Gravitational waves from phase transitions during inflation

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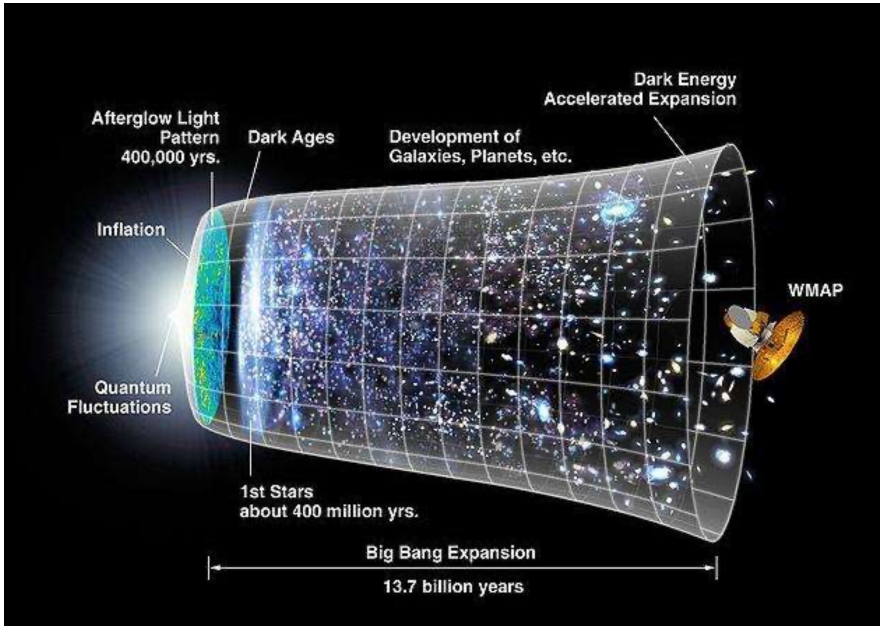
In collaboration with Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou

2009.12381, 2201.05171

Very brief introduction of inflation

1. Solves the causality problem
2. Solves the flatness problem
3. Solves the magnetic monopole problem
4. Generates the seed of large scale structure



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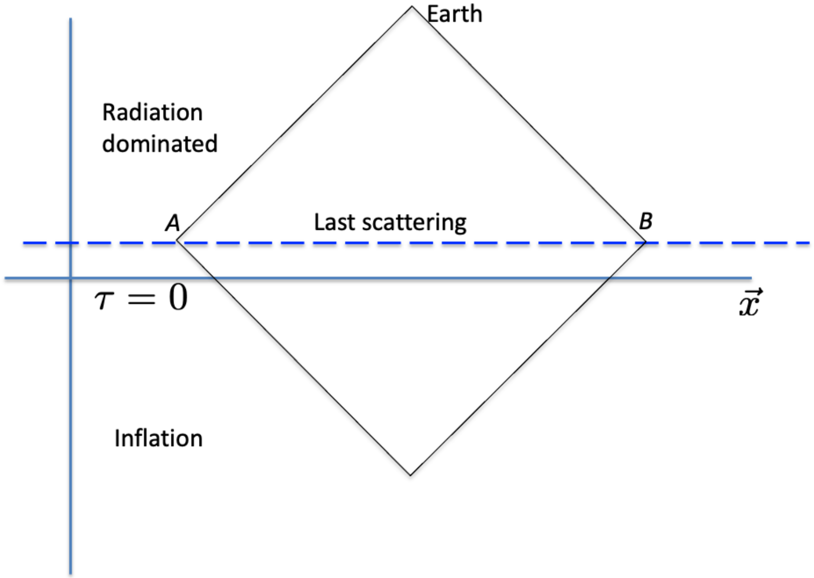
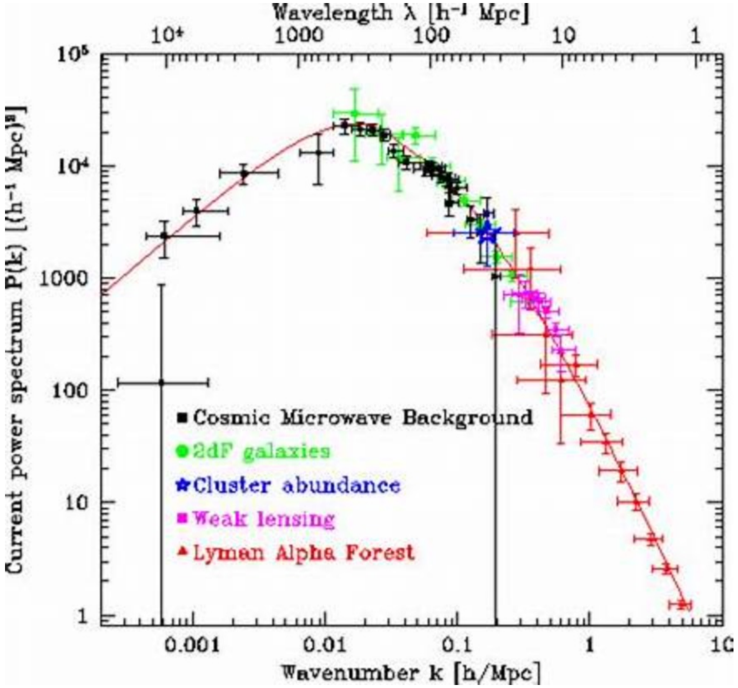
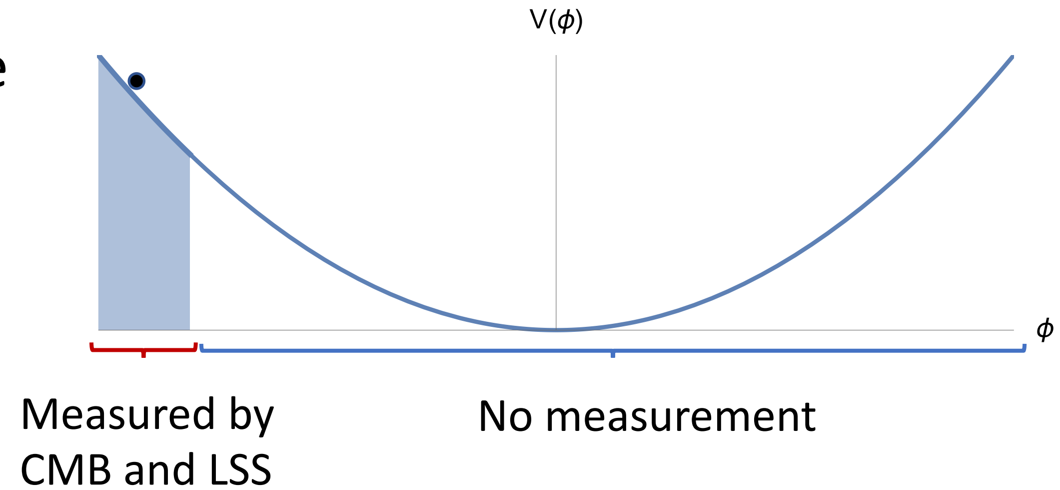




- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

Slow roll models

- We usually assume a potential.
- Use it to calculate n_s, r ...



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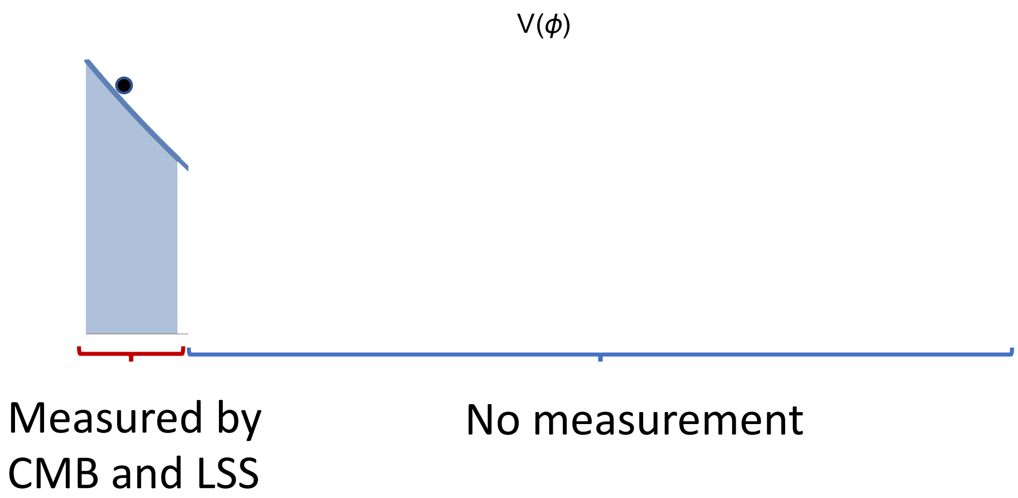

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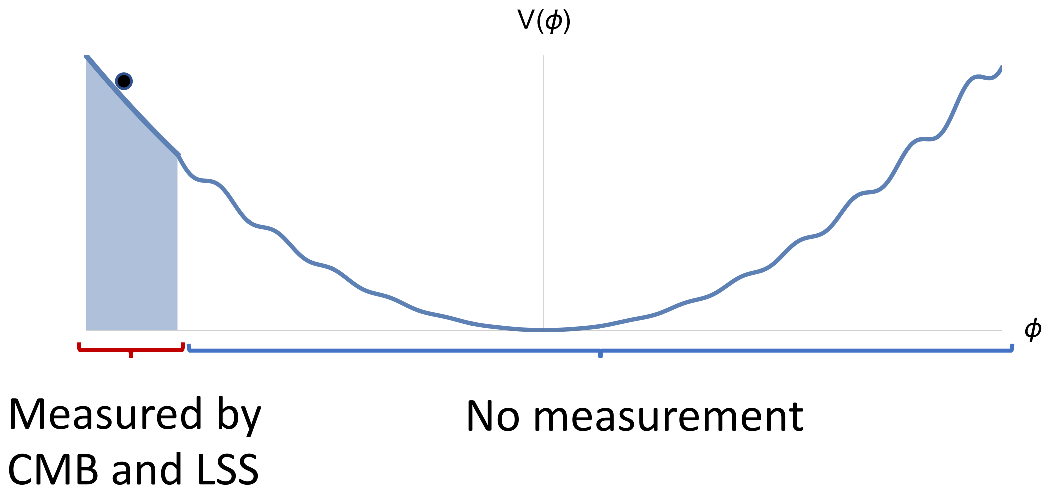

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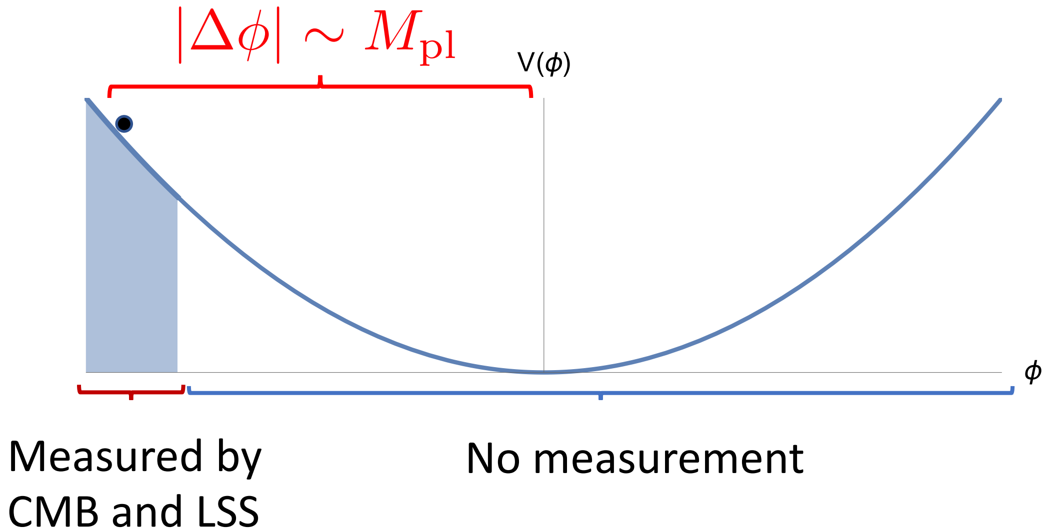
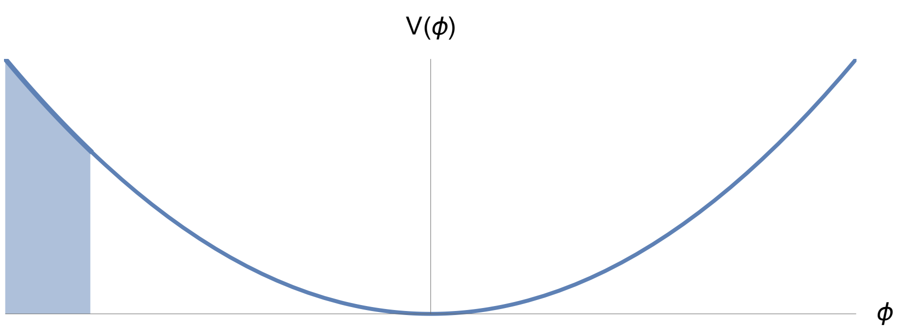


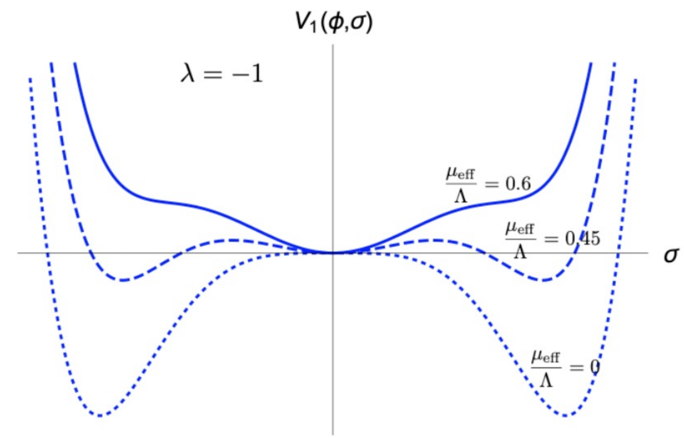
- The inflaton must couple to some spectator field.
- The masses or couplings in the spectator sector can be changed drastically due to the evolution of the inflaton field.

Phase transitions in the spectator sector

- ϕ: inflaton field σ: spectator field

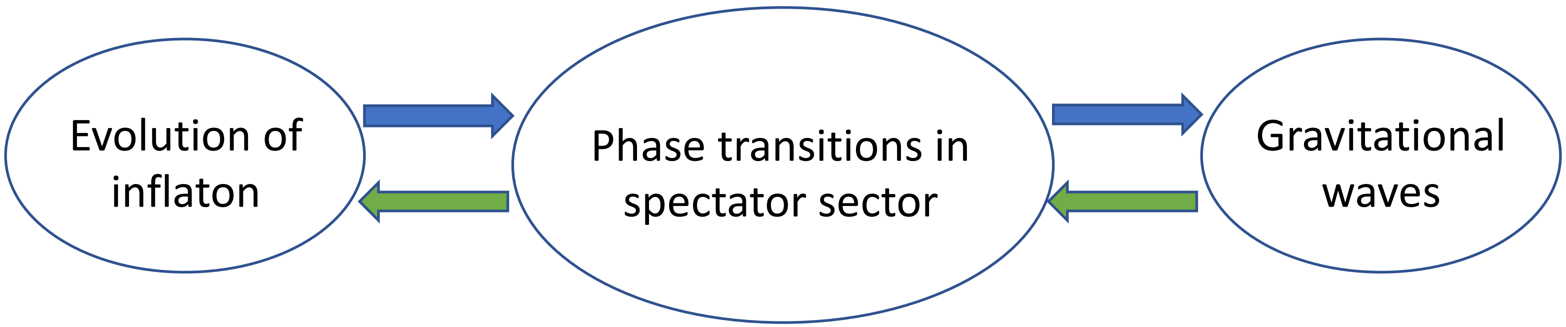
Example 1: $$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$





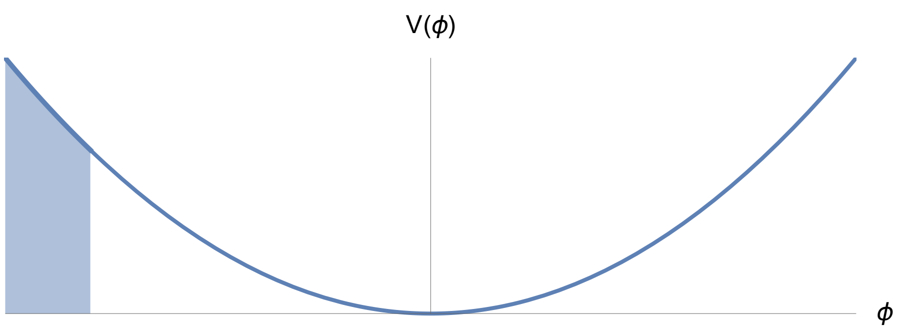
Example 2: $$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

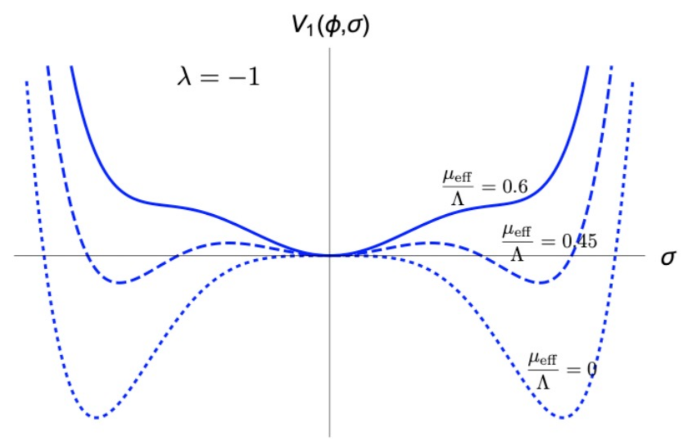
Phase transitions in the spectator sector



ϕ: inflaton field

σ: order parameter in the spectator sector





We focus on first-order phase transitions in this talk.

Models of first order phase transition during inflation

- Models in the literature:
  - Open inflation

    K. Sugimura, D. Yamauchi, M. Sasaki, 1110.4773
  - GUT phase transition at the beginning of inflation

    H. Jiang, T. Liu, S. Sun, Y. Wang, 1512.07538
  - Obtained the correct UV behavior of the GW spectrum

    Y.-T. Wang, Y. Cai, Y.-S. Piao, 1801.03639

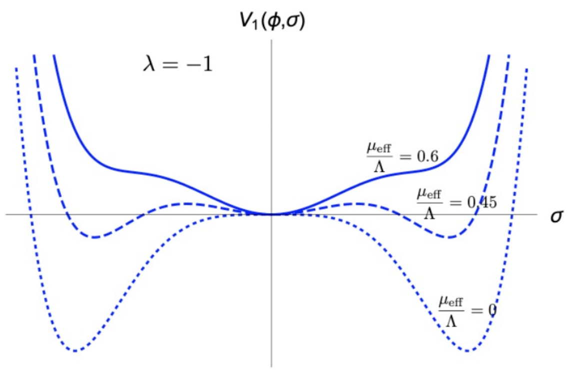
Outline

- Conditions for first-order phase transitions to complete during inflation
- Properties of GWs from first-order phase transition during inflation
- Possible detections
- Summary and outlook

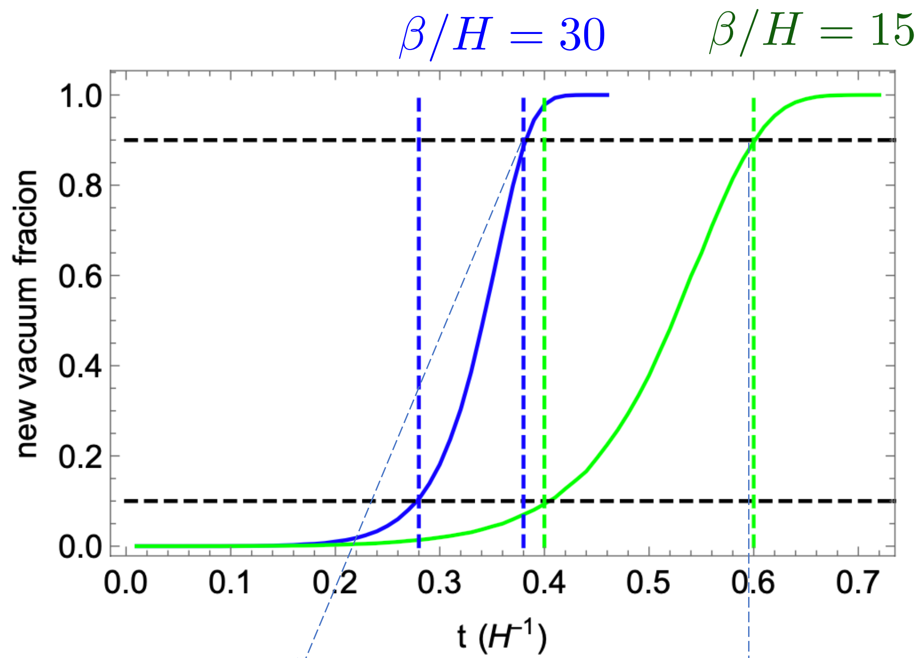
First-order phase transition during inflation

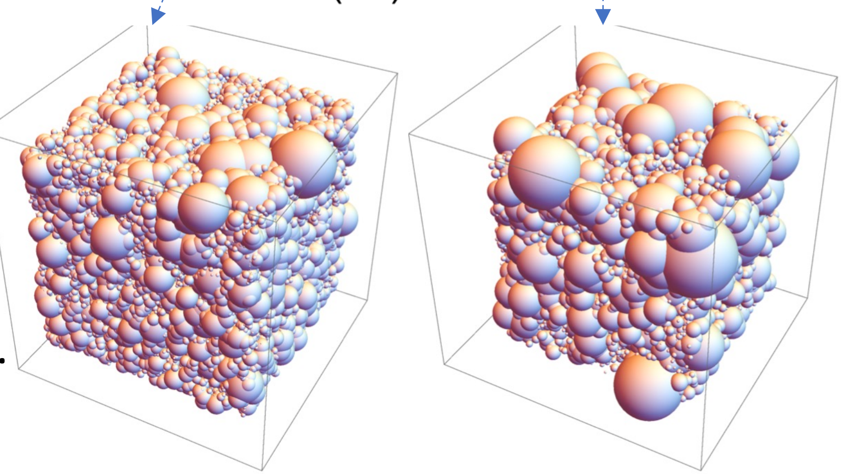
$$\frac{\Gamma}{V} = I_0 m_\sigma^4 e^{-S_4}$$

S_4 becomes smaller during



- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.

