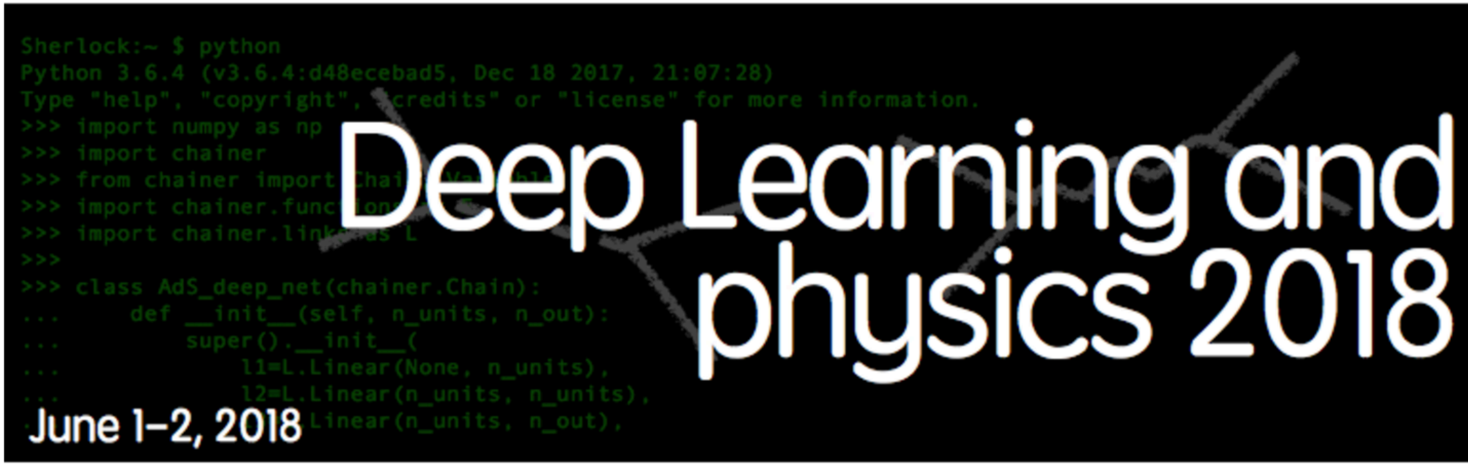
KIAS, 26 March, 2018
CQuest, Sogang u., 29 March, 2018
MIT, CTP, 4 Apr, 2018
MPI, AEI, 13 Apr, 2018
HET group, Osaka, 30 May, 2018
DLAP2018 workshop, Osaka, 1 June, 2018
Paris QCD workshop, 11 June, 2018
Machine learning workshop, TSiMF, China, 15 June, 2018

Deep Learning and AdS/CFT

Koji Hashimoto (Osaka u)

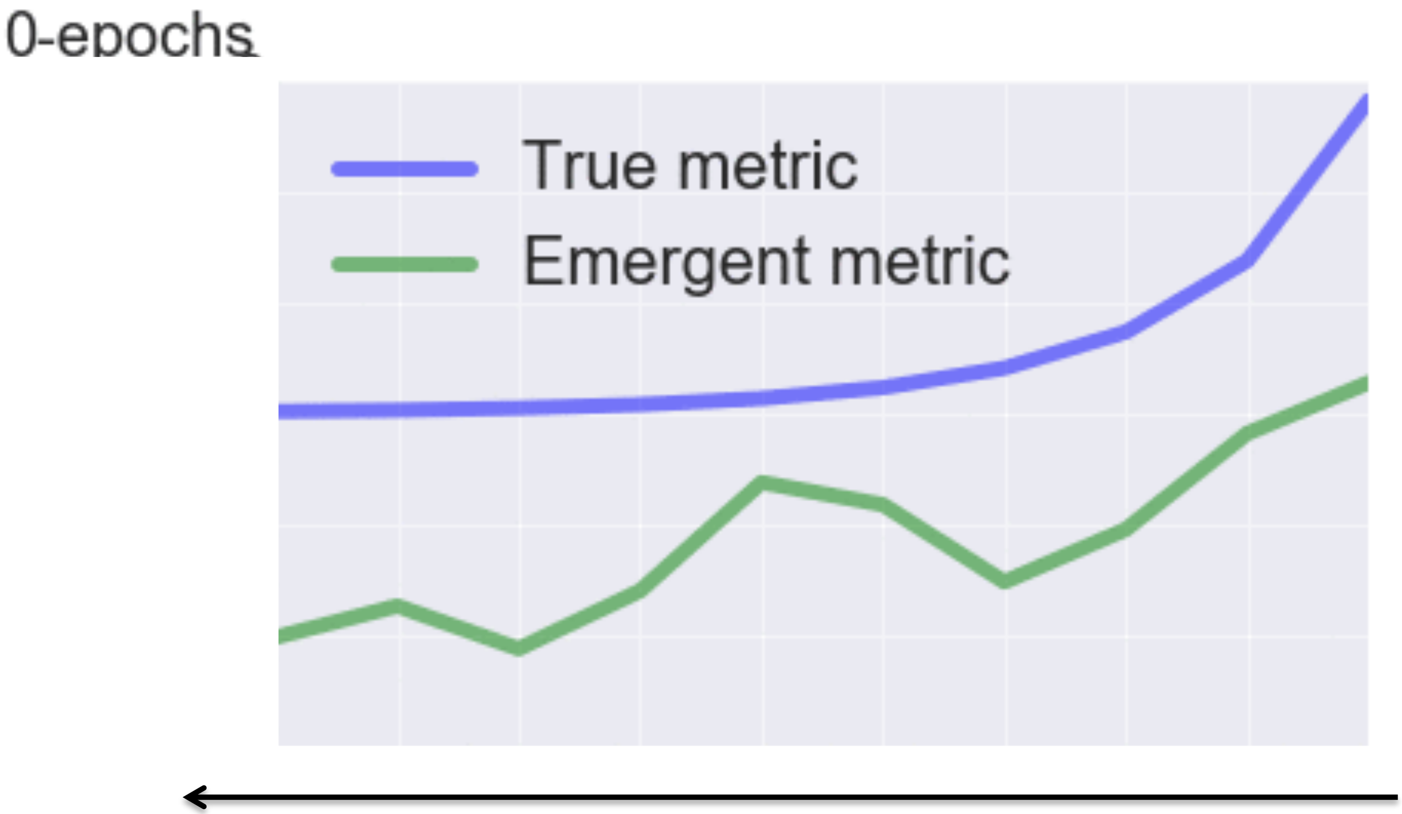
ArXiv:1802.08313 w/ S. Sugishita (Osaka),
A. Tanaka (RIKEN AIP),
A. Tomiya (CCNU)



Target scope of Workshop

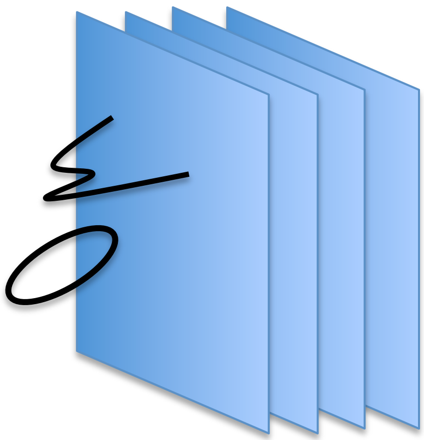
Deep learning plays a central role in recent developments in research in artificial intelligence (AI). Various ideas based on physics are found in the research of deep learning, and consequently, deep learning and physics are related intimately. This workshop is dedicated to (1) applications of deep learning to physics, (2) discovering similarities among deep learning and physics, and (3) leading to new paradigm in physics motivated by deep learning. Researchers in related fields are welcome to attending discussions at our workshop.

