Physics of (ultra)light scalars

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Dark matter and energy

- Dark matter and dark energy has been discovered a while ago through gravitational interactions. Dark staff contributes total ~ 95% of the energy budget of the universe

- How does dark matter and dark energy fit in our fundamental understanding (QM + relativity) of matter and energy?

Dark matter and energy

- Theoretical embarrassment:

$$\frac{\rho_{DE}^{(theo.)}}{\rho_{DE}^{(exp.)}} \sim \frac{M_P^4}{(1\text{meV})^4} \sim 10^{120}.$$
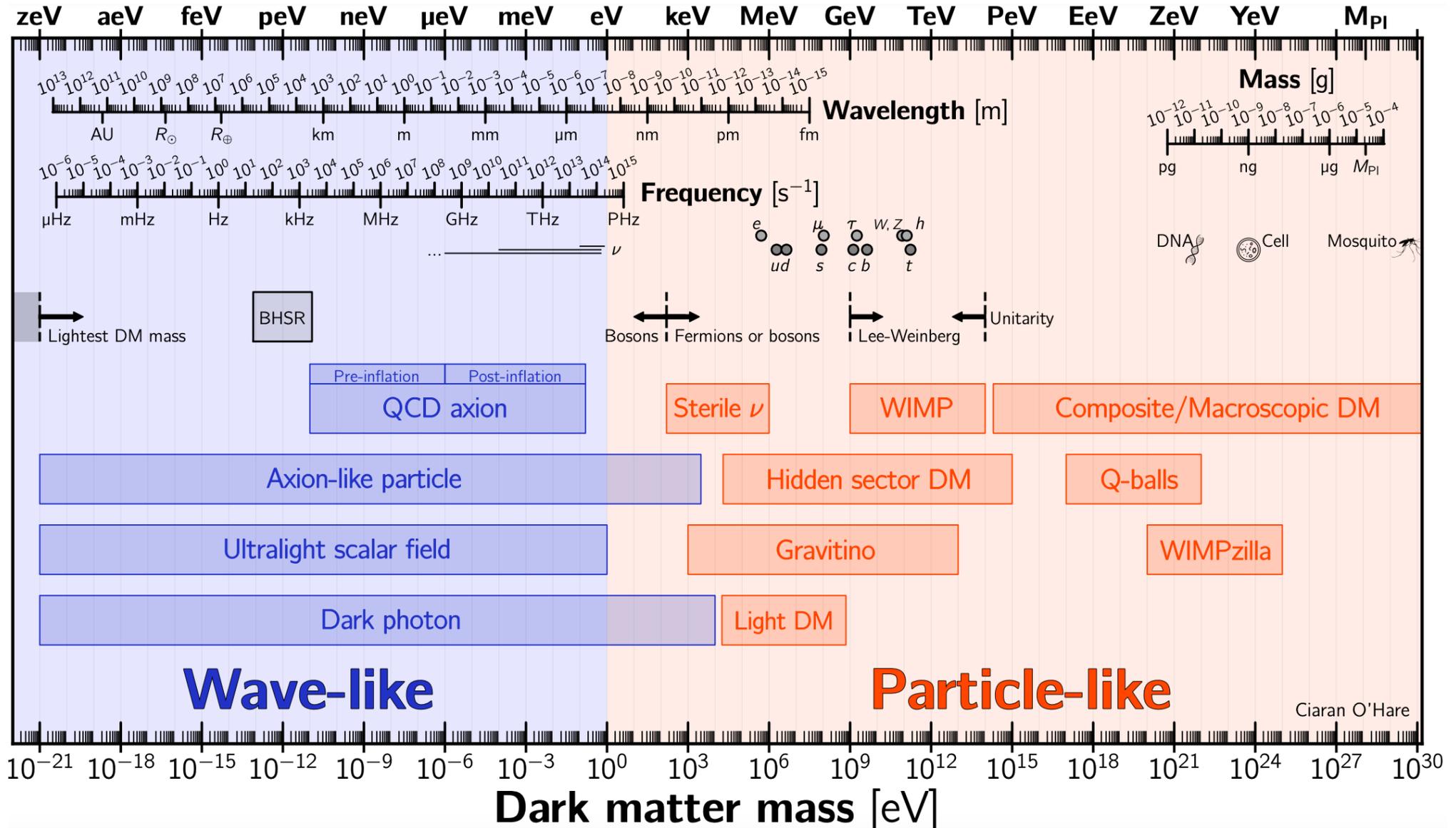
- QFT does not get well along with classical GR. What about quantum gravity? Do we misrepresent the problem?

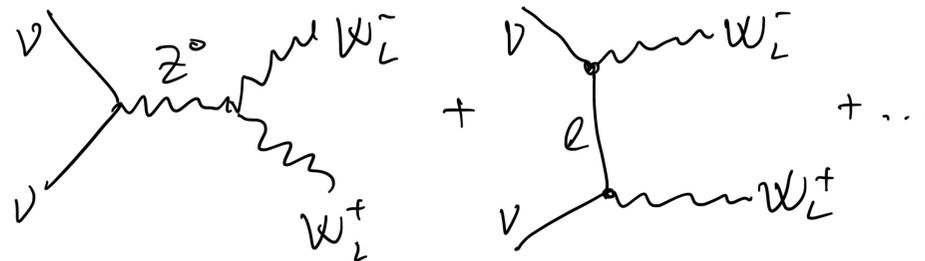(some thoughts in Dvali, AK, Sakhelashvili, 2406.18402)

Dark matter and energy

New physics (local interactions perspectives)

- Theory we know: SM + Majorana neutrino masses

- EW gauge symmetry is nonlinearly realised:

$$\mathcal{L}_{\nu\text{mass}} \propto m_\nu \nu^T \nu + i\frac{m_\nu}{v_{ew}} w^3 \nu^T \nu + ... + \text{h.c.}$$

- Perturbative unitarity breaks down at:

$$\Lambda \simeq \frac{4\pi v_{ew}^2}{\sqrt{3} m_\nu} \sim 10^{15} \left(\frac{m_\nu}{0.1 \text{ eV}}\right) \text{ GeV}$$

New physics (topology of vacuum states)

In addition to local interactions, SM is defined through its lowest energy (vacuum) state:

- The global topological properties of the quantum vacuum state lead to novel non-perturbative interactions and affect the spectrum of particle states. This necessitates the existence of *(ultra)light scalar particles* (and potentially heavy bound states), indicating a new scale of non-perturbative physics.

