Coupling a Cosmic String to a TQFT

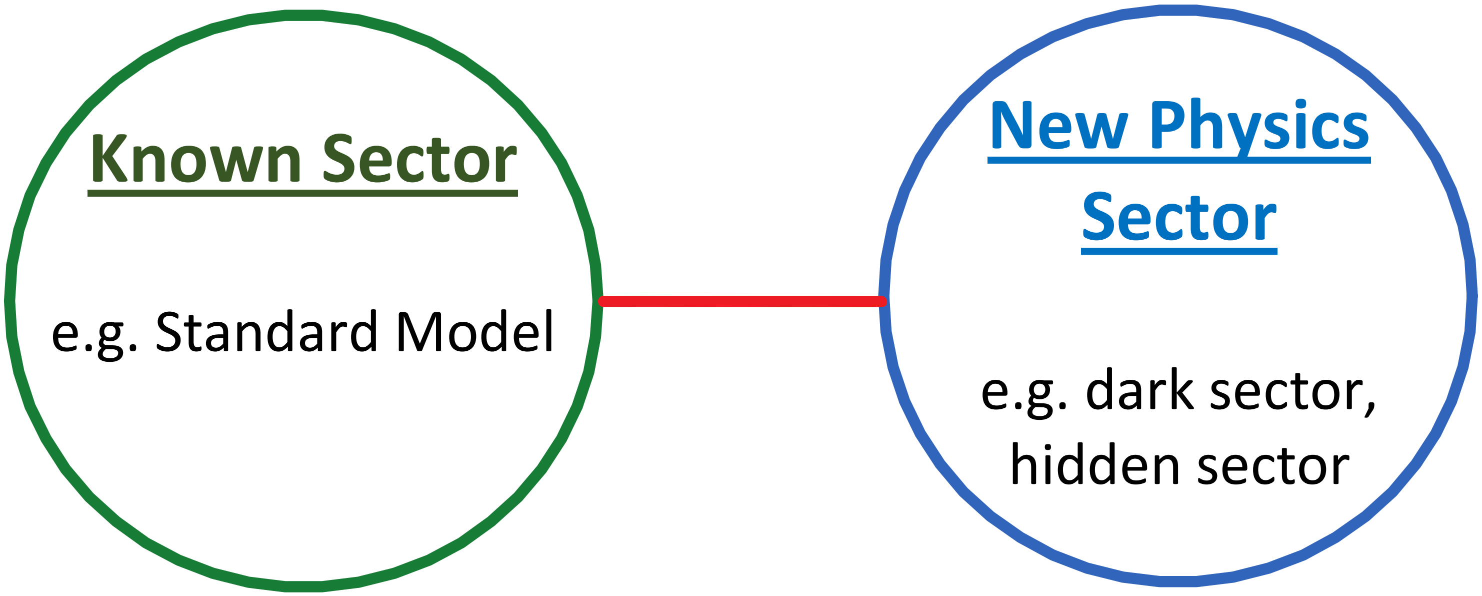
Sungwoo Hong

KAIST

(2302.00777: T.D Brennan, SH, LT Wang)

IAS Program on High Energy Physics

A General Setup in Particle Physics

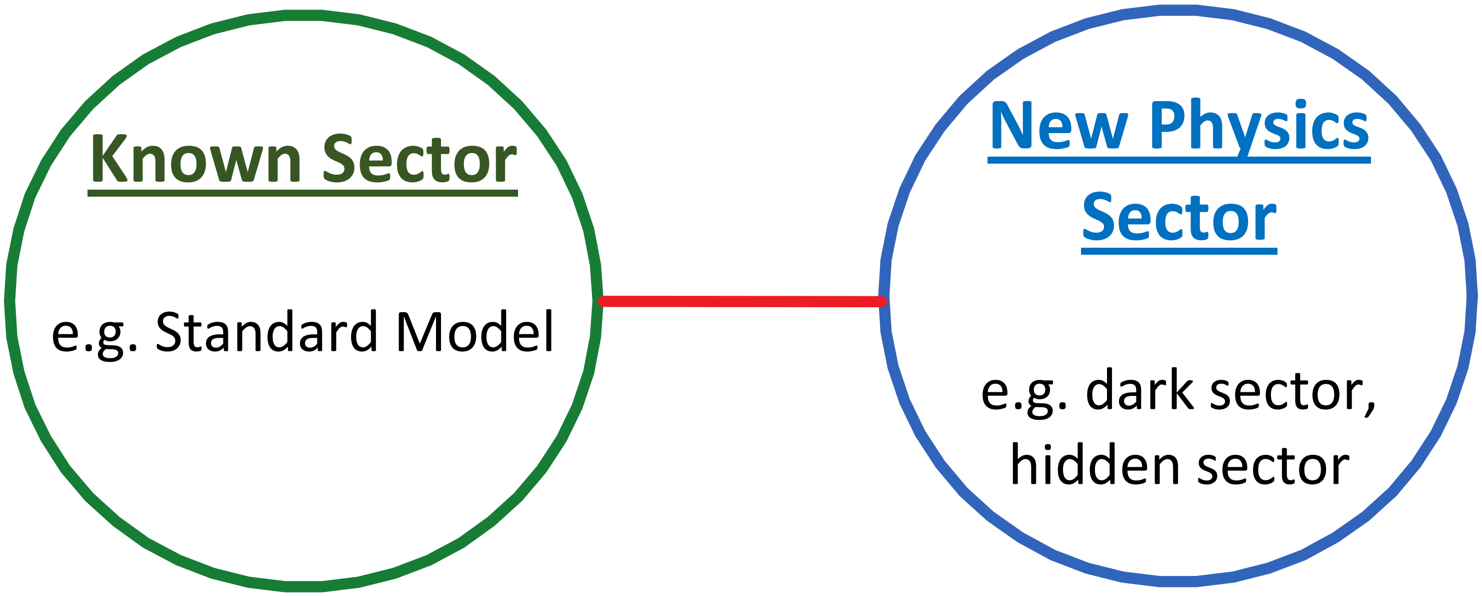


E.g. Dark (matter) sector,

SUSY breaking sector and SUSY breaking mediation,

Composite-Elementary sector, ...

A General Setup in Particle Physics



In all the cases considered so far,

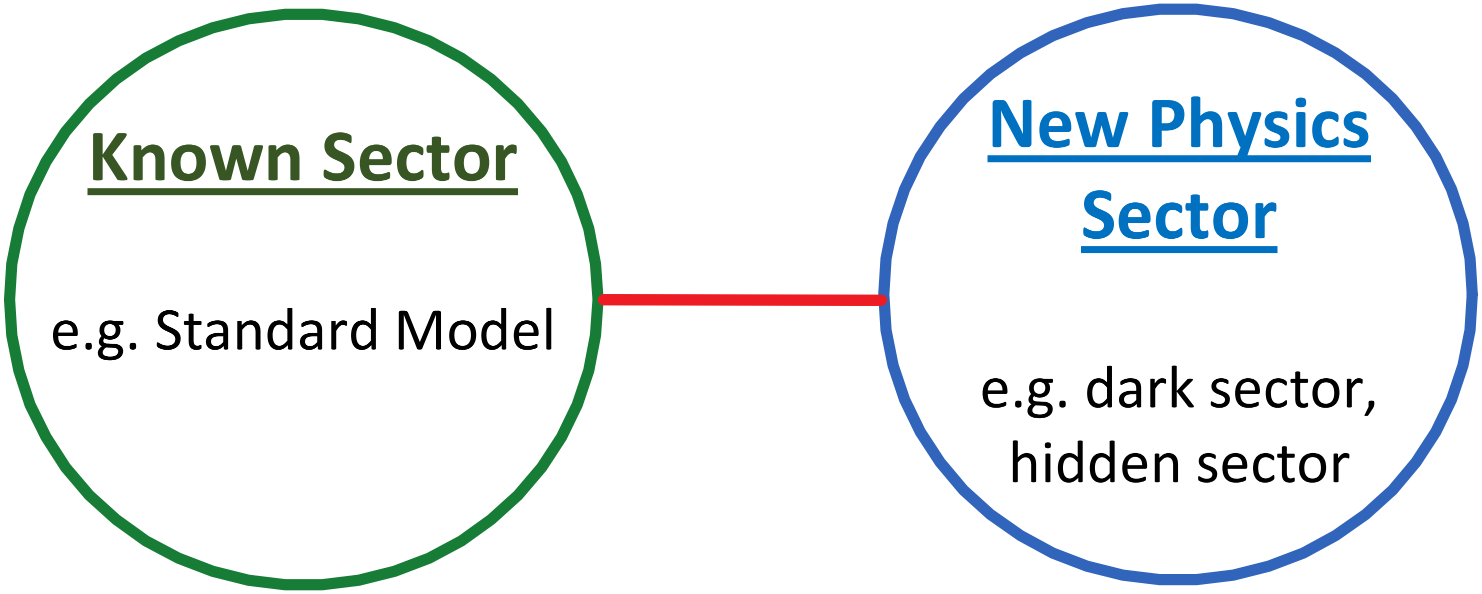
New Physics Sector described by a local QFT

