Knotted Cosmic Strings and Gravitational Wave

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based on arXiv:2407.11731 w/

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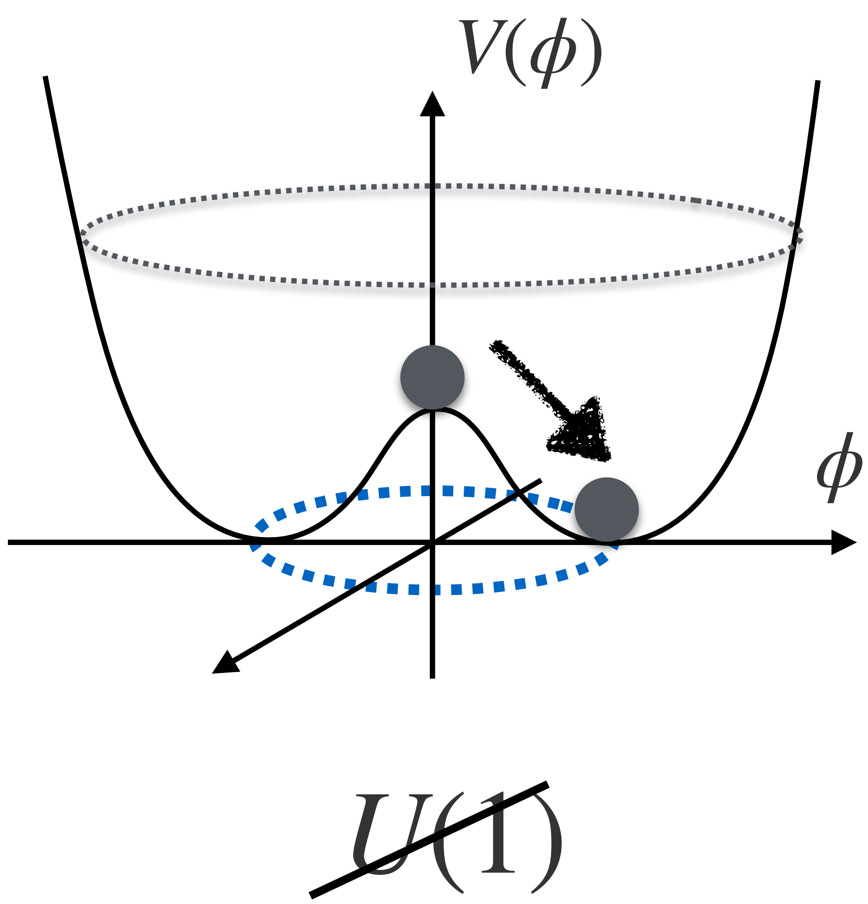
IAS Program on Fundamental Physics, 15th Jan 2025@HKUST

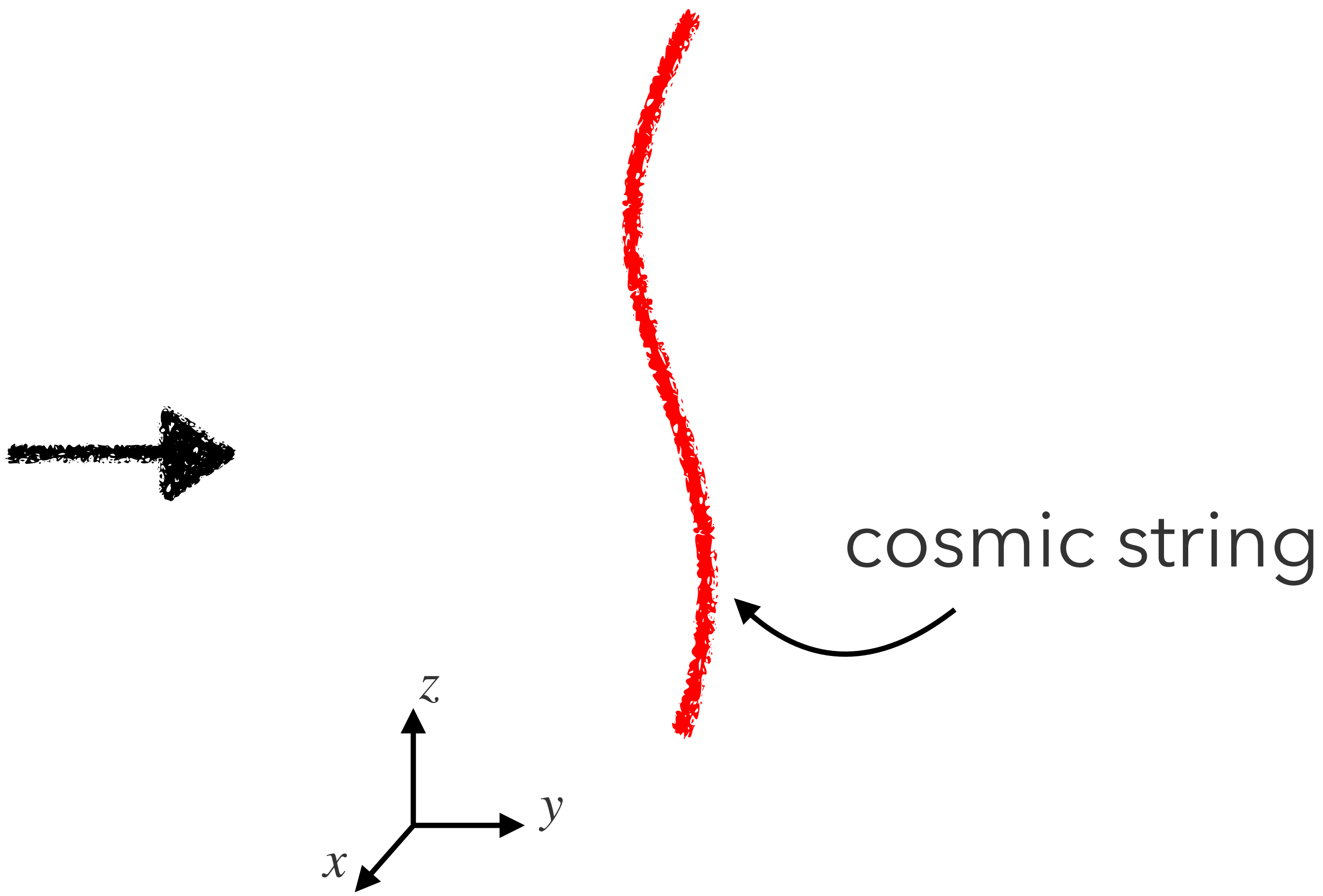
Introduction

Cosmic string

[Abrikosov '58]

[Nielsen-Olesen '73]

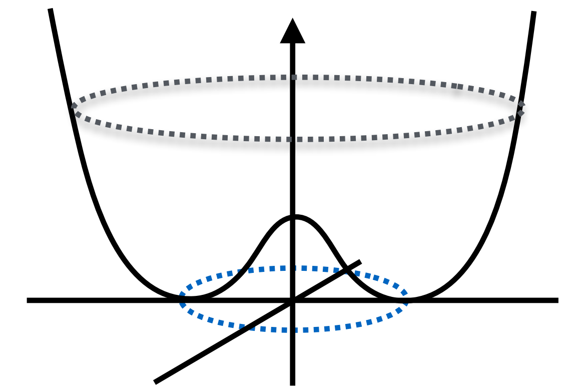




Example of cosmic string

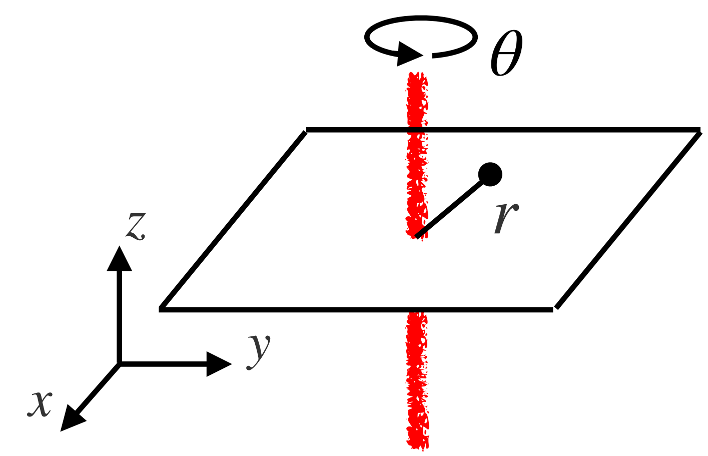
[Abrikosov '58]
[Nielsen-Olesen '73]

- Abelian-Higgs model



$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$

- Field configuration in polar coordinate:



$$\phi(x) = v f(r) e^{i\theta} \qquad \overrightarrow{A}(x) = g^{-1} a(r)\, \overrightarrow{e_\theta}$$

ϕ's phase has winding # = 1

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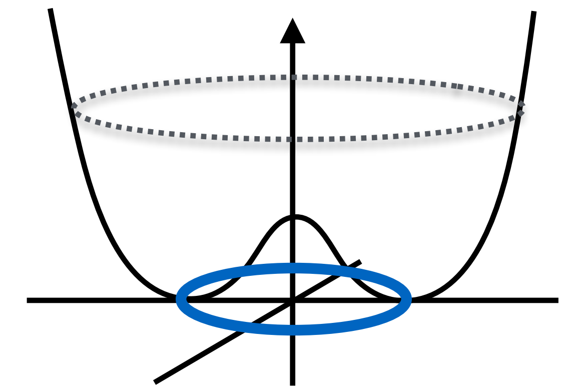

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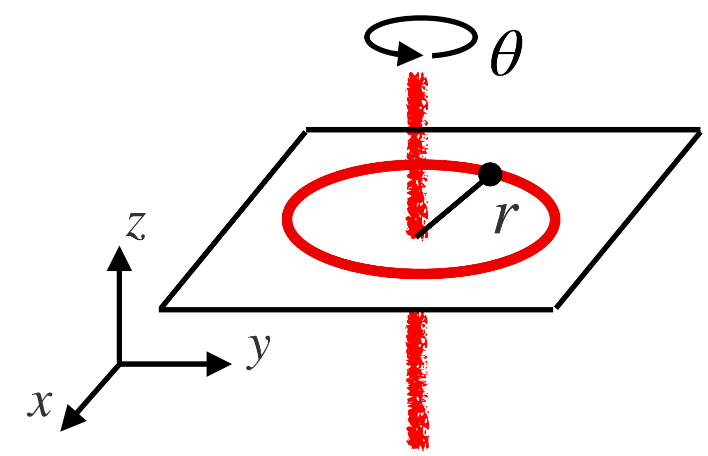

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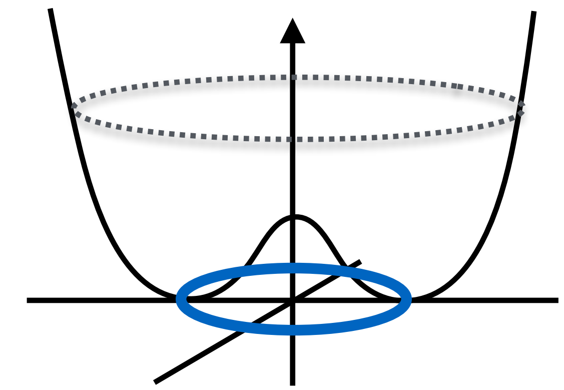

