UNIVERSITÀ DEGLI STUDI DI MILANO

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Parity violation studies at colliders: a few theoretical aspects

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Topics to be discussed

- High-precision measurements at low-energy facilities

  What is the main interest?

  a) high-precision (stress) test of the SM at the level of its quantum corrections
  at energy scales much different than those probed at high-energy colliders
  e.g. independent determination $\sin^2\theta_W$ might help to solve long-standing discrepancies

  b) New Physics searches
  several urgent questions (e.g. existence of dark matter, the matter-antimatter asymmetry) point out that the SM can not be the final theory of the fundamental interactions

  how should we look for New Physics, relevant at high-energy scales, in low-energy experiment ?
  → Precision (experimental and theoretical)

  we look for any possible significant discrepancy between the data and the best SM predictions
  if a significant tension appears, it can be interpreted as a first indirect hint towards New Physics states
  which contribute at the quantum level via virtual corrections

Topics to be discussed

- Complementarity with high-energy experiments

the search for BSM signals benefits of a very precise understanding of the energy dependence of the observables

one single deviation from the SM is not conclusive evidence of New Physics. (e.g. the CDF result for m_W)

a systematic pattern of deviations from the SM, at different energies, would be a more significant signal

the determination of the running with energy of the fundamental couplings of the SM lagrangian
is a complex program of studies which can yield such evidence

How can we access the value of the gauge couplings ?

- the EW charged current interaction has a clear V-A structure
  identification of the e.m. current in the neutral sector → prediction of a new current coupling to the Z boson
  the weak mixing angle parameterises which combination of $SU(2)_L$ and $U(1)_Y$ enter in the Z field

$$g \sin\theta_W = g' \cos\theta_W = e \qquad \rightarrow \qquad \tan\theta_W = \frac{g'}{g}$$

(only two parameters (g, g') are independent!)

the vector and axial-vector couplings of the Z boson to fermions depend on $\sin^2\theta_W$

$$v_f = T_f - 2Q_f \sin^2\theta_W \qquad a_f = T_f$$

→ their measurement allows the $\sin^2\theta_W$ determination

the non-vanishing axial-vector coupling leads to parity violation in the processes mediated by the weak interaction
→ observables sensitive to parity violation are thus useful to determine $\sin^2\theta_W$

e.g. in e- p scattering $$A_{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha_{em}}\left(1 - 4\sin^2\theta_W - F(E_i, Q^2)\right)$$

where $\sigma_\pm$ are the electron-proton cross sections with polarised electrons in a given fiducial volume

Parity violation: what can be learned from precision e- p measurements?

The asymmetry $A_{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha_{em}} \left(Q_W - F(E_i, Q^2) \right)$ is obtained polarising the electron beam

$$A_{PV}(P2) \sim -40 \cdot 10^{-9}$$

- A_{PV} is proportional to the weak charge of the proton, accidentally suppressed in the SM: $Q_W(p) = 1 - 4\sin^2\theta_W \sim 0.09$

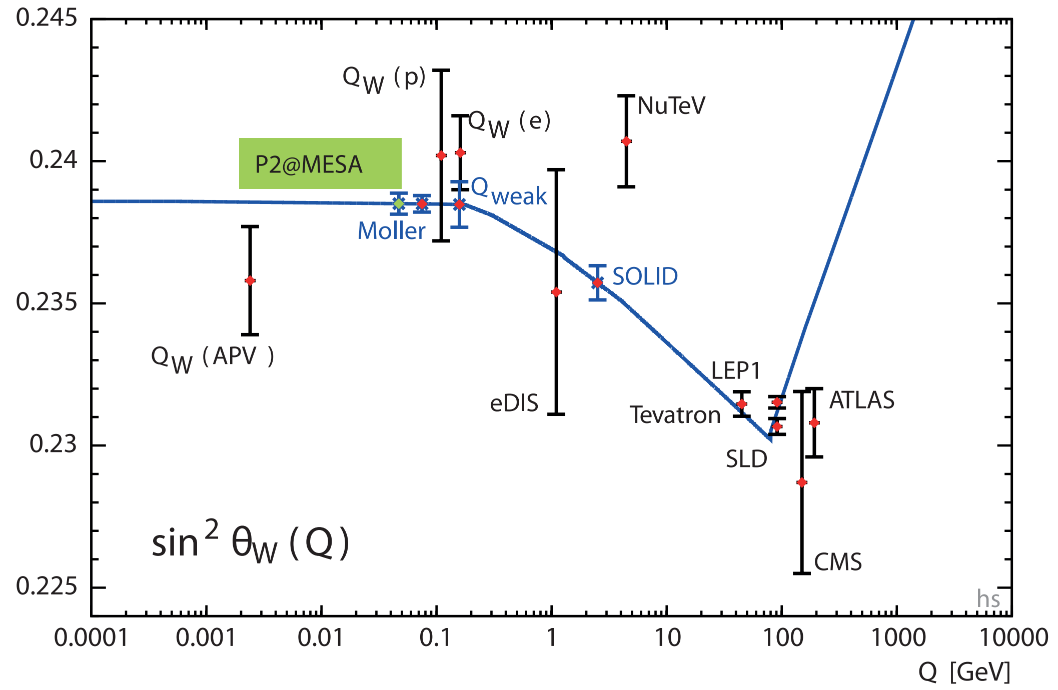
- the tree-level suppression of $Q_W(p)$ i) enhances the sensitivity to $\sin^2\theta_W$: $\Delta Q_W / Q_W \sim 0.09 \; \Delta\sin^2\theta_W / \sin^2\theta_W$

  → a measurement at the 1.4% level of $A_{PV}(P2)$ allows a determination of $\sin^2\theta_W$ with an error $\Delta\sin^2\theta_W \sim 33 \cdot 10^{-5}$ (cfr. LEP error $\Delta\sin^2\theta_W \sim 16 \cdot 10^{-5}$)

  ii) enhances the impact of the radiative corrections (e.g. -39% in Møller scattering)

- radiative corrections contribute to the precise value of the asymmetry A_{PV} (→ $\sin^2\theta_W$ determination)
  may include BSM contributions (tree-level suppression of $Q_W(p)$ →enhanced sensitivity to BSM effects)

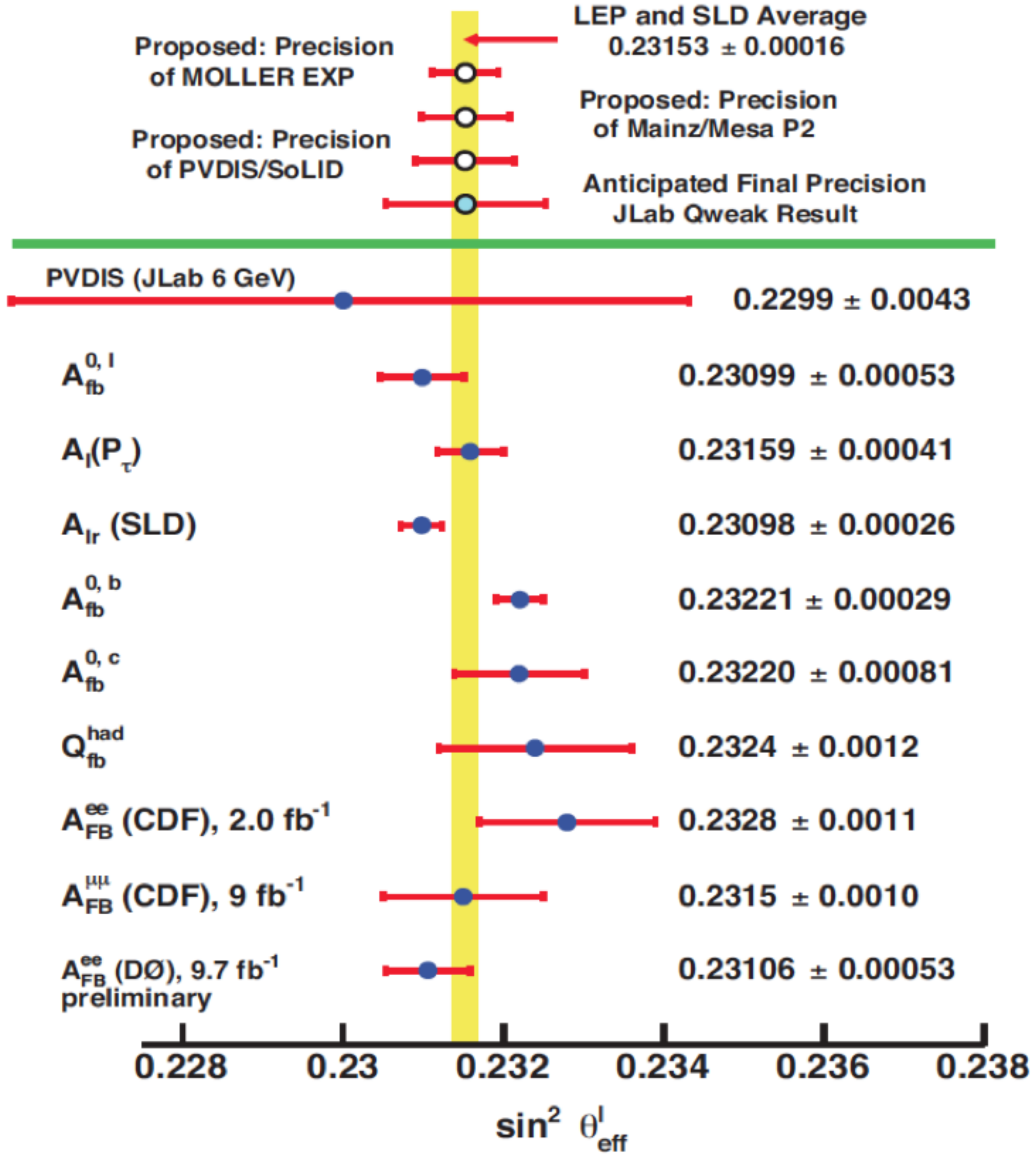
- the value of the effective weak mixing angle at $q^2 = 0$ is about 3% larger than at $q^2 = m_Z^2$
  this SM prediction has to be tested and it might reveal BSM effects