new particles + new interactions

⇒ new/novel dynamics

⇒ solutions to problems in particle physics

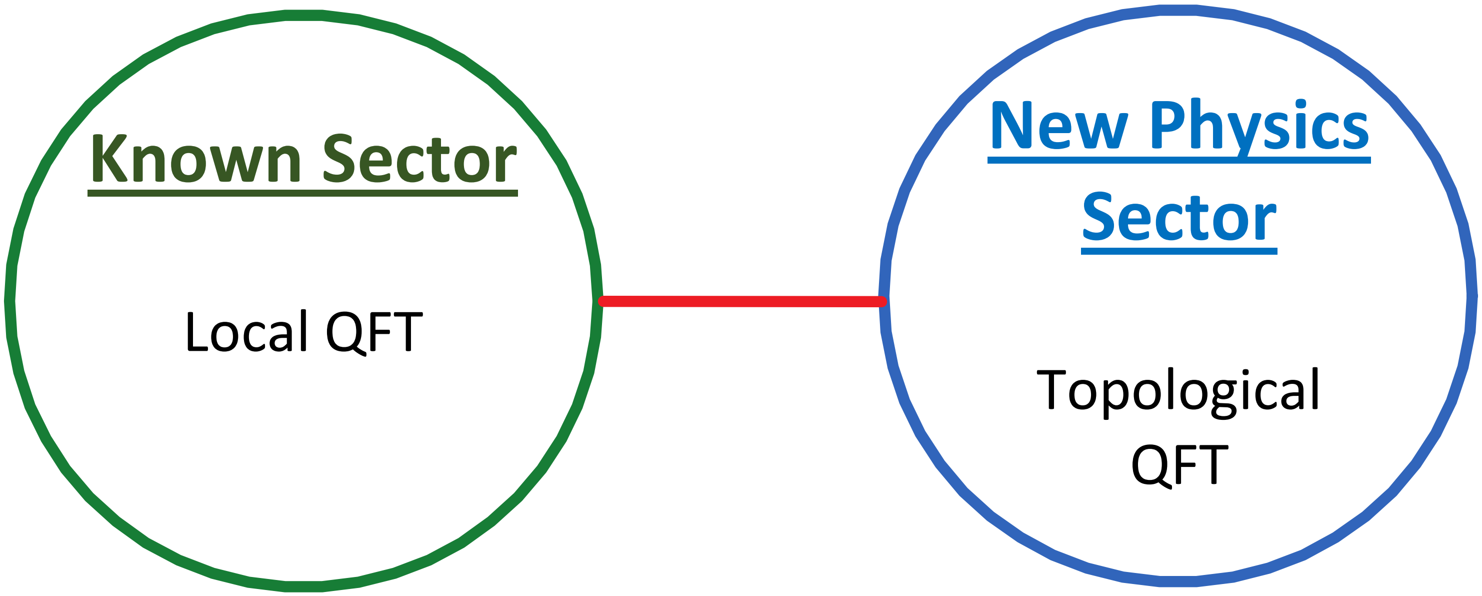
A General Setup in Particle Physics



Symmetry

provides an extremely powerful tool.

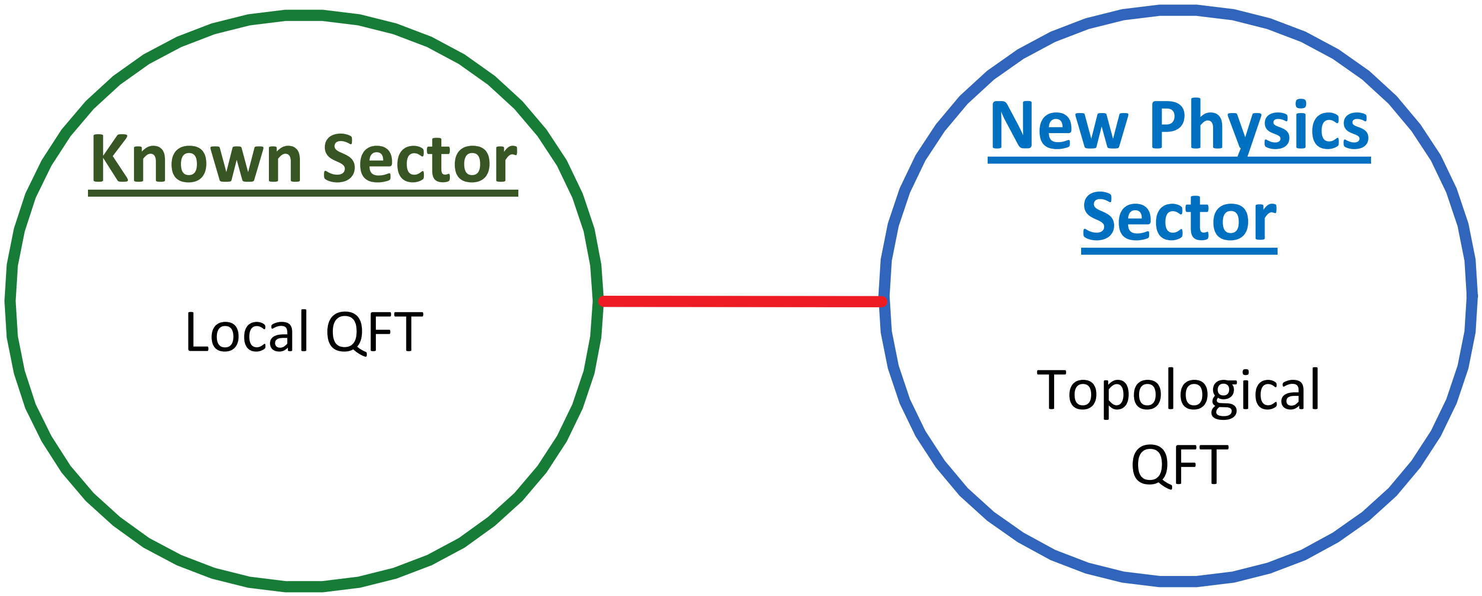
In this talk,



Symmetry

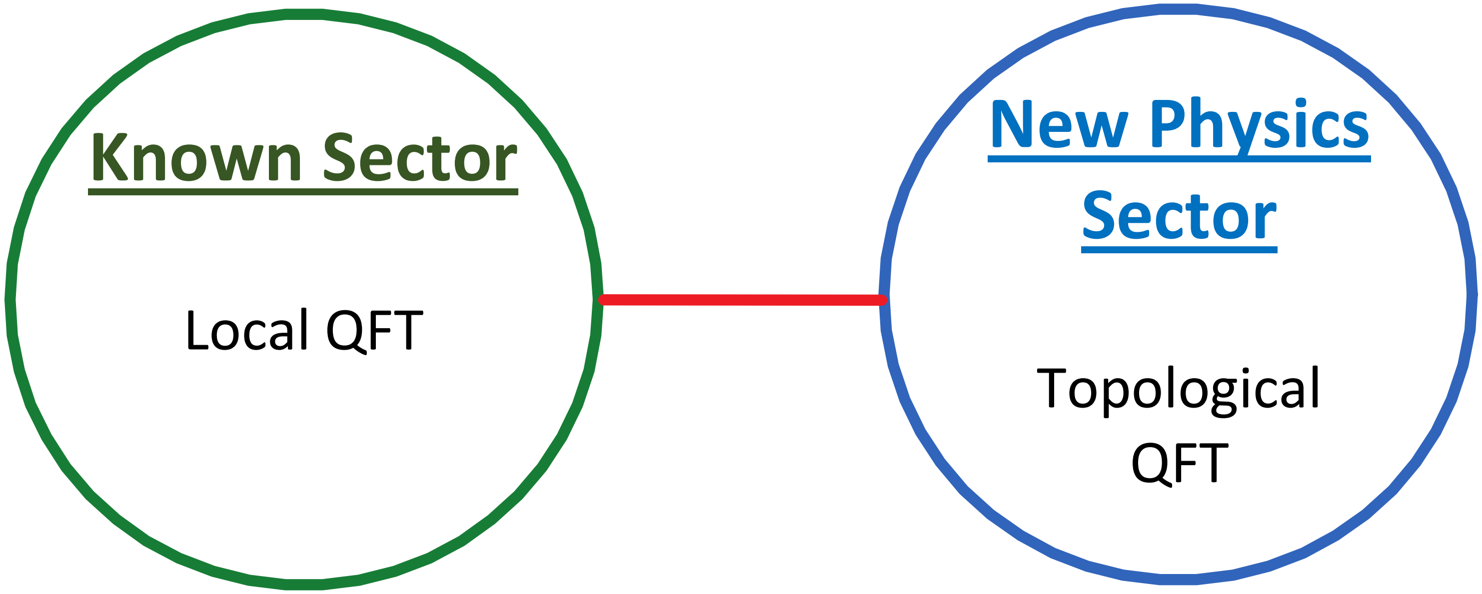
provides an extremely powerful tool.

In this talk,



Generalized Global Symmetries
provides an extremely powerful tool.

In this talk,



Generalized Global Symmetries

provides an extremely powerful tool.

(Q1) Implications of TQFT-couplings

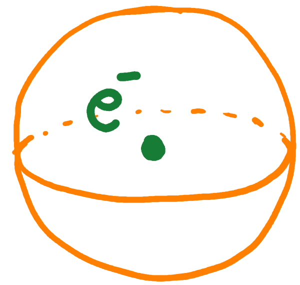
(Q2) Observable consequences (even in principle)

(Q3) show that TQFT-couplings can exist rather ubiquitously.

Generalized Global Symmetries!

