Cosmic Tensions
and
Cracks in the Standard Model of Cosmology

Waleed El Hanafy

waleed.elhanafy@bue.edu.eg

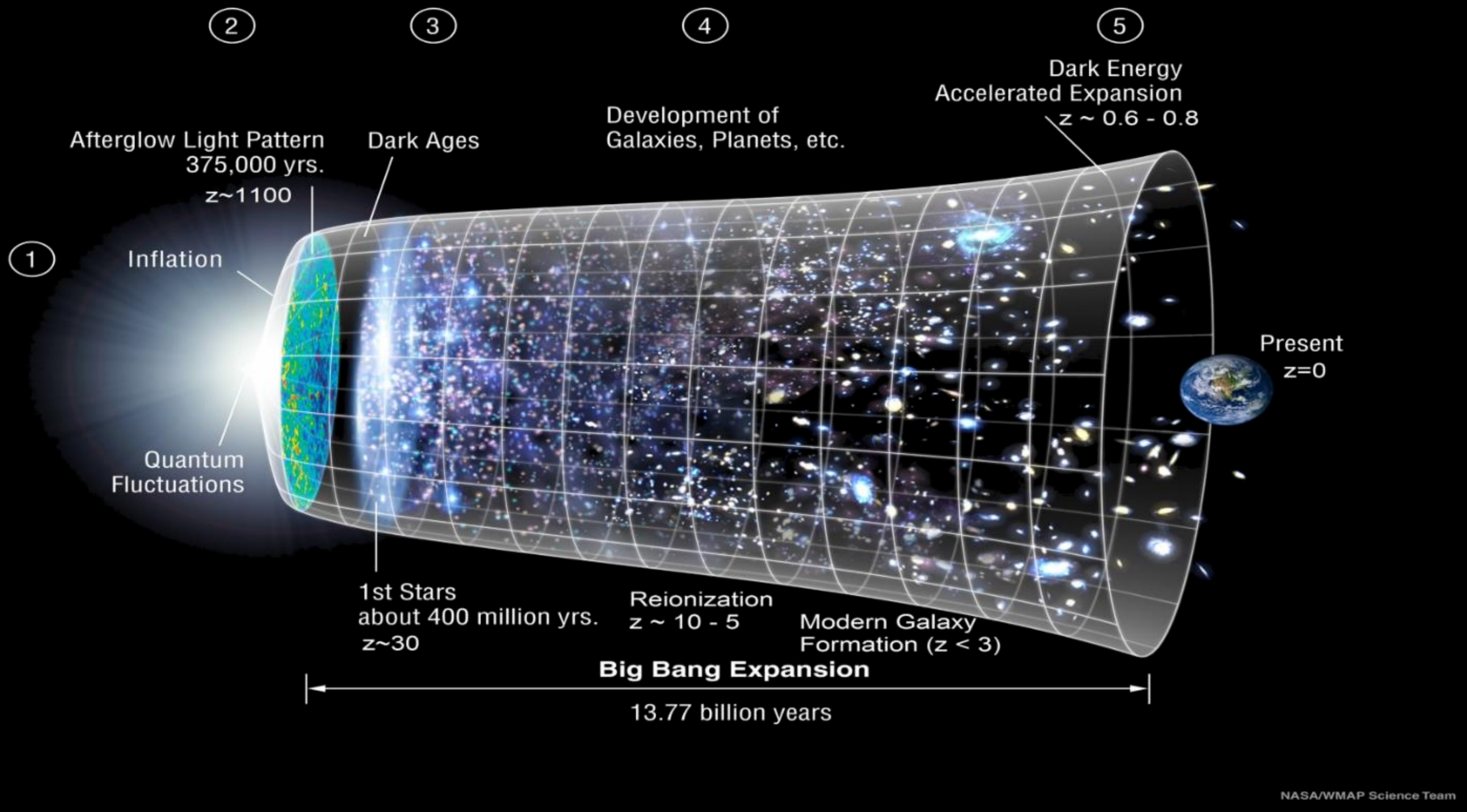
Centre for Theoretical Physics, The British University in Egypt

Workshop on Astro-particles and Gravity
Sep 20 – 22, 2022
Cairo University

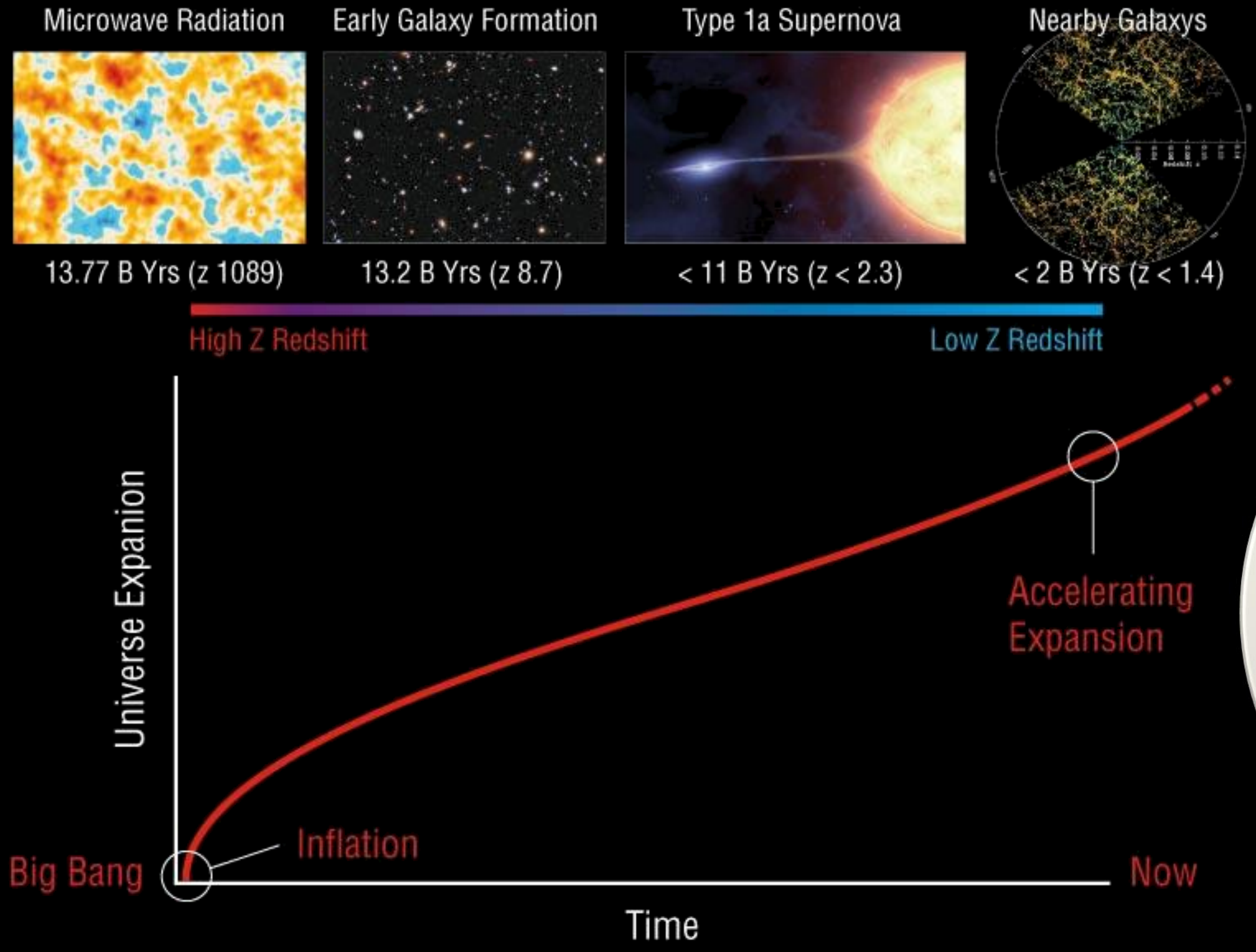
Abstract

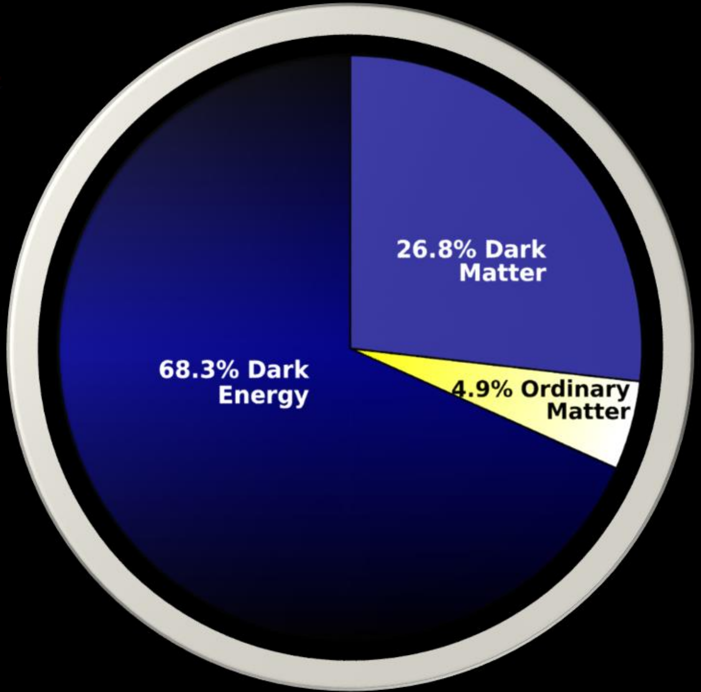
The standard Λ Cold Dark Matter cosmological model amazingly fits a wide range of astrophysical and astronomical data. However, the increase of the experimental sensitivity emerges some cracks in the standard scenario due tensions between different independent cosmological datasets. The Planck mission estimation of Hubble constant H_0 is at 4-6σ tension with its measured value by SH0ES and H0LiCOW collaborations. Also, the tension between Planck data and weak lensing measurements and redshift surveys about the value of the matter energy density Ω_m, and the amplitude or rate of growth of structure (σ_8, $f\sigma_8$) becomes significant. New physics could be in action to resolve these cosmic tensions. We give an outline of the different approaches to solve these tensions with some interesting models.

The Standard Model of Cosmology (ΛCDM)



Relevant Cosmological Observations





Standard Candle

If two sources have the same intrinsic luminosity (“standard candles”), from the ratio of their apparent brightness we can derive the ratio of their luminosity distances.

Standard Ruler

If two sources have the same physical size (“standard rulers”), from the ratio of their apparent angular sizes we can derive the ratio of their angular diameter distances.

SNIa Standard Candle

Distance modulus

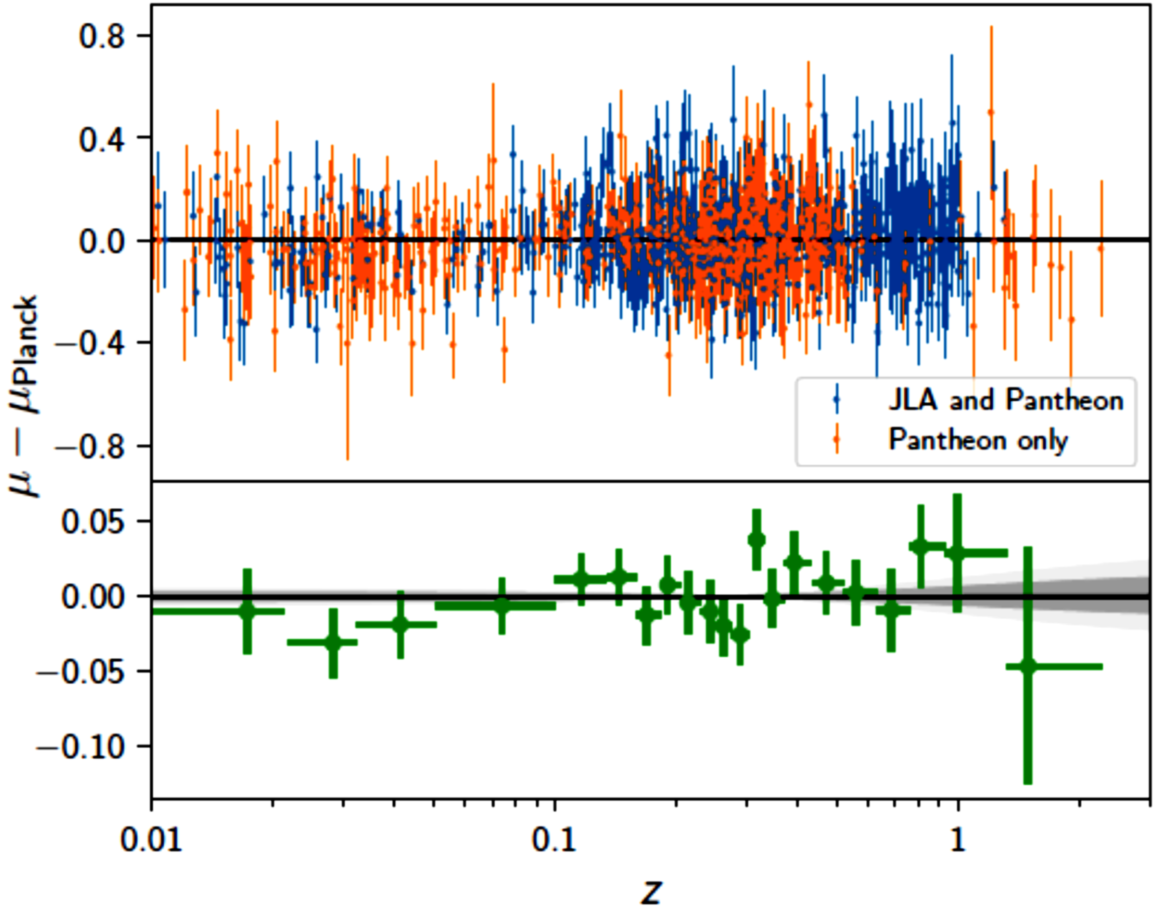
$$\mu(z) = m - M = 5\log_{10}\left(\frac{D_L(z)}{1\ \mathrm{Mpc}}\right) + 25,$$

Obs

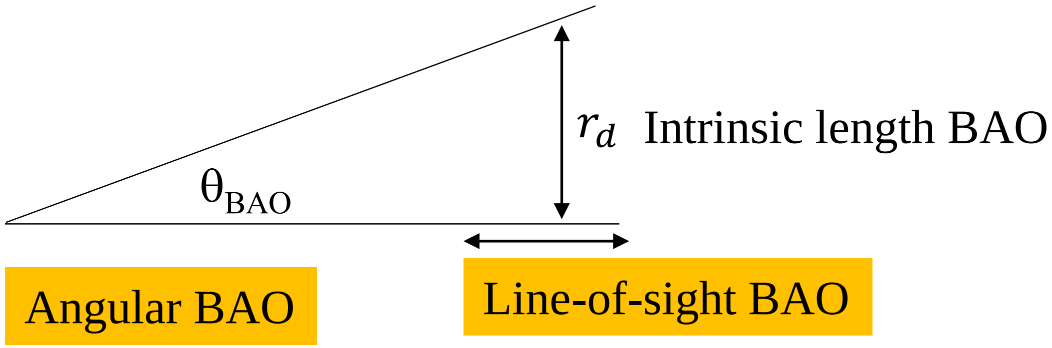
Theor

Luminosity distance

$$D_L = c(1+z)\int_0^z \frac{dz'}{H(z')}.$$



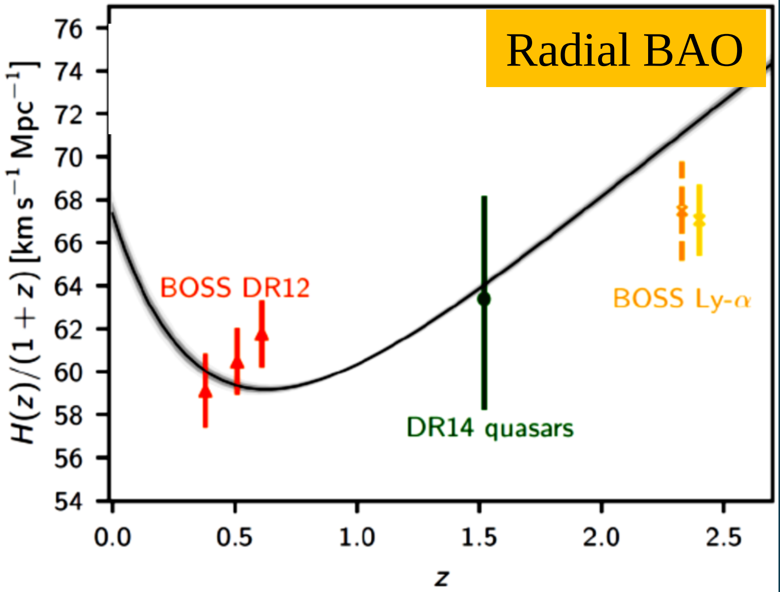
BAO Standard Ruler

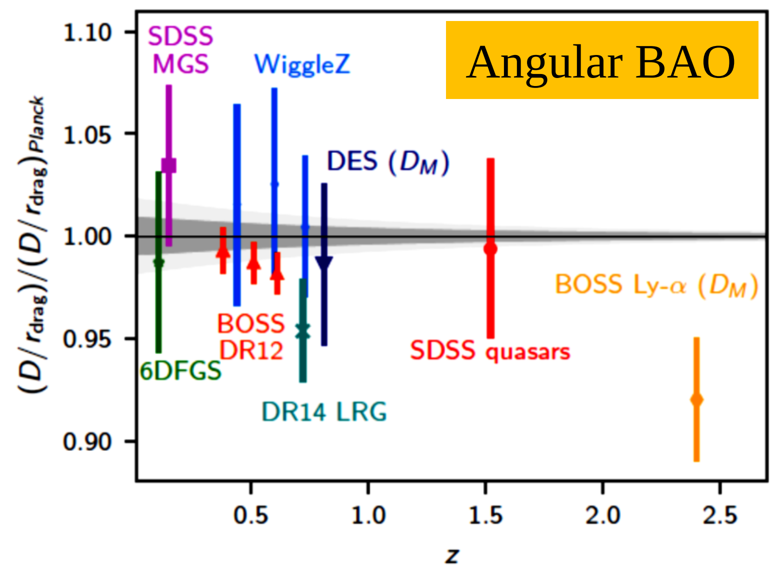


$$\delta\theta_{BAO} = \frac{r_d}{D_M(z)}$$

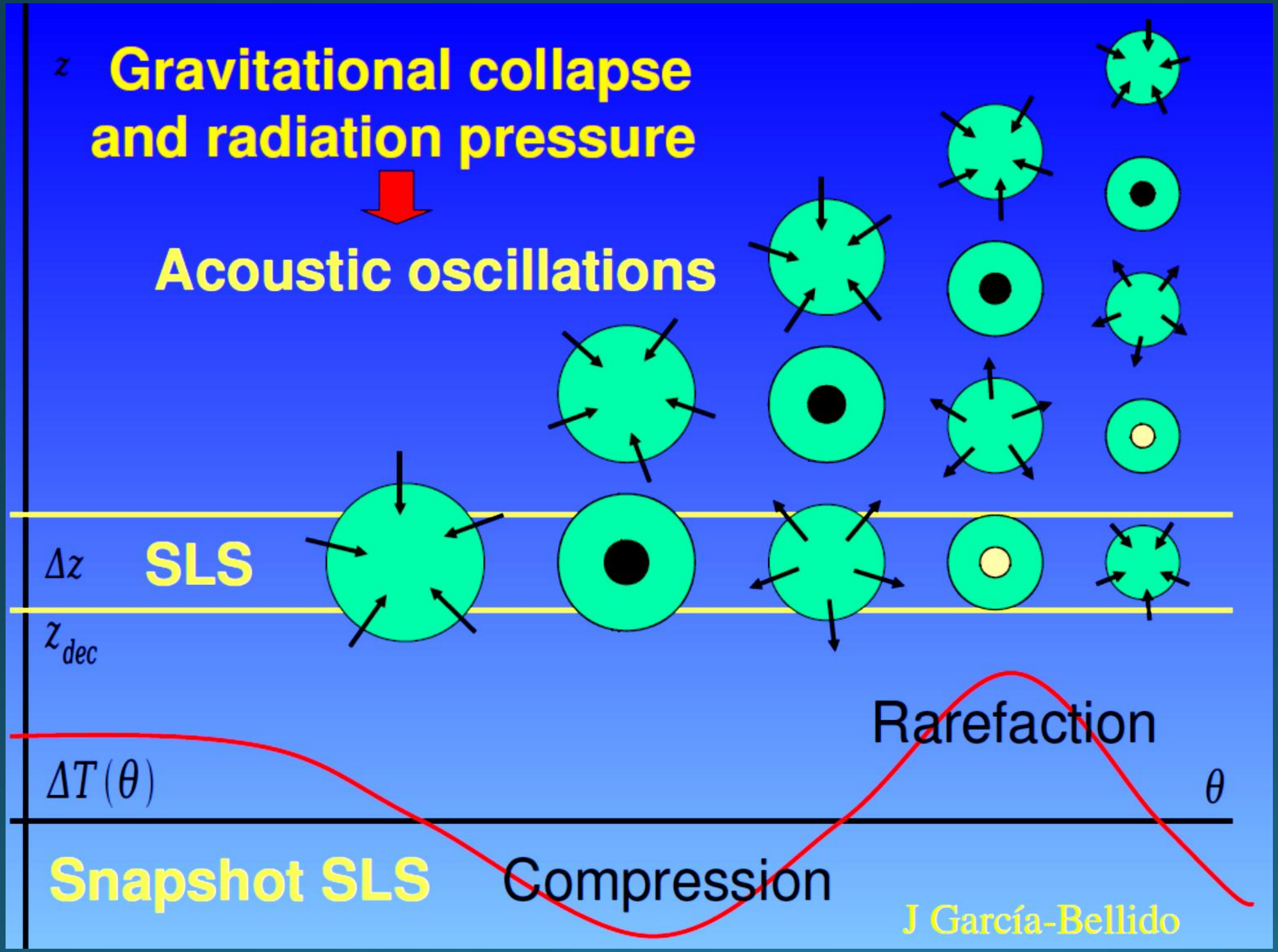
$$\delta z = r_d H(z)$$

$$D_M(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

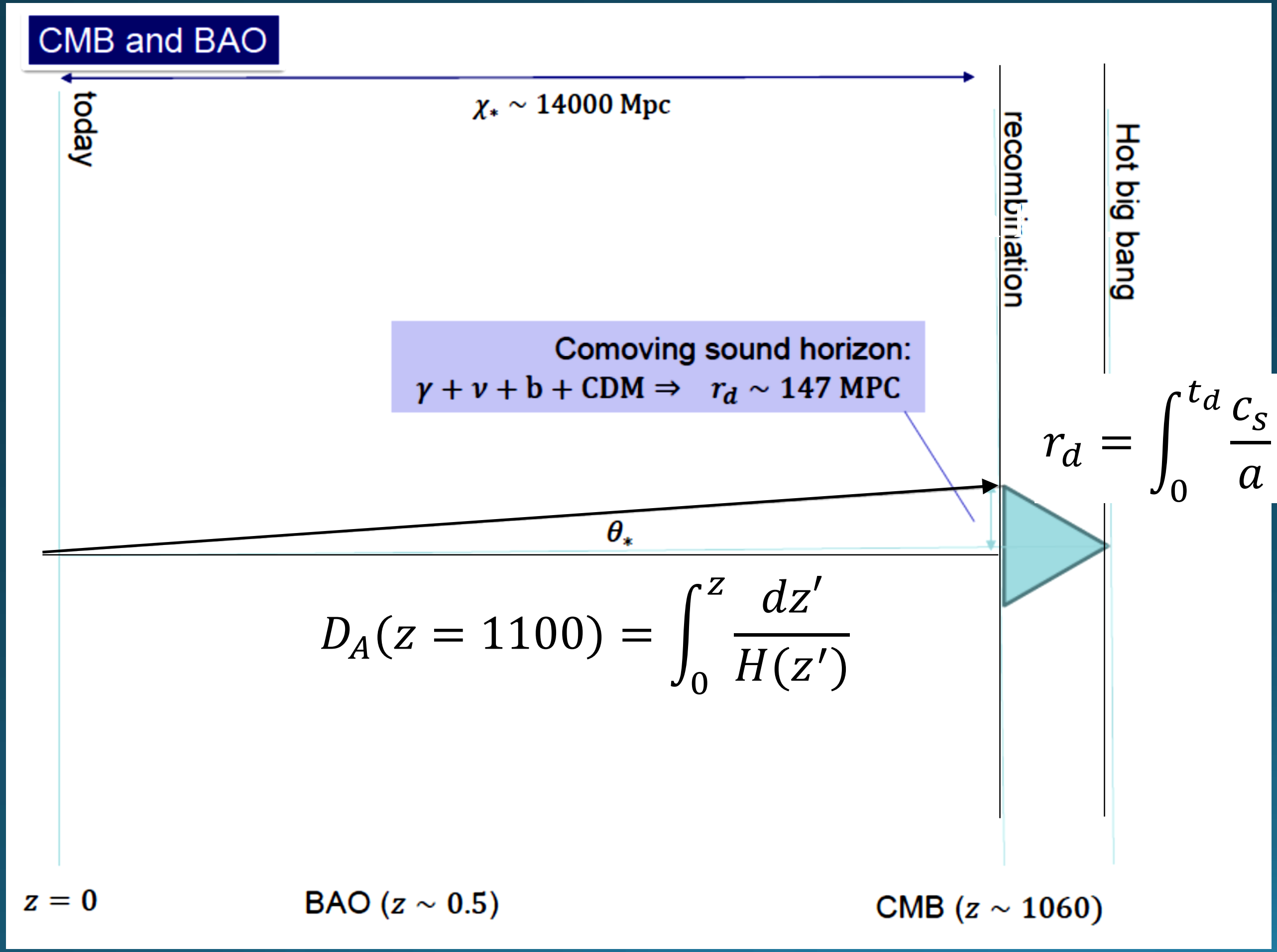




BAO Standard Ruler



Sound horizon scale



$$D_A(z = 1100) = \int_0^z \frac{dz'}{H(z')}$$

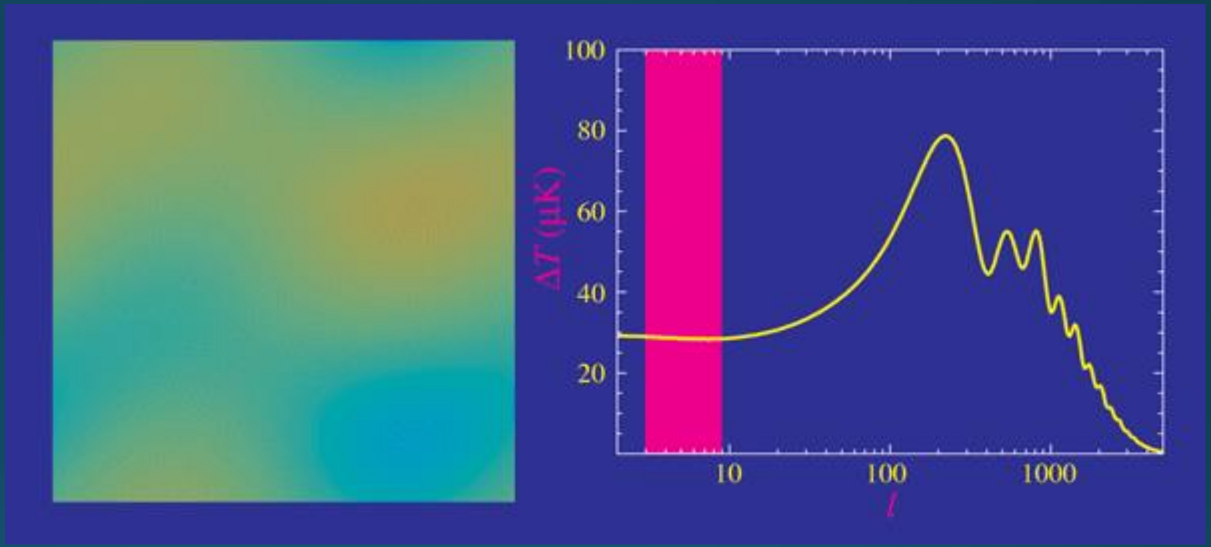
$$r_d = \int_0^{t_d} \frac{c_s}{a} dt = \int_0^{a_d} c_s \frac{da}{a^2 H(a)}$$