- Gravity plays important role in particle physics: it contributes in shaping the vacuum state of SM

(Ultra)Light scalars – general considerations

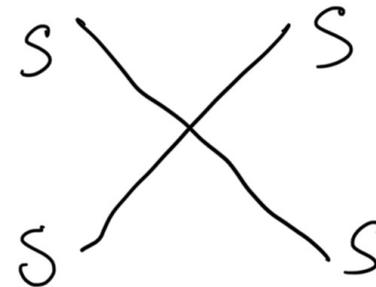- Scalars want to be heavy, e.g.,

$$m_h^2 = m_0^2 + c\Lambda^2 >> m_0^2 \text{(infamous hierarchy problem)}$$

- *Massless scalar* protected by a shift symmetry: $s'(x) = s(x) + c$

$$\mathcal{L}_s = 1/2 \left(\partial_\mu s\right)^2 - \frac{\lambda_4}{f^4} \left(\partial_\mu s\right)^4 - ... - \frac{\lambda_{2n}}{f^{2n}} \left(\partial_\mu s\right)^{2n} + \frac{\lambda'_k}{f^k} \left(\partial_\mu s\right)^k \hat{O}_{SM}$$

- This theory breaks down at

$$\Lambda \sim \lambda_4^{1/4} \cdot f$$

$\sim \lambda_4 \dfrac{E^4}{f^4}$

(Ultra)Light scalars – general considerations

- An ultraviolet completion – spontaneously broken global symmetry:

$$s(x) \to \phi(x) = \rho(x)\mathrm{e}^{is(x)/f}; \quad \phi'(x) = \mathrm{e}^{ic}\phi$$

$$\mathcal{L}_\phi = |\partial_\mu\phi|^2 - V(|\phi|) - f(|\phi|)\hat{O}_{\mathrm{SM}} , \quad \langle 0|\hat{\phi}|0\rangle = f$$

Integrating out massive $\rho(x)\,[m_\rho \sim f]$ gives effective low-energy theory for the massless scalar

- *SSB*: vacuum structure and relativistic invariance implies the existence of massless particle(s) [the Goldstone theorem]

(Ultra)Light scalars – general considerations

- Dual description of massless scalars: $\partial^\mu s = \dfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma}$

$$\mathcal{L}_s \to \mathcal{L}_B = \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}, \quad H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]}$$

- Gauge redundancy:

$$B'_{\mu\nu} = B_{\mu\nu} + \partial_{[\mu}\alpha_{\nu]} \text{ (analogue of } A'_\mu = A_\mu + \partial_\mu\alpha)$$

- At low energies two theories describe the same physics. UV completions differ.

Gauge-invariant coupling to strings - $B_{\mu\nu}J^{\mu\nu}$ [analogue of $A_\mu J^\mu$]

(Ultra)Light scalars – general considerations

- How to generate mass for (pseudo)Goldstone scalar:
    - Explicitly break global symmetry
    - Gauge the global symmetry (the Higgs mechanism)

- Explicit breaking:
    - Explicit breaking at classical level, e.g.,

$$\mathcal{L}_\phi + \epsilon\, \hat{O}(\phi) \ [\text{soft breaking if } d_{\hat{O}} < 4]$$

    - Exact classically, broken upon quantisation (quantum anomaly)

$$m_s \sim \frac{\mu^2}{f} <<< f, \quad \mu^4 \sim \begin{cases} \epsilon \langle 0|\hat{O}(\phi)|0\rangle <<< f^4 \\ \\ f^4\, e^{-\frac{2\pi}{\alpha}} <<< f^4 \end{cases}$$

(Ultra)Light scalars – general considerations

- (pseudo)Goldstone nature dictates also couplings to SM fields:

  - All couplings scale with powers of 1/f
  - Symmetry-preserving couplings are momentum dependent (completely irrelevant at very low energies, important when a scalar vary in time/space)
  - In the case of explicit breaking, the couplings are proportional to the typical breaking parameter

- Parity assignment:

  - Parity odd, aka *axion:* $\quad s \equiv a, \quad \dfrac{c_{a\gamma}}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} = -\dfrac{c_{a\gamma}}{f_a} a \vec{E} \cdot \vec{B}; \quad \dfrac{c_{af}}{f_a} \partial_\mu a \bar{f} \gamma^\mu \gamma_5 f$

  - Parity even, aka *dilaton:* $s \equiv \varphi, \quad \dfrac{c_{\varphi\gamma}}{f_\varphi} a F_{\mu\nu} F^{\mu\nu} = -\dfrac{2 c_{\varphi\gamma}}{f_\varphi} \varphi (\vec{E}^2 + \vec{B}^2); \quad \dfrac{c_{\varphi f}}{f_\varphi} \varphi \bar{f} f$

(Ultra)Light scalars – general considerations

- Gauging the global shift symmetry

  - Scalar field $\quad \mathcal{L}_m \propto \dfrac{g^2 f^2}{2} \left(A_\mu - \dfrac{1}{g^2 f^2} \partial_\mu s \right)^2 + ... [\delta A_\mu = \partial_\mu c(x), \ \delta s = c(x)]$

  - $m^2 = g^2 f^2 <<< f^2$ if $g <<< 1.$ Almost global symmetry (g -> 0), but with 3 dof (vs 1) – light dark photon

  - Dual 2-form picture: $\quad \mathcal{L}_m \propto \dfrac{m^2}{12} \left(C_{\mu\nu\rho} - \dfrac{1}{m^2} \partial_{[\mu} B_{\nu\rho]} \right)^2 + ... [\delta C_{\mu\nu\rho} = \partial_{[\mu} \alpha_{\nu\rho]}, \ \delta B_{\mu\nu} = \alpha_{\mu\nu}]$

  This theory still describes 1 massive dof! 3-form gauge field is topological (no propagating dof).

  Powerful alternative view on θ-vacua and the solution of the strong CP problem [Dvali '15]

(Ultra)Light scalars – general considerations

- In what form light scalars exist?

$$N_s = \frac{\rho_s}{m_s}\lambda_s^3 = \frac{(2\pi)^3 \rho_s}{m_s^4 v^3} \simeq 10^6 \left(\frac{1 \text{ eV}}{m_s}\right)^4 \left(\frac{250 \text{ km/s}}{v}\right)^3 \left(\frac{\rho_s}{0.4 \text{ GeV/cm}^3}\right)$$

- This implies that light scalars exist as dark matter in a form of collective excitations (aka *classical waves* (BEC description, Sikivie, Yang '09)).