Watch how machine learns AdS black hole

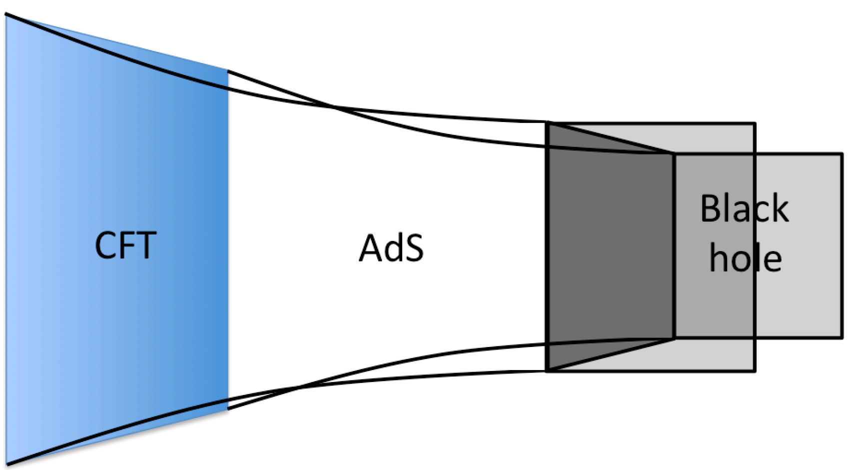


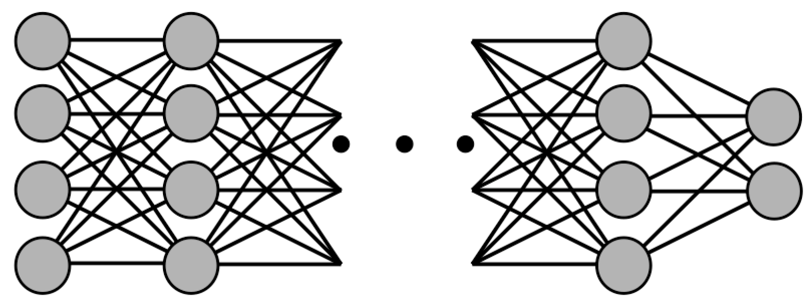
AdS radial direction

Brane
(Superstring theory)

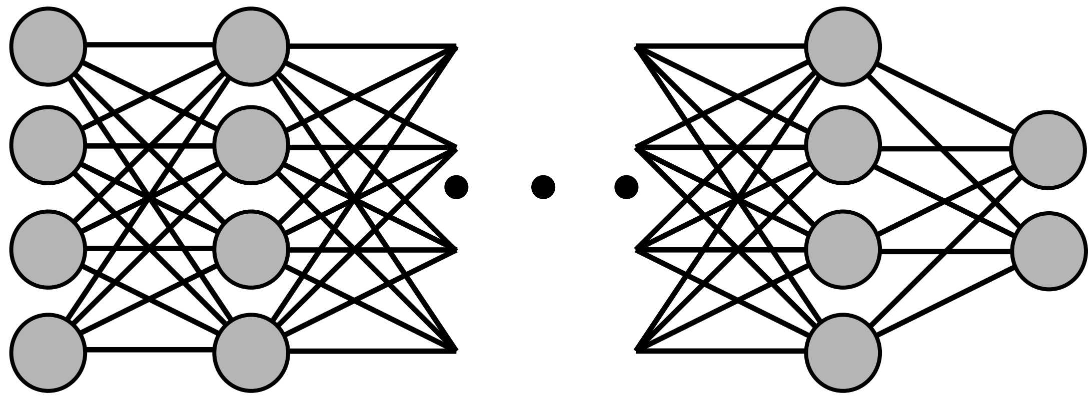


Brain
(Neuroscience)





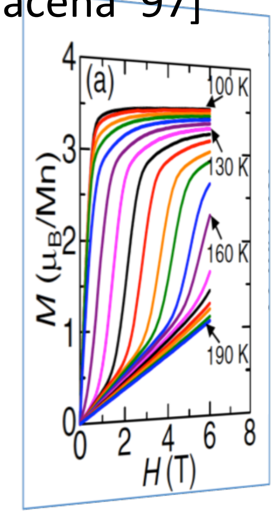
Deep Learning

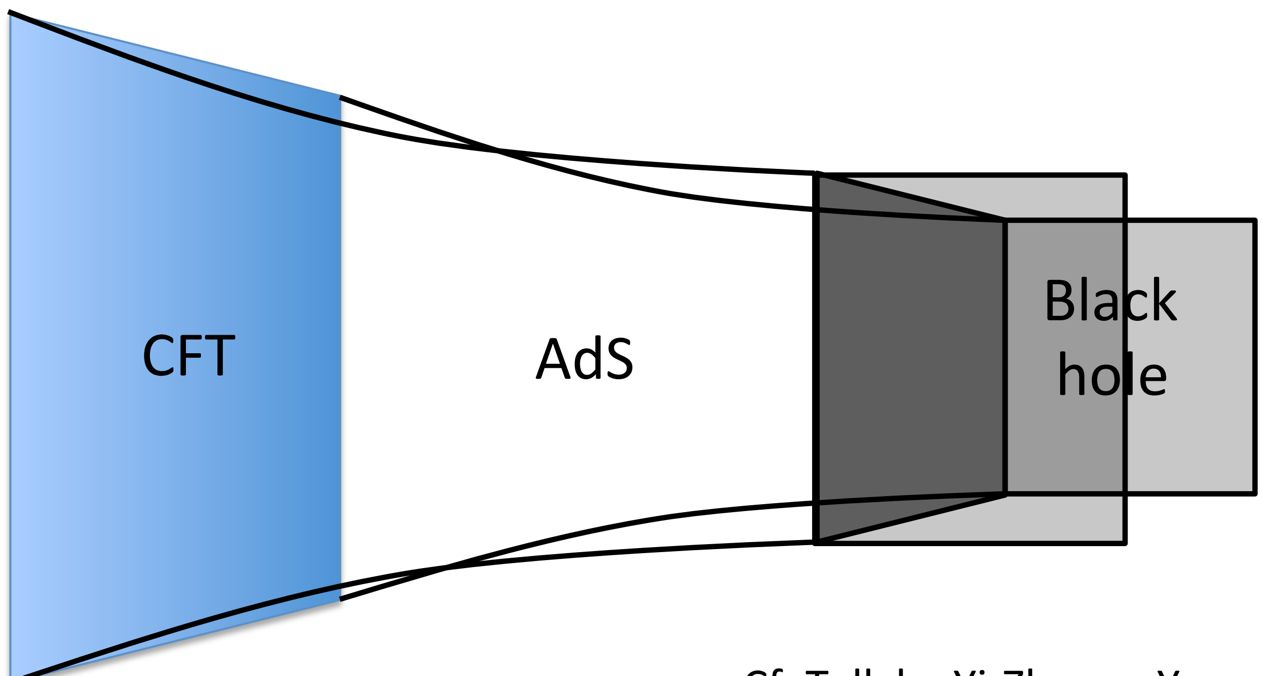


"cat"