Power Spectra

- Power Spectra

- Baryons
- Matter
- Curvature DarkEnergy
- Reionization Tensors
- Damping Tail
- Baryon Fraction
- EB Polarization
- Scalar
- Vector
- Tensor

Decompose the anisotropy (random Gaussian distribution) in spherical harmonics

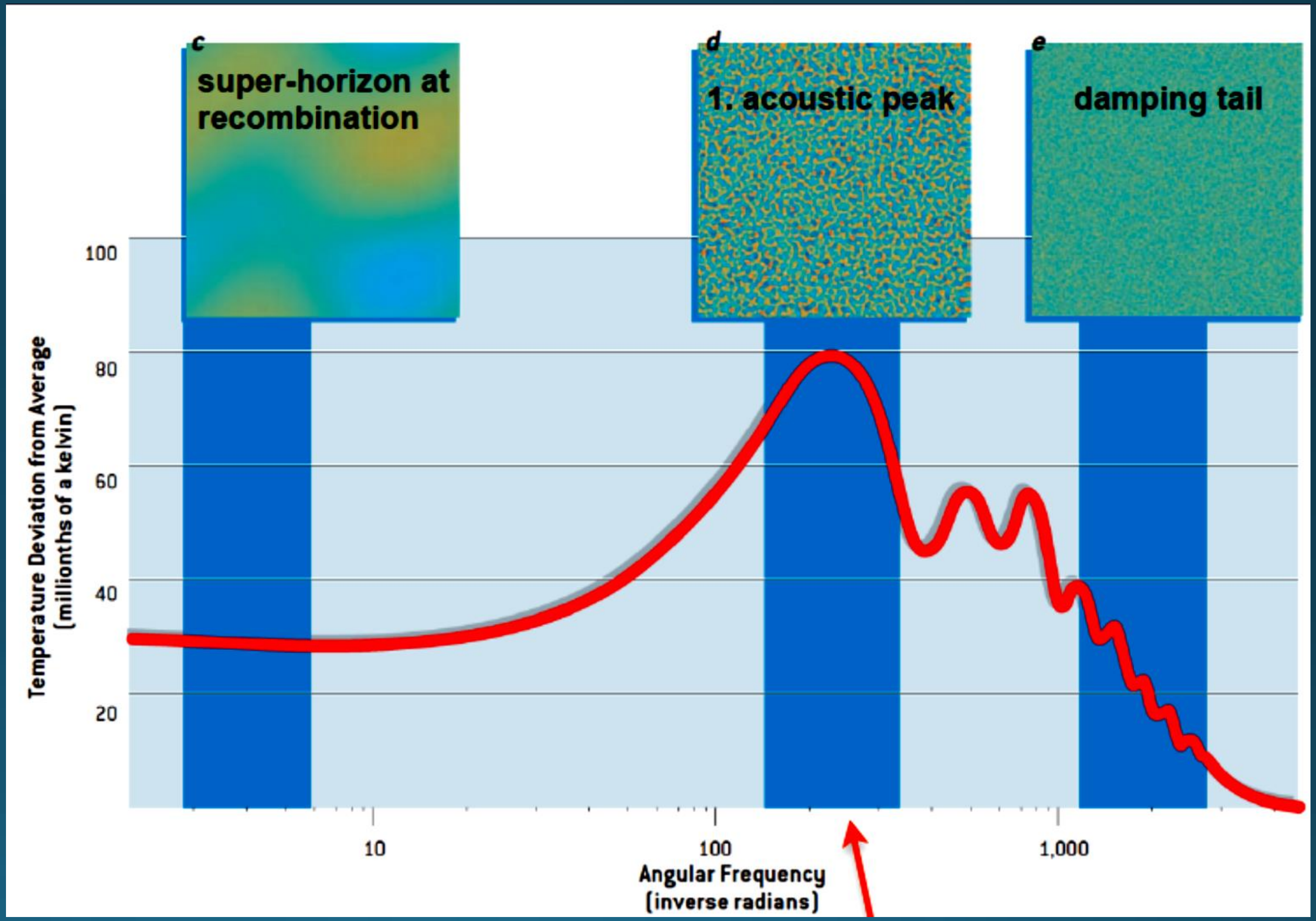


Wayne hu
Professor, Department of Astronomy and Astrophysics, University of Chicago

http://background.uchicago.edu/~whu/metaanim.html

Power Spectra

CMB anisotropies are mainly formed at redshift z ~ 1000 when either dark energy or modifications to GR appear to be negligible. However, while CMB photons travel to us, they are affected and distorted by other, low redshift, mechanisms, that could help in understanding the nature of the accelerating universe.



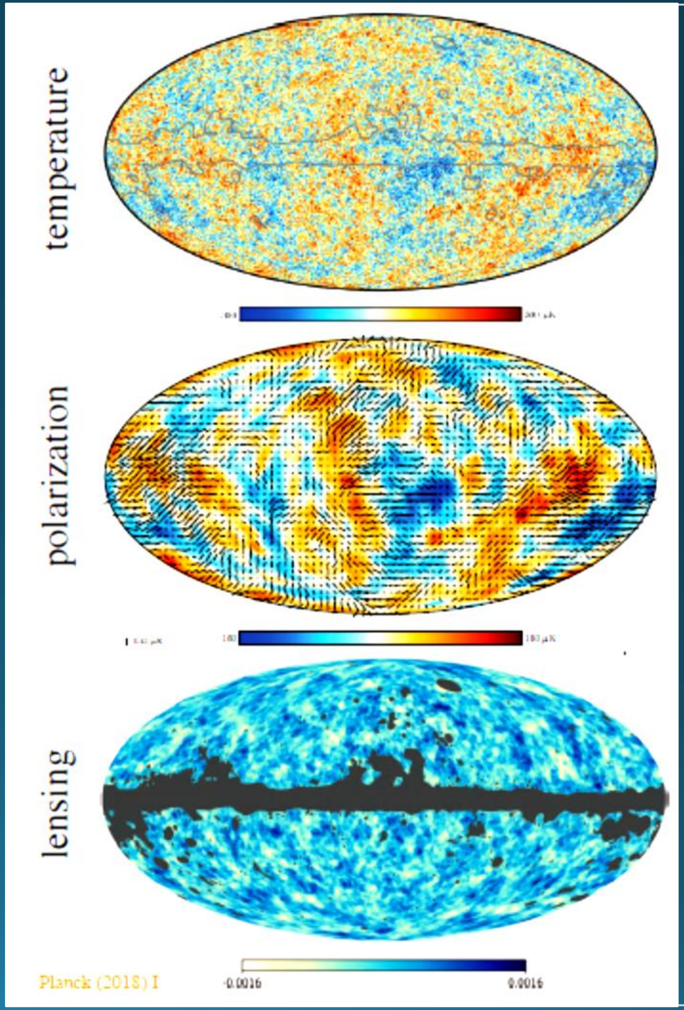
Precession Cosmology (Planck 2018)

Precision measurements and maps
– temperature
– polarization
– lensing
– 9 frequencies

Control over systematics
– most recently polarization

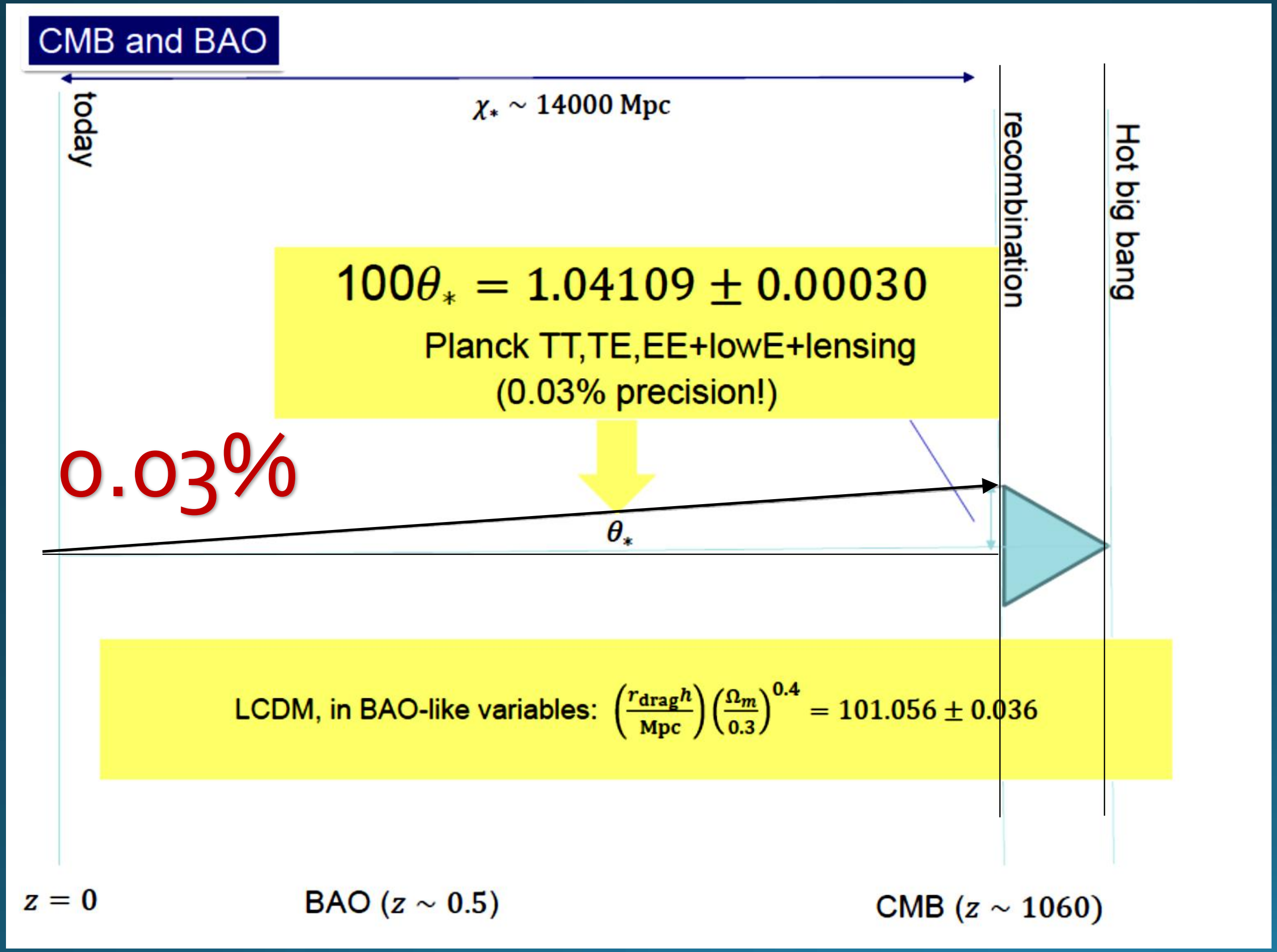
Accurate and precise

theoretical predictions
– Gaussian, adiabatic
– LCDM



Planck Legacy

Sound horizon scale



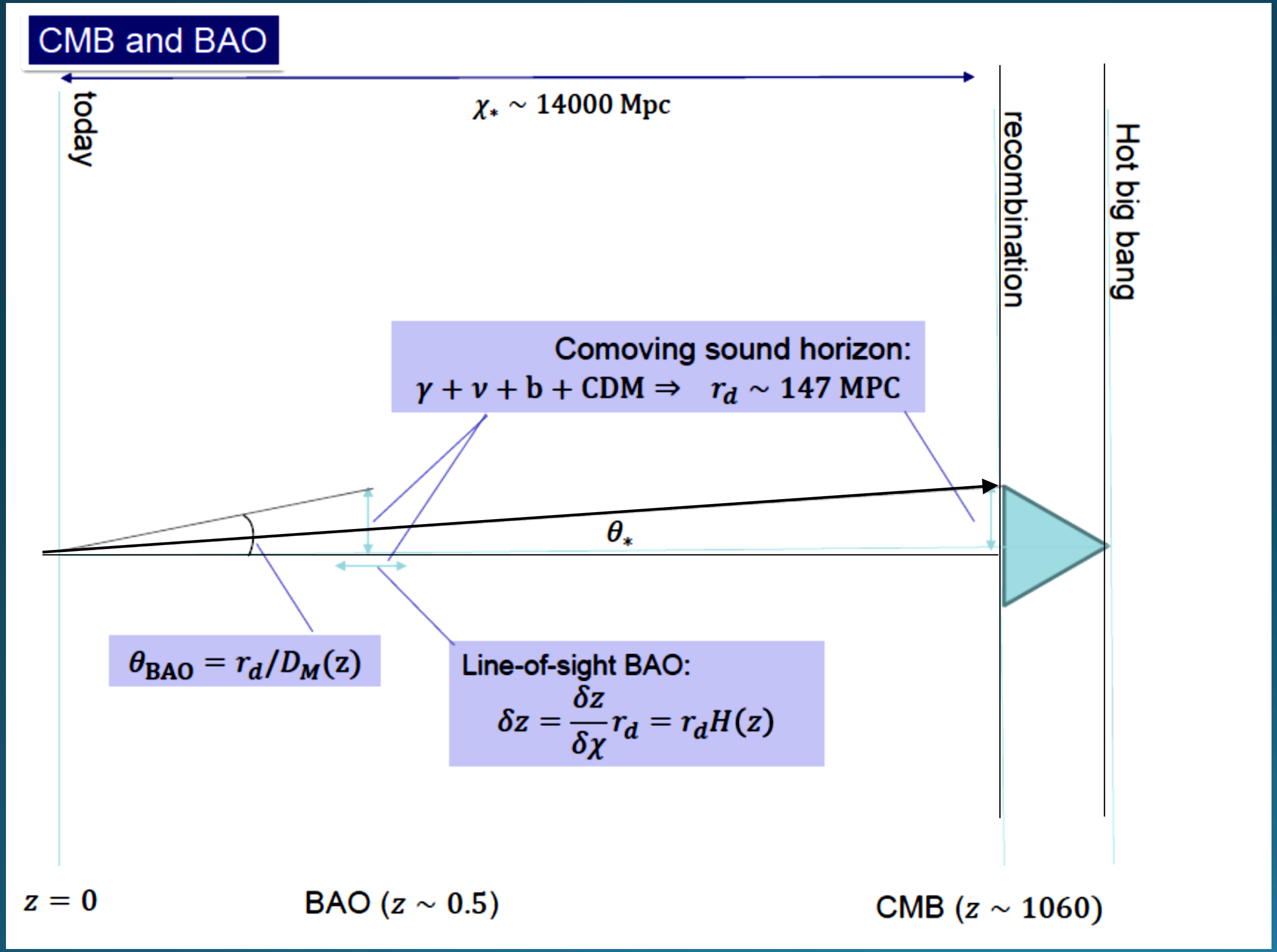
$2 < \ell < 2500$

Lots of
peaks & troughs

Temperature	7 peaks
Polarization	5 peaks
TE correlation	6 peaks

High
precession measurement

Sound horizon scale



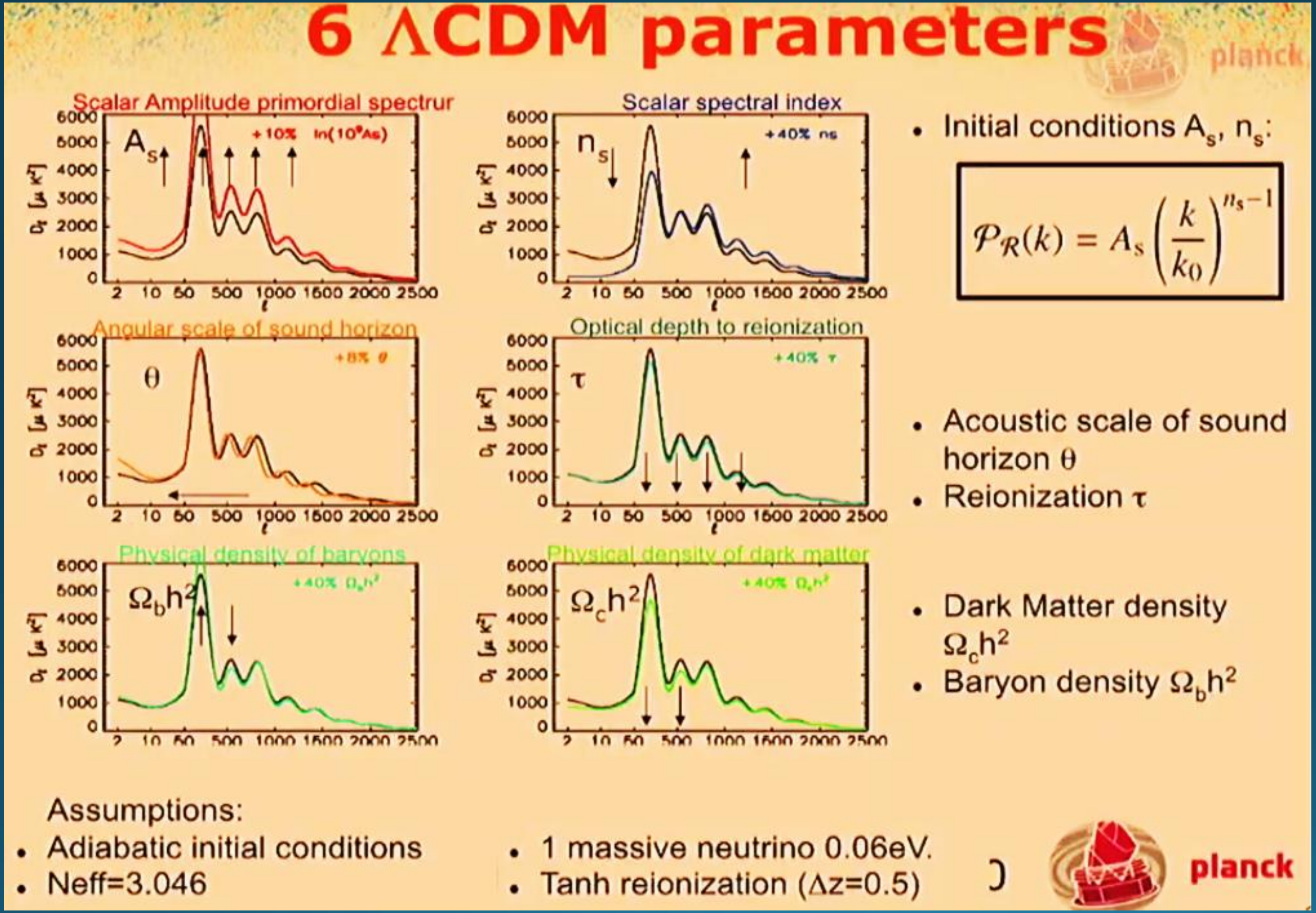
2 < ℓ < 2500

Lots of
peaks & troughs

Temperature	7 peaks
Polarization	5 peaks
TE correlation	6 peaks

High
precession measurement

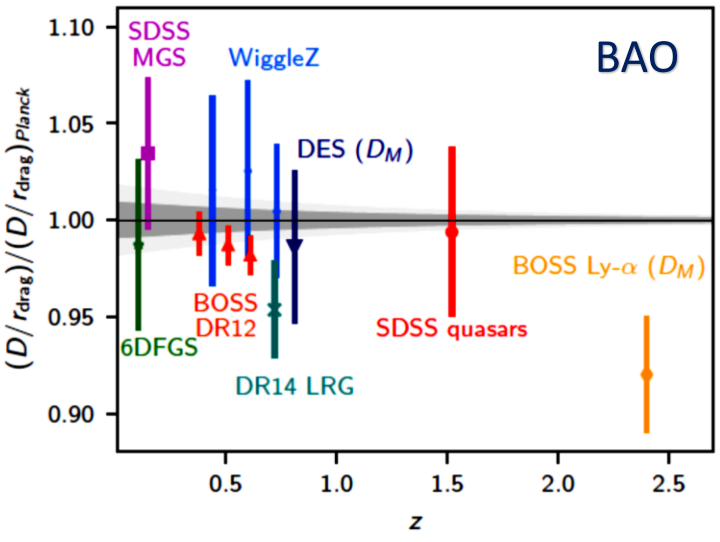
LCDM 6 parameters Space

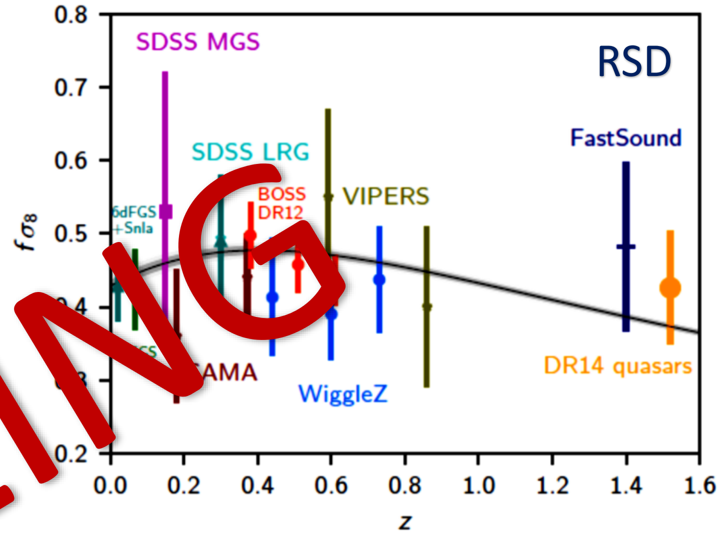


LCDM 6 PARAMETERS SPACE

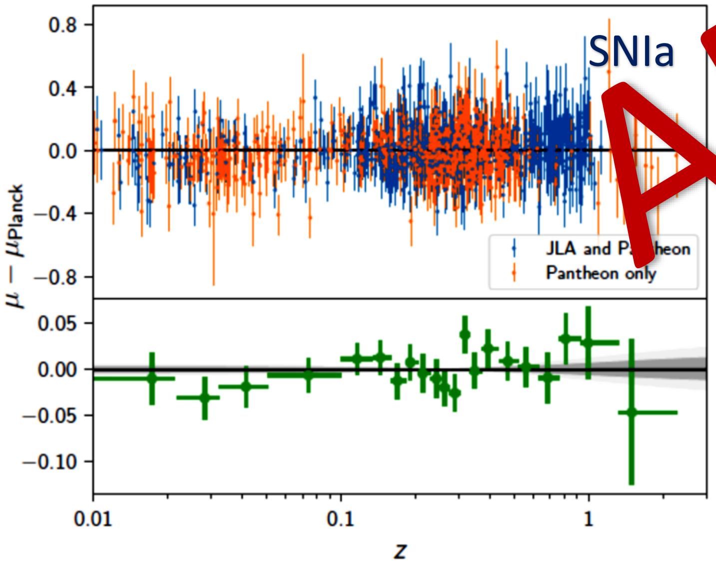
Parameter	Plik best fit	Plik [1]	CamSpec [2]	$([2]-[1])/\sigma_1$	Combined
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	−0.5	0.02233 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	−0.3	0.1198 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031	1.04087 ± 0.00031	−0.2	1.04089 ± 0.00031
τ	0.0543	0.0544 ± 0.0073	$0.0536^{+0.0069}_{-0.0077}$	−0.1	0.0540 ± 0.0074
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014	3.041 ± 0.015	−0.3	3.043 ± 0.014
n_s	0.96605	0.9649 ± 0.0042	0.9656 ± 0.0042	+0.2	0.9652 ± 0.0042
$\Omega_m h^2$	0.14314	0.1430 ± 0.0011	0.1426 ± 0.0011	−0.3	0.1428 ± 0.0011
H_0 [km s^{-1}Mpc^{-1}]	67.32	67.36 ± 0.54	67.39 ± 0.54	+0.1	67.37 ± 0.54
Ω_m	0.3158	0.3153 ± 0.0073	0.3142 ± 0.0074	−0.2	0.3147 ± 0.0074
Age [Gyr]	13.7971	13.797 ± 0.023	13.805 ± 0.023	+0.4	13.801 ± 0.024
σ_8	0.8120	0.8111 ± 0.0060	0.8091 ± 0.0060	−0.3	0.8101 ± 0.0061
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.8331	0.832 ± 0.013	0.828 ± 0.013	−0.3	0.830 ± 0.013
z_{re}	7.68	7.67 ± 0.73	7.61 ± 0.75	−0.1	7.64 ± 0.74
$100\theta_*$	1.041085	1.04110 ± 0.00031	1.04106 ± 0.00031	−0.1	1.04108 ± 0.00031
r_{drag} [Mpc]	147.049	147.09 ± 0.26	147.26 ± 0.28	+0.6	147.18 ± 0.29

