## الAS Program on Fundamental Physics (FP 2025) المعادية (FP 2025)

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion

## Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU



Jan 15, 2025

2411.16304 with S. Girmohanta, Y. Nakai, Z. Zhang



Wiggly dilaton/radion a landscape o spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion

# I. Introduction

◆□ ▶ ◆□ ▶ ◆三 ▶ ◆三 ▶ ◆□ ▶ ◆○ ◆



Wiggly dilaton/radion a landscape o spontaneously broken scale invariance

Yu-Cheng QIU

Introduction Setup Landscape?

Summary an Discussion



Figure: Sheldon Lee GLASHOW at HKUST 2018.



Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion On a quest of a naturally light dilaton, we accidentally found a wiggly dilaton potential.

うせん 正則 ふゆやえゆや (日本)

#### 

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary and Discussion

• "These are transformations that would be exact invariances of the world if all elementary particle masses vanishes. ... " (Dilatations by Coleman (1971))

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

- "These are transformations that would be exact invariances of the world if all elementary particle masses vanishes. · · · " (Dilatations by Coleman (1971))
- Dilaton naturally arise in the string theory.
- Naturally light dilaton is tightly related to the cosmological constant. (Sundrum (2003))
- The existence of a naturally light dilaton, as the pNGB of spontaneous symmetry breaking of dilatation, is the question.

## **Spontaneous breaking of scale invariance (SBSI)**

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion Dilatation is scale transformation.

The Goldstone theorem indicates that the SI manifest itself nonlinearly after SBSI by

$$au(x) o au(\lambda x) + \log \lambda$$
 .

The SI potential of a canonically normalized dilaton  $\chi = f e^{\tau}$  is  $V = \lambda \chi^4$ . (2)



## الله المعاملة المعاملة المعاملة (SBSI) المعاملة المعاملة (SBSI)

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion Dilatation is scale transformation.

The Goldstone theorem indicates that the SI manifest itself nonlinearly after SBSI by

$$au(x) o au(\lambda x) + \log \lambda$$
 .

The SI potential of a canonically normalized dilaton  $\chi = f e^{\tau}$  is  $V = \lambda \chi^4$  . (2)

If  $\lambda \neq 0$ ,  $\chi$  cannot be stabilized unless  $f \rightarrow 0$ . Thus, the exact SBSI only happens if  $\lambda \rightarrow 0$ .

• Dilaton mass measures the explicit breaking of SI.

## Naturally light dilaton

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

- Dilatation is part of conformal transformations.
- SBSI in 4D  $\rightarrow$  Stabilization of the *radion* in 5D. (AdS/CFT Correspondence Maldacena (1997))
- $\bullet~\textit{Radion} \rightarrow$  the size of the compactified 5th dim.
- $\bullet$  A naturally light dilaton  $\rightarrow$  a naturally light radion

#### 🛞 हैक्षे अन्द्रम 5D Radion stabilization

Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion



#### 

## 5D Radion stabilization

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary and Discussion

- The 5D model is based on RS1 model. (Randall & Sundrum (1999))  $V(\chi) = 0$  under the tuning  $\Lambda_{\rm UV} = -\Lambda_{\rm IR} = -\Lambda_{\rm bulk}/k$ .
- The RS1 geometry can be stabilized via GW mechanism. (Goldberger & Wise (1999))

A bulk scalar  $\phi$  with Dirichlet boundary conditions has a nontrivial profile  $\phi(y)$  along the 5th-dim.

Backreation to the metric is neglected.

• The backreaction of the GW scalar to the metric can be included. (Csaki, Erlich, Grojean, Hollowood (2000))

#### 

## 5D Radion stabilization

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

- The 5D model is based on RS1 model. (Randall & Sundrum (1999))  $V(\chi) = 0$  under the tuning  $\Lambda_{\rm UV} = -\Lambda_{\rm IR} = -\Lambda_{\rm bulk}/k$ .
- The RS1 geometry can be stabilized via GW mechanism. (Goldberger & Wise (1999))

A bulk scalar  $\phi$  with Dirichlet boundary conditions has a nontrivial profile  $\phi(y)$  along the 5th-dim.