- With sufficiently attractive self-interactions, excitations in BEC may exhibit superfluid behaviour (Berezhiani, Khouri '15)

- Different behaviour at small scales, resolves core-cusp problem; MOND-like behaviour in a superfluid state,

$$M_b \sim v_c^4 \text{ (Tully–Fisher relation)}$$

(Ultra)Light scalars – general considerations

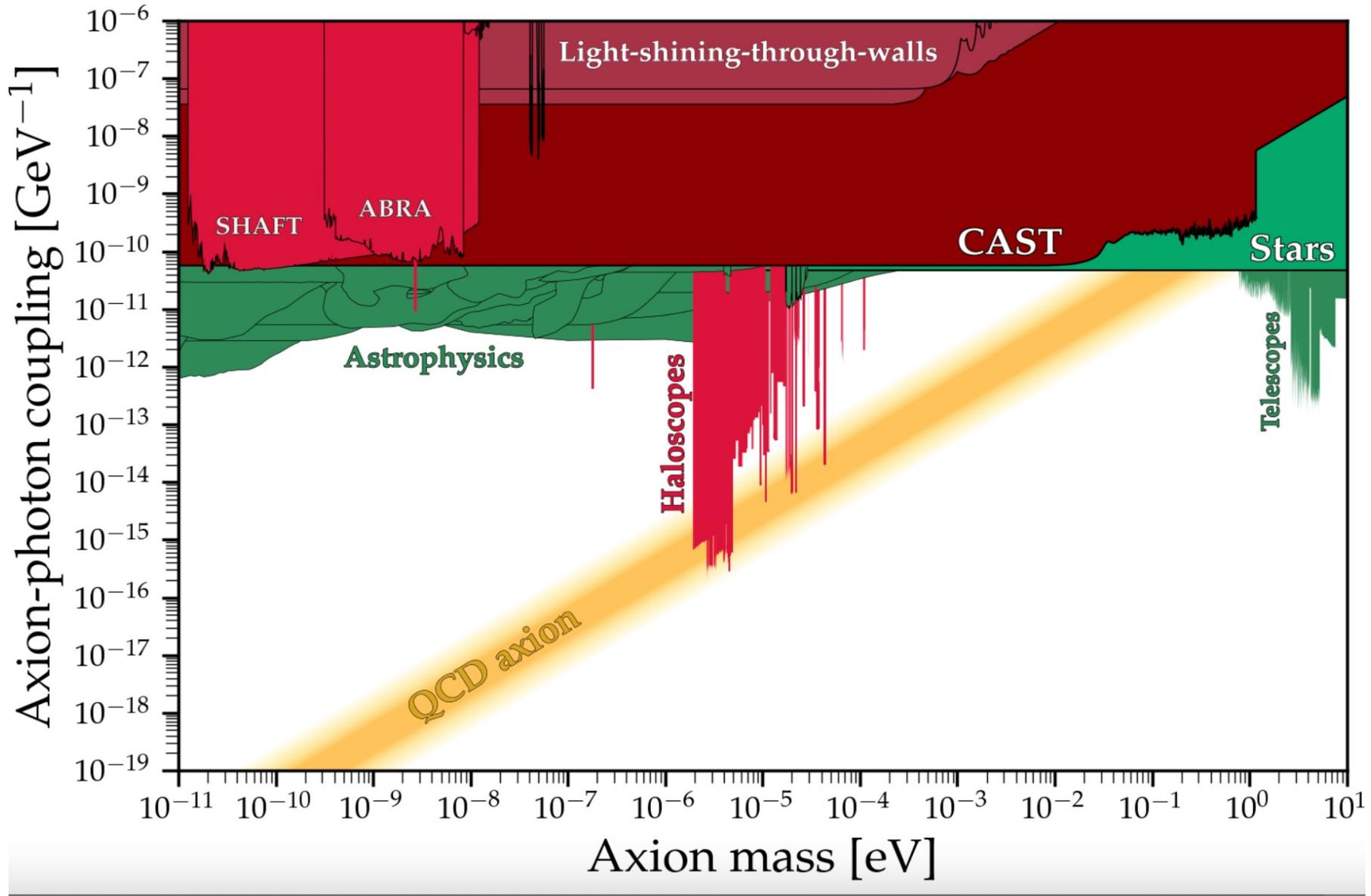- How were they produced?
    - Axion in FLRW universe:

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0 \ [a = a(t)]$$

  - Initially, $H >> m_a, \ a(t) = a_0 = \text{conts.}$
  - As universe expands, $H << m_a, \ a(t) = A\cos(m_a t)$
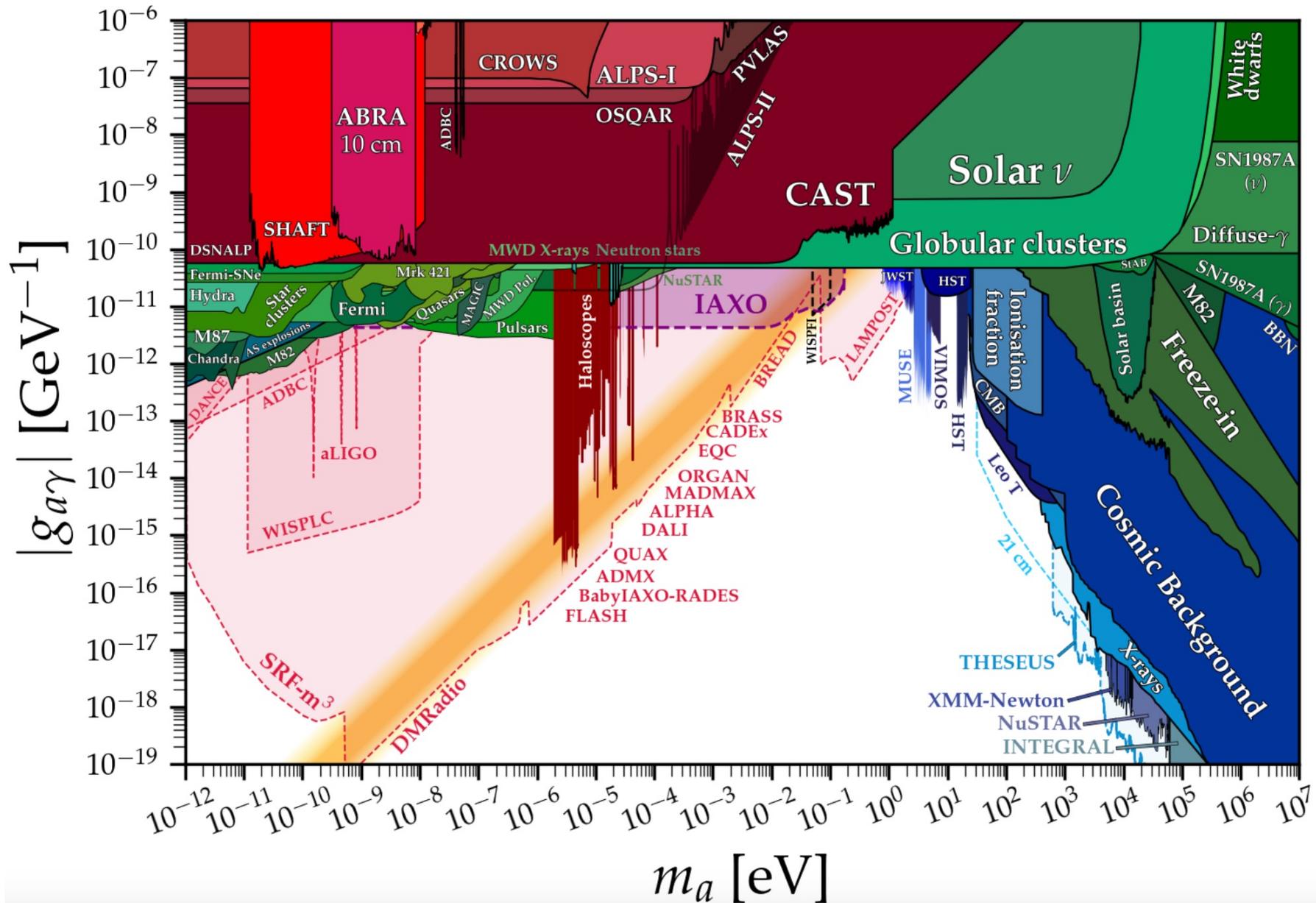 - Matching when $H = 1/2t \approx m_a, \ A \approx a_0$

