quad$ matrices $\qquad$ basis $\quad |\underline{j}\rangle \qquad\qquad j = 0, 1, \ldots, \chi - 1 \qquad\qquad$ qudit $\qquad$ "bond space"

$\chi \quad$ "bond dimension" $\qquad\qquad |L\rangle, |R\rangle \quad$ boundary states $\quad$ "auxiliary", "ancilla", "memory", "virtual"

Can represent any state $|\psi\rangle$ this way, for sufficiently large $\chi \qquad$ — can be useful if χ is not too large

Entanglement entropy $\quad S \sim \log \chi \qquad\qquad$ Energy of first excited state - Energy of ground state > 0 for $n \to \infty$

For the ground state of a gapped Hamiltonian: $\quad S \sim$ area $\qquad$ (i.e., $S \sim n^{D-1}$, D = # space dimensions)

$\Rightarrow \quad$ In 1 space dimension, S = constant (independent of n)

$\Rightarrow \qquad \chi$ = constant (independent of n)

$$|\psi\rangle = \sum_{\vec{m}} a_{\vec{m}} |\vec{m}\rangle \qquad\qquad a_{\vec{m}} = \langle L| A_n^{m_n} \dots A_2^{m_2} A_1^{m_1} |R\rangle$$

For GHZ state:

$$|\text{GHZ}_n\rangle = \frac{1}{\sqrt{2}} (|0\rangle^{\otimes n} + |1\rangle^{\otimes n})$$

$$\langle L| = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$A_i^0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad A_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad\qquad |R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = H|0\rangle \qquad \chi = 2$$

translational invariant

Note

$$\left(A^0\right)^2 = A^0, \quad \left(A^1\right)^2 = A^1, \quad A^0 A^1 = 0 = A^1 A^0 \qquad \Rightarrow \quad \textcolor{red}{\text{m's are either all 0's or all 1's!}}$$

$\Rightarrow$

$$a_{\vec{m}} = \begin{cases} \frac{1}{\sqrt{2}}, & \text{if} \quad \vec{m} = 0\dots0 \quad \text{or} \quad \vec{m} = 1\dots1 \\ 0, & \text{otherwise} \end{cases}$$

✓

Also

$$A^m |j\rangle = \delta_{m,j} |j\rangle$$