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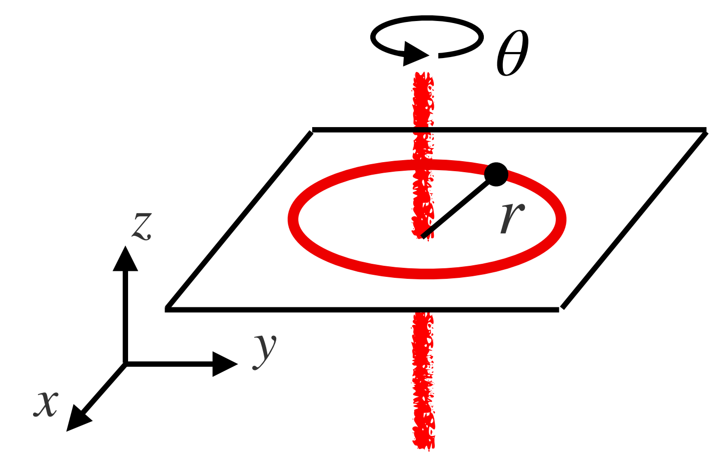
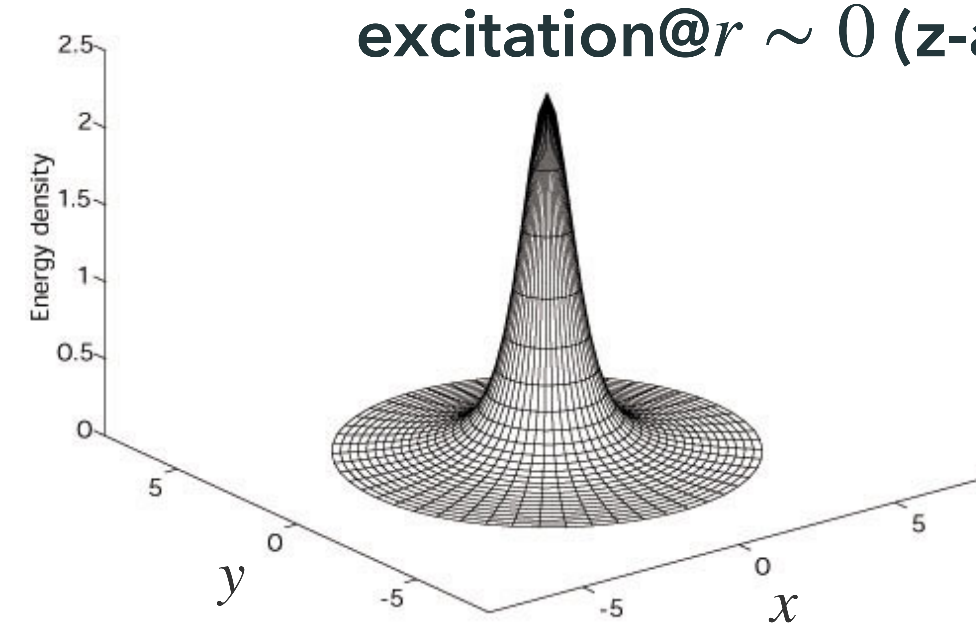


excitation@$r \sim 0$ (z-axis)

- **Non-trivial solution of EOMs exist**



(from Hindmarsh-Kibble '95)

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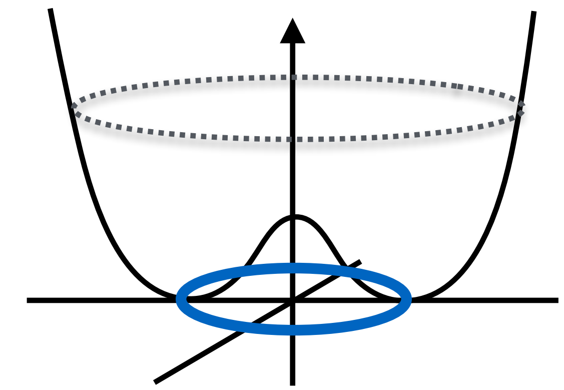

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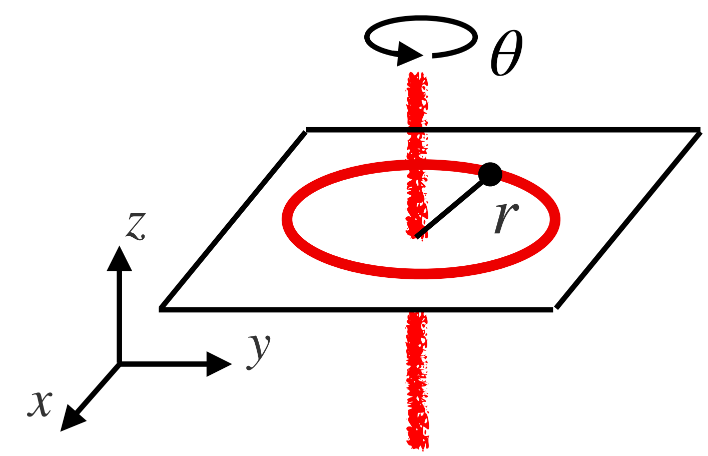

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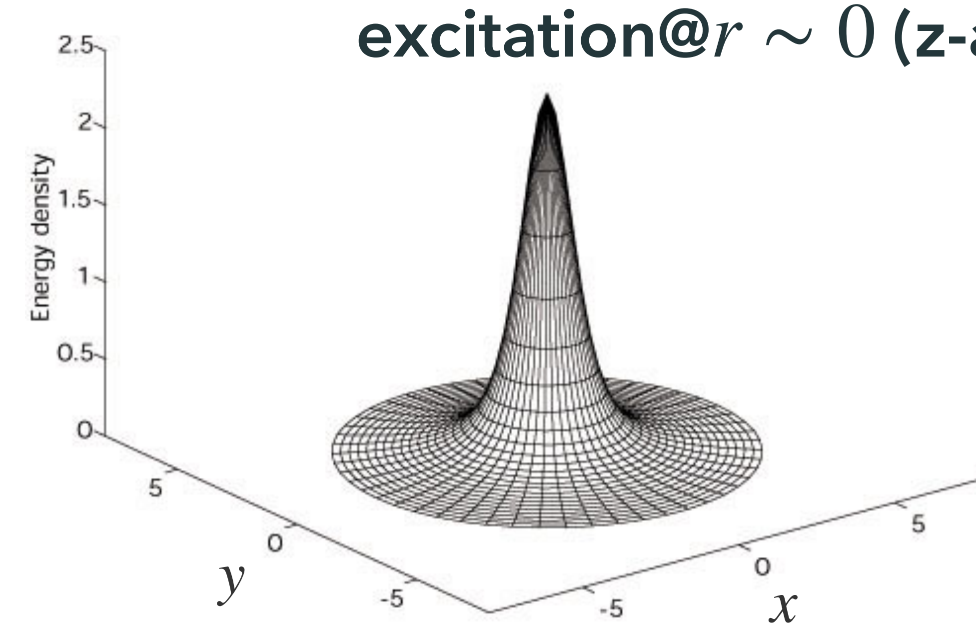

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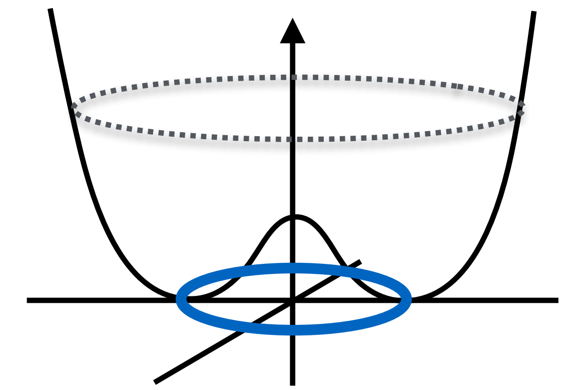

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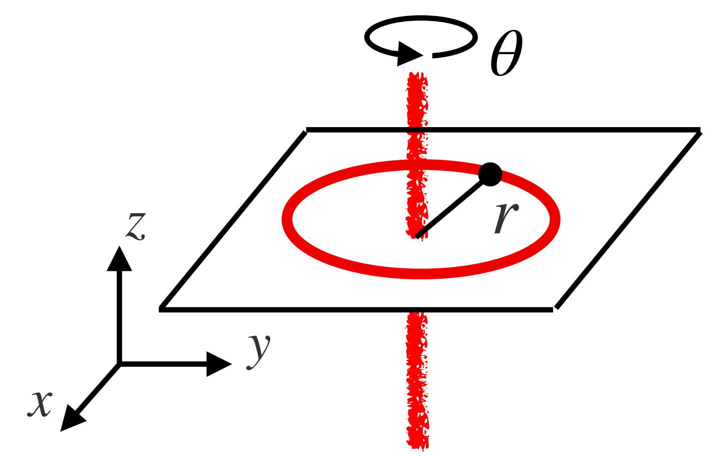

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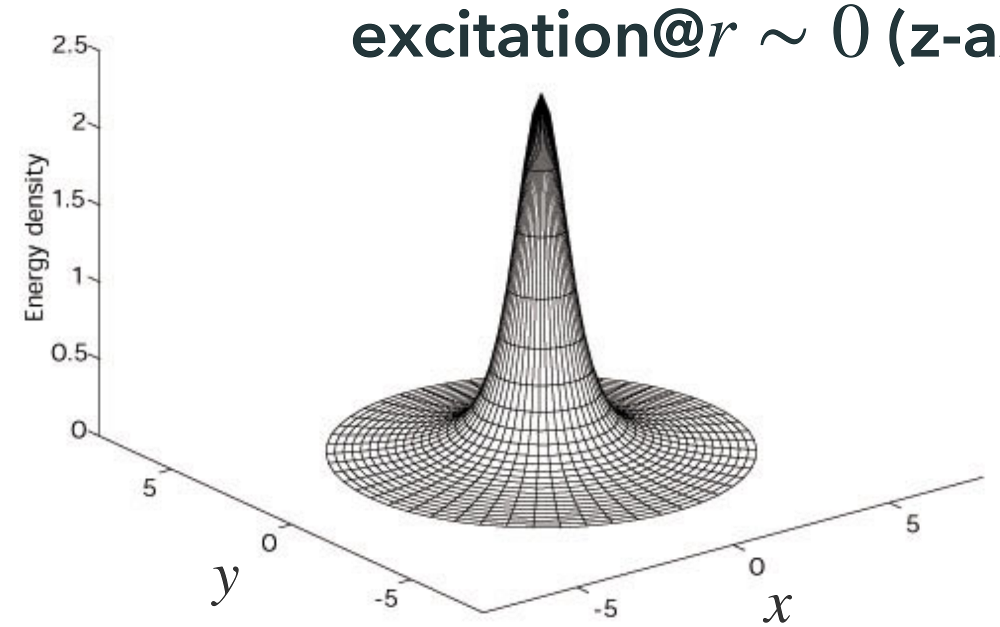


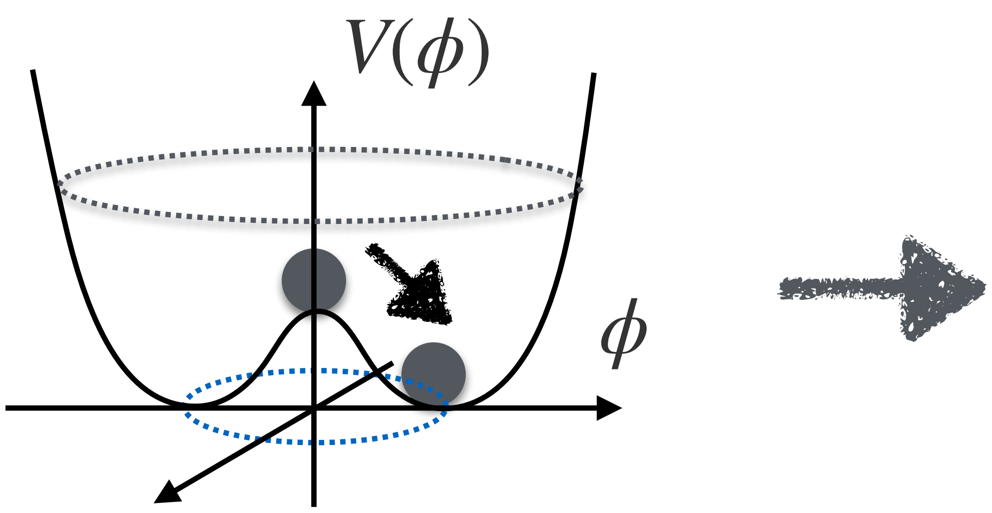
- **Magnetic flux: $B = \frac{2\pi}{g}$ (supercond)**