Coherently oscillating scalar field behaves as cold dark matter:
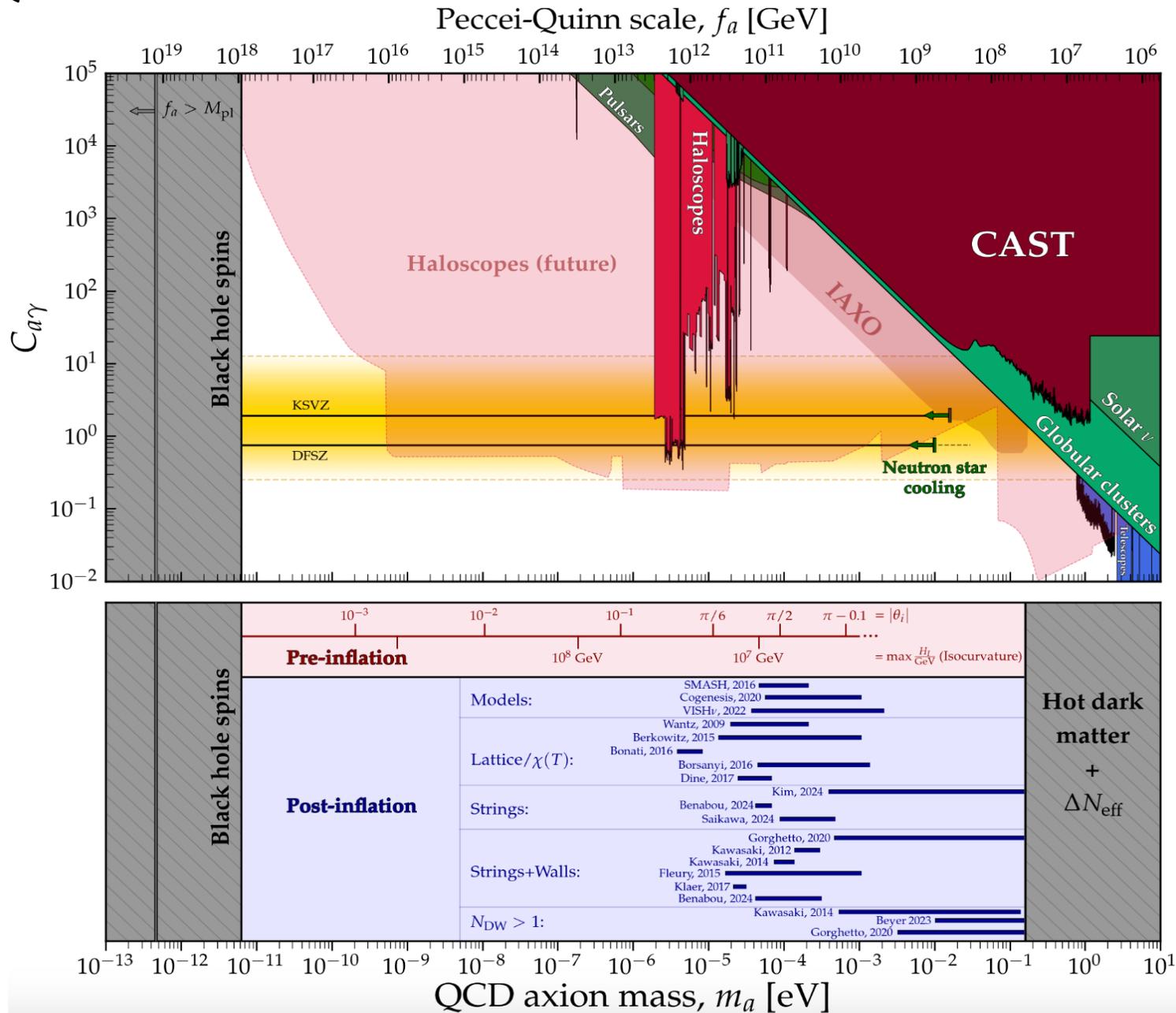$$\rho_a \sim m_a^2 a_0^2, \ \ p_a \approx 0 \ \ [\text{the misalignment mechanism}]$$