AdS/CFT

[Maldacena '97]





Cf. Talk by Yi-Zhuang You

1. Formulation of AdS/DL correspondence

2. Implementation of AdS/DL and emerging space

1. Formulation of AdS/DL correspondence

1-1 **Solving inverse problem**

review **AdS/CFT: quantum response from geometry**

review **Deep learning: optimized sequential map**

1-2 **From AdS to DL**

1-3 **Dictionary of AdS/DL correspondence**

1-1 Brief history of quantum gravity

1974 Yoneya, Scherk-Schwarz: String = quantum gravity.

Yoneya, Prog.Theor.Phys. 51 (1974) 1907.
Scherk, Schwarz, Nucl.Phys. B81 (1974) 118.

1976 Hawking: Information loss problem of black holes.

Hawking, Phys.Rev.D14(1976)2460.

1997 Maldacena: Discovery of AdS/CFT.
A quantum gravity is nonperturbatively defined.

Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231.

2002 Holographic QCD.

Karch, Katz, JHEP 0206:043.
Kruczenski,Mateos,Myers,Winters JHEP 0405:041.
Sakai, Sugimoto, PTP 113 (2004) 843.

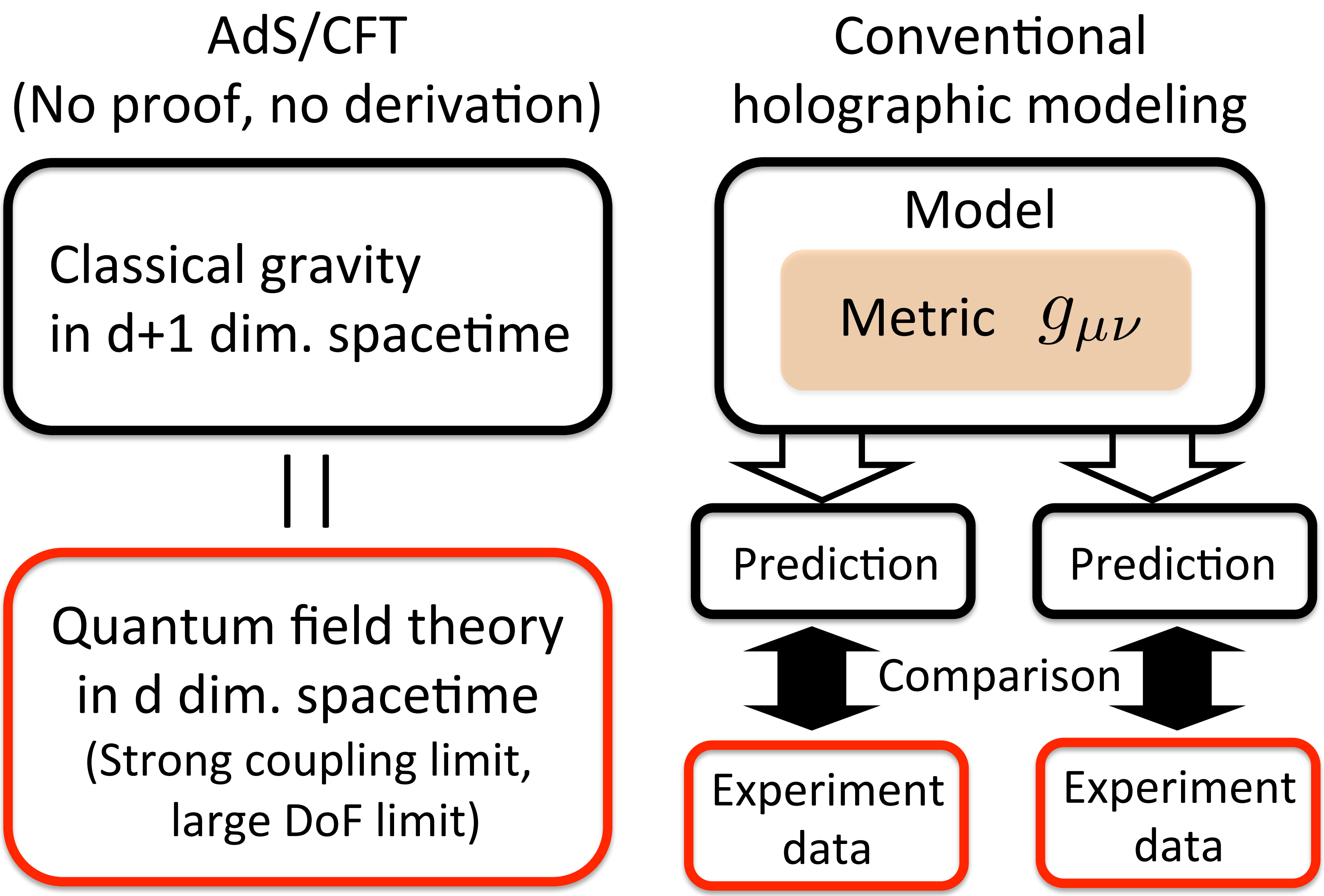
2008 Holographic superconductor.

Hartnoll, Herzog, Horowitz, PRL 101(2008)031601.