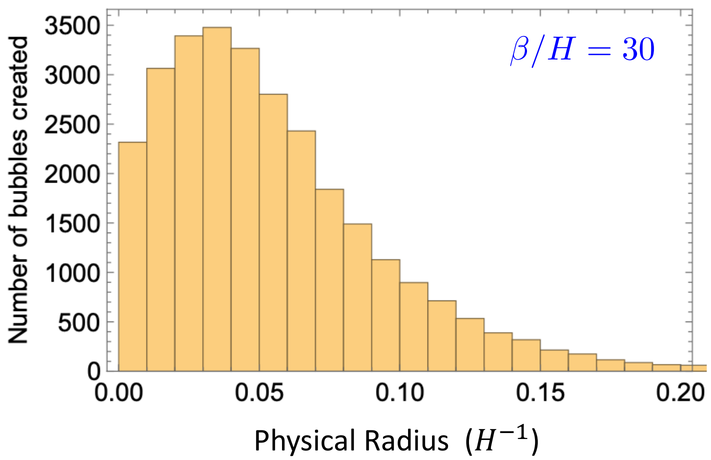


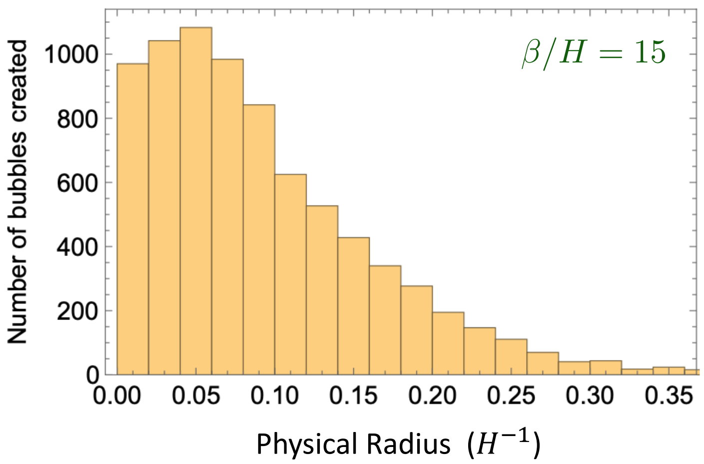


First-order phase transition during inflation

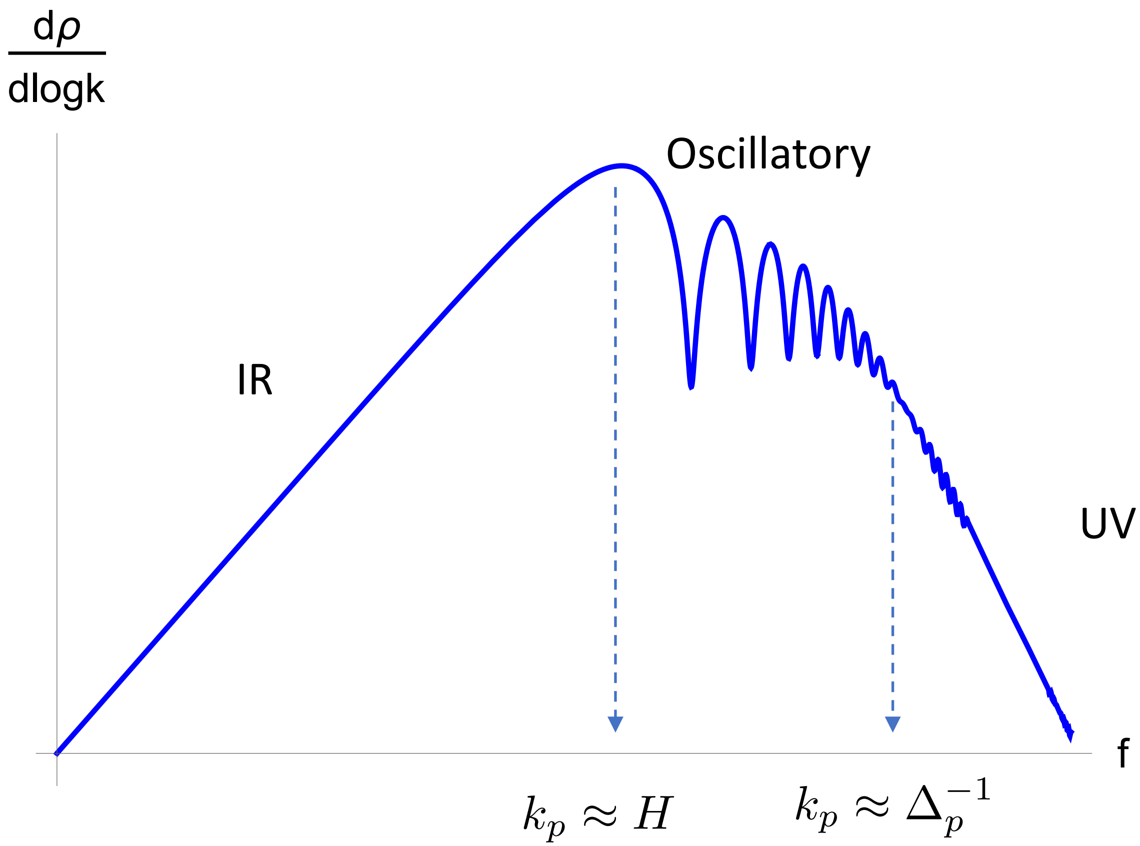
- Bubble radius also determined by β.

$$R_{\text{bubble}} \approx \beta^{-1} \ll H^{-1}$$





Generic features of GW spectrum



k_p: Physical momentum when it is produced.

Δ_p: Duration of the phase transition.

How to calculate GW?

- In E&M: $\partial_\mu F^{\mu\nu} = J^\nu$
  - We solve the Green's function first.
  - We convolute the Green's function with the source.

- In GR: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
  - We solve the Green's function first. (instantaneous and local source)
  - We convolute the Green's function with the source.

GW from instantaneous and local sources (qualitative study)

- E.O.M. of GW $$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

- For an instantaneous and local source, the source can be seen as delta funtion in both space and time.

$$\sigma_{ij} \sim \delta(\mathbf{x})\delta(\tau - \tau')$$

- E.O.M. in Fourier space

$$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$

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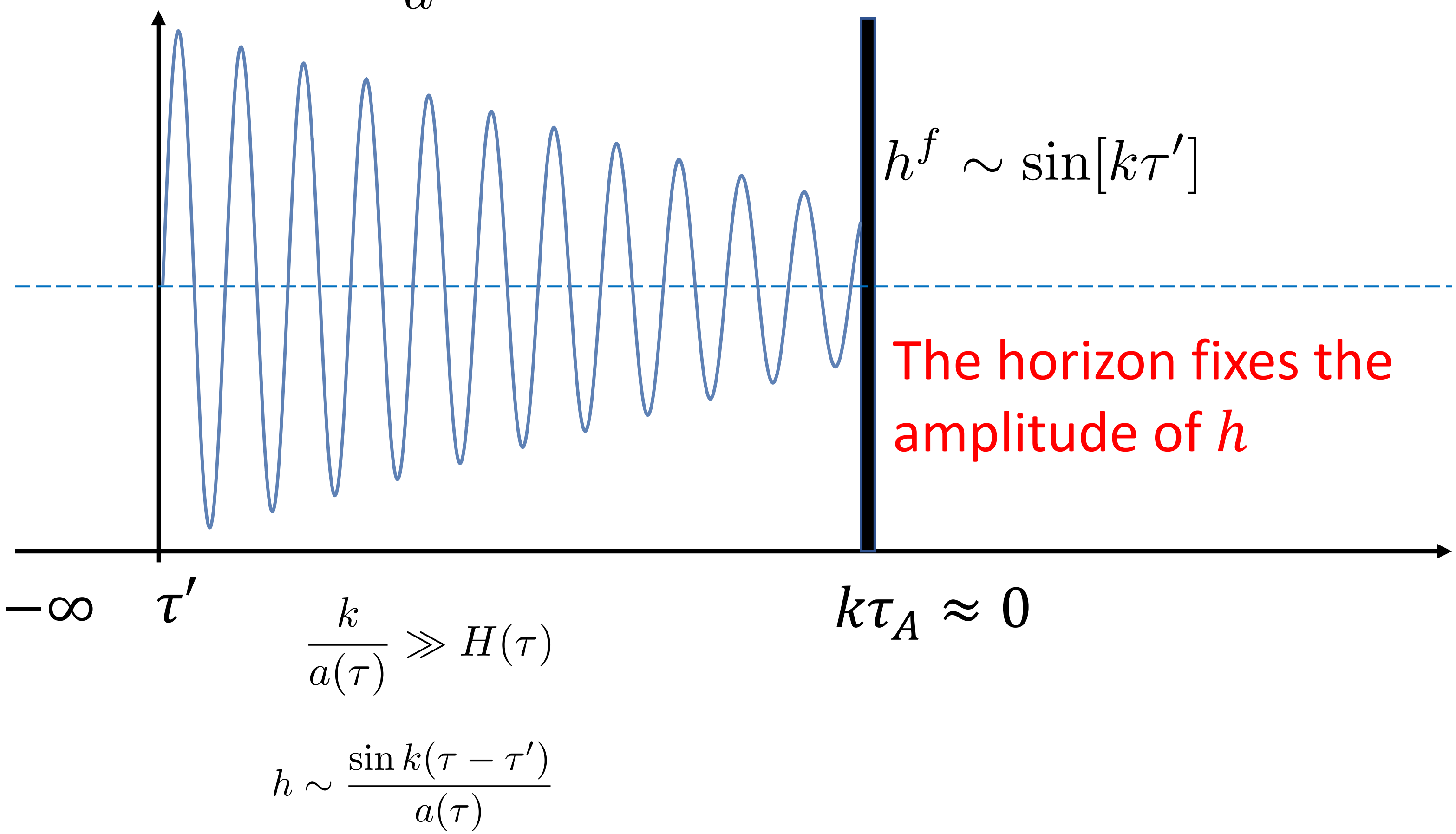

GW from instantaneous and local sources (qualitative study)

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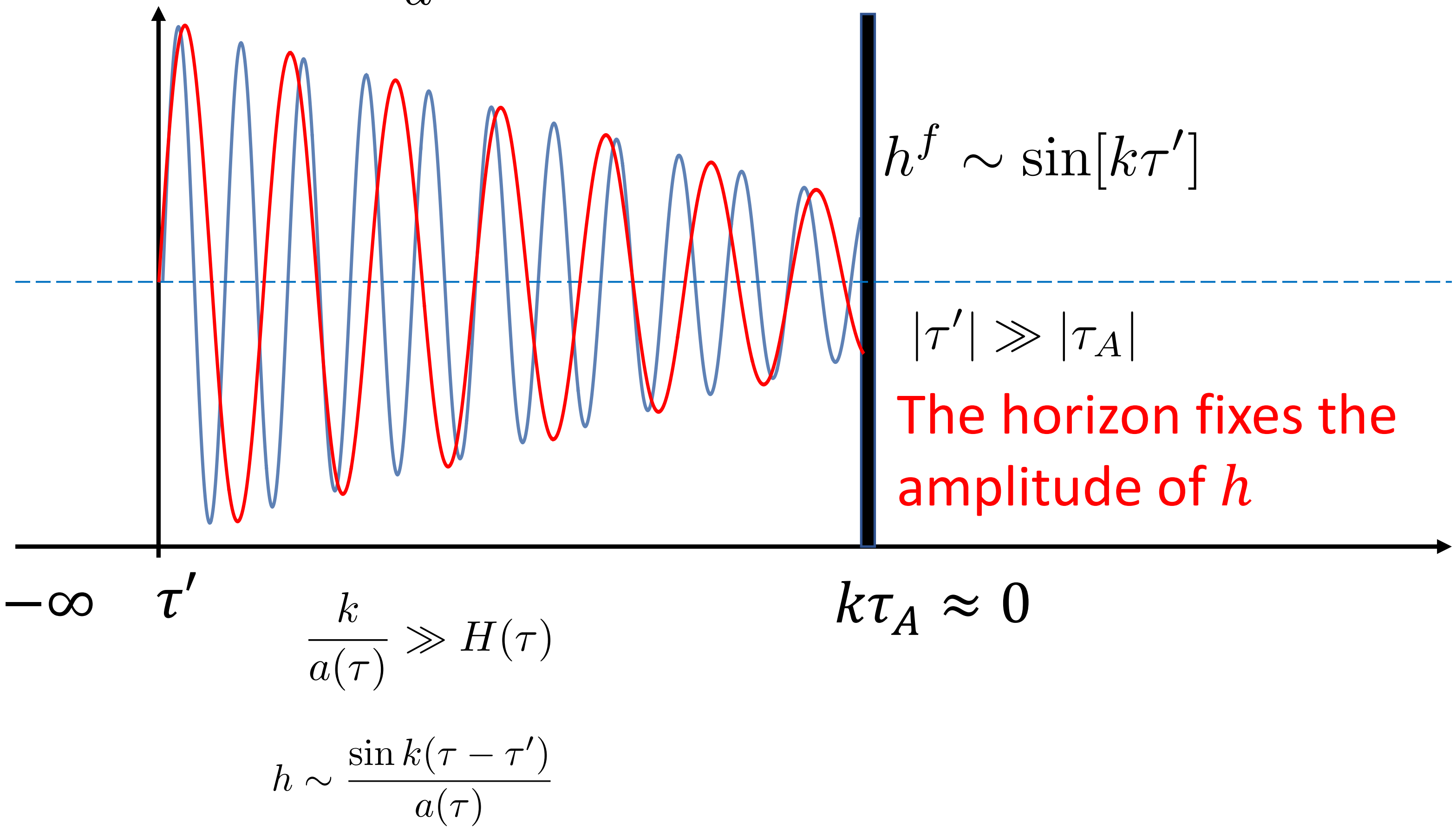
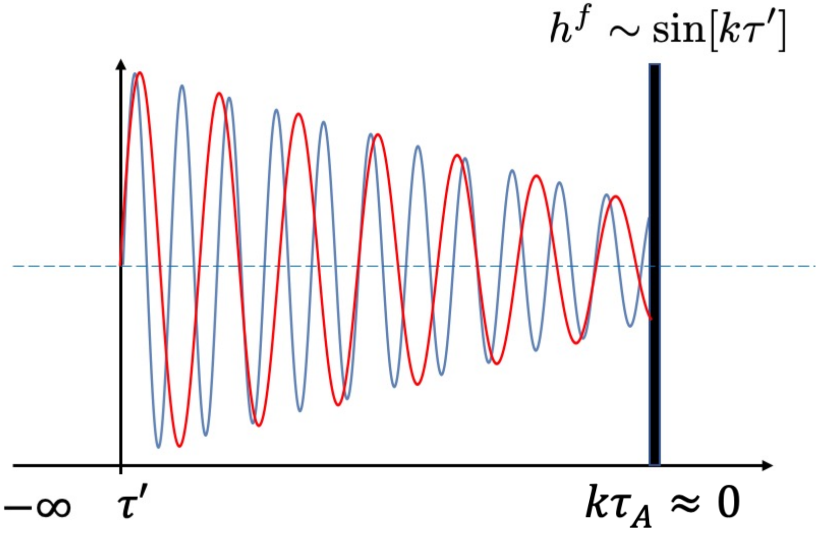


GW from instantaneous and local sources (qualitative study)

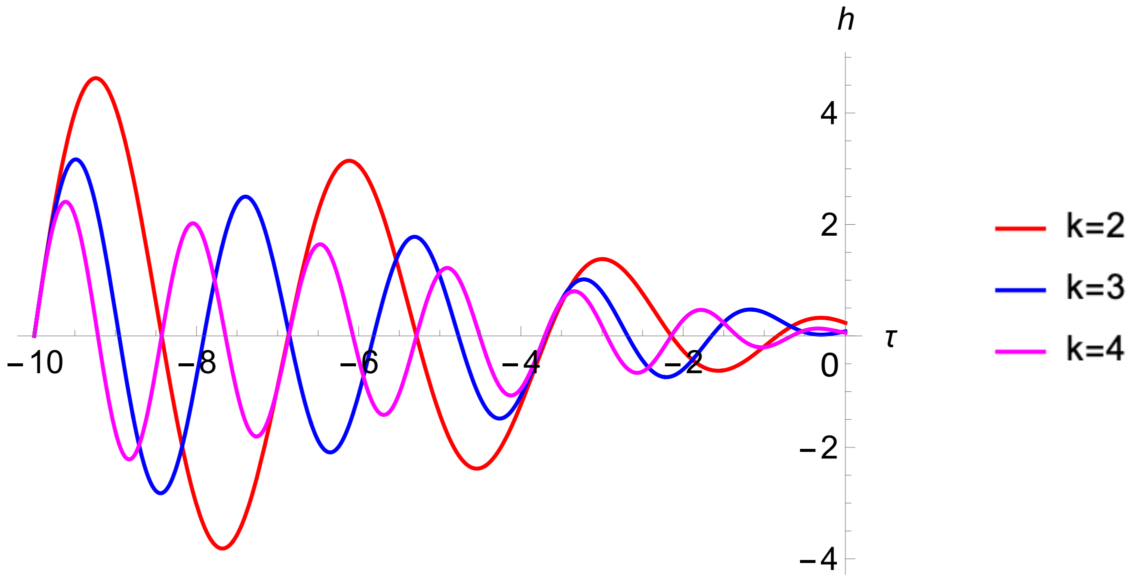
- The conformal time between the source and the horizon is fixed.
- The phase of h at the source is fixed.
- The value of h^f at the horizon oscillates with k.
- h^f is the initial condition for later evolution.



Quasi-de Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$

- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



De Sitter inflation as an example

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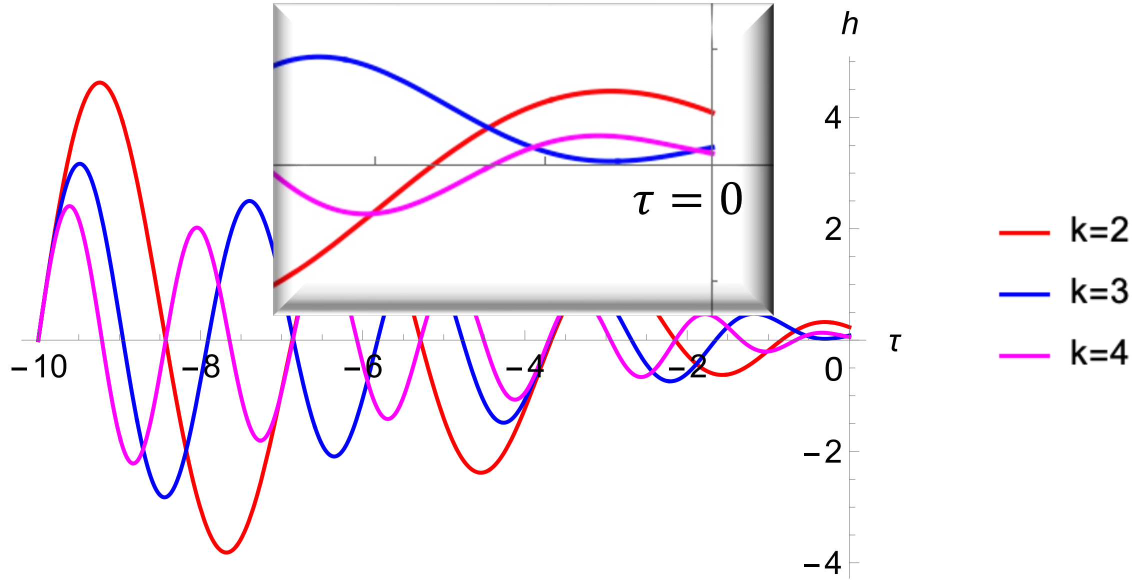

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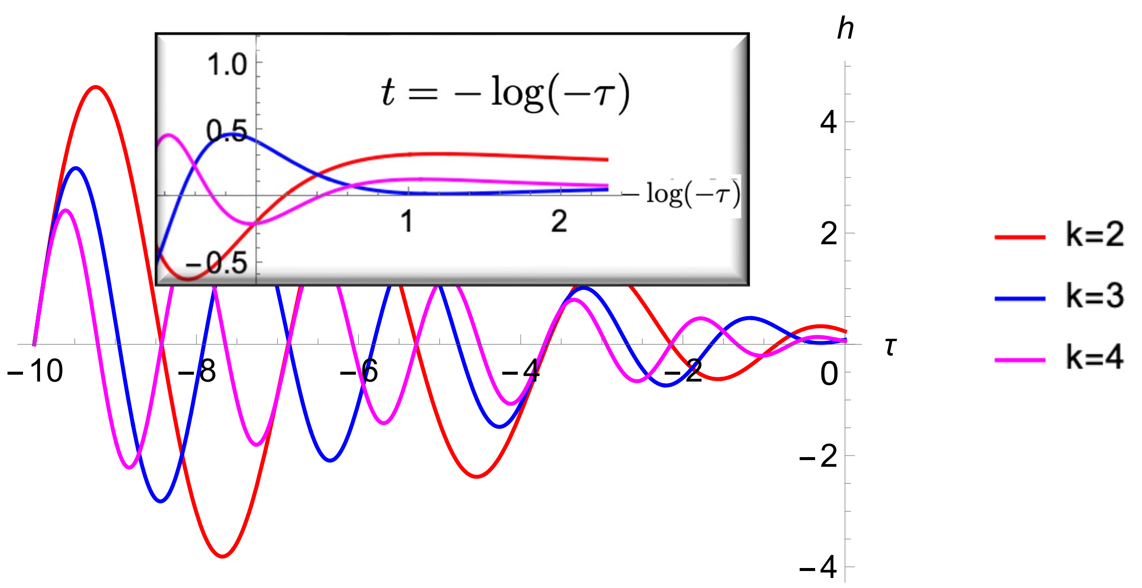
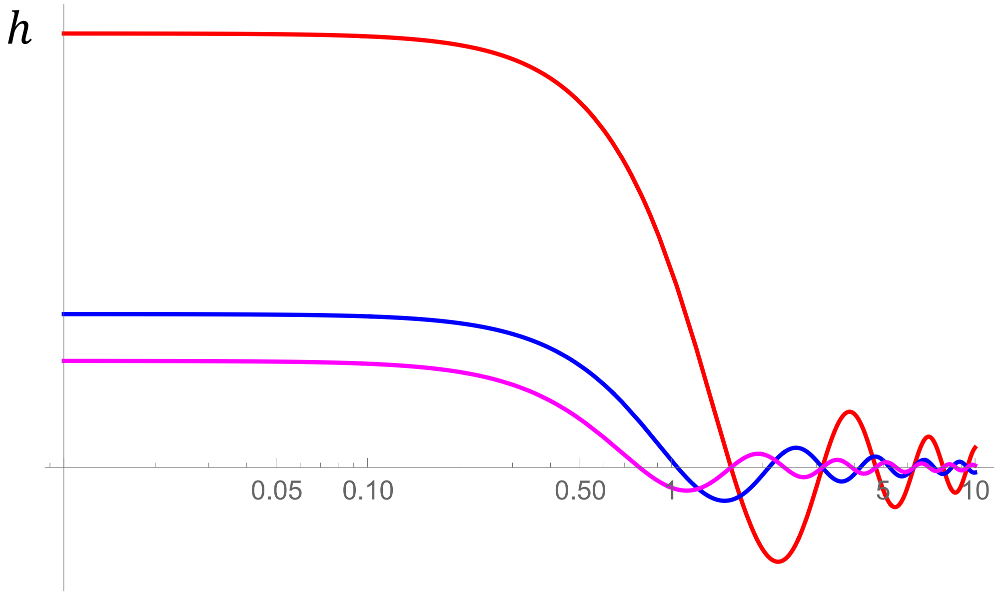