Most Symmetries in particle physics act on local operators

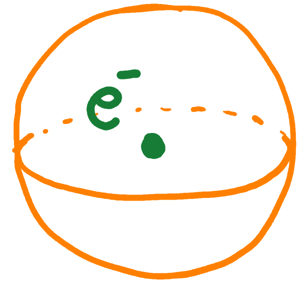
$$\psi(x) \rightarrow e^{i\alpha Q}\,\psi(x)$$



Generalized Global Symmetries!

Most Symmetries in particle physics act on local operators

$$\psi(x) \to e^{i\alpha Q}\, \psi(x)$$



Recently, concept of symmetry has gone through explosive generalizations!

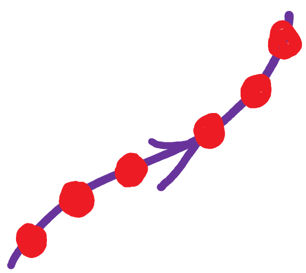
"Generalized Global Symmetries (GGS)"

Generalized Global Symmetries!

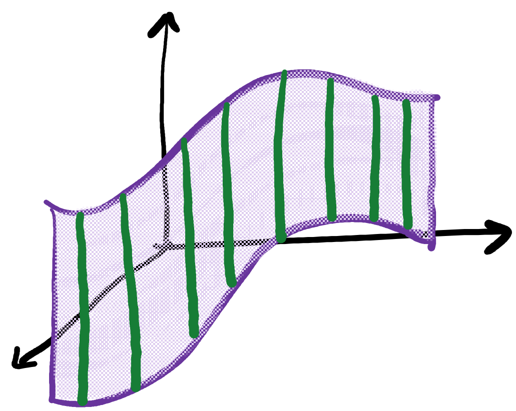
I. Higher-form symmetries

Various **extended objects** appear in broad class of theories.

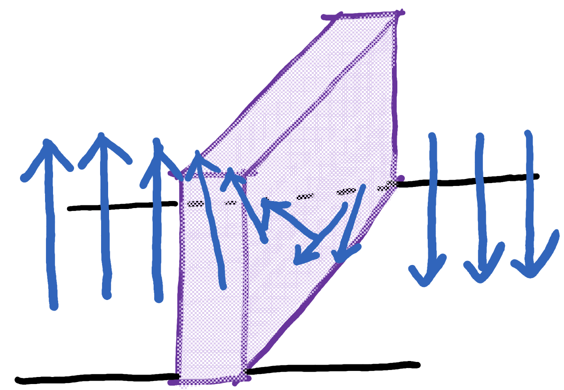
Local operator
e.g. particle
0-form symmetry



Line operator
e.g. Wilson loop
't Hooft loop
1-form symmetry



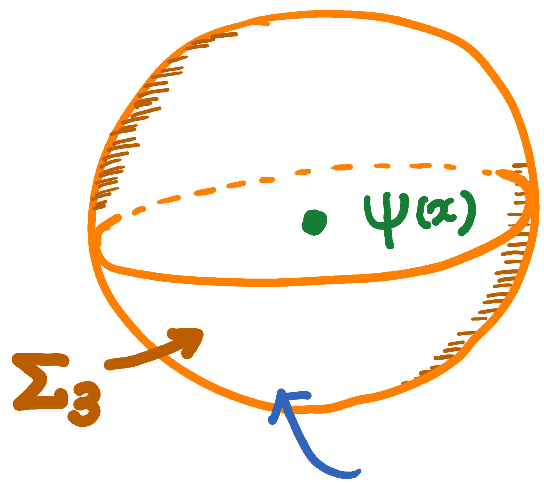
Surface operator
e.g. Cosmic string
2-form symmetry



Volume operator
e.g. Domain Wall
3-form symmetry

Generalized Global Symmetries!

II. Non-Invertible Symmetries



$$S_{defect} = \frac{iN}{4\pi}\int_{\Sigma_3} C \wedge dC$$

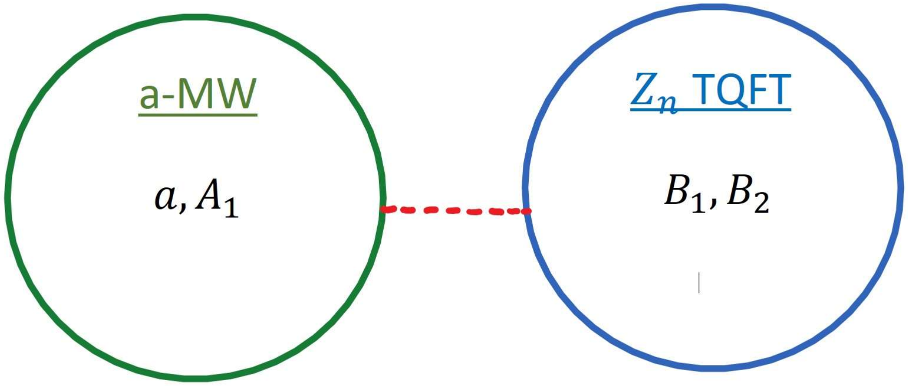
$$C \to C + \frac{1}{N}\epsilon_1, \int \frac{\epsilon_1}{2\pi} \in Z$$

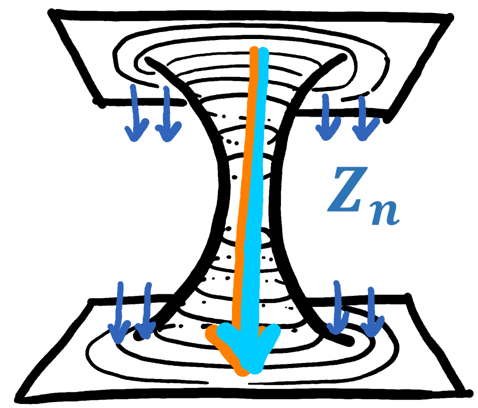
$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \to D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\left(\frac{F}{2\pi}\right) \; with \; \frac{p}{N} = \frac{N_f}{k}$$

Outline

Coupling a Cosmic String to a TQFT

(with T. Daniel Brennan and Liantao Wang)





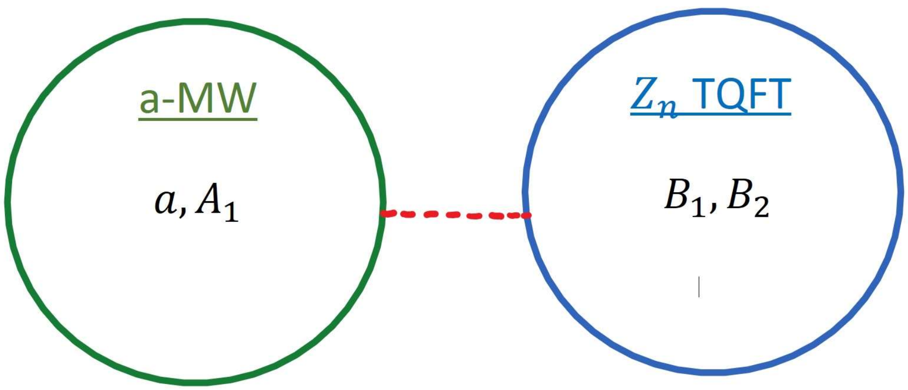
I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT

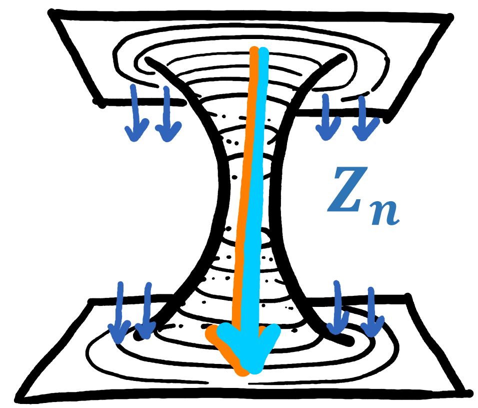
II. TQFT-Coupling 2: Z_M Discrete Gauging

Outline

Coupling a Cosmic String to a TQFT

(with T. Daniel Brennan and Liantao Wang)