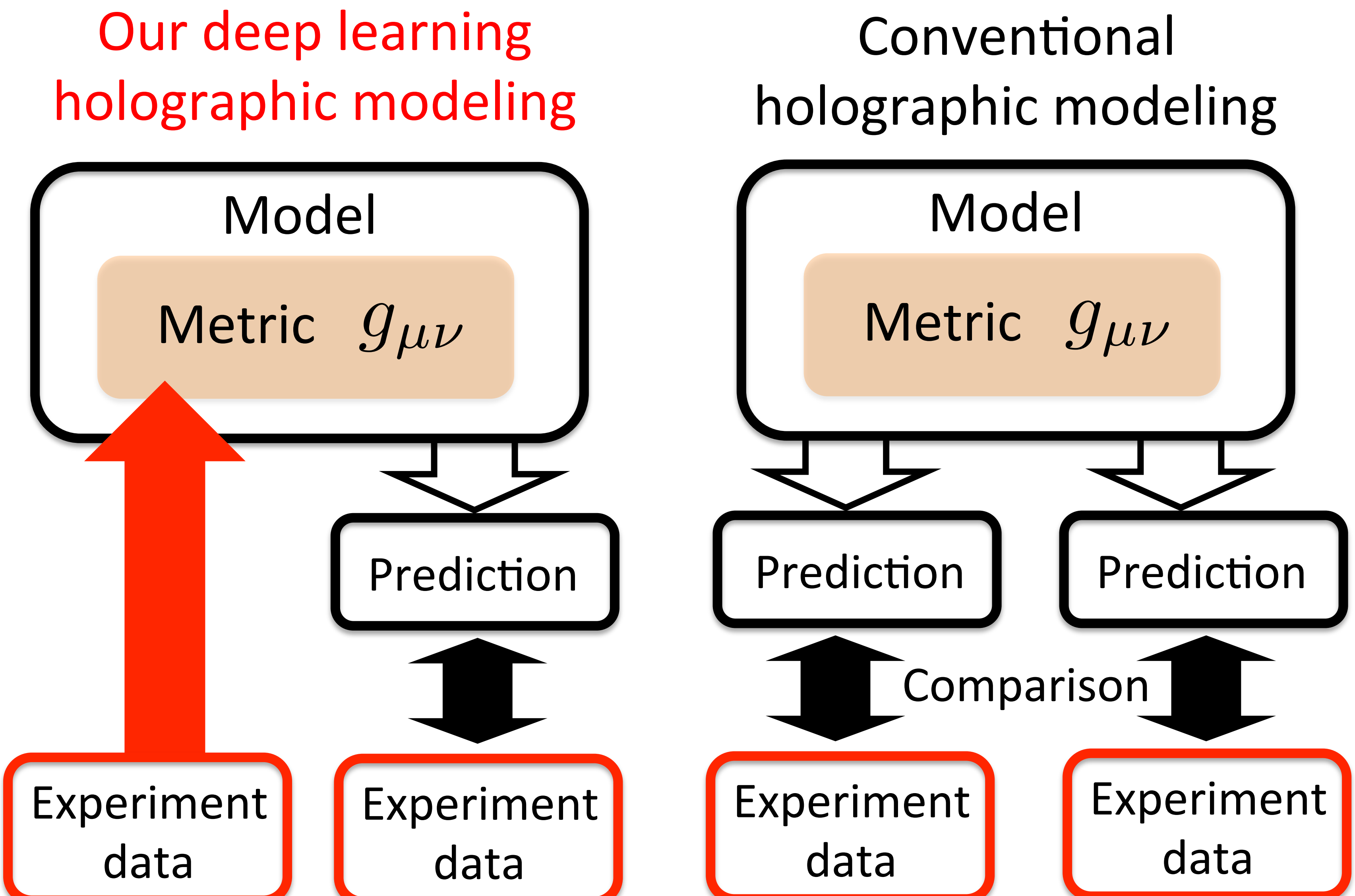
2009 Bulk reconstruction.

Heemskerk,Penedones,Polchinski,Sully, JHEP 0910:079.

1-1 Solving inverse problem



1-1 Solving inverse problem



review

AdS/CFT: quantum response from geometry

[Klebanov, Witten]

Classical scalar field theory in (d+1) dim. geometry

$$S = \int d^{d+1}x \sqrt{-\det g}\left[(\partial_\eta \phi)^2 - V(\phi)\right]$$

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2)$$

- AdS boundary ($\eta \sim \infty$) : $f \sim g \sim \exp[2\eta/L]$
- Black hole horizon ($\eta \sim 0$) : $f \sim \eta^2,\ g \sim \text{const.}$

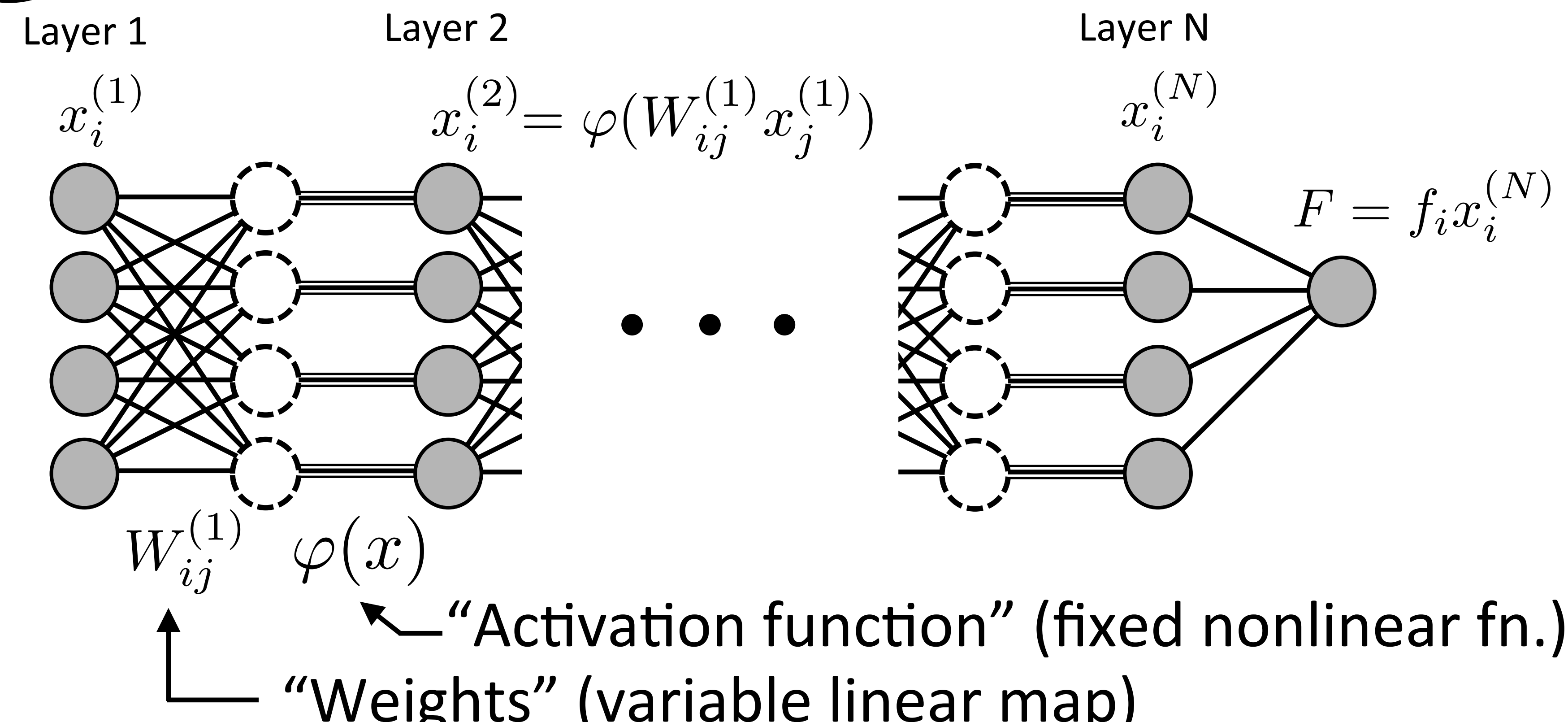
Solve EoM, get response $\langle \mathcal{O} \rangle_J$. Boundary conditions:

- AdS boundary ($\eta \sim \infty$) :

$$\phi = J e^{-\Delta_- \eta} + \frac{1}{\Delta_+ - \Delta_-}\langle \mathcal{O} \rangle e^{-\Delta_+ \eta}$$

- Black hole horizon ($\eta \sim 0$) : $\partial_\eta \phi \,\Big|_{\eta=0} = 0$

review Deep learning : optimized sequential map



1) Prepare many sets $\{x_i^{(1)}, F\}$: input + output
2) Train the network (adjust W_{ij}) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f_i(\varphi(W_{ij}^{(N-1)} \varphi(\cdots \varphi(W_{lm}^{(1)} x_m^{(1)})))) - F \right|$

1-2 From AdS to DL

Bulk EoM $\partial_\eta^2 \phi + \underline{\underline{h(\eta)}}\partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$

metric $h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}} \right]$

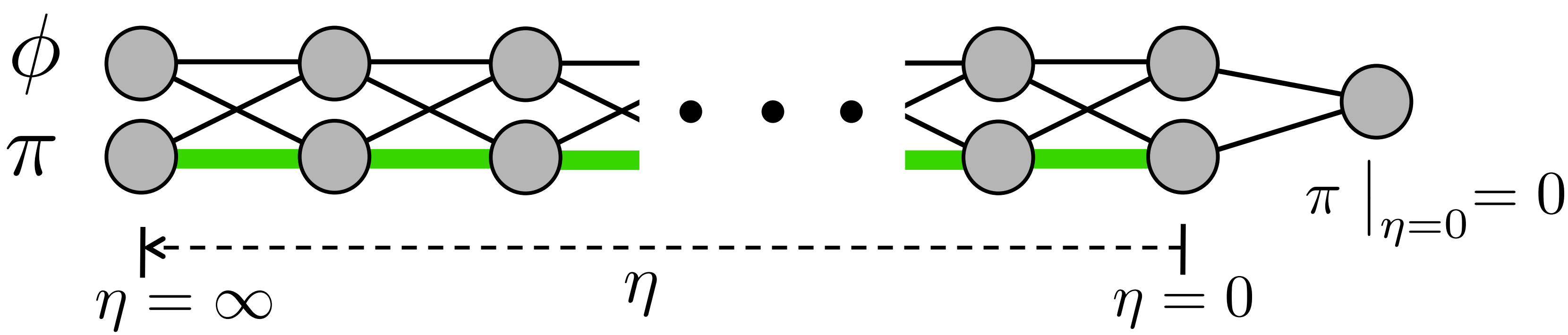
⇩

Discretization, Hamilton form

$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta)\pi(\eta) - \frac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$$

⇩

Neural-Network representation



1-3 Dictionary of AdS/DL correspondence

AdS/CFT	Deep learning
Emergent space $\infty > \eta \geq 0$	Depth of layers $i = 1, 2, \cdots, N$
Bulk gravity metric $h(\eta)$	Network weights $W_{ij}^{(a)}$
Nonlinear response $\langle \mathcal{O} \rangle_J$	Input data $x_i^{(1)}$
Horizon condition $\partial_\eta \phi \big|_{\eta=0} = 0$	Output data F
Interaction $V(\phi)$	Activation function $\varphi(x)$

1. Formulation of AdS/DL correspondence

1-1 **Solving inverse problem**

review **Deep learning : optimized sequential map**

review **AdS/CFT: quantum response from geometry**

1-2 **From AdS to DL**

1-3 **Dictionary of AdS/DL correspondence**

1. Formulation of AdS/DL correspondence

2. Implementation of AdS/DL and emerging space

2. Implementation of AdS/DL and emerging space

- 2-1 **Emergent geometry in deep learning**
- 2-2 **Can AdS Schwarzschild be learned?**
- 2-3 **Emergent space from real material?**
- 2-4 **Numerical experiment summary**
- 2-5 **Machines learn…, what do we learn?**

2-1 Emergent geometry in deep learning

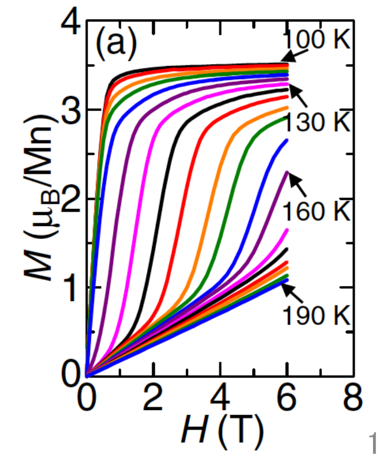
Experiment 1: "Can AdS Schwarzschild be learned?"

1) Use AdS Schwarzschild and generate input data.
2) Prepare network with unspecified metric.
3) Let the network learn it by the data.
4) Check if AdS Schwarzschild is reproduced.

Experiment 2: "Emergent space from real material?"

1) Use material experimental data.
   Ex) Magnetization curve of strongly correlated material
2) 3) (same as above.)
4) Watch how space emerges!

$Sm_{0.6}Sr_{0.4}MnO_3$



2-2 Exp1: Can AdS Schwarzschild be learned?

1) Use AdS Schwarzschild and generate input data.
2) Prepare network with unspecified metric.
3) Let the network learn it by the data.
4) Check if AdS Schwarzschild is reproduced.

$$\partial_\eta^2 \phi + h(\eta)\partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$$

$$h(\eta) = 3\coth(3\eta) \qquad V[\phi] = -\phi^2 + \frac{1}{4}\phi^4$$

AdS Schwarzschild metric
in the unit of AdS radius $L = 1$

2-2 Exp1: Can AdS Schwarzschild be learned?

1) Use AdS Schwarzschild and generate input data.
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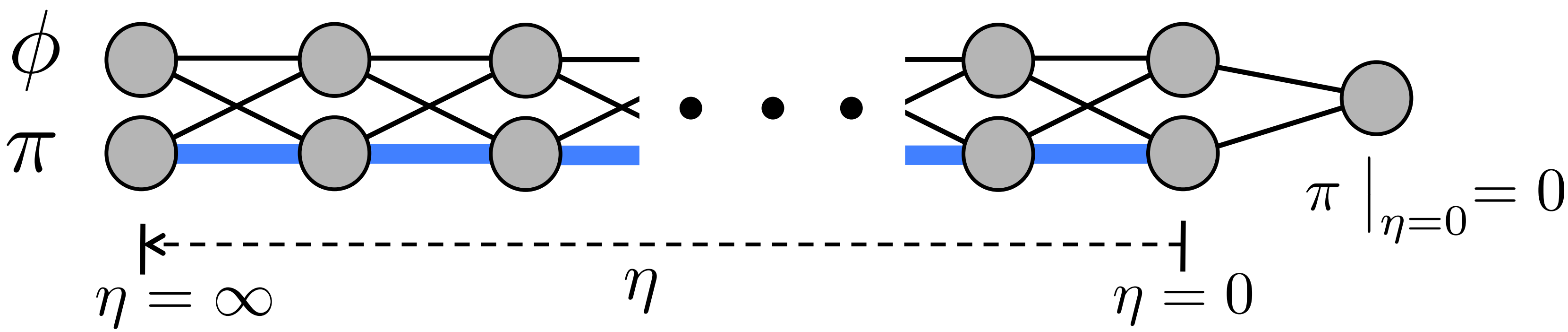
$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta)\pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$$