After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



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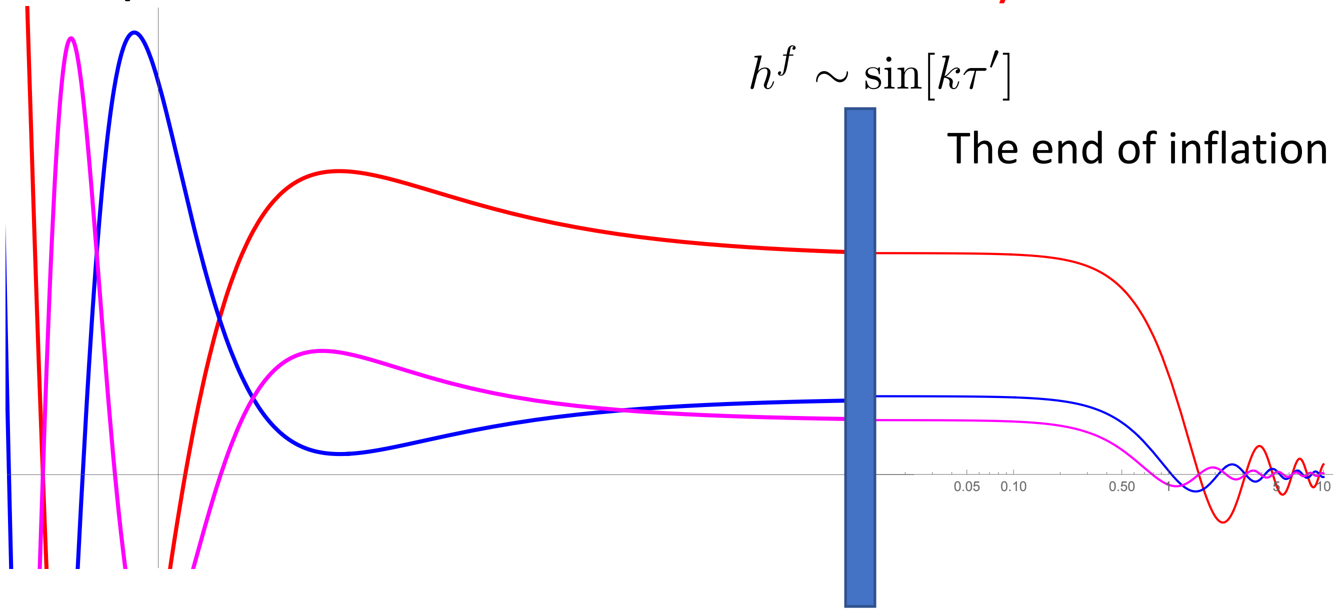


Spectrum of GW from a real source

- $|k\tau'| > \eta_A$ (the mode produced inside horizon)
  - At the end of inflation

$$\tilde{h}^f_{ij}(\mathbf{k}) = \frac{16\pi G_N \tilde{\mathcal{G}}^f_0(k)}{k} \int d\tau' \tilde{T}_{ij}(\tau', \mathbf{k_p}) \ \cos[k(\tau - \tau')]$$

  - After inflation (damped oscillation)

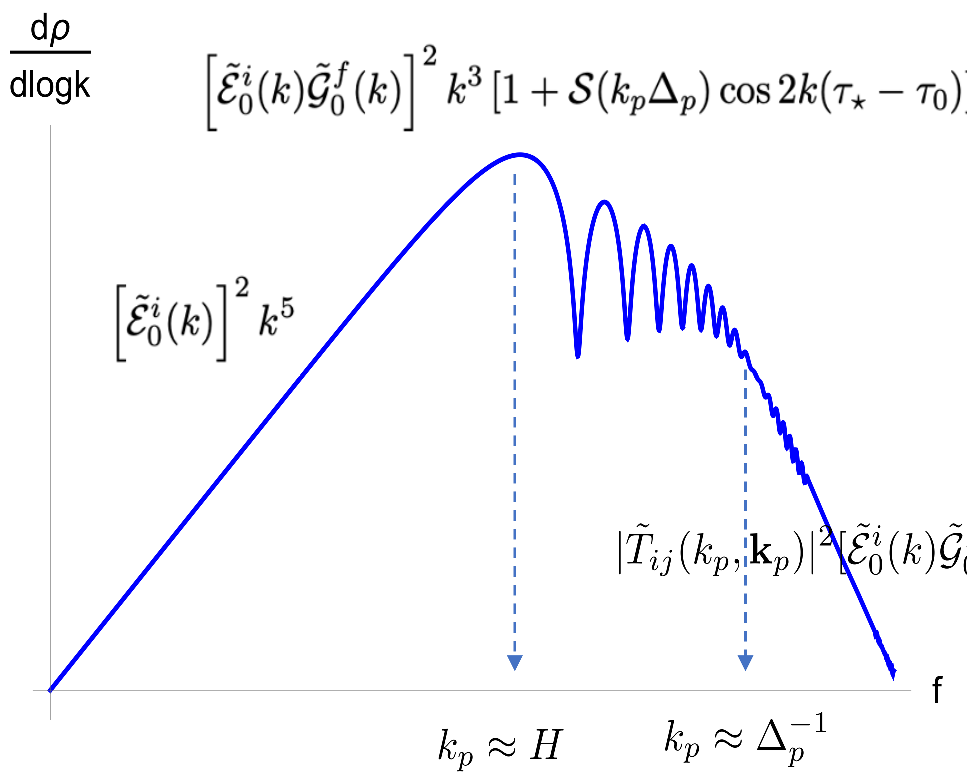
$$\tilde{h}_{ij}(\tau, \mathbf{k}) = \tilde{h}^f_{ij}(\mathbf{k}) \mathcal{E}(k\tau)$$

$$\mathcal{E}(\eta) = \tilde{\mathcal{E}}^i_0(k) a^{-1} \sin(\eta + \phi)$$

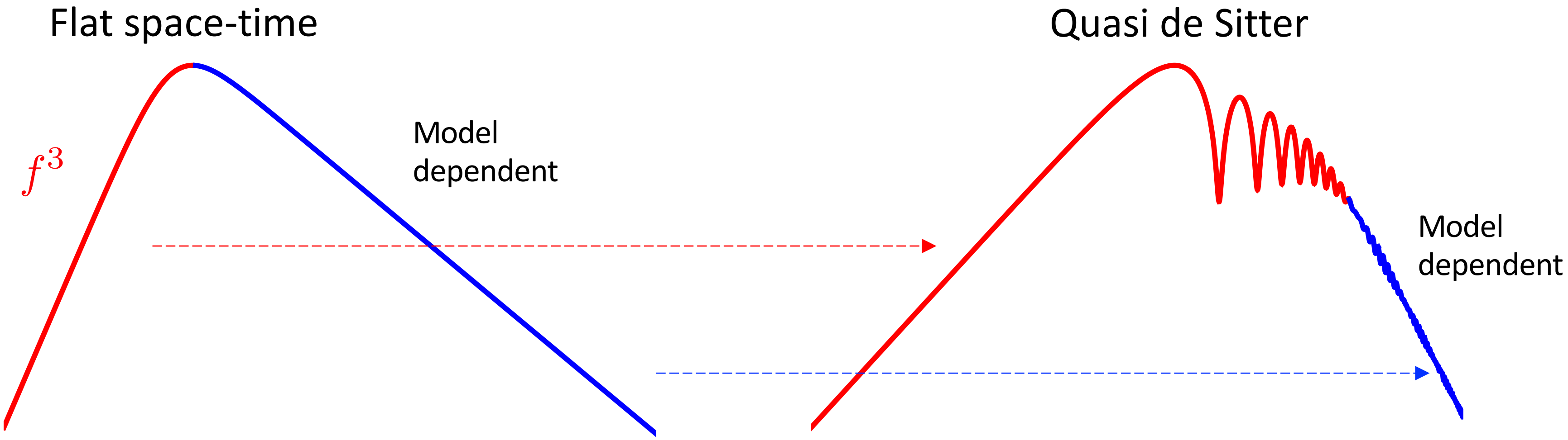
  - $\rho_{\rm GW} = \dfrac{1}{16\pi G_N a^2} \langle h'^2_{ij}(\tau, \mathbf{x}) \rangle$

Generic features of GW spectrum

- Inflation models
  - de Sitter inflation
    $$\tilde{\mathcal{G}}_0^f \sim \frac{1}{k}$$
  - t^p inflation
    $$\tilde{\mathcal{G}}_0^f \sim k^{\frac{p}{1-p}}$$
- Evolution after inflation
  - In RD, $\tilde{\mathcal{E}}_0^i \sim k^{-1}$
  - In MD, $\tilde{\mathcal{E}}_0^i \sim k^{-2}$
  - In $t^{\tilde{p}}$, $\tilde{\mathcal{E}}_0^i \sim k^{\tilde{p}/(\tilde{p}-1)}$

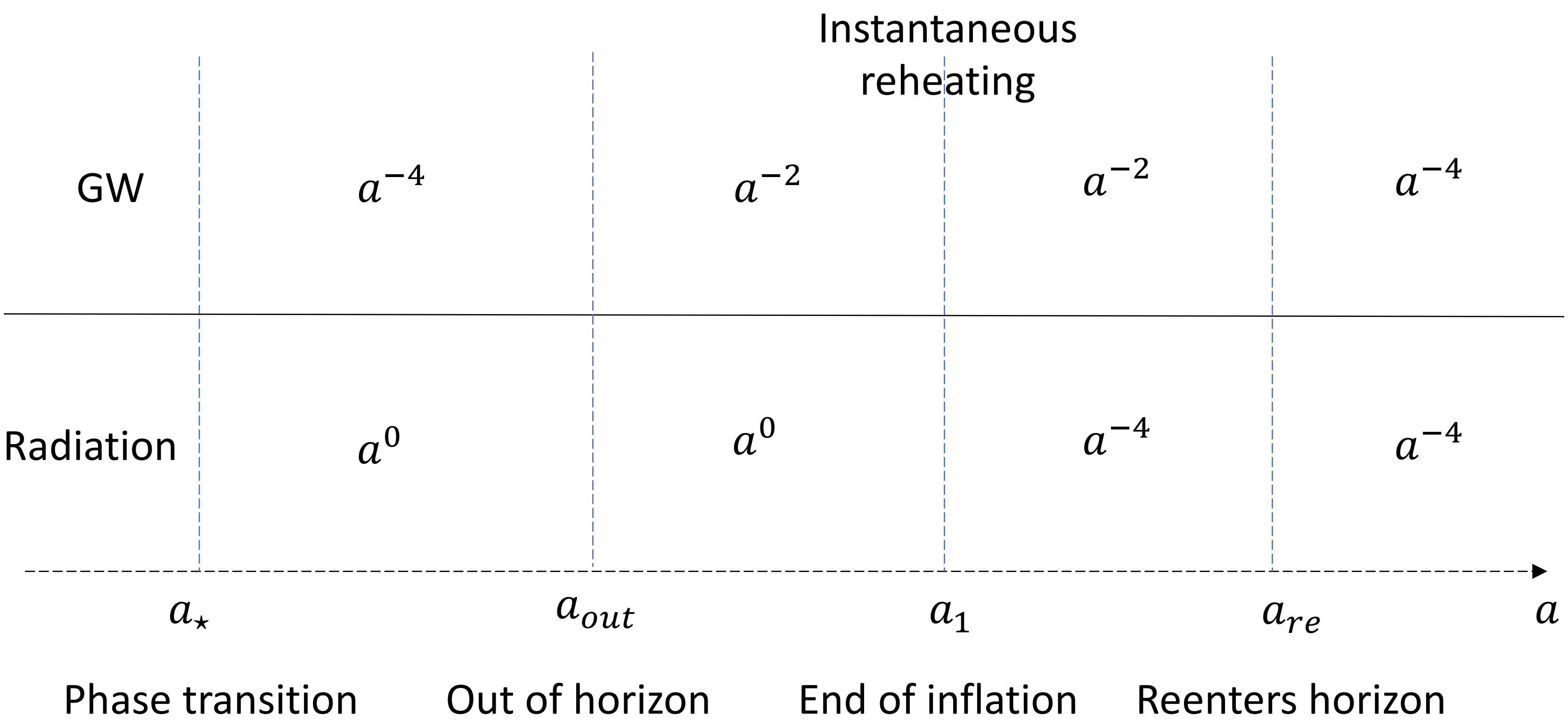


Spectrum distortion by inflation



Cai, Pi, Sasaki, 1909.13728

Redshifts of the GW signal



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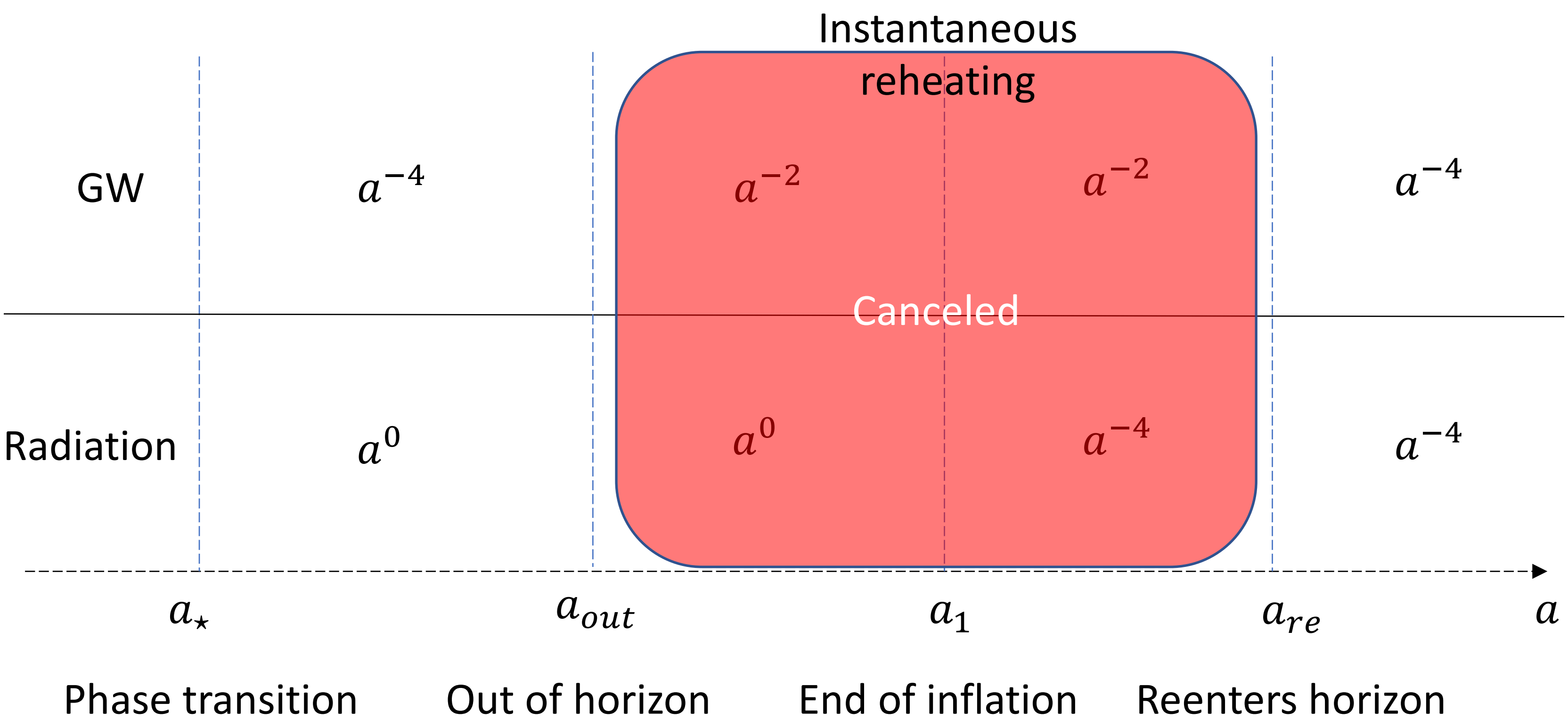
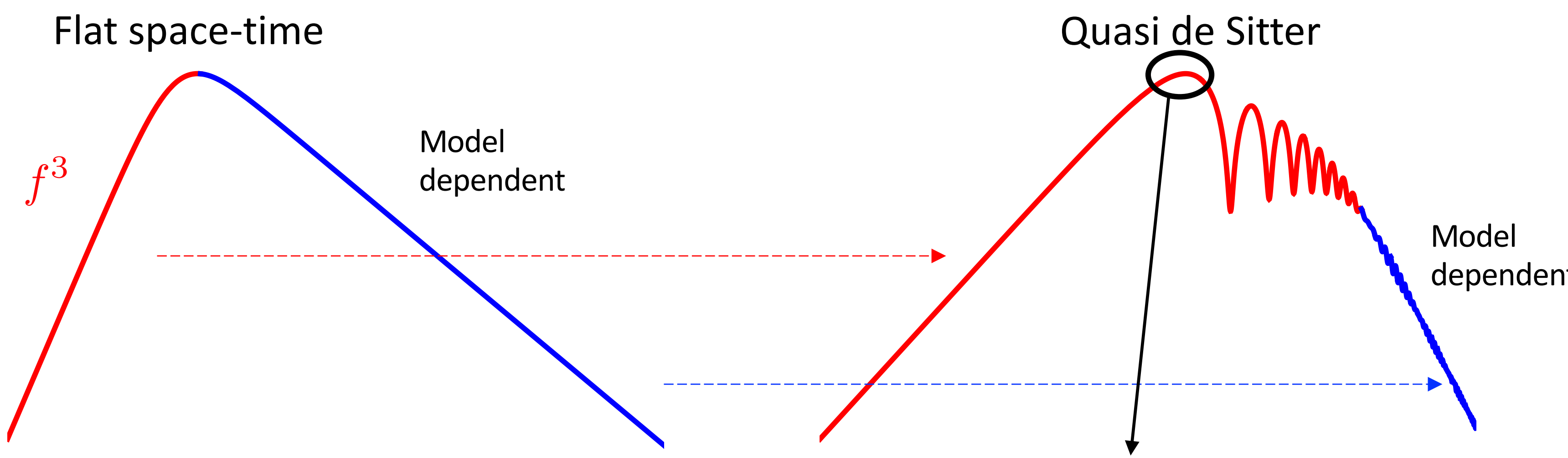


$$\frac{\Omega_{\rm GW}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\rm out}}\right)^4 \sim \left(\frac{H}{\beta}\right)^4$$

Spectrum distortion by inflation



$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-12} \times \left(\frac{H_{\rm inf}}{0.1\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-17} \times \left(\frac{H_{\rm inf}}{0.01\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

First order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \frac{H_{\rm inf}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right]\left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2 \frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac}\, d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction

Redshift

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

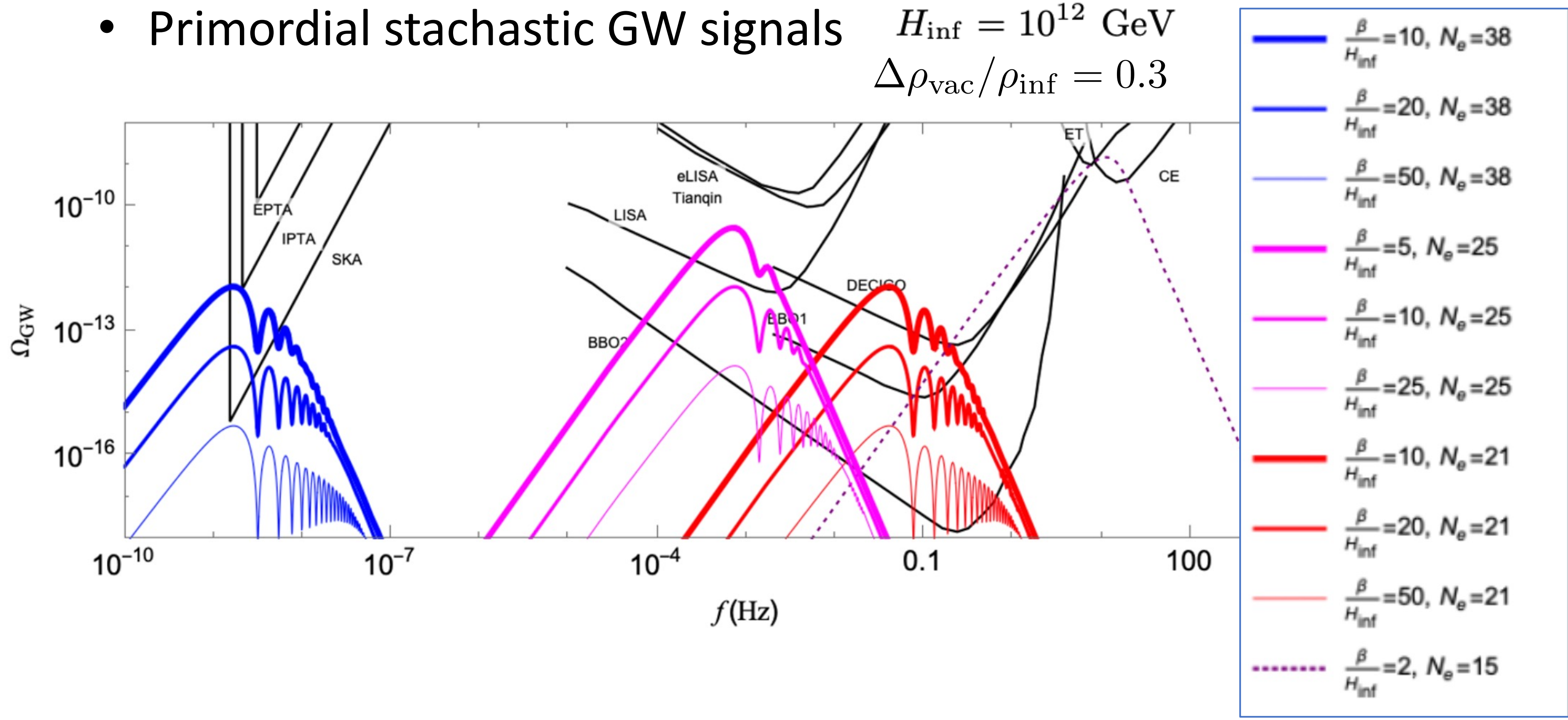
$$e^{-N_e}$$

N_e: e-folds before the end of inflation

First order phase transition during inflation

- Primordial stachastic GW signals

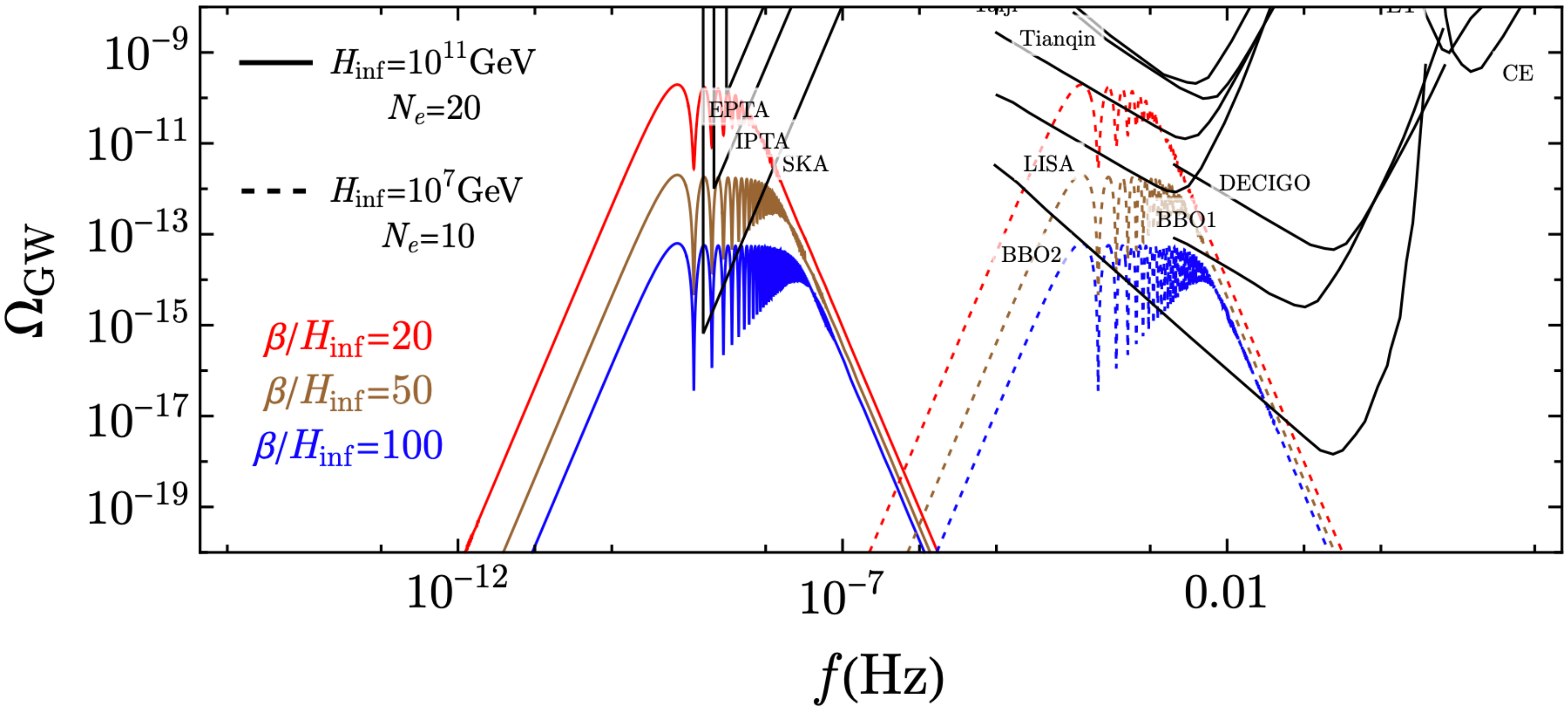
$$H_{\rm inf} = 10^{12}\ \mathrm{GeV}$$
$$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.3$$



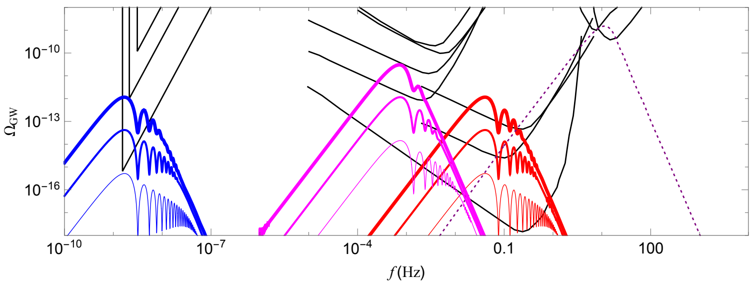
First order phase transition during inflation

- Signal strength is also sensitive to intermediate stages

Kination domination



Summary



- First-order phase transitions can happen in a spectator sector during inflation.
- We show that there is an oscillatory feature in the spectrum.
- The slopes of the spectrum can tell us information about the inflation model and evolution of the universe when the modes re-enter the horizon.
- If we are lucky enough, such a signal can be detected by future GW detectors.