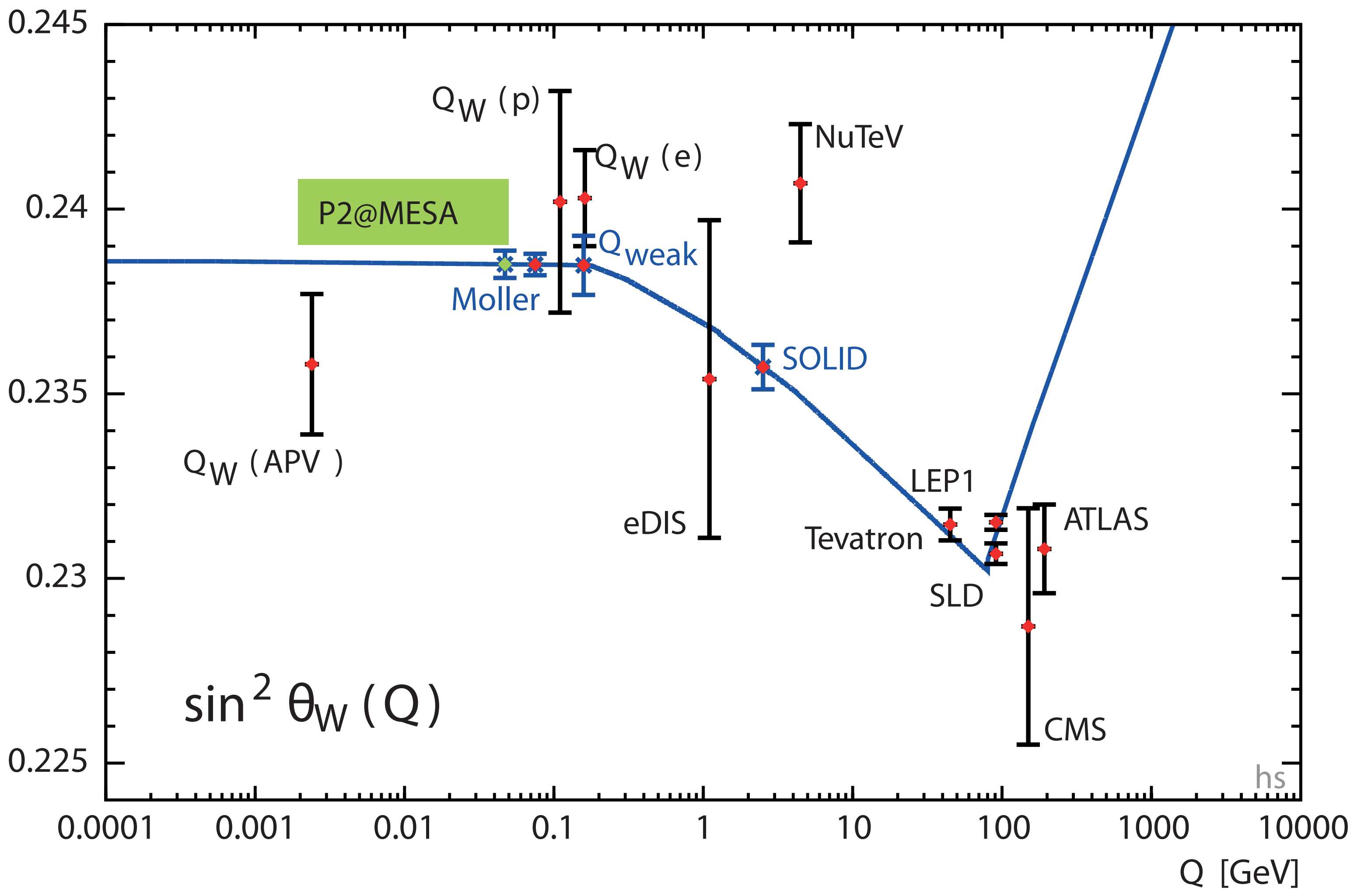
Comparison of different weak mixing angle determinations

The sensible comparison of different determinations of $\sin^2\theta_W$ offers a test of the SM

- the values extracted at e+e- and hadron colliders are based on observables with different systematics but also use different definitions to fit the data
- for a meaningful test, it is important to compare the same weak mixing angle (different definitions appear when discussing the quantum corrections)



LEP/SLD longstanding discrepancies might be clarified



The renormalisation of the SM and a framework for precision tests

- The Standard Model is a renormalizable gauge theory based on SU(3) x $SU(2)_L$ x $U(1)_Y$
- The gauge sector of the SM lagrangian is assigned specifying (g, g', v, λ) in terms of 4 measurable inputs
- More observables can be computed and expressed in terms of the input parameters, including the available radiative corrections, at any order in perturbation theory
- The validity of the SM can be tested comparing these predictions with the corresponding experimental results

- The input choice $(g, g', v, \lambda) \leftrightarrow (\alpha(0), G_\mu, m_Z, m_H)$ minimises the parametric uncertainty of the predictions

$$\begin{aligned} \alpha(0) &= 1/137.035999139(31) \\ G_\mu &= 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2} \\ m_Z &= 91.1876(21)\ \mathrm{GeV}/c^2 \\ m_H &= 125.09(24)\ \mathrm{GeV}/c^2 \end{aligned}$$

- with these inputs, MW and the weak mixing angle are predictions of the SM, to be tested against the experimental data

The weak mixing angle(s): theoretical prediction(s) at $q^2 = m_Z^2$

- the prediction of the weak mixing angle can be computed in different renormalisation schemes differing for the systematic inclusion of large higher-order corrections

- on-shell definition: $$\sin^2\theta_{OS} = 1 - \frac{m_W^2}{m_Z^2}$$ definition valid to all orders

  Sirlin, 1980

- MSbar definition: $$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8 m_{W,0}^2} \longrightarrow \hat{s}^2\hat{c}^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2(1-\Delta\hat{r})}$$ $$\hat{s}^2 \equiv \sin^2\hat{\theta}(\mu_R = m_Z)$$

  Marciano, Sirlin, 1980; Degrassi, Sirlin, 1991

  weak dependence on top-quark corrections

- the effective leptonic weak mixing angle enters in the definition of the effective Z-f-fbar vertex at the Z resonance ($q^2 = m_Z^2$)

$$\mathcal{M}^{eff}_{Zl^+l^-} = \bar{u}_l\gamma_\alpha\left[\mathcal{G}_v^f(m_Z^2) - \mathcal{G}_a^f(m_Z^2)\gamma_5\right] v_l\,\varepsilon_Z^\alpha \qquad 4\,|Q_f|\sin^2\theta_eff^f = 1 - \frac{\mathcal{G}_v^f}{\mathcal{G}_a^f}$$

and can be computed in the SM (or in other models) in different renormalisation schemes using (α_0, G_μ, m_Z) as input parameters of the calculation

$$\sin^2\theta_{eff}^{lep} = \kappa(m_Z^2)\sin^2\theta_{OS} = \hat{\kappa}(m_Z^2)\sin^2\hat{\theta}$$

it is crucial to verify at which energy scale the predictions are defined

The weak mixing angle at different energy scales

Goal: testing the parity-violating structure of the weak interactions at different energy scales

Problems: a) define an observable quantity, analogous to $\sin^2\theta_{eff}^{lep}$ at $q^2 = m_Z^2$,

now e.g. at $q^2 = 0$ for the t-channel processes like e-p or e-e- scattering

b) given the large size of the NLO corrections at $q^2 = 0$, the fixed-order result is not sufficient

we have to resum to all orders large classes of radiative corrections in the definition of a running parameter

Solution 1: introduction of $\sin^2\theta_{eff}^{e^-e^-}$ at $q^2 = 0$ to describe Møller scattering   Ferroglia, Ossola, Sirlin, hep-ph/0307200

it absorbs the effect of the EW corrections to the Møller amplitude

in a new effective parameter $\sin^2\theta_{eff}^{e^-e^-}$, via a gauge-invariant form factor $\kappa(q^2 = 0)$,

in a tree-level-like structure

this parameter is a physical observable which can be i) predicted and ii) measured → comparison with $\sin^2\theta_{eff}^{lep}$

Solution 2: the definition of $\sin^2\hat{\theta}(\mu_R)$ in the MSbar scheme is strictly bound to the presence of a renormalisation scale μ_R

$\sin^2\hat{\theta}(\mu_R)$ satisfies the RGE (→ it needs a boundary condition computed at one given scale q^2)

this quantity can be predicted in the SM using $(\alpha(0), G_\mu, m_Z)$ as basic input parameters

the scale μ_R allows to probe the size of resummed radiative correction to the couplings at different scales

The running of $\sin^2\hat\theta(\mu_R)$ and the prediction of $\sin^2\hat\theta(0)$ Erler,Ramsey-Musolf, hep-ph/0409169

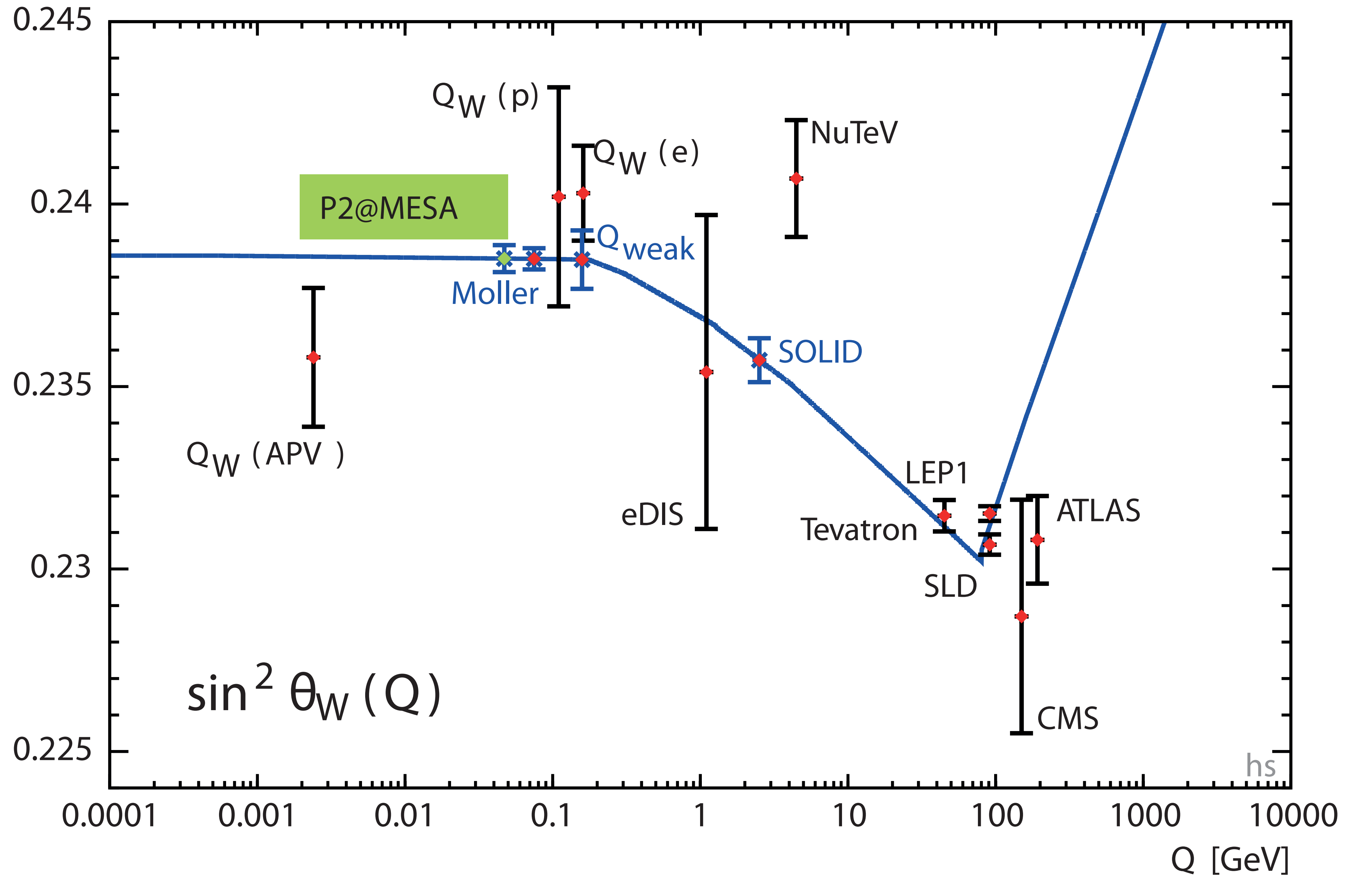
given $\sin^2\hat\theta(m_Z^2)$, we want to study a process with $Q^2 \ll m_Z^2$ → the radiative corrections contain large $\log(Q^2/m_Z^2)$ factors

