Renormalization and Hierarchical Knowledge Representations

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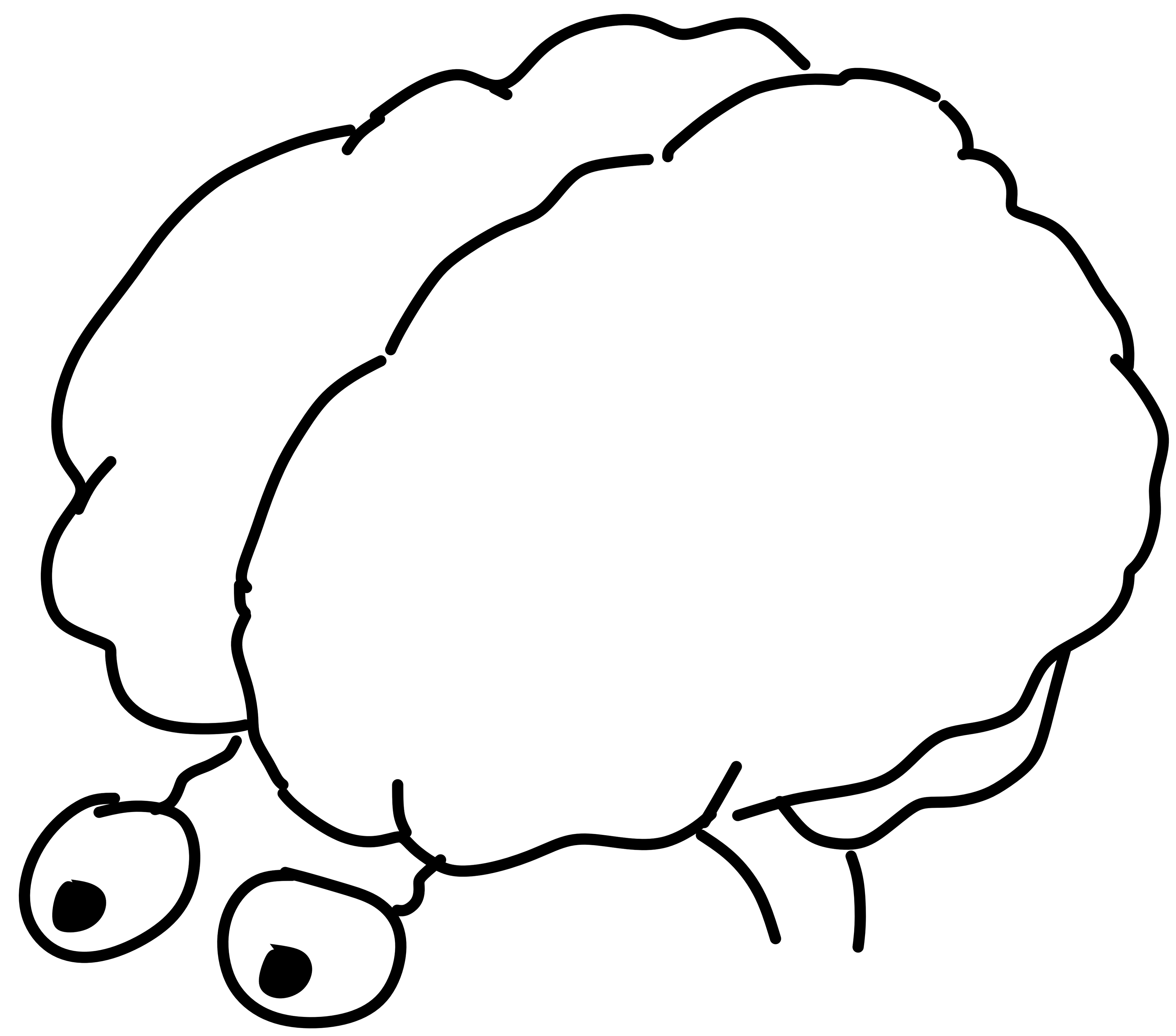
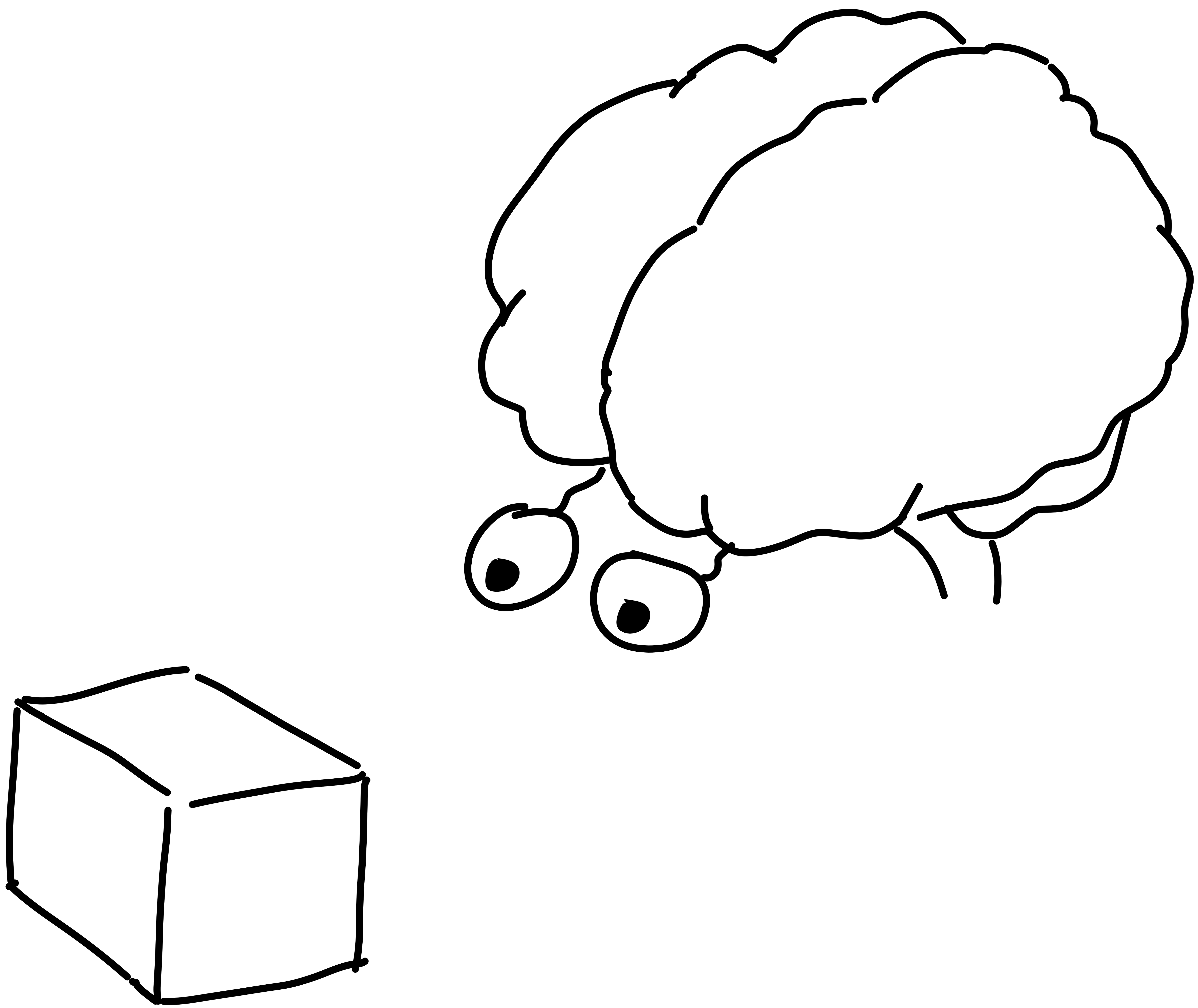