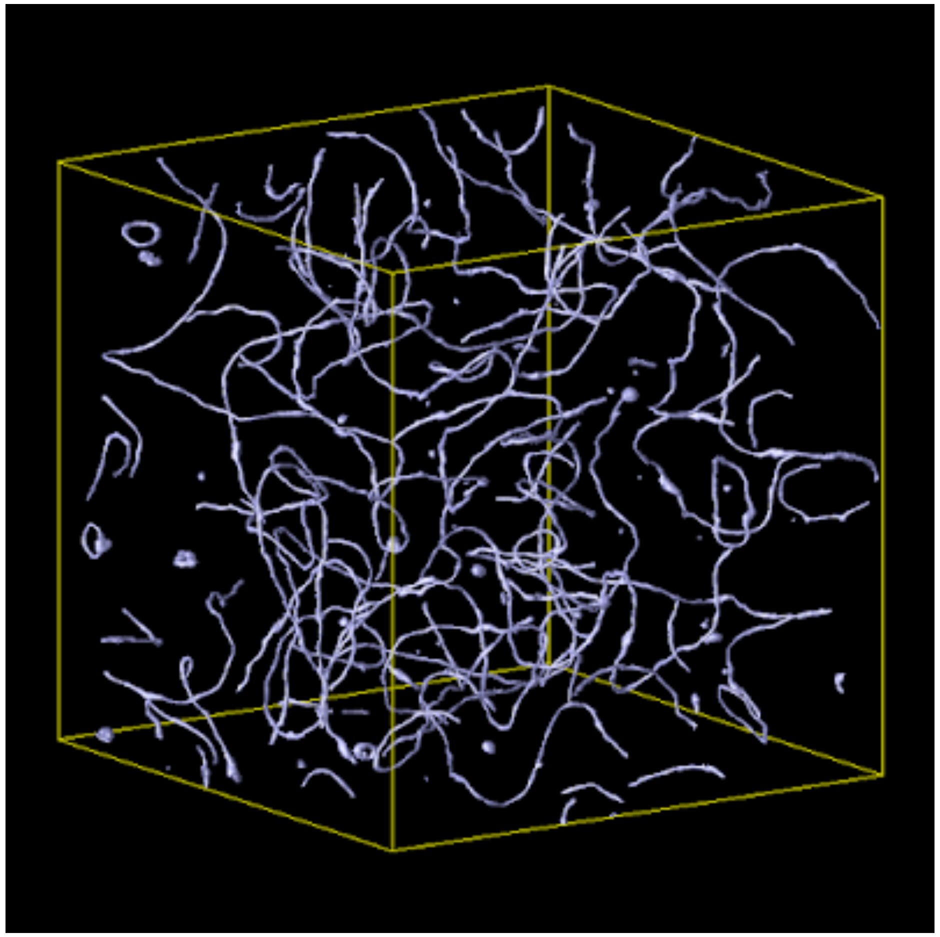
- **No magnetic flux for global $U(1)$**

(from Hindmarsh-Kibble '95)

Cosmic string

- SSB in early universe → network of cosmic string in universe



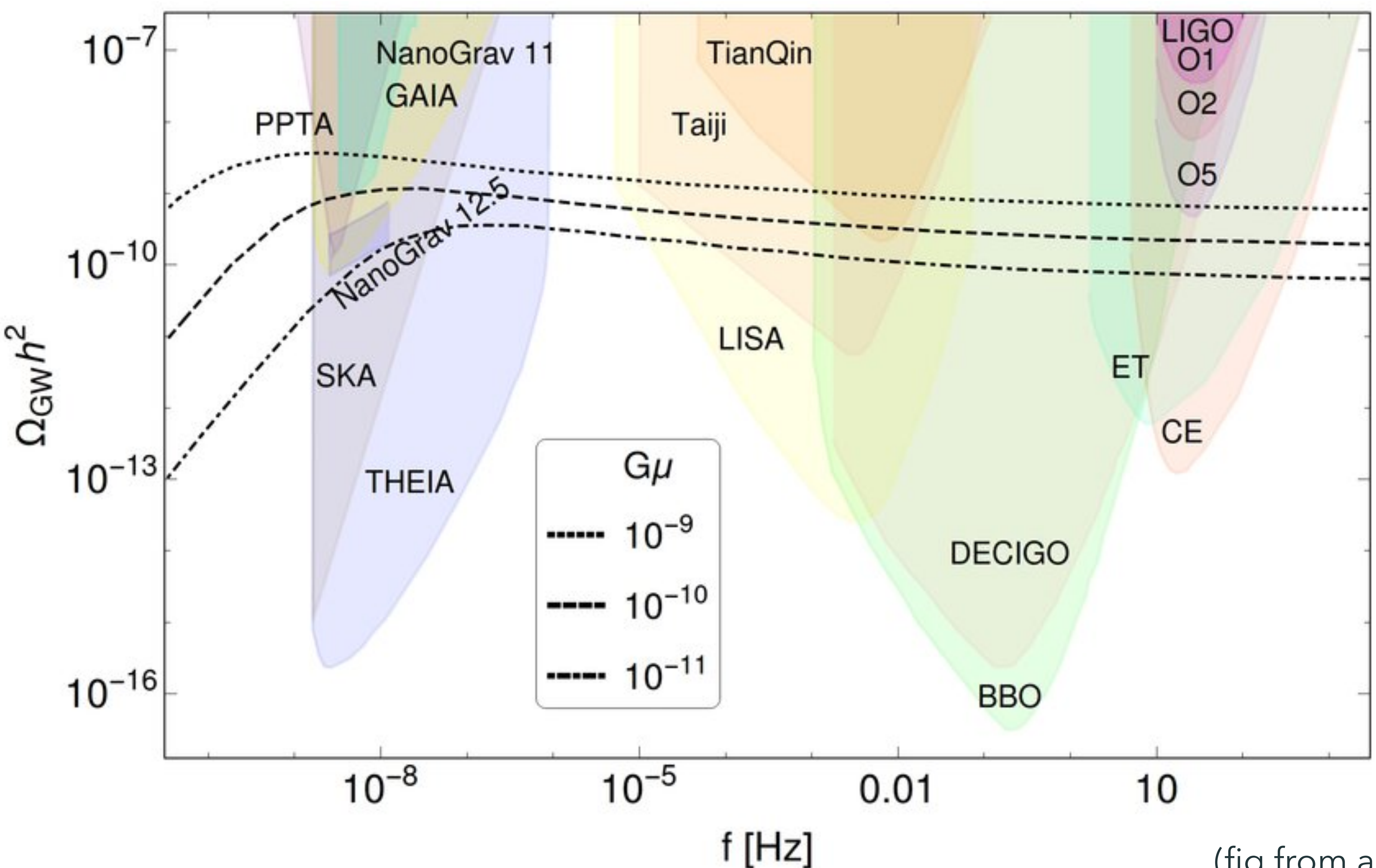


from slide by Takashi Hiramatsu

- continuously **radiates gravitational wave**

Future prospect of GW

$G\mu \simeq v_{\rm st.}^2/M_{\rm pl.}^2$



(fig from arXiv: 2212.05291)

age of GW & cosmic string!?

Common knowledge: $\cancel{U(1)}$ → **cosmic string**

What about two $\cancel{U(1)}$ **?**

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What about two $\cancel{U(1)}$?

→ new stable soliton appears!

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More is different!

Our result in a nutshell

Model: SSB of global $U(1)_{\mathrm{PQ}}$ and gauge $U(1)_{B-L}$ symmetries

→ two kinds of cosmic strings

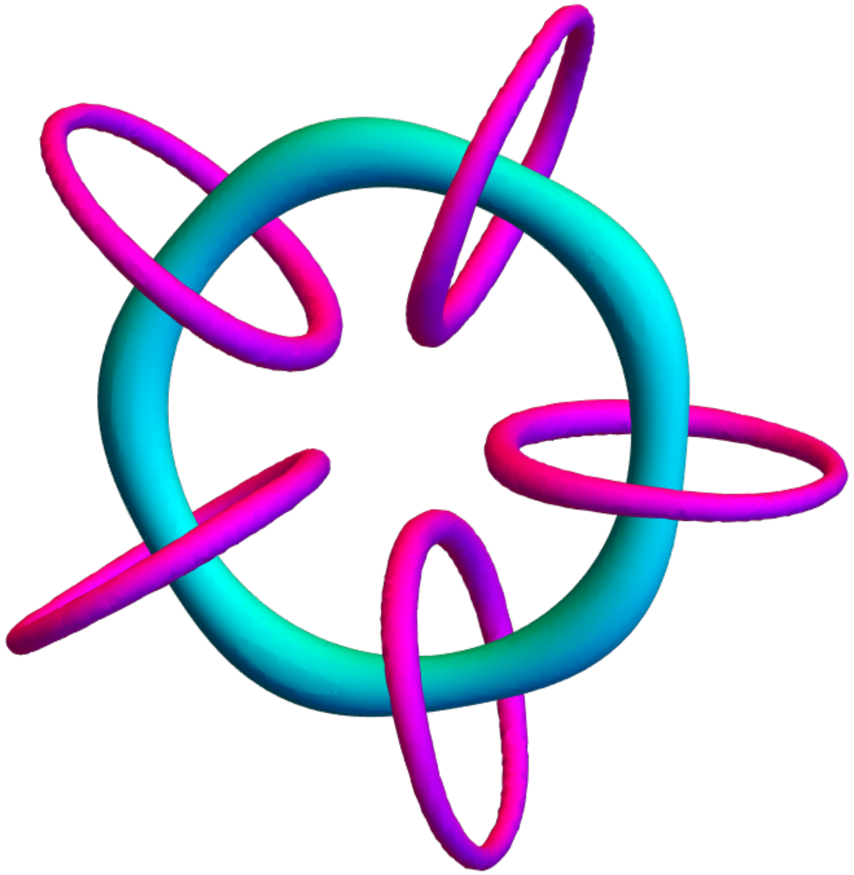
→ combining them to obtain a new soliton

knotted cosmic strings

= Knot soliton!

They could have been abundant

in the early universe.