in the MSbar scheme, the RGE allows to compute the coupling at an arbitrary scale μ^2, once the value at a given Q^2 is known

$$\sin^2\hat\theta(Q^2) = \hat\kappa(Q^2,\mu^2)\ \sin^2\hat\theta(\mu^2)$$

setting $\mu^2 = Q^2$ resums the large $\log(Q^2/\mu^2)$ in $\sin^2\theta(\mu^2)$
the behaviour at the physical thresholds is fixed via matching conditions

$$\sin^2\theta_W(\mu)_{\overline{\rm MS}} = \frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}}\sin^2\theta_W(\mu_0)_{\overline{\rm MS}} + \lambda_1\left[1-\frac{\alpha(\mu)}{\alpha(\mu_0)}\right] + \frac{\alpha(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}} + \tilde\sigma(\mu_0) - \tilde\sigma(\mu)\right].$$



we predict $\sin^2\hat\theta(0) = \hat\kappa(0)\ \sin^2\hat\theta(m_Z^2)$
resumming large perturbative corrections in $\hat\kappa(0)$

non-perturbative contributions enter via $\Sigma_{\gamma Z}(\mu \sim \Lambda_{QCD})$
and are treated along with the e.m. coupling

gauge invariance is respected in the MSbar $\hat\kappa$ factor

$$\hat\kappa(0) = 1.03232 \pm 0.00029$$
$$\sin^2\hat\theta(m_Z^2) = 0.23124(6) \to \sin^2\hat\theta(0) = 0.23871(9)$$

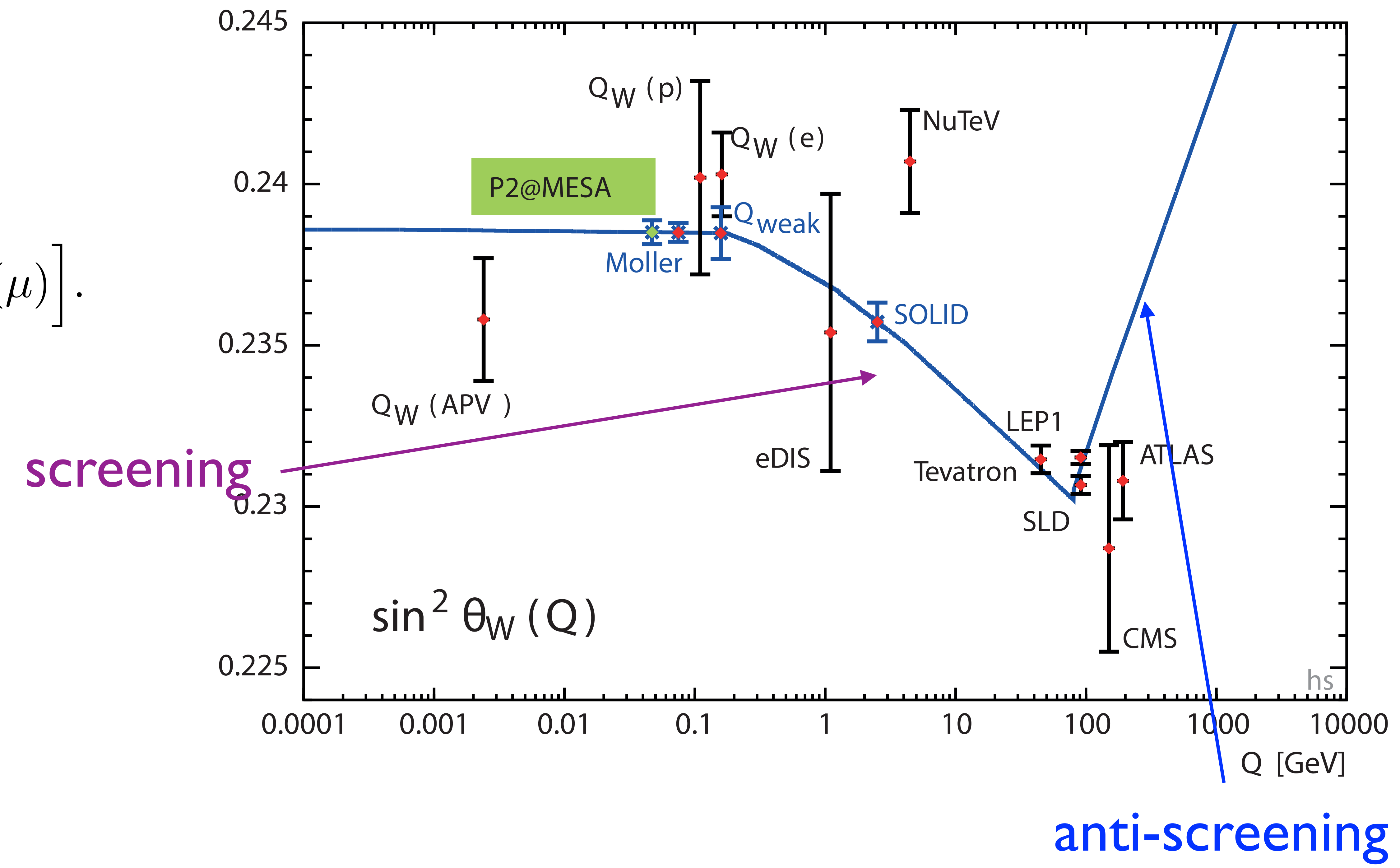
Kumar, Mantry, Marciano, Soudry, arXiv:1302.6263

The running of $\sin^2\hat{\theta}(\mu_R)$ and the prediction of $\sin^2\hat{\theta}(0)$ Erler,Ramsey-Musolf, hep-ph/0409169

The running of the MSbar parameter depends

on the particles active in the theory at a given scale μ^2 and the sign of the associated beta function coefficient

$$\sin^2\theta_W(\mu)_{\overline{\rm MS}} = \frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}}\sin^2\theta_W(\mu_0)_{\overline{\rm MS}} + \lambda_1\left[1-\frac{\alpha(\mu)}{\alpha(\mu_0)}\right] + \frac{\alpha(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2}+\frac{3\lambda_3}{4}\ln\frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}}+\tilde{\sigma}(\mu_0)-\tilde{\sigma}(\mu)\right].$$



The large lever arm (3 orders of magnitude) and the high precision of the P2 determination might possibly emphasise the presence of non-SM contributions.

Alternatively , significant compatibility with the SM prediction would be a striking success of the SM

what about the experimental determination of $\sin^2\hat{\theta}(\mu_R)$?

Fit of observables, parameter determination and EW input schemes

An experimental procedure measures observables $\mathcal{O}$, i.e. cross section and asymmetries

These observables $\mathcal{O}$ can be computed in a given model, e.g. the SM, with an input scheme, e.g. (α_0, G_μ, m_Z)
and expressed in terms of only the input parameters of the lagrangian $\mathcal{O} = \mathcal{O}(\alpha_0, G_\mu, m_Z)$.

If we want to determine the value of one input parameter of the lagrangian,
we fit the experimental observable with its theoretical prediction, letting the input parameter free to vary.
→ for a given EW input scheme, only the input parameters can be "measured", all the other parameters are predictions
in the (α_0, G_μ, m_Z) scheme we can predict $\sin^2\hat{\theta}(\mu_R)$, but we can not "measure" it

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→ we need to compute the asymmetries and all the relevant observables (in e-p, e-e-, e+e- scattering)
using an EW scheme with $\sin^2\hat\theta(\mu_R)$ as one of the input parameters, e.g. $(\alpha_0, \sin^2\hat\theta(\mu_R), m_Z)$
Setting the scale $\mu_R^2 = q^2$ at a value q^2 typical of the process defines that $\sin^2\hat\theta(\mu_R)$ is renormalised at that scale,
the fit will then choose the best value for $\sin^2\hat\theta(q^2)$

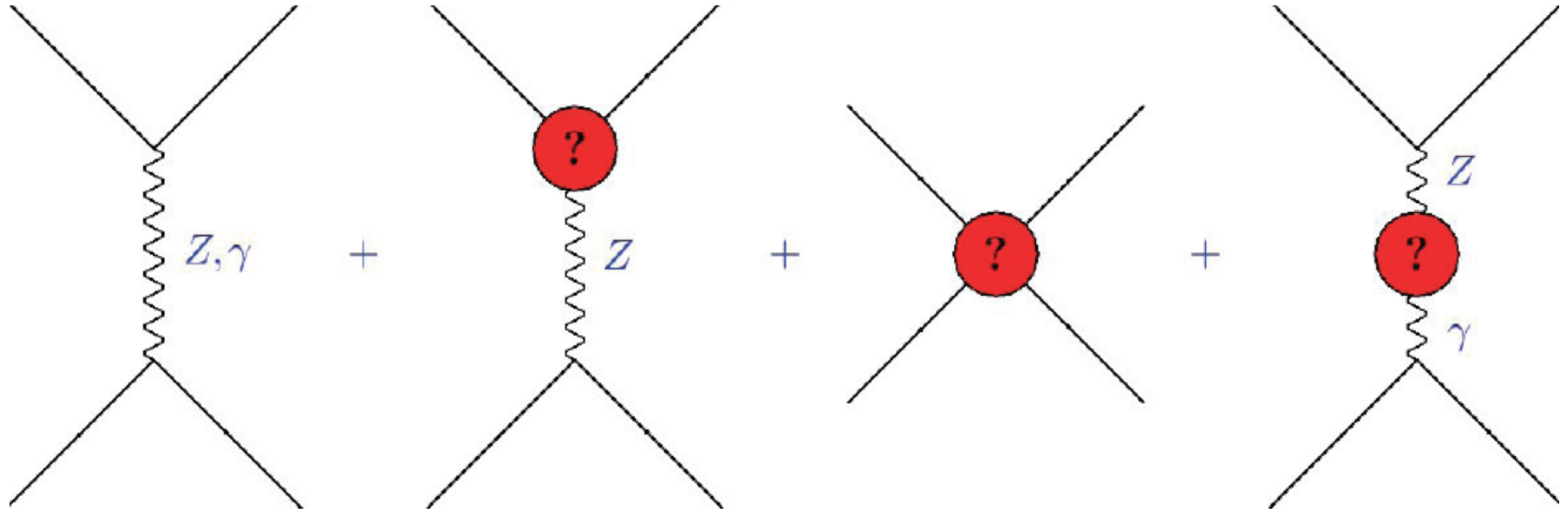
The determination of $\sin^2\hat\theta(q^2)$ at different q^2 values allows then
a test of the predicted running of this parameter (predicted in the (α_0, G_μ, m_Z) input scheme).