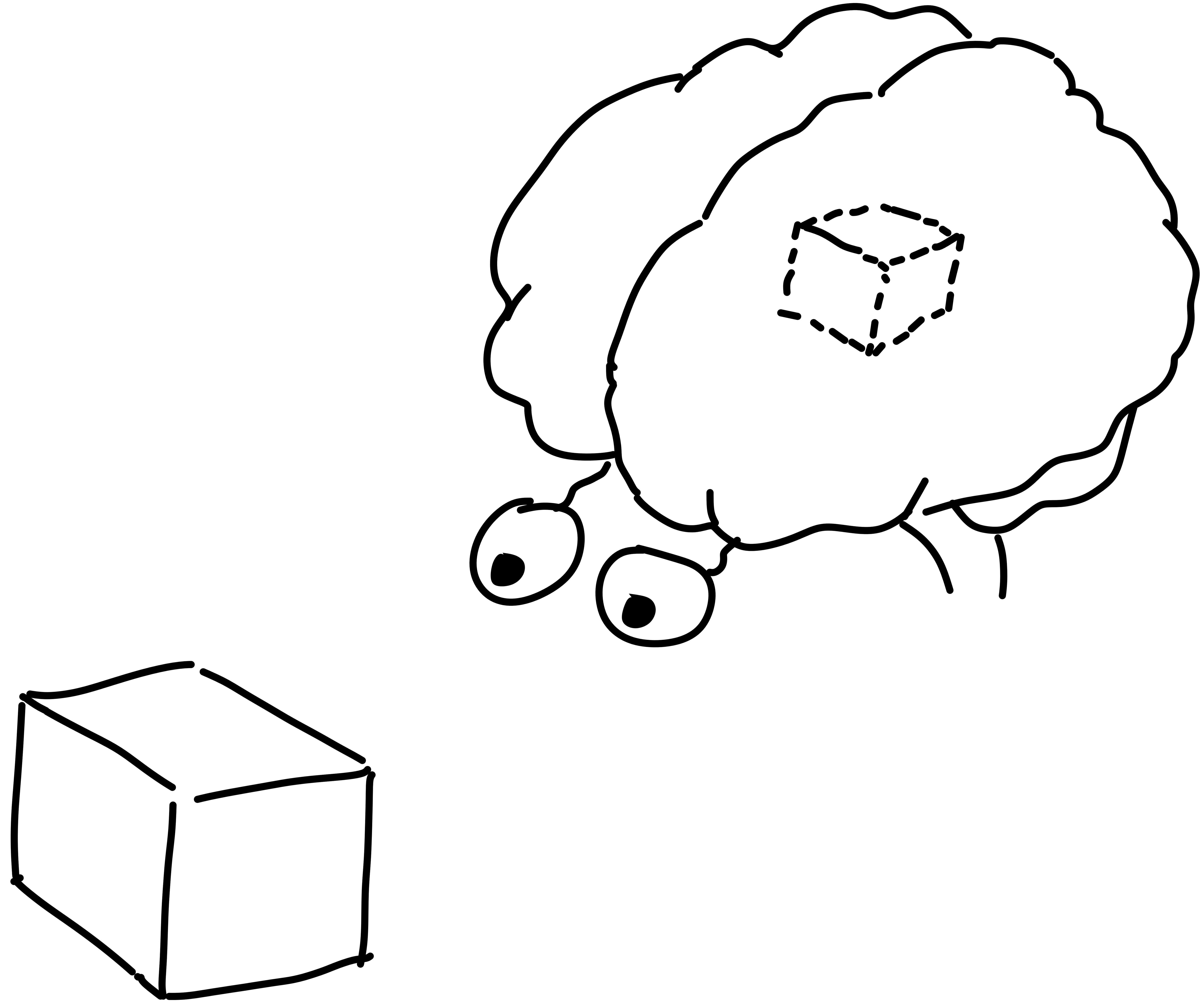
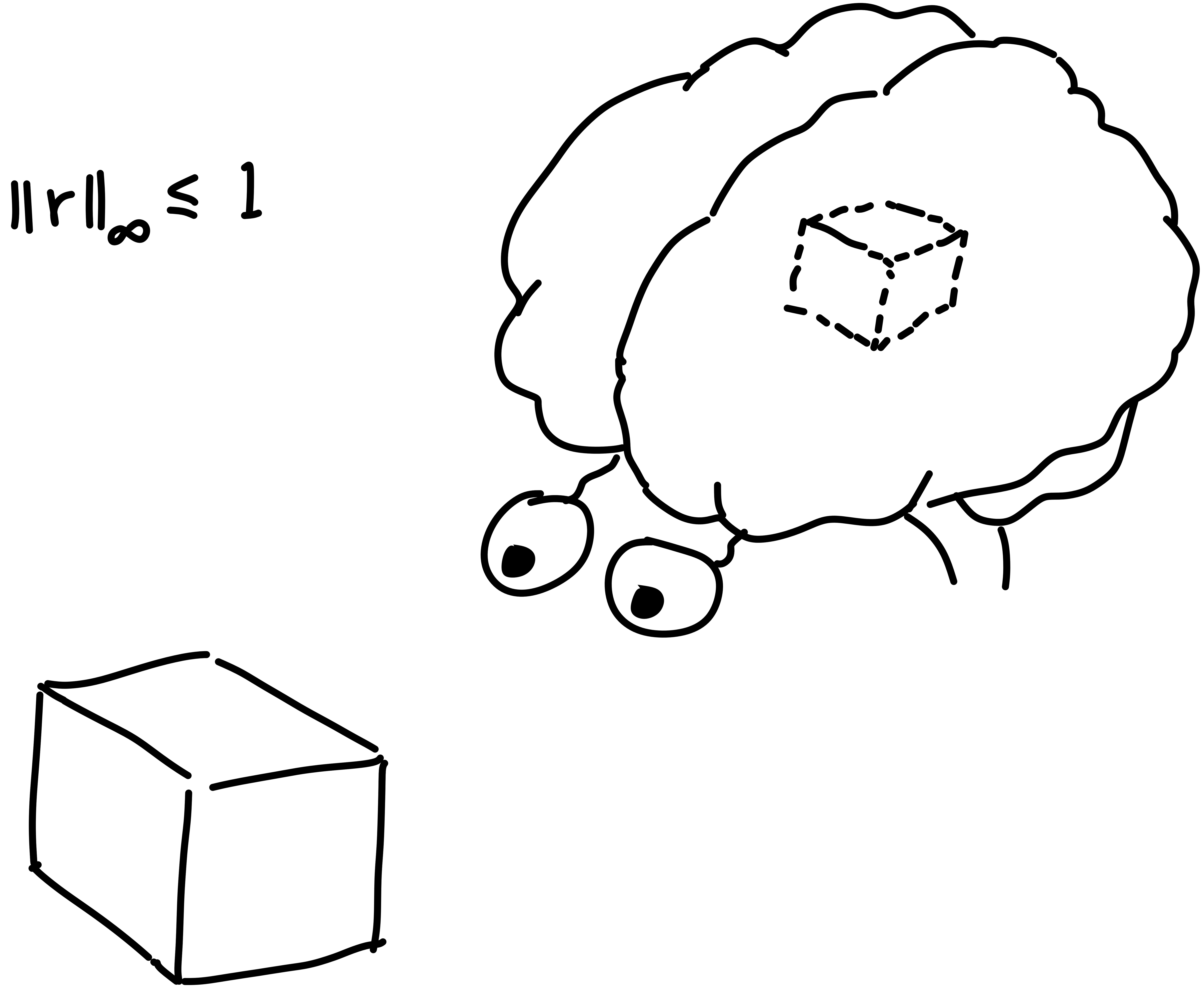
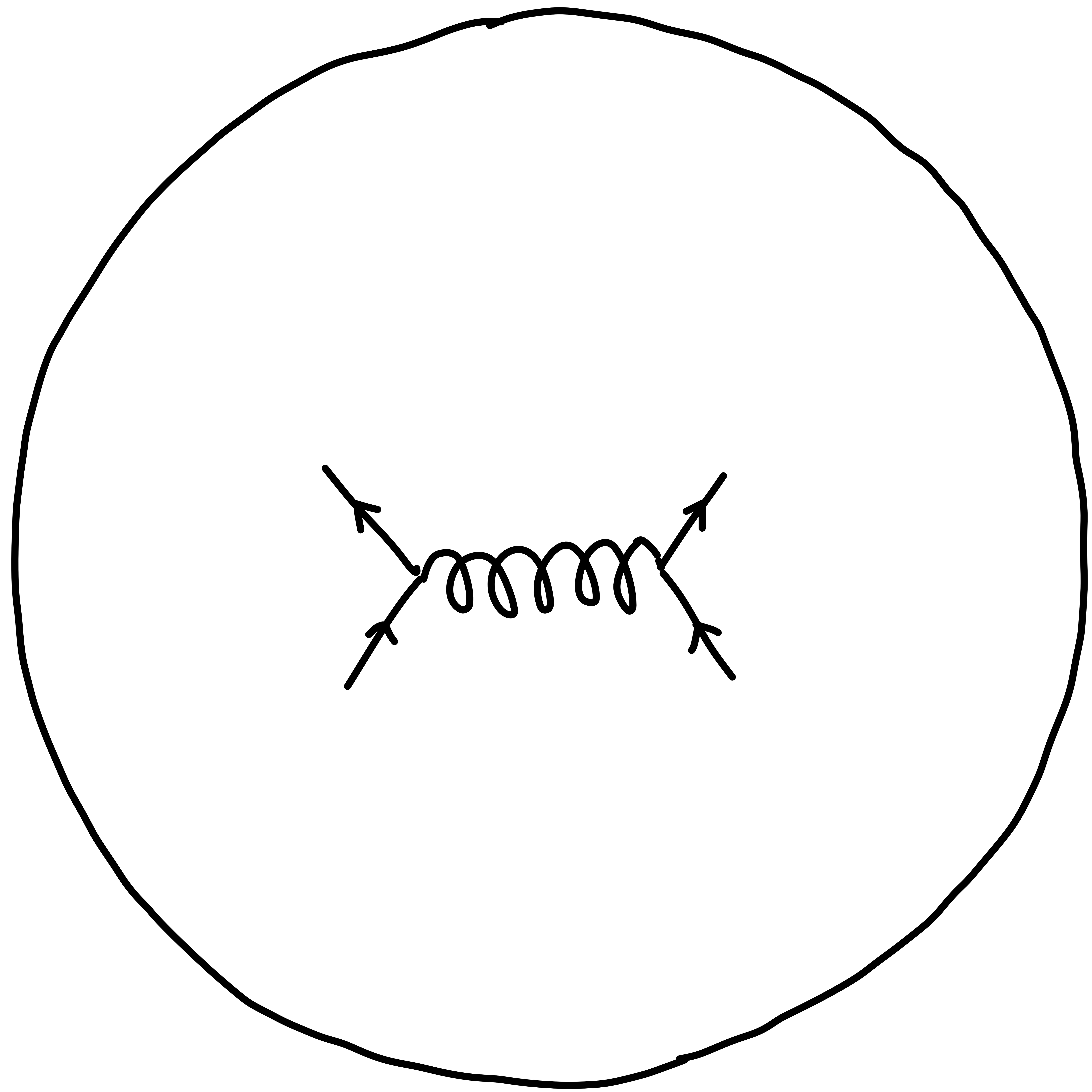
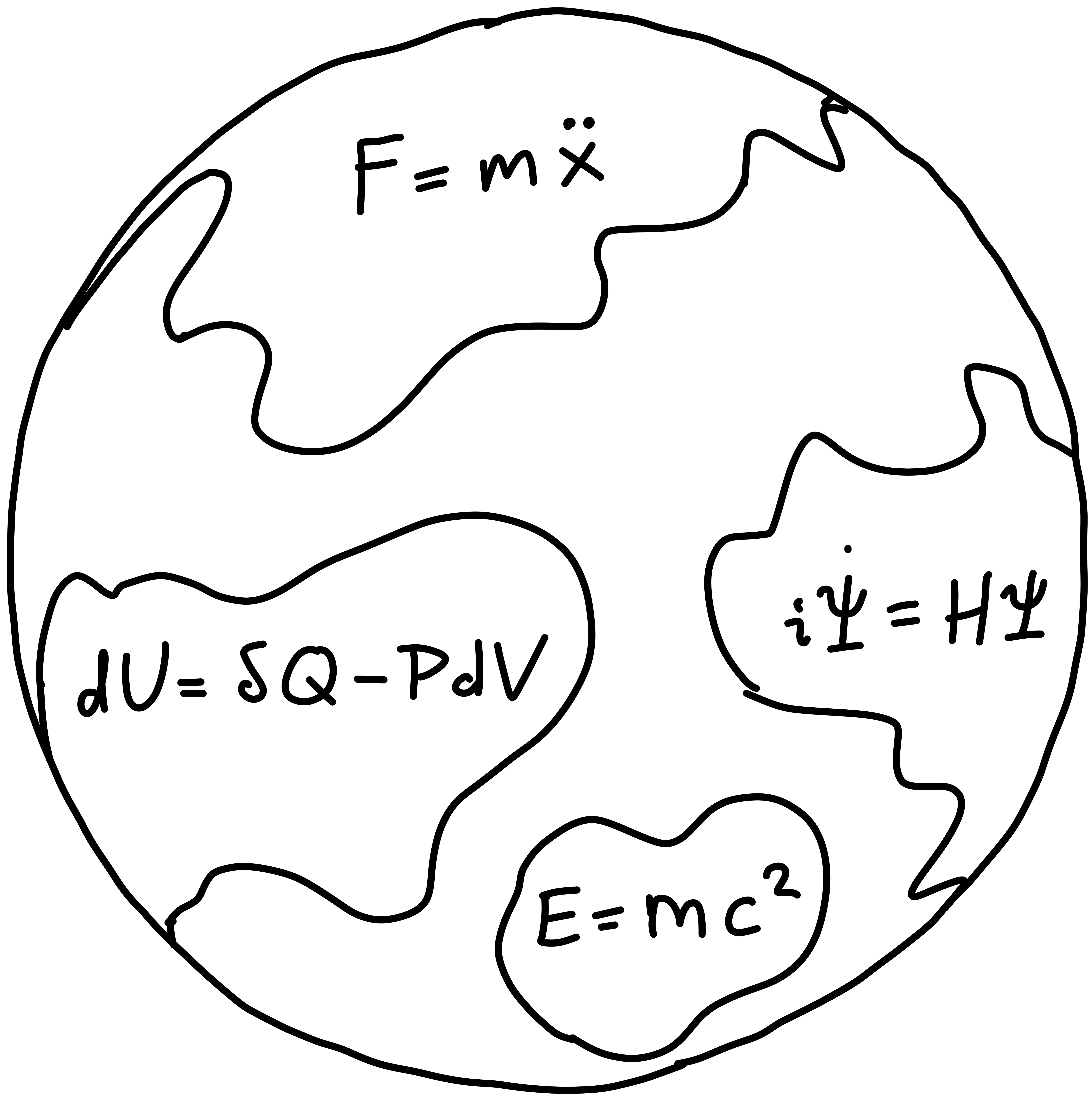