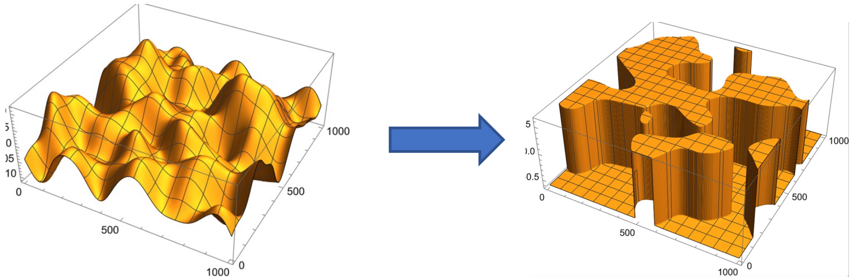
Outlook

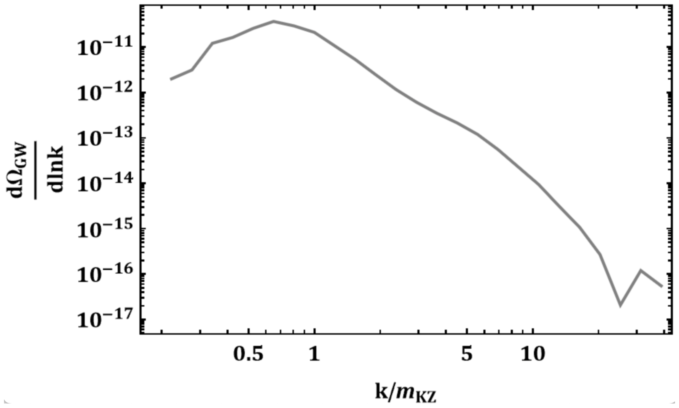
- Back reaction to the inflaton field $\sigma \to \delta\phi$
  - Primordial black holes from first-order phase transition during inflation
  - Secondary GWs from $\delta\phi$

with Boye Su, Hanwen Tai, Xi Tong, Liantao Wang, Chen Yang, Siyi Zhou

- GWs from second-order phase transition during inflation
  - Collision of pseudo-bubbles
  - Topological defects

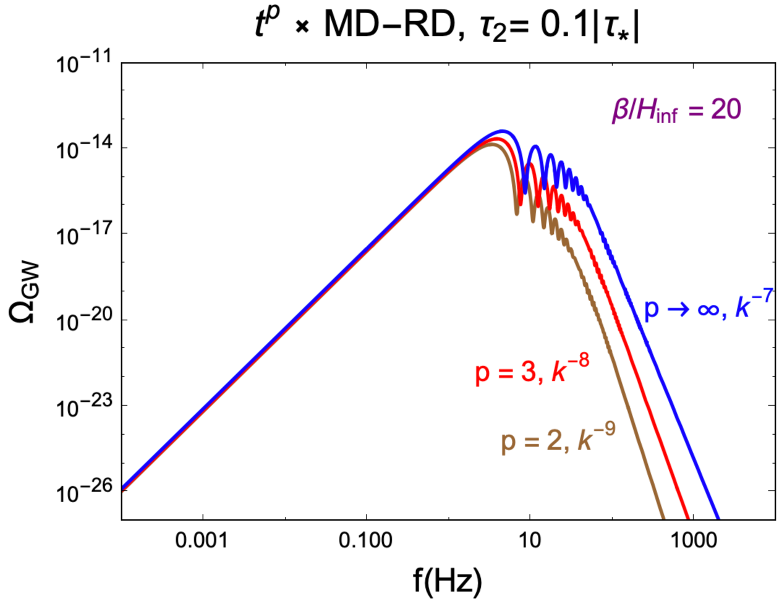
with Tingyu Li and Chen Yang

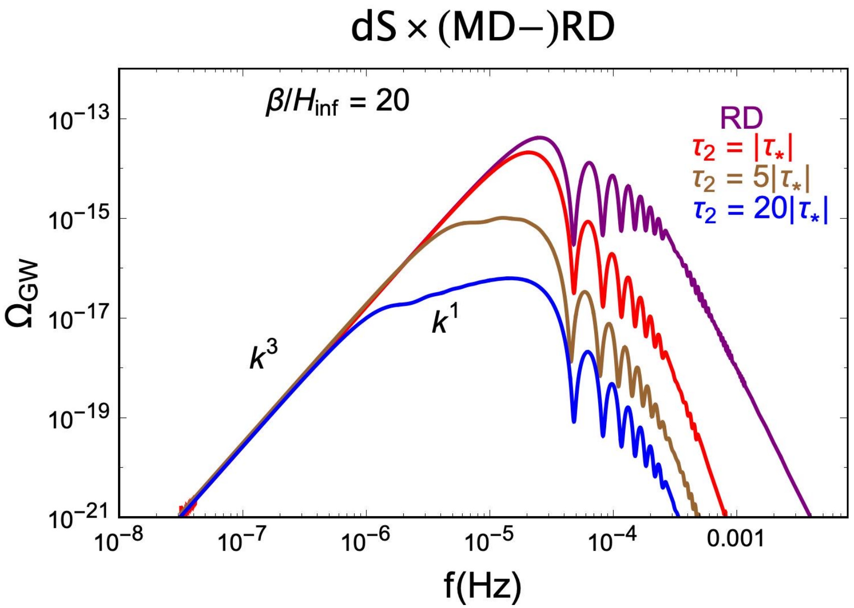




Backups

Comparing scenarios





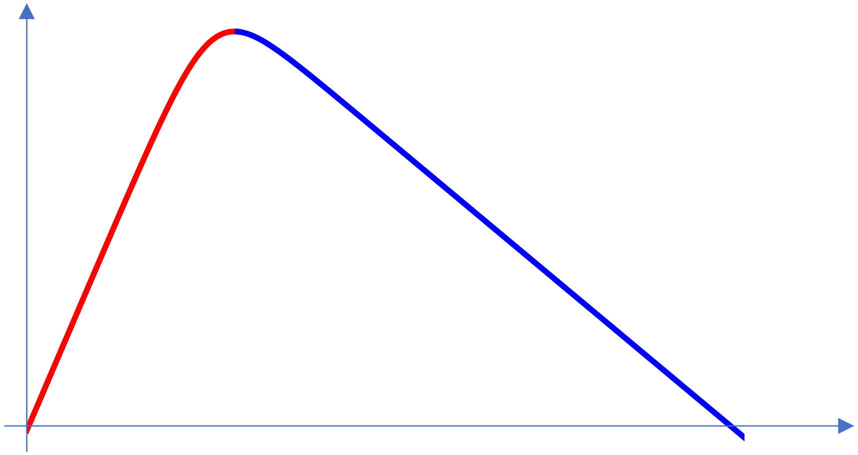
Different inflation scenarios

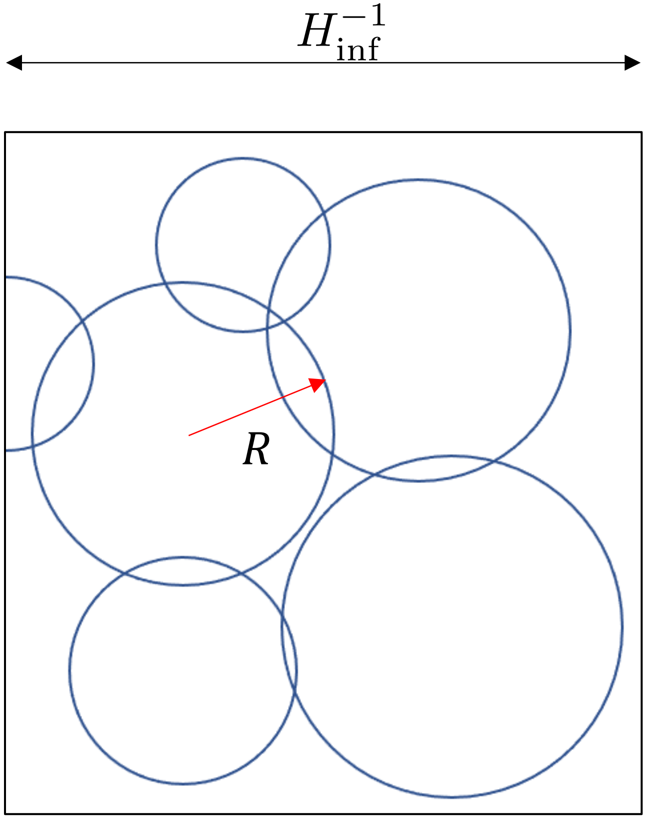
→ Different slopes in the UV and oscillatory parts

Temporary MD betwteen inflation and RD

τ_2: MD-RD transition

GWs produced in flat space-time



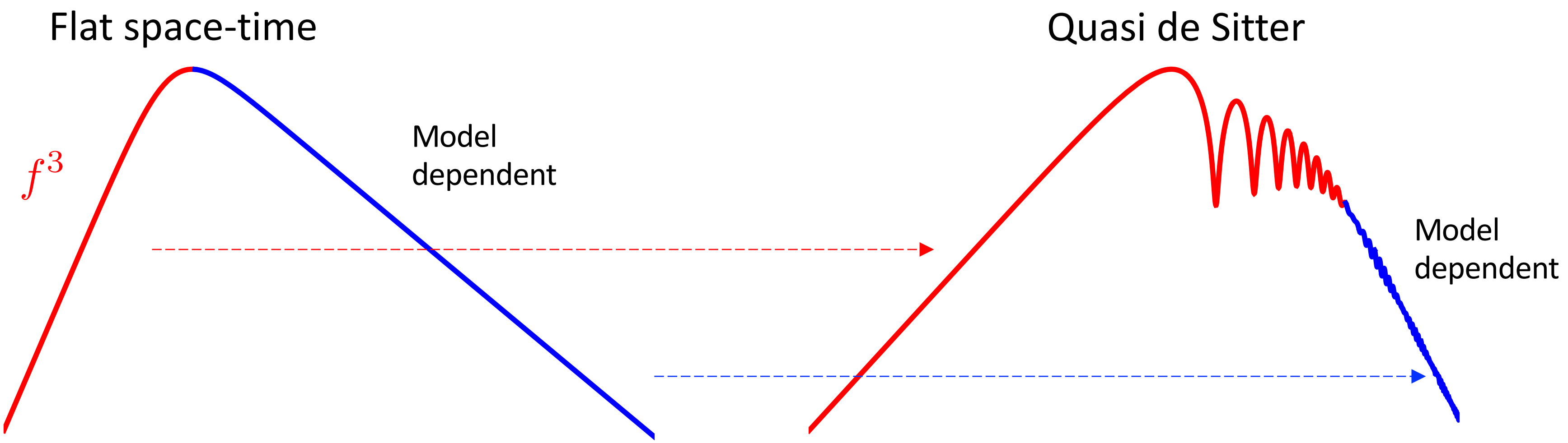


$$\frac{d\rho_{\rm GW}^{flat}}{\Delta\rho_{\rm vac} d\log k_p} \approx \left(\frac{H_{\rm inf}}{\beta}\right)^2 \times \frac{\beta\ k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$

Huber and Konstandin, 0806.1828

$$\Omega_{\rm GW}^{(0)} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^2 \frac{\beta k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$

Spectrum distortion by inflation



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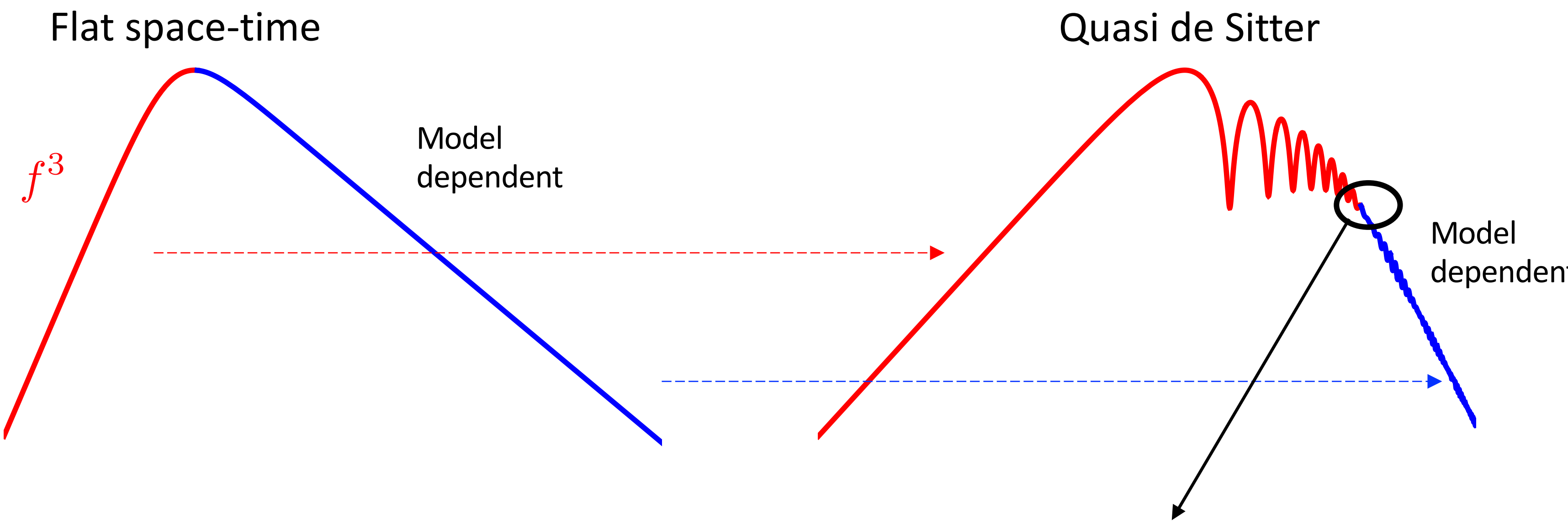
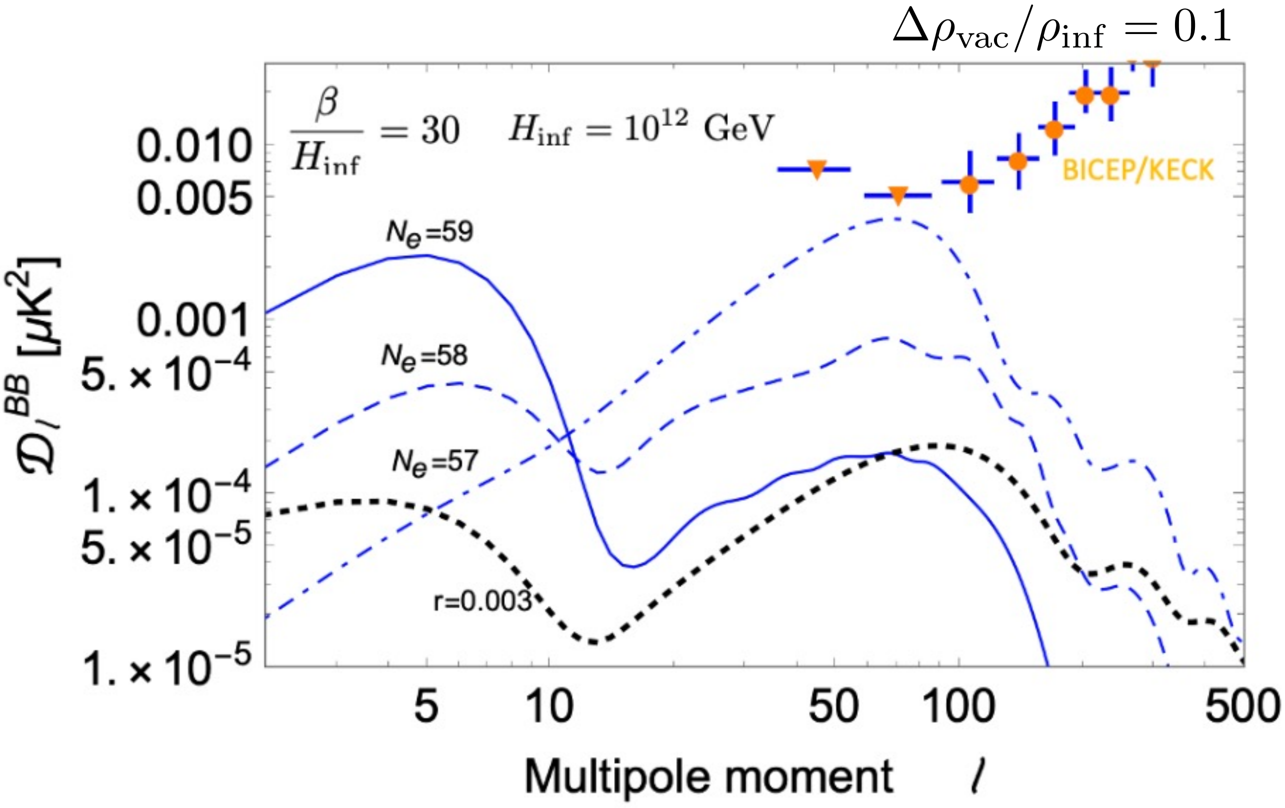


$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

First order phase transition during inflation

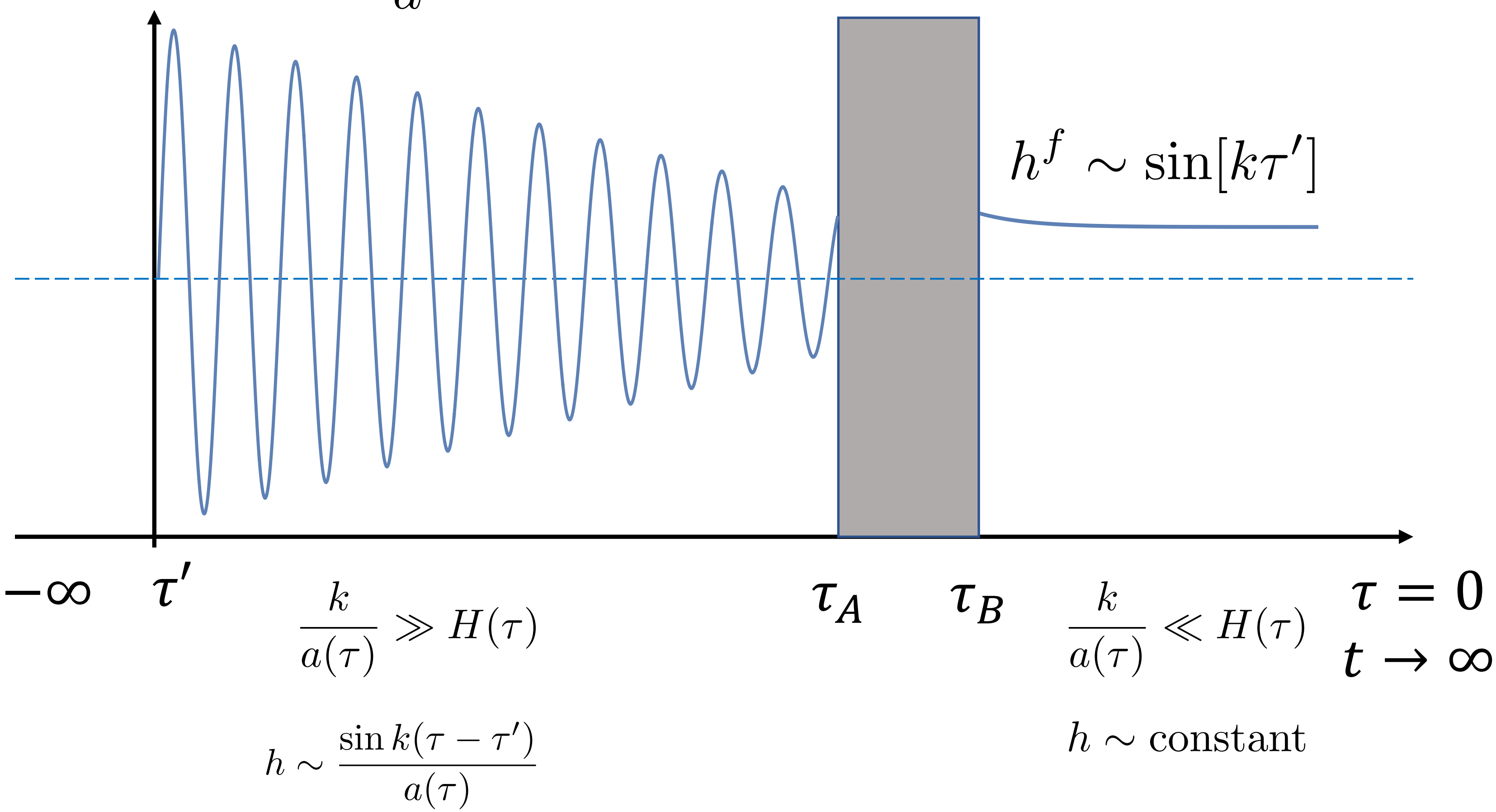
- CMB B modes



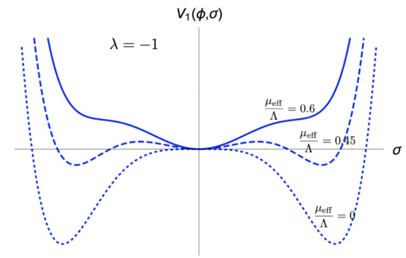
Simulation with the CLASS package.