Knot soliton

The model

Lagrangian:

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$V(\phi_1, \phi_2) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- Symmetries:

$$U(1)_{gauge} : \phi_1 \to e^{i\theta_1}\phi_1 \qquad U(1)_{global} : \phi_2 \to e^{i\theta_2}\phi_2$$

$$D_\mu \phi_1 \equiv (\partial_\mu - igA_\mu)\phi_1$$

- Both symmetries are broken at the vacuum:

$$\langle \phi_1 \rangle = v_1, \ \langle \phi_2 \rangle = v_2$$

→ co-existence of two cosmic strings from $U(1)_{gauge}$ and $U(1)_{global}$

The model

- Natural setup: $U(1)_{gauge} = U(1)_{B-L}$ & $U(1)_{global} = U(1)_{PQ}$

- requires right-handed neutrino coupled w/ ϕ_1: $y_R \phi_1^* \bar{\nu}_R \nu_R^c$

  → $\langle \phi_1 \rangle$ gives Majorana mass → type-I seesaw

  [Minkowski '77] [Yanagida '79] [Gell-Mann+ '79] [Mohapatra-Senjanovic+ '80]

- phase of ϕ_2 (a) is identified as QCD axion

  [Peccei-Quinn '77] [Weinberg '78] [Wilczek '78]

  → solution of strong CP problem & Dark matter

$$\Rightarrow v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$$

Chern-Simons coupling

Lagrangian:

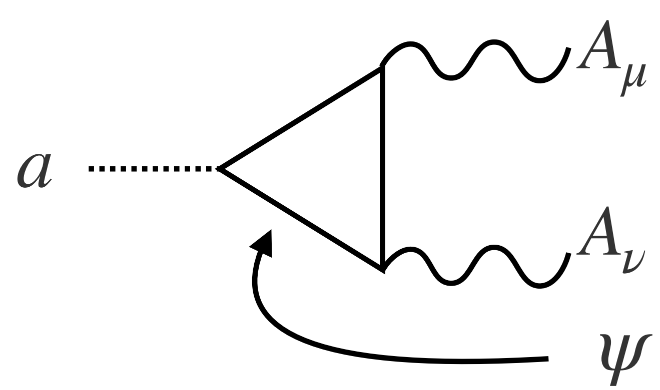
Chern-Simons coupling

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi_1, \phi_2) + \frac{c}{16\pi^2} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$V(\phi_1, \phi_2) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$a \equiv -i \arg(\phi_2)$ $D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$

- At the broken phase, CS coupling is induced by triangle anomaly.



The coefficient c depends on matter sector,
but we take it as free parameter in this talk.

Linking configuration

- Rewriting CS coupling $\frac{c}{16\pi^2} aF_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\frac{c}{16\pi^2} \left(\partial_i a\right) A_0 B^i$

$B_i \equiv \epsilon_{ijk}\partial^j A^k$

"magnetic $B - L$ field"

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→ Gauss law:

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \frac{g^2 c}{16\pi^2}\vec{\nabla} a \cdot \vec{B} = 0$$

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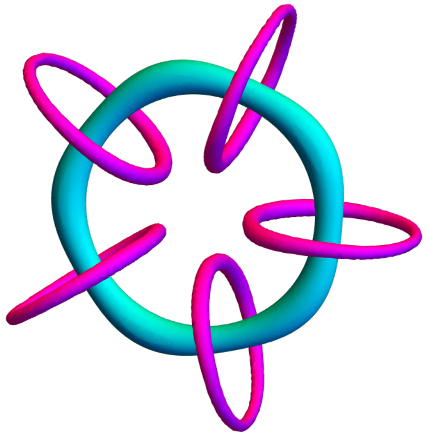
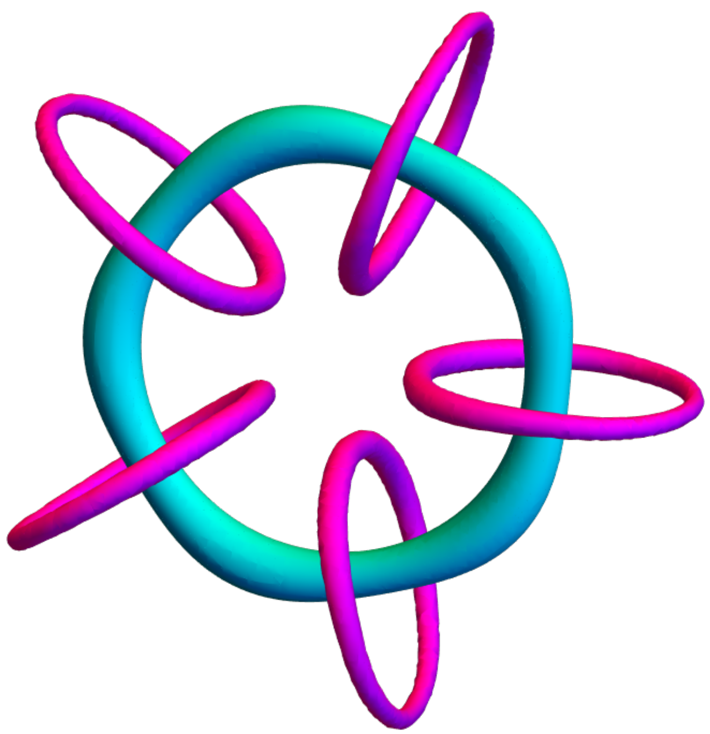
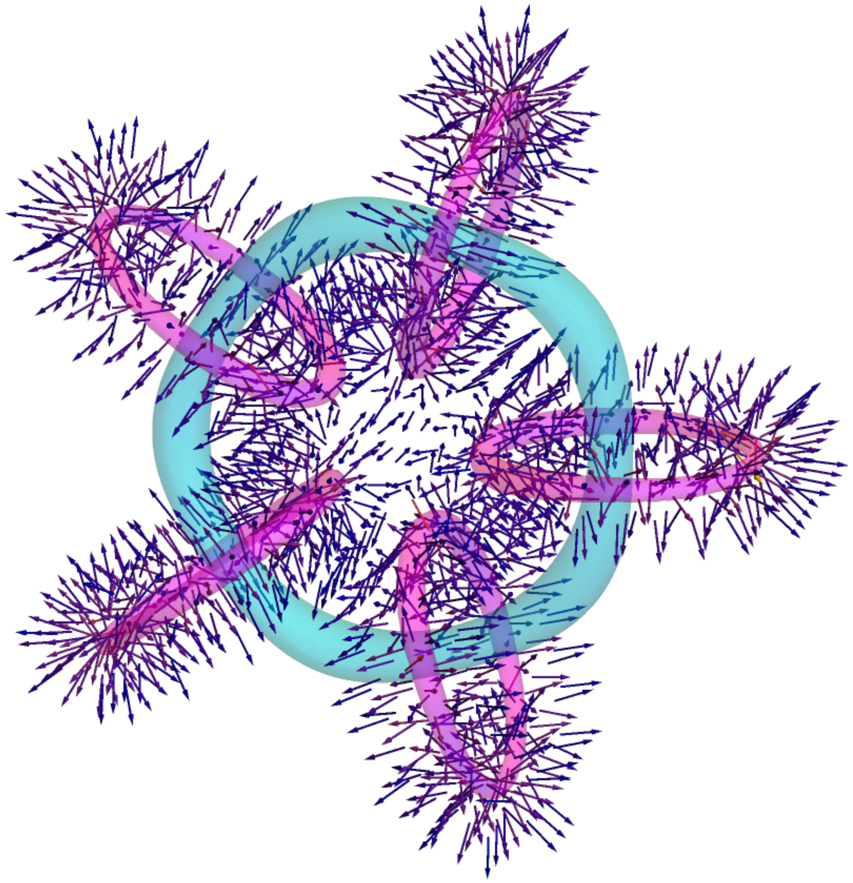
→ **linking strings acquires "electric $B-L$ charge"**
→ **stabilizes the linking configuration**



Numerical solution



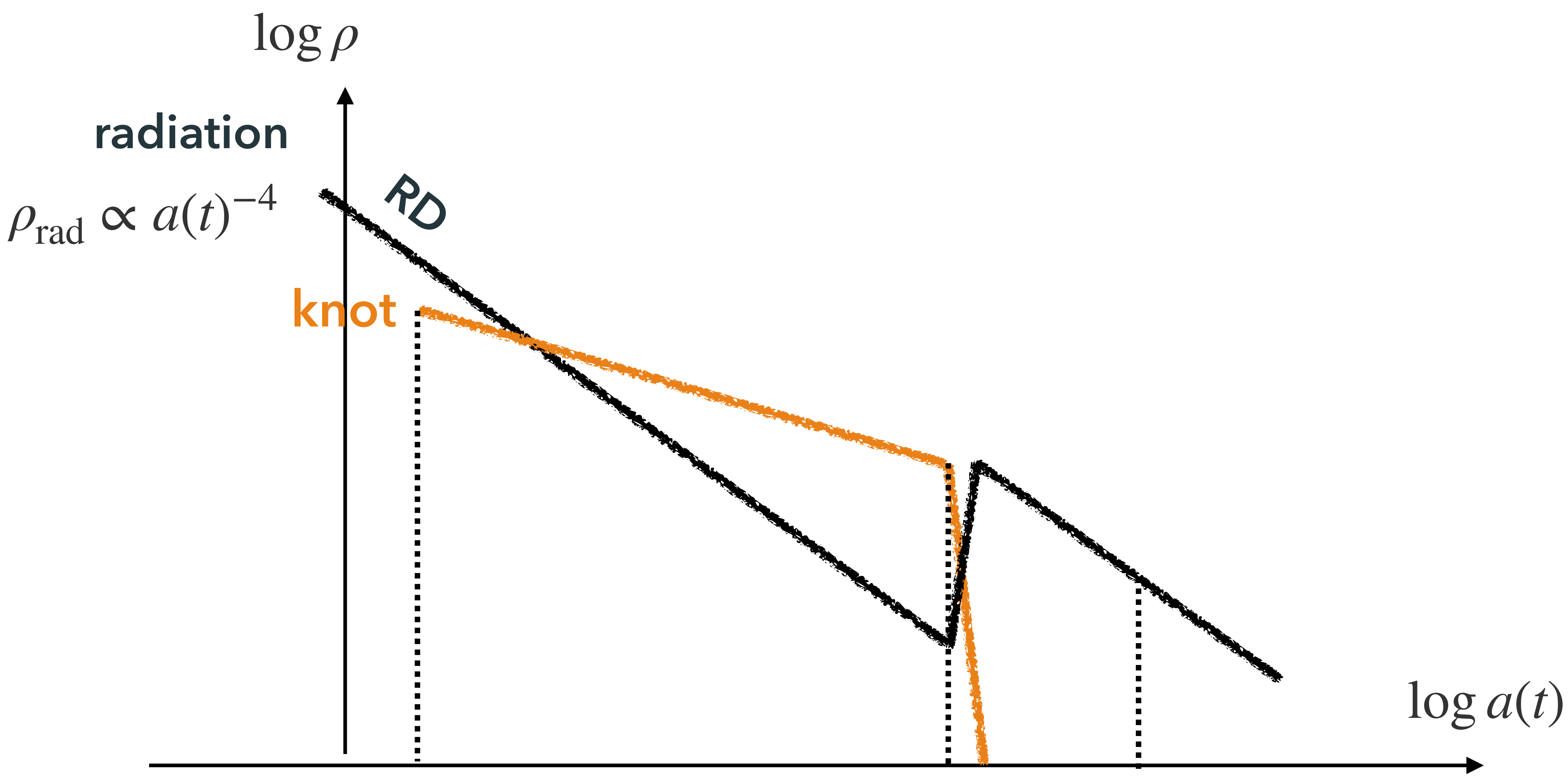
$$\overrightarrow{E} = \overrightarrow{\nabla} A_0$$