Backreation to the metric is neglected.

- The backreaction of the GW scalar to the metric can be included. (Csaki, Erlich, Grojean, Hollowood (2000))
- A holographic formulation of a naturally light dilaton emerge (?). Named CPR framework.

([unpublished] Contino, Pomarol, Rattazzi (2010)) (Coradeschi, Lodone, Pappadopulo, Rattazzi, Vitale (2013)) (Bellazzini, Csaki, Hubisz, Serra, Terning (2013))

## العندة Dilaton/radion potentials المعندة ال

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion



Figure: From 1305.3919 (Bellazzini, Csaki, Hubisz, Serra, Terning).

포네크

· 李世道将京可 Ison, Duotee Issurere	The CPR framework				
Yu-Cheng QIU		CFT4		AdS5	
ntroduction Setup	marginal	$\lambda$	$\longleftrightarrow$	$\phi$	GW scalar with $\epsilon$ mass
	near marginal	$eta(\lambda) \ll 1$	$\longleftrightarrow$	$ \delta \phi(y)  \ll 1$	slow-varying profile
		$F[\lambda]$	$\longleftrightarrow$	$V_{IR}(\phi)$	IR brane tension



## The CPR framework in 5D

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

The 5D formulation of CPR scenario includes:

- 1. a bulk scalar  $\phi$  whose mass is parametrically small  $\epsilon,$
- 2. Dirichlet boundary conditions  $\phi(y_{UV/IR}) = v_{UV/IR}$ , and
- 3. it backreacts to the metric.

The resulting dilaton/radion  $\chi$  has

$$\frac{m_{\chi}}{\langle \chi \rangle} \propto \sqrt{\epsilon} , \quad \langle \chi \rangle = \left(\frac{v_{\mathsf{UV}}}{v_{\mathsf{IR}} + \xi}\right)^{1/\epsilon} + \mathcal{O}(\epsilon) \tag{3}$$



The CPR framework in 5D

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion The 5D formulation of CPR scenario includes:

- 1. a bulk scalar  $\phi$  whose mass is parametrically small  $\epsilon,$
- 2. Dirichlet boundary conditions  $\phi(y_{UV/IR}) = v_{UV/IR}$ , and
- 3. it backreacts to the metric.

The resulting dilaton/radion  $\chi$  has

$$\frac{m_{\chi}}{\langle \chi \rangle} \propto \sqrt{\epsilon} , \quad \langle \chi \rangle = \left(\frac{v_{\rm UV}}{v_{\rm IR} + \xi}\right)^{1/\epsilon} + \mathcal{O}(\epsilon) \tag{3}$$

Two questions/comments:

- 1. what is the origin of  $\epsilon$ ?  $\phi$  can be a pNGB.
- 2. one has to fine-tune the  $v_{\rm UV}$  and  $v_{\rm IR}$  to get a reasonable  $\langle\chi\rangle$  for small  $\epsilon.$



#### Our journey

Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion The idea:

The bulk profile back-reacts to the boundary.

Hopefully, some relaxation may happen to resolve the fine-tuning in  $\langle \chi \rangle$  between  $v_{\rm UV}$  and  $v_{\rm IR}.$ 



#### Our journey

Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion The idea:

The bulk profile back-reacts to the boundary.

Hopefully, some relaxation may happen to resolve the fine-tuning in  $\langle \chi \rangle$  between  $v_{\rm UV}$  and  $v_{\rm IR}.$ 

The result:

A wiggly dilaton !