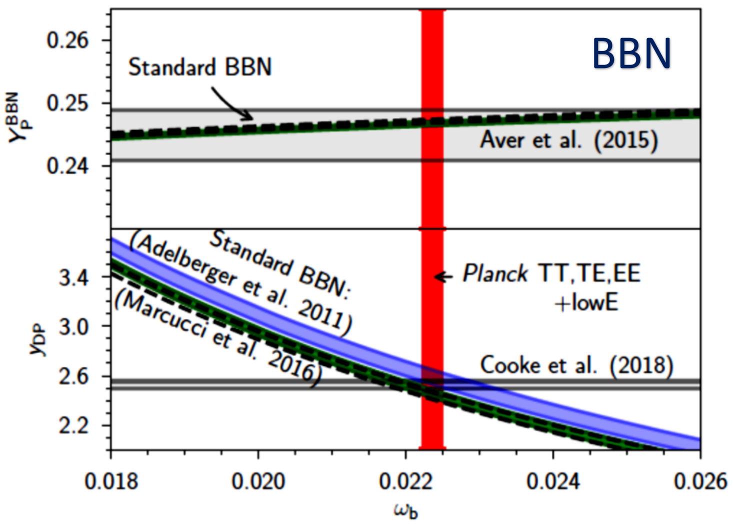
Planck + LCDM vs. External Datasets





AMAZING





Cracks in Standard Model

- H_0-tension
- σ_8-tension
- No concordance model

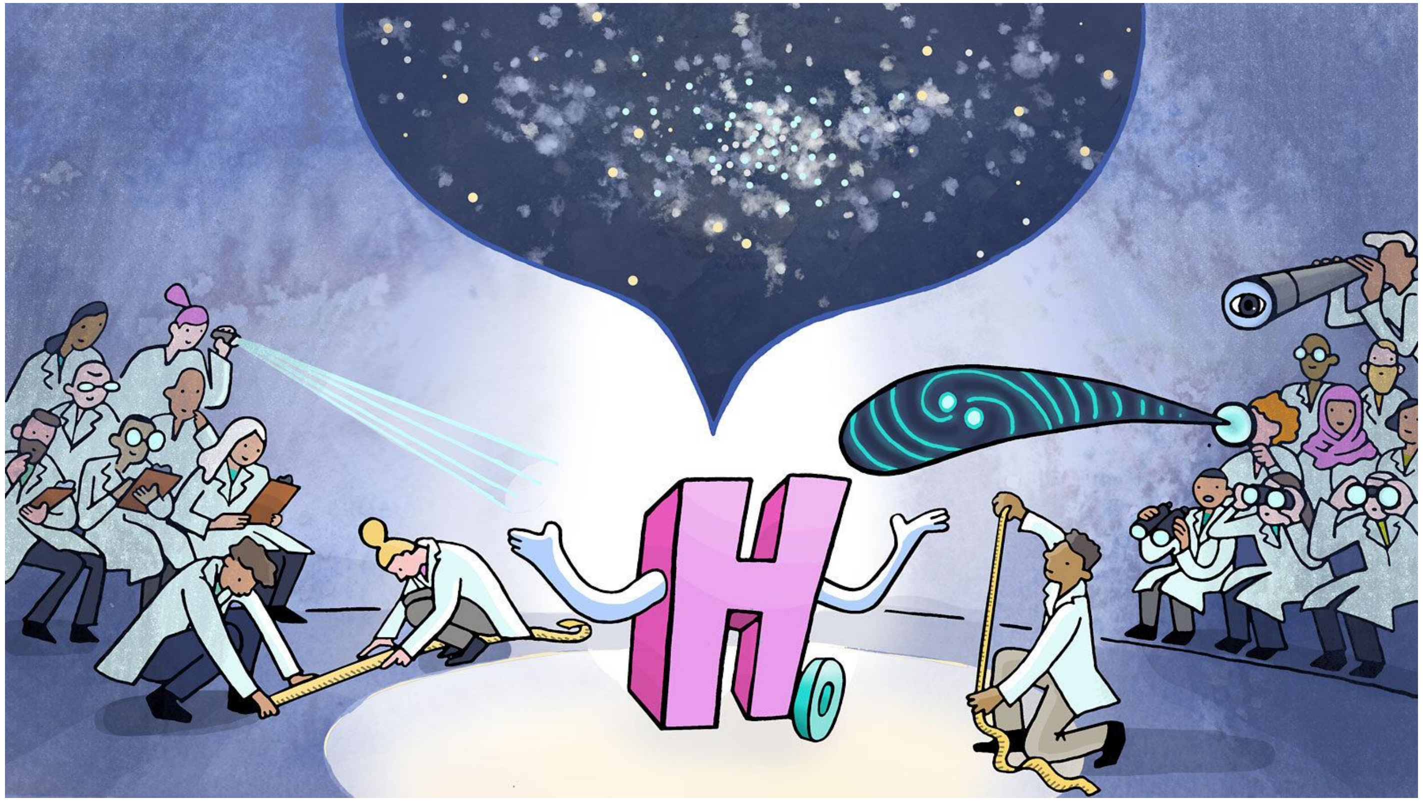
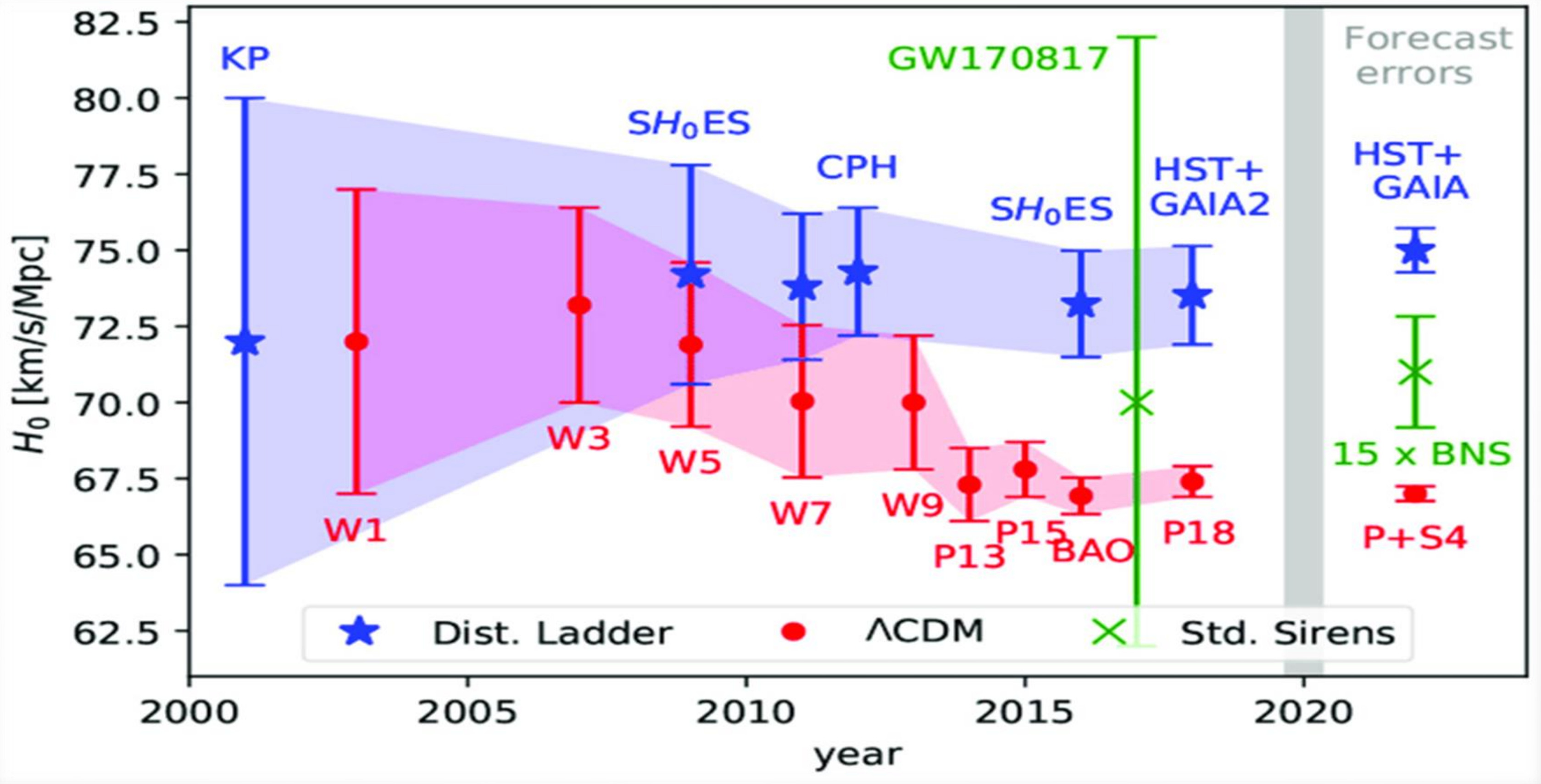
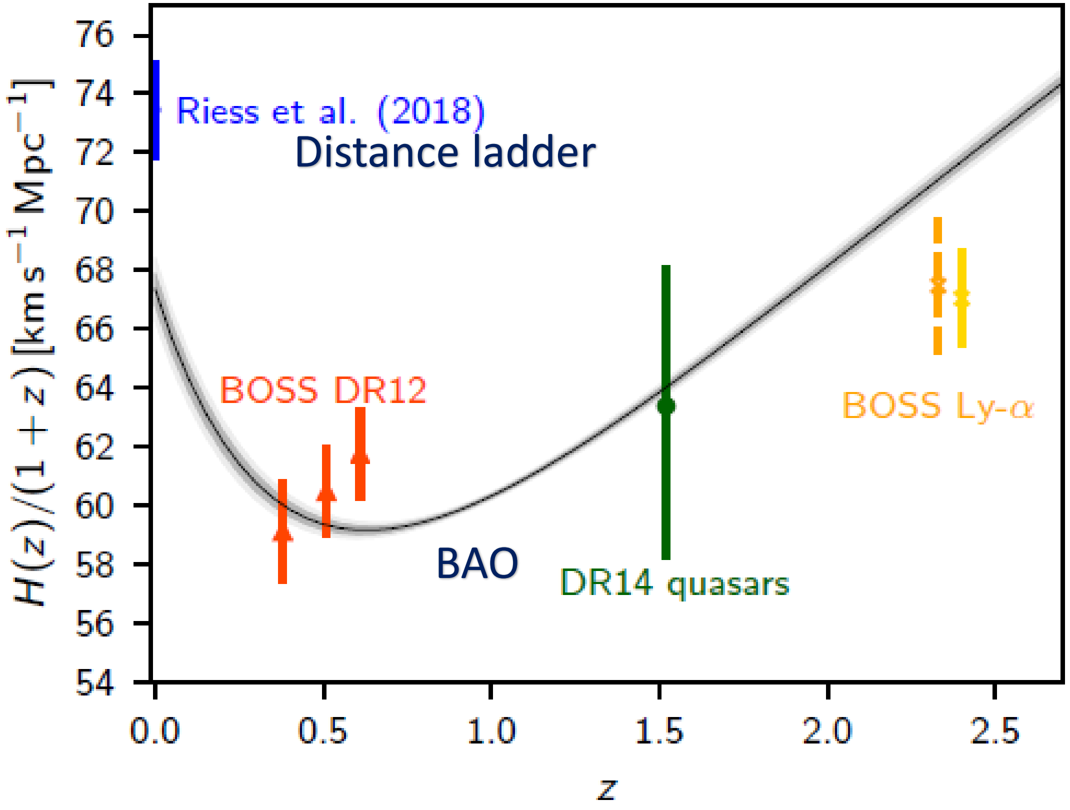


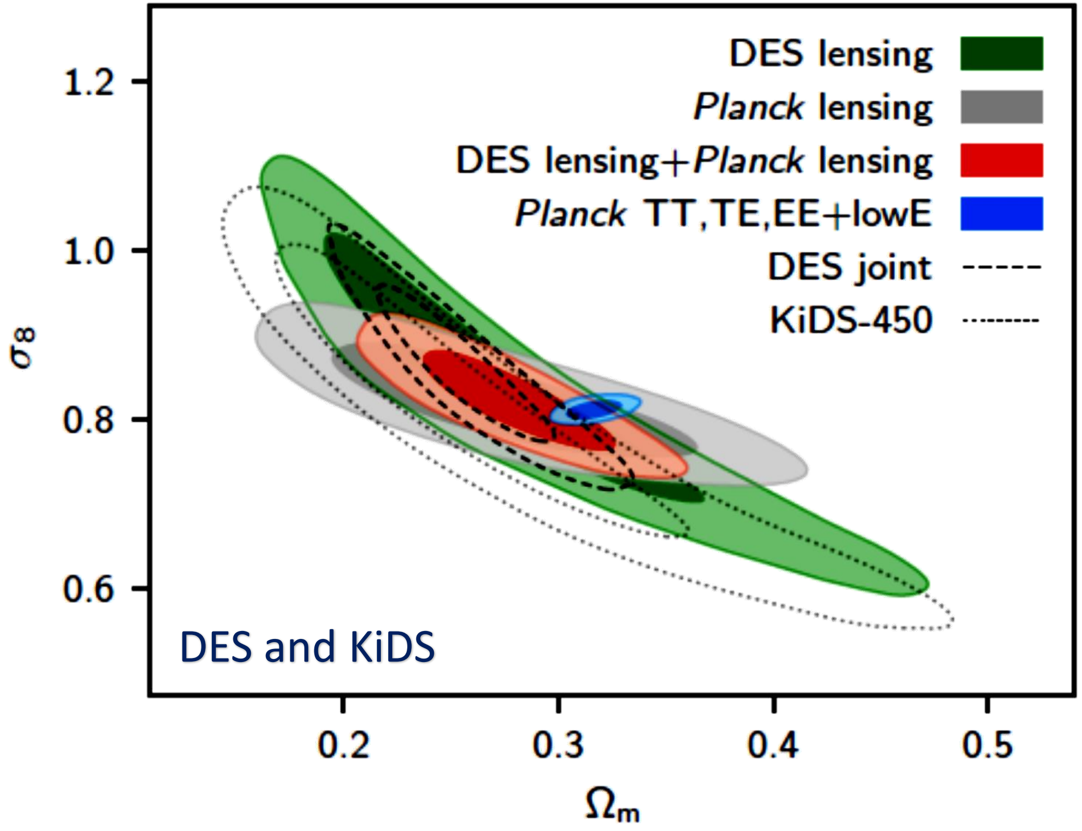
Illustration by Sandbox Studio, Chicago with Corinne Mucha

HOW FAST IS THE UNIVERSE EXPANDING?

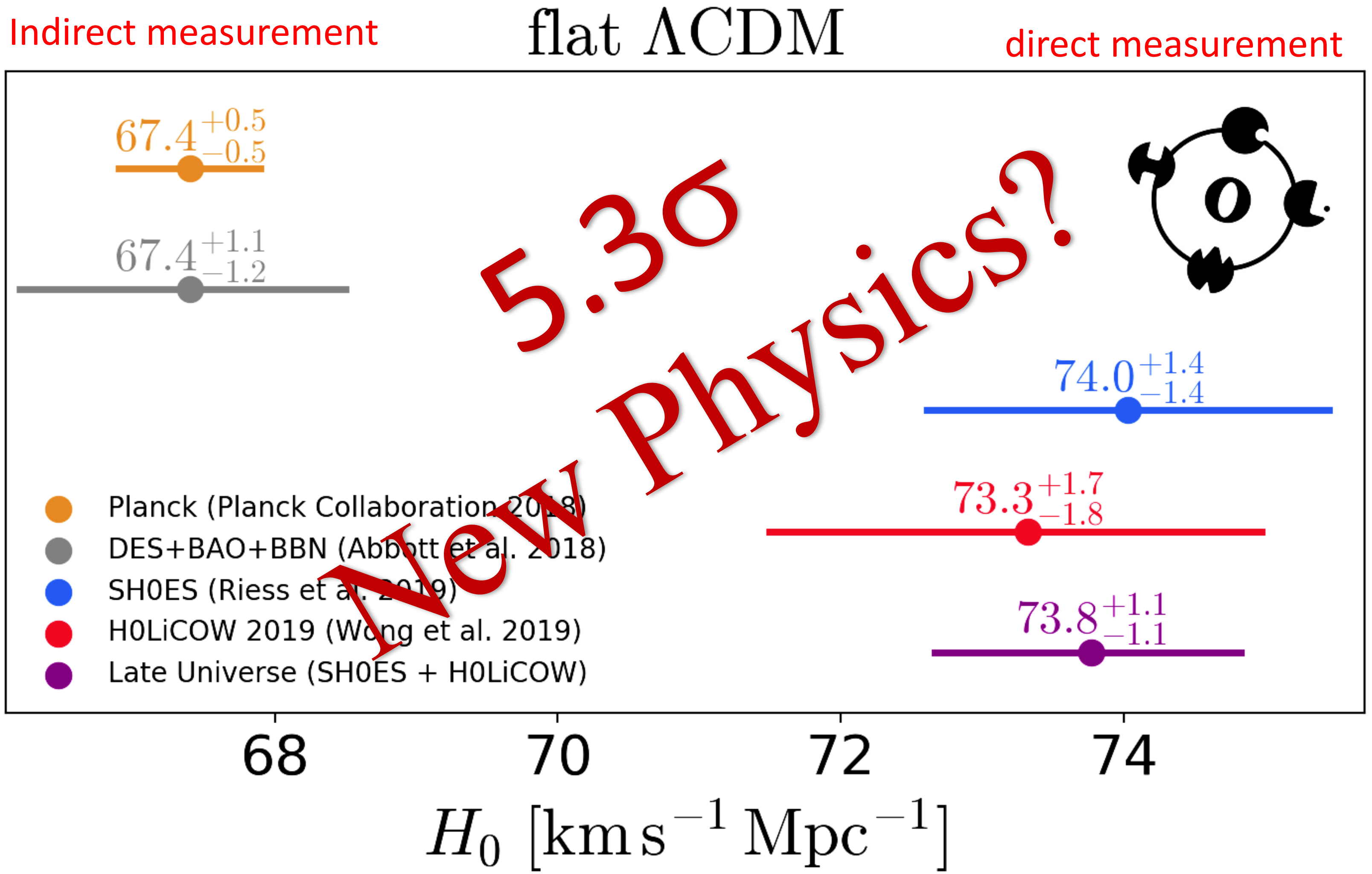


Planck + LCDM vs. External Datasets





H_0 Tension

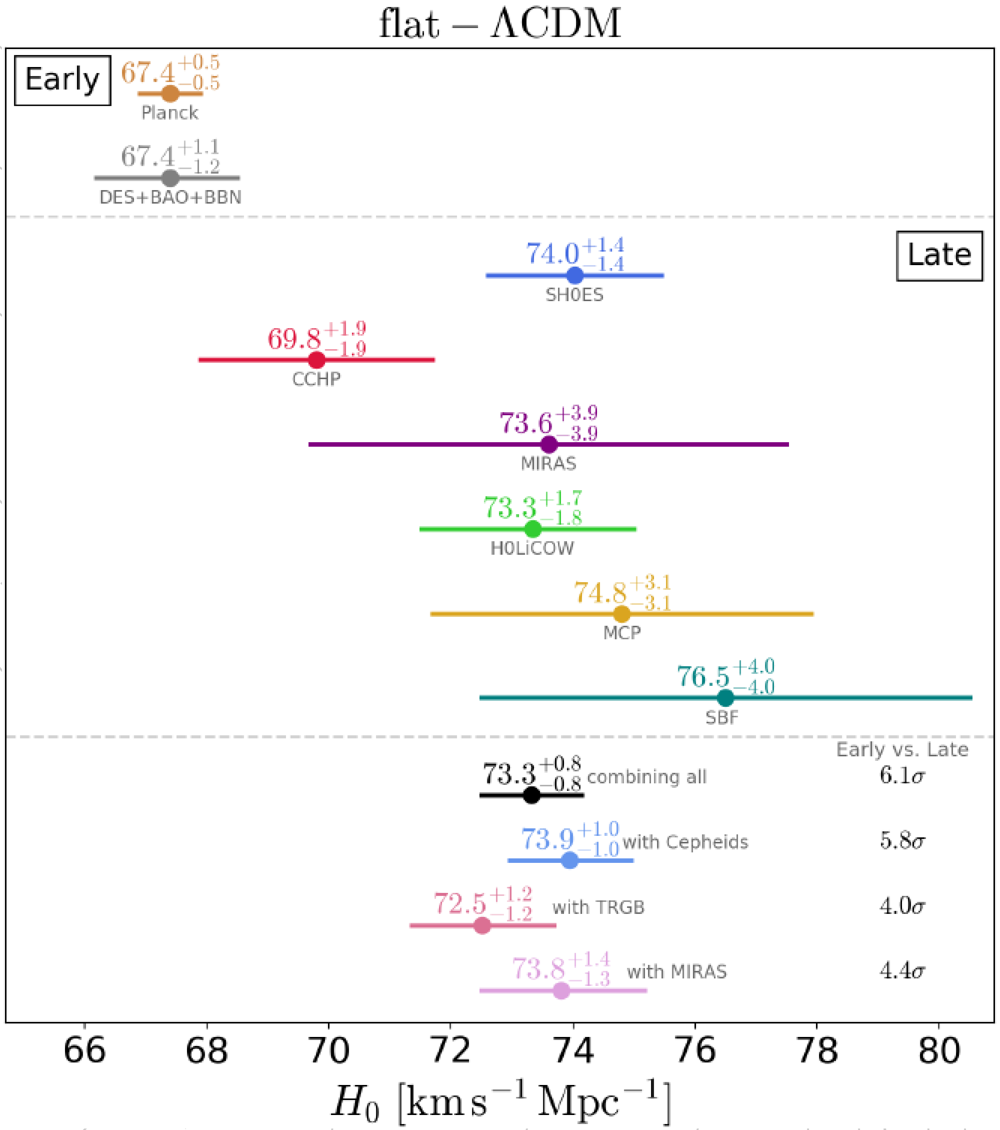

Indirect measurement
flat ΛCDM
direct measurement
$67.4^{+0.5}_{-0.5}$
$67.4^{+1.1}_{-1.2}$
$74.0^{+1.4}_{-1.4}$
$73.3^{+1.7}_{-1.8}$
$73.8^{+1.1}_{-1.1}$
5.3σ
New Physics?
Planck (Planck Collaboration 2018)
DES+BAO+BBN (Abbott et al. 2018)
SH0ES (Riess et al. 2019)
H0LiCOW 2019 (Wong et al. 2019)
Late Universe (SH0ES + H0LiCOW)
68
70
72
74
H_0 [km s^{-1} Mpc^{-1}]

Tensions between the Early and the Late Universe

Discrepancies developing between observations at early and late cosmological time may require an expansion of the standard model, and may lead to the discovery of new physics.

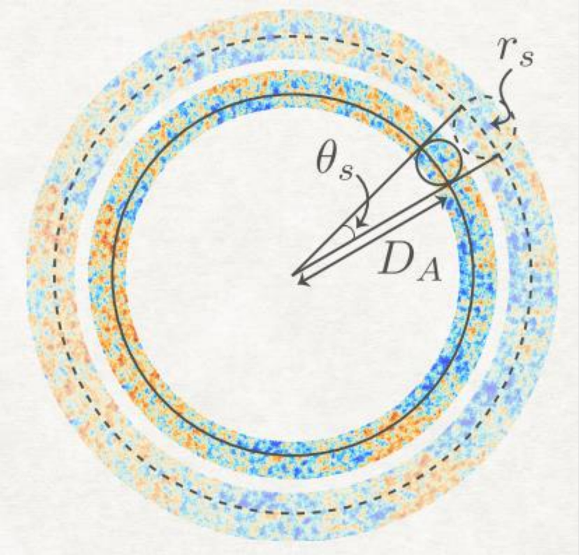
arXiv:1907.10625

L. Verde, T. Treu and A. G.~Riess, Nature Astron. **3**, 891



HOW TO SOLVE?

- Planck analysis of the CMB power spectra provides a precise measurement of the acoustic angular scale of the sound horizon at last scattering (recombination).
- z_* is the redshift at last scattering (at which the CMB photon optical depth equals unity). Planck measures $100\theta_* = 1.04109 \pm 0.00030$ (68 %, TT, TE, EE+lowE), with 0.03 % precision.
- Any solution should keep this scale fixed.



$$\theta_* = \frac{r_s(z_*)}{D_A(z_*)}$$

HOW TO SOLVE?