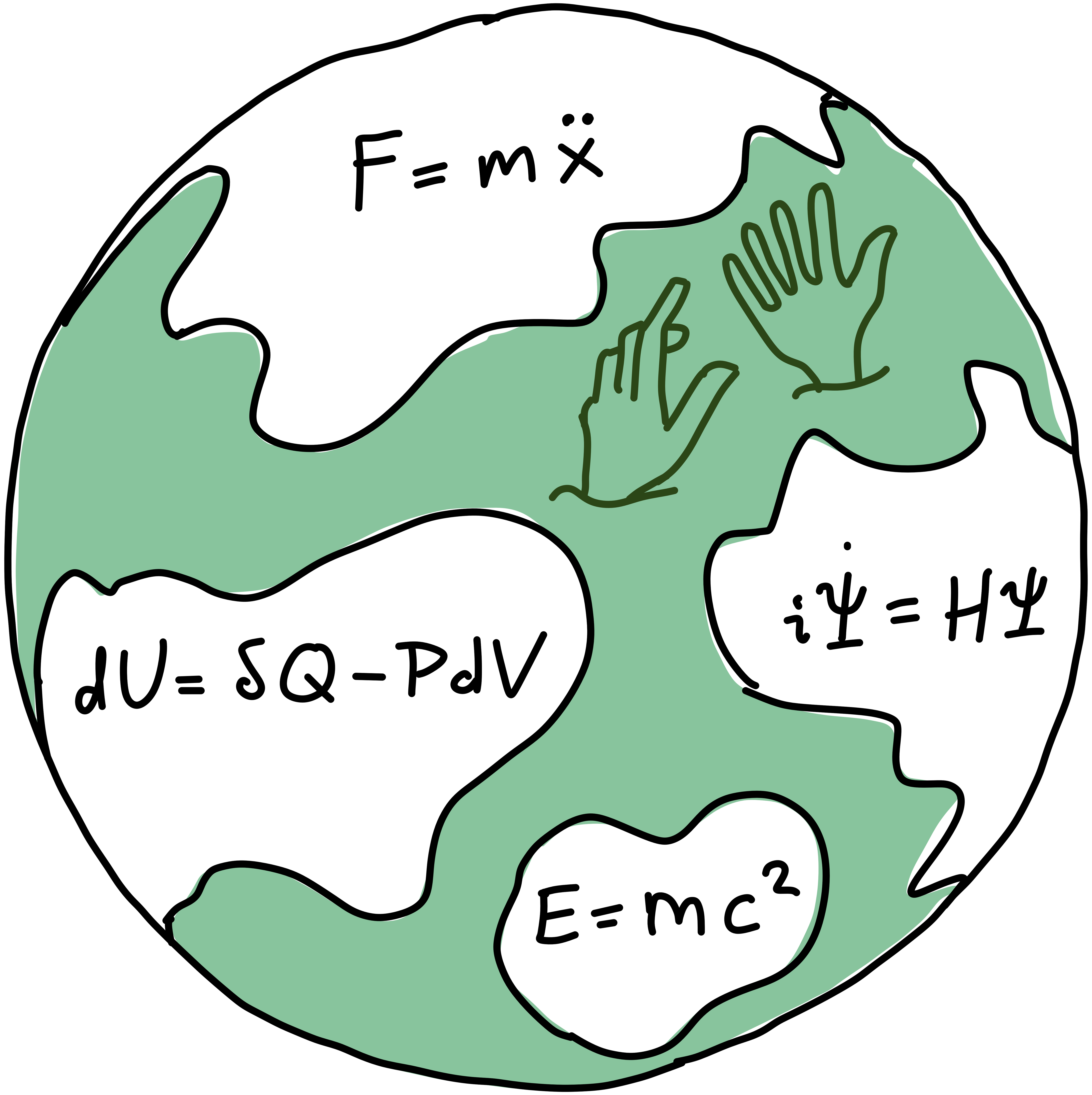




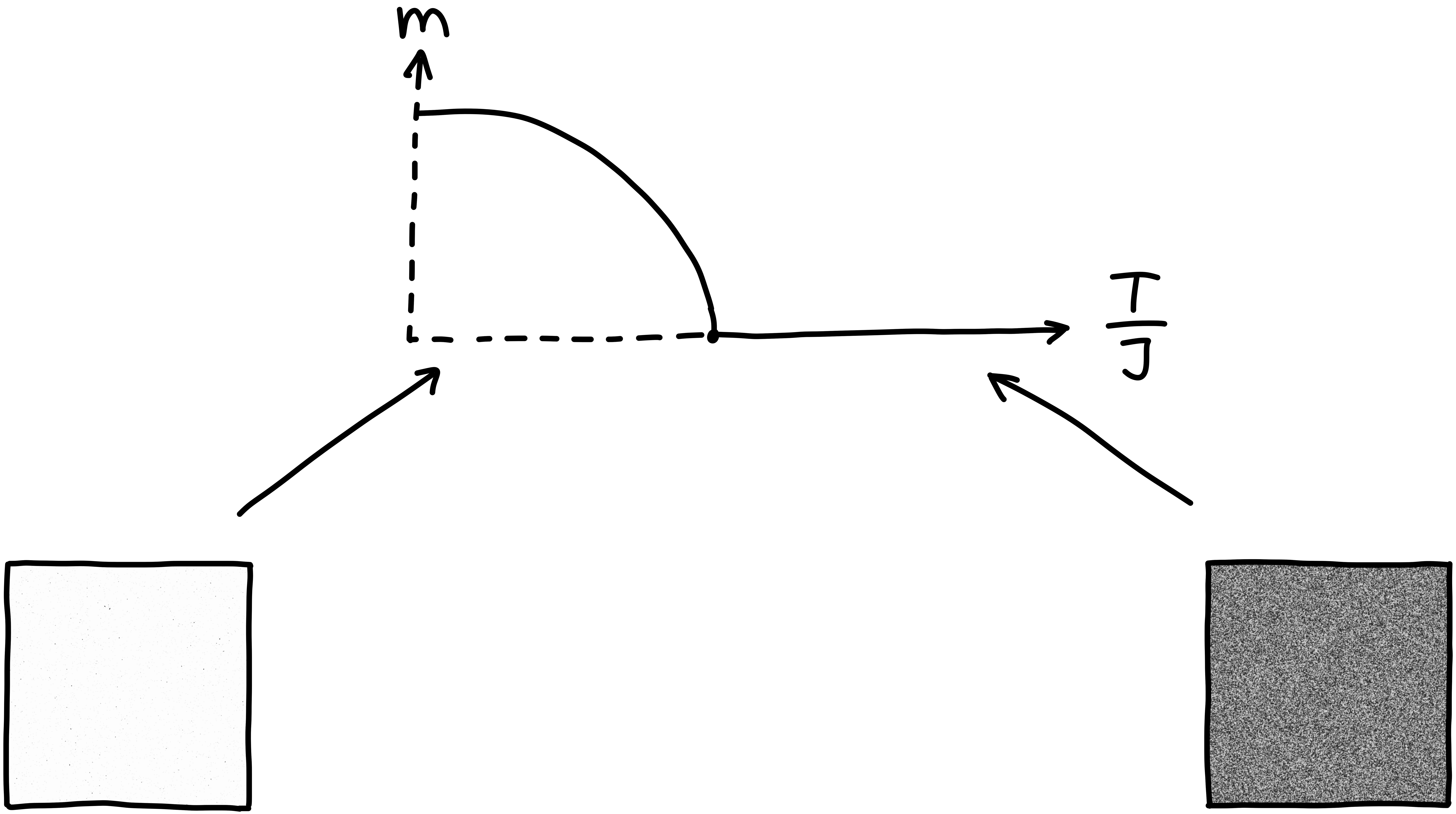

$\|r\|_\infty \leq 1$

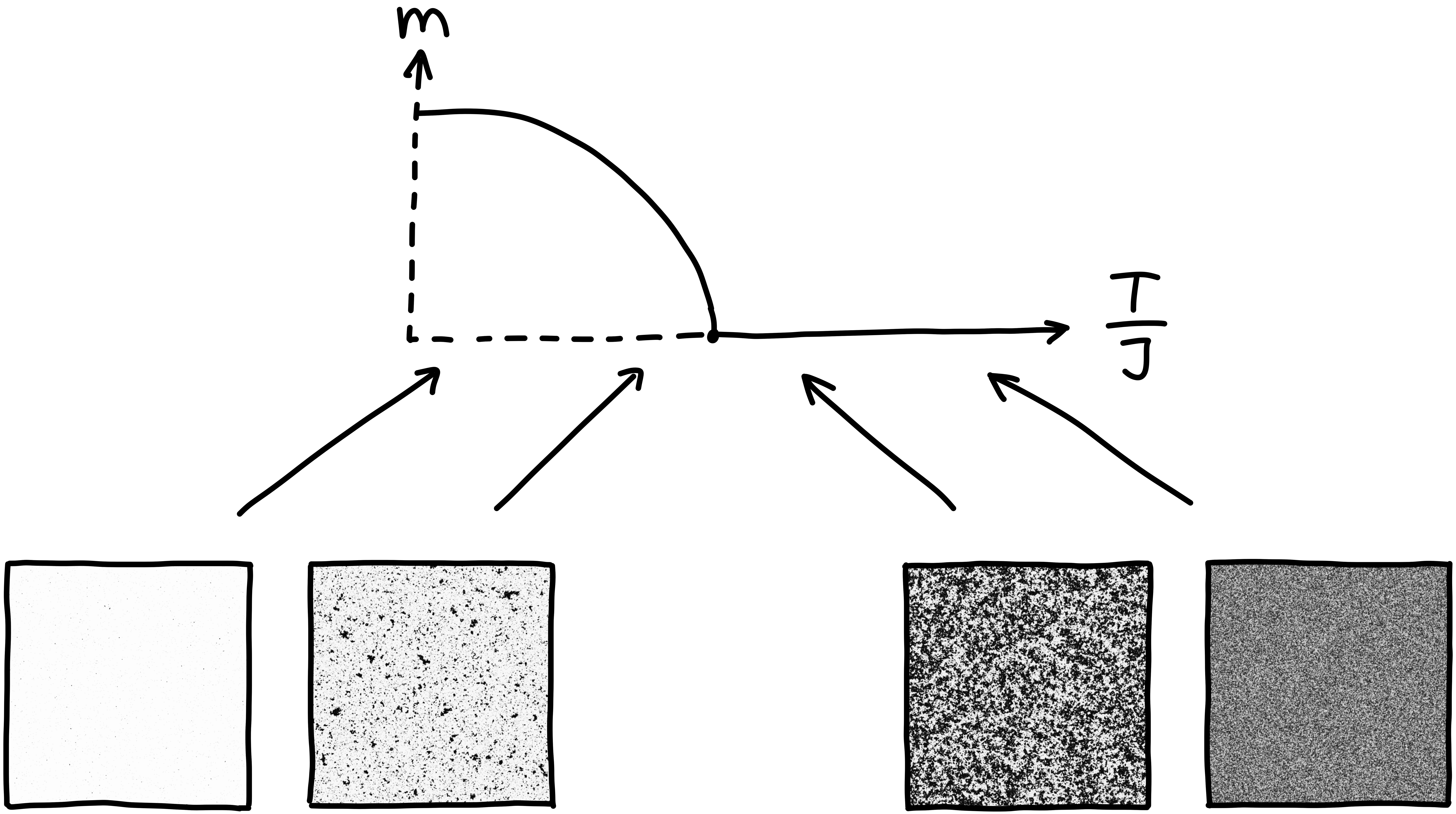



F = mẍ
dU = δQ − PdV
iΨ̇ = HΨ
E = mc²


$F = m\ddot{x}$
$dU = \delta Q - PdV$
$i\dot{\Psi} = H\Psi$
$E = mc^2$

Renormalization “group”