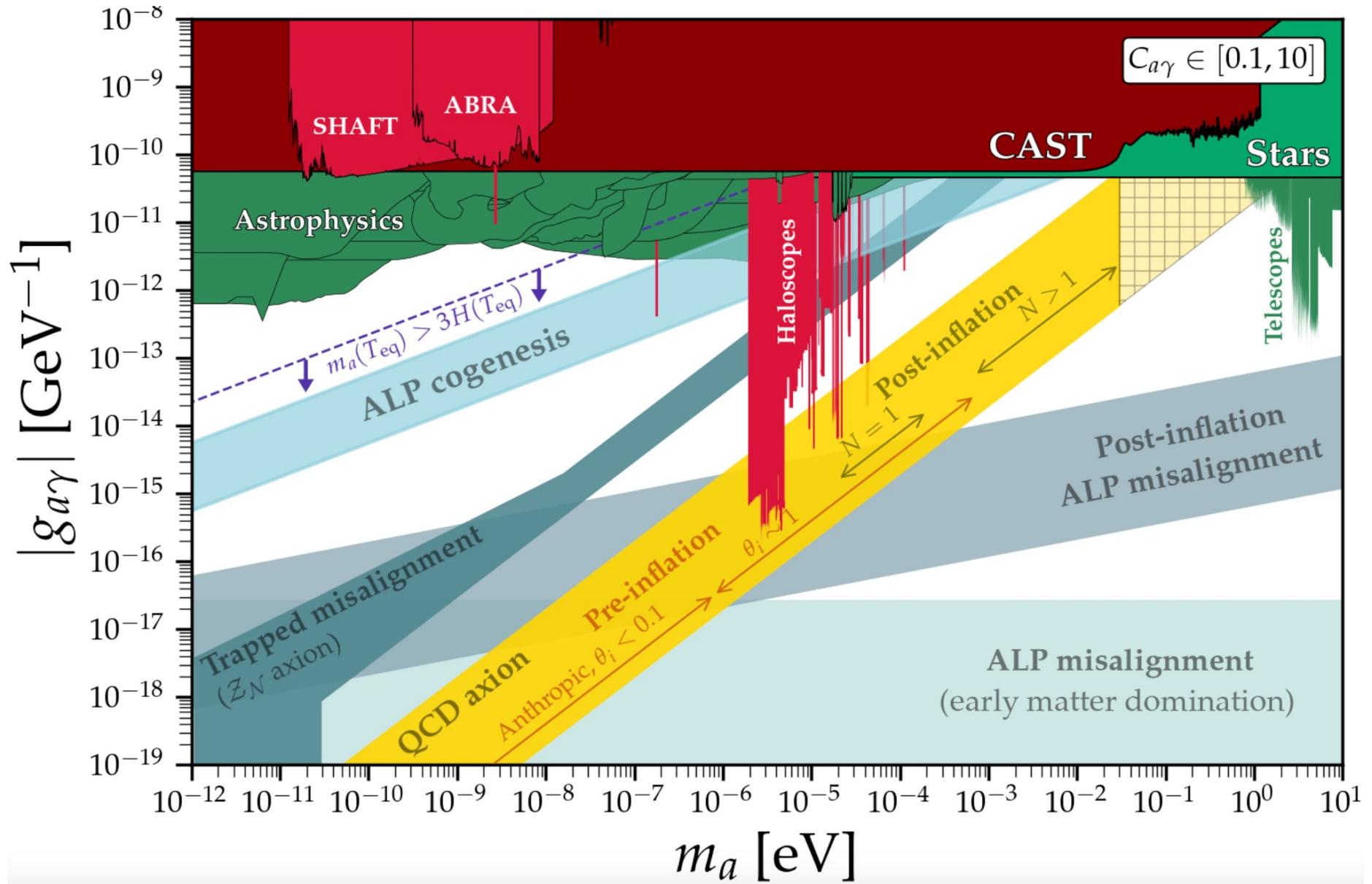# Searching for axions through coupling with photons