The sound horizon size at recombination

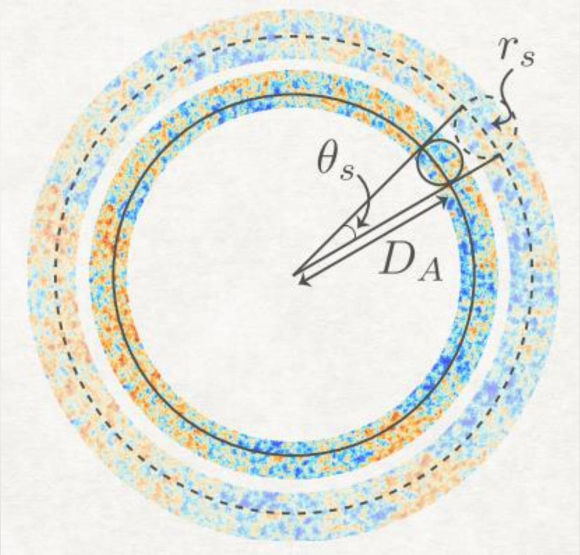
$$r_s(z_*) = \frac{1}{H_0}\int_{z_*}^{\infty} \frac{c_s(z)}{E(z)}dz = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)}dz.$$

The sound speed

$$c_s^2(z) = \frac{1}{3}\left[1 + \frac{3}{4}\frac{\omega_b(z)}{\omega_r(z)}\right]^{-1}.$$

The angular distance to recombination

$$D_A(z_*) = \frac{1}{(1+z_*)H_0}\int_0^{z_*} \frac{dz}{E(z)} = \frac{1}{1+z_*}\int_0^{z_*} \frac{dz}{H(z)}.$$



$$\theta_* = \frac{r_s(z_*)}{D_A(z_*)}$$

HOW TO SOLVE?

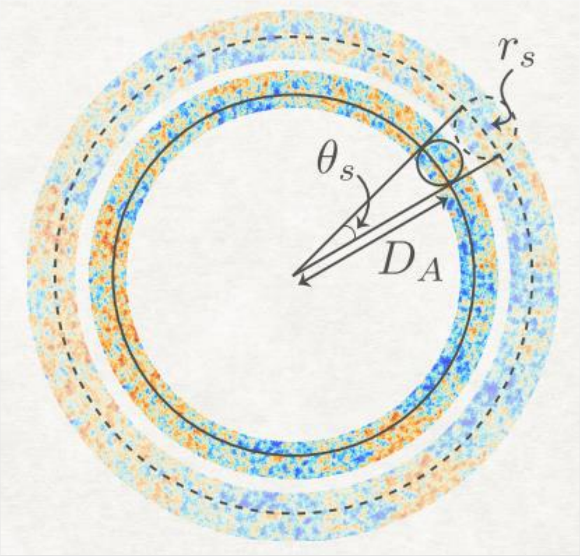
Early Universe

(PRE-RECOMBINATION)

- Decrease $r_s(z^*)$ at fixed θ_* To decrease $D_A(z^*)$ and increase H_0.
- Late universe observables are unaffected.

Hubble rate (LCDM)

$$H(z)^{\Lambda CDM} = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda},$$



$$\theta_* = \frac{r_s(z_*)}{D_A(z_*)}$$

Late Universe

(POST-RECOMBINATION)

- Keep $r_s(z^*)$ and $D_A(z^*)$ fixed and break the relationship between D_A and H_0.
- Early universe physics is left unaffected.

EARLY UNIVERSE SOLUTION

One way to solve H_0-tension is to reduce the sound horizon size at recombination rs(z∗) by ~ 10 Mpc,

$$r_s(z_*) = \frac{1}{H_0}\int_{z_*}^{\infty} \frac{c_s(z)}{E(z)}dz = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)}dz.$$

The sound speed cs is sensitive to the physical densities of baryon (ωb) and radiation (ωr)

$$c_s^2(z) = \frac{1}{3}\left[1 + \frac{3}{4}\frac{\omega_b(z)}{\omega_r(z)}\right]^{-1}.$$

The physical densities are well constrained by BBN, therefore it is not recommended to reduce the sound horizon via changing c_s. On the other hand, the sound horizon can be reduced by increasing the Hubble expansion rate H(z) before recombination

EARLY UNIVERSE SOLUTION

This can be achieved by introducing an early dark energy component which should decay just before recombination when the sound horizon is decreased as required. This keeps post-recombination just as ΛCDM. However, in order to restore θ∗, one expect the angular distance to recombination DA to decrease similar to rs

$$D_A(z_*) = \frac{1}{(1+z_*)H_0} \int_0^{z_*} \frac{dz}{E(z)} = \frac{1}{1+z_*} \int_0^{z_*} \frac{dz}{H(z)}.$$

Since the expansion rate E(z) after recombination is just as in ΛCDM, the H_0 should be increased to solve the tension.

EARLY UNIVERSE

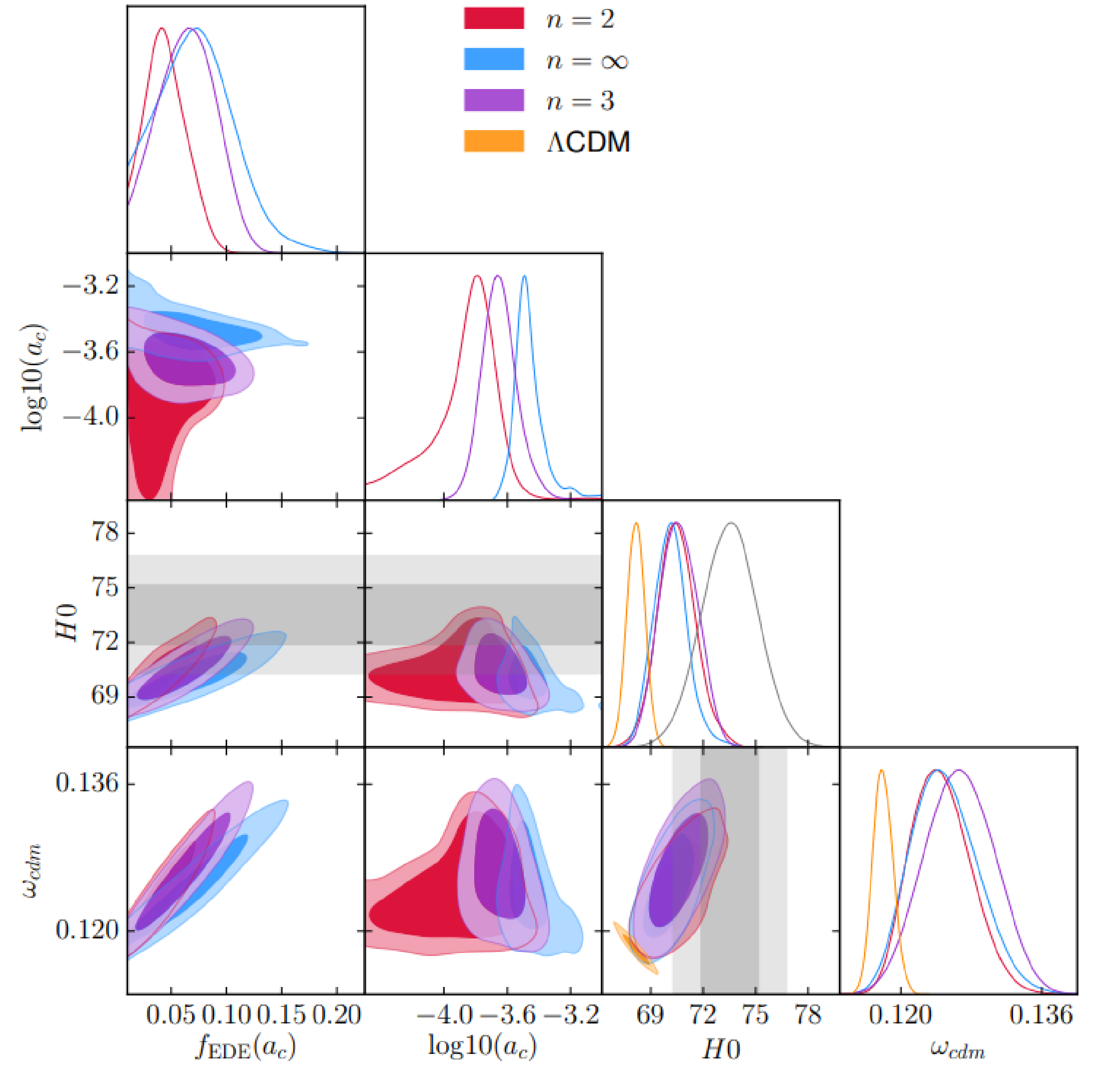
- Early Dark Energy
- Energy injection
- Extra free streaming neutrinos
- Self-interacting neutrino

EARLY UNIVERSE

Early Dark Energy

This is realized by introducing an extra scalar field with an oscillating potential $V(\phi) \propto (1 - \cos\phi)^n$ during the epoch between the matter-radiation equality and the recombination to be diluted faster than radiation. It has been shown that this type of early universe modification can give $H_0 = 70.6 \pm 1.3$ km/s/Mpc (Poulin et al. 2019) including a prior H_0 value from SH0ES.

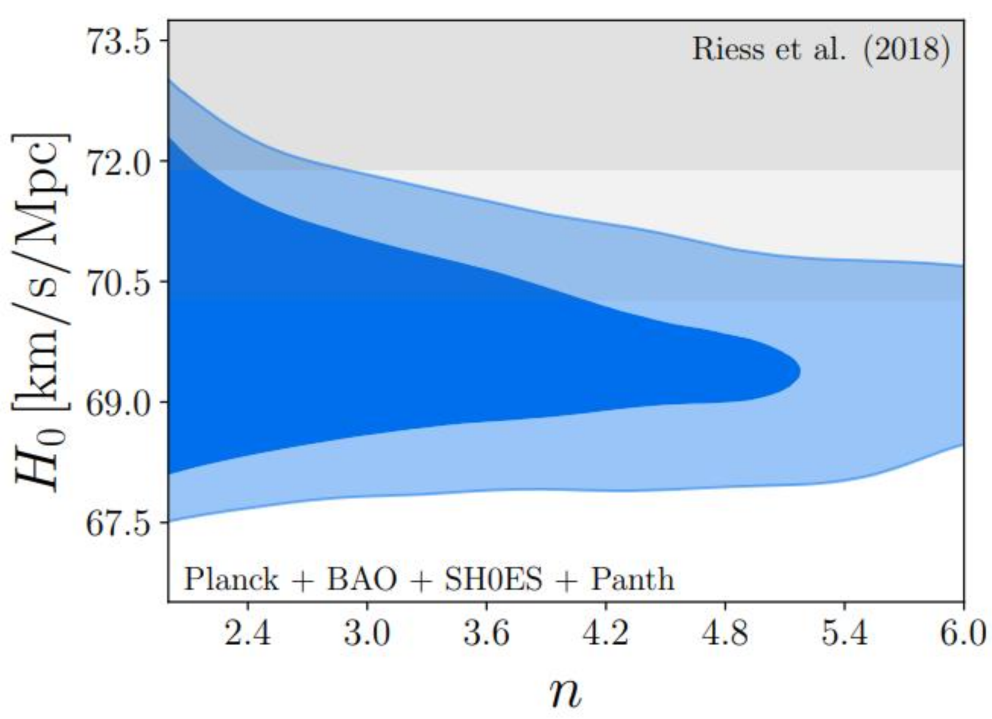
There is no evidence for EDE when CMB data is used alone (Hill et al. 2020) .



EARLY UNIVERSE SOLUTION

Energy injection (Rock 'n' Roll Solutions)

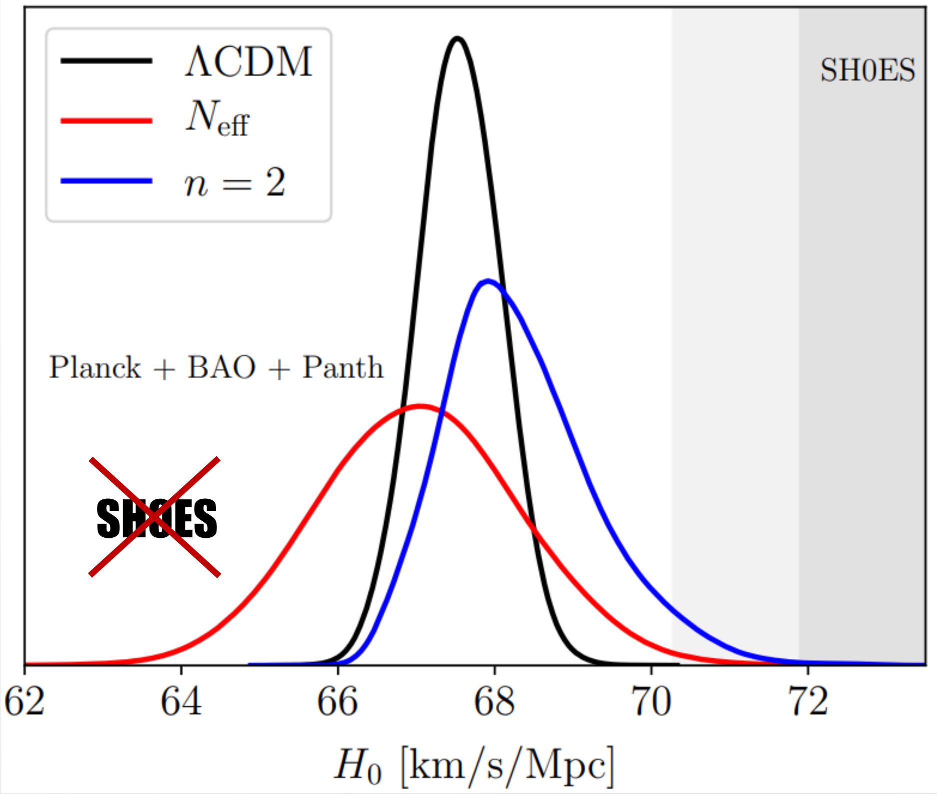
This is realized by introducing an extra scalar field $V(\phi) \propto \phi^n$ localized at the recombination with simple asymptotic behavior, both oscillatory (rocking) and rolling. It has been shown that this type of modifications (n=2) can give higher H_0 value (Agrawal et al. 2019).



EARLY UNIVERSE SOLUTION

Energy injection (Rock 'n' Roll Solutions)

This is realized by introducing an extra scalar field $V(\phi) \propto \phi^n$ localized at the recombination with simple asymptotic behavior, both oscillatory (rocking) and rolling. It has been shown that this type of modifications (n=2) can give higher H_0 value (Agrawal et al. 2019).

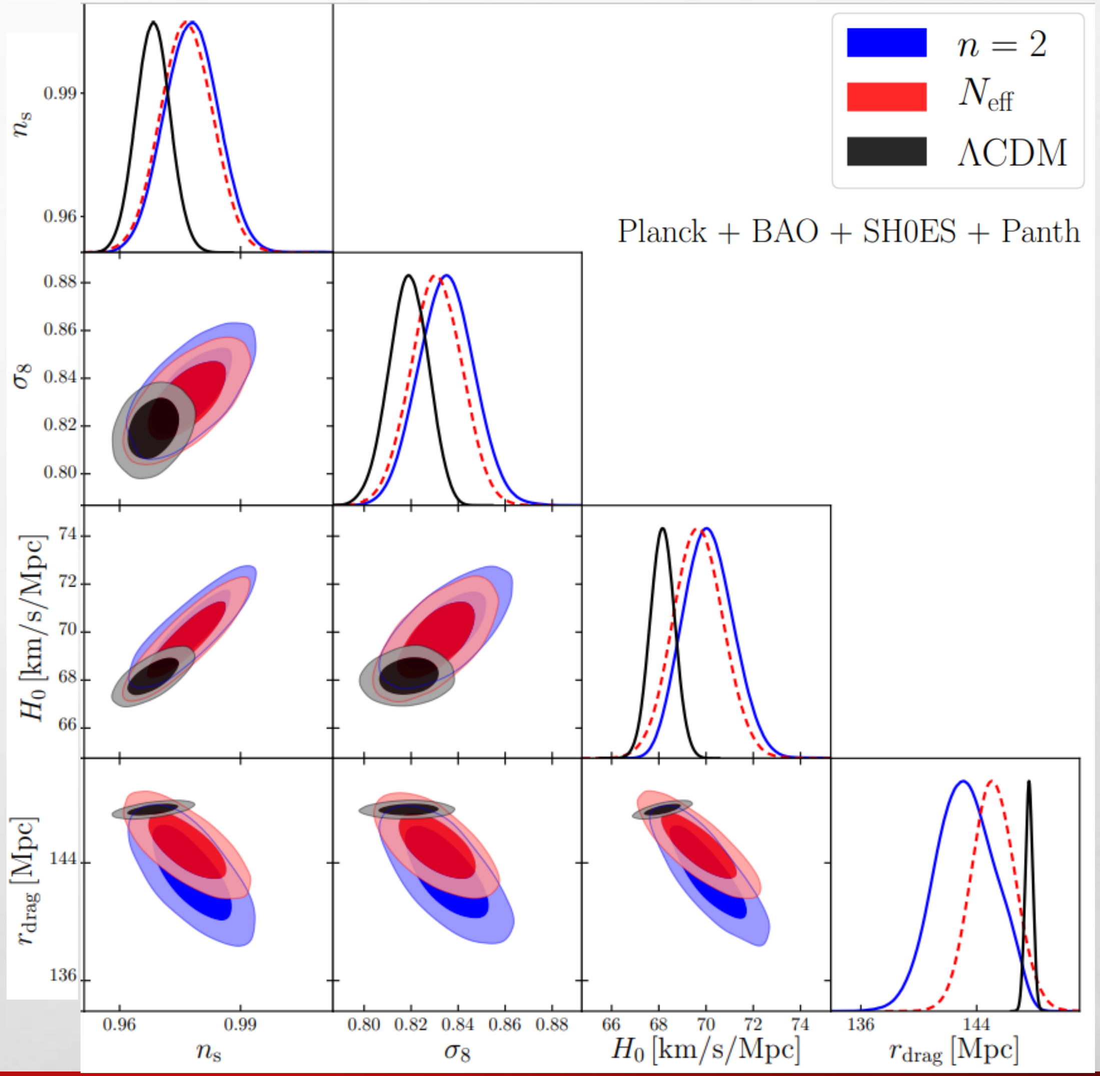


EARLY UNIVERSE

Energy injection (Rock 'n' Roll Solutions)

This is realized by introducing an extra scalar field $V(\phi) \propto \phi^n$ localized at the recombination with simple asymptotic behavior, both oscillatory (rocking) and rolling. It has been shown that this type of modifications (n=2) can give higher H_0 value (Agrawal et al. 2019).

To bring the CMB damping tail in agreement with the data, it requires a change to the primordial spectrum of fluctuations (As and ns). This leads larger values of σ8.



EARLY UNIVERSE

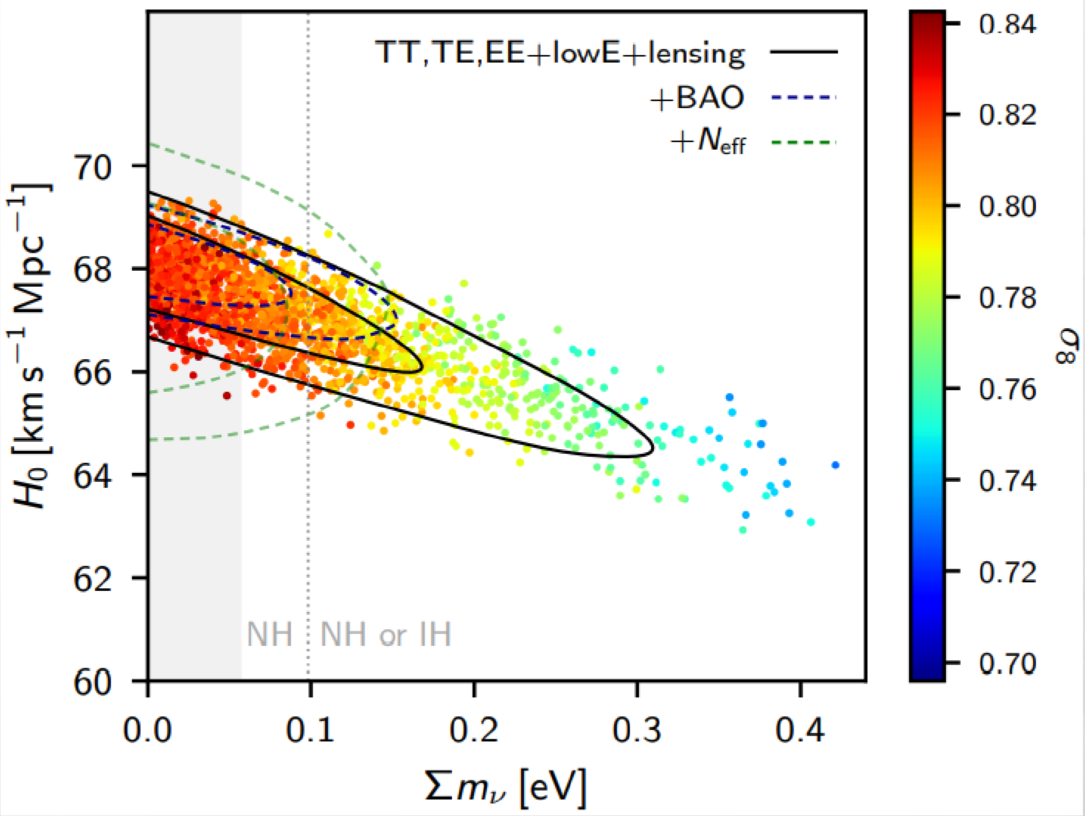
Extra free streaming neutrinos

(dark radiation)

This can be realized by introducing extra free streaming neutrinos with a specific gravitational coupling.

Planck 2018 results

- Solid black contours (Planck TT, TE, EE + lowE + lensing + BAO)
- Dashed lines the joint constraint also including Riess et al. (2018).



EARLY UNIVERSE

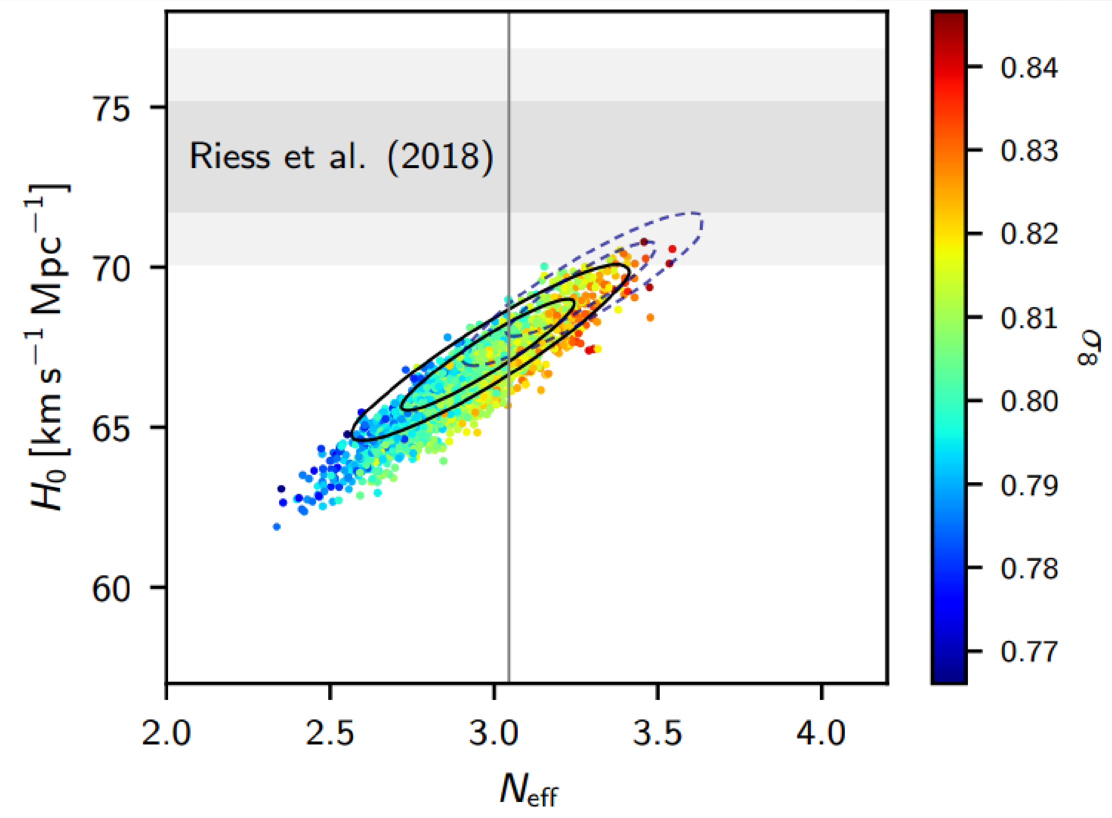
Extra free streaming neutrinos

(dark radiation)

This can be realized by introducing extra free streaming neutrinos with a specific gravitational coupling.

Planck 2018 results

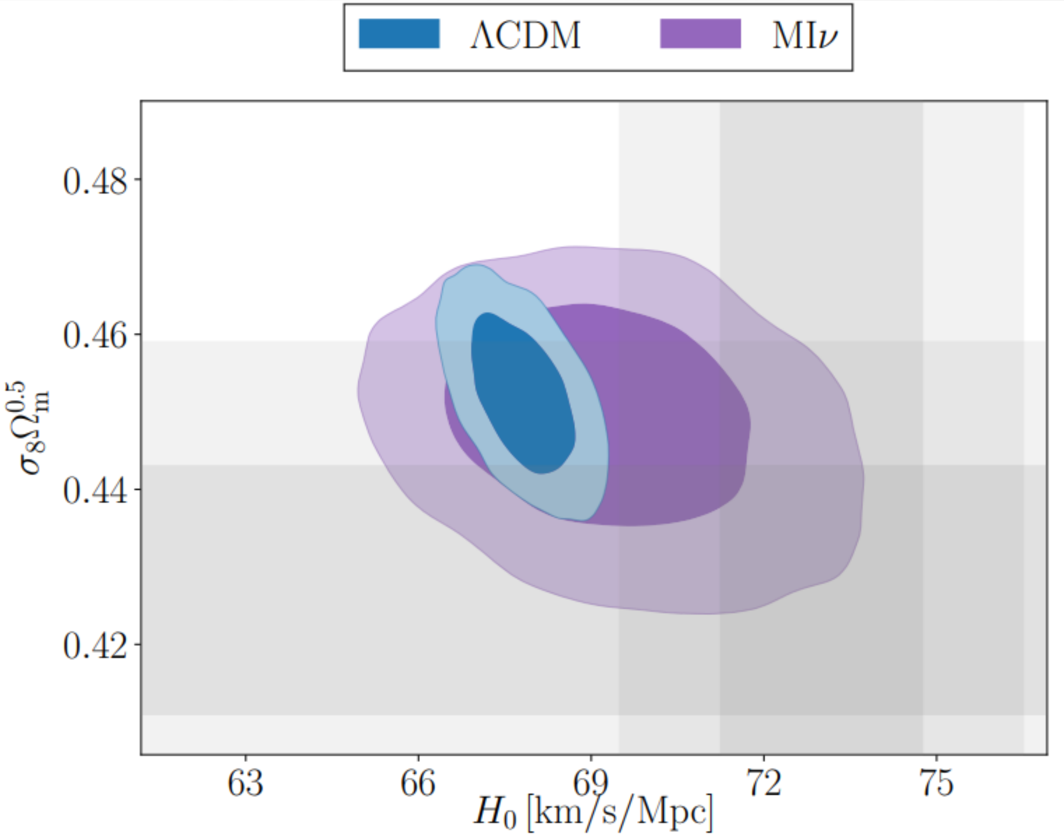
- Solid black contours (Planck TT, TE, EE + lowE + lensing + BAO)
- Dashed lines the joint constraint also including Riess et al. (2018).