I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT

II. TQFT-Coupling 2: Z_M Discrete Gauging

Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2} \frac{a}{f} F \wedge F$$

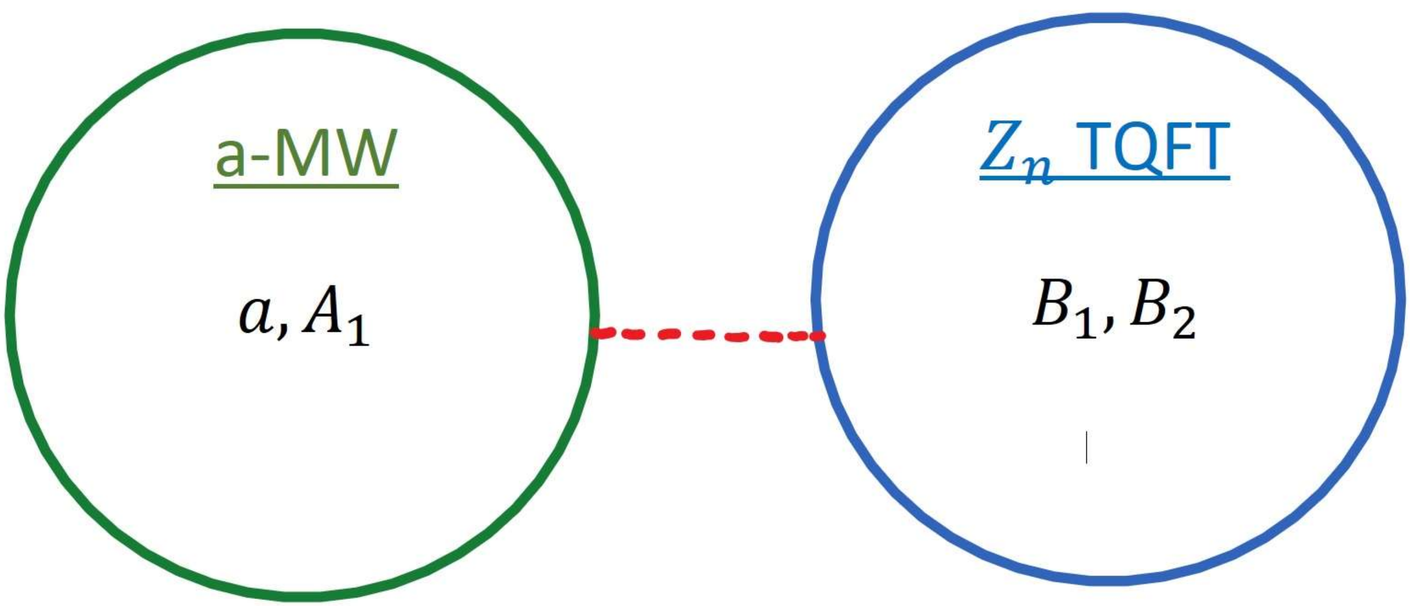
Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \ + \int \frac{1}{2g^2}\ F \wedge * \, F \ - \int \frac{iK}{8\pi^2}\frac{a}{f} F \wedge F$$

- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
  - 1-form electric
  - 1-form magnetic
  - ✸ 3-group
  - ✸ Non-invertible symmetries (Cordova, Ohmori '22)

I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT [Brennan,**Hong**,Wang '23]

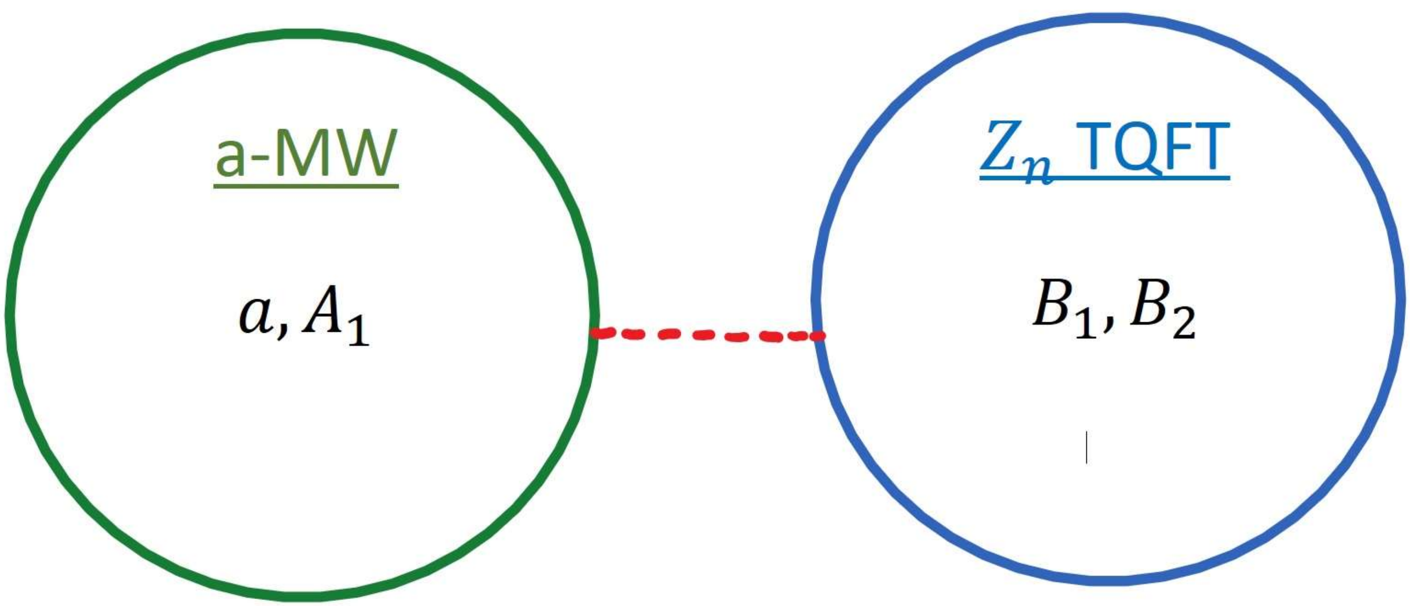
$$S = \int \frac{1}{2}\, da \wedge {*}\, da + \int \frac{1}{2g_A^2}\, F_A \wedge {*}\, F_A - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$



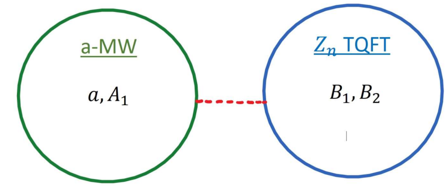
I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT [Brennan,**Hong**,Wang '23]

$$S = \int \frac{1}{2}\, da \wedge^* da \; + \int \frac{1}{2g_A^2}\, F_A \wedge^* F_A \; - \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{4\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$



I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT



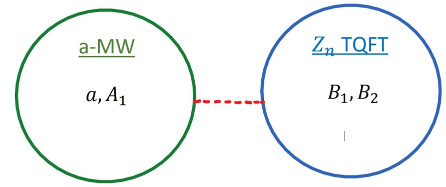
$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{4\pi^2} \frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2} \frac{a}{f} F_B \wedge F_B$$

(Q1) Can there be any **IR-Universal** (local) observable effect?

(Q2) Is this very exotic / pure academic setup?
Or can this arise as IR-EFT of some **standard UV QFT** relevant for particle physics?

I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT



$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{4\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$

(Q1) Can there be any **IR-Universal** (local) observable effect?

(Q2) Is this very exotic / pure academic setup?
Or can this arise as IR-EFT of some **standard UV QFT** relevant for particle physics?

✓ Illustrate importance of studying carefully the effects of remnant TQFT-couplings (GGS = essential tools)

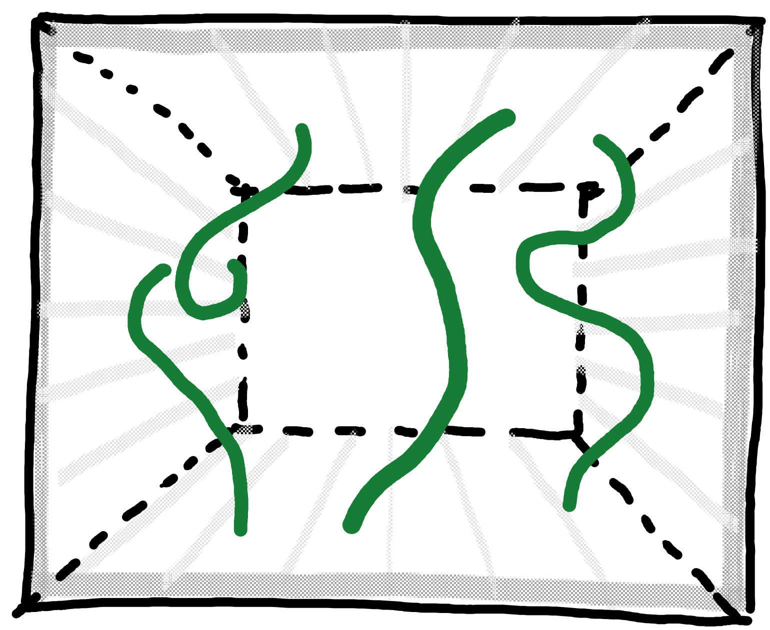
IR-Universal Observables from TQFT-Coupling

- **Anomaly Inflow**

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling [Callan and Harvey '85]

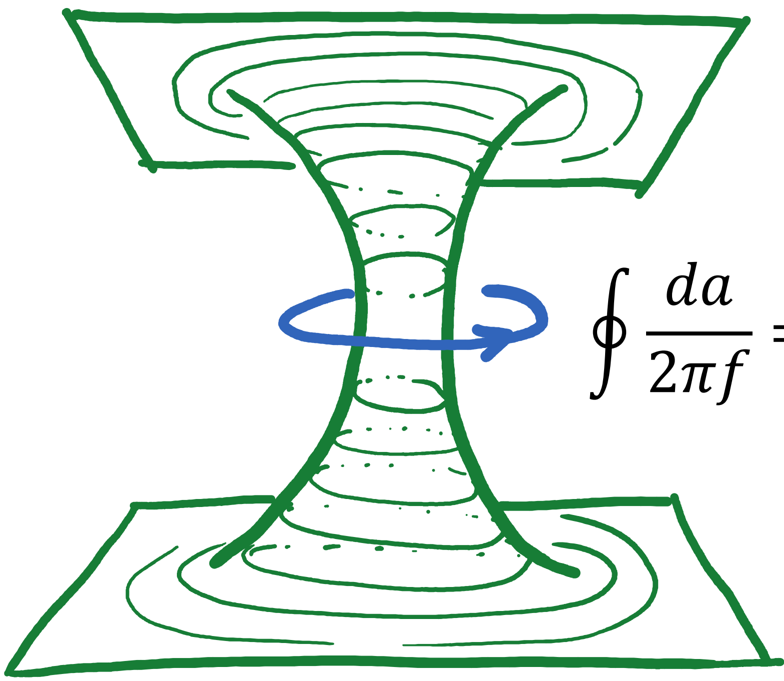
$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$