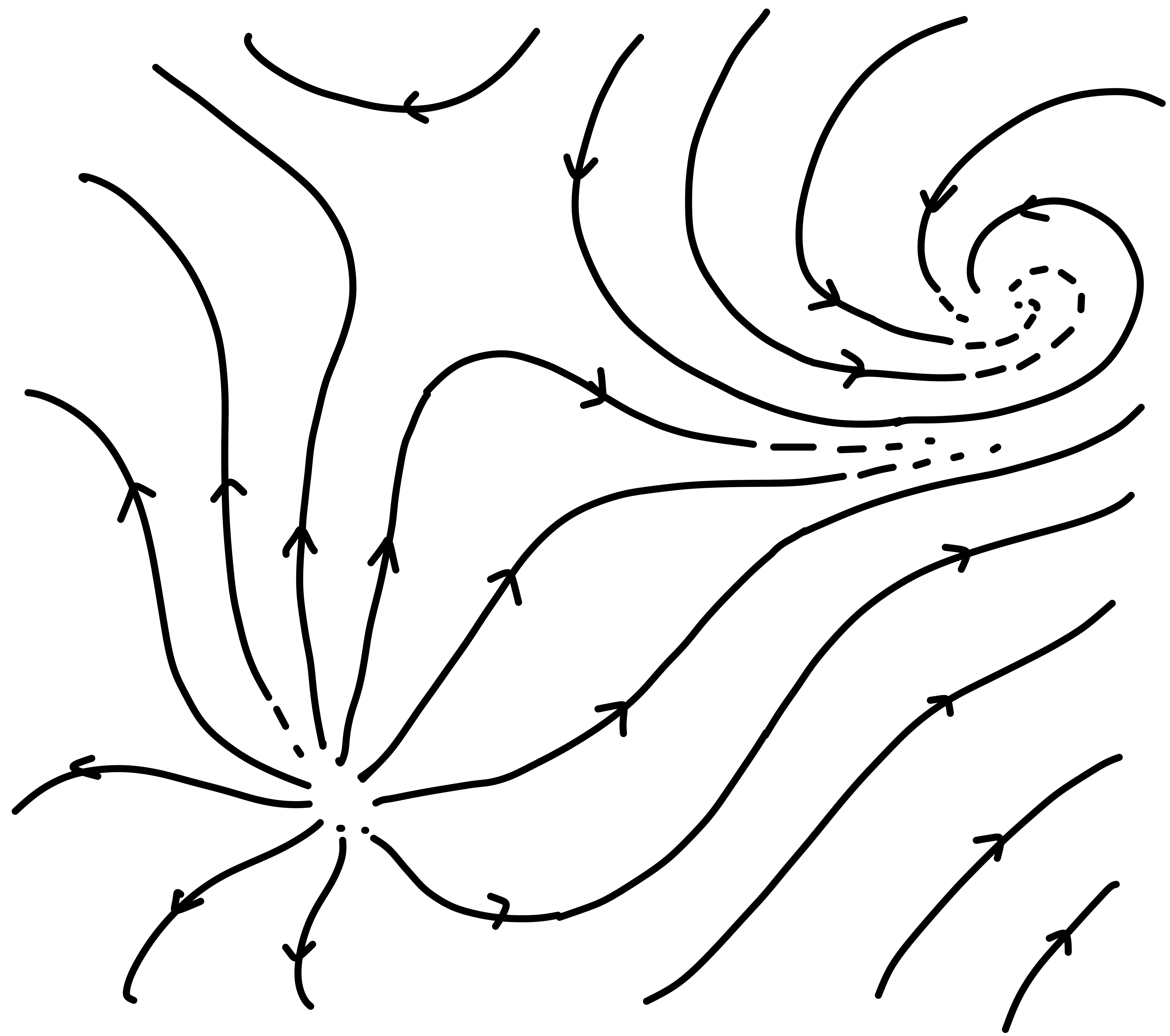

m
T
J


m
T
J

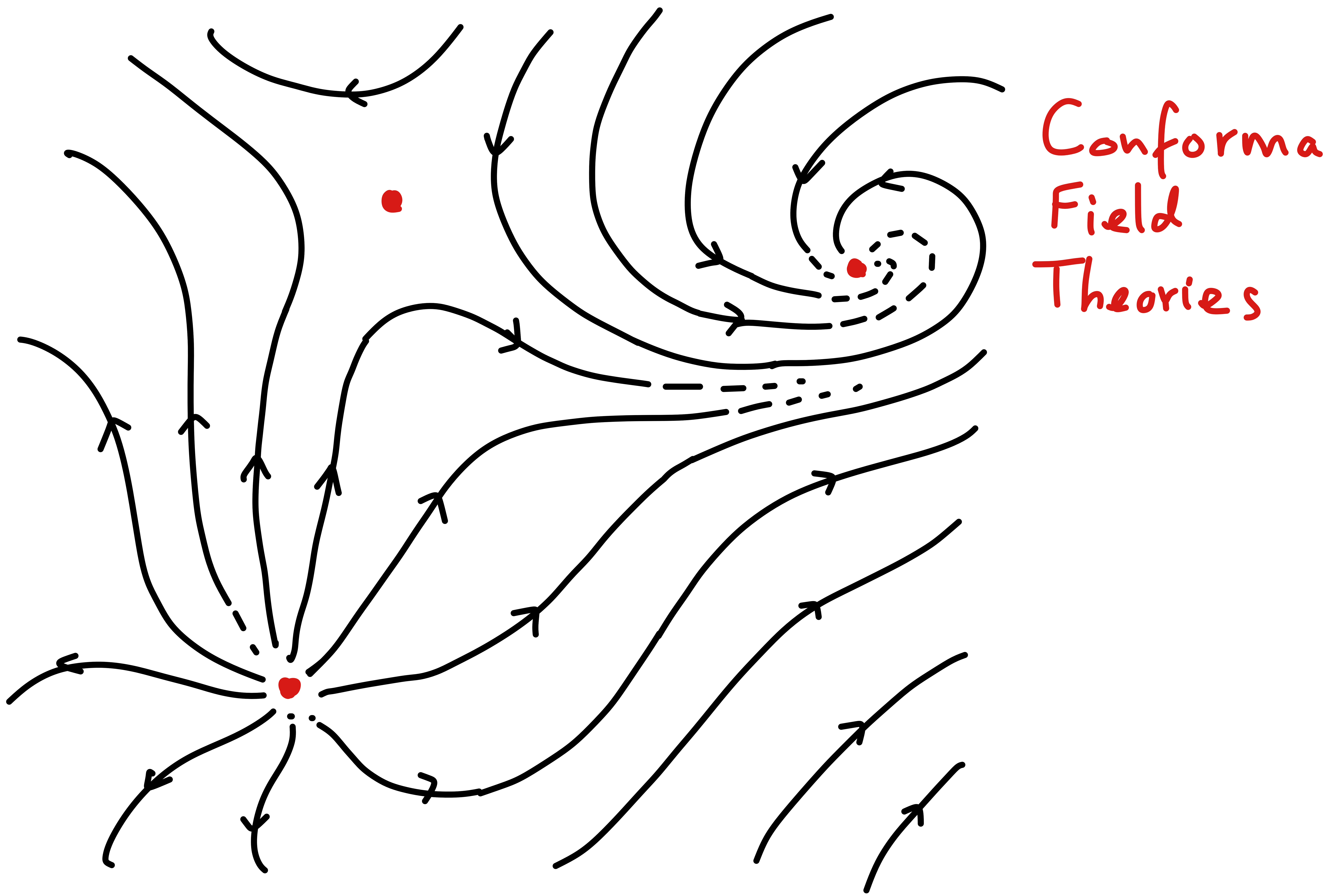

m
T/J
T/J = 0
T/J_c
RG flow
T/J = ∞


m
T/J
T/J = 0
T/J_c
RG flow
T/J = ∞