BSM searches

Any significant tension of A_{PV}^{SM} with the data might be interpreted as a BSM signal

Different kinds of new interaction might yield the same observable effect:

new parity-violating contact interaction operators
new dark bosons
new additional gauge bosons (Z')



The P2 potential to discover new physics is enhanced by :

a) accidental suppression of the proton weak charge at tree level → BSM effects have stronger impact on A_{PV}

$$A_{PV} = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha_{em}} \left(Q_W - F(E_i, Q^2) + \Delta_{SM\, rad.corr.}(Q^2) + \Delta_{BSM}(Q^2) \right)$$

b) absence of suppression of the interferences of BSM with SM tree level amplitudes (at variance with the Z pole)

at the Z pole the SM amplitude is purely imaginary and the interference with real BSM amplitudes vanishes

The P2 high precision makes its discovery potential comparable to the one of high-energy experiments

BSM searches

New contact interactions

$$\mathcal{L}_{\rm SM}^{\rm PV} = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\mu\gamma_5 e \sum_q C_{1q}\ \bar{q}\gamma^\mu q,$$

$$\mathcal{L}_{NEW}^{\rm PV} = \frac{g^2}{4\Lambda^2}\bar{e}\gamma_\mu\gamma_5 e \sum_f h_V^q\ \bar{q}\gamma^\mu q,$$

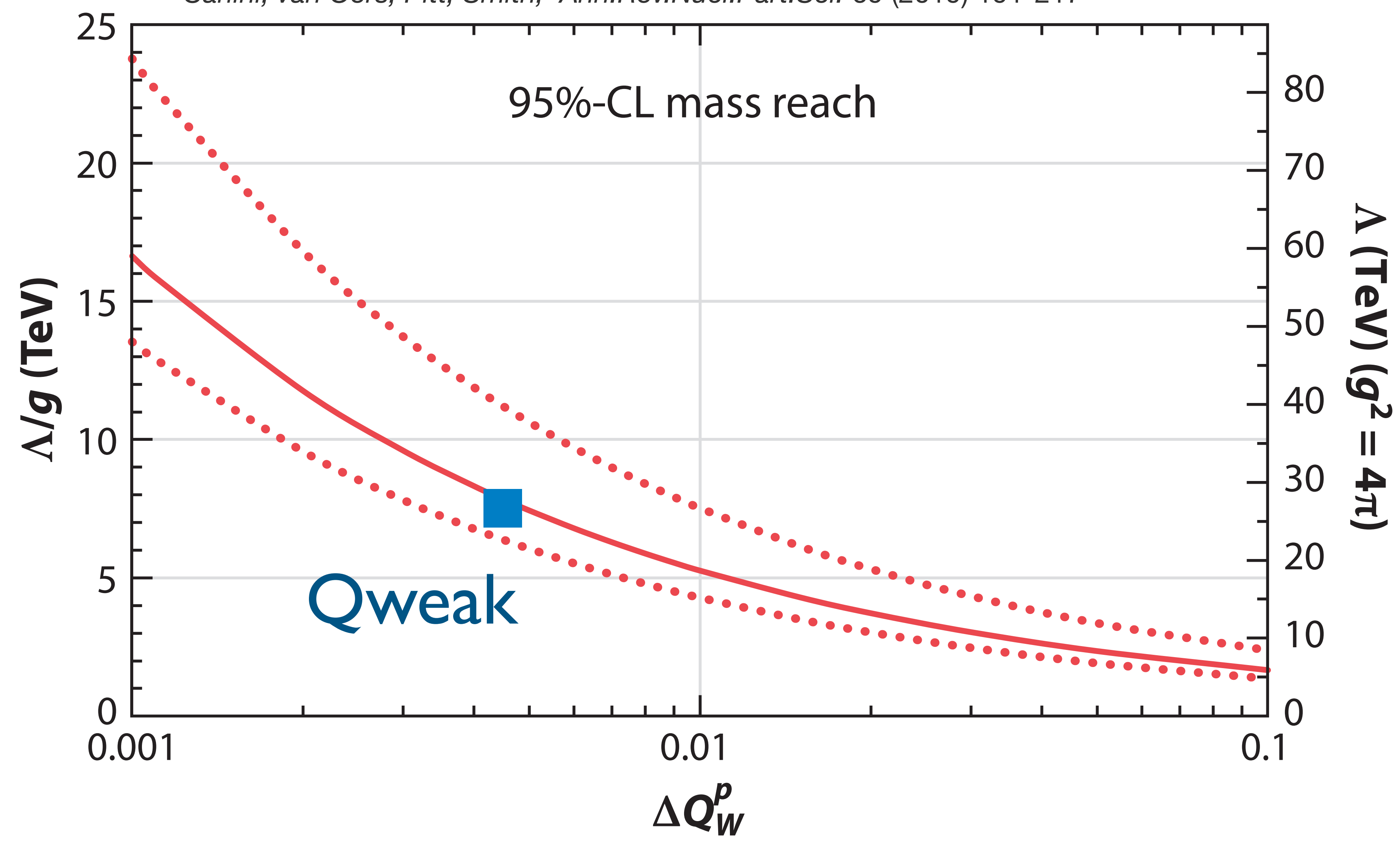
$$\frac{\Lambda}{g} \sim \frac{1}{\sqrt{\sqrt{2}G_F|\Delta Q_W^p|}}$$

Limits on the scale of New Physics can be set in the strong coupling ($g^2 = 4\pi$) assumption or for the Wilson coefficient

The exclusion range is computed
about a SM central value hypothesis for Q_W^p (solid line) with ±1σ

The expected $\Delta Q_W^p(P2) \sim 0.0011$ will push the exclusion limit
up to the 80 TeV level
in the strong coupling scenario and in the most favoured configuration

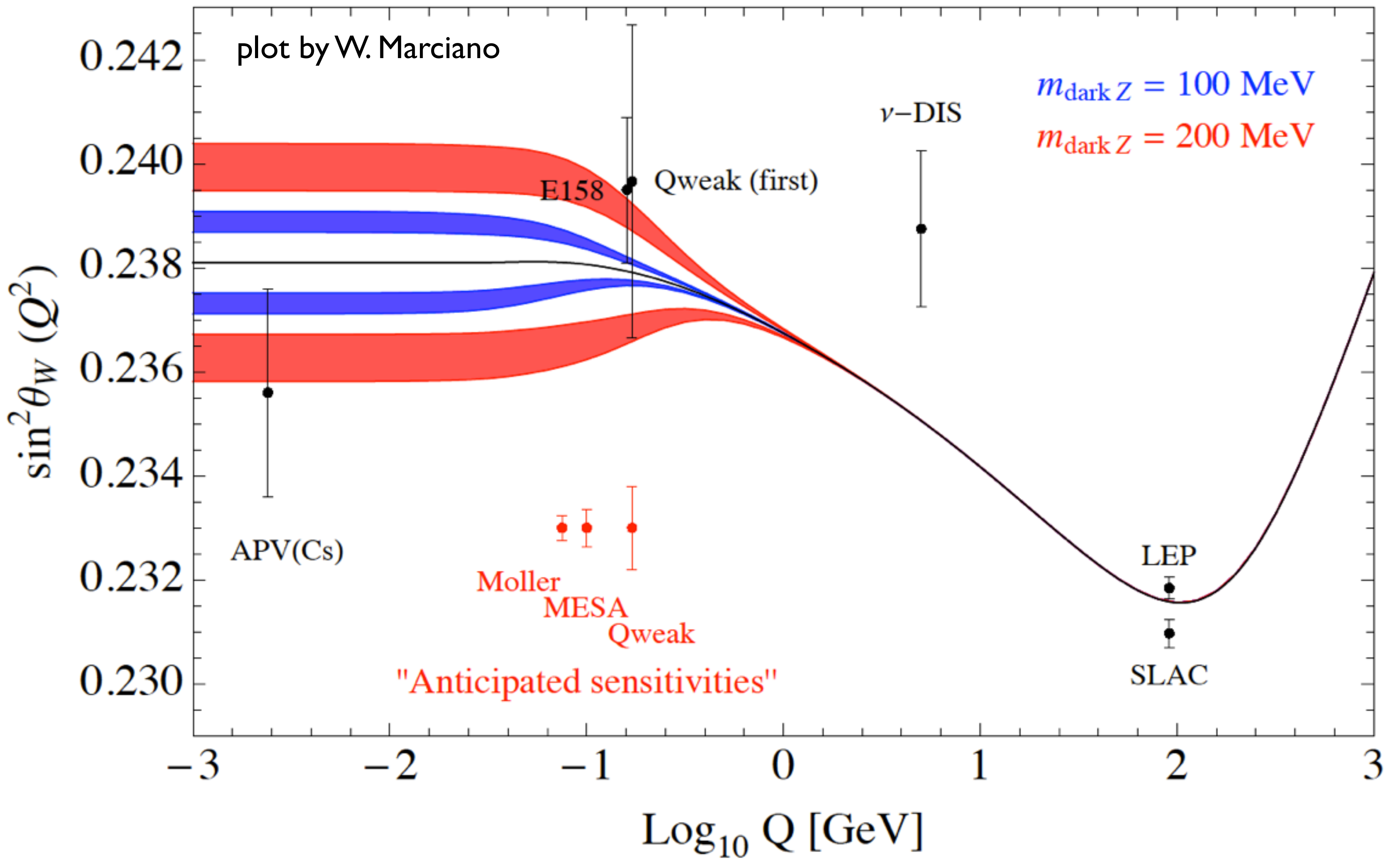
Carlini, van Oers, Pitt, Smith, Ann.Rev.Nucl.Part.Sci. 69 (2019) 191-217



The limits will be stronger than at LEP2 thanks to the higher precision of the weak charge determination

BSM searches

New dark parity-violating bosons



A new dark bosons, mixing with the SM Z boson, may modify the strength of the parity-violating couplings

The effects can be completely absent at the Z resonance, where the SM amplitude is purely imaginary.