$$\lambda/g^2 = 10^3,\ \kappa/g2 = 0.0008,\ \chi = 0,\ C = 400$$

Application to cosmology

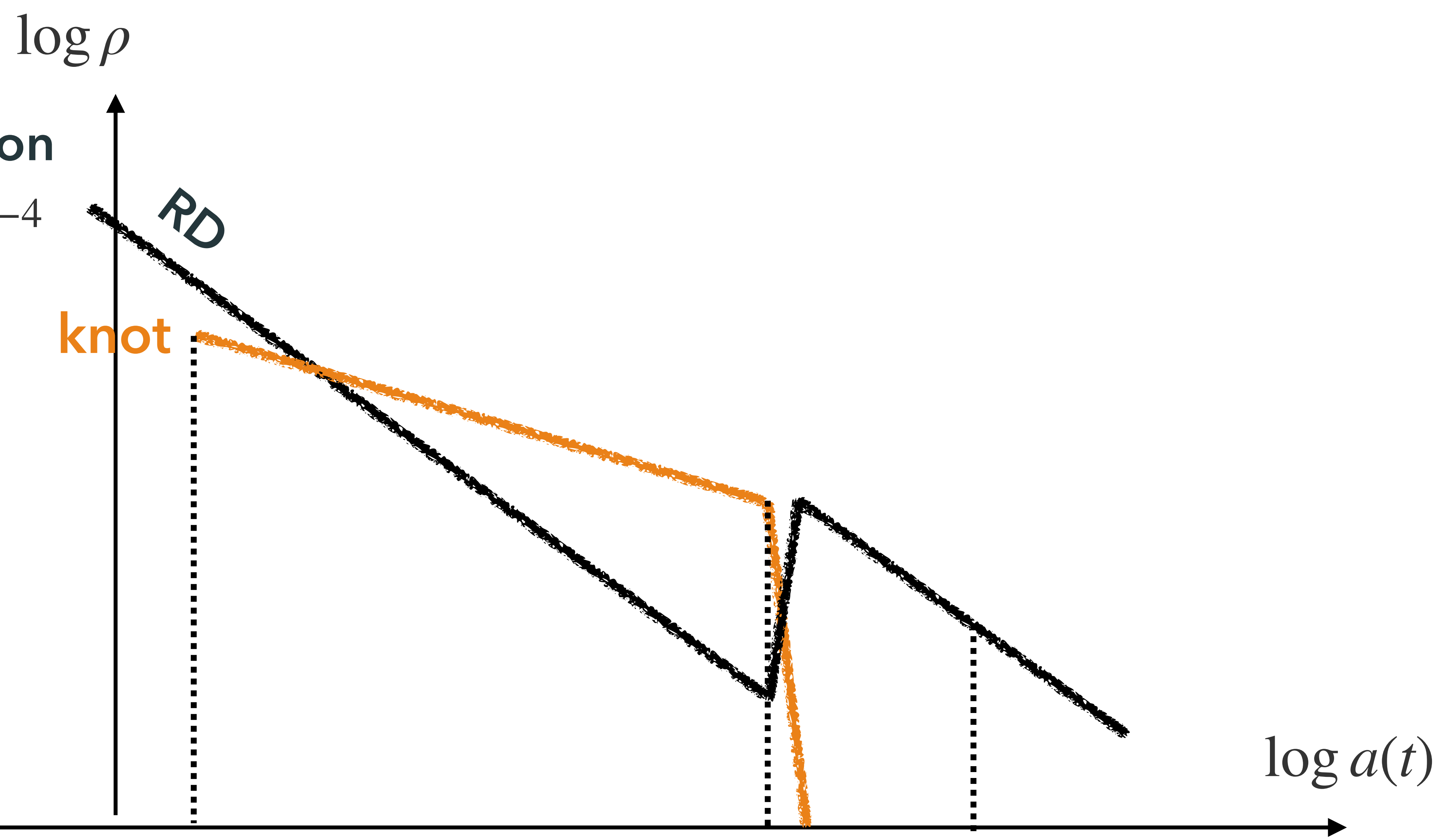
Fate of knot soliton

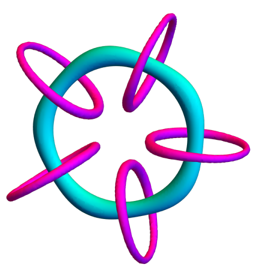


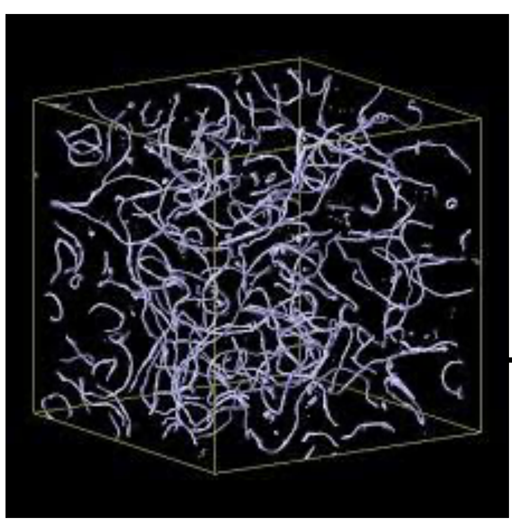
Fate of knot soliton

radiation

$\rho_{\rm rad} \propto a(t)^{-4}$





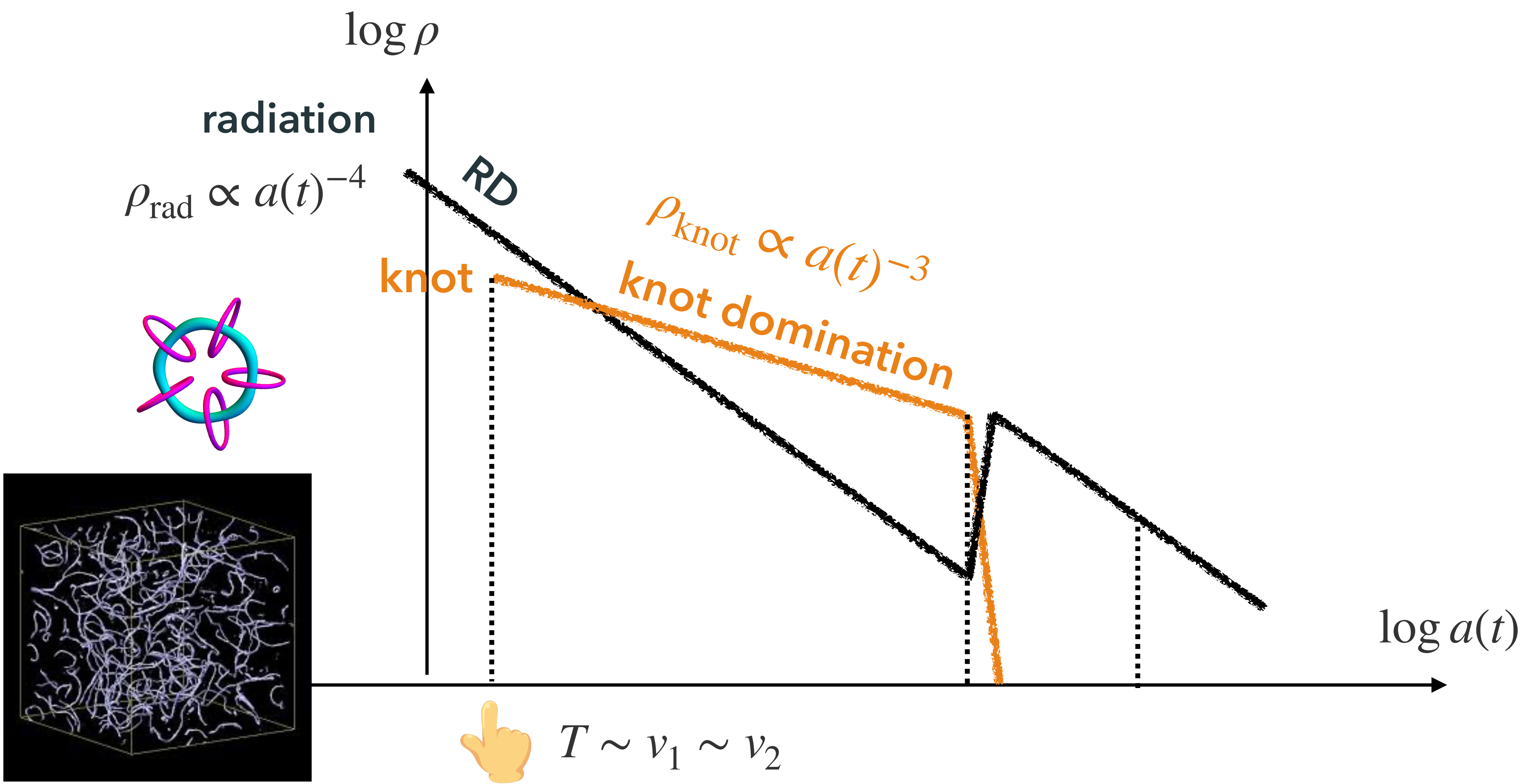


$T \sim v_1 \sim v_2$

SSB: ~~$U(1)_{B-L} \times U(1)_{\rm PQ}$~~

→creation of **knots & cosmic strings**

Fate of knot soliton