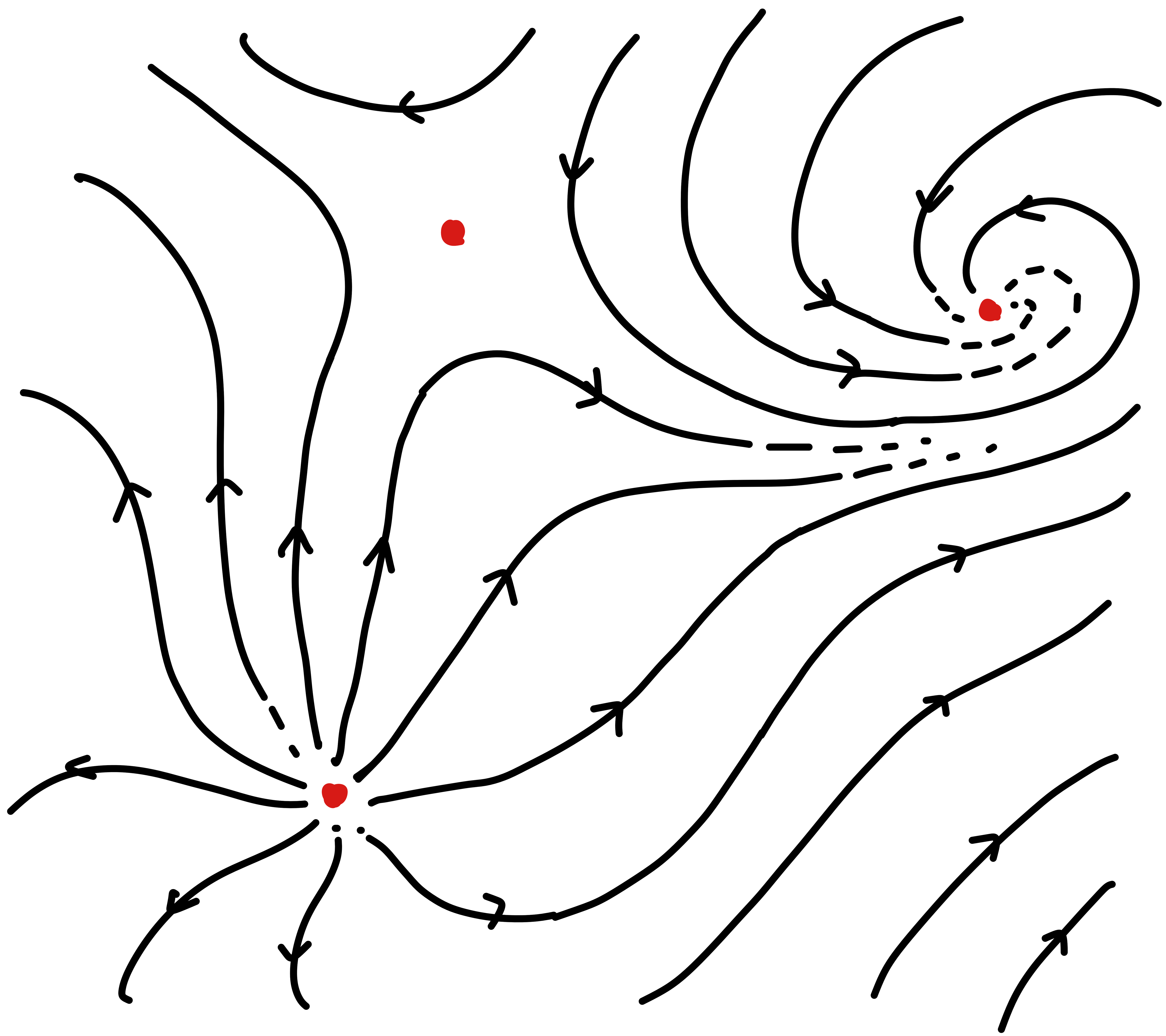
space of theories

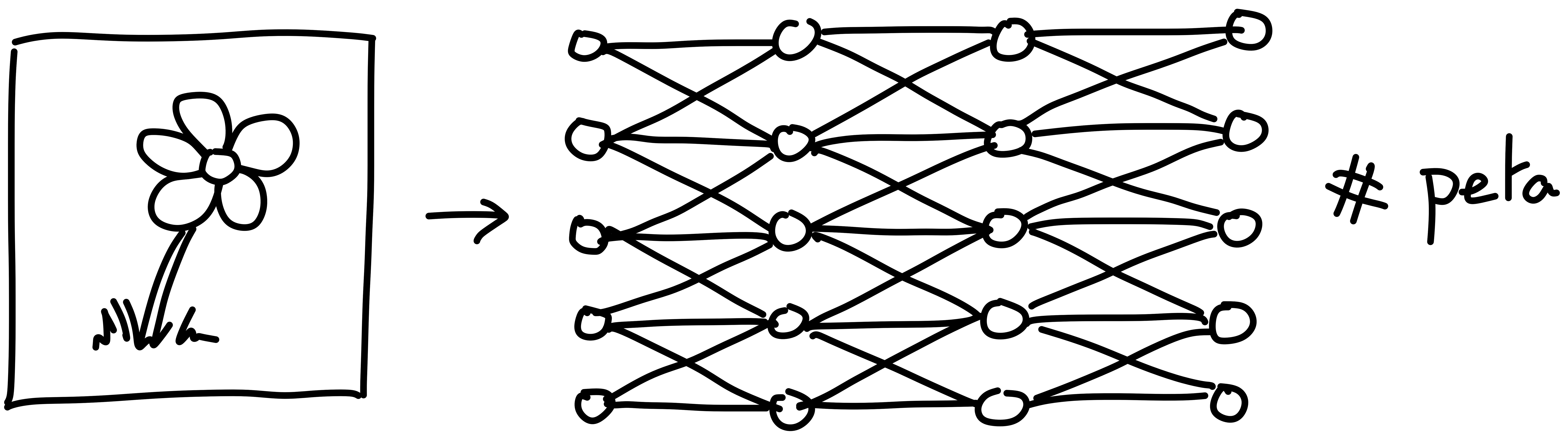


space of theories

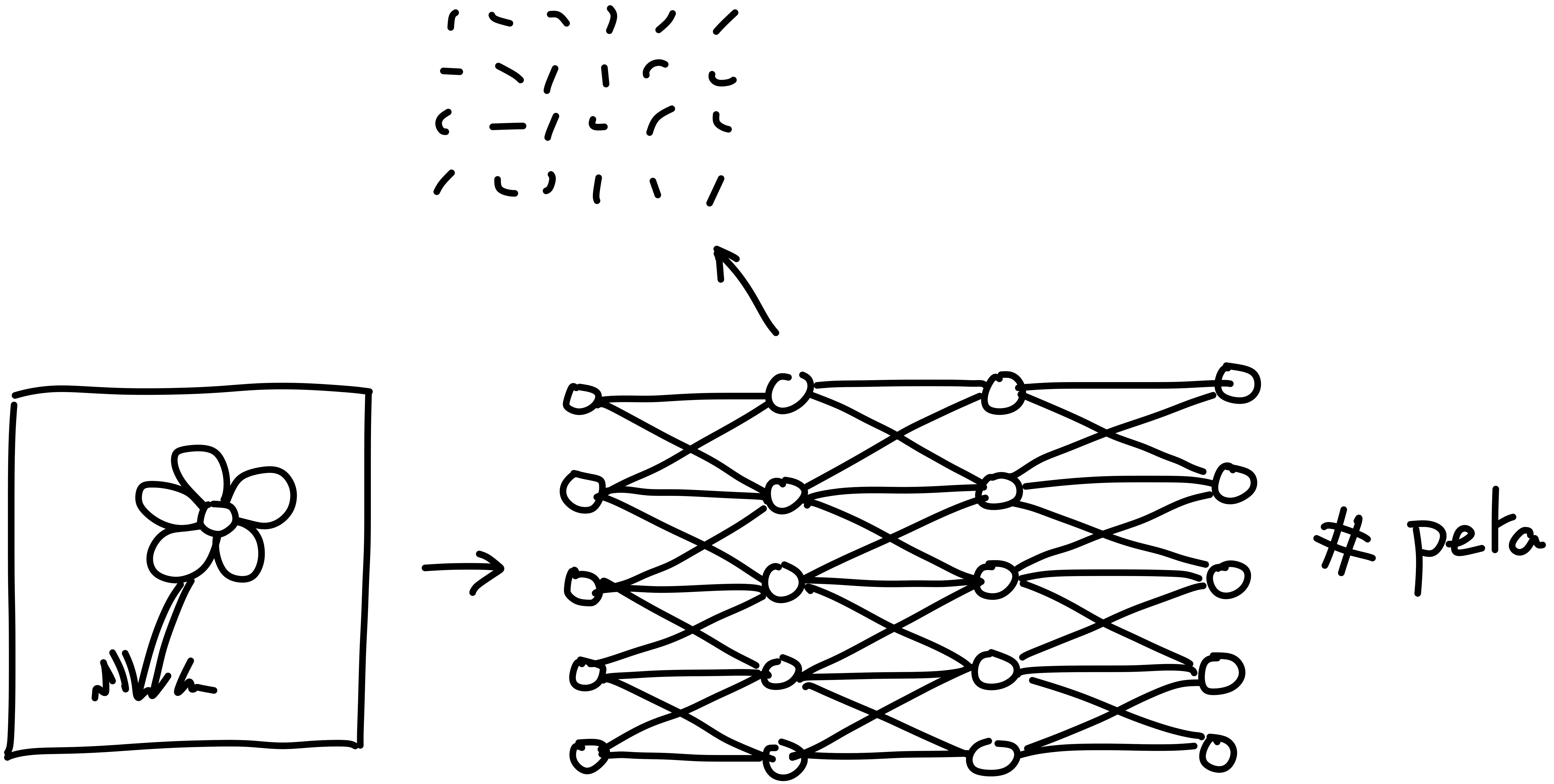


space of theories ?