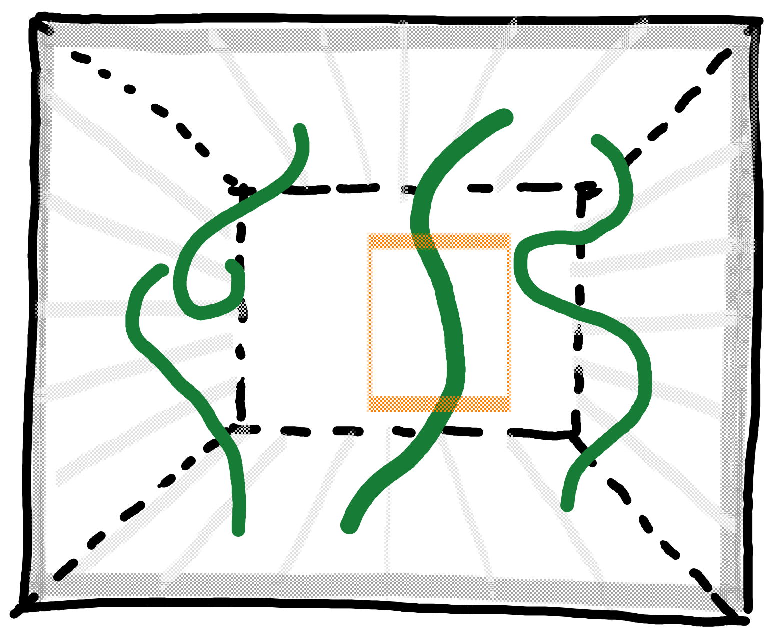
IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling [Callan and Harvey '85]

$$S = \int \frac{1}{2}\, da \wedge {*}\, da \;+ \int \frac{1}{2g_A^2}\, F_A \wedge {*}\, F_A \;- \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$



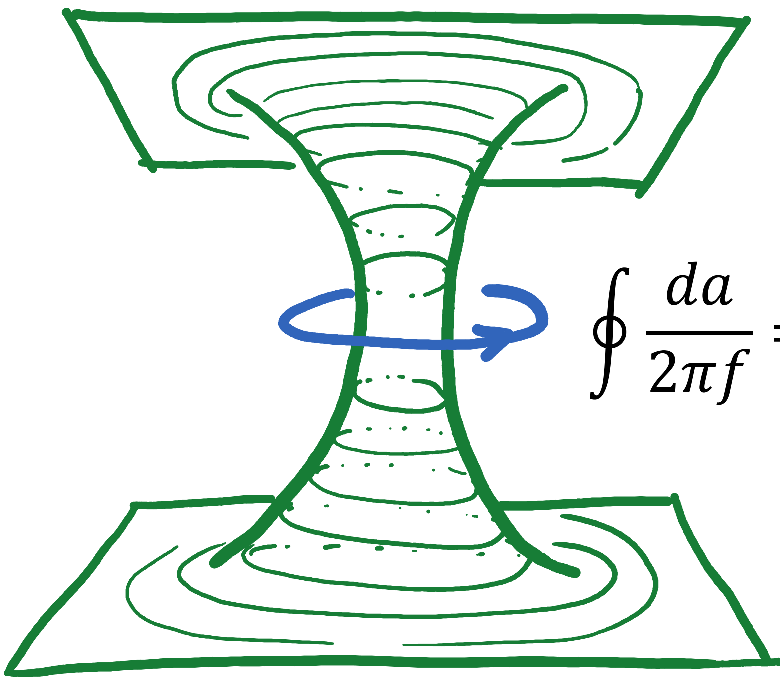
$$\oint \frac{da}{2\pi f} = 1$$



IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling [Callan and Harvey '85]

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \;\; - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$



$$\oint \frac{da}{2\pi f} = 1$$

○ Consistency with $U(1)_A$ invariance :

$$A_1 \to A_1 + d\lambda$$

○ $S \supset \frac{iK_A}{8\pi^2} \int da \wedge A_1 \wedge F_A$

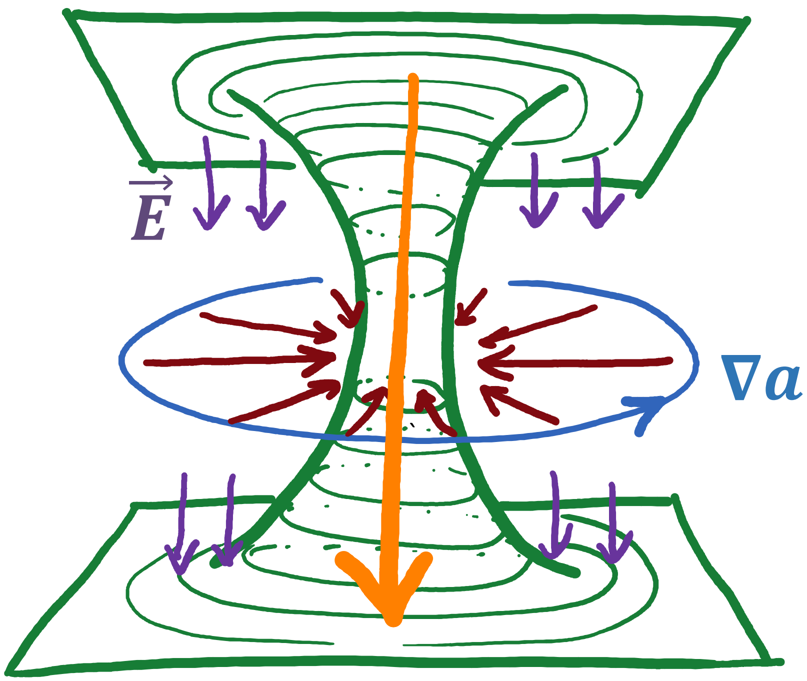
$$\downarrow$$

○ $\delta S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \left(\lambda \frac{K_A}{4\pi} F_A\right)$

IR-Universal Observables from TQFT-Coupling

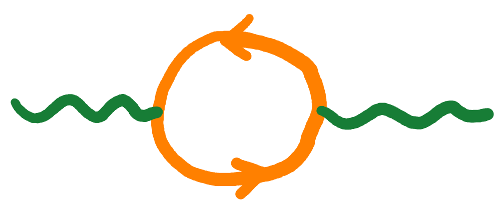
✸ **Anomaly Inflow** : W/O TQFT-Coupling [Callan and Harvey '85]

$$S \supset \frac{iK_A}{8\pi^2}\int da \wedge A_1 \wedge F_A = i\int A_1 \wedge * J_1$$



- $* J_1 = \frac{K_A}{4\pi}\, da \wedge F_A$
- $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi}\, F_A \wedge \delta^{(2)}\left(M_2^{st}\right)$
- $\vec{j}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)
- 2d chiral fermions $\{\alpha_i\,(z,t)\}$

$$\mathrm{d} * J_1(2d) = -\frac{K_A}{4\pi}\, F_A$$



$$\sum_i Q_i^2 = K_A$$

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{4\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi} B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{4\pi^2} \frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2} \frac{a}{f} F_B \wedge F_B$$

1. $A_1 \to A_1 + d\lambda_A$

$$\delta_A S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$$

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A \; - \int \frac{iK_A}{8\pi^2}\frac{a}{f} F_A \wedge F_A$$

$$+ \int \frac{in}{2\pi} B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{4\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$

1. $A_1 \to A_1 + d\lambda_A$

$$\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$$

2. $B_1 \to B_1 + d\lambda_B \,, \qquad \lambda_B = \frac{2\pi}{n}\kappa \,, \qquad \kappa = 0,1,\cdots,n-1$

$$\delta_B S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_B \left(\frac{K_{AB}}{2\pi} F_A + \frac{K_B}{4\pi} F_B \right)$$

IR-Universal Observables from TQFT-Coupling

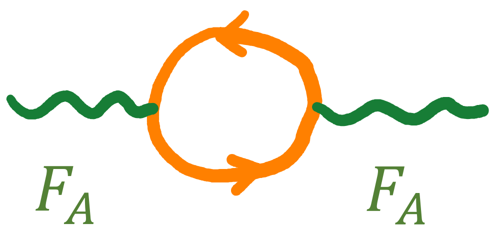
- **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