GW from instantaneous and local sources (qualitative study)

- $h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$



First-order phase transition during inflation

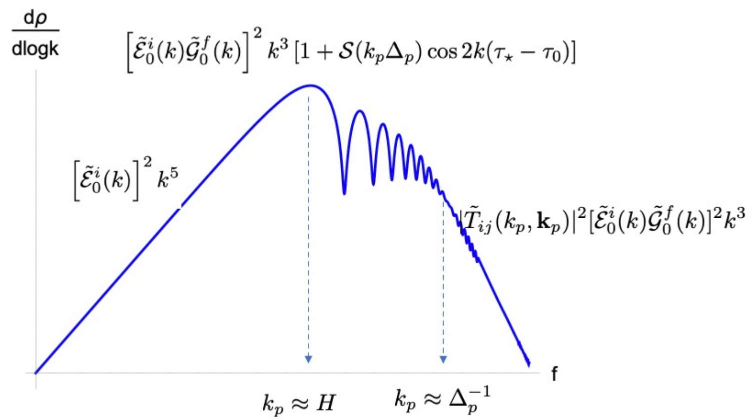


- Bubble nucleation rate: $\frac{\Gamma}{V} = I_0 m_\sigma^4 e^{-S_4}$
- Phase transition starts: $\mathcal{O}(1) = \int_{-\infty}^{t} dt' H^{-3} I_0 m_\sigma^4 e^{-S_4(t')}$
- The bounce: $S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_\sigma^4}{H^4}\right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm pl}} \frac{m_\sigma^4}{H^4}\right)$
- First order phase transiton: $S_4 \gg 1$ ➡ $H^4 \ll m_\sigma^4$
- Total energy density dominated by the inflaton sector:

$$m_\sigma^4 \ll 3 M_{\rm pl}^2 H^2$$

$$\boxed{H^4 \ll m_\sigma^4 \ll 3 M_{\rm pl}^2 H^2}$$

Summary

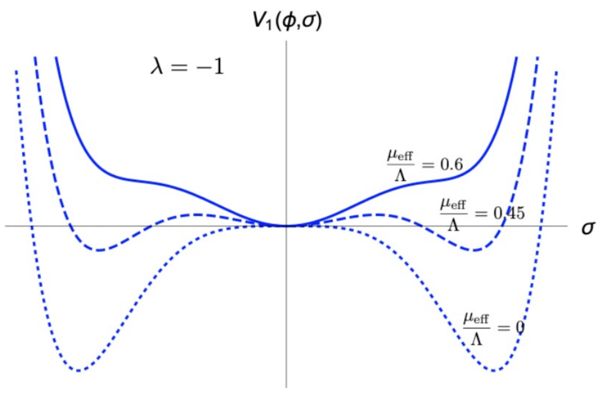


- We study the features of classical GWs produced from instantaneous sources during inflation.
- We show that there is an oscillatory feature in the spectrum.
- The slopes of the spectrum can tell us information about the inflation model and evolution of the universe when the modes re-enter the horizon.
- First order phase transition during inflation can be realized with simple models.
- If we are lucky enough, such a signal can be detected by future GW detectors.

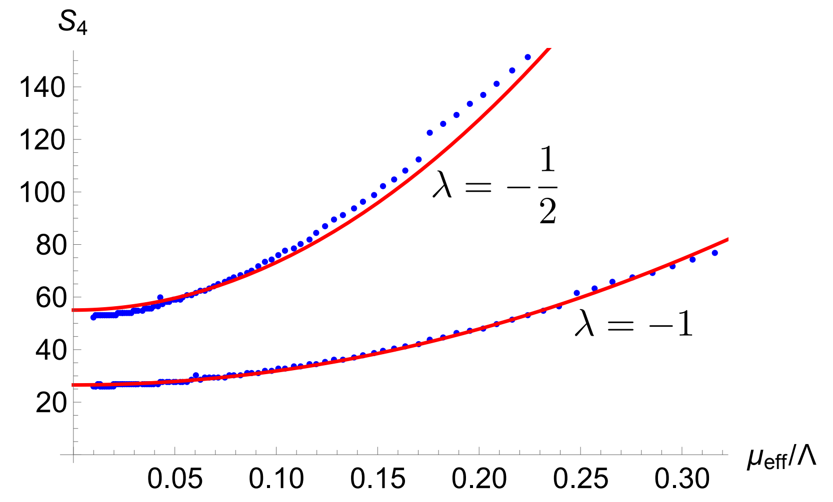
First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot\phi}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right| \qquad \mu_{\rm eff}^2 = -(\mu^2 - c^2\phi^2)$$

$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$



$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



CosmoTransitions

First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot{\phi}}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right| \qquad \mu_{\rm eff}^2 = -(\mu^2 - c^2\phi^2)$$

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$$\int_{\phi_{\rm end}}^{\phi_{\rm PT}} \frac{d\phi}{\sqrt{2\epsilon}M_{\rm pl}} = N_{\rm e}$$

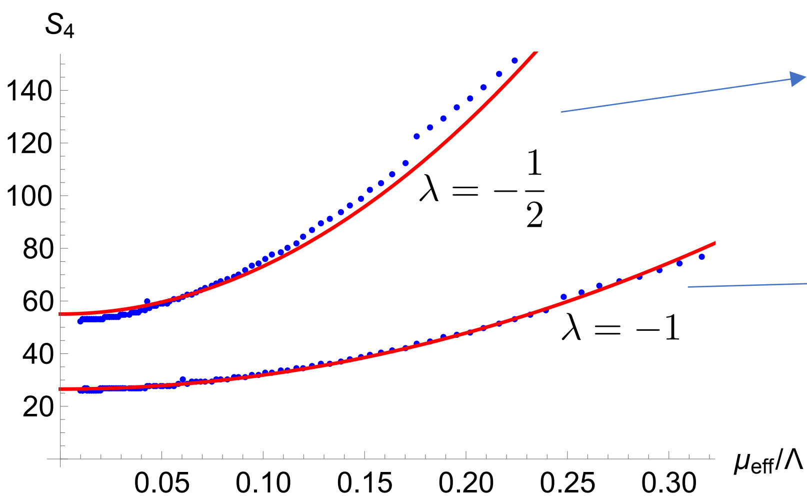
$$\sim \mu_{\rm eff}^2/\Lambda^2$$

$$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$

N_e: e-fold before the end of the inflation.

First order phase transition during inflation

- $$\frac{\beta}{H} \sim \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$



CosmoTransitions

$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

N_e: e-folds before the end of the inflation

$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

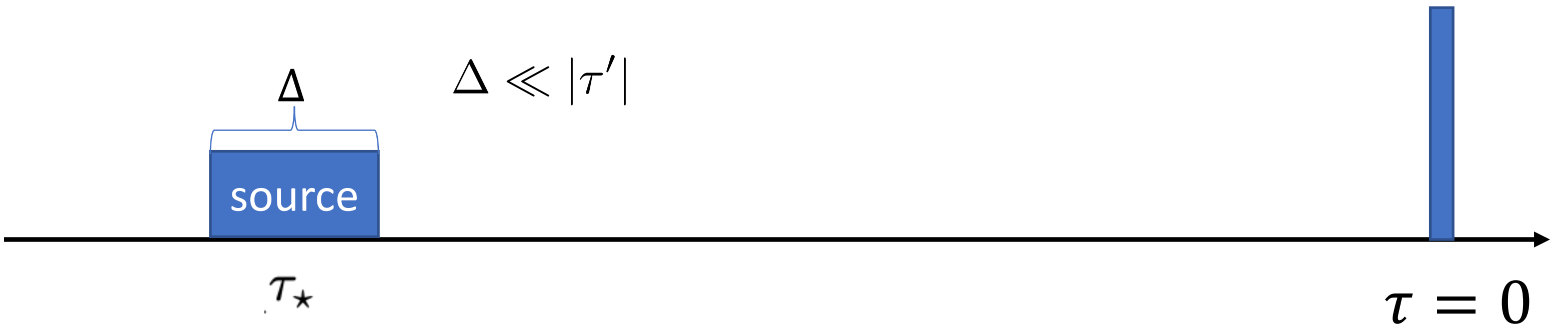
Outline

- Motivations
- GWs from an instantaneous source during inflation
- GWs from a source with finite duration during inflation
- GWs from first order phase transition during inflation
- Summary

Generic features of GW spectrum

- Instantaneous source

$$\Delta \ll |\tau'|$$



$$k\tau' = k\tau_\star + k_p t_p$$

$$k_p = k a^{-1}(\tau_\star) \qquad t_p = a(\tau_\star)(\tau' - \tau_\star)$$

$$\begin{aligned}\cos k(\tau' - \tau_0') &= \cos[k(\tau_\star - \tau_0) + k_p t_p] \\ &= \cos[k(\tau_\star - \tau_0)] \cos k_p t_p - \sin[k(\tau_\star - \tau_0)] \sin k_p t_p\end{aligned}$$

Generic features of GW spectrum

- $k_p \ll \Delta_p^{-1}$ $\quad \cos k_p t_p \to 1\ ,\ \ \sin k_p t_p \to 0$

$$\rho_{\rm GW}(\tau) = \int \frac{d^3k}{(2\pi)^3} \frac{8\pi G_N \left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2}{Va^4(\tau)a^2(\tau_\star)} \cos^2 k(\tau_\star - \tau_0)\tilde{T}_{ij}(0,\mathbf{k}_p)\tilde{T}^*_{ij}(0,\mathbf{k}_p)$$

$$\tilde{T}_{ij}(0,\mathbf{k}_p) = \int dt_p \tilde{T}_{ij}(\tau,\mathbf{k}_p)$$

$\langle \tilde{T}_{ij}\tilde{T}^*_{ij}\rangle_{k_p \ll \Delta_p^{-1}}$ independent of *k*. *Cai, Pi and Sasaki, 1909.13728*

- $k\Delta \ll 1 \ll |k\tau_\star|$, an oscillating feature in the GW spectrum

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{4G_N|\tilde{T}_{ij}(0,0)|^2}{\pi^2 Va^4(\tau)a^2(\tau_\star)} \left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3 \cos^2 k(\tau_\star - \tau_0)\right\}$$

Generic features of GW spectrum

- Finite size effect

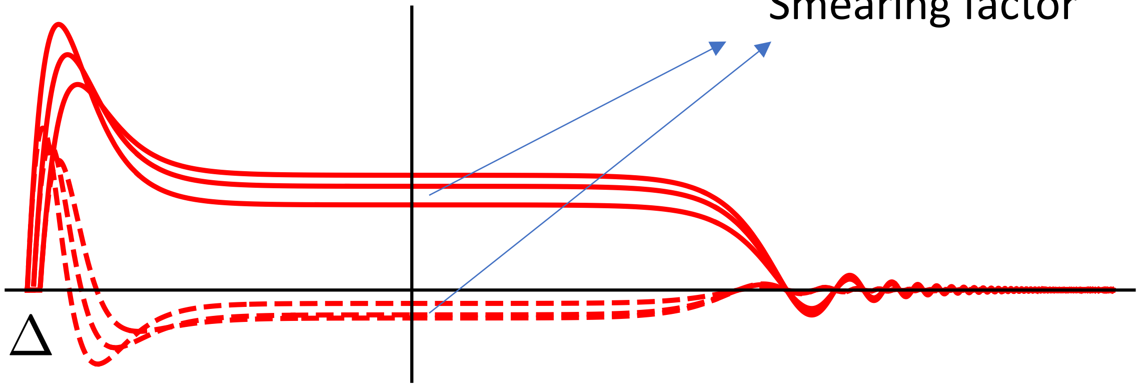
$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{4G_N|\tilde{T}_{ij}(0,0)|^2}{\pi^2 V a^4(\tau)a^2(\tau_\star)}\left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3\cos^2 k(\tau_\star-\tau_0)\right\}$$

$$\frac{1}{2}+\frac{1}{2}\cos 2k(\tau_\star-\tau_0)$$

Oscillating

$$\frac{d\rho_{\rm GW}^{\rm osc}}{d\log k} = \frac{2G_N|\tilde{T}_{ij}(0,0)|^2}{\pi^2 V a^4(\tau)a^2(\tau_\star)}\left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3\left[1+\mathcal{S}(k_p\Delta_p)\cos 2k(\tau_\star-\tau_0)\right]\right\}$$

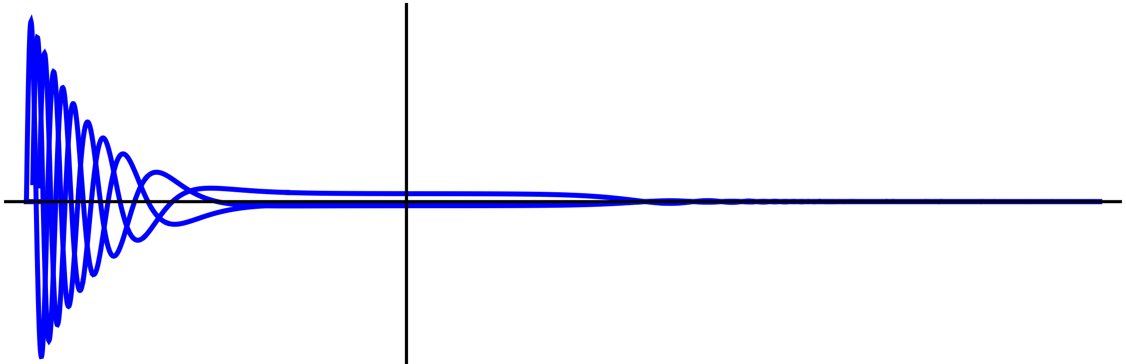
Smearing factor



Generic features of GW spectrum

- The UV part of the spectrum
  - $k_p \Delta_p \gg 1$, the oscillation pattern is completely smeared out.

$$\frac{d\rho_{\rm GW}^{\rm UV}}{d\log k} = \frac{2G_N |\tilde{T}_{ij}(k_p, \mathbf{k}_p)|^2}{\pi^2 V a^4(\tau) a^2(\tau_\star)} \left\{ \left[\tilde{\mathcal{E}}_0^i(k) \tilde{\mathcal{G}}_0^f(k) \right]^2 k^3 \right\}$$

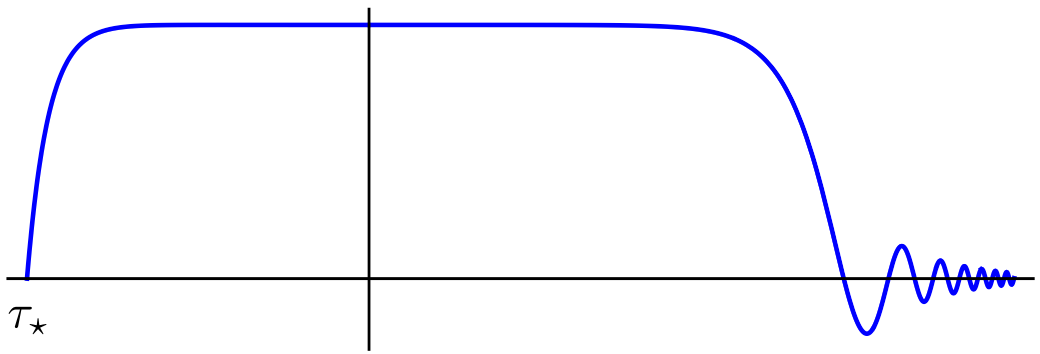


Completely smeared out

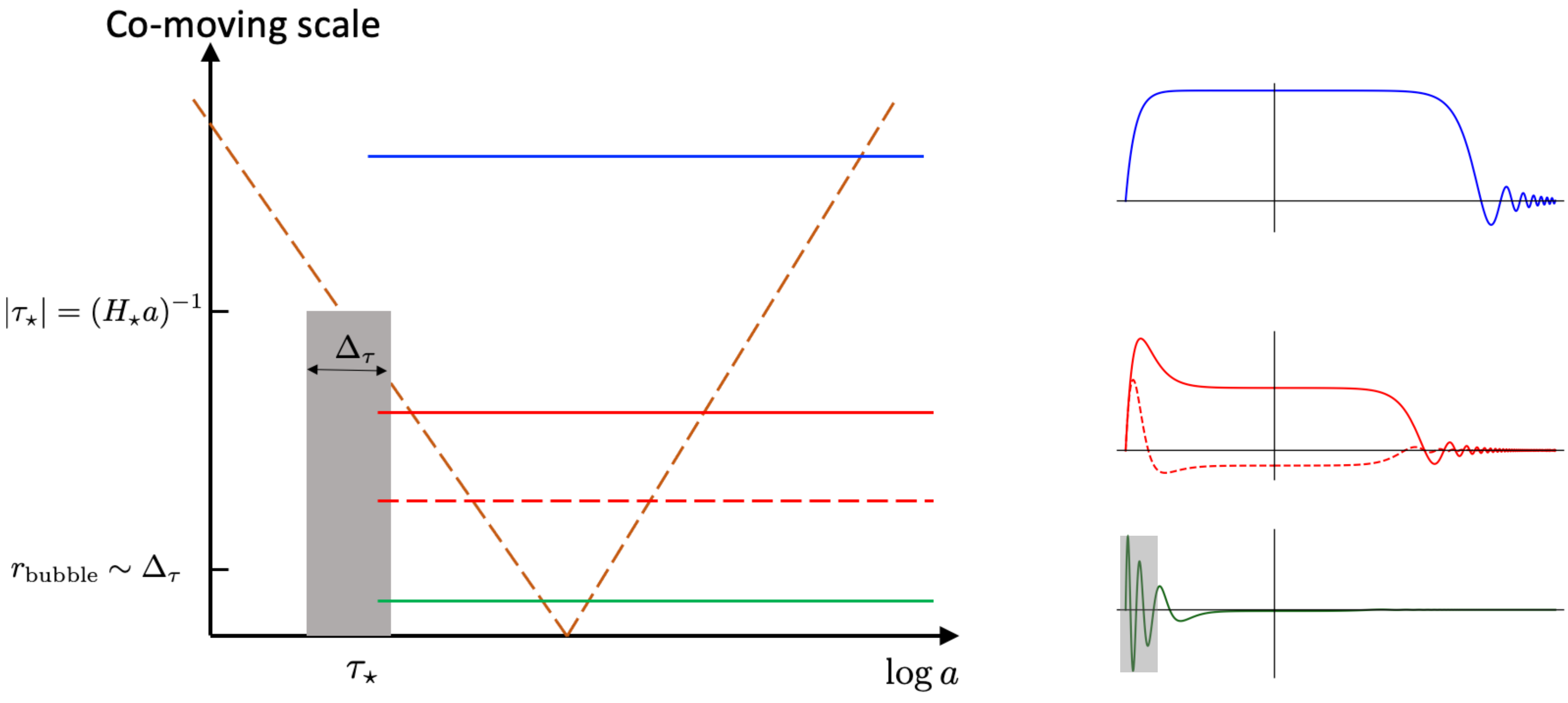
Generic features of GW spectrum

- The IR part of the spectrum $|k\tau_\star| \ll 1$
  - $\tilde{\mathcal{G}}^f$ is flat, no oscillation pattern in the spectrum either,

$$\frac{d\rho_{\rm GW}^{\rm IR}}{d\log k} = \frac{4G_N|\tilde{T}_{ij}(0,0)|^2}{\pi^2 V a^4(\tau)} \left[\int_{\tau_\star}^{0} a^{-2}(\tau_1) d\tau_1\right]^2 \left\{\left[\tilde{\mathcal{E}}_0^i(k)\right]^2 k^5\right\}$$

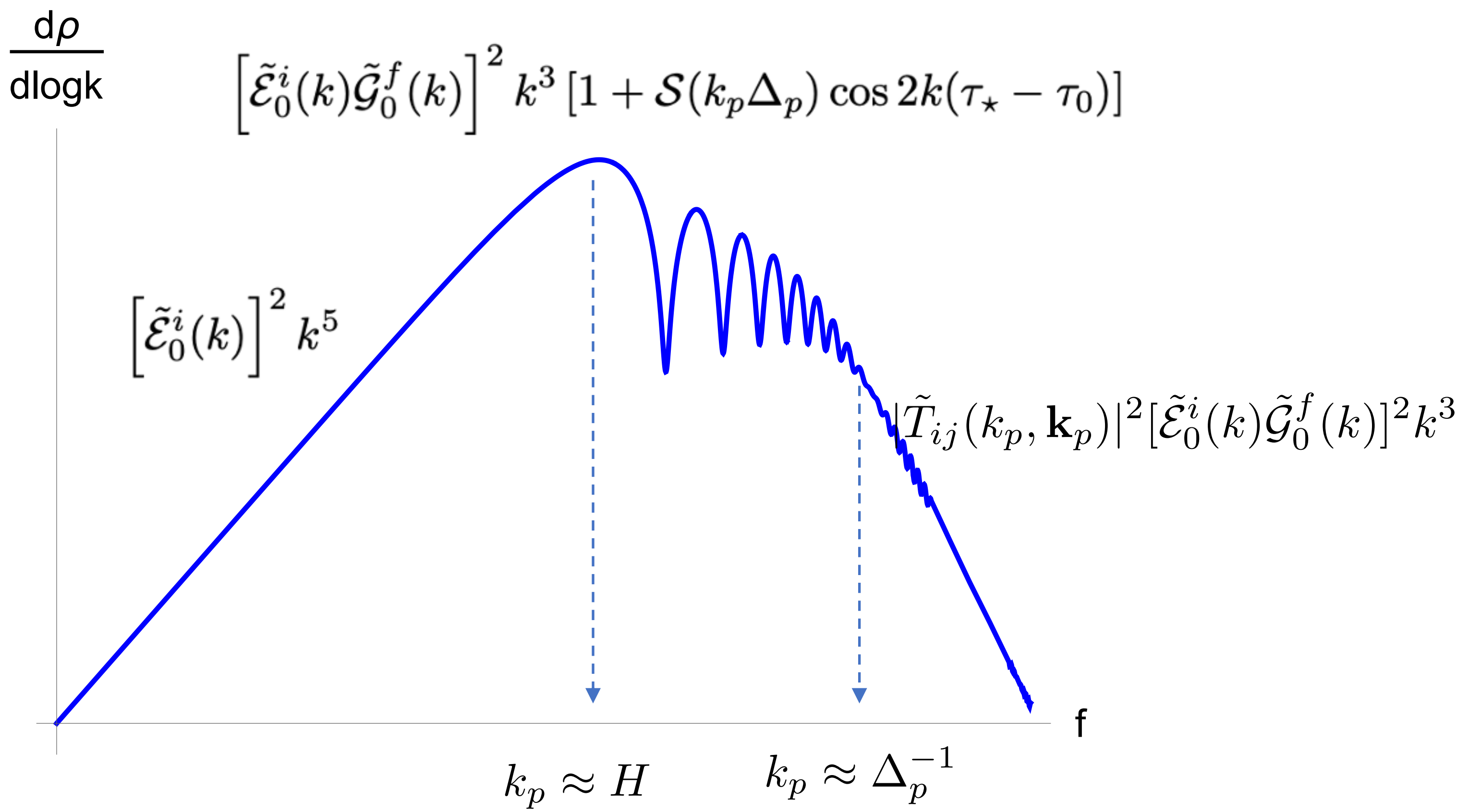


Generic features of GW spectrum



Generic features of GW spectrum

- Shape of the spectrum



Examples

- Inflation models
  - Quasi-de Sitter inflation $\tilde{\mathcal{G}}_0^f = \left(-\frac{H}{k}\right)$, $\eta_0' = 0$
  - t^p inflation $\tilde{\mathcal{G}}_0^f = a_0^{-1}(-k\tau_0)^{\frac{p}{1-p}} \frac{2^{\frac{p}{-1+p}}}{\sqrt{\pi}} \Gamma\left(\frac{3}{2} + \frac{1}{-1+p}\right)$, $\eta_0' = \frac{\pi}{2-2p}$