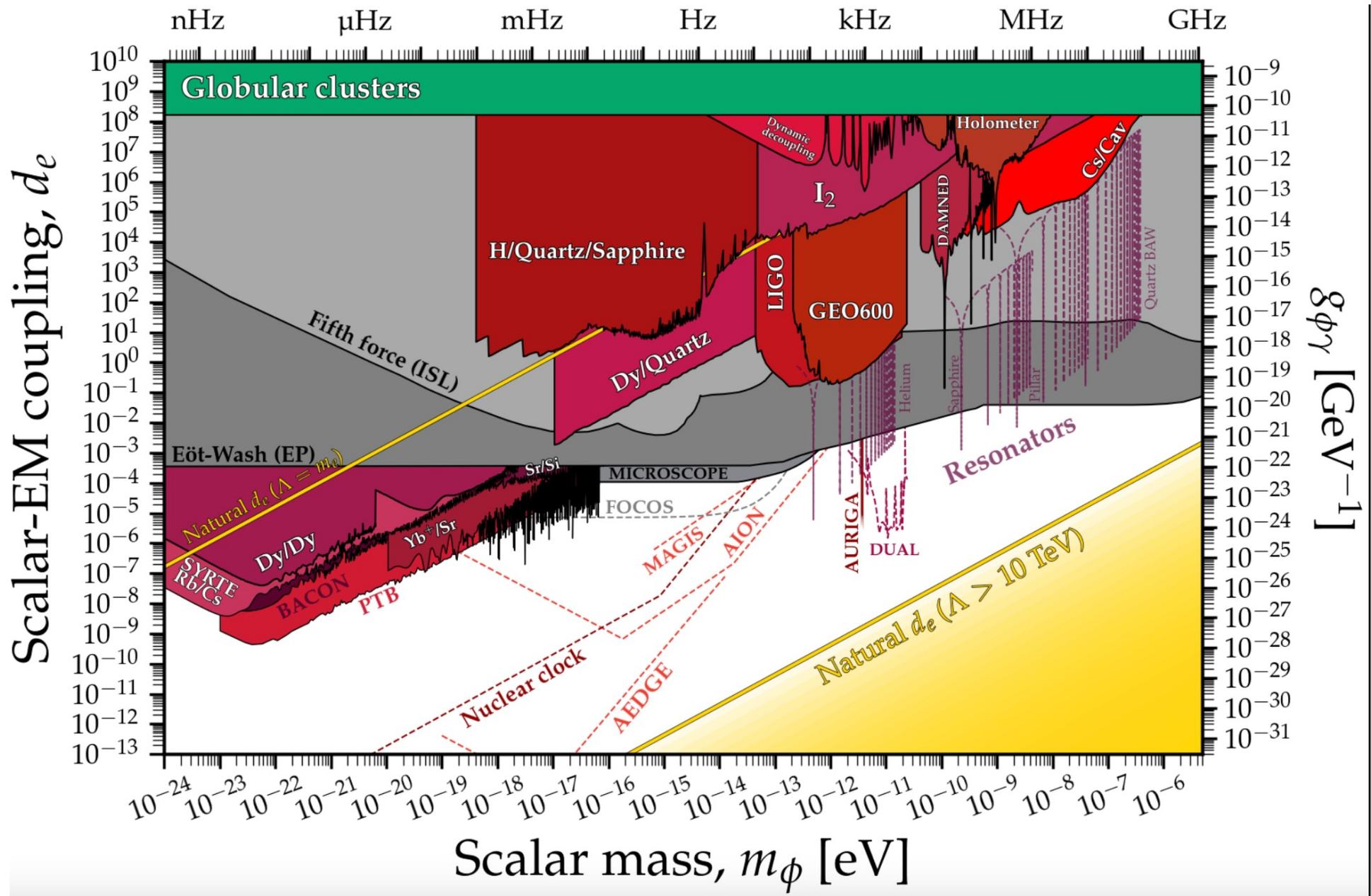# Searching for axions through coupling with photons
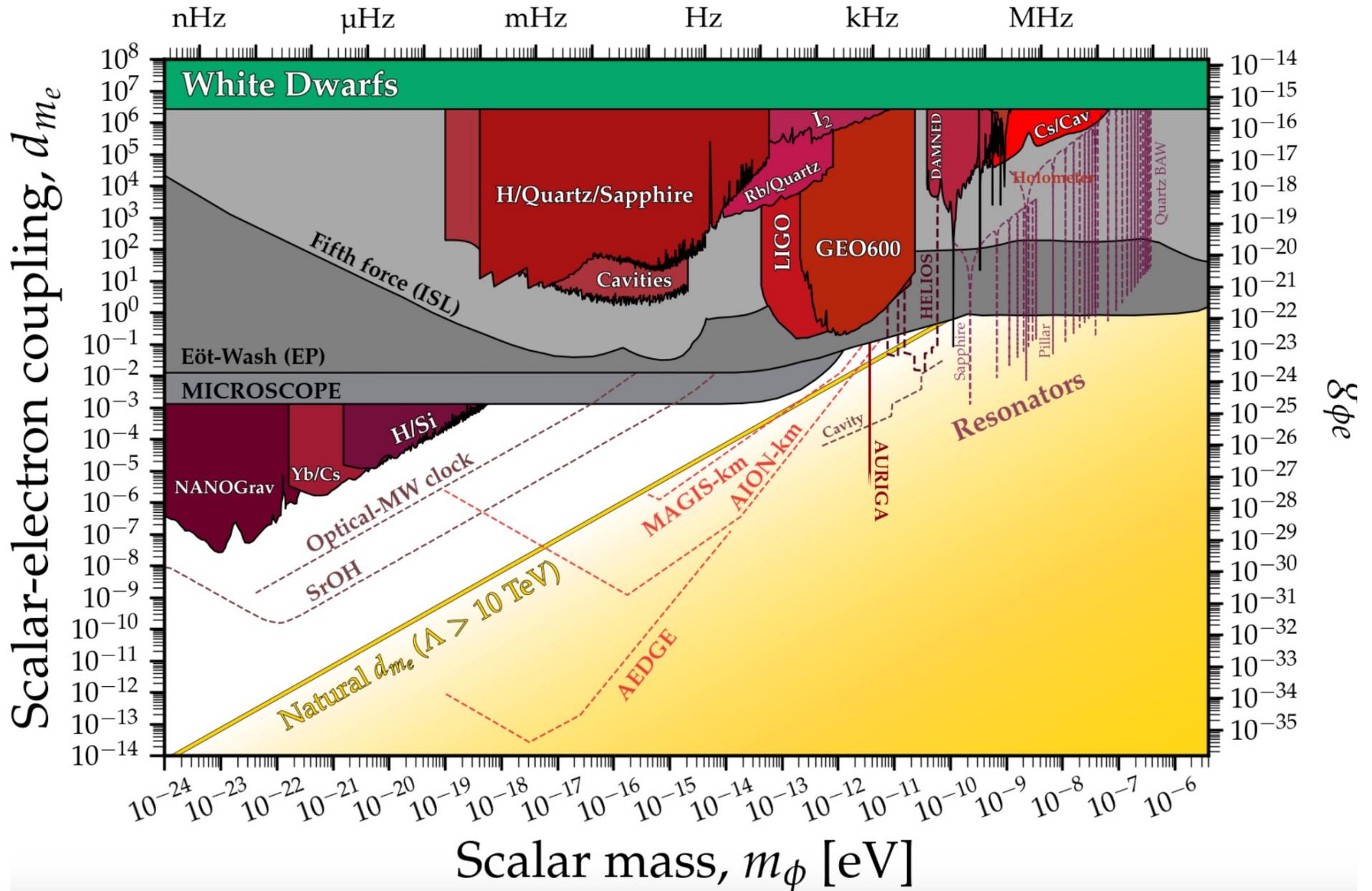
QCD Axion DM searches

Other Axion DM searches

Searching for dilaton through coupling with photons

Searching for dilaton through coupling with electrons

QCD-Gravitational instantons and companion axion

- Chen, AK, 2108.05549
- Chen, AK, O'Hare, Picker, Pierobon, 2109.12920, 2110.11014

QCD-Gravitational instantons and companion axion

FIG. 2. **Left:** Current bounds on the companion-axion model. The colorscale corresponds to the value of the lighter axion's mass, whereas the heavier axion's mass is shown (roughly) by the upper horizontal axis. We can rule out parts of this parameter space using stellar cooling arguments, ADMX, and black hole superradiance. **Right:** As in the left-hand panel, but now showing projected constraints from future experiments: MADMAX [58], IAXO [59] and DMRadio/ABRACADABRA [60, 61].

New light scalar in the SM

The light pseudo-Goldstone emerge when:

A continuous symmetry is spontaneously broken and it is anomalous (small explicit breaking).