Wiggly dilaton/radion a landscape o spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary an Discussion II. Setup



#### The setup

Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

The topology of the compactified extra dimension is  $S^1/Z_2$ . The UV/IR brane locates at  $y_0/y_1$ . The 5D action is

$$S = \overbrace{\int d^4 x dy \left( -\frac{R}{2\kappa^2} + \frac{1}{2\kappa^2} (\partial a)^2 - V(a) \right)}^{\text{bulk}} - \underbrace{\int d^4 x \sqrt{g_0} V_0(a)}_{\text{UV brane}} - \underbrace{\int d^4 x \sqrt{g_1} V_1(a)}_{\text{IR brane}} .$$

Warped metric ansatz is

$$ds^{2} = e^{-2T(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} - dy^{2} .$$
 (5)

<□> <圕> <필> < 글> < 글> < 글> < 글|= のへ(~ 15/25

(4)



#### The EOMs

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

#### Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

#### The equations of motion are

$$4T'^{2} - T'' + \frac{2\kappa^{2}}{3}V = 0$$
(6)  

$$T'^{2} - \frac{1}{12}a'^{2} + \frac{\kappa^{2}}{6}V = 0$$
(7)  

$$a'' - 4T'a' - \kappa^{2}\frac{\partial V}{\partial a} = 0$$
(8)

The boundary condition for a are

$$2T'|_{y_0,y_1} = \pm \frac{\kappa^2}{3} V_{0,1}(a)|_{y_0,y_1} , \quad 2a'|_{y_0,y_1} = \pm \kappa^2 \frac{\partial V_{0,1}}{\partial a}|_{y_0,y_1}$$
(9)



### The dilaton/radion potential

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion The bulk and brane potentials for the axion are

$$V = \Lambda_5 - \epsilon \frac{2k^2}{\kappa^2} a^2$$
(10)  
$$V_i = \Lambda_i + \epsilon_i \frac{k}{\kappa^2} \left[1 - \cos(a - v_i)\right]$$
(11)

Note that

- For  $\epsilon_i \to \infty$ , one recovers Dirichlet boundary conditions.
- For  $\epsilon_i \rightarrow 0$ , one has Neumann boundary conditions.

The dilaton/radion is  $\chi = e^{-ky_1}$ .

The dilaton/radion potential is obtained by performing  $\int dy(\cdots)$ .



### The boundary condition

dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

The effective dilaton/radion potential is determined by the IR boundary potential

$$V_{\rm eff} = \chi^4 F \;, \quad F = F(V_1(a(y_1)))$$
 (12)

Suppose at the boundaries  $a(y_0) = \tilde{v}_0$  and  $a(y_1) = \tilde{v}_1$ , the BCs give

$$2\epsilon \tilde{\nu}_0 = \epsilon_0 \sin(\tilde{\nu}_0 - \nu_0) \tag{13}$$

$$-4\sqrt{3}\sinh(2\beta) = \epsilon_1\sin(\tilde{\nu}_1 - \nu_1) \tag{14}$$

$$eta = rac{1}{\sqrt{3}} \left( ilde{ extbf{v}}_1 - ilde{ extbf{v}}_0 \chi^{-\epsilon} 
ight)$$

#### 一 李世道将充于 ISCNO DUCTER INSTITUTE

## The dialton/radion potential

Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape?

Summary and Discussion

$$V_{\text{eff}}(\chi) = \chi^4 F[\beta(\chi)]$$
$$F[\beta(\chi)] = [-1 + \xi \Delta(\beta) + \xi \cosh(2\beta)] \operatorname{sech}^2 \beta$$
$$\Delta(\beta) = \frac{\epsilon_1}{6} \left[ 1 - \eta \sqrt{1 - \frac{48}{\epsilon_1^2} \sinh^2(2\beta)} \right]$$

The function  $\beta$  can be given in the small  $\epsilon_1$  limit,

$$\beta(\chi) = \frac{\epsilon_1}{8\sqrt{3}} \sin\left(v_1 - \tilde{v}_0 \chi^{-\epsilon}\right) + \mathcal{O}(\epsilon_1^2) .$$
 (16)

•  $|\tilde{v}_0| \le |\epsilon_0/2\epsilon|$  is essentially a free parameter.

The potential is determined by parameters  $\{\epsilon, \epsilon_1, v_1, \tilde{v}_0\}$ .