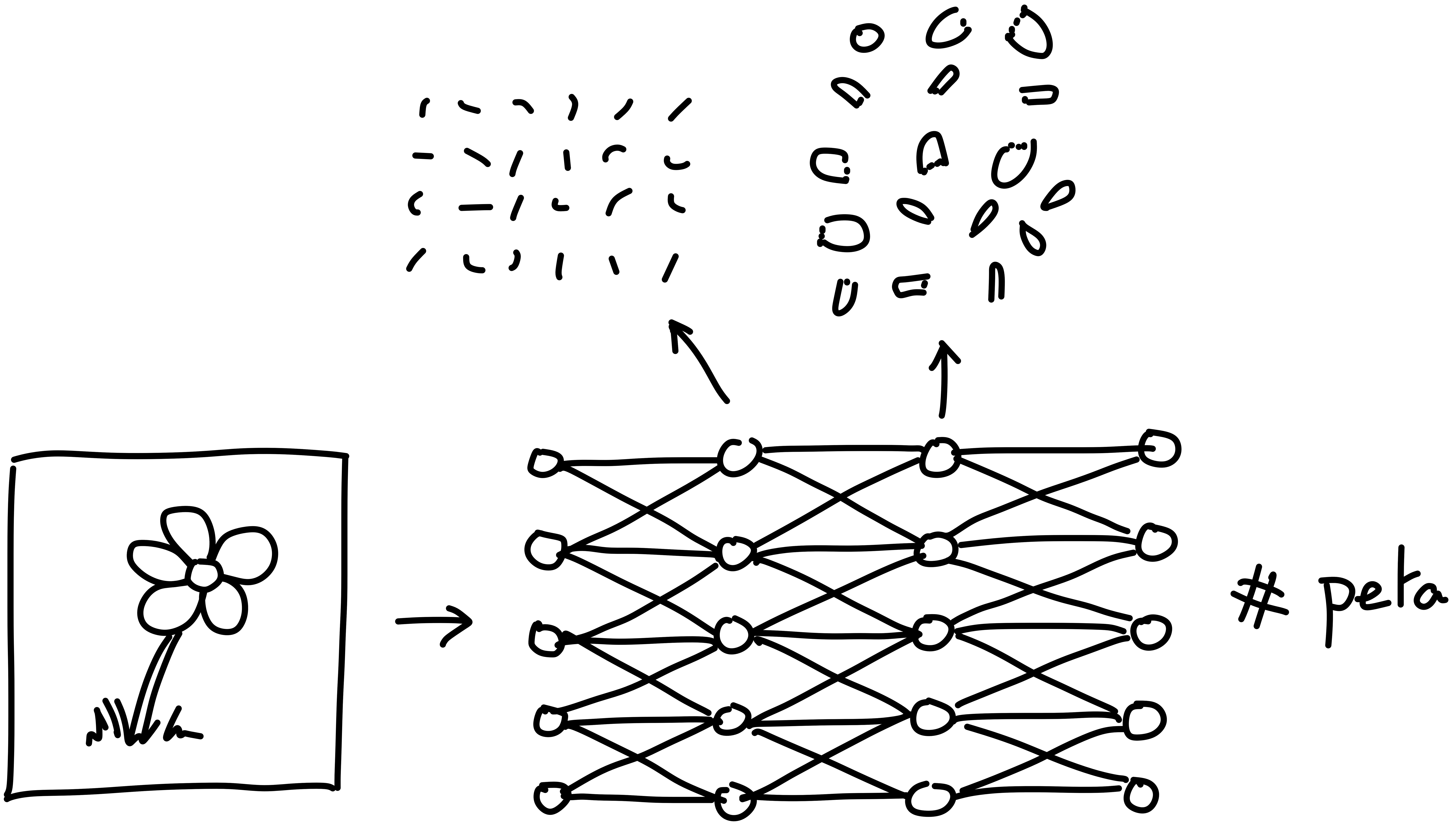


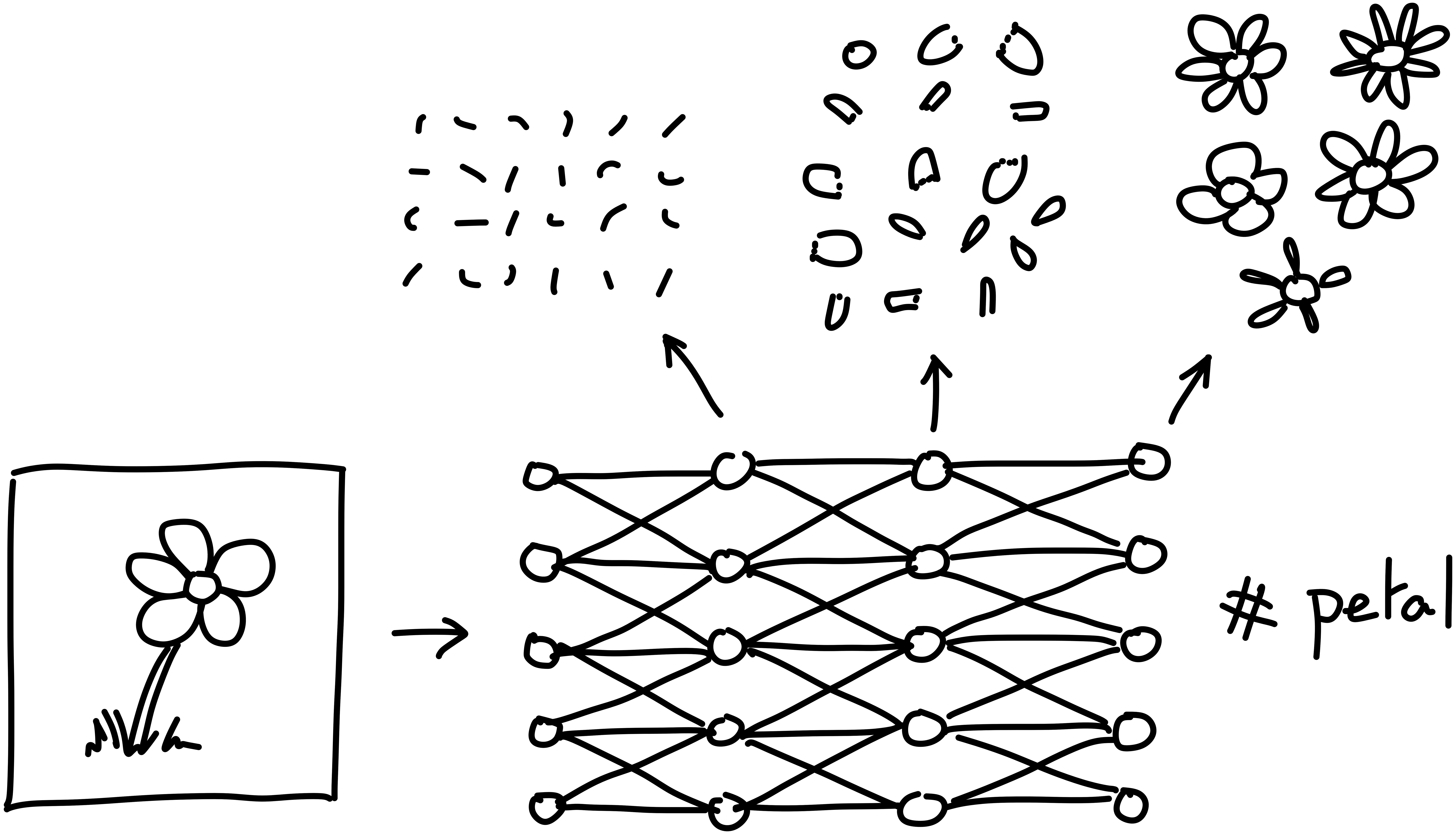


petal

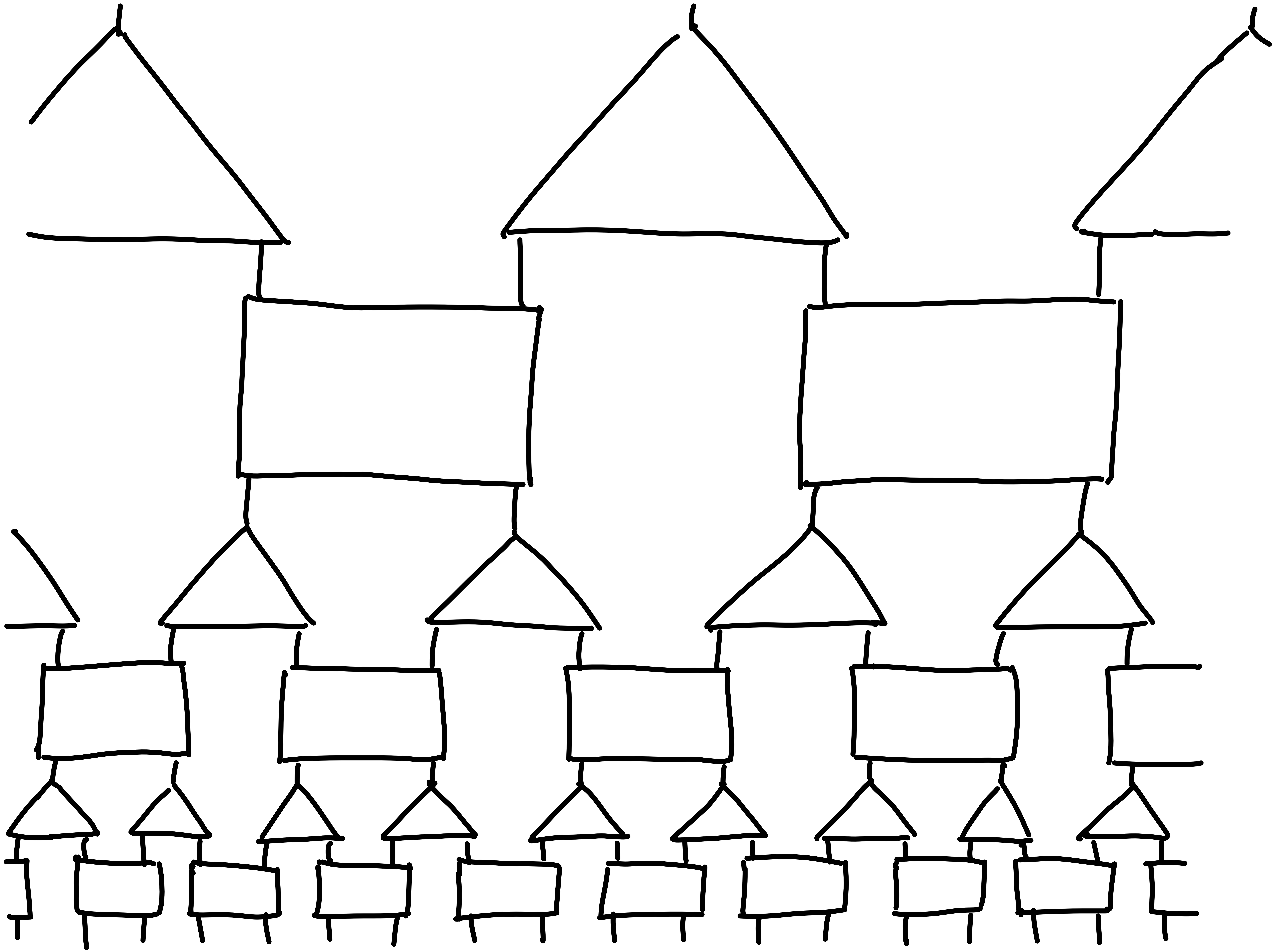