In SM:

- $U(1)_{B+L}$ symmetry is exact at classical level
- $U(1)_{B+L}$ explicitly broken through electroweak anomaly (EW instantons)
- Topological features of EW vacuum implies that $U(1)_{B+L}$ must also be broken spontaneously

We predict a (ultra)light scalar (the electroweak η_w) within SM

$$m_\eta^2 \sim \frac{\mu^2}{f} \approx v_{ew} \left(\frac{2\pi}{\alpha_{ew}} \right)^2 e^{-\pi/\alpha_{ew}} \approx 10^{-24} \text{ eV [crude estimate!]}$$

Dvali, AK, Sakhelashvili, 2408.07535

Topology, θ-vacua, anomalies and condensates: general considerations

- Consider SU(2) pure Yang-Mills theory (prototype for QCD and the EW sectors of the standard model or even GR)

$$\mathcal{L} = -\frac{1}{2g^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{16\pi^2}\mathrm{Tr}F_\mu \tilde{F}^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right], \ \ A_\mu = A_\mu^a \sigma^a/2, \ \ \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}$$

- θ-term is topological:

$$\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv \partial_\mu K^\mu, \ \ K^\mu = \frac{1}{6}\epsilon^{\mu\nu\rho\sigma}C_{\nu\rho\sigma}$$

$$C_{\nu\rho\sigma} = \mathrm{Tr}\left(A_{[\nu}\partial_\rho A_{\sigma]} - \frac{2i}{3}A_{[\nu}A_\rho A_{\sigma]} \right)$$ - Chern-Simons 3-form

Topology, θ-vacua, anomalies and condensates: general considerations

- **θ-term** is mandatory (!) to preserve causality (cluster property) and results from topological properties of the ground (vacuum) state

- Topology of vacuum states :

$$A_0 = 0, \ A_i = g \partial_i g^{-1}, \ g(\vec{x}) = \mathrm{e}^{i\alpha^a(\vec{x})\sigma^a/2} \in SU(2)$$

$$g(\vec{x}): \ S^3 \to SU(2), \ \pi_3(SU(2)) = \mathbb{Z}$$

$$n = \frac{1}{16\pi^2} \int d^4x \, \mathrm{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} = -\frac{g^3}{96\pi^2} \int d^3\vec{x} \ \epsilon_{0ijk}\epsilon^{abc} A_i^a A_j^b A_k^c$$

Topology, θ-vacua, anomalies and condensates: general considerations

- The ground state

$$|n\rangle = \int \mathcal{D}\alpha^a(\vec{x}) \; |\mathrm{e}^{i\alpha^a(\vec{x})\sigma^a/2} A_i^a(n)\rangle$$

  - Invariant under "small" gauge transformations [$\alpha^a(\vec{x})$ is smooth on S^3]
  - "Large" gauge transformations: $|n\rangle \to |n+\nu\rangle$ [ν winding #]

  Fully gauge invariant ground state:

$$|\theta\rangle = \sum_n \mathrm{e}^{i\theta n}|n\rangle, \quad H_\theta|\theta\rangle = E_{\theta,\min}|\theta\rangle$$

Topology, θ-vacua, anomalies and condensates: general considerations

- Gauge invariance implies that θ = const., and different θ-vacua are orthogonal, $\langle \theta' | \theta \rangle = 0 \; [\theta' \neq \theta + 2\pi k]$

- Fock space of states $\qquad \mathcal{F} = \coprod_{\theta \in \mathbf{R}} \mathcal{F}_\theta$

- Generating functional

$$Z[J] = \langle \theta | \theta \rangle_J = \int \mathcal{D}A_\mu^{(\nu)} \exp\left(-S[A] - \int d^4x \, J_\mu A^\mu + \int d^4x \frac{i\theta}{16\pi^2} \mathrm{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- Quantum transition between different $|n\rangle$ vacua are 'mediated' by instantons   Belavin, Polyakov, Schwarz and Tyupkin '75

Topology, θ-vacua, anomalies and condensates: general considerations

- Add (spin-1/2) fermions:

$$Z = \int \mathcal{D}A_\mu^{(\nu)} \mathrm{Det}\left(\slashed{D}^{(\nu)} + M\right) \exp\left(-S[A] + \int d^4x \frac{i\theta}{16\pi^2} \mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

- Vacuum energy:

$$E(\theta) = -\frac{1}{V}\ln Z \geq 0 \ \left[\mathrm{Det}\left(\slashed{D} + M\right) \geq 0, \ S[A] \geq 0\right]$$

$$E(\theta = 0) = 0.$$

Vafa and Witten '84

- $\theta \neq 0$ is incompatible with S-matrix formulation of gravity

Dvali and Gomez '16

Topology, θ-vacua, anomalies and condensates: general considerations

- If the Dirac operator admits zero-mode solutions

$$\left(\slashed{D}^{(\nu)} + M \right) \Psi_0^{(\nu)} = 0$$

the vacuum transitions with $\nu \neq 0$ are halted and θ becomes unobservable

- Fermion zero-modes are mandated by the topological index theorem

Atiyah and Singer '63

$$\partial_\mu J_5^\mu = \frac{2N}{16\pi^2} \mathrm{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$Q_5(t = +\infty) - Q_5(t = -\infty) \equiv n_+ - n_- = 2N\nu$$

Topology, θ-vacua, anomalies and condensates: general considerations

- Anomalous U(1) symmetry must be spontaneously broken

$$\mathrm{e}^{i\alpha Q_5/2N} \in U(1), \quad \left[Q_5 = \int d^3\vec{x}(J_5^0 + 2K^0) \right]$$

- Action on the vacuum state:

$$\mathrm{e}^{i\alpha Q_5/2N}|\theta\rangle = |\theta + \alpha\rangle$$

$$\langle\theta|\theta + \alpha\rangle = 0 \implies Q_5|\theta\rangle \neq 0$$

Pseudo-Goldstone particle in the spectrum

3-form gauge theory formulation of θ-vacua

$$\theta \operatorname{Tr} F \tilde{F} \rightarrow \theta \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}$$

$$F_{\mu\nu\rho\sigma} = \partial_{[\mu} C_{\nu\rho\sigma]}, \quad C'_{\nu\rho\sigma} = C_{\mu\nu\rho} + \partial_\nu \omega_{\rho\sigma} + \partial_\sigma \omega_{\nu\rho} + \partial_\rho \omega_{\sigma\nu}$$