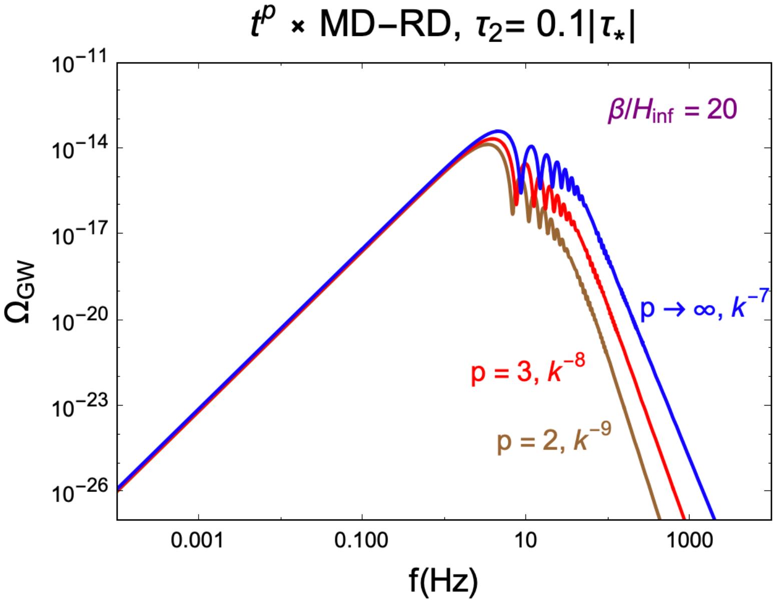
  In t^p inflation, we have the slow-roll parameter $\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{p}$

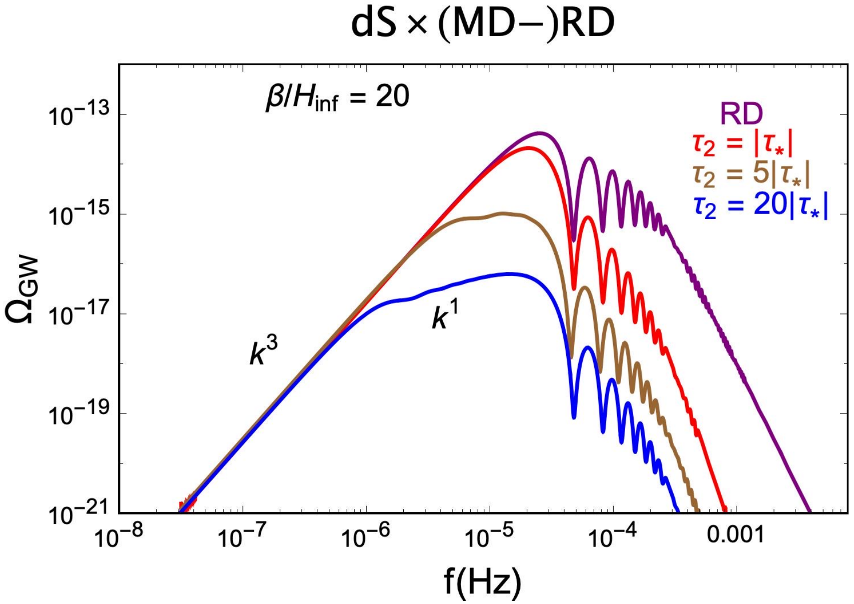
$$\tilde{\mathcal{G}}_0^f \sim k^{-\frac{1}{1-\epsilon}}$$

- Evolution after inflation
  - In RD, $\tilde{\mathcal{E}}_0^i \sim k^{-1}$
  - In MD, $\tilde{\mathcal{E}}_0^i \sim k^{-2}$
  - In $t^{\tilde{p}}$, $\tilde{\mathcal{E}}_0^i \sim k^{\tilde{p}/(\tilde{p}-1)}$

	w	$\rho(a)$	$\tilde{p}$
MD	0	a^{-3}	2/3
RD	1/3	a^{-4}	1/2
Λ	-1	a^0	∞
Cosmic string	-1/3	a^{-2}	1
Domain wall	-2/3	a^{-1}	2
kination	1	a^{-6}	1/3

Comparing scenarios





Different inflation scenarios

➡ Different slopes in the UV and oscillatory parts

Temporary MD betwteen inflation and RD

τ_2: MD-RD transition

Outline

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Why first order phase transition during inflation?

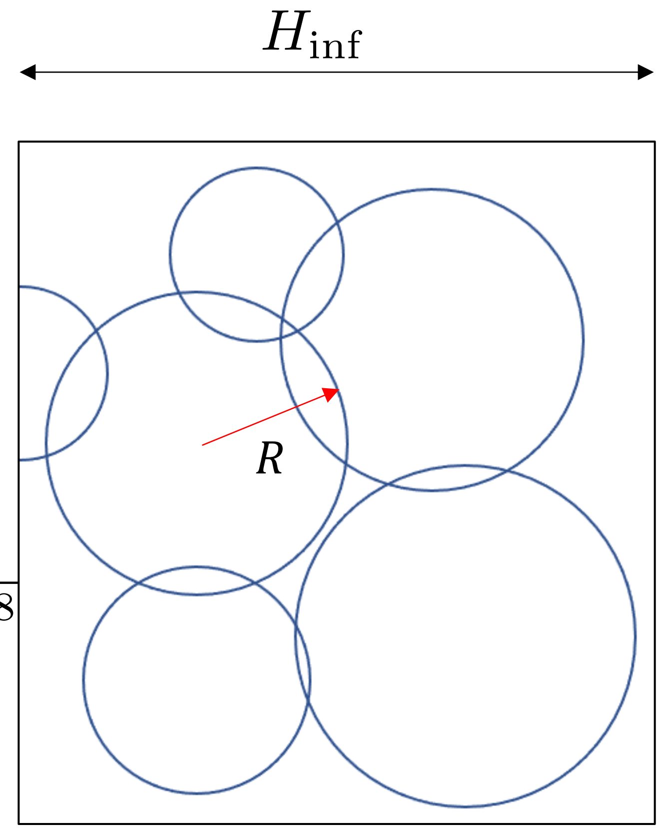
- For phase transition to finish

$$R = \Delta_p \ll H_{\rm inf}^{-1}$$

$$\beta = \frac{dS_4}{dt} \sim \Delta_p^{-1} \gg H_{\rm inf}$$

$$\frac{d\rho_{\rm GW}^{flat}}{\Delta\rho_{\rm vac} d\log k_p} \approx \left(\frac{H_{\rm inf}}{\beta}\right)^2 \times \frac{\beta\ k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$

Huber and Konstandin, 0806.1828



Models of first order phase transition during inflation

- Models in the literature:
  - Open inflation

    K. Sugimura, D. Yamauchi, M. Sasaki, 1110.4773
  - GUT phase transition at the beginning of inflation

    H. Jiang, T. Liu, S. Sun, Y. Wang, 1512.07538
  - Obtained the correct UV behavior of the GW spectrum

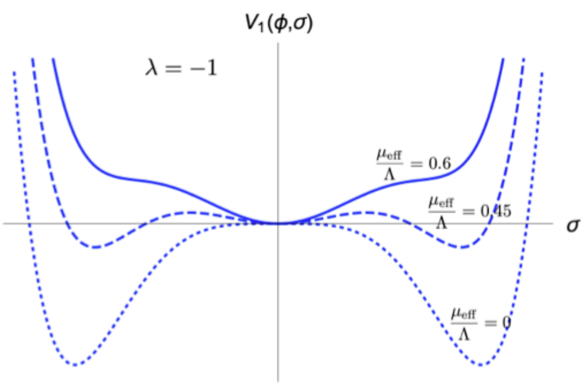
    Y.-T. Wang, Y. Cai, Y.-S. Piao, 1801.03639

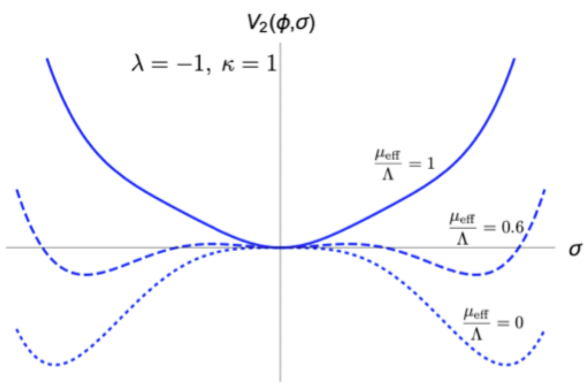
Models of first order phase transition during inflation

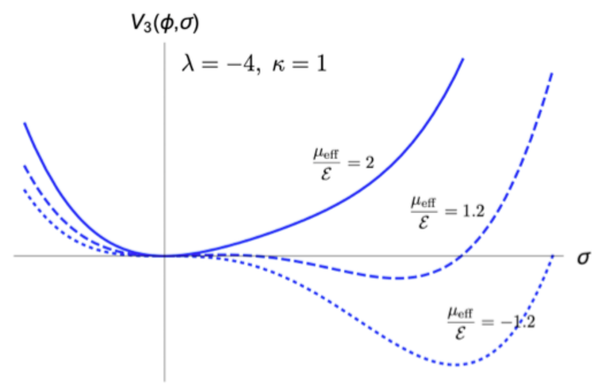
- Simple models: $\phi:\ \text{inflaton field},\ \ \sigma:\ \text{spectator}$

$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$
$$V_2(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\kappa}{4}\sigma^4\log\frac{\sigma^2}{\Lambda^2}$$
$$V_3(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{3}\mathcal{E}\sigma^3 + \frac{\kappa}{4}\sigma^4\ .$$

$$\mu_{\rm eff}^2 = -(\mu^2 - c^2\phi^2)$$







First order phase transition during inflation

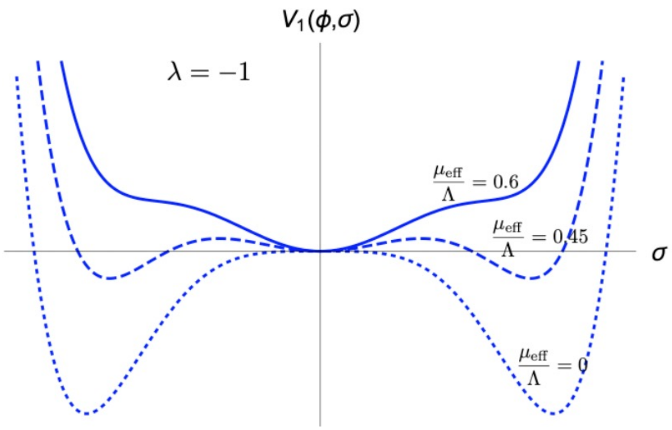
- Bubble nucleation rate: $\frac{\Gamma}{V} = I_0 m_\sigma^4 e^{-S_4}$
- Phase transition starts: $\mathcal{O}(1) = \int_{-\infty}^{t} dt' H^{-3} I_0 m_\sigma^4 e^{-S_4(t')}$
- The bounce: $S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_\sigma^4}{H^4}\right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm pl}} \frac{m_\sigma^4}{H^4}\right)$
- First order phase transiton: $S_4 \gg 1$

$$H^4 \ll m_\sigma^4 \ll 3 M_{\rm pl}^2 H^2$$

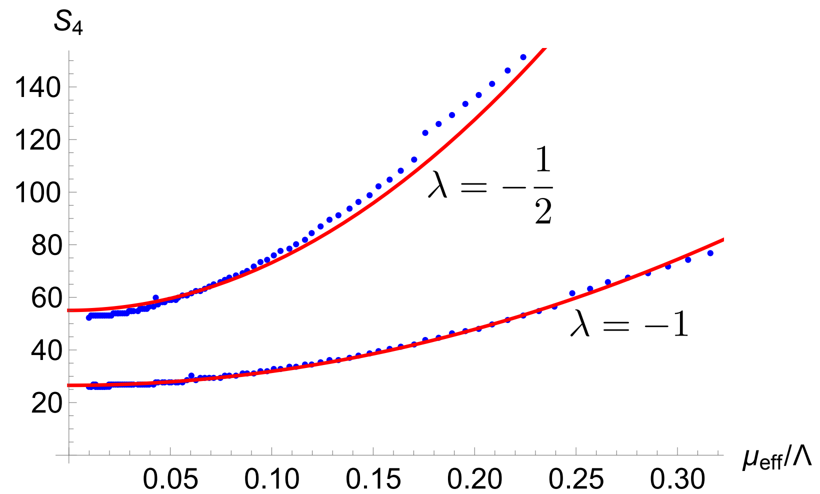
First order phase transition during inflation

- $\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot\phi}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right|$ $\qquad \mu_{\rm eff}^2 = -(\mu^2 - c^2\phi^2)$

$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$

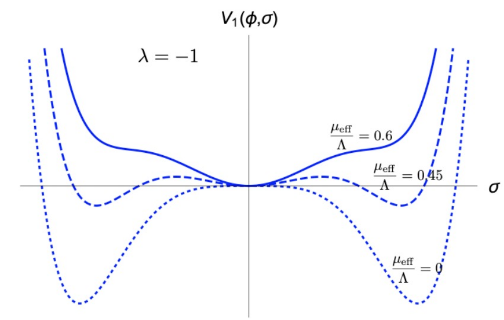


$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



CosmoTransitions

First order phase transition during inflation



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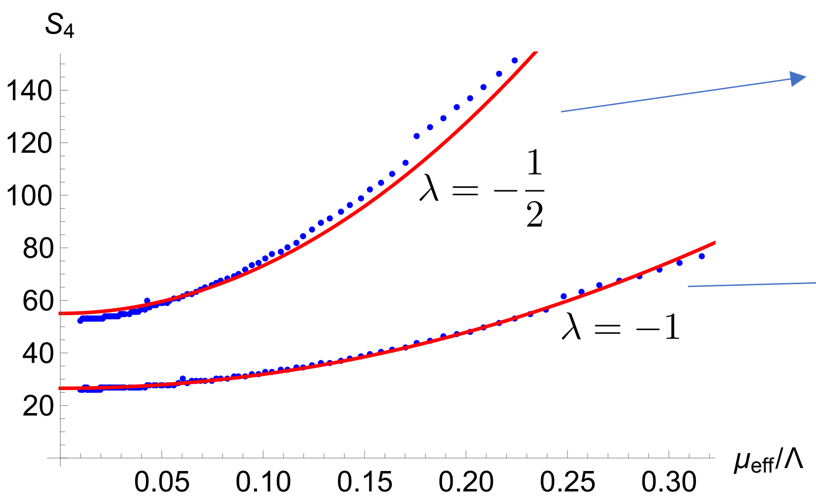
$$\sim \mu_{\rm eff}^2/\Lambda^2$$

$$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$

First order phase transition during inflation

- $$\frac{\beta}{H} \sim \left| \frac{dS_4}{d\log \mu_{\rm eff}^2} \right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

N_e: e-folds before the end of inflation

$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

First order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \frac{H_{\rm inf}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right]\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac}\, d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction

Redshift

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

$$e^{-N_e}$$

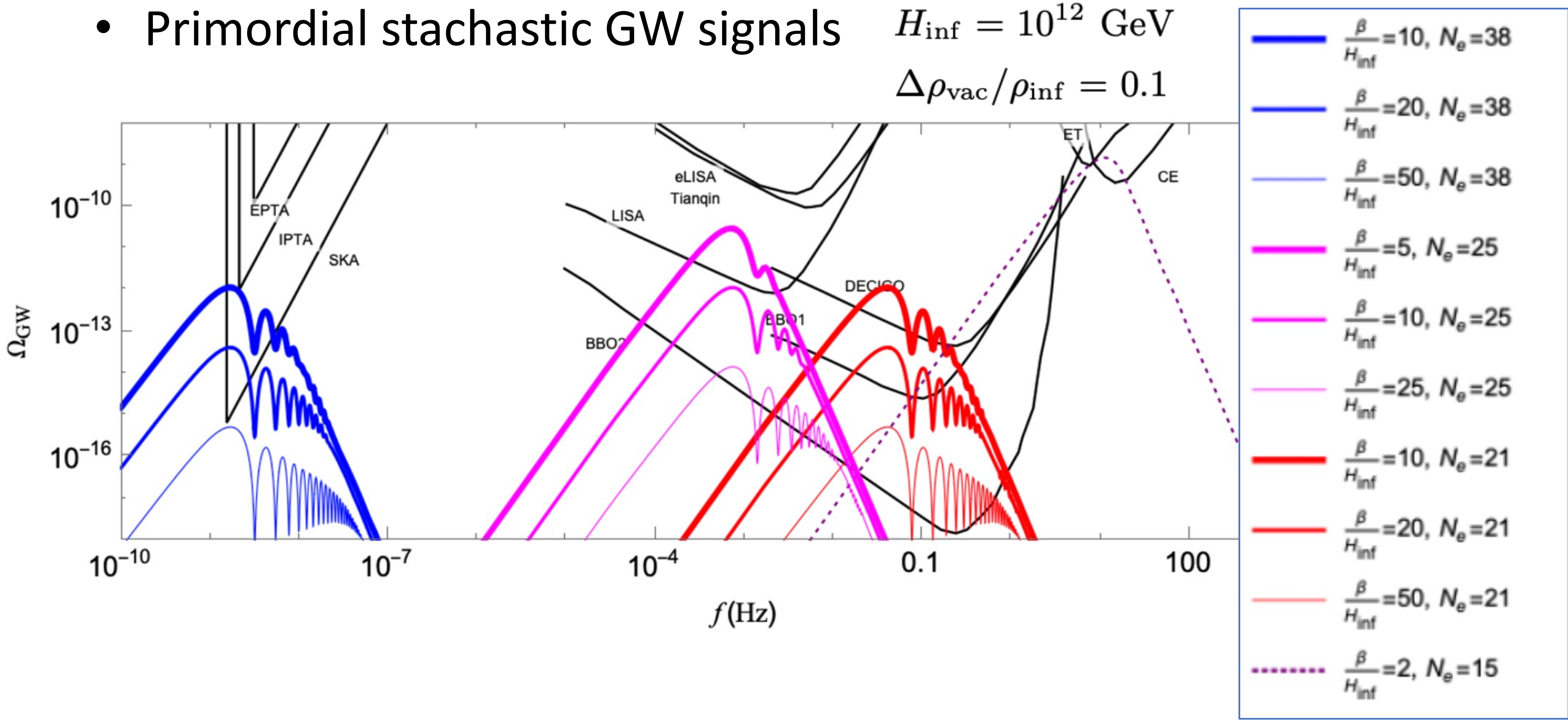
N_e: e-folds before the end of inflation

First order phase transition during inflation

- Primordial stachastic GW signals

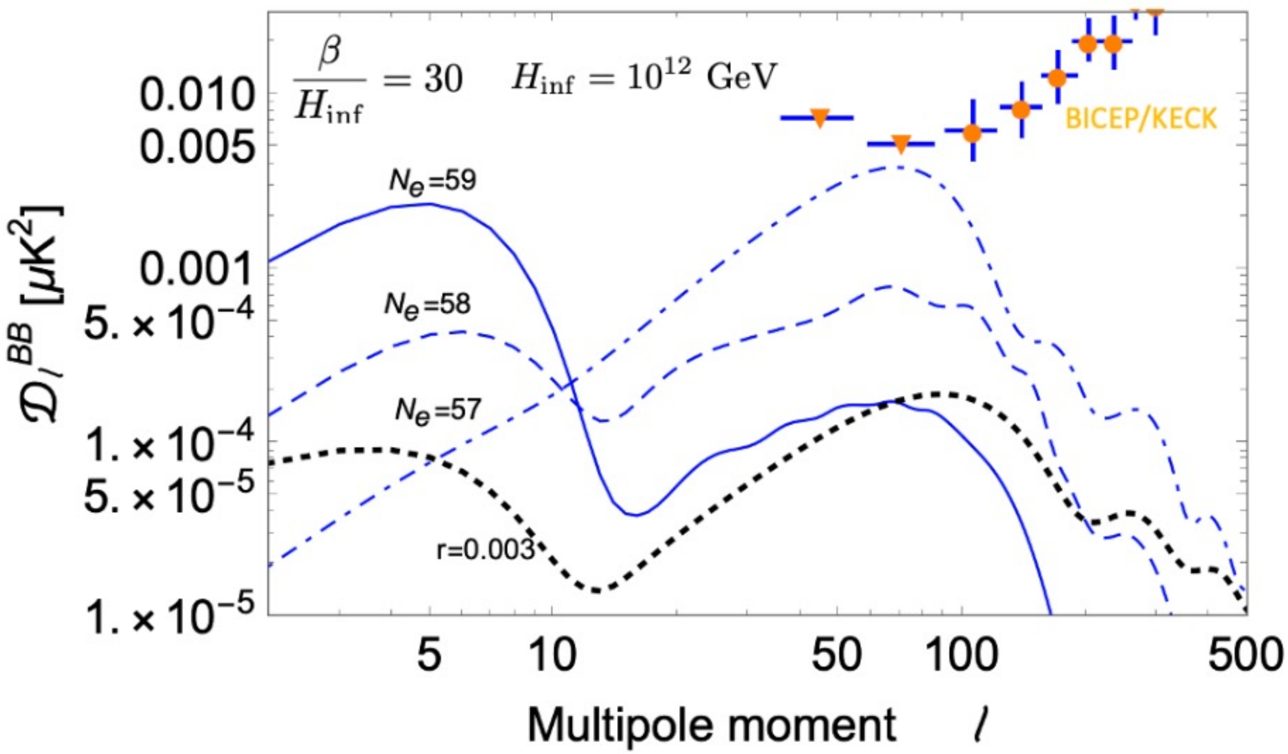
$H_{\rm inf} = 10^{12}$ GeV

$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.1$

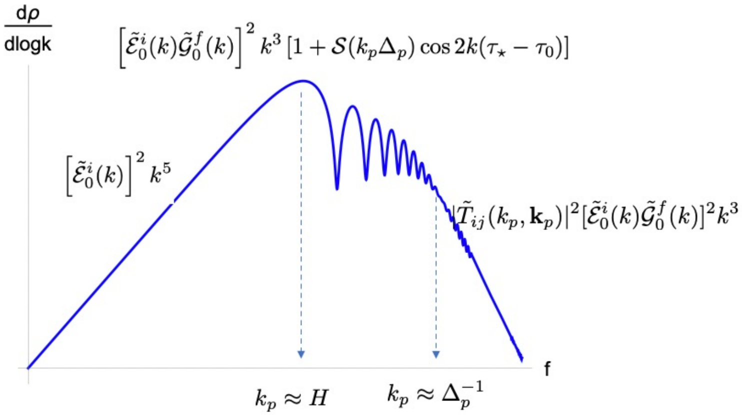


First order phase transition during inflation

- CMB B modes



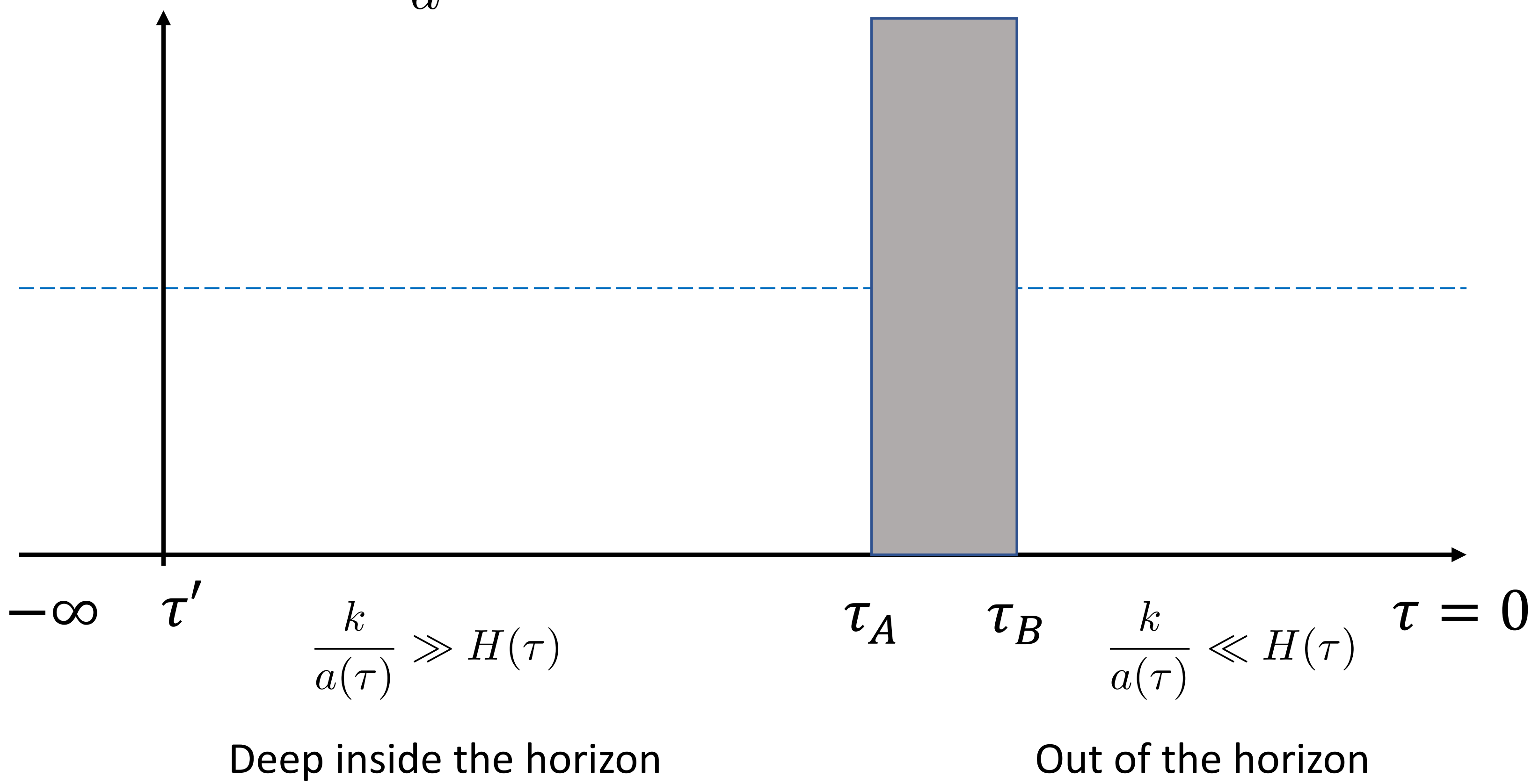
Summary



- We study the features of classical GWs produced from instantaneous sources during inflation.
- We show that there is an oscillatory feature in the spectrum.
- The slopes of the spectrum can tell us information about the inflation model and evolution of the universe when the modes re-enter the horizon.
- First order phase transition during inflation can be realized with simple models.
- If we are lucky enough, such a signal can be detected by future GW detectors.