The presence of the extra boson modifies the running of $\sin^2\hat{\theta}(\mu_R)$,
with a modulation due to the assumed boson mass and couplings

The sensitivity to this kind of interaction is quite unique to the low-energy electron-scattering experiments

Complementarity of different $\sin^2\theta_W$ determinations

- With low-energy facilities we have the opportunity to test the SM prediction of the weak mixing angle at two energy scales and in very different experimental environments: low-energy electron(-positron) scattering, the Z resonance at LEP/SLD, the Z line shape (up to TeV scale) at hadron colliders

The errors of these determinations are comparable and much smaller than the radiative effects due to SM running

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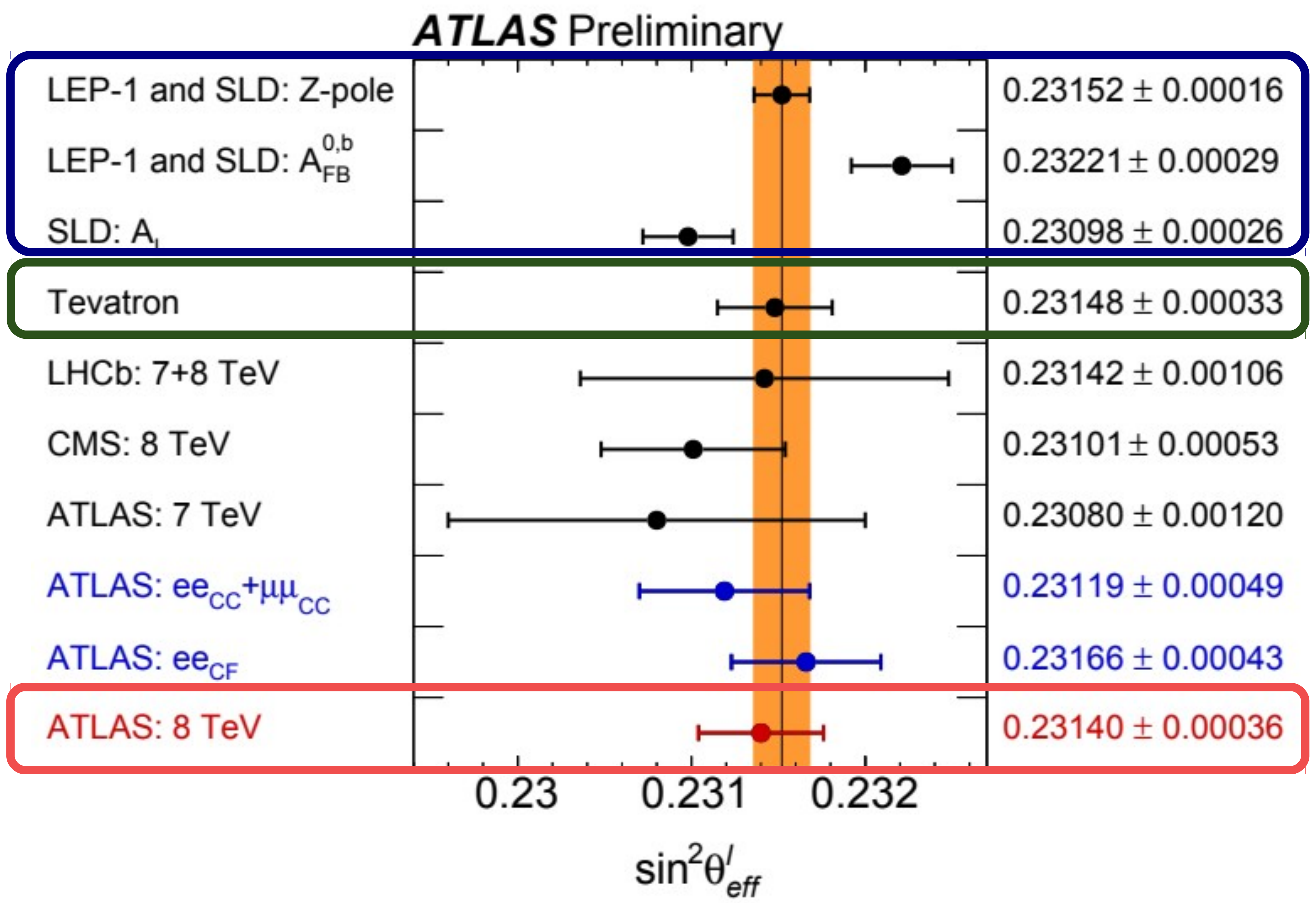

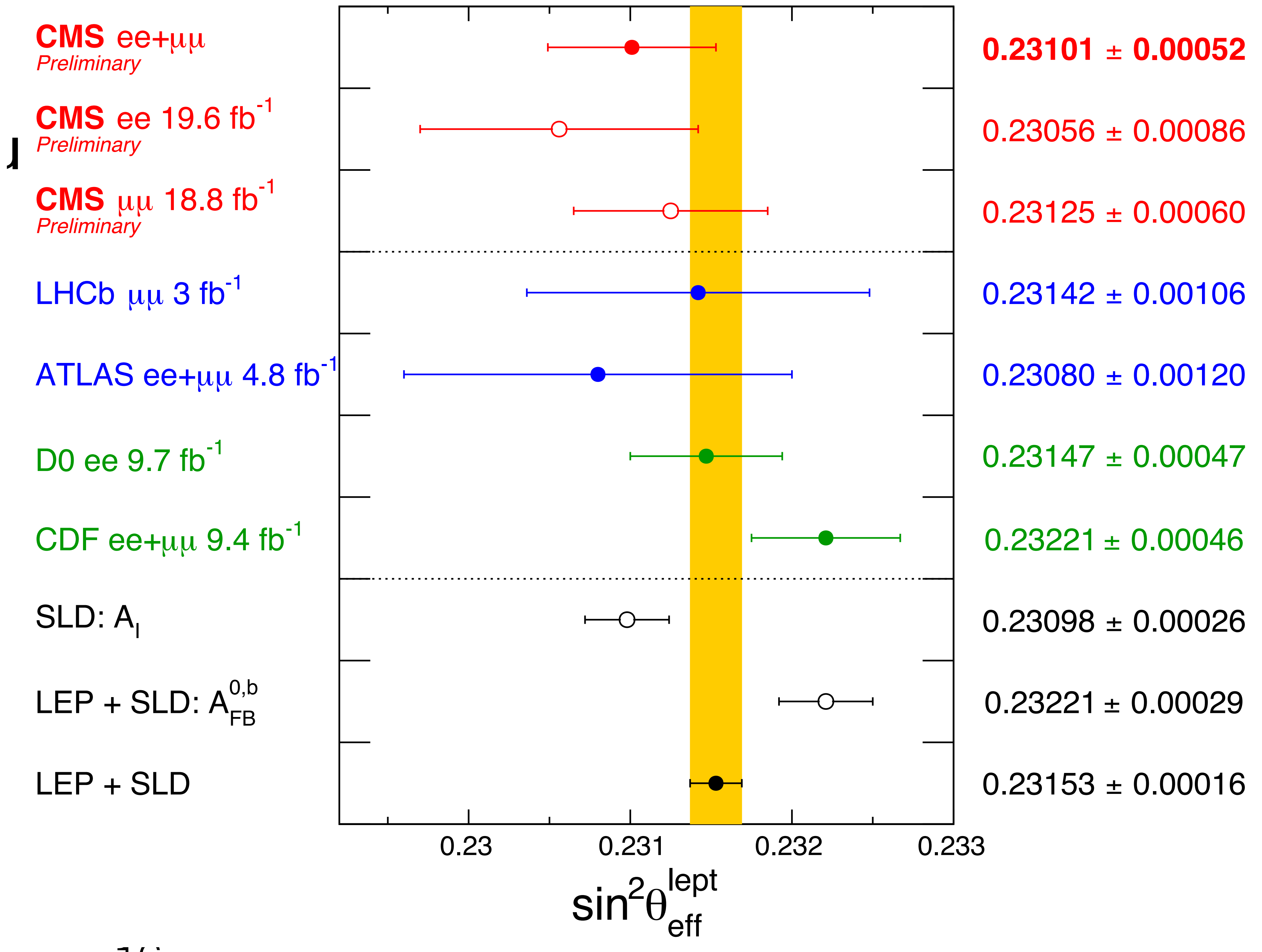

Complementarity of different $\sin^2\theta_W$ determinations

- The comparison/combination of these different results is valuable if we consider exactly the same quantity:
  a popular example is $\sin^2\theta_{eff}^{lep}$, but in view of the current discussion it could be $\sin^2\hat{\theta}(m_Z^2)$
- for each collider/observable we have to "access" the hard scattering process (proportional to $\sin^2\theta_{eff}^{lep}$ or to $\sin^2\hat{\theta}(m_Z^2)$)
  by deconvoluting standard QED/QCD effects, dealing with the proton (lepton) PDFs, and considering higher-order corrections

→ different strategies and input schemes are adopted in the literature; their consistency has to be checked