EARLY UNIVERSE

Self-interacting neutrino

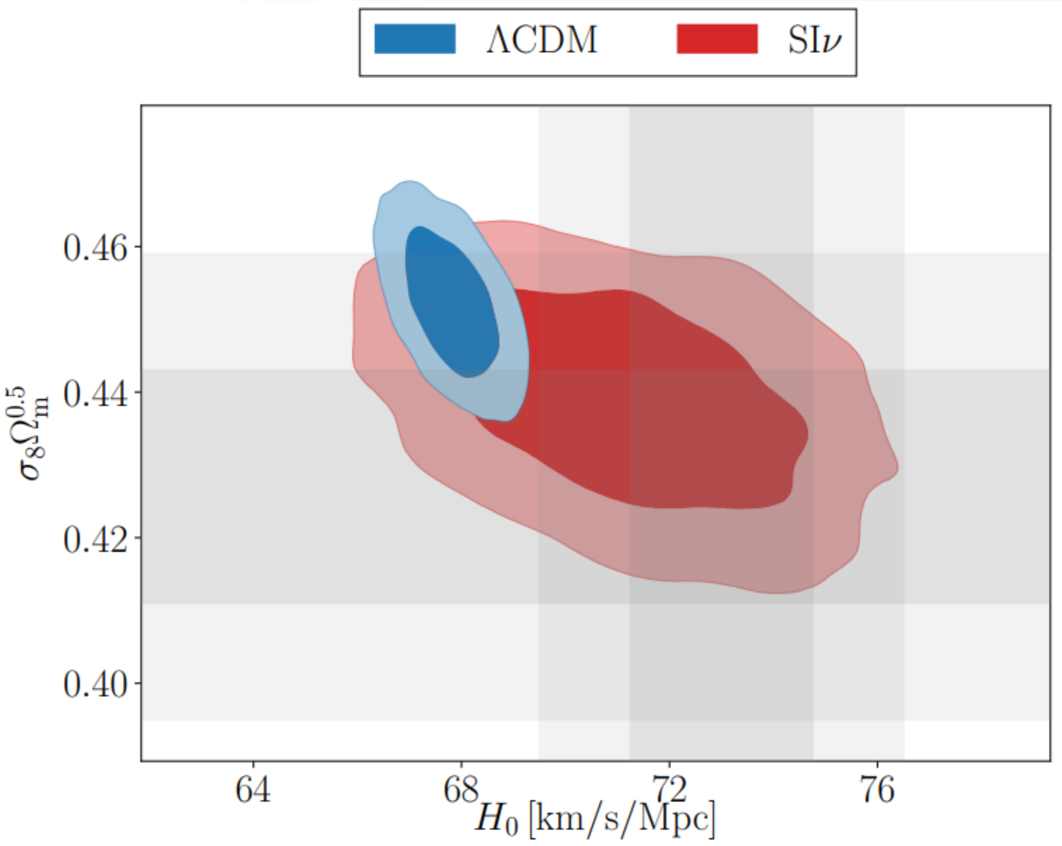
This is realized by allowing neutrino to act as tightly coupled radiation instead of free streaming and also by allowing extra species of neutrinos $N_{eff} \approx 4$ (Kreisch et al. 2019).This offers a possible solution to the H0 and σ8. We show here that delaying the onset of neutrino free streaming until close to the epoch of matter-radiation equality can naturally accommodate a larger value for the Hubble constant H0 = 72.3 ± 1.4 km s−1Mpc−1 and a lower value of the matter fluctuations σ8 = 0.786 ± 0.020, while not degrading the fit to the cosmic microwave background (CMB) damping tail.



EARLY UNIVERSE

Self-interacting neutrino

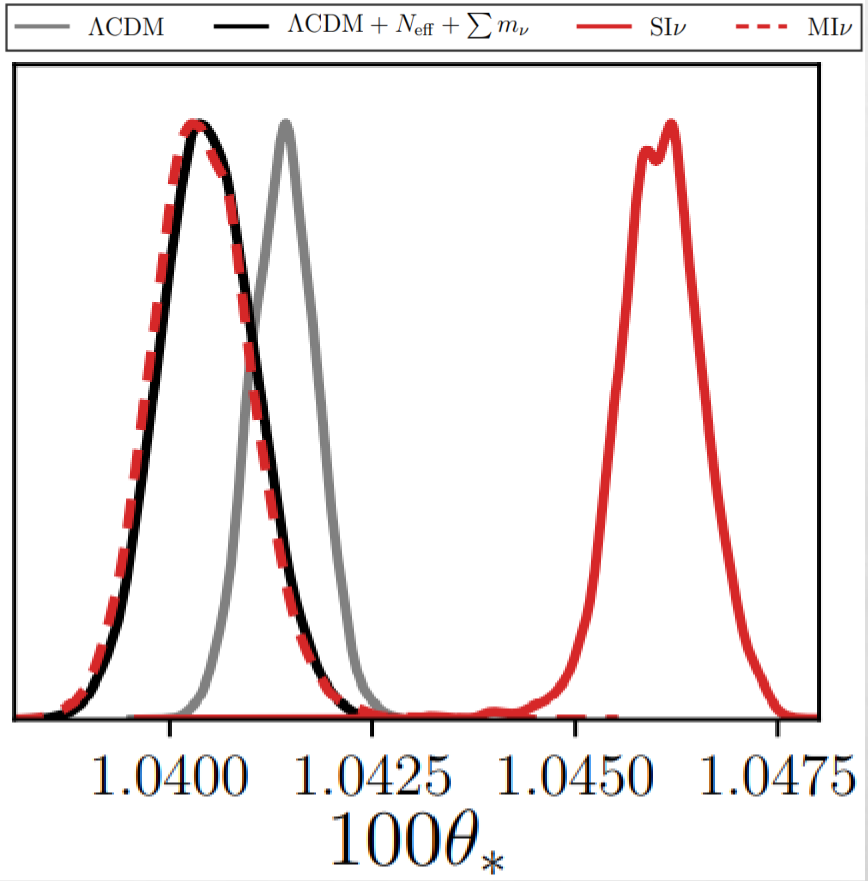
This is realized by allowing neutrino to act as tightly coupled radiation instead of free streaming and also by allowing extra species of neutrinos $N_{eff} \approx 4$ (Kreisch et al. 2019).This offers a possible solution to the H0 and σ8. We show here that delaying the onset of neutrino free streaming until close to the epoch of matter-radiation equality can naturally accommodate a larger value for the Hubble constant H0 = 72.3 ± 1.4 km s−1Mpc−1 and a lower value of the matter fluctuations σ8 = 0.786 ± 0.020, while not degrading the fit to the cosmic microwave background (CMB) damping tail.



EARLY UNIVERSE

Self-interacting neutrino

Self interacting neutrinos provide a better framework for solving both tensions simultaneously. But the tightly-coupled neutrinos scenario, unlike free streaming, does not phase shift the photon-baryon fluctuations. Hence, the CMB power spectra are slightly displaced towards high-ℓ, and **the acoustic scale θ_* at recombination must take larger values in order to restore the fit with observed CMB**. It remains a challenge to construct and verify viable models with requirements beyond standard model physics with **very large couplings**.



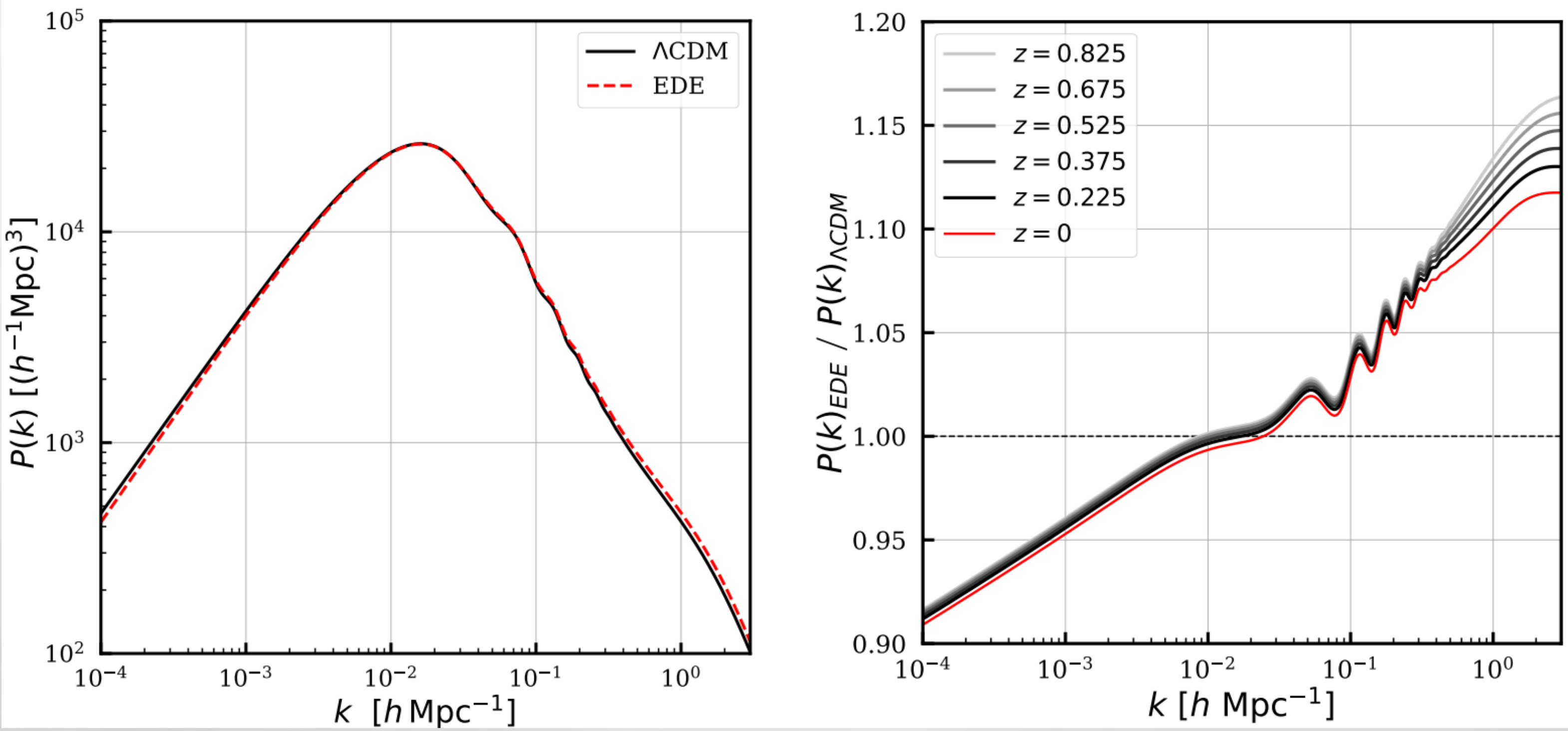
DOES EARLY UNIVERSE SOLVE?

These models suffer fine tuning problems at eV scales and also lead to severe scale dependent changes in the matter spectrum, which worsen the tension. They also shift some standard CDM parameters, in particular the spectral index n_s and the physical baryon density $\Omega_b h^2$ ($h = H_0/100$ km/s/Mpc) (Smith et al. 2020).

However, it has been shown that such modification leads to scale dependent severe changes in the matter spectrum p(k) across a decade in k-space and also to worsen the density fluctuation amplitude tension (Hill et al. 2020). In addition, the same study shows that the there is no evidence for EDE when CMB data is used alone.

In general, any sort of early dark energy slightly affects the growth of perturbations during its acting period. This implies an increase in the CDM density to compensate the loss in the perturbation growth in order to fit the CMB data (Hill et al. 2020).

DOES EARLY UNIVERSE SOLVE?



Hill et al. 2020

HOW TO SOLVE?

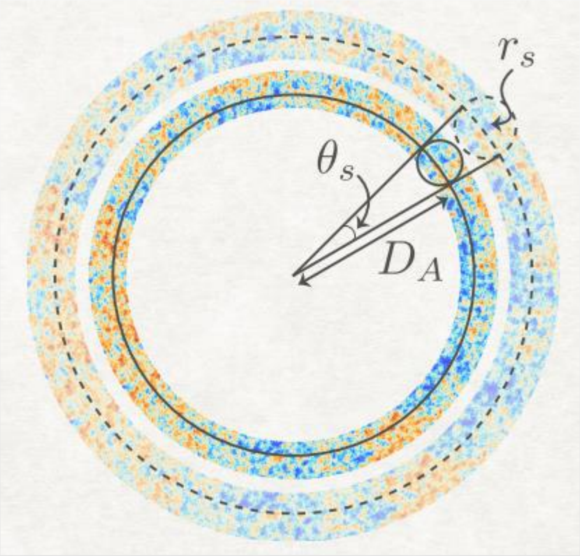
Early Universe

(PRE-RECOMBINATION)

- Decrease $r_s(z^*)$ at fixed θ_* to decrease $D_A(z^*)$ and increase H_0.
- Late universe observables are left unaffected.

Hubble rate (LCDM)

$$H(z)^{\Lambda CDM} = H_0\sqrt{\Omega_{m,0}(1+z)^3+\Omega_\Lambda},$$



$$\theta_* = \frac{r_s(z_*)}{D_A(z_*)}$$

Late Universe

(POST-RECOMBINATION)

- Keep $r_s(z^*)$ and $D_A(z^*)$ fixed and break the relationship between D_A and H_0.
- Early universe physics is left unaffected.

LATE UNIVERSE SOLUTION

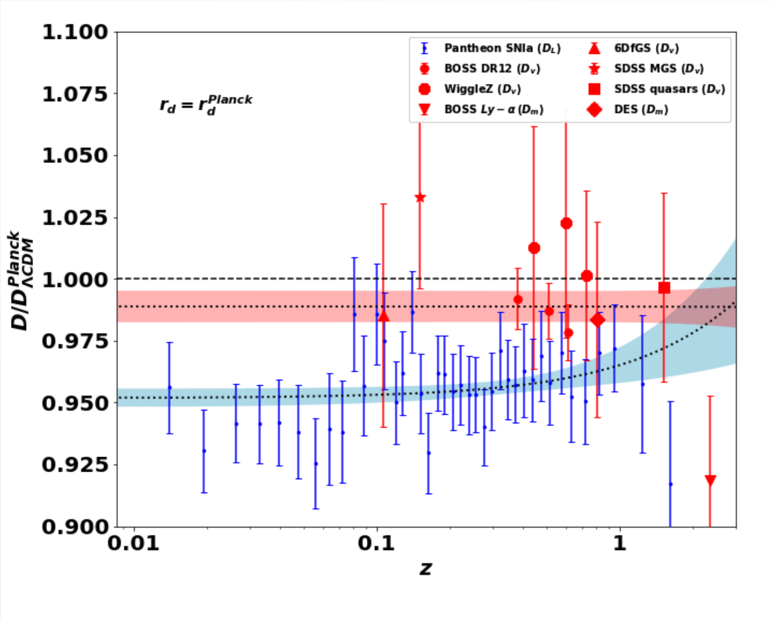
Another way to solve the H_0 tension is to change the late universe by considering dark energy different for the cosmological constant.

$$H(z) = H_0 E(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{de} y(z)},$$

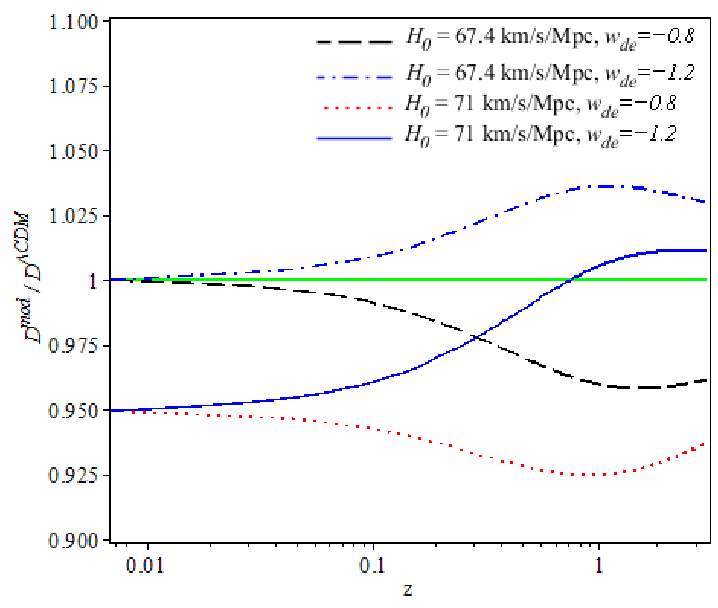
$$\Omega_{de} = 1 - \Omega_{m,0},\; y(z) = \exp\left[3 \int_0^z \frac{1 + w_{de}(z')}{1 + z'} dz'\right].$$

For ΛCDM model, $w_{de} = w_\Lambda = -1$, we have $y(z) = 1$. In order to account for larger H_0 value, the expansion rate $E(z)$ should be lower than the ΛCDM to keep $D_A(z*)$ fixed to Planck measurement. As clear, this can be achieved by considering phantom dark energy $w_{de} = w_{ph} < -1$ (for simplicity we take w_{ph} fixed). Clearly, one can see that $y(z = 0) = 1$ while $y(z > 0) < 1$, which in return finds $E(z)_{ph} < E(z)_{\Lambda CDM}$ for redshifts $z > 0$.

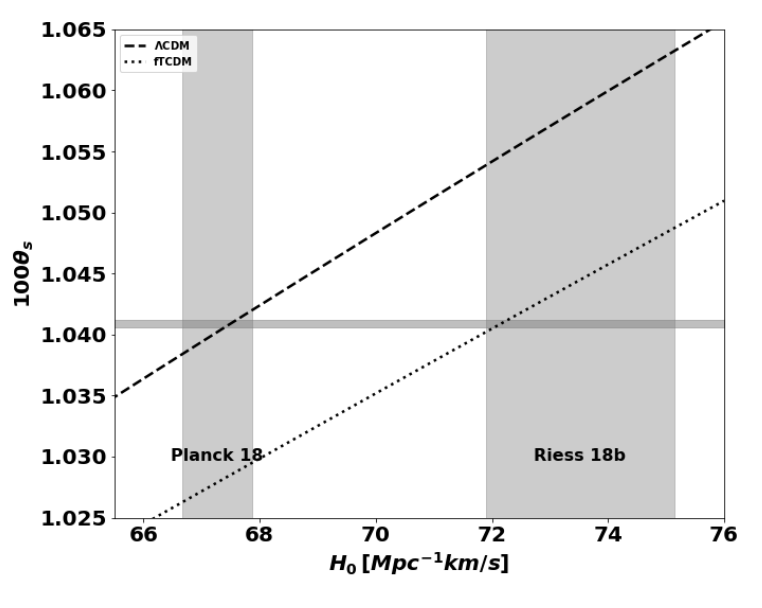
LATE UNIVERSE SOLUTION



Two distance indicators: (a) The SNIa luminosity distance anchored to the H0 value as inferred by the distance ladder method. (b) The BAO measurements of the angular distance anchored to the rd value as inferred by the inverse distance ladder method.



Distance indicators of four different models according to the choices of H0 and wde . By comparison to the observed distance indicators, it is clear that the phantom dark energy can fit better.



The horizontal grey band shows the CMB 1σ constraints on the angular scale of the sound horizon at recombination θ∗, while the vertical bands show the 1σ constraints on the H0 as derived by CMB and as measured by distance ladder method.

LATE UNIVERSE SOLUTION

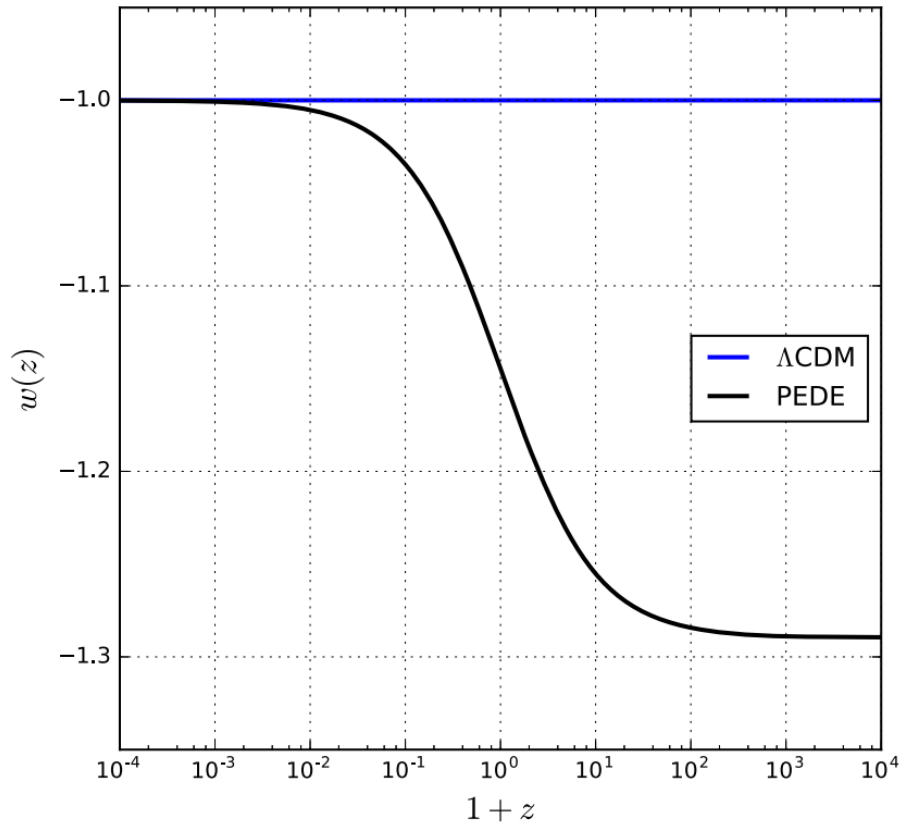
- Phenomenological Emergent Dark Energy
- Interacting dark energy
- Modified Gravity

LATE UNIVERSE SOLUTION

Phenomenological Emergent Dark Energy

Another approach which has been proposed recently is the dynamical dark energy, e.g. phenomenological emergent dark energy model (PEDE) is suggested (Li & Shaeloo 2019). This approach is motivated by a specific parameterization of the DE density parameter evolution $\Omega_{DE} \propto 1 - \tanh(1+z)$ which results in a forever phantom DE equation of state.

$$w(z) = \frac{1}{3}\frac{d\ln\Omega_{\rm DE}}{dz}(1+z) - 1$$

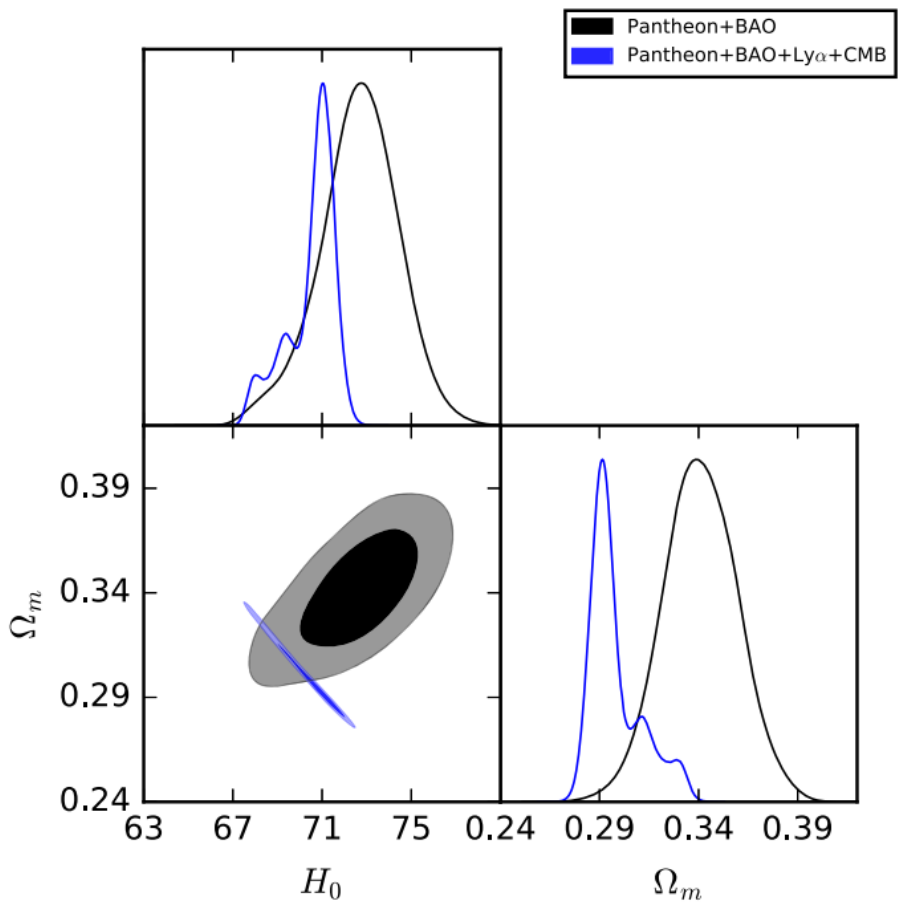


LATE UNIVERSE SOLUTION

Phenomenological Emergent Dark Energy

Another approach which has been proposed recently is the dynamical dark energy, e.g phenomenological emergent dark energy model (PEDE) is suggested (Li & Shaeloo 2019). This approach is motivated by a specific parameterization of the DE density parameter evolution $\Omega_{DE} \propto 1 - \tanh(1+z)$ which results in a forever phantom DE equation of state.