GW from instantaneous and local sources (qualitative study)

- $h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$

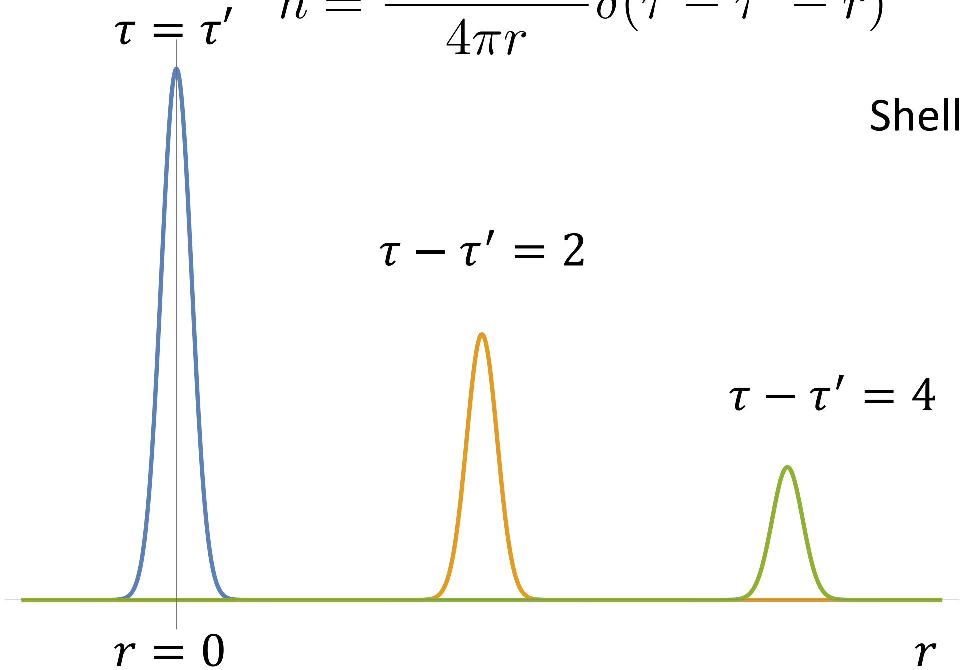


de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In Minkovski space

$$h = \frac{16\pi G_N T}{4\pi r}\delta(\tau - \tau' - r)$$

Shell with radius $|\tau - \tau'|$



de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} + \left(\frac{1}{k^2\tau} - \frac{1}{k^2\tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3\tau\tau'} \sin k(\tau - \tau') \right]$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$\frac{\tau}{4\pi x}\delta(\tau - \tau' - |\mathbf{x}|)$$

$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij}\tau\Theta(\tau - \tau')\left[\frac{\sin k(\tau - \tau')}{k} + \left(\frac{1}{k^2\tau} - \frac{1}{k^2\tau'}\right)\cos k(\tau - \tau') + \frac{1}{k^3\tau\tau'}\sin k(\tau - \tau')\right]$$

$$\frac{1}{4\pi}\Theta(\tau - \tau' - |\mathbf{x}|)$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h(\tau, \mathbf{x}) \sim \frac{\tau}{4\pi x}\delta(\tau - \tau' - x) + \frac{1}{4\pi}\Theta(\tau - \tau' - x)$$

Similar to Minkovski	Intrinsic in de Sitter
Decreases with both x and τ	constant
Vanishes out of horizon	

de Sitter inflation as an example

- At $\tau \to 0$ $$h(\tau, \mathbf{x}) \sim \frac{1}{4\pi}\Theta(|\tau'| - x)$$

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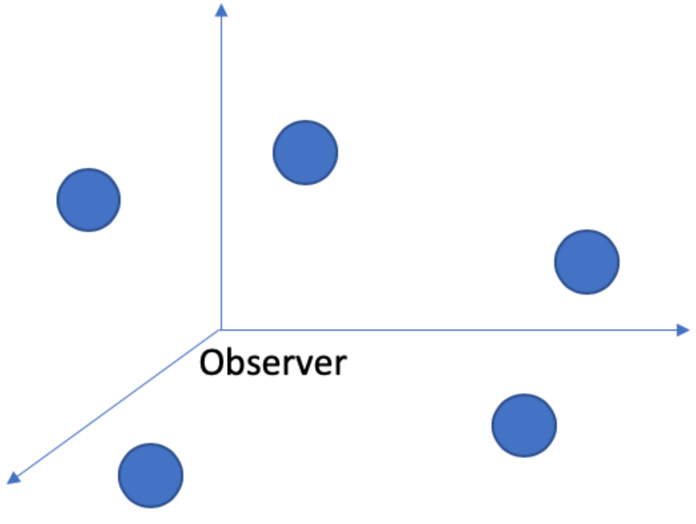
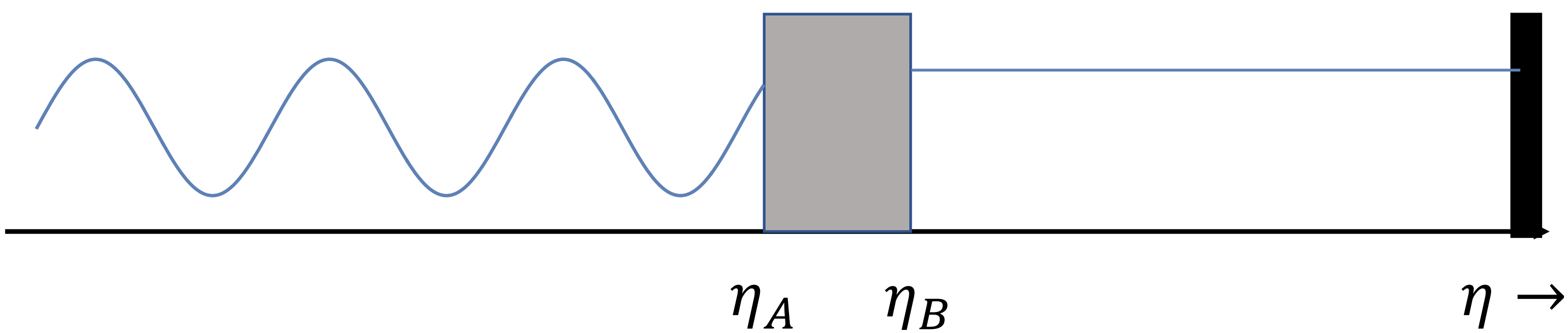


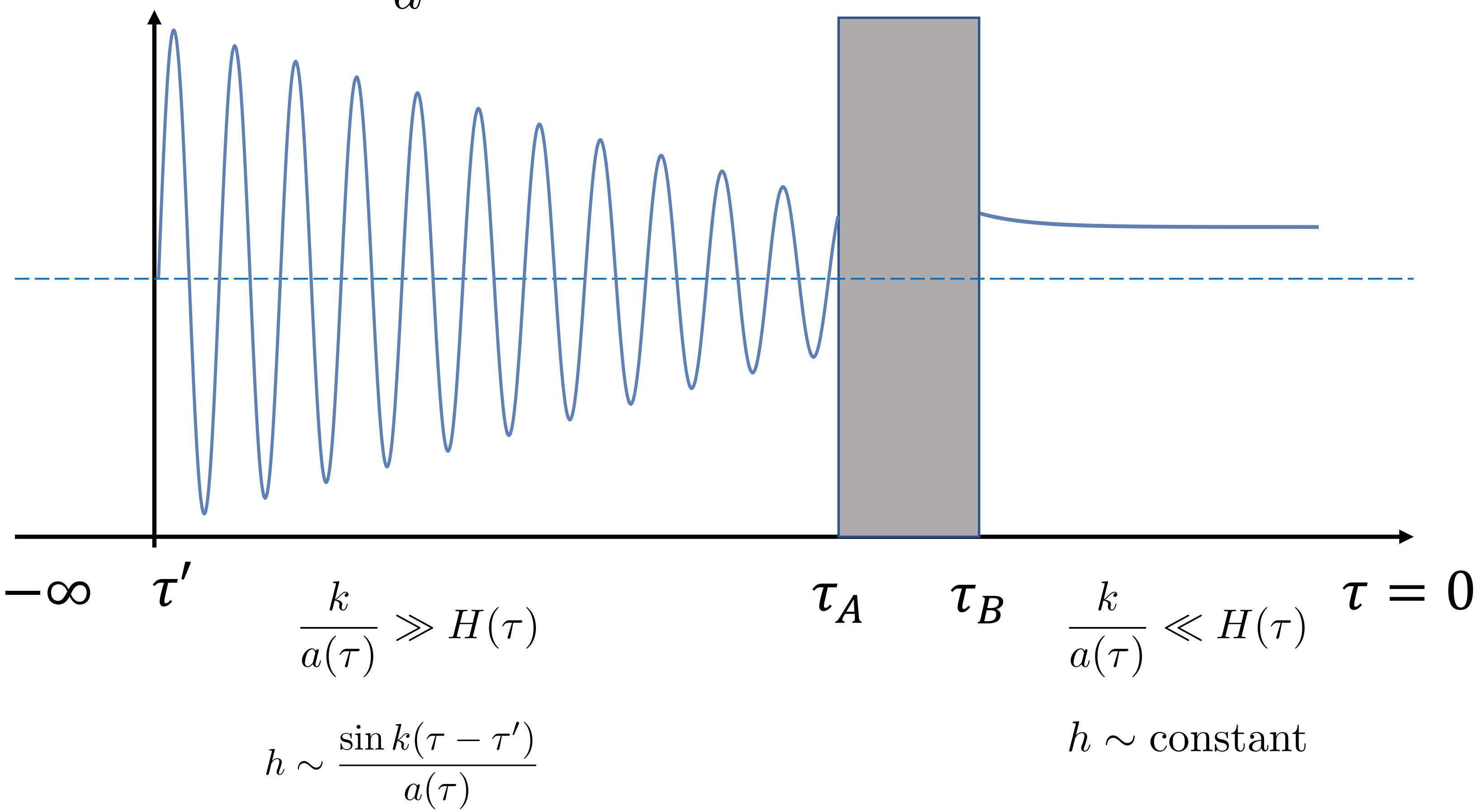
Green's function of GW in inflation

- Generic features
  - For $|k\tau'| > \eta_A$, $h^f = k^{-1}\cos k(\tau' - \tau_0')\tilde{\mathcal{G}}_0^f(k)$
  - For $|k\tau'| < \eta_B$, $h^f = \left[a(\tau')\int_{\tau'}^0 a^{-2}(\tau_1)d\tau_1\right]$



GW from instantaneous and local sources (qualitative study)

- $h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$



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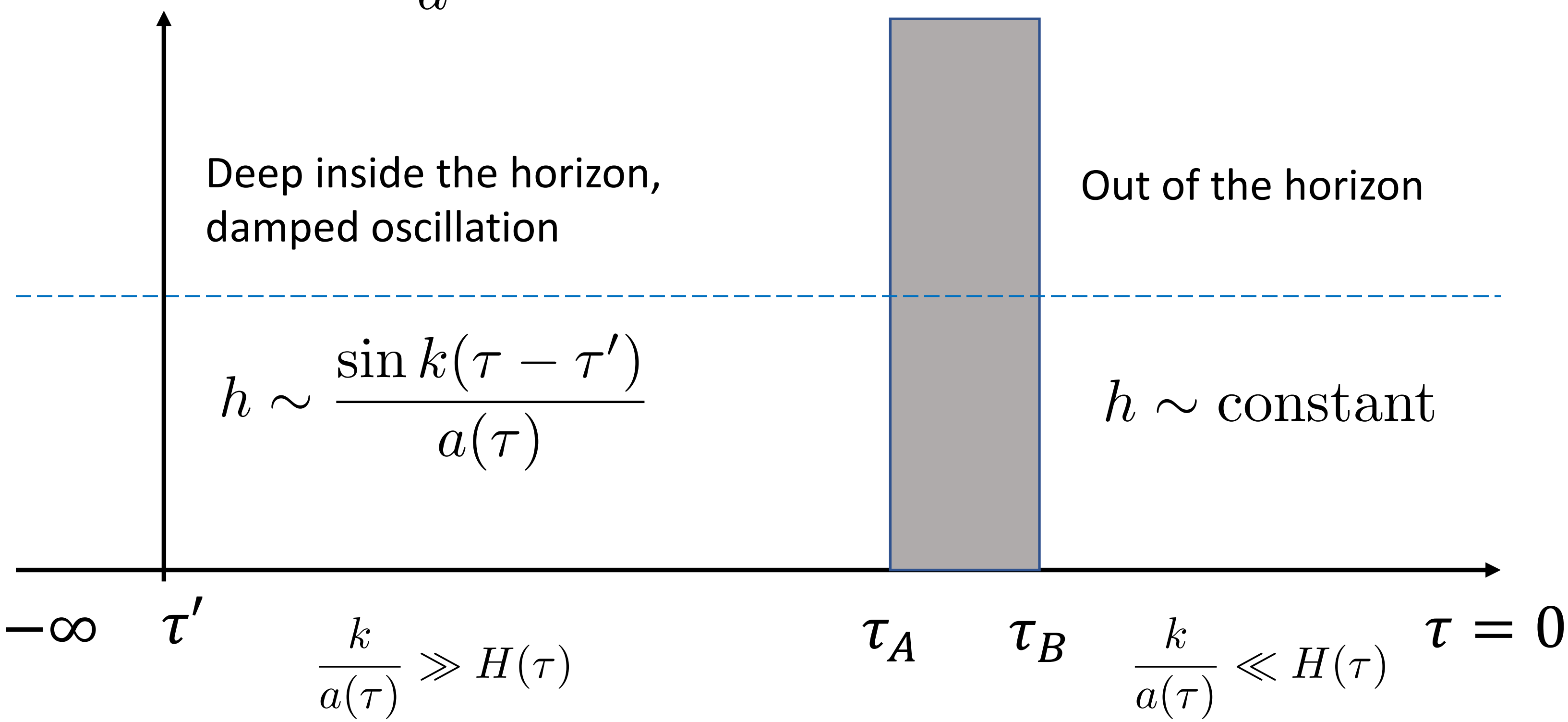

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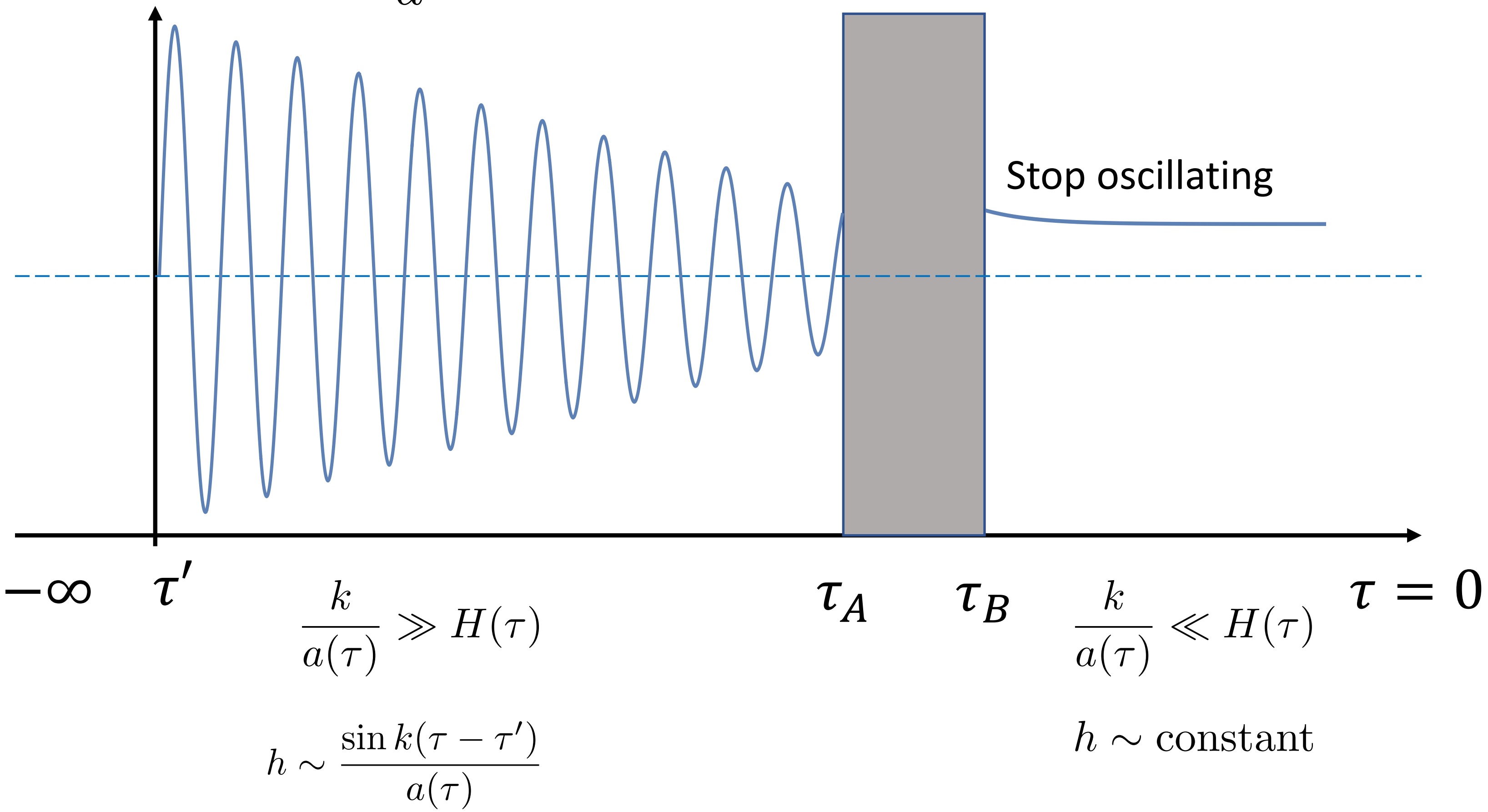

h^f in a generic inflation model

- Generic features

source

Model dependent

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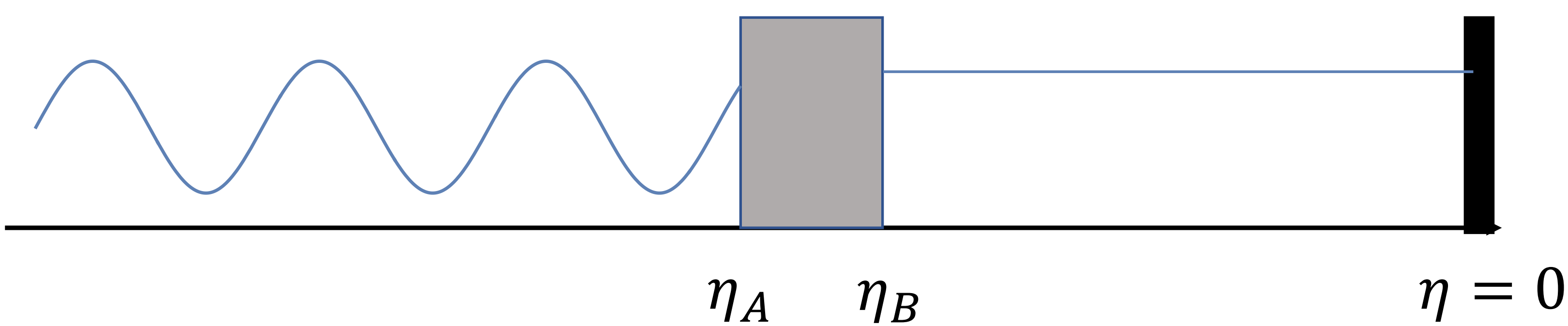
Independent of k



Outline

- Motivations
- Phase transi
- GWs from an instantaneous source during inflation
- GWs from a source with finite duration during inflation
- GWs from first order phase transition during inflation
- Summary

Properties of universe undergoing an accelerated expansion

- The metric $$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

GW in transverse traceless part of *h*

- Conformal time

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

- Inflation
  - $\ddot{a} > 0$, $d\tau = a^{-1}(t)dt$, τ has a finite upper bound. We shift τ so that $\tau \leq 0$.
  - $|\tau|$ is the size of the comoving horizon.

GW from instantaneous and local sources (qualitative study)

- E.O.M. of GW $$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

- For an instantaneous and local source, the source can be seen as delta funtion in both space and time.