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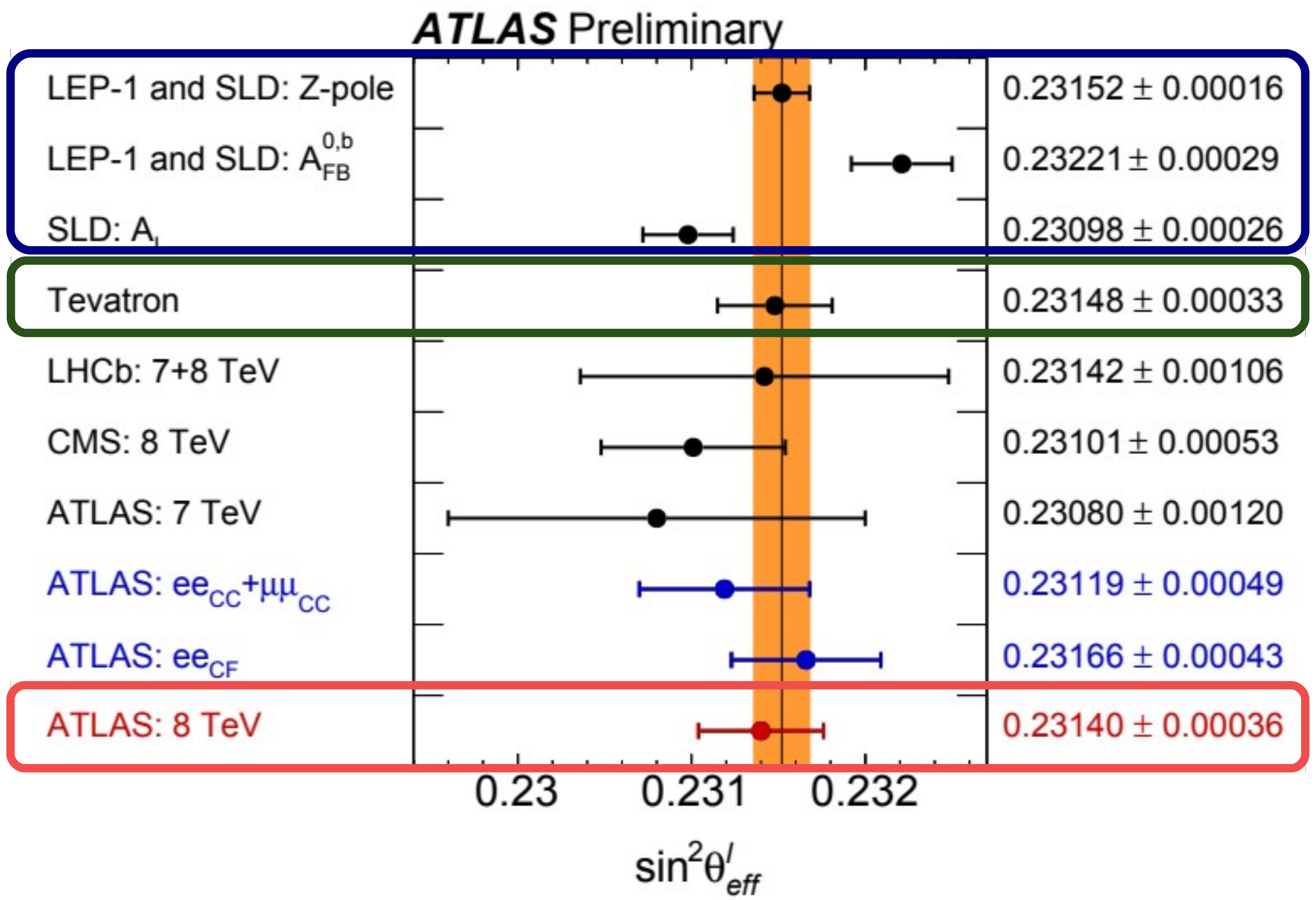
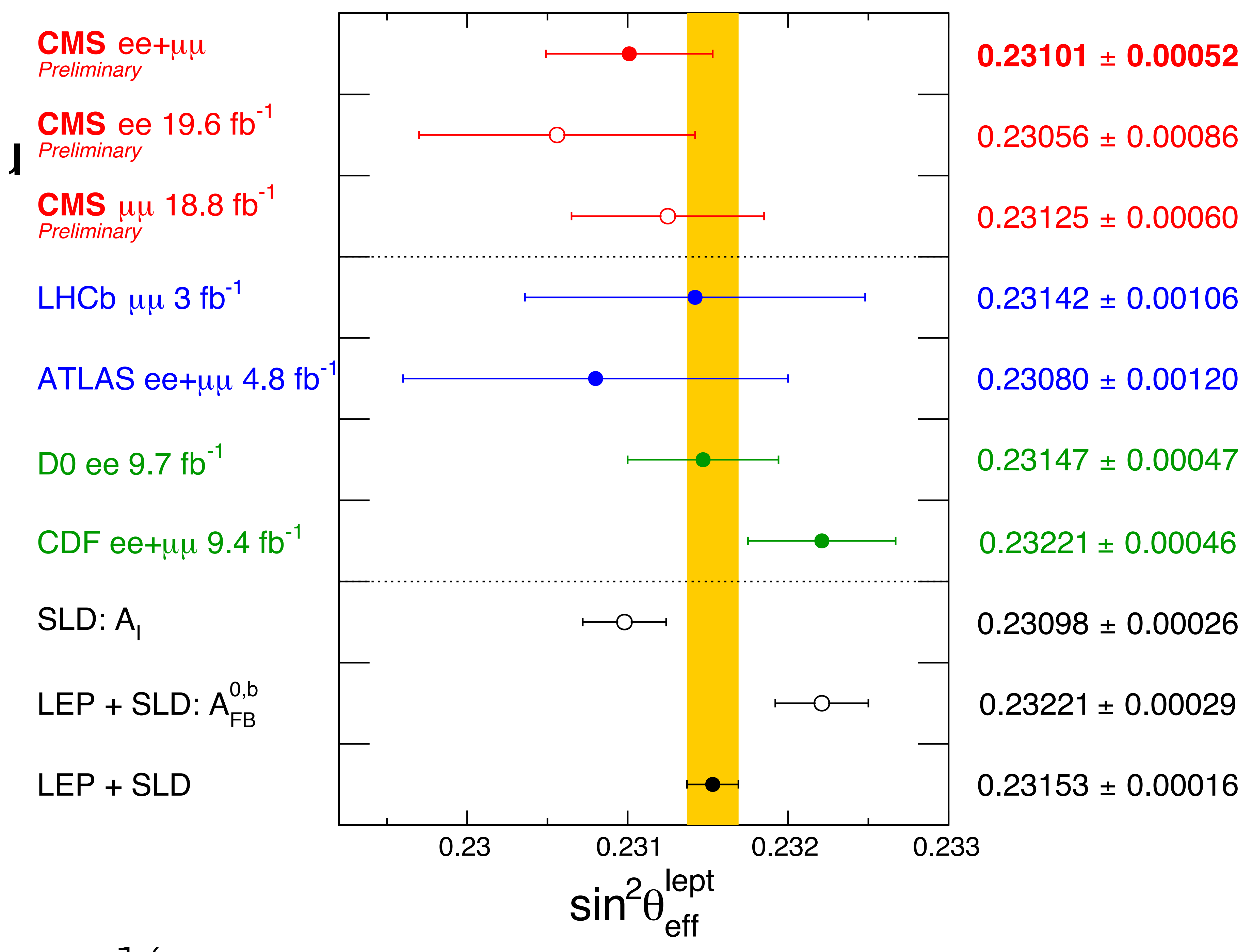




Possible critical points in view of a global fit

example 1:

- determination of $\sin^2\theta_{eff}^{lep}$ at the Z resonance (LEP1, FCC-ee, CEPC) $A_{FB}^{exp}(m_Z^2) - \mathscr{A}_{nonfact} = \frac{3}{4}\mathscr{A}_e\mathscr{A}_f$
- after this factorisation, the interpretation of the form factors in terms of $\sin^2\theta_{eff}^{lep}$ is straightforward
- the factorisation in (initial)x(final) form factors requires the subtraction of the $\mathscr{A}_{nonfact}$ term
  with a residual uncertainty better than the precision goal of the LEP measurements;
  this procedure, acceptable at LEP1, should be verified at FCC-ee / CEPC, but also at low-energies

example 2:

- determination of $\sin^2\theta_{eff}^{lep}$ from neutral-current Drell-Yan at hadron colliders
- parity violating observables ($A_{FB}(M_{ll}),\ A_4(p_\perp^{ll})$) are kinematical distributions and $\sin^2\theta_{eff}^{lep}$ is related to their shapes
- the convolution of PDF x (Parton Shower) x (hard partonic xsec) "shields" the access to $\sin^2\theta_{eff}^{lep}$
- in the absence of a simple analytical formulation, only a numerical template fit procedure is viable
- template fits have been performed in some cases in the (G_μ, m_W, m_Z) scheme, m_W has been determined, not $\sin^2\theta_{eff}^{lep}$!!!
  eventually translating the best m_W in terms of the corresponding $\sin^2\theta_{eff}^{lep}$ in the SM!!!

the estimate of the residual theoretical uncertainties assigned to the fitted value can be a delicate point

Possible improvements in view of a global fit

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

Alternative EW scheme, using $(G_\mu, \sin^2\theta_{eff}^{lep}, m_Z)$ as inputs of the gauge sector

first developed in the framework of the LHC analyses

(extended lepton-pair invariant mass intervals with non-factorisable corrections much more important than at $q^2 = m_Z^2$)

it can be immediately applied to any e+e- collider study

it allows to express any observable and templates as $\mathcal{O} = \mathcal{O}(G_\mu,\, \sin^2\theta_{eff}^{lep}, m_Z,)$

→ direct $\sin^2\theta_{eff}^{lep}$ central value estimate

→ direct MC determination of the systematic uncertainties

A completely analogous approach is in progress (to appear soon)
for a clean determination of the MSbar weak mixing angle at hadron colliders

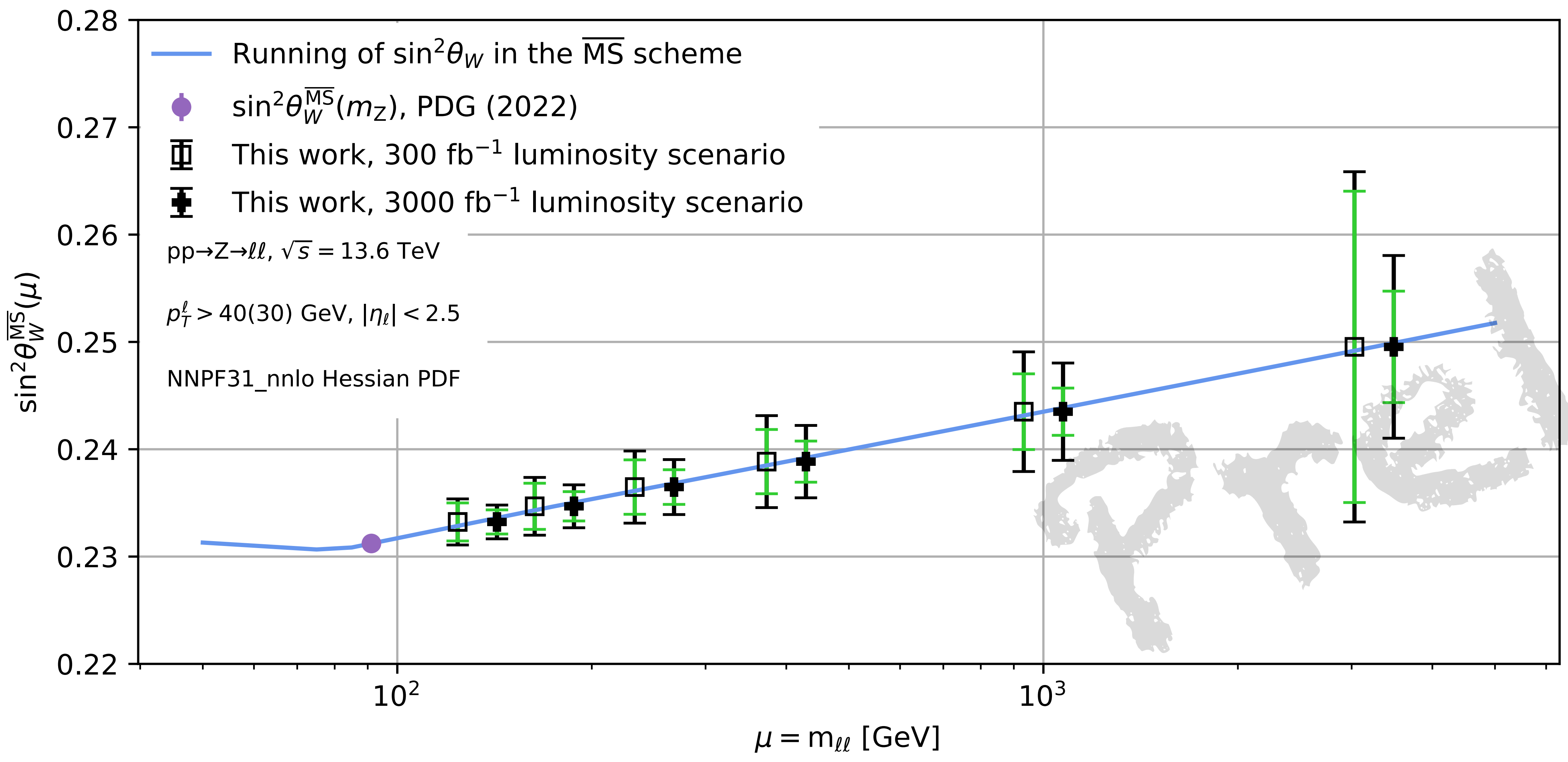
Toward a determination of $\sin^2\hat{\theta}(\mu_R)$ at large invariant masses