$$w(z) = \frac{1}{3}\frac{d \ln \Omega_{\rm DE}}{dz}(1+z) - 1$$

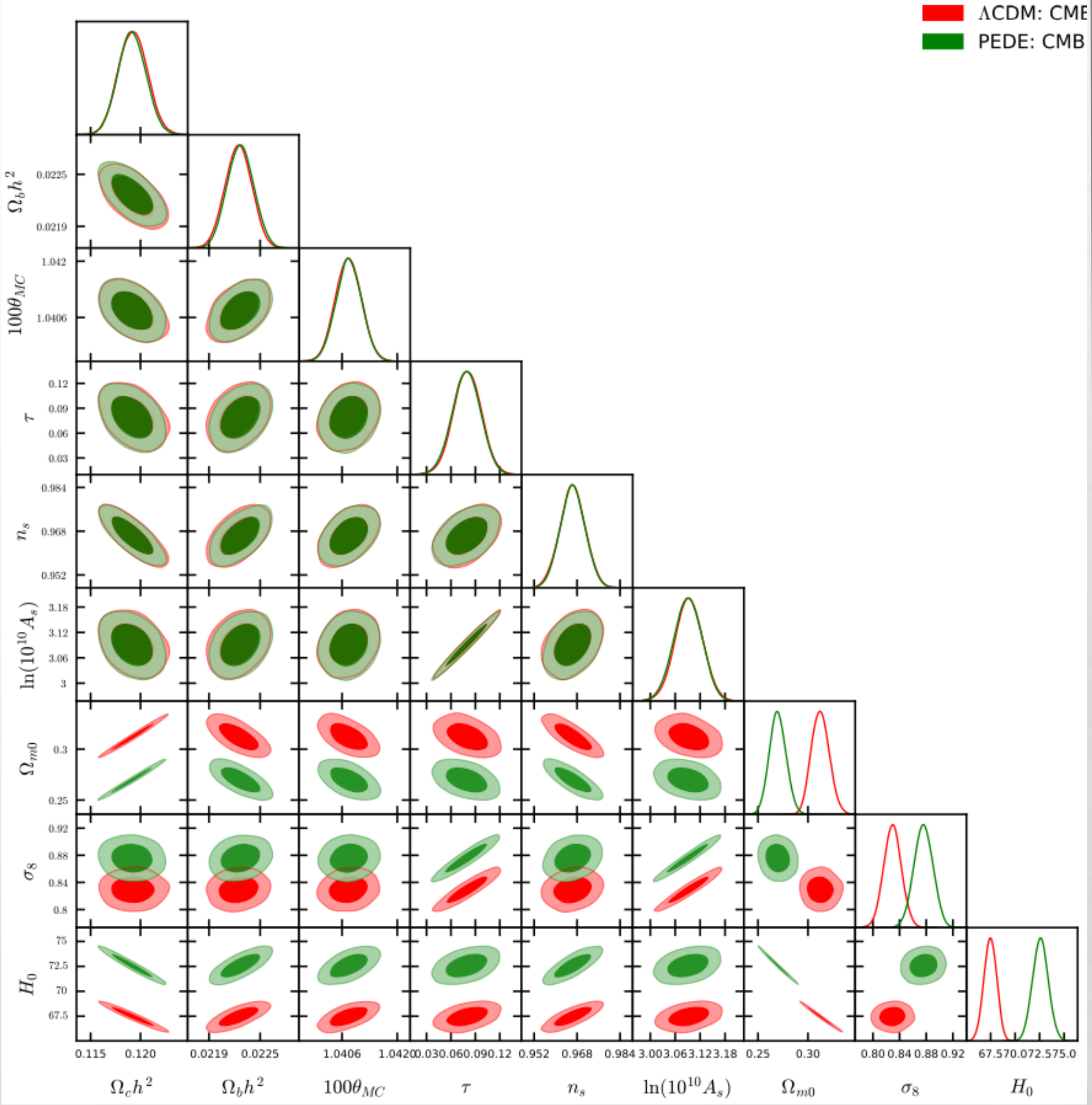


LATE UNIVERSE

Phenomenological Emergent Dark Energy

Another approach which has been proposed recently is the dynamical dark energy, e.g phenomenological emergent dark energy model (PEDE) is suggested (Li & Shaeloo 2019). This approach is motivated by a specific parameterization of the DE density parameter evolution $\Omega_{DE} \propto 1 - \tanh(1+z)$ which results in a forever phantom DE equation of state.

$$w(z) = \frac{1}{3}\frac{d\ln\Omega_{\rm DE}}{dz}(1+z) - 1$$



LATE UNIVERSE SOLUTION

Phenomenological Emergent Dark Energy

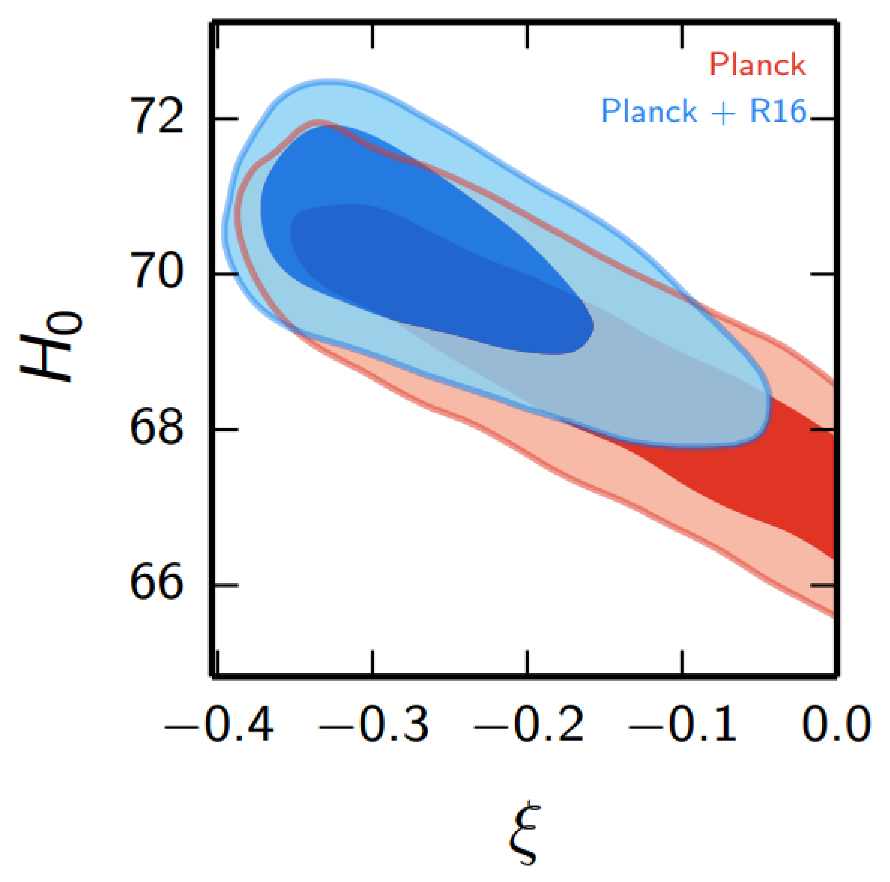
Although the model is still having only six parameters similar to LCDM, the PEDE has been introduced via the physical matter sector. However, such types of modifications are excluded because they have a negative squared sound speed $c_s^2 < 0$, while they should abide the stability and causality physical conditions $0 \leq c_s^2 \leq 1$ and to be compatible with the null energy condition (NEC). Otherwise, any small perturbations of the background energy density derive the cosmological model to be unstable, that is known as a Laplacian (or gradient) instability problem (Quiros et al. 2018).

LATE UNIVERSE SOLUTION

Interacting dark energy

In principle, if energy ow from dark matter to dark energy is allowed (negative matter-energy coupling ξ < 0), the dark matter density parameter is reduced and then H0 can have higher values satisfying the CMB constraint $\Omega_m h^2$. The H0 tension has been investigated within the interacting dark energy scenario (Di Valentino et al. 2017).

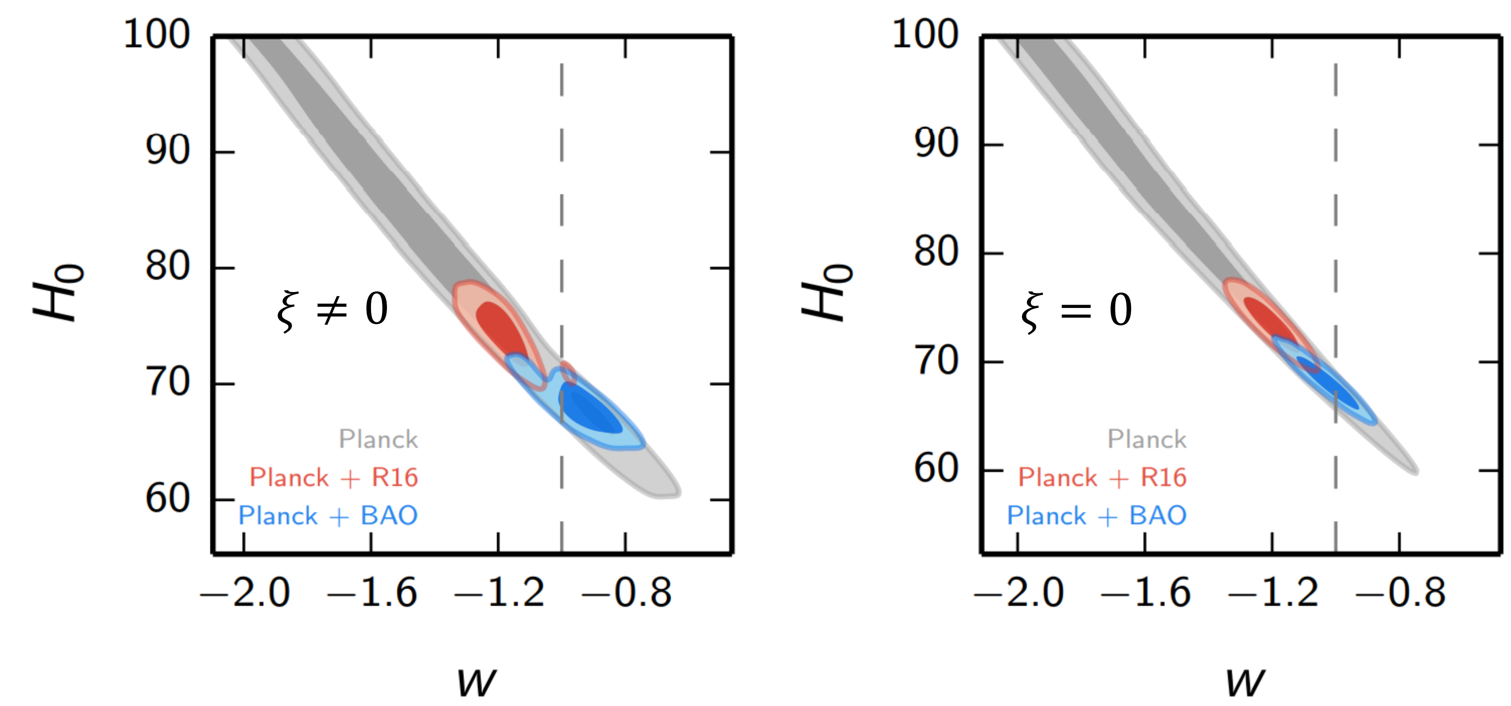
$$\dot{\rho}_{dm} + 3\mathcal{H}\rho_{dm} = \xi\mathcal{H}\rho_{de}$$
$$\dot{\rho}_{de} + 3\mathcal{H}(1+w)\rho_{de} = -\xi\mathcal{H}\rho_{de}$$



LATE UNIVERSE SOLUTION

Interacting dark energy

It has been shown that the combination of CMB and R18 gives an evidence for non-zero coupling with $\xi < 0$ along with phantom DE $w_{de} < -1$.



LATE UNIVERSE SOLUTION

Interacting dark energy

- Again, the NEC violation is still the main defect in addition to the presence of two extra free parameters and w_{de}.
- When BAO datasets are included, the analysis strongly enforces the model to CDM (w_{de}= -1) with no evidence for interacting DE.

LATE UNIVERSE SOLUTION

Infrared Gravity suggests some corrections by weakening gravity at the infrared scales. These appear only on large distances keeping the GR hold on solar system and below scales. Therefore, it is a reasonable choice of dark energy approach and it needs be investigated.

Infrared gravity effectively acts as a phantom dark energy which is recently favored to resolve the H0-tension between early and late universe. However, in modified gravity we can avoid the major problems which face phantom dark energy (Null Energy Conditions/instability problems) if assumed to be a canonical scalar field.

f(T) Cosmology

The f(T) Teleparallel Gravity

The Action

$$S = \frac{1}{2\kappa^2}\int d^4x\, e f(T) + S_m,$$

The Field Equations

$$\frac{1}{e}\partial_\mu\left(e S_a{}^{\mu\nu}\right) f' - e_a^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} f' + S_a{}^{\mu\nu}\partial_\mu T f'' + \frac{1}{4} e_a^\nu f = \frac{\kappa^2}{2} e_a^\rho \mathfrak{T}_\rho{}^\nu,$$

Einstein frame

$$G_{\mu\nu} = \mathcal{T}_{\mu\nu} + \tilde{\mathcal{T}}_{\mu\nu},$$

The torsional stress-energy tensor

$$\tilde{\mathcal{T}}_{\mu\nu} = \left(f_T T - f\right)\frac{1}{2} g_{\mu\nu} - 2S_{\mu\nu}{}^\sigma \partial_\sigma f_T + \left(1 - f_T\right) G_{\mu\nu}.$$

Modified Friedmann equations

FLRW $e_\mu{}^a = \mathrm{diag}(1, a(t), a(t), a(t)).$

$$T = -6H^2.$$

$$\rho_m = \frac{1}{2\kappa^2}\left[f(H) - H f_H\right],$$

$$p_m = \frac{-1}{2\kappa^2}\left[f(H) - H f_H - \frac{1}{3}\dot{H} f_{HH}\right] = \frac{1}{6\kappa^2}\dot{H} f_{HH} - \rho,$$

Phase Portraits of general f(T) Cosmology

The f(T) phase portrait

$$\dot{H} = 3(1+\omega_m)\frac{f(H) - H f_H(H)}{f_{HH}}.$$

one-dimensional autonomous system

f(T)= T = – 6H^2 (TEGR)

$$\dot{H} = -\frac{3}{2}(1+\omega_m)H^2 = \mathcal{F}(H).$$

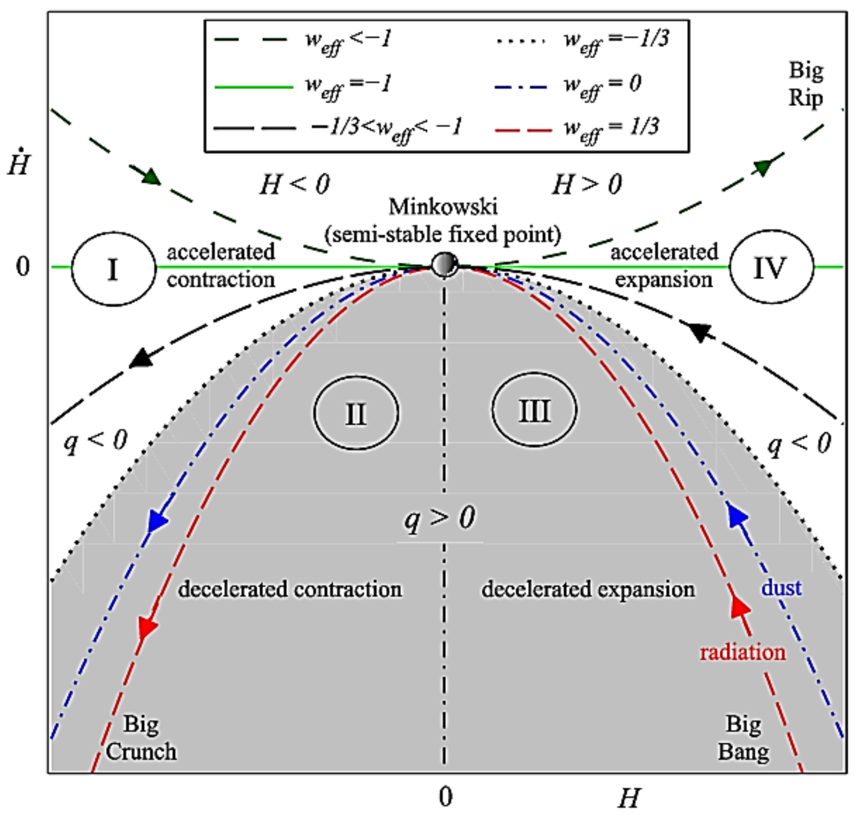
FOUR DYNAMICAL REGIONS OF THE PHASE SPACE

(I)-region represents an accelerated contracting universe as q<0 and H<0.

(II)-region represents an decelerated contracting universe as q>0 and H<0.

(III)-region represents an decelerated expanding universe as q>0 and H>0 which characterizes the standard thermal history (BBN).

(IV)-region represents an accelerated expanding universe as q<0 and H>0 which characterizes the so-called inflation (early universe) or dark energy (late universe).

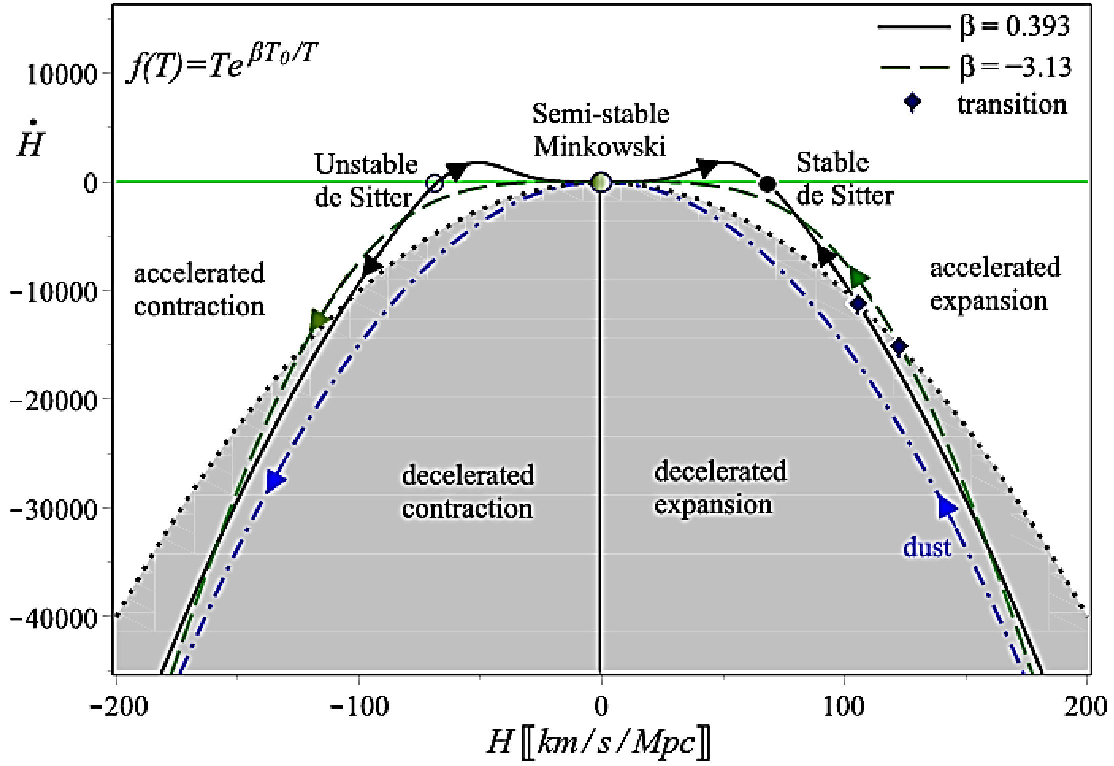


A. Awad, et. al., JCAP 02 (2018), 052 [arXiv:1710.10194]

Exponential Infrared f(T) Gravity

New Suggested Model
Exponential Infrared f(T) Gravity

The model: $f(T) = Te^{\beta T_0/T}$



The f(T) phase portrait

$$\dot{H} = -\frac{3}{2}(1+w)\frac{(H^2 - 2\beta H_0^2)H^4}{H^4 - \beta H_0^2 H^2 + 2\beta^2 H_0^4}.$$

The parameter is not independent

$$\beta = \frac{1}{2} + W\left(-\frac{1}{2}e^{-1/2}\,\Omega_{m,0}\right),$$

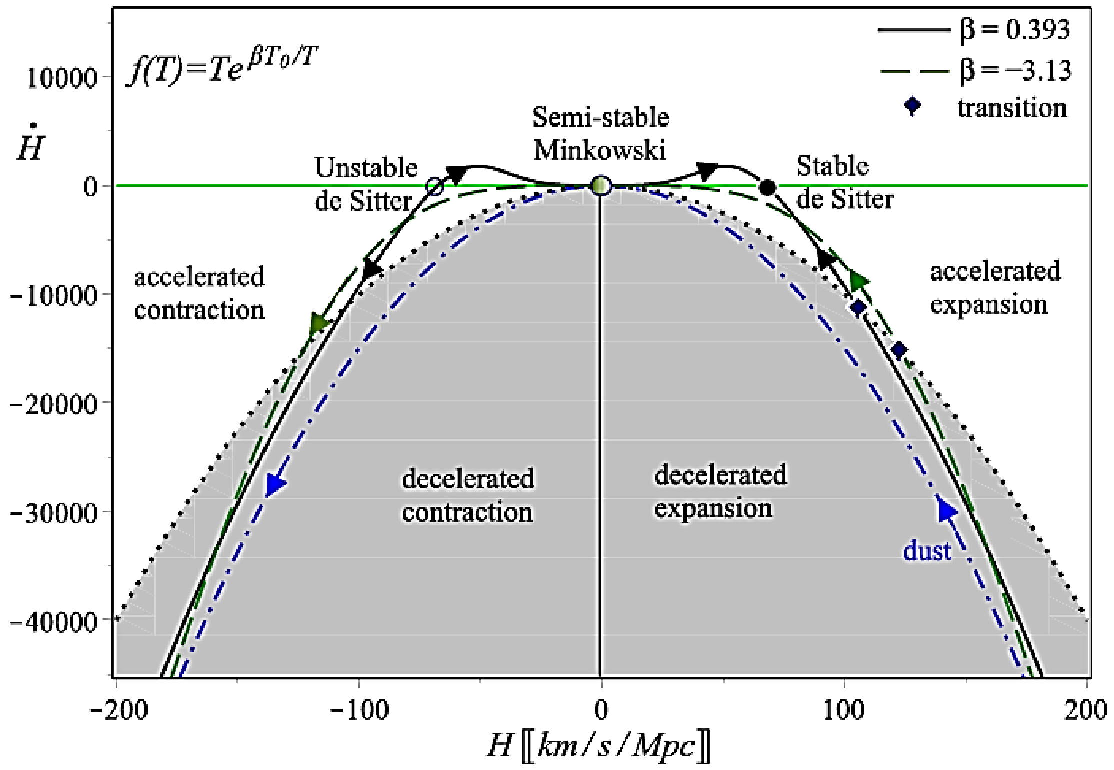
No extra free parameter

Exponential Infrared f(T) Gravity

The exponential correction reduces to unity at large T-regimes (early universe/strong gravity), and therefore the model does not change early universe at last scattering (CMB) or at solar system. It affects the evolution at late universe only on cosmic scales.

In infrared gravity models, usually each additional term in the Lagrangian introduces an extra free parameter. Generally these modifications introduce new free parameters which may require further explanation and interpretation. Finally, it favours the LCDM model statistically when confronting with observations. In addition, in our choice, we can describe a complete spectrum of infrared corrections without introducing new parameters which is not an easy task.