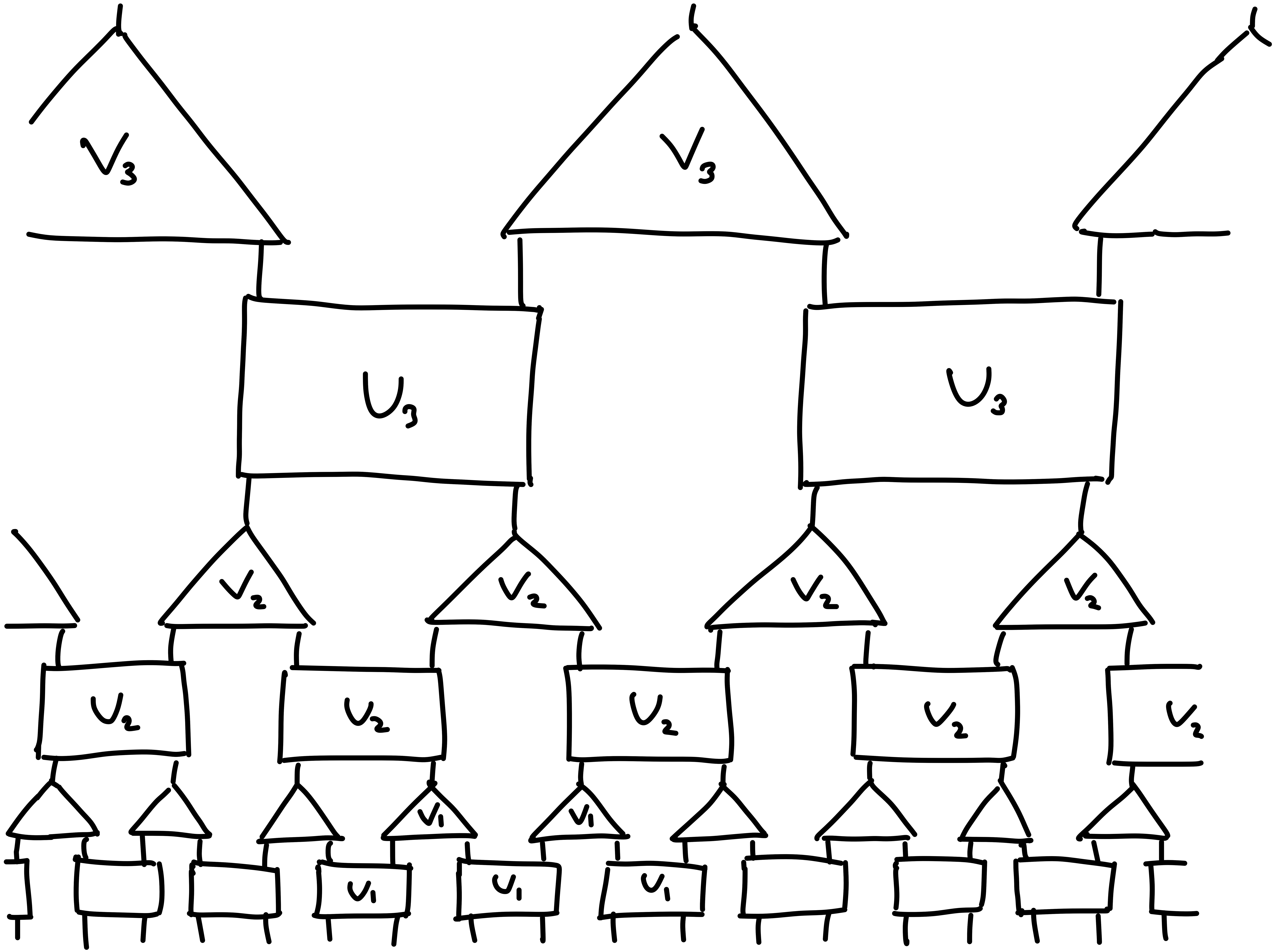
petal

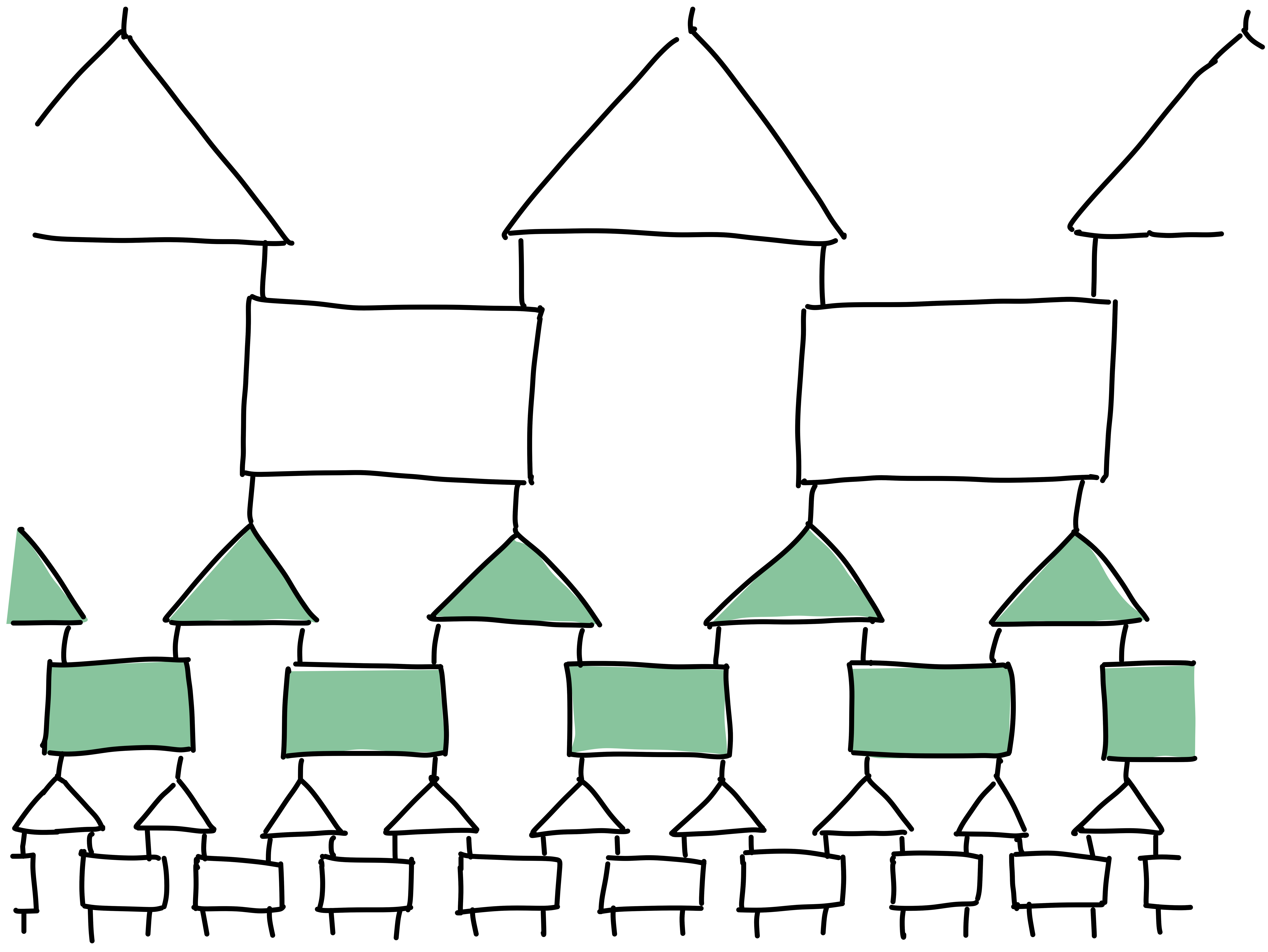

petal

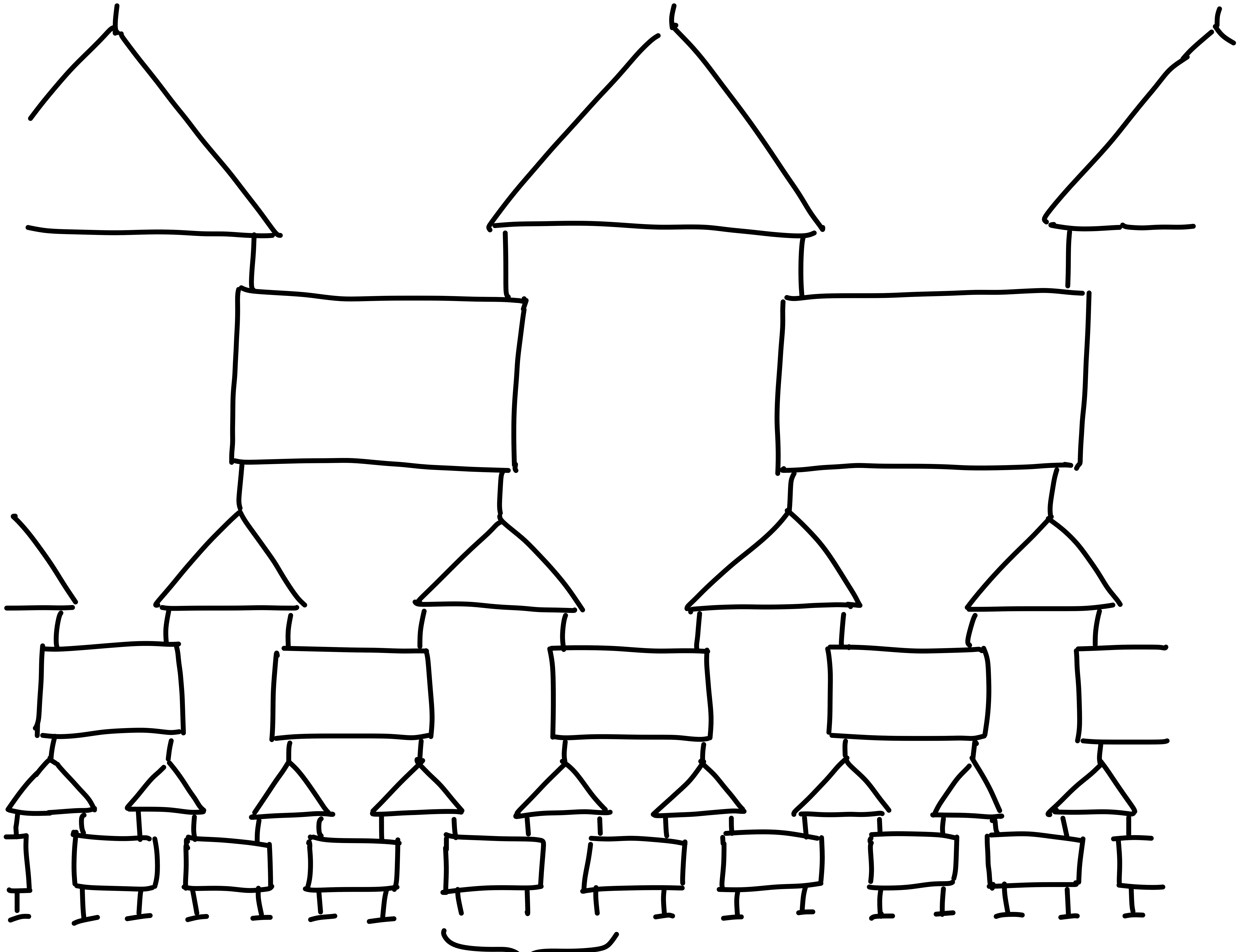

petal