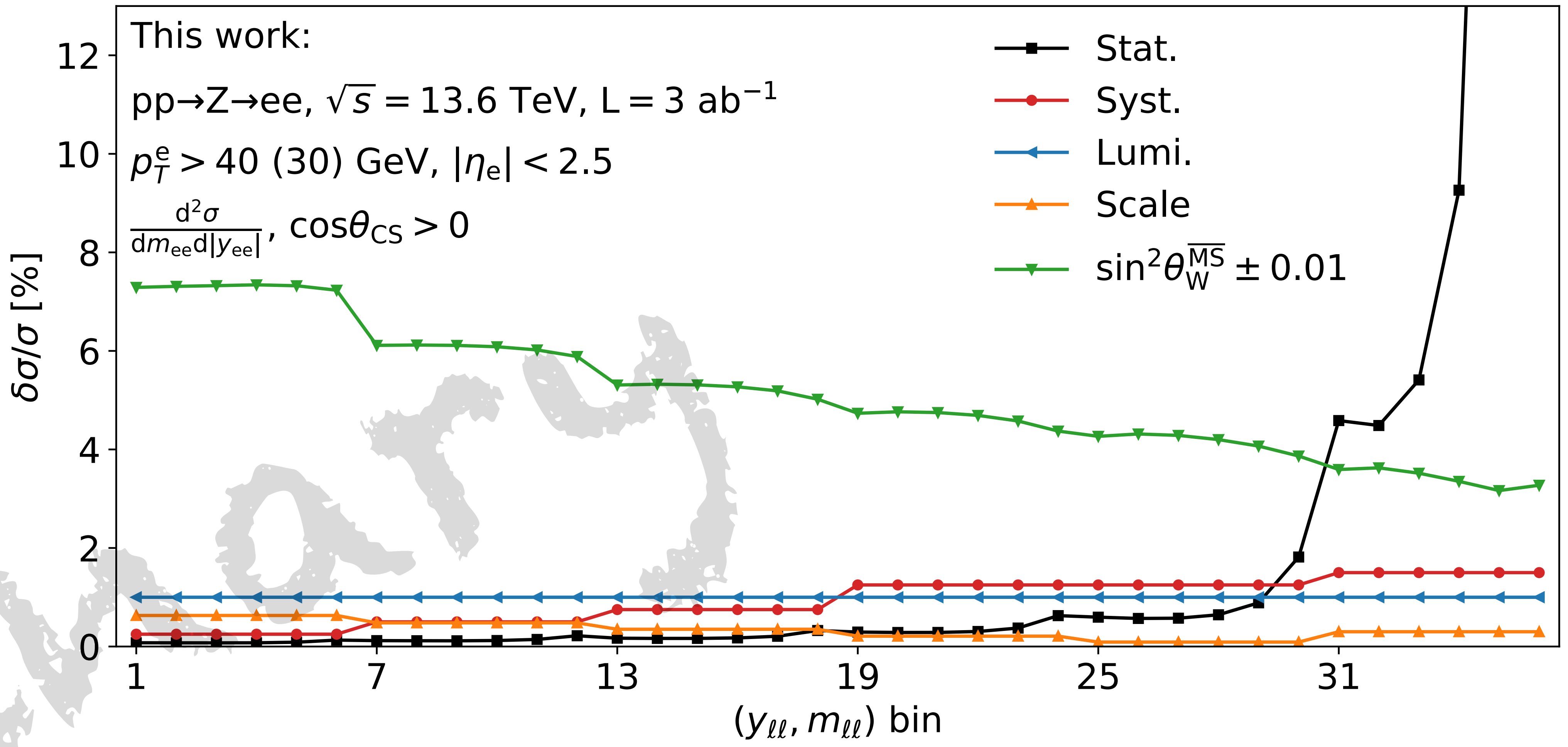
S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.xxxxx

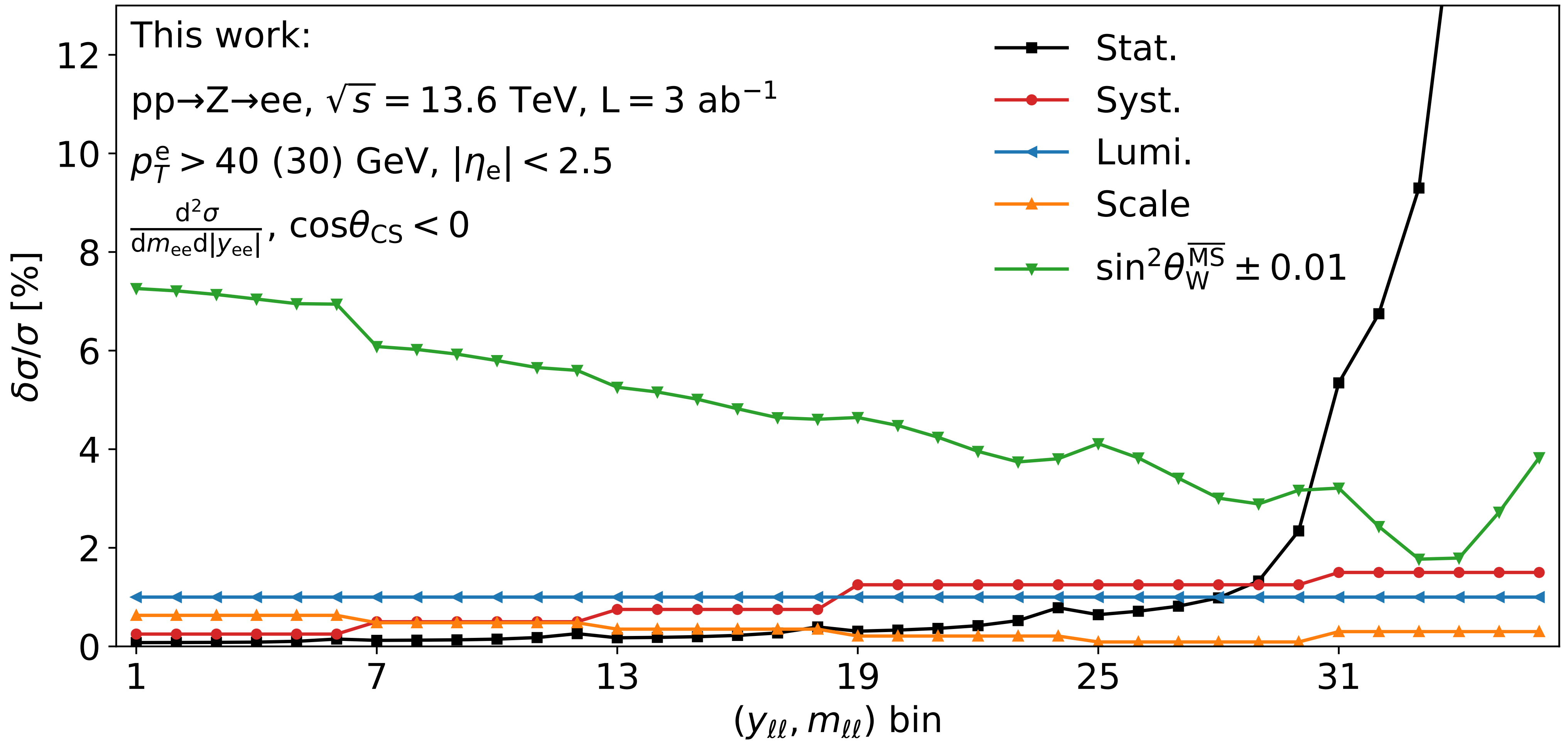
Alternative EW scheme, using $(G_\mu, \sin^2\theta_{\overline{MS}}(\mu_R), m_Z)$ as inputs of the gauge sector, M.Chiesa, C.L. Del Pio, F.Piccinini, arXiv:2302.xxxxx

The Drell-Yan lepton-pair invariant mass distribution has been studied
with POWHEG at NLO QCD+EW + parton shower
in this new input scheme, at NLO-EW → clear distinction between
- the effects of running of the weak mixing angle
- the other EW radiative corrections

sensitivity to variations of the input parameter $\sin^2\hat{\theta}(\mu_R)$







The running of the MSbar angle can be established at LHC in Run III and at HL-LHC with percent precision

Conclusions

Experiments at different energy scales offer the great opportunity to perform a high-precision test of the SM

- testing the SM at the quantum level
- for $\sin^2\theta_W$, with a precision comparable or higher than the LEP benchmark