1. $$\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$$

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$



IR-Universal Observables from TQFT-Coupling

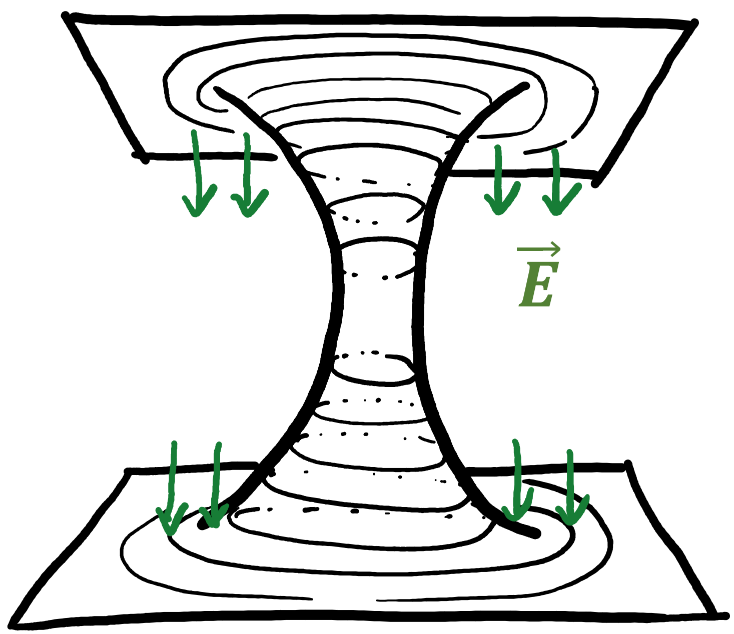
✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

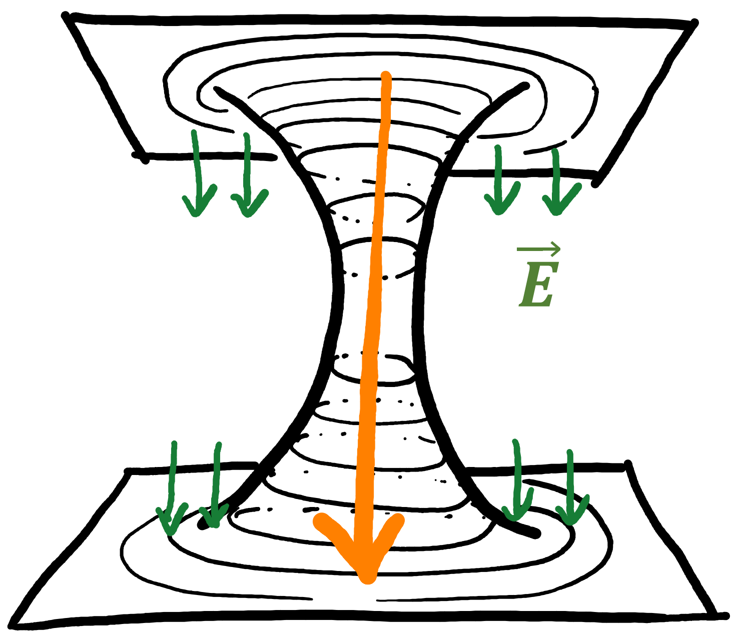
1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$



IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

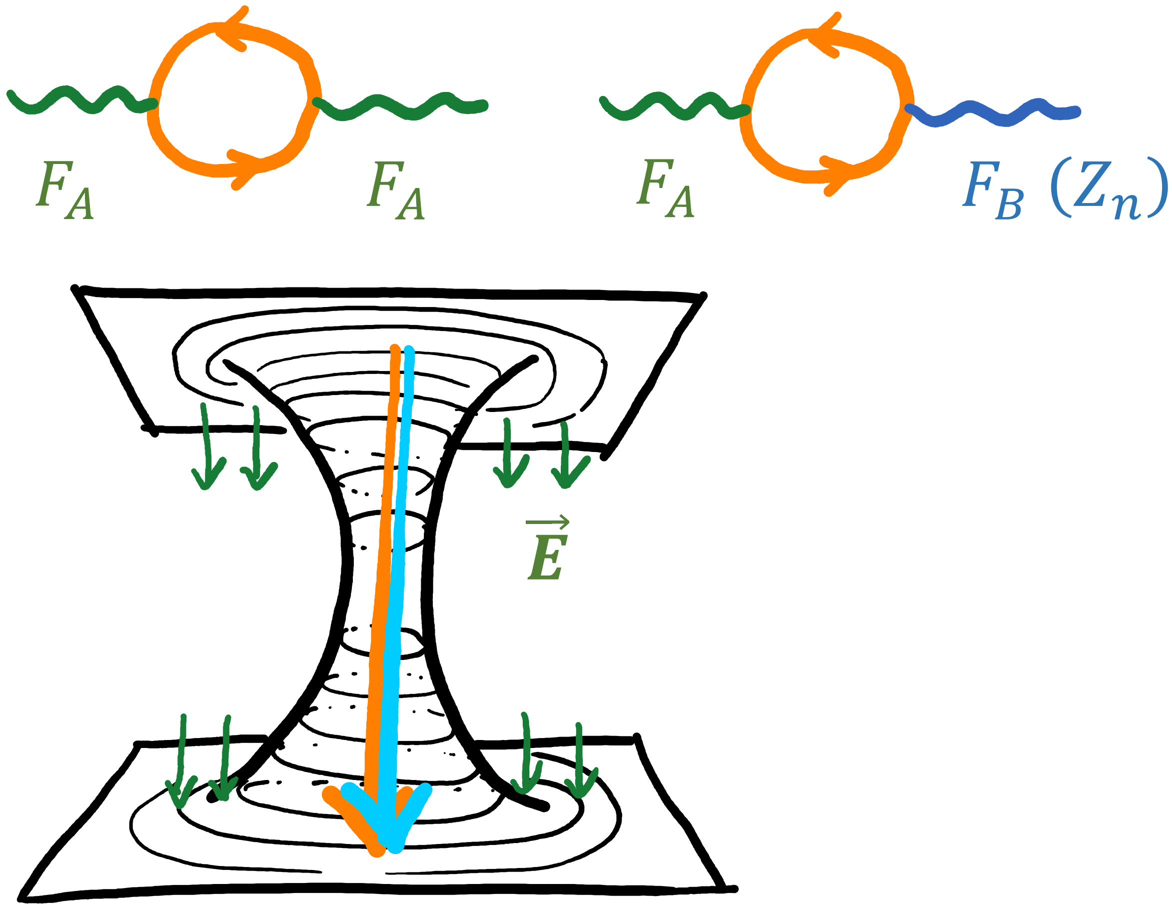
1. $\delta_A S = i\int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$



IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

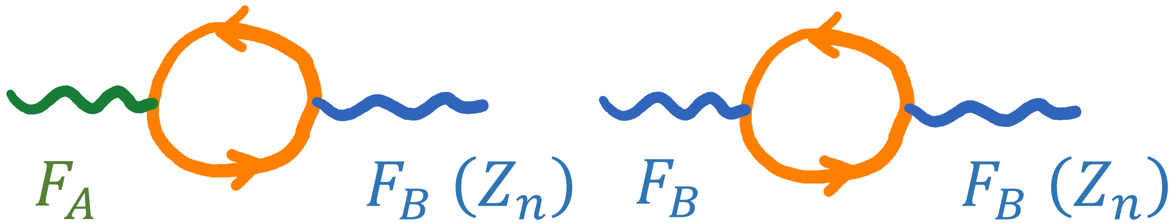
1. $\delta_A S = i \int \delta^{(2)}(M_2^{st}) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B \right)$

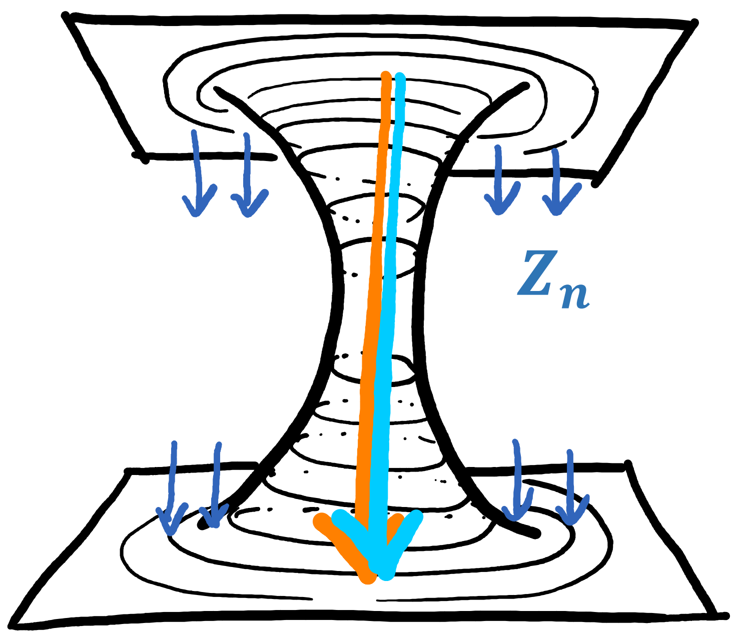


IR-Universal Observables from TQFT-Coupling

- **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

2. $$\delta_B S = i\int \delta^{(2)}(M_2^{st}) \wedge \lambda_B\left(\frac{K_{AB}}{2\pi}F_A + \frac{K_B}{4\pi}F_B\right)$$