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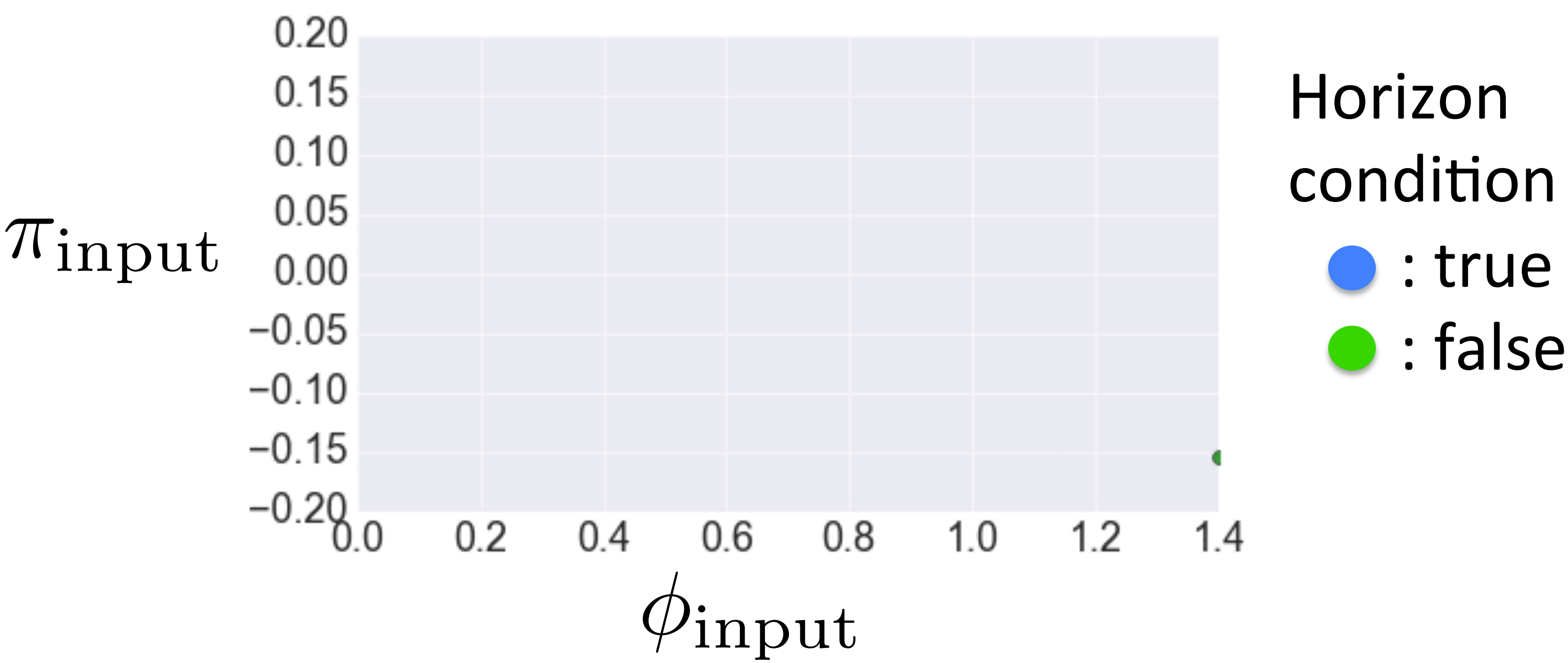

2-2 Exp1: Can AdS Schwarzschild be learned?

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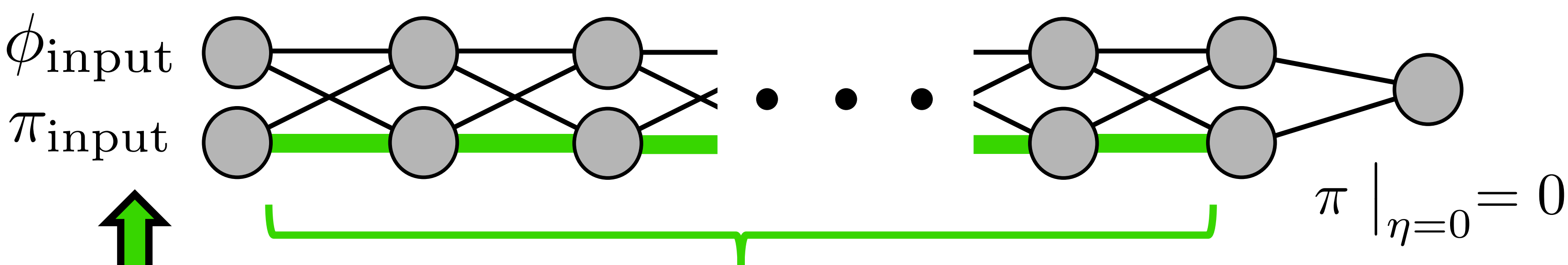
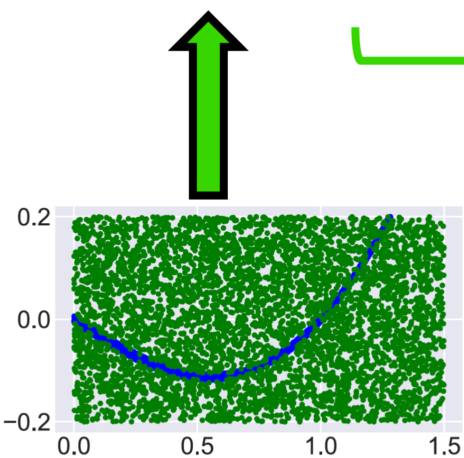