(15)



Wiggly dilaton/radion a landscape o spontaneously broken scale invariance

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary and Discussion

# III. Landscape ?

◆□ ▶ ◆□ ▶ ◆三 ▶ ◆三 ▶ ◆□ ▼ ● ◆

## A detailed investigation on the potential

Wiggly dilaton/radion a landscape o spontaneously broken scale invariance

Yu-Cheng QIU

Introductio

Landscape?

Summary an Discussion The mistune of IR brane tension and the bulk CC can be parametrized by  $\xi = \Lambda_5/k\Lambda_1 \equiv 1 + \sigma$ .

 $V_{\text{eff}}(\chi) = \chi^4 F[\beta(\chi)]$  $\epsilon = 0.3$ ,  $\epsilon_1 = 0.2$ ,  $\tilde{v}_0 = 1$ ,  $v_1 = 3.3$  $--- \sigma = 0.1$ 0.15 $\sigma = 0$ 0.10  $\cdots \sigma = -0.02$  $F[\beta(\chi)]$ 0.05  $--- \sigma = -0.1$ 0.0-0.05-0.10 $10^{-5}$  $10^{-4}$ 0.001 0.010 0.100 $\chi/\mu_0$ < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

#### Dilaton landscape

Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

· 李达道将完所 ISUNDWITE INSTITUTE

Yu-Cheng QIU

Introduction Setup

Landscape?

Summary and Discussion



$$\langle \chi \rangle^{(p)} \simeq \left( \frac{\tilde{v}_0}{v_1 - 2p\pi} \right)^{1/\epsilon} \left[ 1 - \frac{24\sigma}{\epsilon^2 \epsilon_1 (v_1 - 2p\pi)^2} + \mathcal{O}(\sigma^2) \right]$$
(17)

$$\left(\frac{m_{\chi}}{\langle\chi\rangle}\right)^{(\nu)} \simeq \frac{\epsilon\epsilon_1^{\nu/2}}{\sqrt{6}}|\nu_1 - 2p\pi| + \mathcal{O}(\sigma)$$
(18)

$$\langle V_{\text{eff}} \rangle^{(p)} \simeq \sigma \left( \frac{\tilde{v}_0}{v_1 - 2p\pi} \right)^{4/\epsilon} + \mathcal{O}(\sigma^2) \qquad p \in \mathbb{Z}$$
(19)



#### Dilaton landscape

Wiggly dilaton/radion a landscape of spontaneously broken scale invariance

Yu-Cheng QIU

Introductior

Setup

Landscape?

Summary and Discussion



Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

@ 李改道研究所

Yu-Cheng QIU

Introduction

Setup

Landscape

Summary and Discussion

- 1. We constructed a wiggly dilaton potential  $V_{\rm eff}(\chi)$ .
- 2. For  $\sigma = 0$ , it has infinite number of degenerate ground states, thus a landscape.
- 3. For  $\sigma > 0$ , the only true ground state is  $\chi = 0$ .
- 4. For  $\sigma<$  0, the only true ground state (if exists) is the local minimum that is closest to  $\chi\rightarrow$  1.
- 5. The limitation of its application is our imagination.
  - 5.1 relaxion
  - 5.2 PBH production
  - 5.3 · · ·

Summarv



Wiggly dilaton/radion a landscape o spontaneously broken scale invariance

Yu-Cheng QIU

Introductio

Setup

Landscape<sup>1</sup>

Summary an Discussion

#### Thank you

#### 

Yu-Cheng QIU

#### The CPR framework

 $V(\chi) = \chi^4 F[\lambda(\chi)]$ <sup>(20)</sup>

To have a naturally light dilaton:

- 1. The CFT should be able to sample a direction with F = 0.
  - 2. The coupling  $\lambda$  should stays 'naturally' close to marginality throughout the RG evolution.
    - imagine g exactly marginal over finite range: manifold of fixed points
    - $V(\varphi) = e^{4\varphi} V_0(g)$
    - generically  $\exists V_0(g_*) = 0$



Figure: From Rattazzi's talk (2010)



#### $\sigma_{\rm crit}$

Yu-Cheng QIU