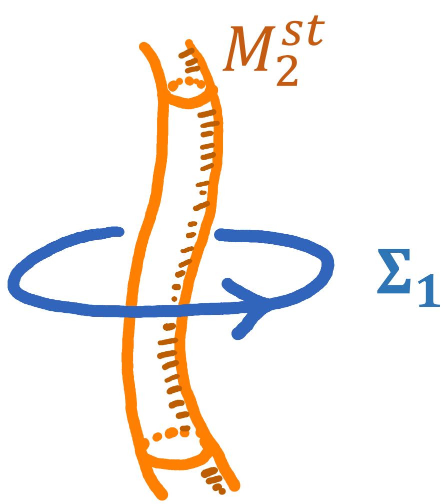




IR-Universal Observables from TQFT-Coupling

W/O TQFT-Coupling

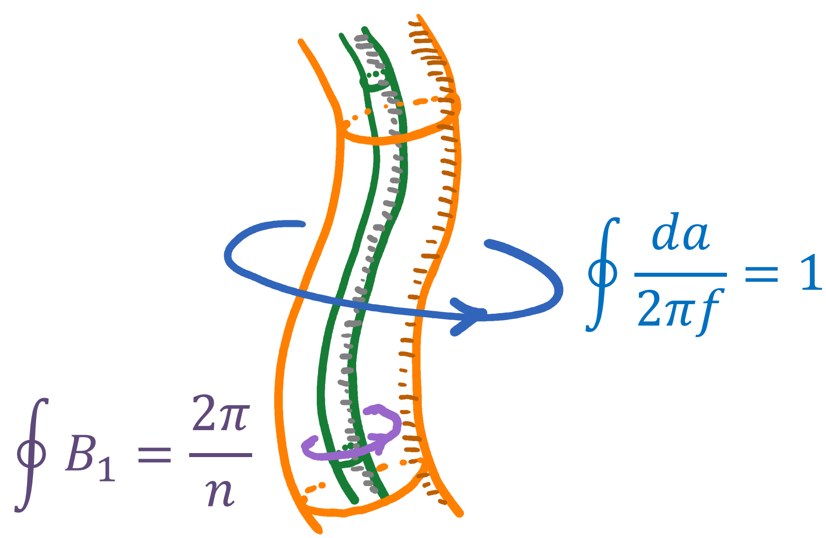
- Axion strings: Global strings



With TQFT-Coupling

- Axion strings: Global strings
- BF strings: $W_2(\Sigma_2, \ell) = e^{i\ell \oint_{\Sigma_2} B_2}$

  Local or (Quasi) Aharonov-Bohm
- Coaxial Hybrid strings ?



Extended KSVZ with TQFT-Coupling [Brennan,**Hong**,Wang '23]

$$\mathcal{L} = -\frac{1}{2g_A^2}\, F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1$$

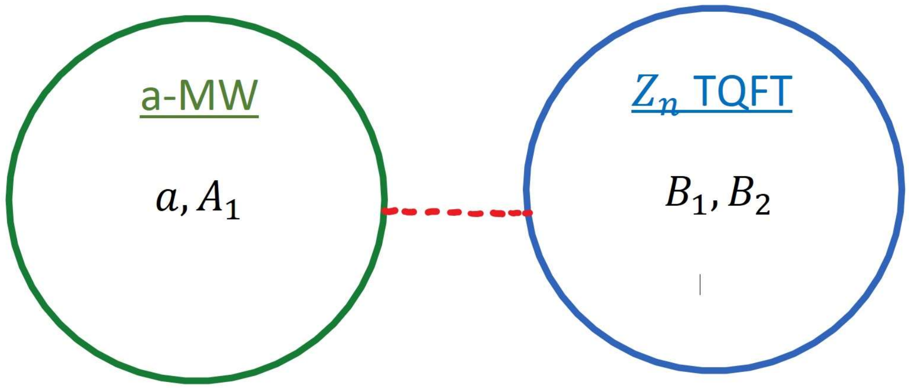
$$-\frac{1}{2g_B^2}\, F_B \wedge * F_B + \overline{\psi_2} i\gamma^\mu D_\mu \psi_2 + \overline{\chi_2} i\gamma^\mu D_\mu \chi_2 - \lambda_2 \Phi_2 \psi_2 \chi_2 + V(\Phi_1, \Phi_2)$$

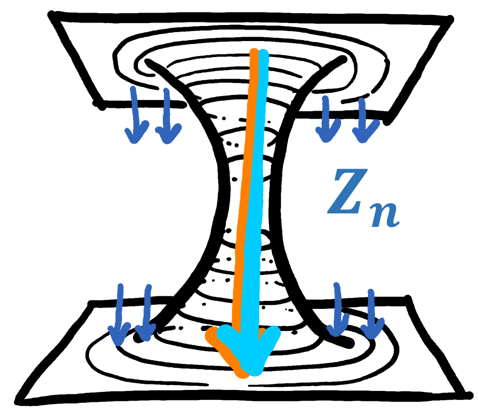
	$U(1)_{PQ}$	$U(1)_A$	$U(1)_B$
Φ_1	1	0	n
Φ_2	0	0	n
ψ_1	1	1	q
χ_1	0	-1	n-q
ψ_2	0	1	q-n
χ_2	0	-1	-q

Outline

Coupling a Cosmic String to a TQFT

(with T. Daniel Brennan and Liantao Wang)





I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT

II. TQFT-Coupling 2: Z_M Discrete Gauging

II. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan,**Hong**,Wang '23]

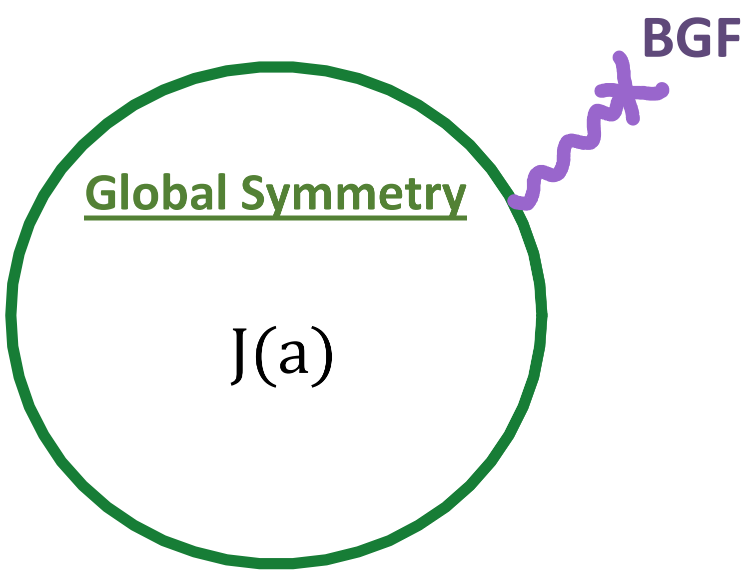
(i) Recall : 0-form axion shift

- $\theta = \frac{a}{f} \rightarrow \theta + c$
- EoM(a): $d * da = 0 \rightarrow d * j_1 = 0, \quad * j_1 = if * da$
- With coupling: $d(if * da) = \frac{K}{8\pi^2} F \wedge F \rightarrow \left[U(1)^{(0)} \rightarrow {Z_K}^{(0)}\right]$
- ABJ-anomaly free : ${Z_K}^{(0)}$
- We can gauge a subgroup : ${Z_M}^{(0)} \subset {Z_K}^{(0)}$

II. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan,**Hong**,Wang '23]

(ii) Gauging a discrete group = Coupling to a TQFT

$$S \supset -i \int A_1 \wedge * J_1 = \frac{1}{2} \int (da - f\mathcal{A}_1) \wedge * (da - f\mathcal{A}_1)$$



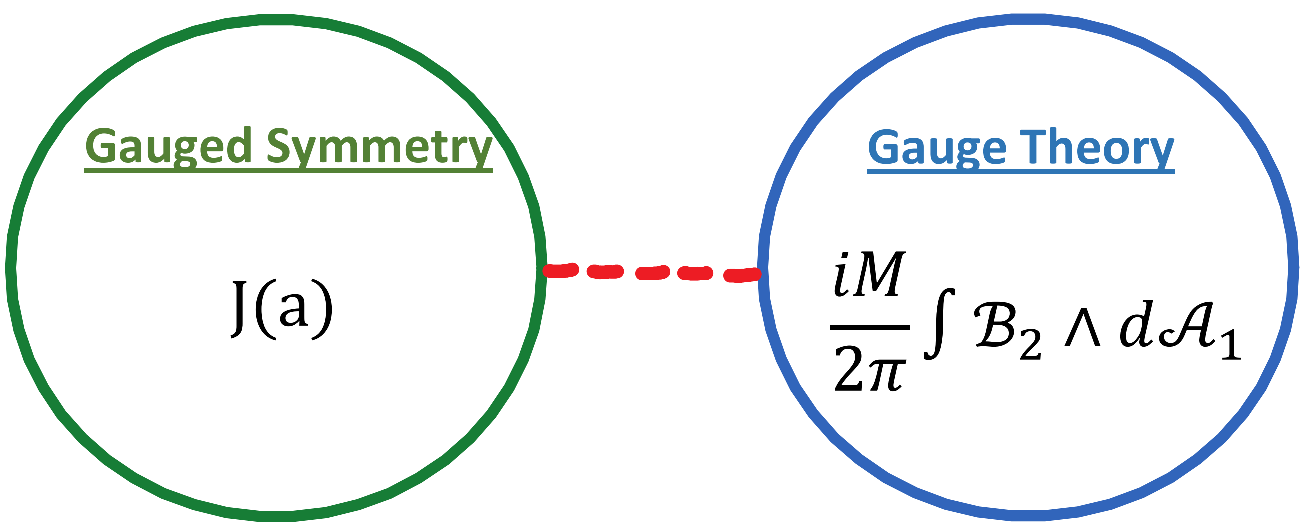
II. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan,**Hong**,Wang '23]

(ii) Gauging a discrete group = Coupling to a TQFT

$$S \supset -i \int A_1 \wedge * J_1 = \frac{1}{2} \int (da - f\mathcal{A}_1) \wedge * (da - f\mathcal{A}_1)$$

$$\Downarrow$$

$$S \supset \frac{1}{2} \int (da - f\mathcal{A}_1) \wedge * (da - f\mathcal{A}_1) + \frac{iM}{2\pi} \int \mathcal{B}_2 \wedge d\mathcal{A}_1$$



III. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan,**Hong**,Wang '23]

(iii) Physical effects of discrete gauging?