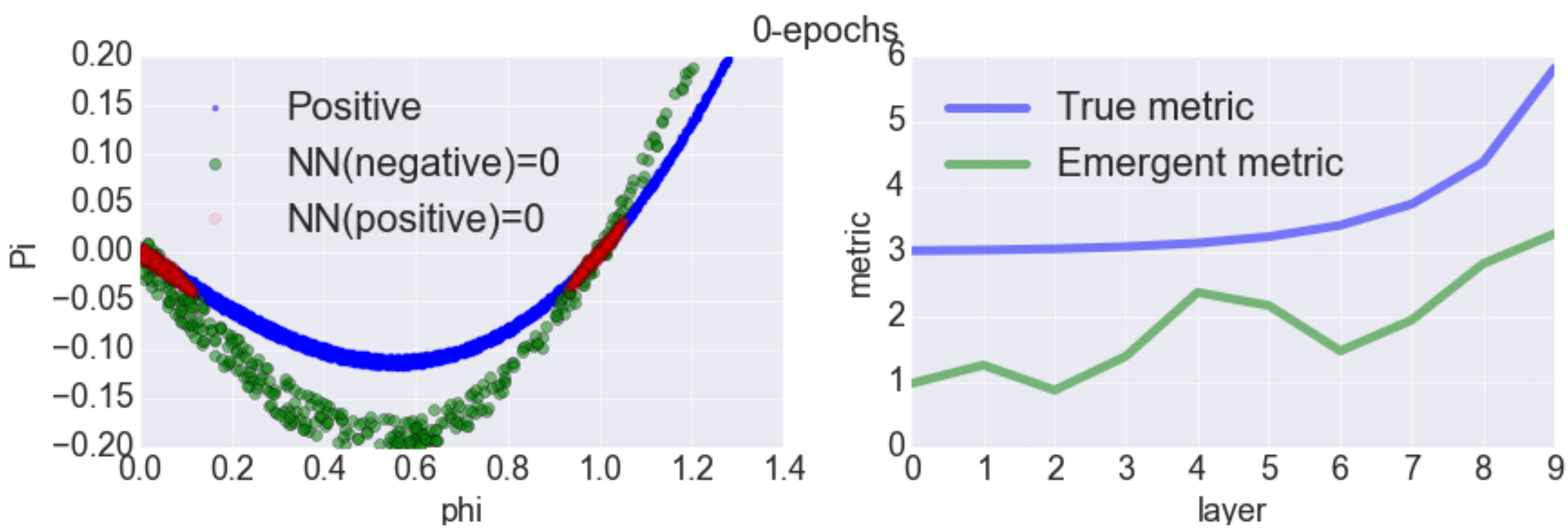
Unspecified metric
(10 layers, to be trained)



Generated data from AdS Schwarzschild
(10000 data points)

2-2 Exp1: Can AdS Schwarzschild be learned?

1) Use AdS Schwarzschild and generate input data.
2) Prepare network with unspecified metric.
3) Let the network learn it by the data.
4) Check if AdS Schwarzschild is reproduced.



With a regularization

2-1 Emergent geometry in deep learning

Experiment 1: "Can AdS Schwarzschild be learned?"

1) Use AdS Schwarzschild and generate input data.
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Experiment 2: "Emergent space from real material?"

1) Use material experimental data.
   Ex) Magnetization curve of
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2) 3) (same as above.)
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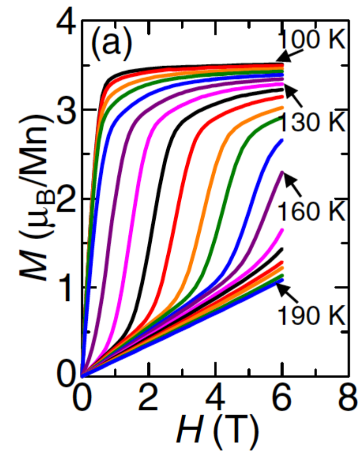
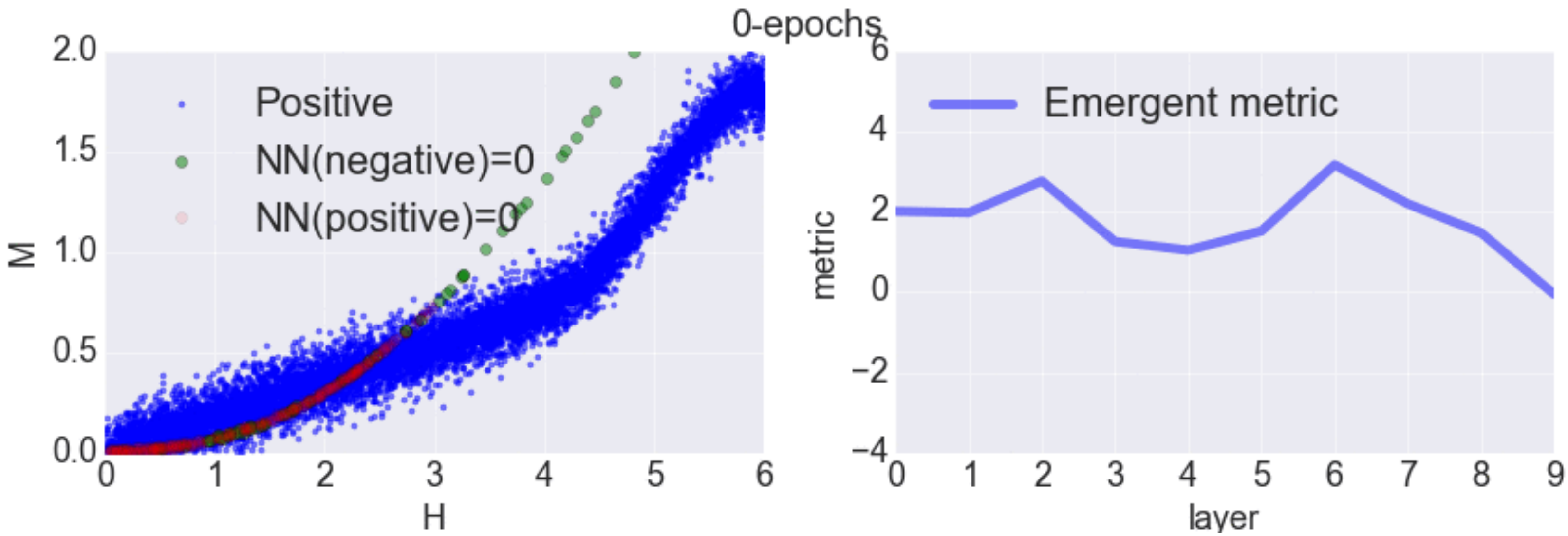
$Sm_{0.6}Sr_{0.4}MnO_3$



2-3 Exp2: Emergent space from real material?



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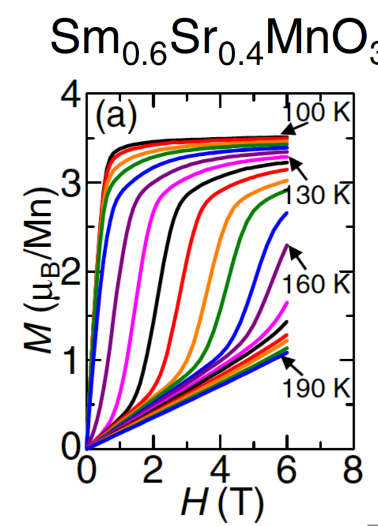
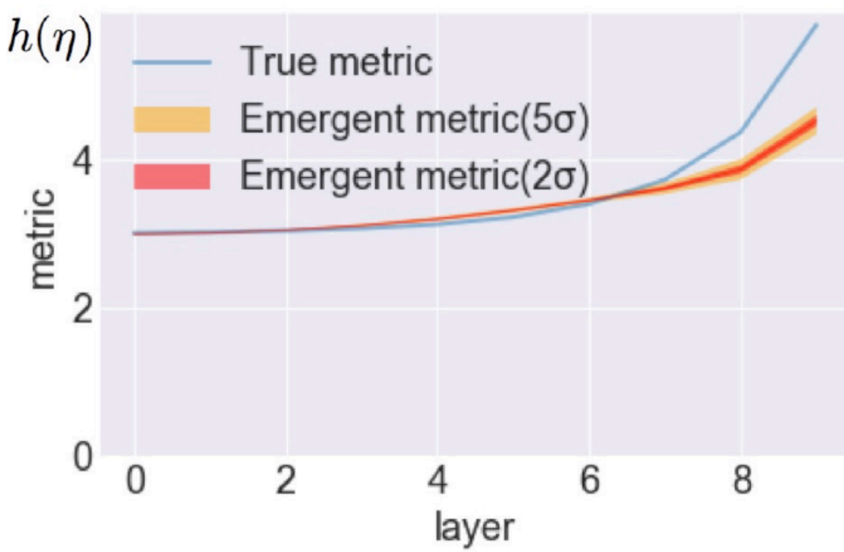