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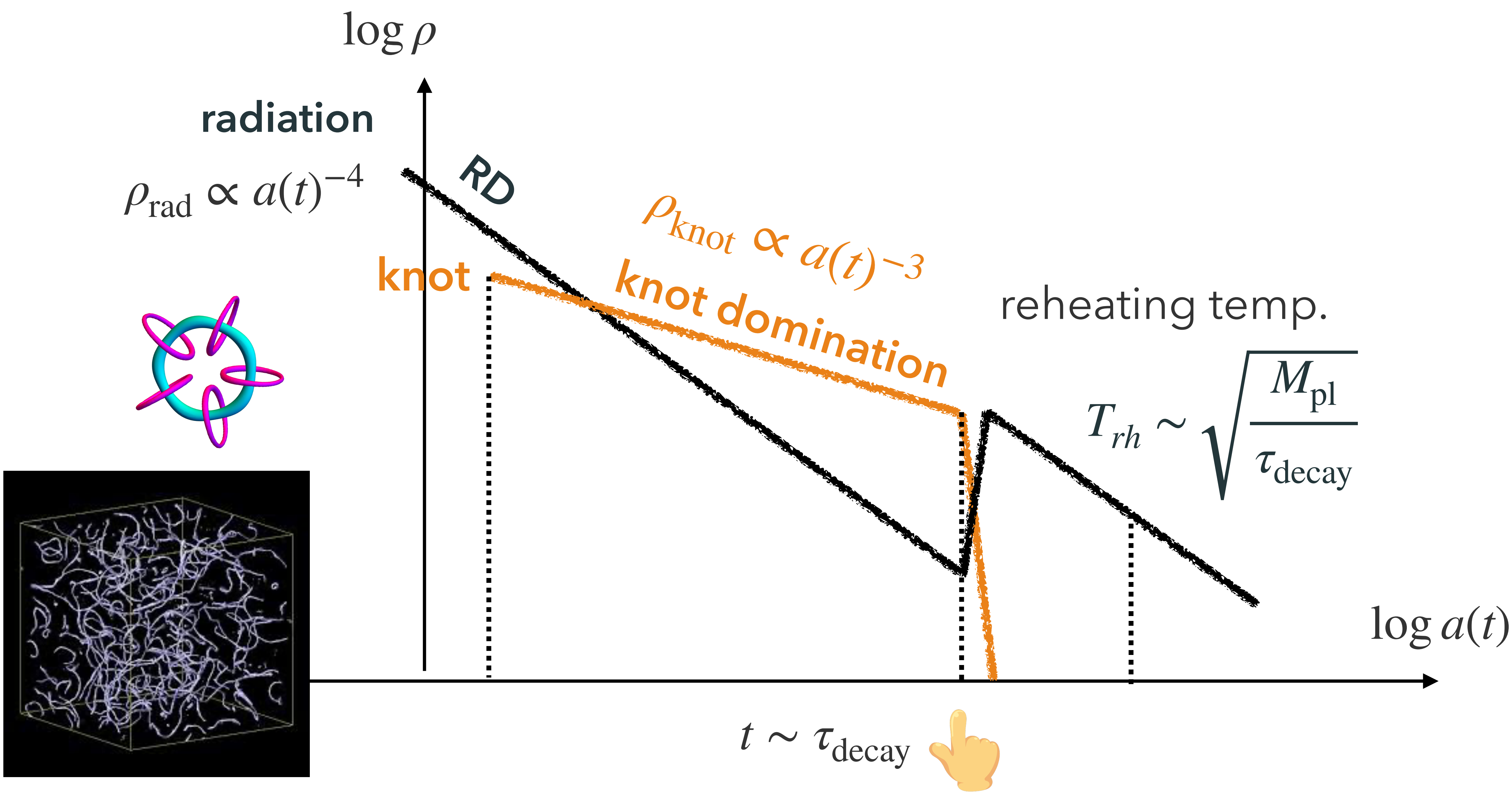
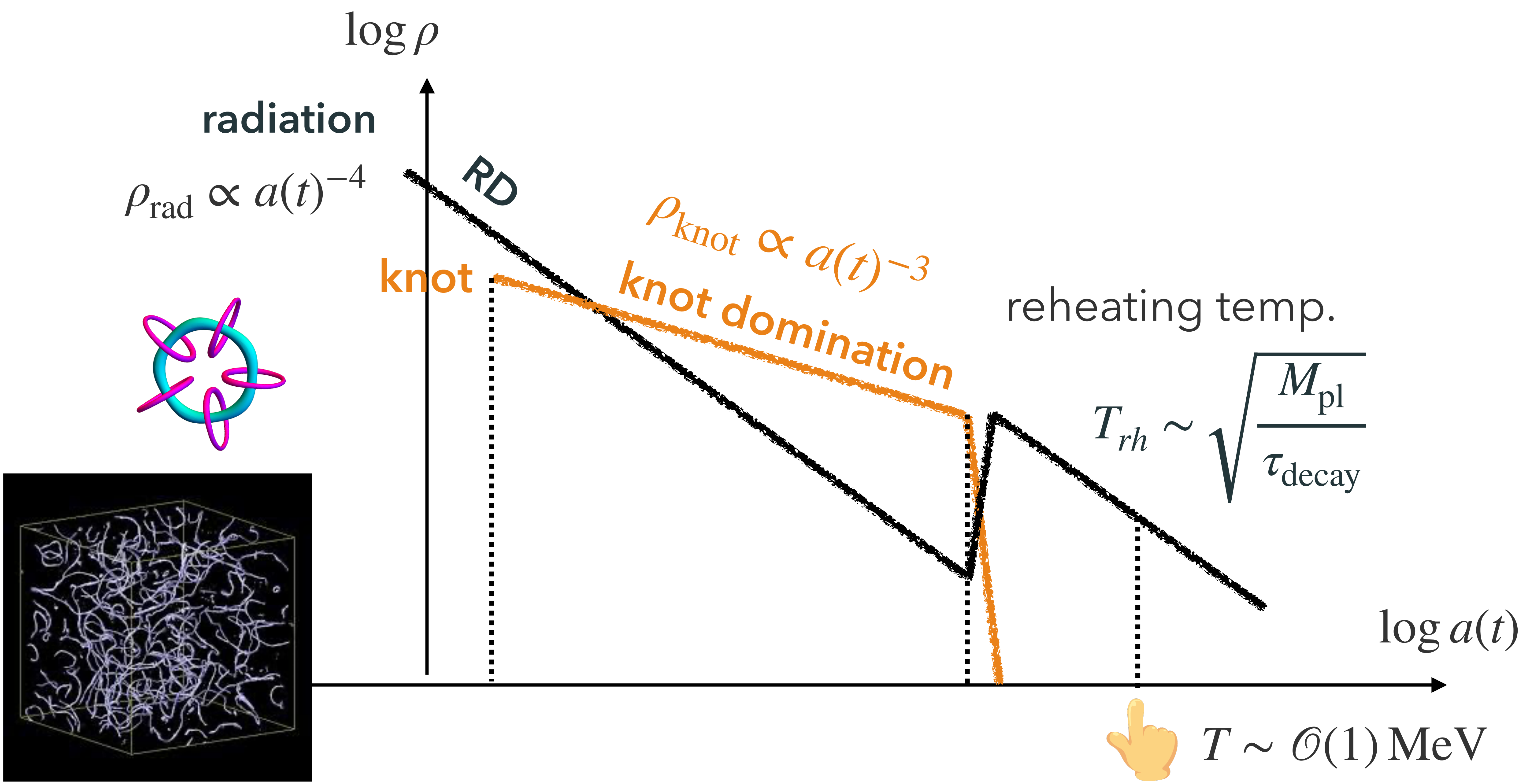


Knot solitons decay due to quantum effects into SM particles

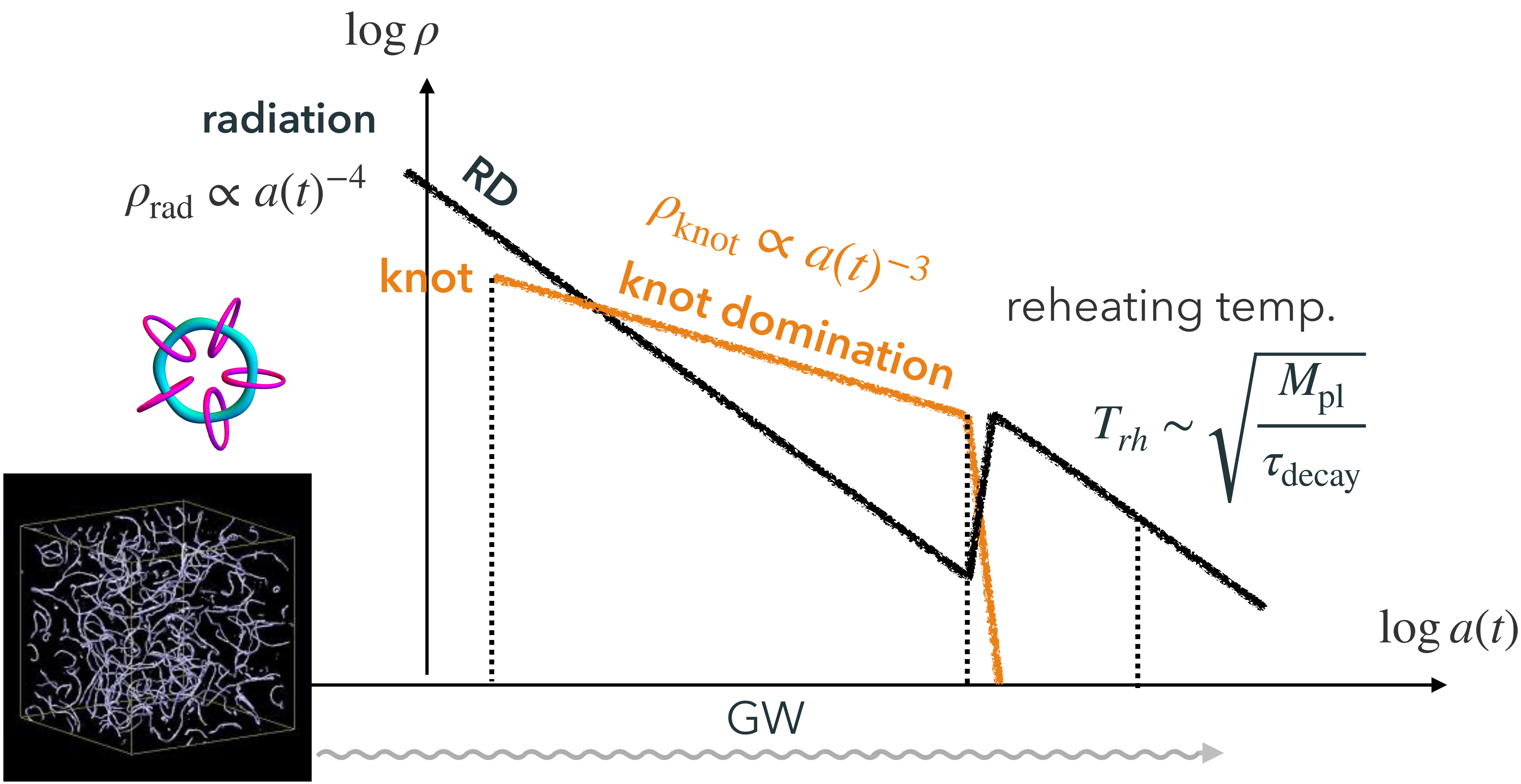
→ **reheat thermal bath again** (secondary reheating T_{rh})

Fate of knot soliton



BBN as in the standard cosmology

Fate of knot soliton



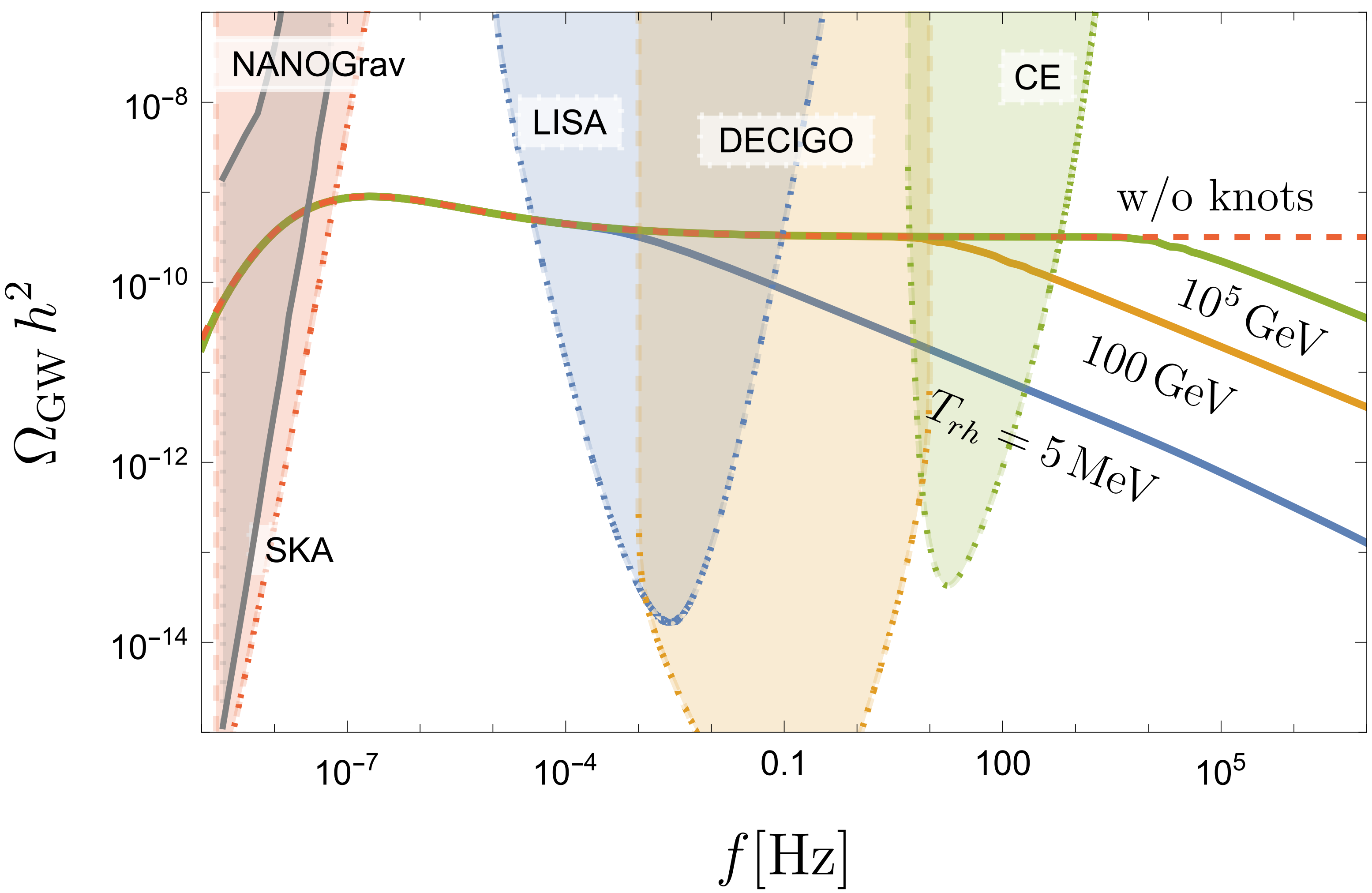
GW spectrum from cosmic strings is affected by knot domination