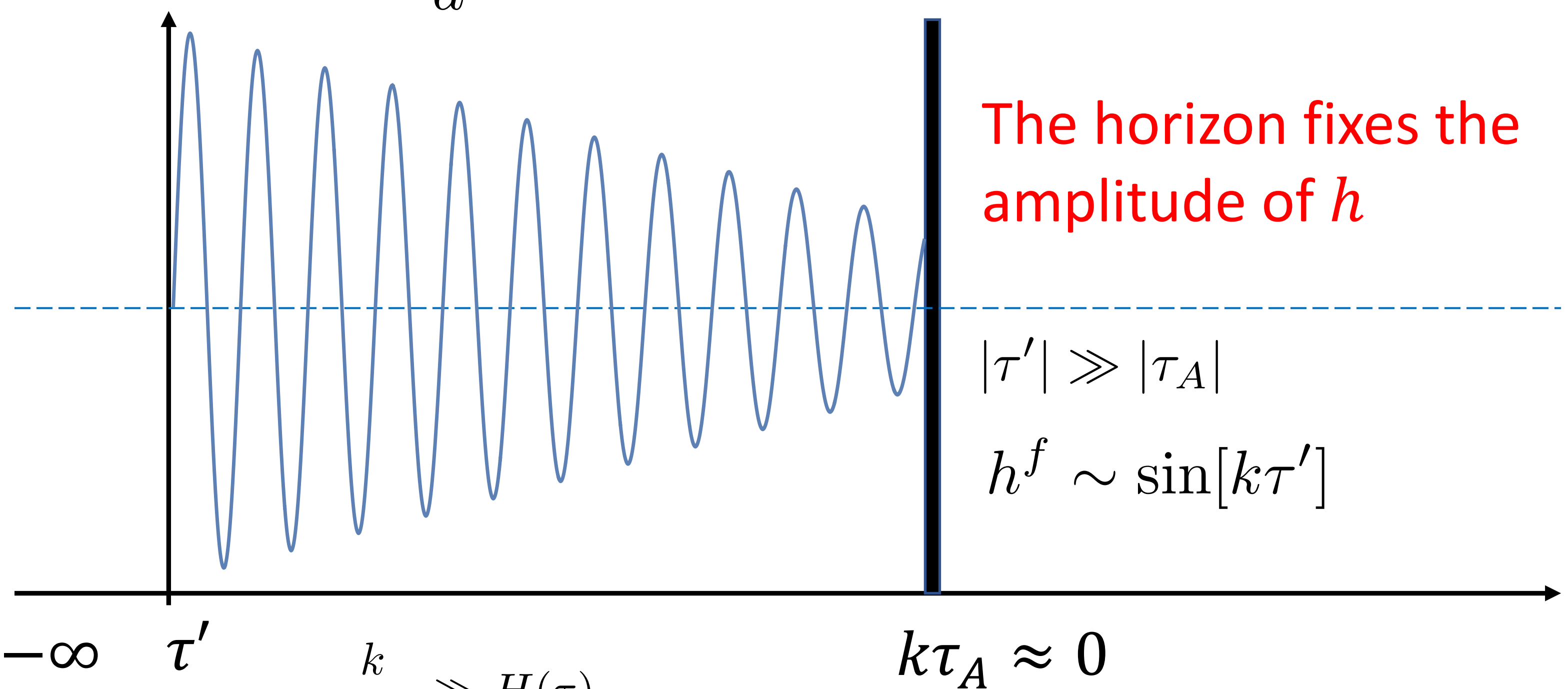
$$\sigma_{ij} \sim \delta(\mathbf{x})\delta(\tau - \tau')$$

- E.O.M. in Fourier space

$$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$

GW from instantaneous and local sources (qualitative study)

- $$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$



The horizon fixes the amplitude of h

$$|\tau'| \gg |\tau_A|$$

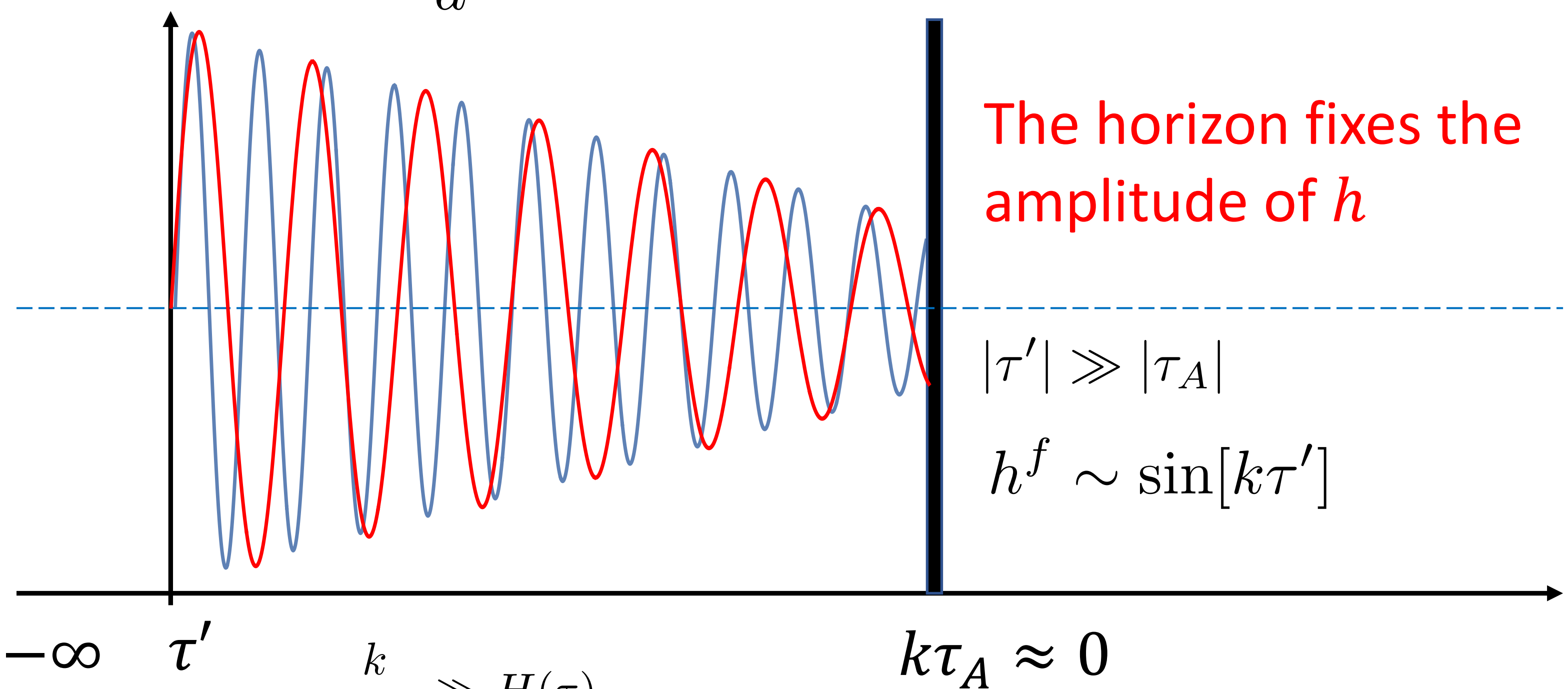
$$h^f \sim \sin[k\tau']$$

$$\frac{k}{a(\tau)} \gg H(\tau)$$

$$h \sim \frac{\sin k(\tau-\tau')}{a(\tau)}$$

GW from instantaneous and local sources (qualitative study)

- $h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$



The horizon fixes the amplitude of *h*

$|\tau'| \gg |\tau_A|$

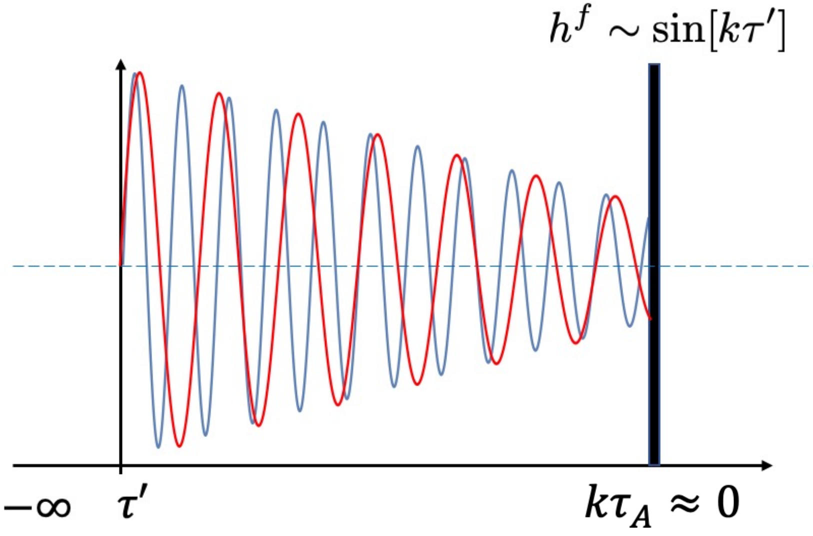
$h^f \sim \sin[k\tau']$

$\frac{k}{a(\tau)} \gg H(\tau)$

$h \sim \frac{\sin k(\tau-\tau')}{a(\tau)}$

GW from instantaneous and local sources (qualitative study)

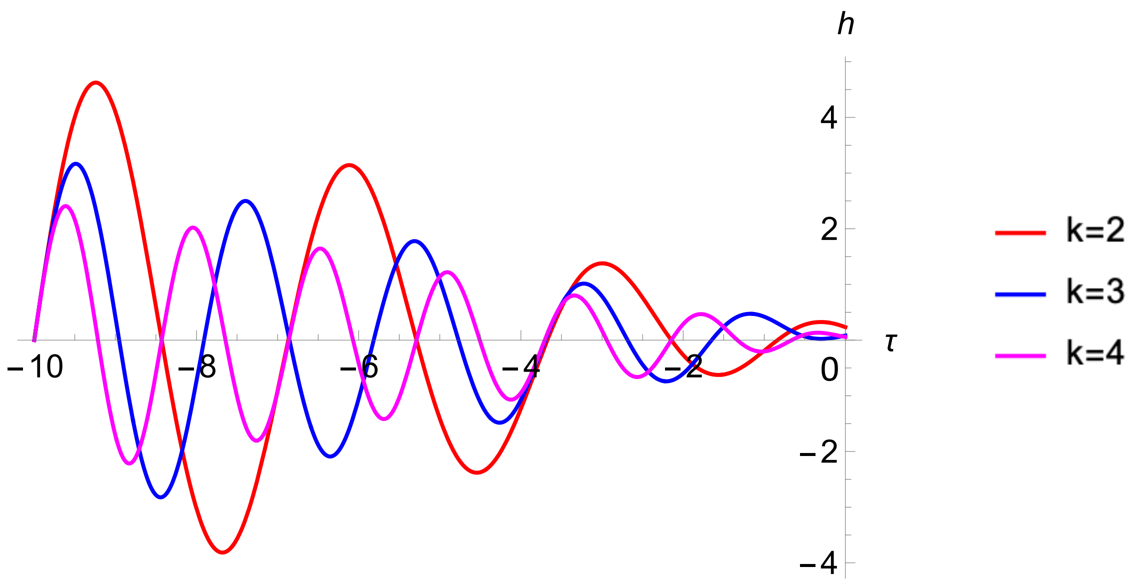
- The conformal time between the source and the horizon is fixed.
- The phase of h at the source is fixed.
- The value of h at the horizon oscillates with k.
- The amplitude h^f oscillates with k.
- h^f is the initial condition for later evolution.



Quasi-de Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$

- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau-\tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau-\tau')\right]$$



De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$

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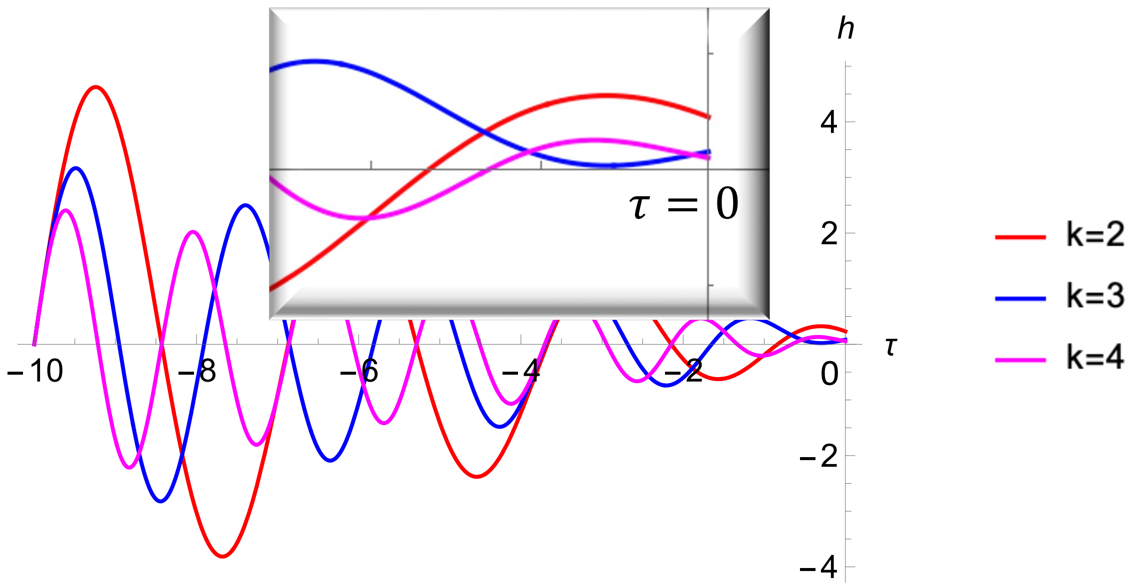

De Sitter inflation as an example

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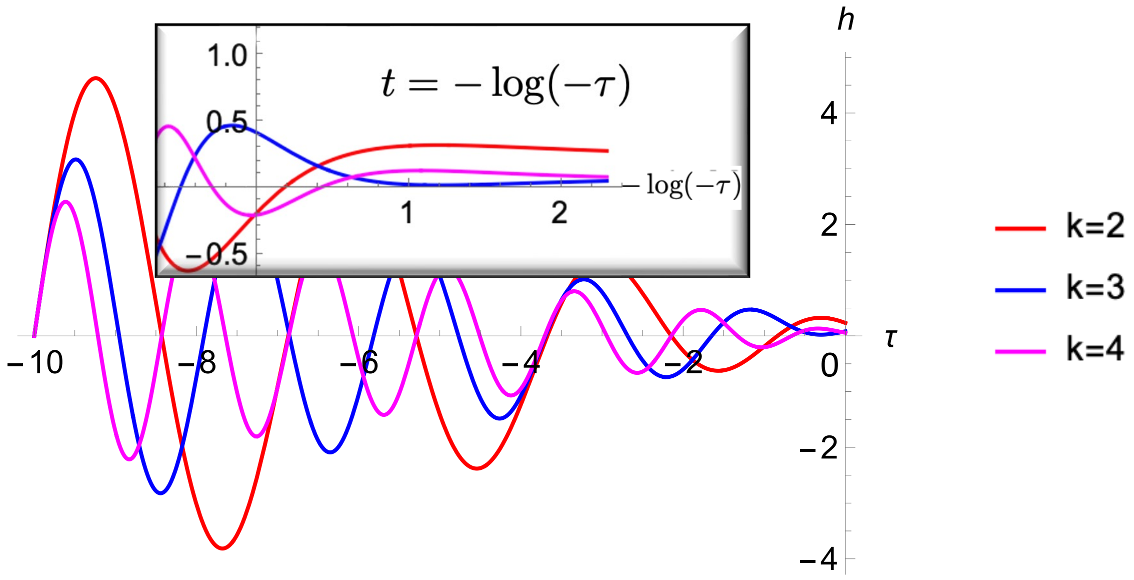
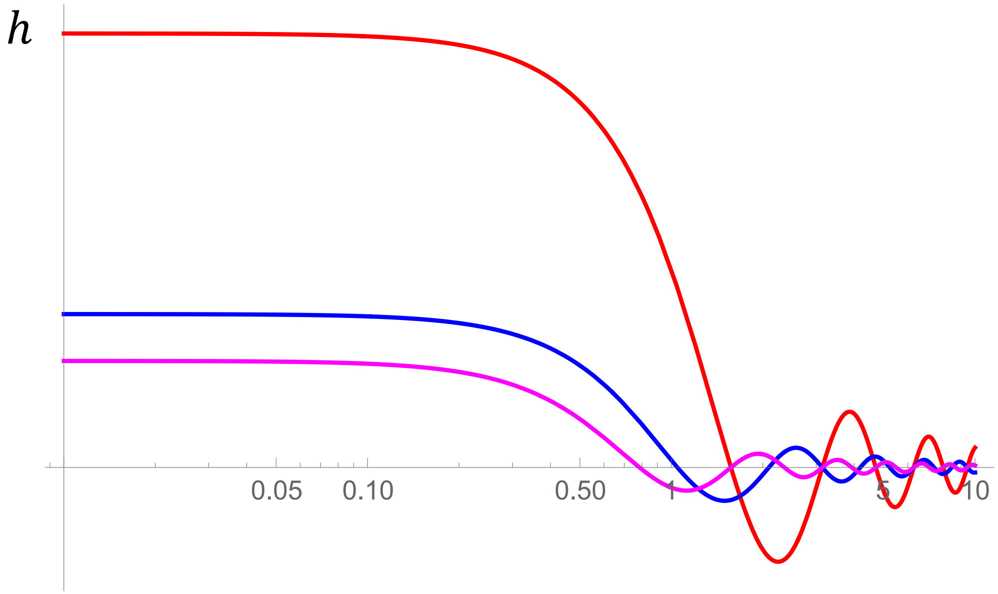


After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



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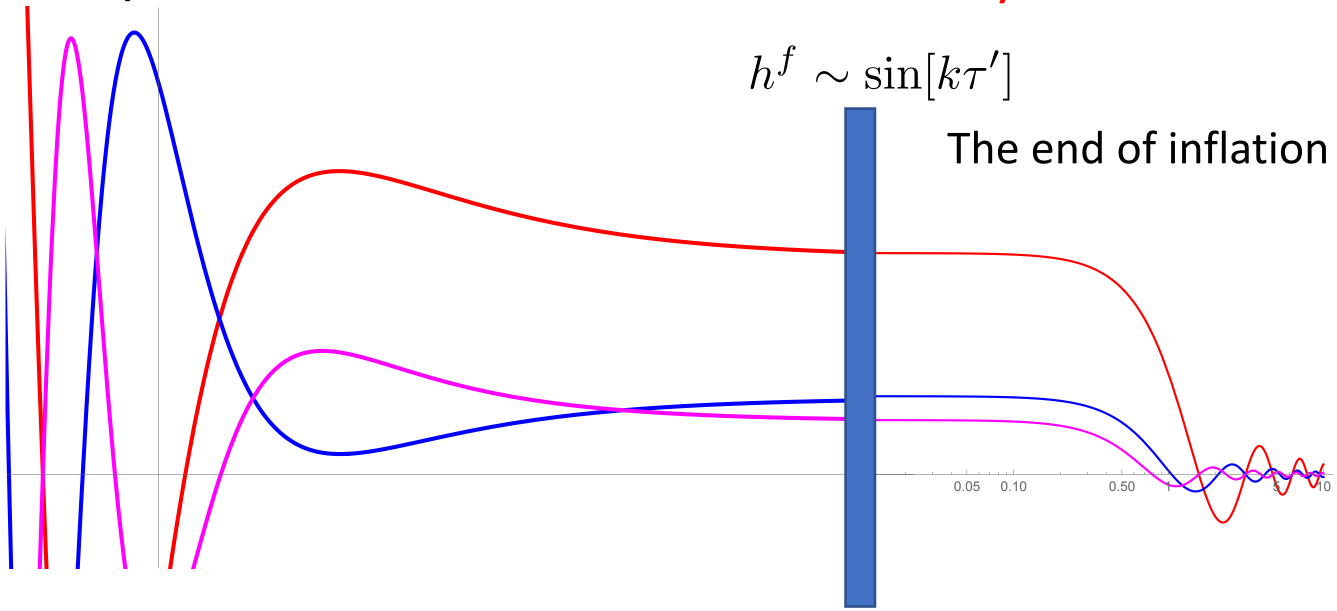
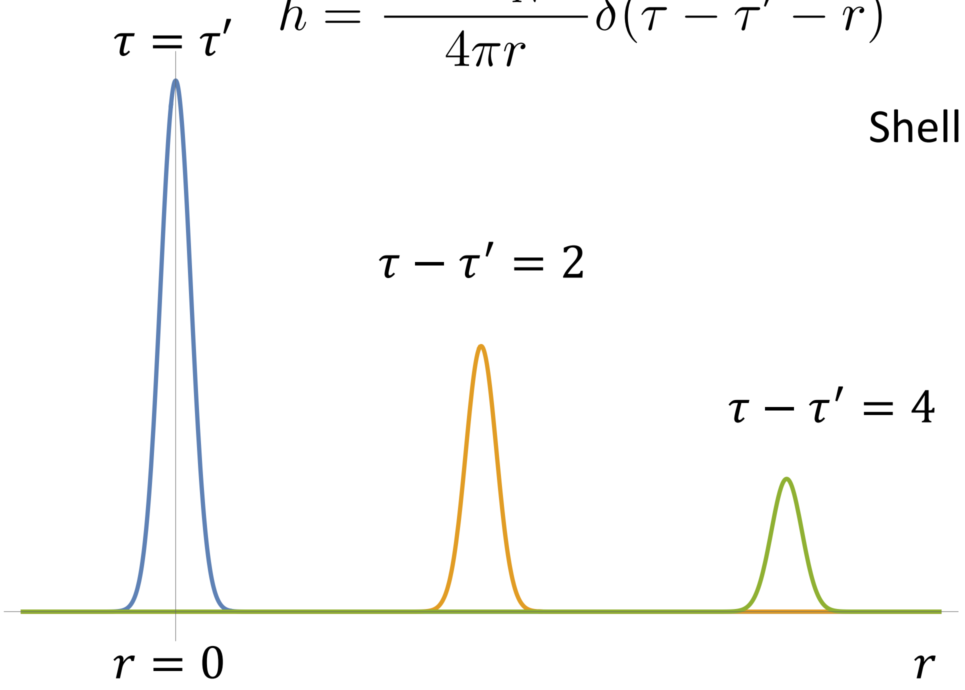


Another way to see the oscillatory pattern

- What is the spatial configuration of h from an instantaneous and local source?
- In Minkovski space

$$h = \frac{16\pi G_N T}{4\pi r}\delta(\tau - \tau' - r)$$

Shell with radius $|\tau - \tau'|$



Another way to see the oscillatory pattern

- What is the spatial configuration of h from an instantaneous and local source?
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$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} \right.$$

$$\left. + \left(\frac{1}{k^2 \tau} - \frac{1}{k^2 \tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3 \tau \tau'} \sin k(\tau - \tau') \right]$$

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$$\frac{\tau}{4\pi x}\delta(\tau - \tau' - |\mathbf{x}|)$$

$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} \right.$$

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$$\frac{1}{4\pi}\Theta(\tau - \tau' - |\mathbf{x}|)$$

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- What is the spatial configuration of h from an instantaneous and local source?
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$$h(\tau, \mathbf{x}) \sim \frac{\tau}{4\pi x}\delta(\tau - \tau' - x) + \frac{1}{4\pi}\Theta(\tau - \tau' - x)$$

Similar to Minkovski	Intrinsic in de Sitter
Decreases with both x and τ	constant
Vanishes out of horizon	

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- At $\tau \to 0$ $$h(\tau, \mathbf{x}) \sim \frac{1}{4\pi}\Theta(|\tau'| - x)$$
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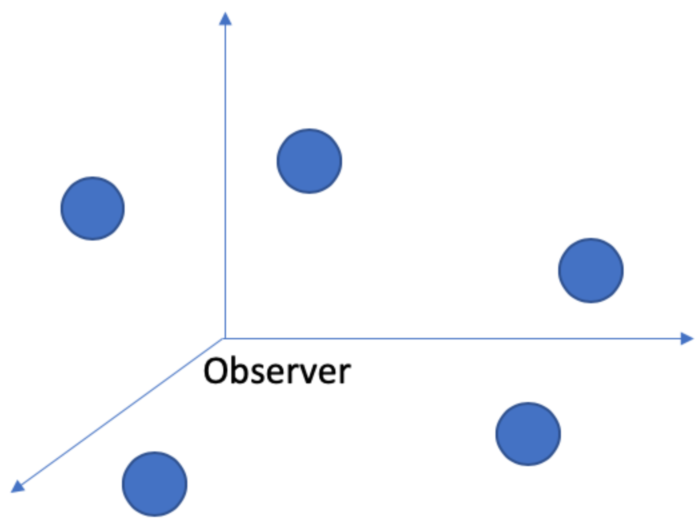

Spectrum of GW from a real source