2-4 Numerical experiment summary

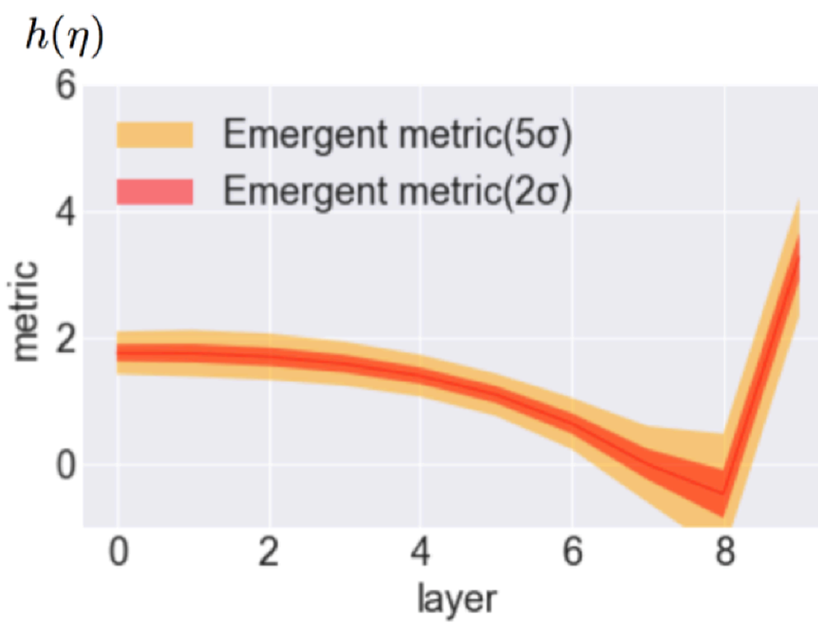
Experiment 1

AdS Schwarzschild is successfully learned.

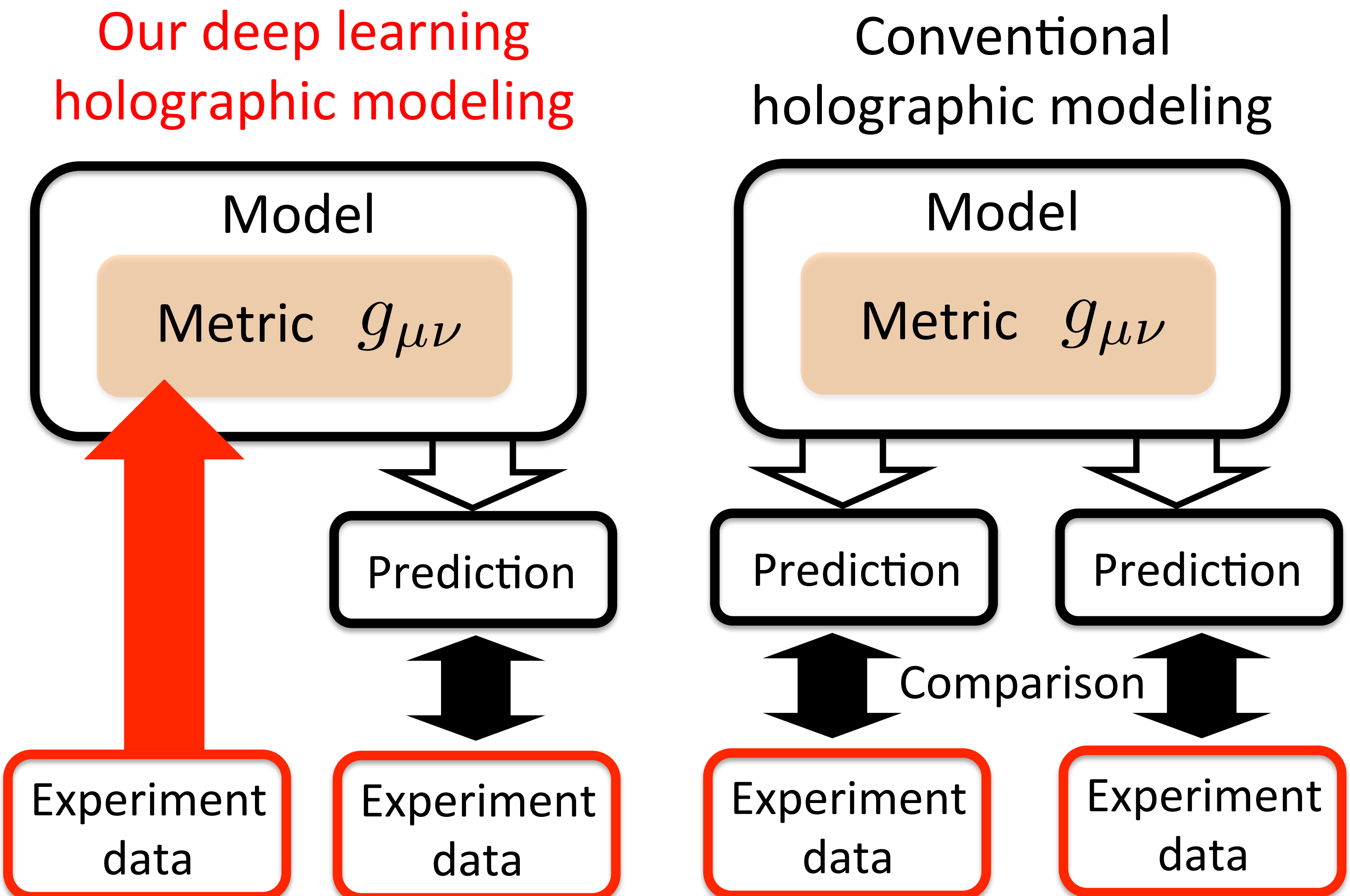


Experiment 2

Experimental data is explained by emergent space.



2-5 Machines learn..., what do we learn?



2-5 Machines learn…, what do we learn?

- Unreproduced region near the horizon?
  → Issue on regularization and data thickness

- Can more components of metric be learned?
  → Bulk gauge fields, inhomogeneous data

- Is Einstein equation satisfied?
  → No, in general.

- Holographic QCD? Prediction?

- Einstein gravity? Emergent space? General network?

1. Formulation of AdS/DL correspondence

2. Implementation of AdS/DL and emerging space