- Gauge redundancy: $\frac{a}{f} \sim \frac{a}{f} + 2\pi \;\rightarrow\; \frac{a}{f} \sim \frac{a}{f} + \frac{2\pi}{M}$
- Project out local operators:

  Local operators charged under 0-form Z_K axion shift: $I(x) = e^{iqa(x)/f}$

  Under gauged Z_M: $I(x) \rightarrow e^{\frac{i2\pi q}{M}} I(x) \;\rightarrow\; I(x),\; q \notin MZ$ projected out

III. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan,**Hong**,Wang '23]

(iii) Physical effects of discrete gauging?

- Gauge redundancy: $\frac{a}{f} \sim \frac{a}{f} + 2\pi \rightarrow \frac{a}{f} \sim \frac{a}{f} + \frac{2\pi}{M}$
- Project out local operators:

  Local operators charged under 0-form Z_K axion shift: $I(x) = e^{iqa(x)/f}$

  Under gauged Z_M: $I(x) \rightarrow e^{\frac{i2\pi q}{M}} I(x) \rightarrow I(x),\ q \notin MZ$ projected out
- Add cosmic strings with "fractional" winding number: $\int_{\Sigma_1} \frac{da}{2\pi f} \in \frac{1}{M} Z$

III. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan,**Hong**,Wang '23]

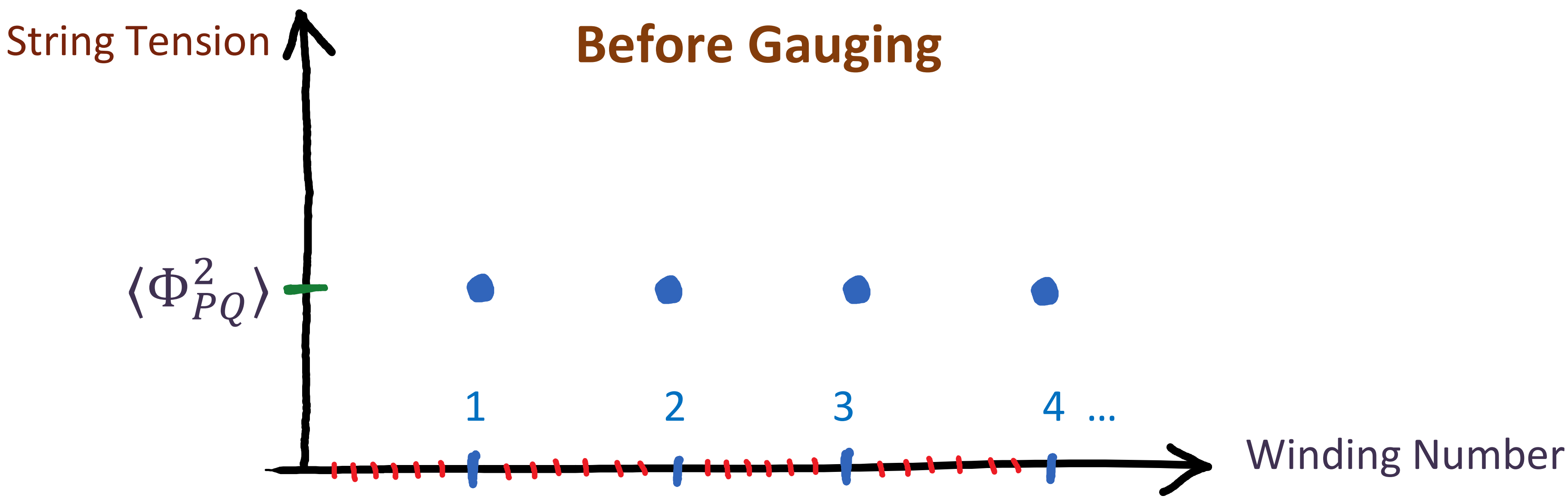
(iii) Physical effects of discrete gauging?

- Gauge redundancy: $\frac{a}{f} \sim \frac{a}{f} + 2\pi \rightarrow \frac{a}{f} \sim \frac{a}{f} + \frac{2\pi}{M}$
- Project out local operators:

  Local operators charged under 0-form Z_K axion shift: $I(x) = e^{iqa(x)/f}$

  Under gauged Z_M: $I(x) \rightarrow e^{\frac{i2\pi q}{M}} I(x) \rightarrow I(x),\ q \notin MZ$ projected out

- Add cosmic strings with "fractional" winding number: $\int_{\Sigma_1} \frac{da}{2\pi f} \in \frac{1}{M} Z$



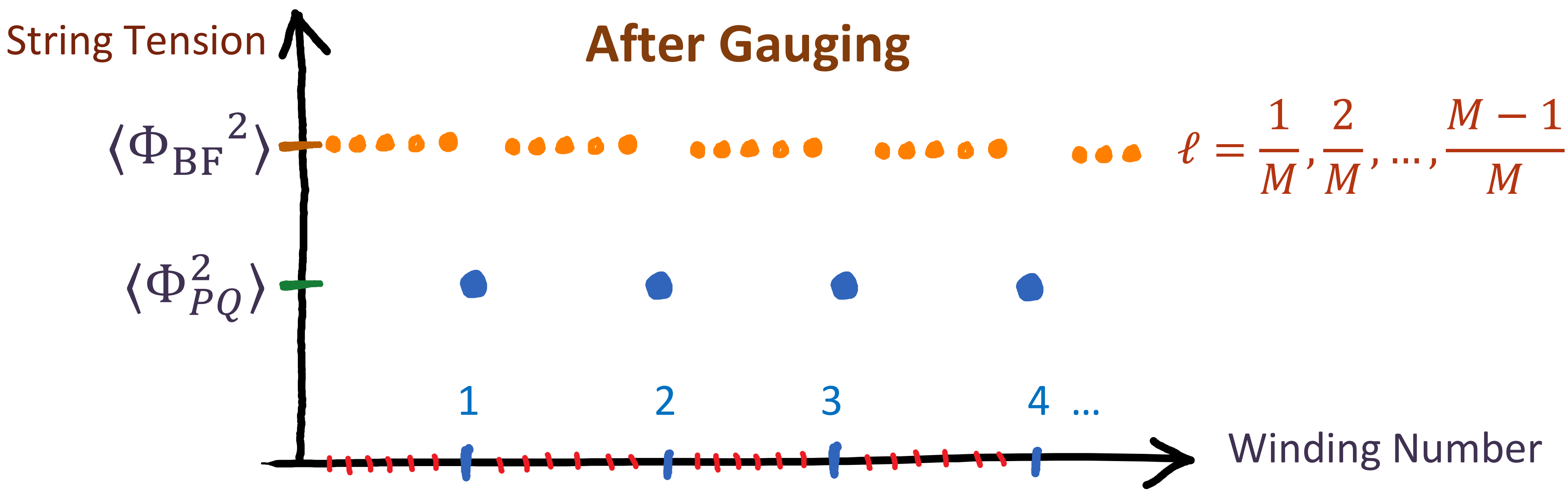
III. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan, **Hong**, Wang '23]

(iii) Physical effects of discrete gauging?

- Gauge redundancy: $\frac{a}{f} \sim \frac{a}{f} + 2\pi \;\rightarrow\; \frac{a}{f} \sim \frac{a}{f} + \frac{2\pi}{M}$
- Project out local operators:

  Local operators charged under 0-form Z_K axion shift: $I(x) = e^{iqa(x)/f}$

  Under gauged Z_M: $I(x) \rightarrow e^{\frac{i2\pi q}{M}} I(x) \;\rightarrow\; I(x),\; q \notin MZ$ projected out
- Add cosmic strings with "fractional" winding number: $\int_{\Sigma_1} \frac{da}{2\pi f} \in \frac{1}{M} Z$



III. TQFT-Coupling 2: Gauging Discrete Subgroup [Brennan,**Hong**,Wang '23]

(iii) Physical effects of discrete gauging?

- Gauge redundancy: $\frac{a}{f} \sim \frac{a}{f} + 2\pi \;\rightarrow\; \frac{a}{f} \sim \frac{a}{f} + \frac{2\pi}{M}$
- Project out local operators:

  Local operators charged under 0-form Z_K axion shift: $I(x) = e^{iqa(x)/f}$

  Under gauged Z_M: $I(x) \rightarrow e^{\frac{i2\pi q}{M}} I(x) \;\rightarrow\; I(x),\; q \notin MZ$ projected out
- Add cosmic strings with "fractional" winding number: $\int_{\Sigma_1} \frac{da}{2\pi f} \in \frac{1}{M} Z$
- Breaks electric 1-form symmetry: seen from 3-group structure.
- 3-group analysis $\Rightarrow$ systematic classification of all possible TQFT-couplings via discrete gauging and associated physical effects

Thank you!