→ **We can probe knot soliton via GW!**

[Cui+, 1711.03104]

Testability with gravitational wave

$$g_X v_1 = 3 \times 10^{13}\,\mathrm{GeV},\;\; G\mu = 10^{-11}$$



- GW spectrum is flat without knot solitons in UV
- Knot domination tilts the spectrum to be $f^{-1/3}$ in UV

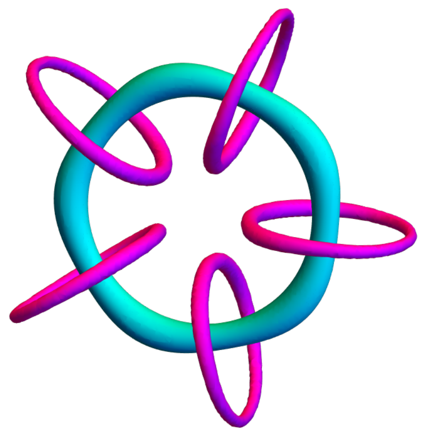
→ discrimination the existence of knot domination via GW

Summary

- showed a new soliton made of two kinds of cosmic strings

knotted cosmic strings

= Knot soliton!



- gauge $U(1)_{B-L}$ & global $U(1)_{\mathrm{PQ}}$ symmetries

→ natural setup

- implying "knot dominated era" in early universe
- **can be probed/tested via GW!**
- **(possible to have non-thermal leptogenesis from knot)**

Backup

Local vs Global strings

- SSB of **gauged** $U(1)$ sym → **local** string

→ **magnetic flux in string**

(eg. magnetic flux in supercond.)

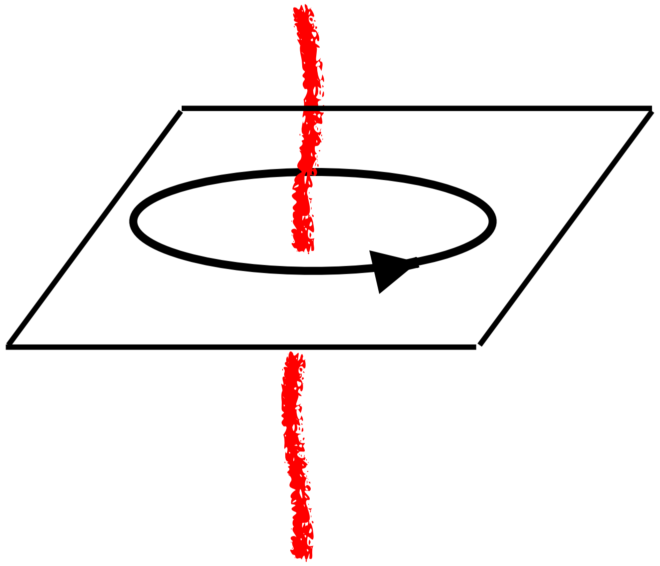
$$\int d^2x\, B = 2\pi/g$$

- SSB of **global** $U(1)$ sym → **global** string

→ **w/o magnetic flux**

The phase of ϕ is physical NG boson

(eg. axion)



$$\phi(x) \sim v\, e^{i\theta}$$

GW & string network

- current GW spectrum:

$$\frac{\rho_{\rm GW,0}(f)}{\rho_{\rm tot,0}} \sim (G\mu)^2 \int_{t_i}^{t_0} dt \left(\frac{a(t)}{a(t_0)}\right)^4 \Delta(t, f_{\rm emit})$$

$$f = \frac{a(t)}{a(t_0)} f_{\rm emit} \qquad ds^2 = -dt^2 + a(t)^2 dr_3^2$$

GW spectrum function

$$G\mu \simeq v_{\rm st.}^2 / M_{\rm pl.}^2$$

depends on cosmology

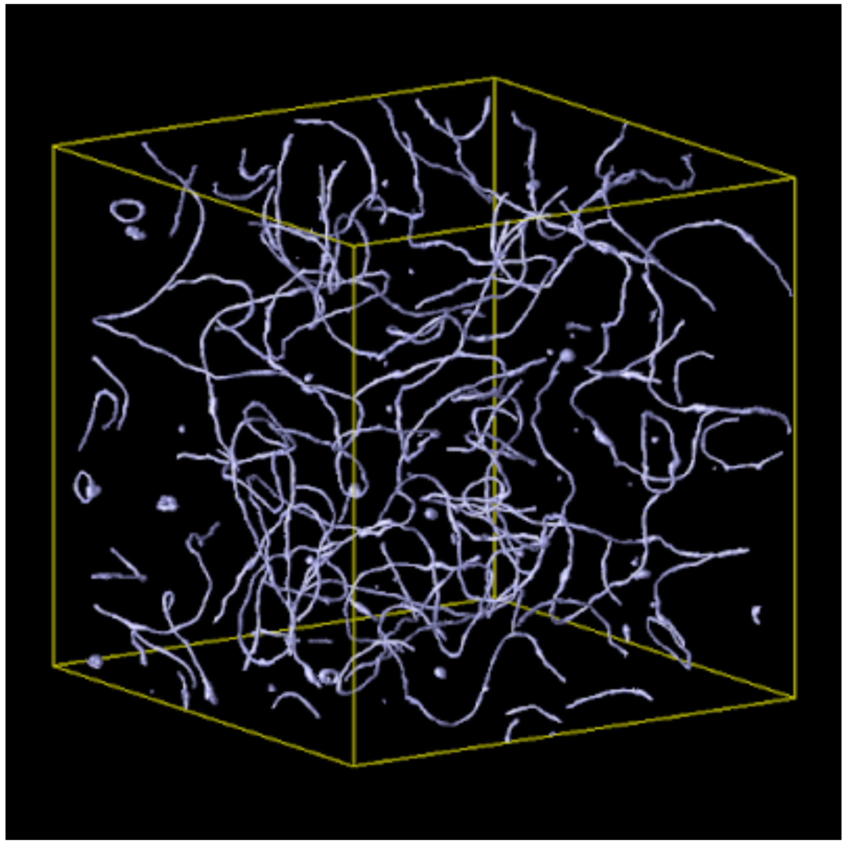
- scale factor $a(t)$: $\dot{a}(t)/a(t) \simeq \sqrt{\rho_{\rm tot}(t)}/M_{\rm pl}$

→ GW from cosmic string "knows" what happened in past universe

→ **if detected, new probe of cosmological history**

Cosmic string network

- Cosmic strings form network in universe
- The network continuously produces small loops of strings
- String loops shrink by emitting GW or light particles (axion etc)



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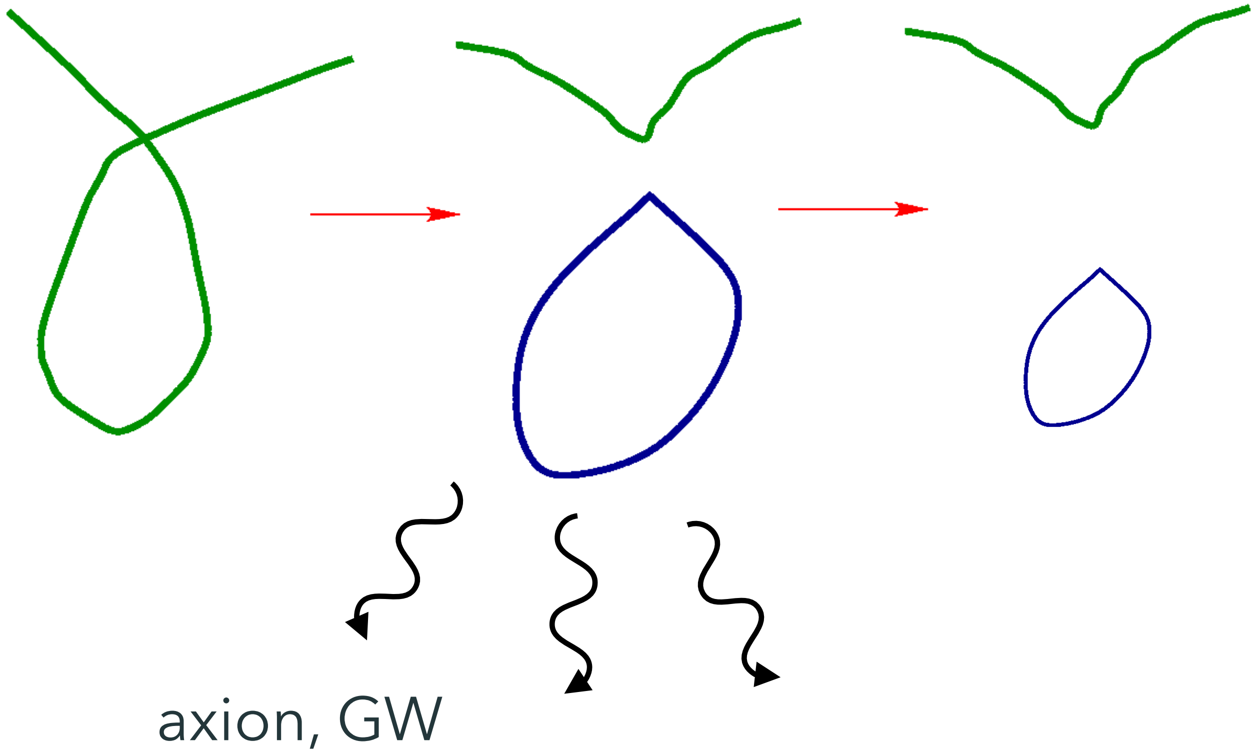


→ No stable object

Knot stability

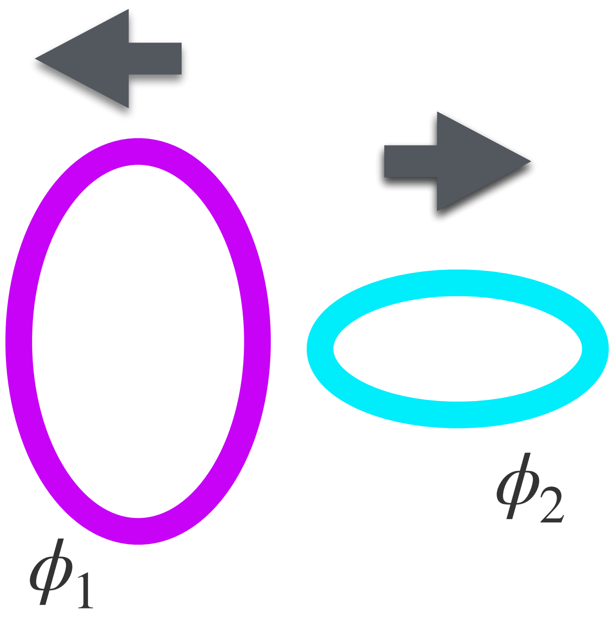
- can decay by delinking?