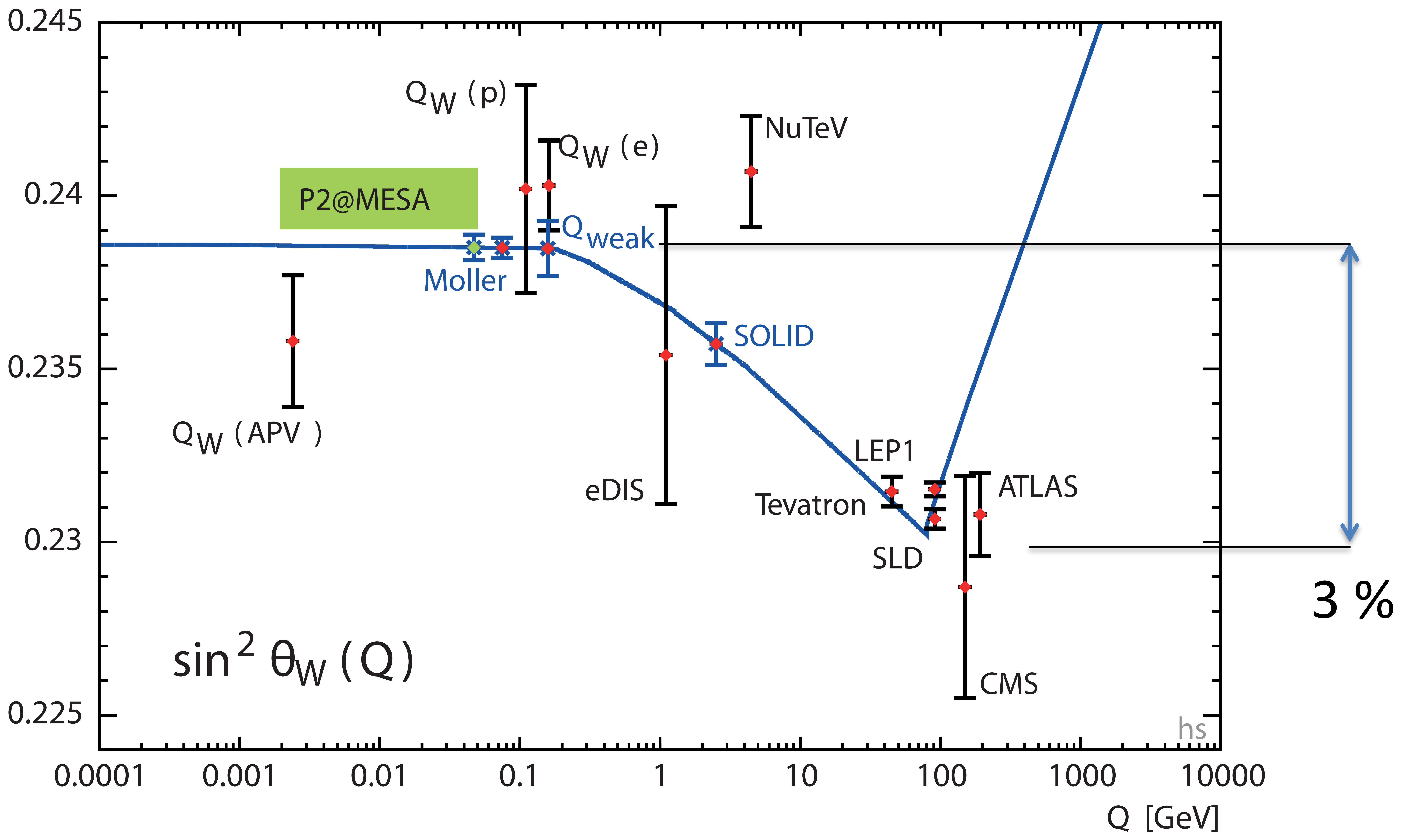
The consistency of the SM at different energy scales would be a very strong indication to formulate BSM searches

In electron-scattering low-energy experiments,
the sensitivity to several BSM models is enhanced by the accidental suppression of the tree-level expression of A_{PV}

The LHC experiments can enlarge the lever arm,
by extending the energy scale, when the weak mixing angle has been measured in the TeV range

The precision tests of the SM and the global EW fit will require a consistent treatment of the radiative corrections
first of all starting from the definition of the fitted parameter, defined in the appropriate input scheme

back-up


0.245
0.24
0.235
0.23
0.225
0.0001
0.001
0.01
0.1
1
10
100
1000
10000
Q [GeV]
Q_W (p)
Q_W (e)
NuTeV
P2@MESA
Q_{weak}
Moller
SOLID
Q_W (APV)
eDIS
LEP1
Tevatron
ATLAS
SLD
CMS
$\sin^2 \theta_W (Q)$
hs
3 %

The effective leptonic weak mixing angle: theoretical prediction

- parameterization of the full two-loop EW calculation + different sets of 3- and 4-loop corrections

I.Dubovyk, A.Freitas, J.Gluza, T.Riemann, J.Usovitsch, arXiv:1906.08815

$$\sin^2\theta^f_{\rm eff} = s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4 \Delta_\alpha + d_5 \Delta_{\rm t} + d_6 \Delta_{\rm t}^2 + d_7 \Delta_{\rm t} L_H + d_8 \Delta_{\alpha_{\rm s}} + d_9 \Delta_{\alpha_{\rm s}} \Delta_{\rm t} + d_{10} \Delta_{\rm Z}$$

$$L_{\rm H} = \log\frac{M_{\rm H}}{125.7\,{\rm GeV}}, \qquad \Delta_{\rm t} = \left(\frac{m_{\rm t}}{173.2\,{\rm GeV}}\right)^2 - 1,$$

$$\Delta_{\alpha_{\rm s}} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.1184} - 1, \qquad \Delta_\alpha = \frac{\Delta\alpha}{0.059} - 1, \qquad \Delta_{\rm Z} = \frac{M_{\rm Z}}{91.1876\,{\rm GeV}} - 1$$

Observable	s_0	d_1	d_2	d_3	d_4	d_5
$\sin^2\theta^\ell_{\rm eff} \times 10^4$	2314.64	4.616	0.539	−0.0737	206	−25.71
$\sin^2\theta^b_{\rm eff} \times 10^4$	2327.04	4.638	0.558	−0.0700	207	−9.554

Observable	d_6	d_7	d_8	d_9	d_{10}	max. dev.
$\sin^2\theta^\ell_{\rm eff} \times 10^4$	4.00	0.288	3.88	−6.49	−6560	< 0.056
$\sin^2\theta^b_{\rm eff} \times 10^4$	3.83	0.179	2.41	−8.24	−6630	< 0.025

Awramik, Czakon, Freitas, hep-ph/0608099

The weak charge of the proton and the determination of $\sin^2\hat\theta(\mu_R)$ in the SM

- The measurement of A_{PV} can be interpreted by comparing it with its SM theoretical expression

$$A_{PV}^{th} = -\frac{G_\mu Q^2}{4\sqrt{2}\pi\alpha}\left[\rho_{ep}\left(1-4\sin^2\hat\theta(0)\right)+\left(\Box_{WW}+\Box_{ZZ}+\Box_{\gamma Z}+\Box_{\gamma\gamma}\right)\right] - \frac{G_\mu Q^2}{4\sqrt{2}\pi\alpha}B(Q^2) = A_{PV}^{exp}$$

- The proton weak charge is defined in the limit $E=0, Q^2\to 0$ of the square bracket, fully known at NLO EW

- Keeping $\sin^2\hat\theta(0)$ as one of the input parameter allows its determination fitting A_{PV}^{th} to the data.
  The theoretical error in the prediction of the whole expression is relevant for the final error on $\sin^2\hat\theta(0)$

- While WW and ZZ boxes contribute large constant terms, safely evaluated in perturbation theory,
  the γZ box carries a not negligible energy dependence and sensitivity to the hadronic structure of the proton
  → may affect the extrapolation to $Q^2\to 0$.

  Several theoretical and computational progresses contributed to bring under control the expression of $\Box_{\gamma Z}$
  a dedicated measurements of the proton anapole moment will further contribute

  Erler, Gorchtein, Koshchii, Seng, Spiesberger, arXiv:1907.07928, Cè et al, arXiv:1910.09525

- QED corrections, up to second order, enter in the experimental determination and
  are necessary to assign the correct Q^2 value to each event Bucoveanu, Spiesberger, arXiv:1903.12229

- The projected values $\langle A_{PV}^{exp}\rangle \sim (-40\pm 0.6)\,10^{-9}$ should allow a determination of $\sin^2\hat\theta(0)$ at the 0.14% level

Becker et al, arXiv:1802.04759

Summary of the SM test

The expected statistical error should allow to perform a determination of the weak mixing angle at the 0.14% level: $\Delta\sin^2\hat{\theta}(0) \sim 33 \cdot 10^{-5}$
competitive with the LEP/Tevatron/LHC ones

The theoretical error on the prediction of $\sin^2\hat{\theta}(0)$

$\hat{\kappa}(0) = 1.03232 \pm 0.00029$

$\sin^2\hat{\theta}(m_Z^2) = 0.23124(6) \to \sin^2\hat{\theta}(0) = 0.23871(9)$

should allow in turn to perform a sensible comparison with the experimental value

The (possible) compatibility of the experimental value with the SM prediction would be a striking feature of the SM,
covering more than 3 orders of magnitude for the energy scale

This single test of the SM should then be merged in a global EW fit with other measurements for the best global determination of $\sin^2\hat{\theta}(m_Z^2)$

Any significant tension with the data might deserve a dedicated BSM study.

E_{beam}	155 MeV
$\bar{\theta}_{\text{f}}$	$35°$
$\delta\theta_{\text{f}}$	$20°$
$\langle Q^2\rangle_{L=600\,\text{mm},\ \delta\theta_{\text{f}}=20°}$	$6 \times 10^{-3}\,(\text{GeV/c})^2$
$\langle A^{\text{exp}}\rangle$	-39.94 ppb
$(\Delta A^{\text{exp}})_{\text{Total}}$	0.56 ppb (1.40 %)
$(\Delta A^{\text{exp}})_{\text{Statistics}}$	0.51 ppb (1.28 %)
$(\Delta A^{\text{exp}})_{\text{Polarization}}$	0.21 ppb (0.53 %)
$(\Delta A^{\text{exp}})_{\text{Apparative}}$	0.10 ppb (0.25 %)
$\langle s_{\text{W}}^2\rangle$	0.231 16
$(\Delta s_{\text{W}}^2)_{\text{Total}}$	3.3×10^{-4} (0.14 %)
$(\Delta s_{\text{W}}^2)_{\text{Statistics}}$	2.7×10^{-4} (0.12 %)
$(\Delta s_{\text{W}}^2)_{\text{Polarization}}$	1.0×10^{-4} (0.04 %)
$(\Delta s_{\text{W}}^2)_{\text{Apparative}}$	0.5×10^{-4} (0.02 %)
$(\Delta s_{\text{W}}^2)_{\Box_{\gamma Z}}$	0.4×10^{-4} (0.02 %)
$(\Delta s_{\text{W}}^2)_{\text{nucl. FF}}$	1.2×10^{-4} (0.05 %)
$\langle Q^2\rangle_{\text{Cherenkov}}$	$4.57 \times 10^{-3}\,(\text{GeV/c})^2$
$\langle A^{\text{exp}}\rangle_{\text{Cherenkov}}$	-28.77 ppb