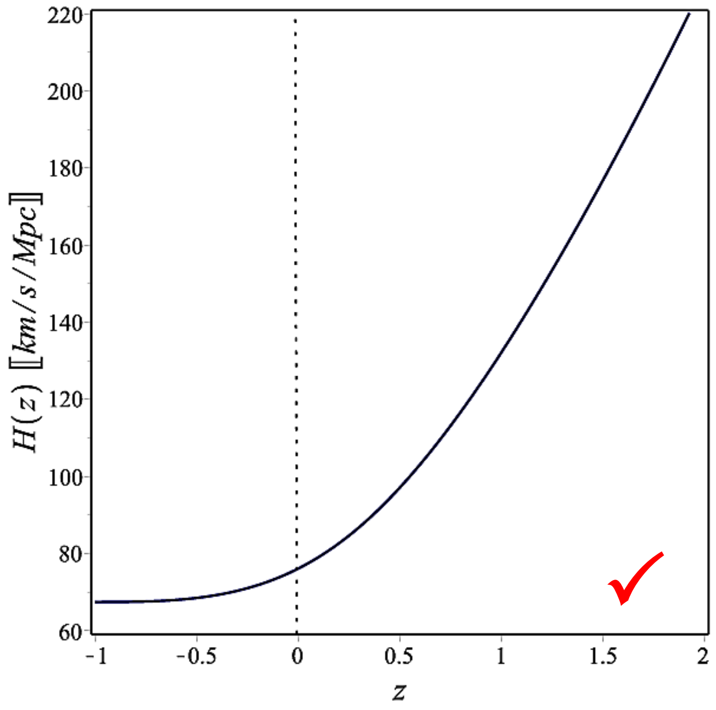
The model: $f(T) = Te^{\beta T_0/T}$



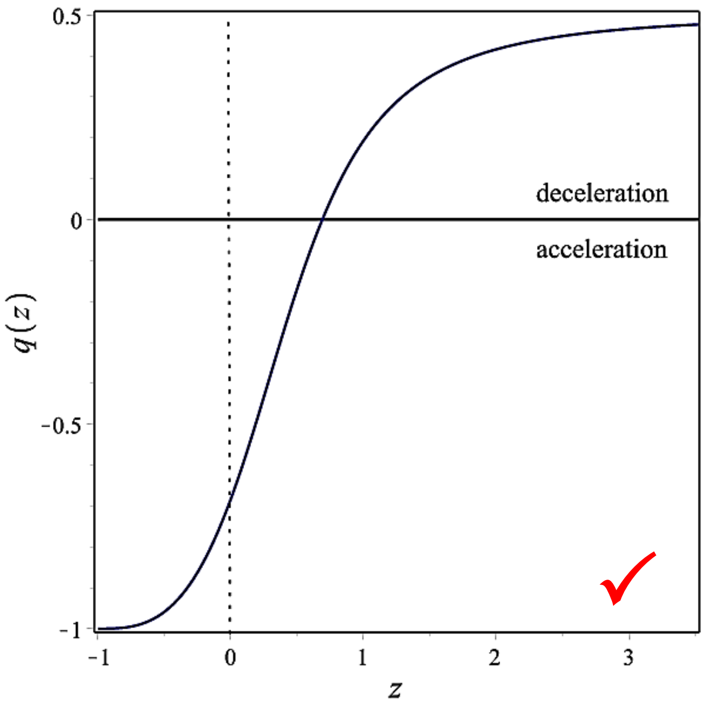
Exponential Infrared f(T) Gravity

Basic requirements

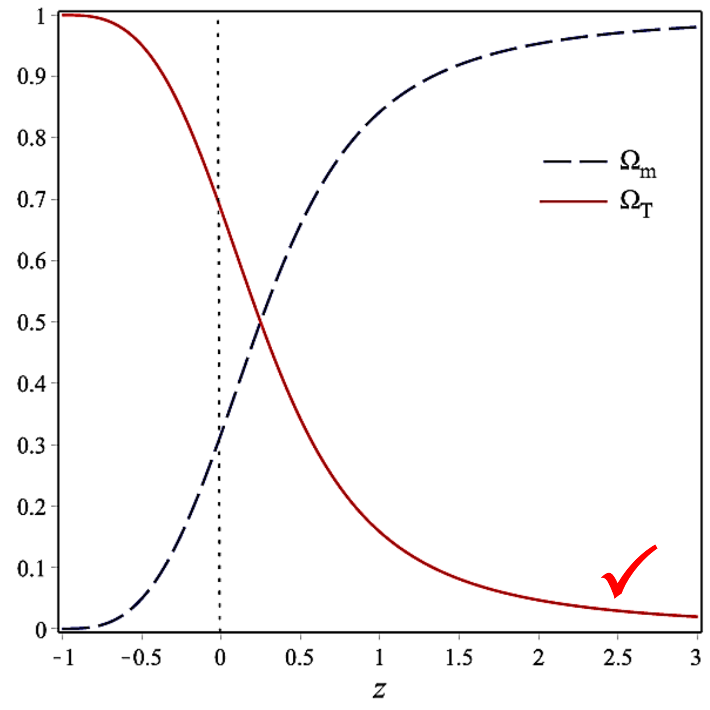
Exponential Infrared f(T) Gravity



Hubble evolution



Deceleration parameter

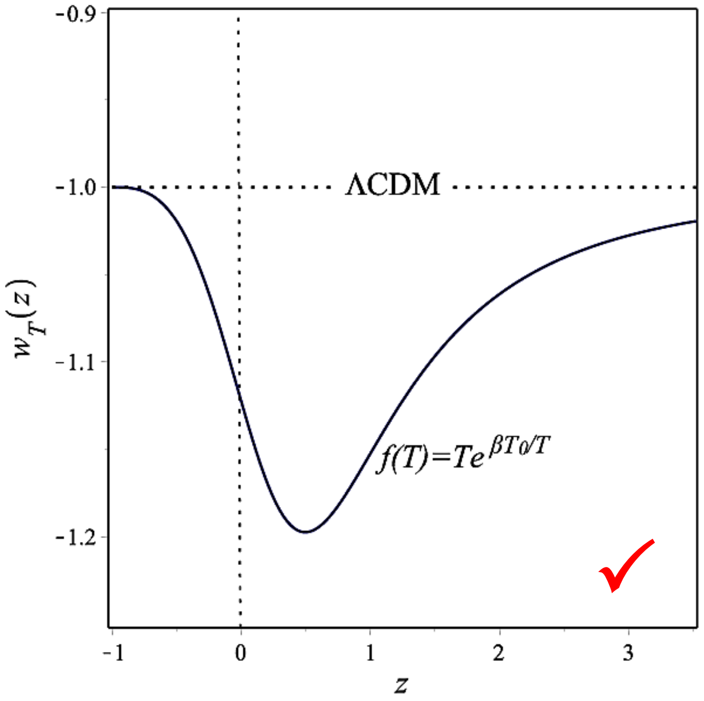


Density parameters

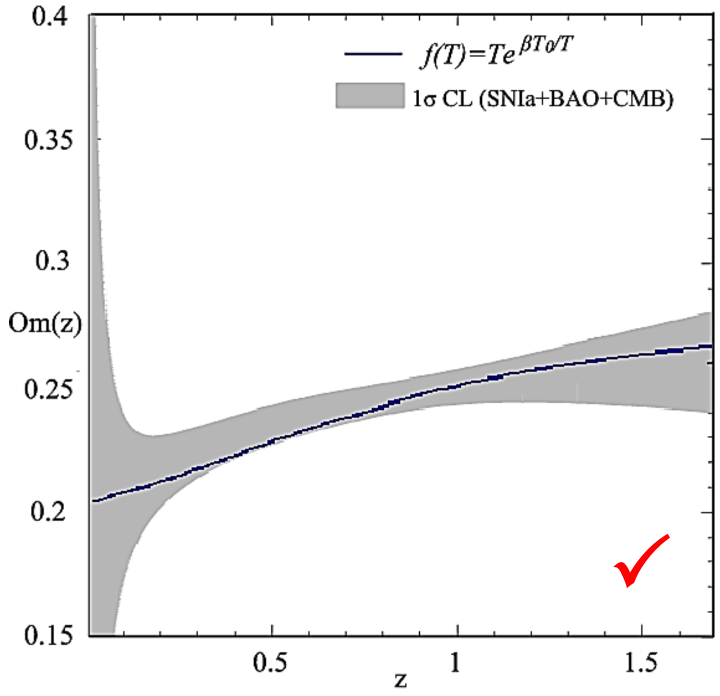
Exponential Infrared f(T) Gravity

Basic requirements

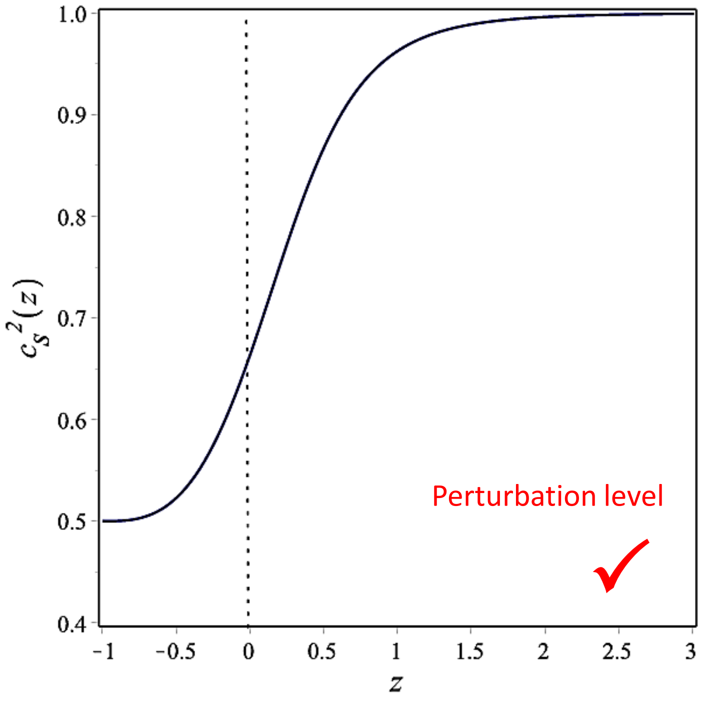
Exponential Infrared f(T) Gravity



Dark torsion



Om(z) parameter



Sound speed

Exponential IR f(T) gravity

Base=TT+TE+EE

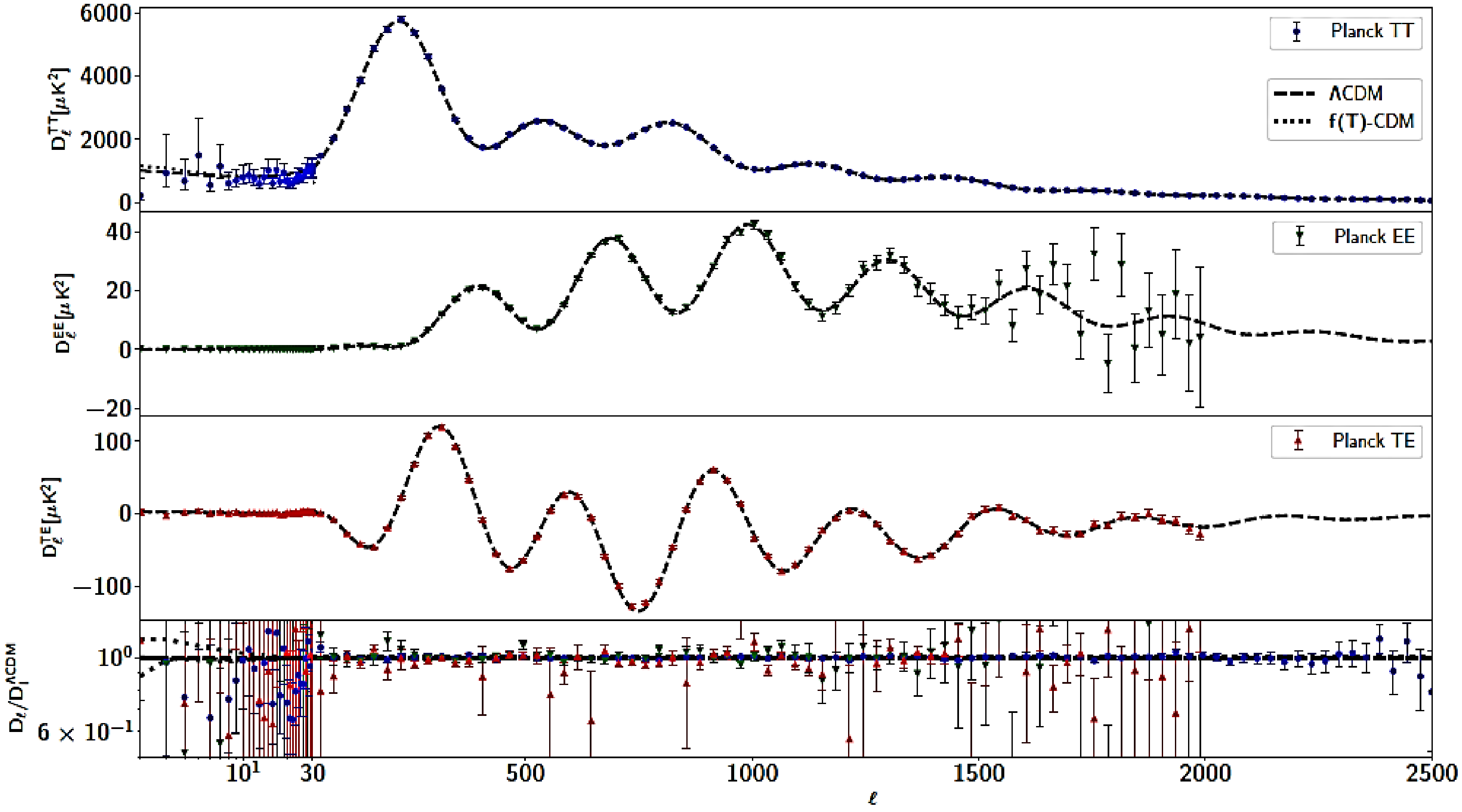
The model: $f(T) = Te^{\beta T_0/T}$

Parameter	ΛCDM Base 68% limits	ΛCDM Base+lensing 68% limits	ΛCDM Base+lensing+BAO 68% limits	$f(T)$-CDM Base 68% limits	$f(T)$-CDM Base+lensing 68% limits	$f(T)$-CDM Base+lensing+BAO 68% limits
$100\Omega_b h^2$	$2.236^{+0.014}_{-0.016}$	2.238 ± 0.015	2.244 ± 0.013	2.238 ± 0.015	2.241 ± 0.015	2.231 ± 0.013
$\Omega_c h^2$	0.1202 ± 0.0014	0.1200 ± 0.0012	0.11934 ± 0.00092	0.1200 ± 0.0014	0.1196 ± 0.0012	0.12106 ± 0.00092
$100\theta_s$	1.04188 ± 0.00029	1.04190 ± 0.00030	1.04197 ± 0.00028	1.04189 ± 0.00030	1.04191 ± 0.00030	1.04179 ± 0.00029
τ_{re}	0.0542 ± 0.0077	0.0545 ± 0.0073	0.0562 ± 0.0074	0.0540 ± 0.0078	0.0535 ± 0.0074	0.0500 ± 0.0069
$\ln(A_s 10^{10})$	3.045 ± 0.016	3.045 ± 0.014	3.048 ± 0.015	3.044 ± 0.016	3.043 ± 0.014	3.038 ± 0.013
n_s	0.9650 ± 0.0044	0.9658 ± 0.0041	0.9673 ± 0.0037	0.9661 ± 0.0044	0.9668 ± 0.0042	0.9636 ± 0.0037
H_0 [km/s/Mpc] ...	$67.31^{+0.57}_{-0.65}$	67.41 ± 0.54	67.72 ± 0.42	72.03 ± 0.70	72.24 ± 0.64	71.49 ± 0.47
Ω_m	0.3162 ± 0.0085	0.3149 ± 0.0074	0.3107 ± 0.0056	0.2758 ± 0.0078	0.2735 ± 0.0069	0.2818 ± 0.0053
σ_8	0.8117 ± 0.0074	0.8116 ± 0.0059	0.8108 ± 0.0060	0.8425 ± 0.0075	0.8412 ± 0.0061	0.8433 ± 0.0058
$S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$	0.833 ± 0.016	0.831 ± 0.013	0.825 ± 0.010	0.808 ± 0.016	0.803 ± 0.013	0.817 ± 0.010
z_{re}	7.66 ± 0.78	7.69 ± 0.73	7.85 ± 0.73	7.62 ± 0.79	7.56 ± 0.75	$7.24^{+0.76}_{-0.68}$
Age[Gyr]	$13.796^{+0.026}_{-0.022}$	$13.793^{+0.025}_{-0.022}$	13.782 ± 0.020	13.706 ± 0.026	13.699 ± 0.025	13.723 ± 0.020
z_s	$1088.91^{+0.23}_{-0.21}$	1088.88 ± 0.21	1088.78 ± 0.17	1088.88 ± 0.22	1088.82 ± 0.21	1089.03 ± 0.17
r_s[Mpc]	144.47 ± 0.30	144.51 ± 0.26	144.64 ± 0.21	144.51 ± 0.30	144.59 ± 0.27	144.30 ± 0.22
z_{drag}	1059.98 ± 0.30	1060.01 ± 0.31	1060.08 ± 0.29	1059.99 ± 0.30	1060.04 ± 0.30	1059.91 ± 0.29
r_{drag}[Mpc]	147.04 ± 0.30	147.08 ± 0.26	147.19 ± 0.23	147.08 ± 0.30	147.14 ± 0.27	146.88 ± 0.23
χ^2_{min}	1386.83	1389.91	1392.83	1386.69	1390.67	1397.25

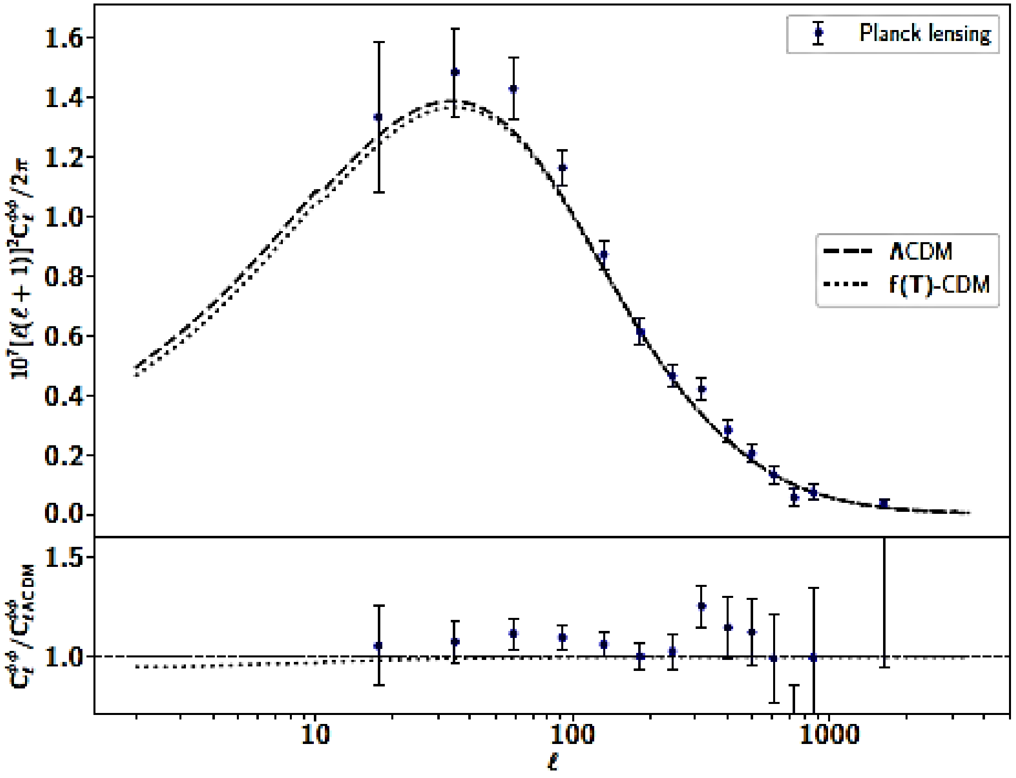
M. Hashim, et. al., JCAP 07 (2021), 053 [arXiv:2104.08311]