Gauge redundancy -> no propagating dof

- Topological susceptibility of vacuum

$$\chi = \text{F.T.} \langle \theta | \operatorname{Tr} F \tilde{F}(x), \operatorname{Tr} F \tilde{F}(0) | \theta \rangle_{p \rightarrow 0} = \begin{cases} \text{const.}, & \theta \text{ observable} \\ 0, & \theta \text{ unobservable} \end{cases}$$

$$\text{F.T.} \langle \theta | C_{\mu\nu\rho}(x), C_{\alpha\beta\gamma}(0) | \theta \rangle_{p \rightarrow 0} \propto \begin{cases} \frac{\rho(0)}{p^2} + ..., & \theta \text{ observable (Coulomb phase, Luscher pole)} \\ & \text{Luscher '78, Veneziano '79} \\ \frac{\rho(0)}{p^2 - m_\eta^2} + ..., & \theta \text{ unobservable (Higgs phase, propagating dof!)} \\ & \text{Dvali '15} \end{cases}$$
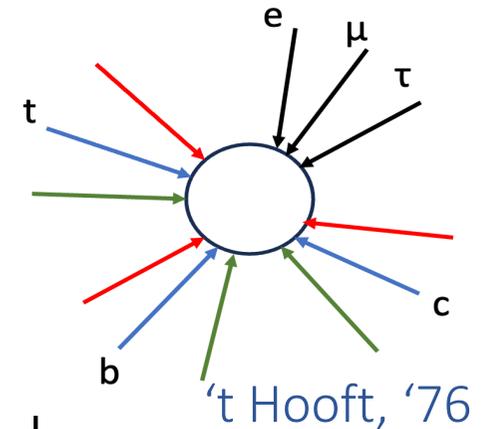
The electroweak η_w meson of the Standard Model

- In the Standard Model U(1)$_{B+L}$ is an exact symmetry in the classical approximation and explicitly broken by quantum anomaly

$$\partial_\mu J^\mu_{B+L} = -\frac{3}{16\pi^2} W^a_{\mu\nu} \tilde{W}^{a\mu\nu} + (\text{hypercharge})$$

$$\Delta Q_{B+l} = 3\nu$$

't Hooft, '76

- U(1)$_{B+L}$ must be broken also spontaneously, the order parameter being 't Hooft's local composite operator comprising of 12 fermionic (quark and lepton) operators. The phase field of the order-parameter is an emergent dof, η_w.

The electroweak η_w meson of the Standard Model

- Toy model: $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$, $\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$, $u_R,\ d_R,\ e_R, \nu_R +$ Higgs doublet

$$\psi = q_L + \ell_R^c \ , \quad \eta = \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \begin{pmatrix} e_L^c \\ -\nu_L^c \end{pmatrix} \ .$$

$$\Psi = (\psi, \eta)^{\mathrm{T}}$$

$$\mathcal{L}_F = \Psi^\dagger \hat{\mathcal{D}} \Psi \ ,$$

$$\hat{\mathcal{D}} = \begin{pmatrix} -i\slashed{D} \ , & -iM_q P_R + i\epsilon M_\ell^* \epsilon P_L \\ -iM_q^\dagger P_L + i\epsilon M_\ell^{\mathrm{T}} \epsilon P_R \ , & -i\slashed{\partial} \end{pmatrix}$$

$$M_q = \begin{pmatrix} y_u \phi^{0*} \ , & y_d \phi^+ \\ -y_u \phi^{+*} \ , & y_d \phi^0 \end{pmatrix} \ , \quad M_\ell = \begin{pmatrix} y_\nu \phi^{0*} \ , & y_e \phi^+ \\ -y_\nu \phi^{+*} \ , & y_e \phi^0 \end{pmatrix}$$

The electroweak η_w meson of the Standard Model

- (B+L) symmetry $\Psi \to e^{i\alpha\Gamma_5/2}\Psi$, $\Psi^\dagger \to \Psi^\dagger e^{i\alpha\Gamma_5/2}$, $\Gamma_5 = \begin{pmatrix} \gamma_5 , & 0 \\ 0 , & -\gamma_5 \end{pmatrix}$

- Despite the fermions are massive, the theory exhibits fermion zero-modes in full agreement with the index theorem

  Krasnikov, Rubakov and Tokarev, '79
  Anselm and Johansen, '93

- Propagator in the instanton vacuum:

$$\frac{1}{\hat{\mathcal{D}} + i\mu} = \frac{P_0}{i\mu} + \Delta - i\mu\Delta^2 + \mathcal{O}(\mu^2)$$

$$\langle x|P_0|x\rangle = \Psi_0^\dagger(x - z)\Psi_0(x - z)$$

The electroweak η_w meson of the Standard Model

- The instanton gas is an excellent approximation to the non-perturbative vacuum because of the Higgs vev provides a natural infrared cutoff:

$$\langle \Psi^\dagger(x)\Psi(x)\rangle = \lim_{\mu\to 0}\frac{1}{i\mu}\int\frac{d^4z\,d\rho}{\rho^5}\,D(\rho)\langle x|P_0|x\rangle \Bigg|_{\displaystyle D(\rho)=\left(\frac{2\pi}{\alpha_2(\rho)}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha_2(\rho)}-2\pi^2 v^2\rho^2}\,\mu\rho}$$

$$\simeq -iv^3\left(\frac{2\pi}{\alpha_2}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha_2}}$$

The electroweak η_w meson of the Standard Model

- Apply the Goldstone theorem with the anomaly contribution:

$$\delta(\Psi^+ \Gamma_5 \Psi) = 2i\Psi^+ \Psi$$

$$\int d^4x \left\langle \left(i\mu \Psi^+ \Gamma_5 \Psi - \frac{\alpha}{4\pi} W\tilde{W} \right)(x) \,,\; \Psi^+ \Gamma_5 \Psi(0) \right\rangle = i\langle \Psi^+ \Psi \rangle \neq 0.$$

$$\int d^4x \langle W\tilde{W}(x) \,,\; \Psi^+ \Gamma_5 \Psi \rangle_{p=0} \propto \langle \Psi^+ \Psi \rangle \implies \langle vac|W\tilde{W}|\eta\rangle = B(p) \neq 0$$

$$\langle \eta|\Psi^+ \Gamma_5 \Psi|vac\rangle = C(p) \neq 0$$

- Electroweak 3-form is Higgsed (hence θ_{ew} is unobservable):

$$FT\langle C^{(\mathrm{CS})}, C^{(\mathrm{CS})}\rangle = \frac{|B(0)|^2}{p^2 - m_\eta^2} + \ldots$$

Summary

- There are compelling theoretical evidence for the existence of (ultra)light particles

- They are manifestations of ultraviolet physics not accessible at colliders and reflect symmetries and topological features of the vacuum state

- There is a growing efforts to detect them through new generation of haloscopes, magnetometers and gravitational wave observatories

- Quantum gravity effects are important in shaping the ground state of the SM – the full solution of the strong CP problem likely requires the second companion axion

- The structure of the nonperturbative electroweak vacuum suggest the existence of light electroweak scalar state, the electroweak η_w (the counterpart of η' of QCD)