  → $\lambda \gg g^2, \kappa, \chi$ prevents delink

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

→ non-linear σ-model w/ $O(4)$ sym. → $O(3)$ sym.

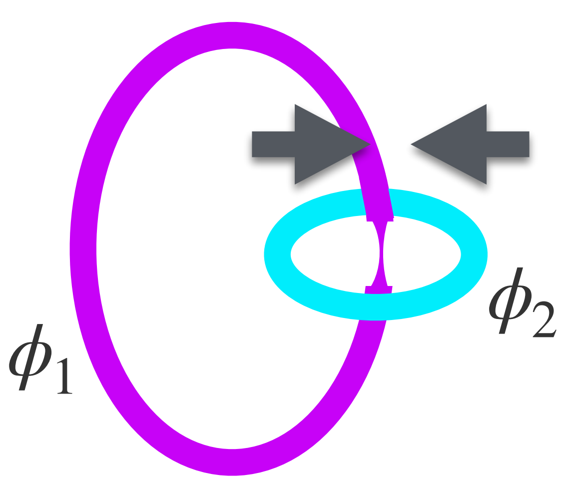
linking # = skyrmion # [Gudnason-Nitta '20]



- Loop of ϕ_2 string can shrink infinitely?

  → $v_2/v_1 \ll 1$ prevents shrinking

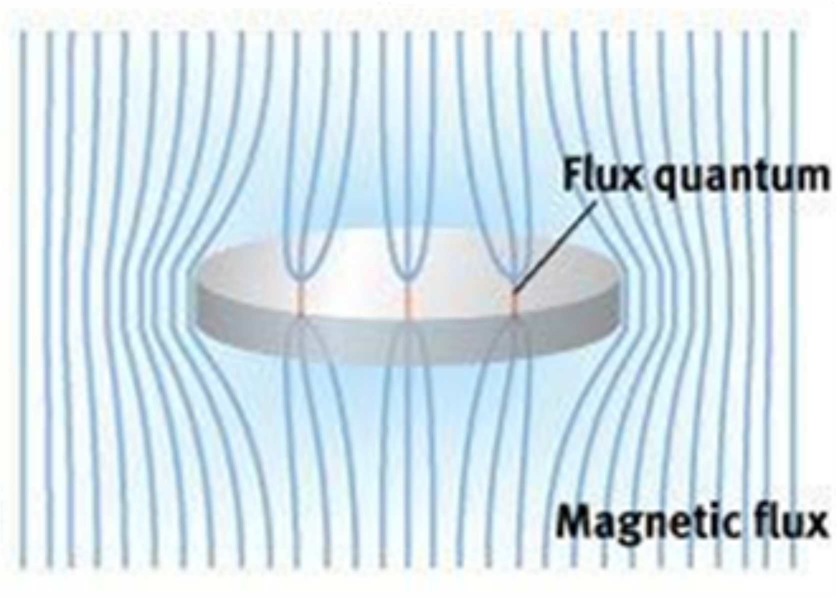
  typically $v_2 \lesssim 0.1 v_1$

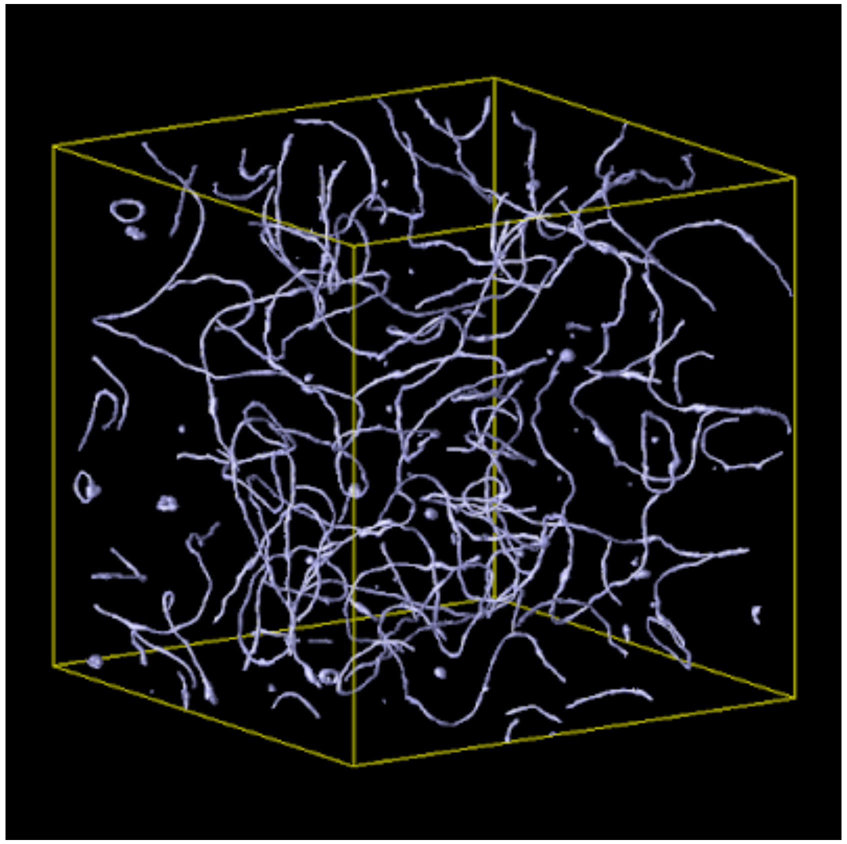


classically stable under these two conditions

Vortex string in many systems

- Magnetic flux tube in **superconductor**
- Superfluid vortex in **neutron star**
- Vortex string in the universe: **Cosmic string**
    - Gravitational wave
    - **strong evidence of new physics**, but haven't yet been discovered.





https://motls.blogspot.com/2017/12/crackpots-lies-about-cosmic-string.html

The model

	$U(1)_{B-L}$	$U(1)_{\rm PQ}$
ϕ_1	2	0
ϕ_2	0	1
KSVZ-like Q	Q^f_{B-L}	$Q^f_{\rm PQ}$
ν_R	-1	0
SM	q: $1/3$ l: -1	0

$v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$

←not specified

$$\Rightarrow c = \sum_f Q^f_{global} (Q^f_{gauge})^2$$

- $\mathscr{L} \supset y_R \phi_1^* \bar{\nu}_R \nu_R^c \rightarrow \langle \phi_1 \rangle$ gives Majorana mass → type-I seesaw

[Minkowski '77] [Yanagida '79] [Gell-Mann+ '79] [Mohapatra-Senjanovic+ '80]

- phase of ϕ_2 (a) is identified as QCD axion

[Peccei-Quinn '77] [Weinberg '78] [Wilczek '78]

→ solution of strong CP problem & Dark matter

Numerical calculation

Static energy in Coulomb gauge:

$$\mathscr{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1,\phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2$$
$$-g^2|\phi_1|^2 A_0^2 - \frac{1}{2g^2}(\partial_i A_0)^2 - \frac{g^2 c}{16\pi^2}\, aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Not positive definite → remove A_0 by solving Gauss law:

$$\frac{\delta\mathscr{L}}{\delta A_0} = \partial_i^2 A_0 - 2g^2|\phi_1|^2 A_0 + \frac{g^2 c}{16\pi^2}(\vec{\nabla} a)\cdot\vec{B} = 0$$

Substitute $A_0 = \frac{g^2 c}{16\pi^2}\frac{(\vec{\nabla} a)\cdot\vec{B}}{-\partial_i^2 + 2g^2|\phi_1|^2}$ into energy functional.

Numerical calculation

Energy in Coulomb gauge:

$$\mathscr{E} = |D_i\phi_1|^2 + |\partial_i\phi_2|^2 + V(\phi_1, \phi_2) + \frac{1}{2g^2}(\partial_i A_j)^2 + \frac{g^2 c}{32\pi^2}(\vec{\nabla} a \cdot \vec{B})A_0$$

$$\text{w/ } A_0 = \frac{g^2 c}{16\pi^2}\frac{(\vec{\nabla} a)\cdot\vec{B}}{-\partial_i^2 + 2g^2|\phi_1|^2}$$

- positive definite -> no obstacle
- Minimizing energy via gradient-flow method
- CPU 3584-cores parallelizing on YITP computer cluster
- lattice spacing $=0.2/gv_1$, $N = 256^3$, converged w/ O(1) days

Relation to Skyrmion

For $\lambda \gg g^2, \kappa, \chi$,

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$$\to \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2$$

→ **non-linear sigma model w/ $O(4)$ symmetry,**

which breaks into $O(3)$

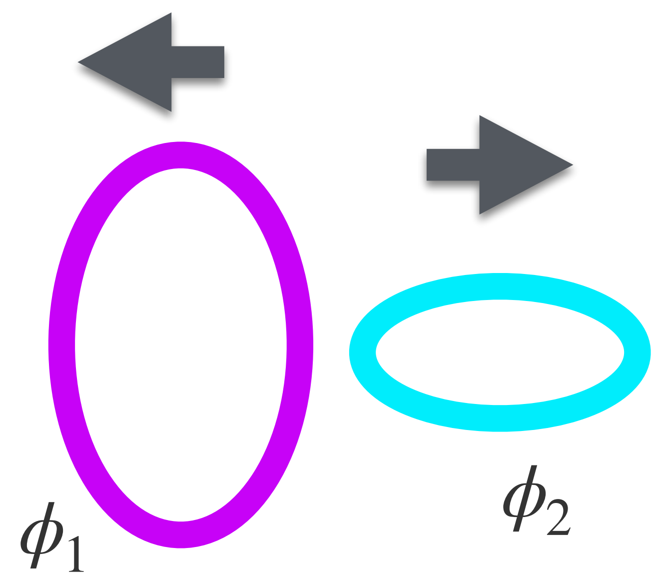
There exists Skyrmion defined by winding number:

$$N_{sk} = \int d^3x \, \epsilon^{ijk} \, \mathrm{Tr} \left[U^\dagger \partial_i U U^\dagger \partial_j U U^\dagger \partial_k U \right] \qquad U \equiv \begin{pmatrix} \mathrm{Re}\,\phi_1 & \mathrm{Im}\,\phi_2 \\ -\mathrm{Im}\,\phi_1 & \mathrm{Re}\,\phi_2 \end{pmatrix}$$

The link is nothing but the Skyrmion!

[Gudnason-Nitta '20]

Decay of link soliton



$$\tau^{-1} \sim \Gamma \sim g v_1 \exp\left[-\frac{4}{3}\sqrt{\frac{\lambda v_1}{g v_2}}\right]$$

$$\frac{M_{\rm pl}}{v_{EW}^2} < \tau < \frac{M_{\rm pl}}{(1{\rm MeV})^2}$$

$$\lambda < 4\pi$$

$$g < \sqrt{24\pi}$$

$$\Leftrightarrow \log\frac{g v_1}{10^{11}{\rm GeV}} + 60 \lesssim \frac{4}{3}\sqrt{\frac{\lambda v_1}{g v_2}} \lesssim \log\frac{g v_1}{10^{11}{\rm GeV}} + 82$$

$$\therefore 100 \lesssim \frac{\lambda}{g} \lesssim 190$$