Exponential IR f(T) gravity

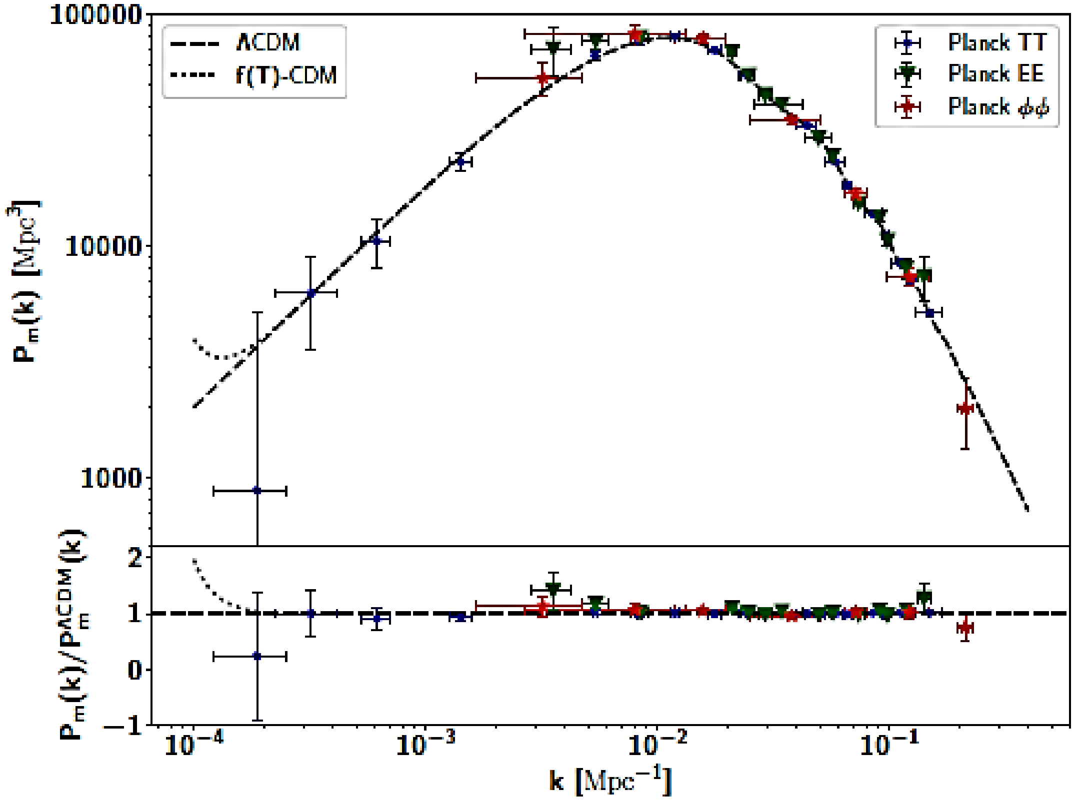
CMB power spectra



Exponential IR f(T) gravity

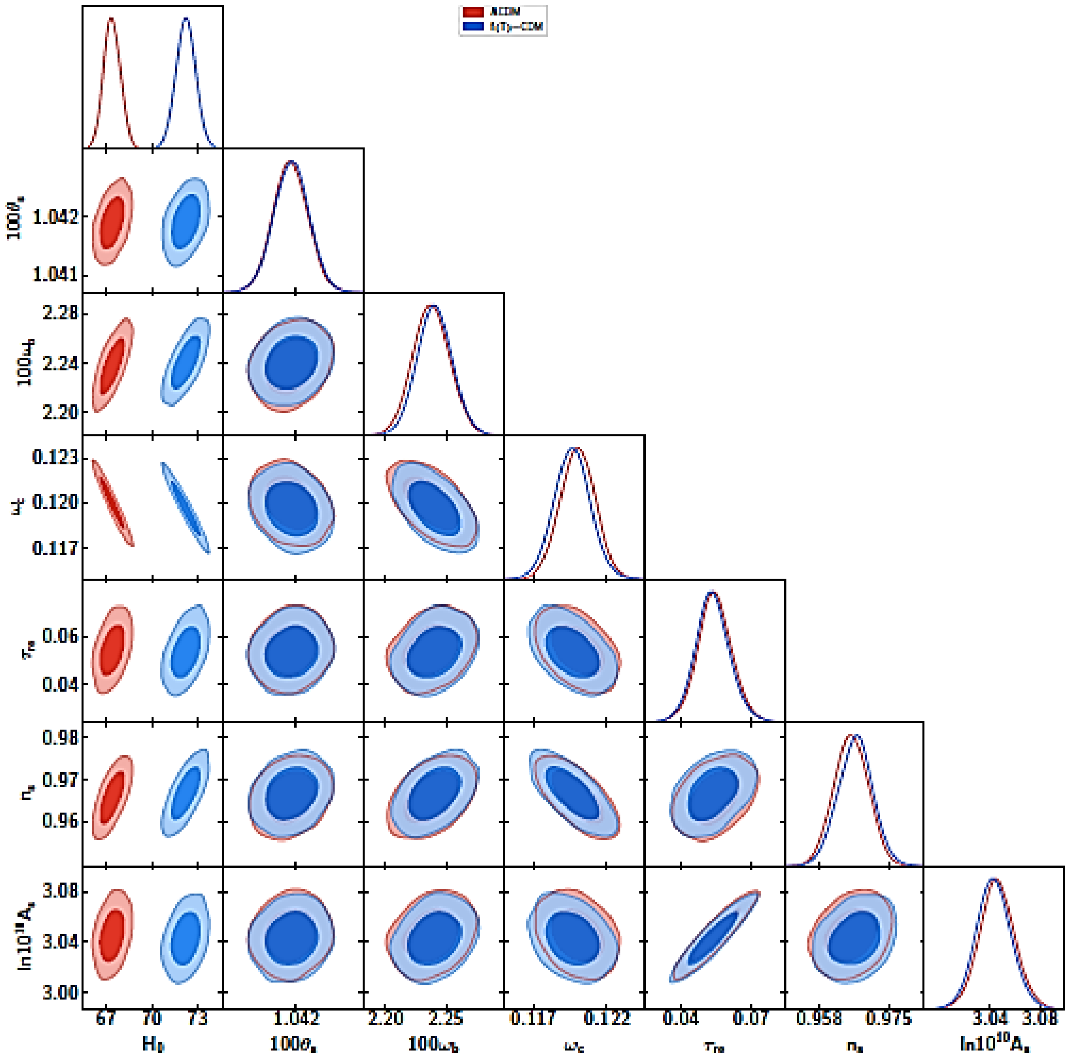


Lensing power spectrum

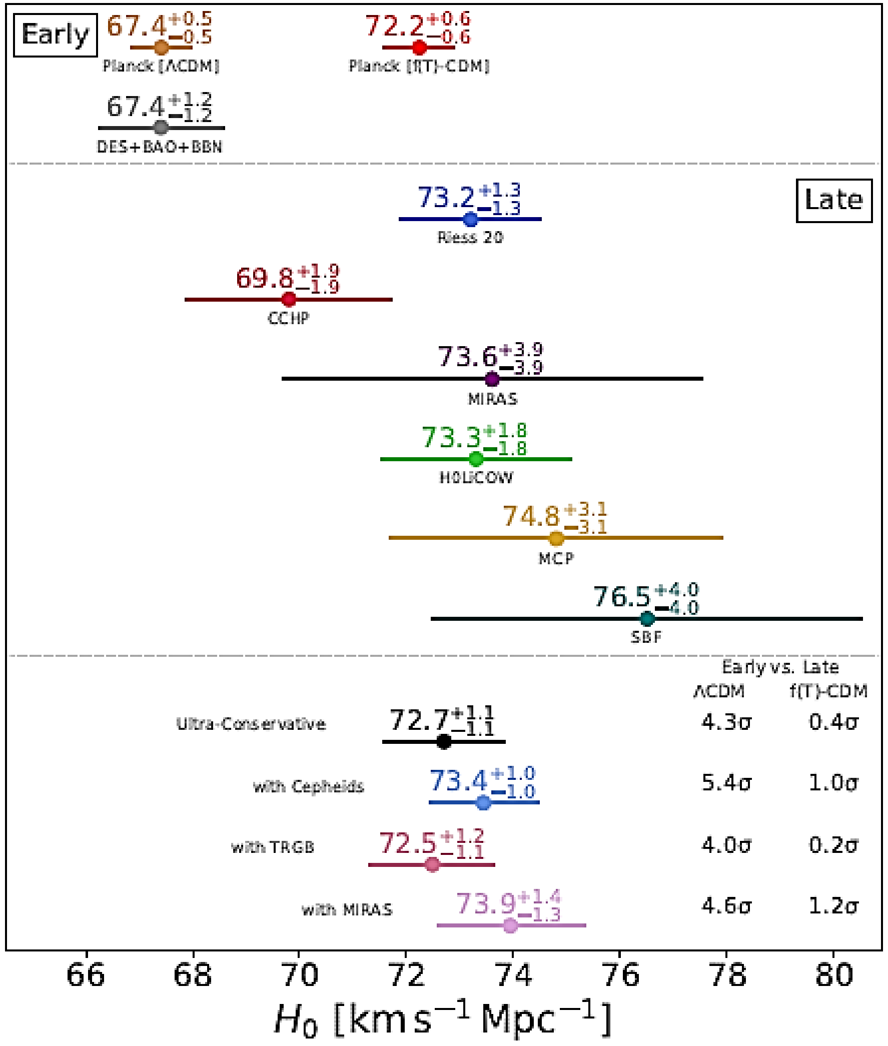


Matter power spectrum

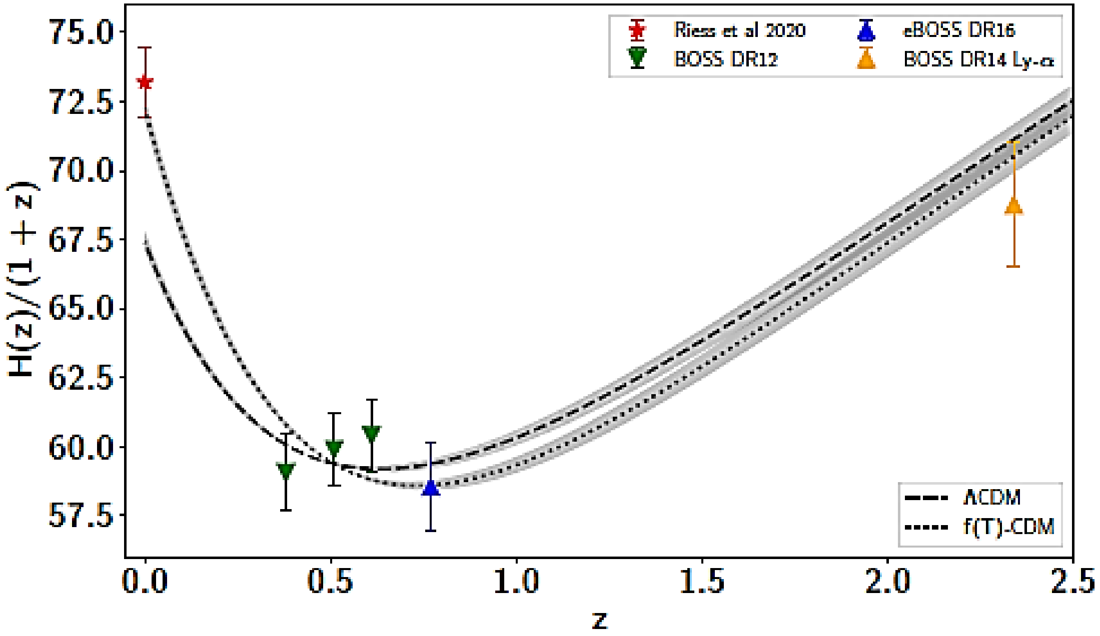
Exponential IR f(T) gravity



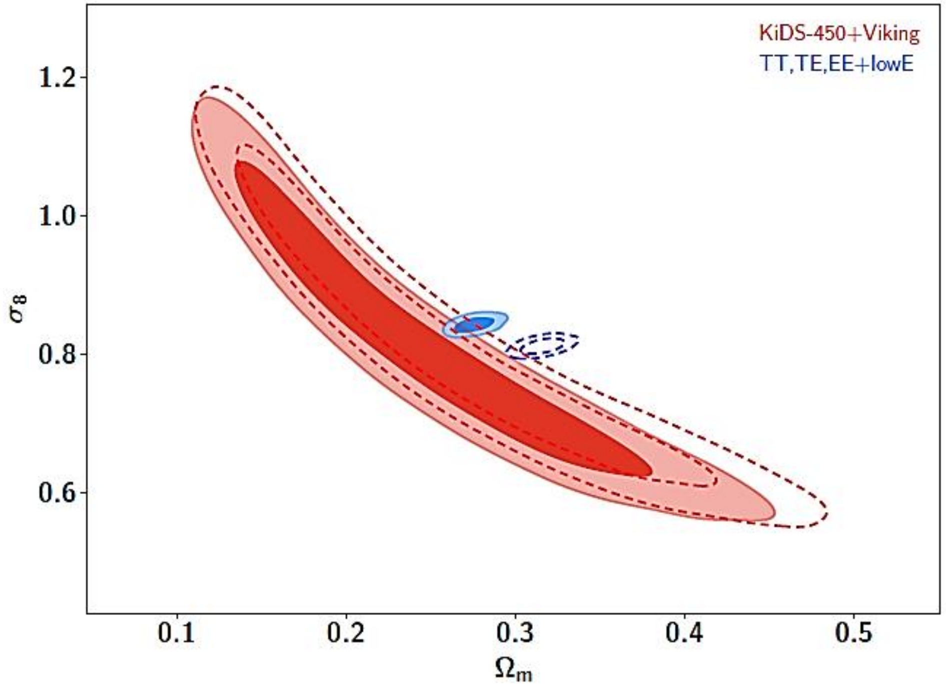
Six parameter space



Exponential IR f(T) gravity



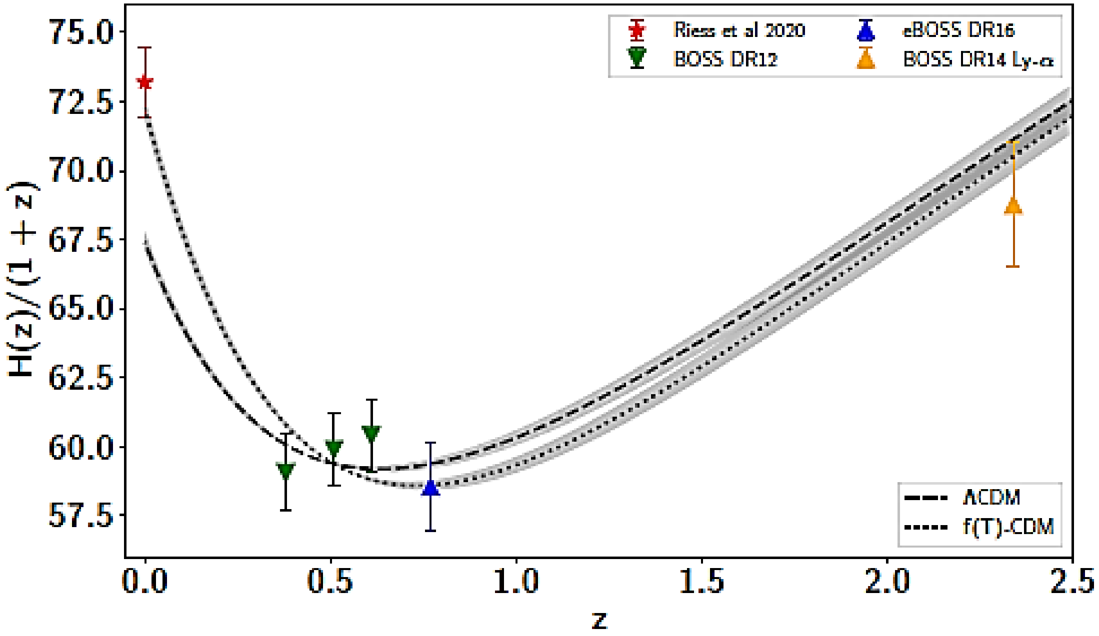
Indeed, using the full CMB spectrum, we find that the Planck (TT+TE+EE+lensing) base-f(T)-CDM alone predicts a Hubble constant of H_0 = 72.24 ± 0.64 km/s/Mpc. This alleviates the H_0 tension with R20+H0LiCOW to the 0.8σ level; in contrast to CDM context, where it reaches 4.8σ.



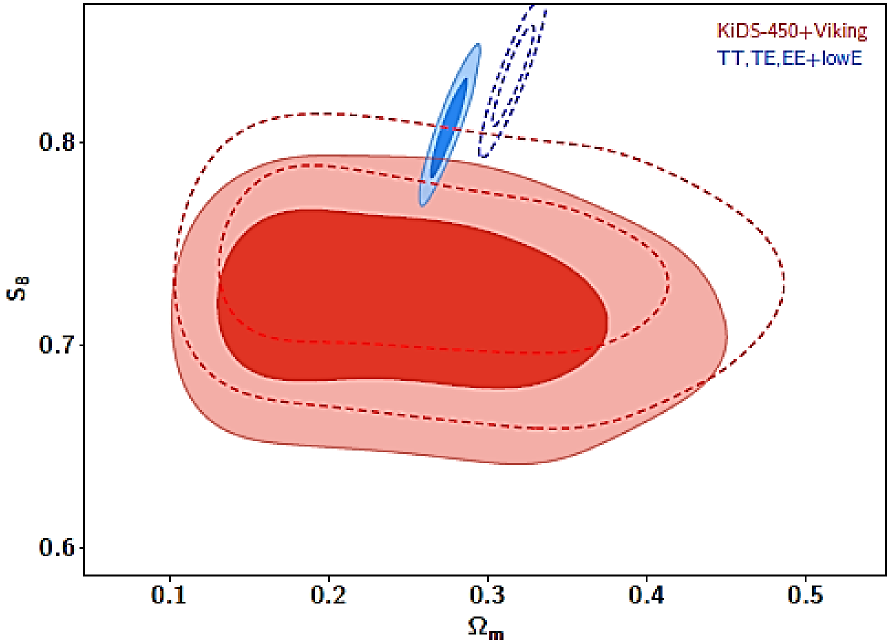
The cosmic shear measurements of σ_8 using VK-450 dataset is at ~ 0.2σ tension higher with our obtained CMB base-f(T)-CDM value.

Exponential IR f(T) gravity

$$S_8 = \sigma_8\sqrt{\Omega_m/0.3}$$

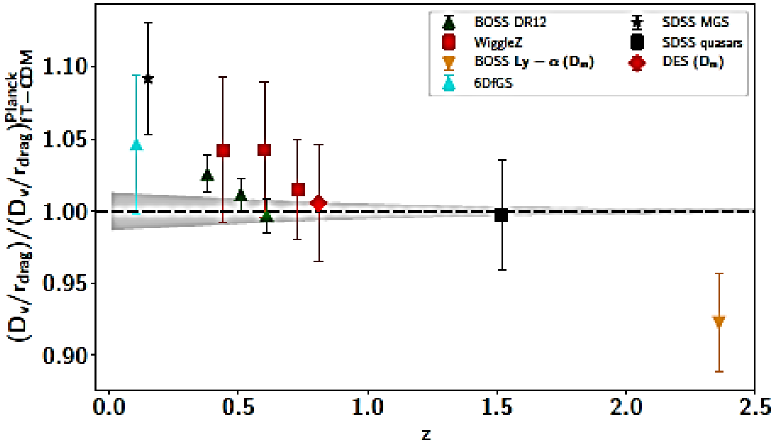


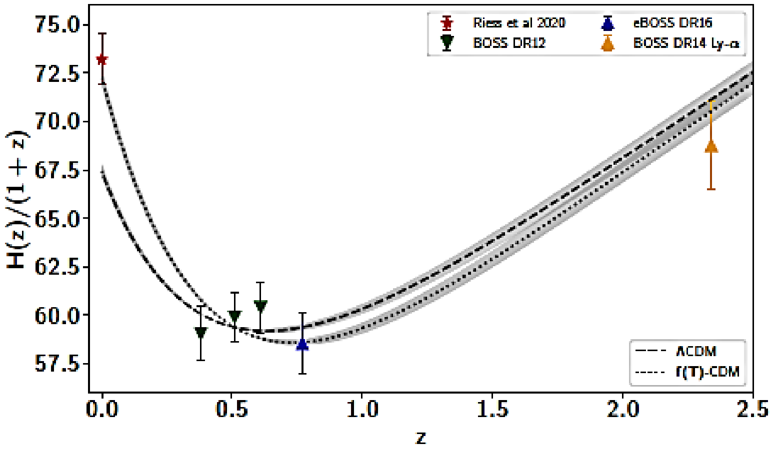
Indeed, using the full CMB spectrum, we find that the Planck (TT+TE+EE+lensing) base-*f*(*T*)-CDM alone predicts a Hubble constant of H_0 = 72.24 ± 0.64 km/s/Mpc. This alleviates the H_0 tension with R20+H0LiCOW to the 0.8σ level; in contrast to CDM context, where it reaches 4.8σ.

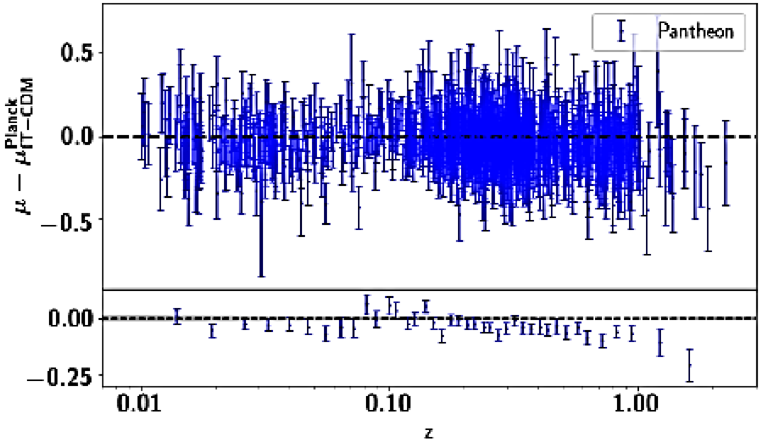


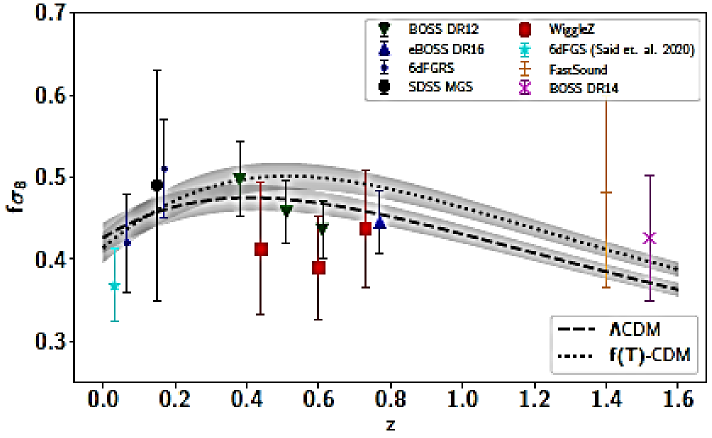
The cosmic shear measurements of S_8 using VK-450 dataset is at 2.4σ tension with our obtained CMB base-f(T)-CDM value, which reduces the S_8 tension by 0.2σ.

Exponential IR f(T) gravity







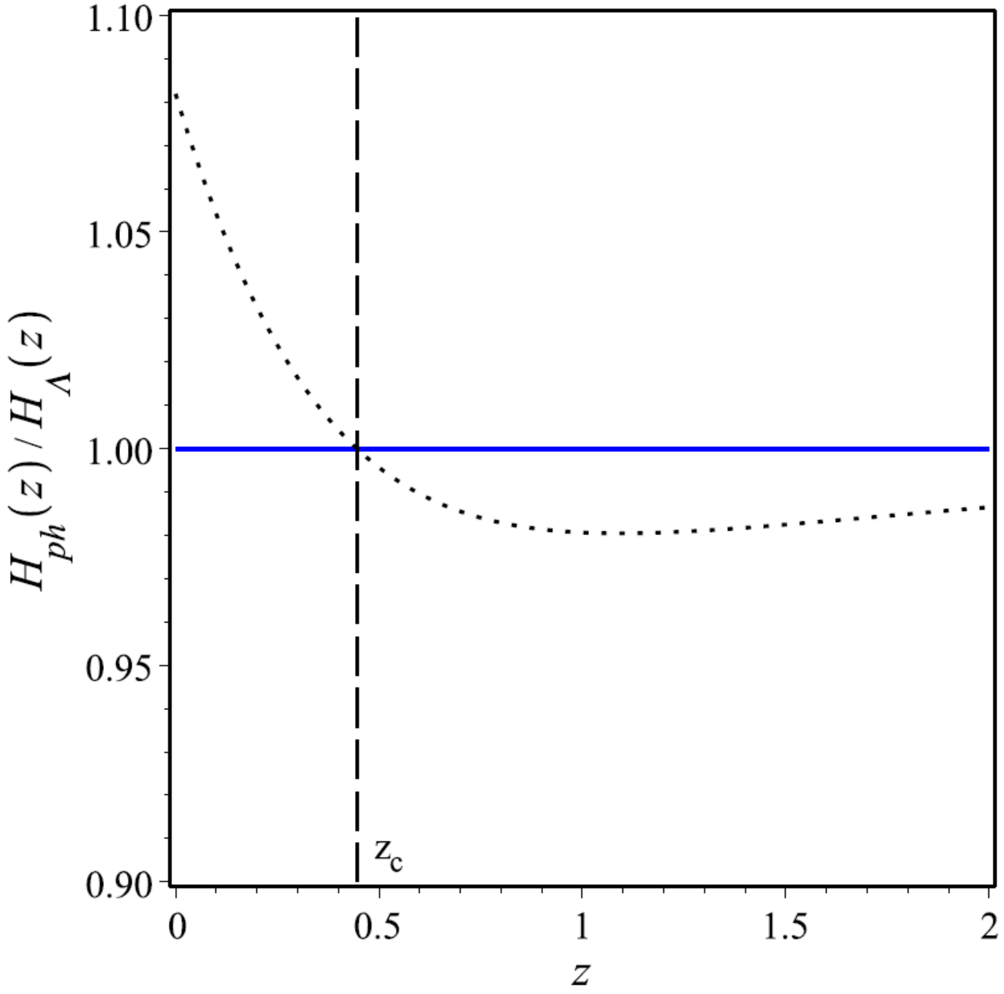


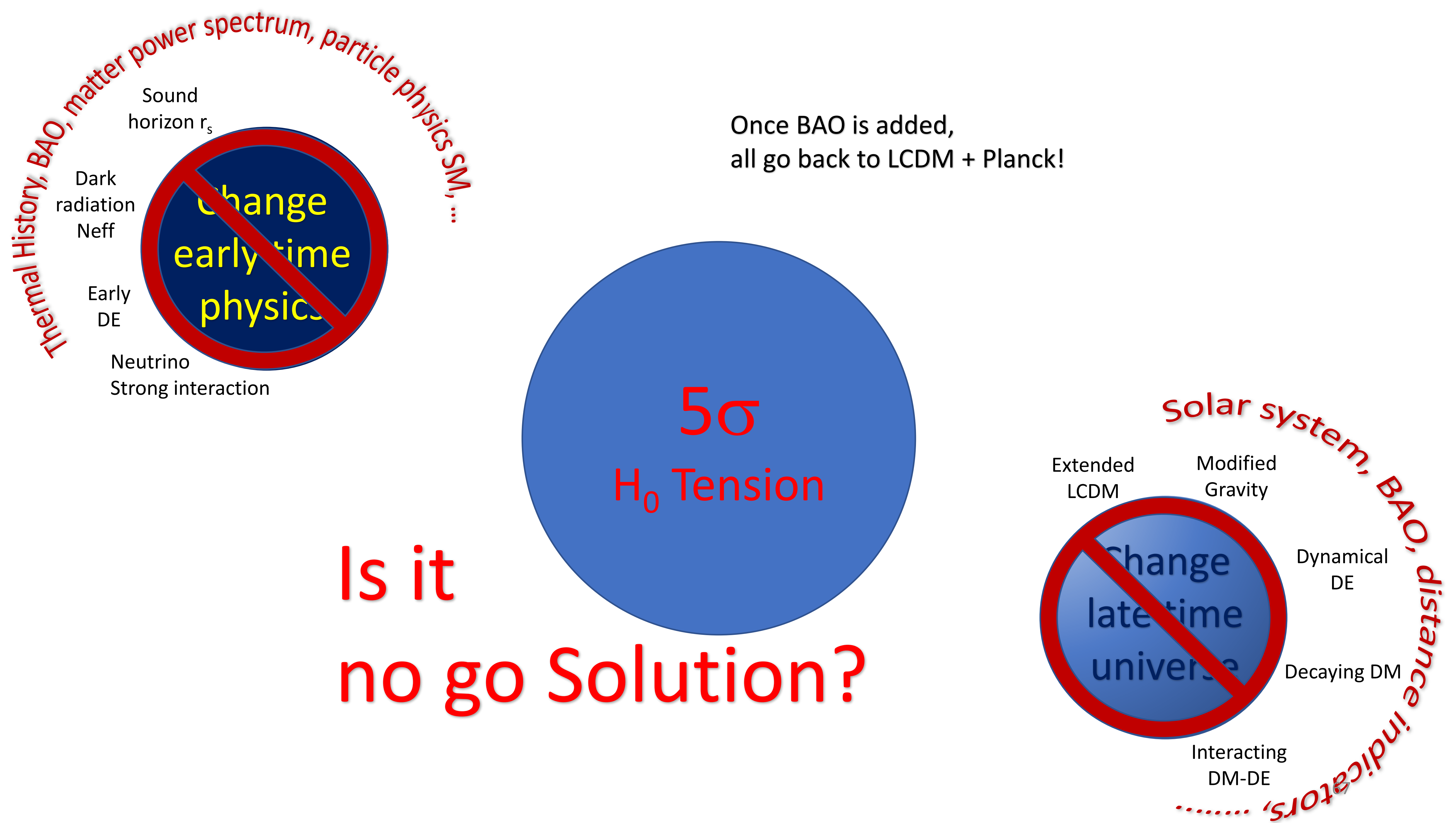
Although the current theory is viable — in the sense that it fits the data with χ^2 values comparable to CDM — it shows systematic deviations with not only the BAO but also the SNIa distances and the growth of structure ($f\sigma_8$), where it is in more severe tension than the standard model.

Phantom regime

It follows that the Hubble expansion rate H(z)
(i) At some critical redshift $z_c > 0$, $H_{ph} = H_{\Lambda CDM}$,
(ii) At redshifts $0 \leq z < z_c$, Hph > HΛCDM (this includes $H_{0,ph} > H_{0,\Lambda CDM}$),
(iii) At redshifts $z > z_c$, $H_{ph} < H_{\Lambda CDM}$.

Since the BAO angular distance is sensitive to the sound horizon size (standard ruler), one does not expect changes in their observed values when early universe is kept unaltered. However, the changes in the Hubble expansion rate in phantom regime at late universe as illustrated above leads to a necessarily systematic deviation with BAO angular distances at low redshift if angular distance to CMB is kept fixed to Planck measurement




Thermal History, BAO, matter power spectrum, particle physics SM, …
Sound horizon r_s
Dark radiation Neff
Early DE
Neutrino Strong interaction
Change early time physics
Once BAO is added,
all go back to LCDM + Planck!
5σ
H_0 Tension
Is it
no go Solution?
Solar system, BAO, distance indicators, …….
Extended LCDM
Modified Gravity
Dynamical DE
Decaying DM
Interacting DM-DE
Change late time universe

HOW TO SOLVE?

The tension level, $n\sigma$, between a measured value $H_0^{obs} \pm \sigma_{obs}^2$ and a model dependent inferred value $H_0^{model} \pm \sigma_{model}^2$ is given as

$$n\sigma = \frac{H_0^{model} - H_0^{obs}}{\sqrt{\sigma_{model}^2 + \sigma_{obs}^2}},$$

assuming linear error propagation, by taking Gaussian approximations to the posterior distribution functions of each H_0 measurement. The tension level $n\sigma$ decreases as the mean values H_0^{model} and H_0^{obs} are getting closer. But, in addition, it can also decrease as a result of an increase in the uncertainty σ_{model} due to inclusion of one or more dataset points with large error bars. In the case when a model includes more relatively free parameters, for the same dataset, the marginalization over these additional parameters increases the uncertainty σ_{model} of the inferred value of H_0. This results in broadening the uncertainties on the inferred value of H_0. In fact, several attempts to address this tension are through a broadening of the posterior distribution due to marginalization over 1 or 2 additional parameters than the six parameters of LCDM.

Recommendations

- **Any alternative/extension to LCDM to solve H0 tension should not worsen the S8 tension**
- **Test your model using CMB data alone and CMB+BAO.**
- **Do not add priors on H0 from late universe observations.**
- **Try to keep the model at six parameter space.**