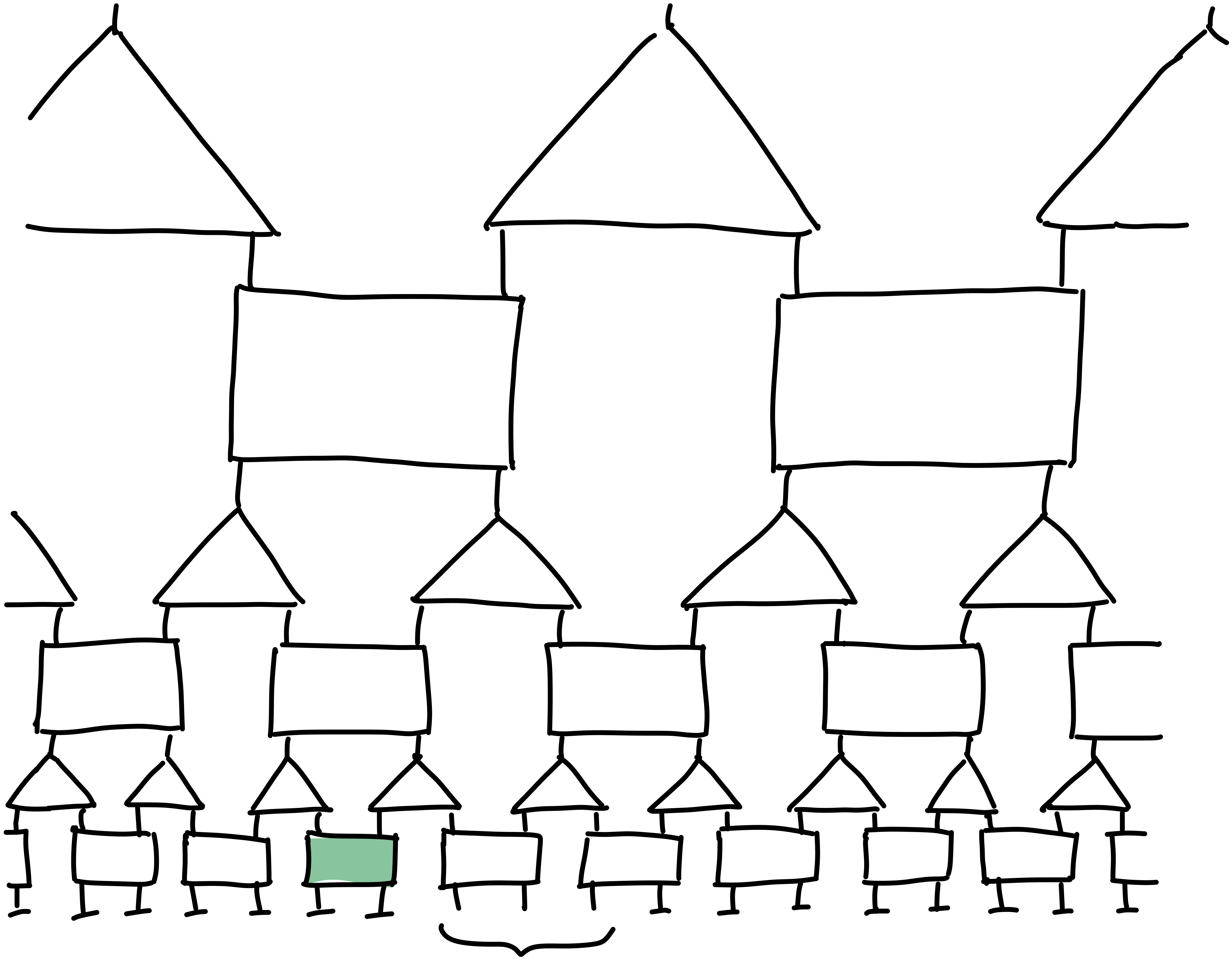
Multiscale Entanglement Renormalization Ansatz (MERA)

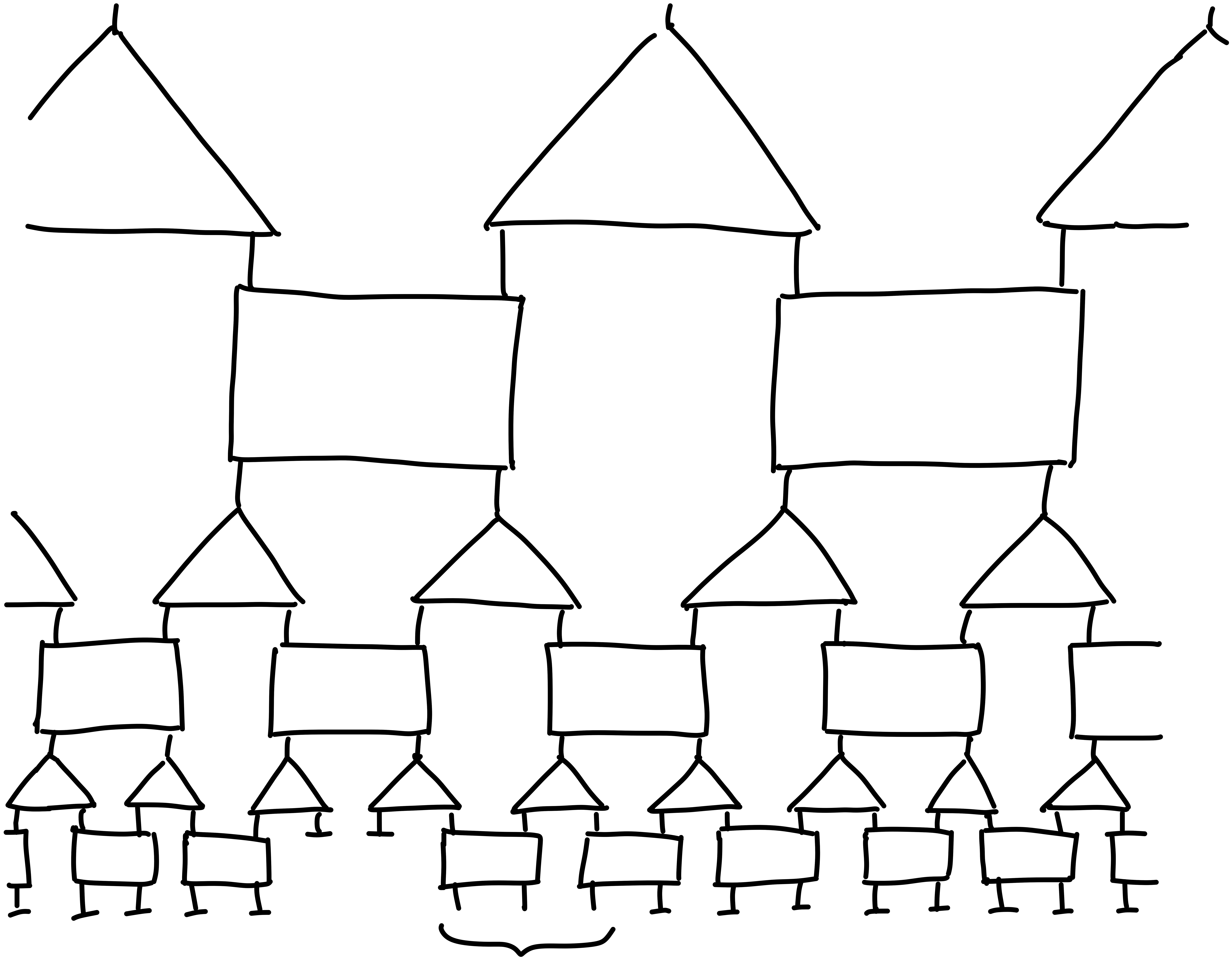


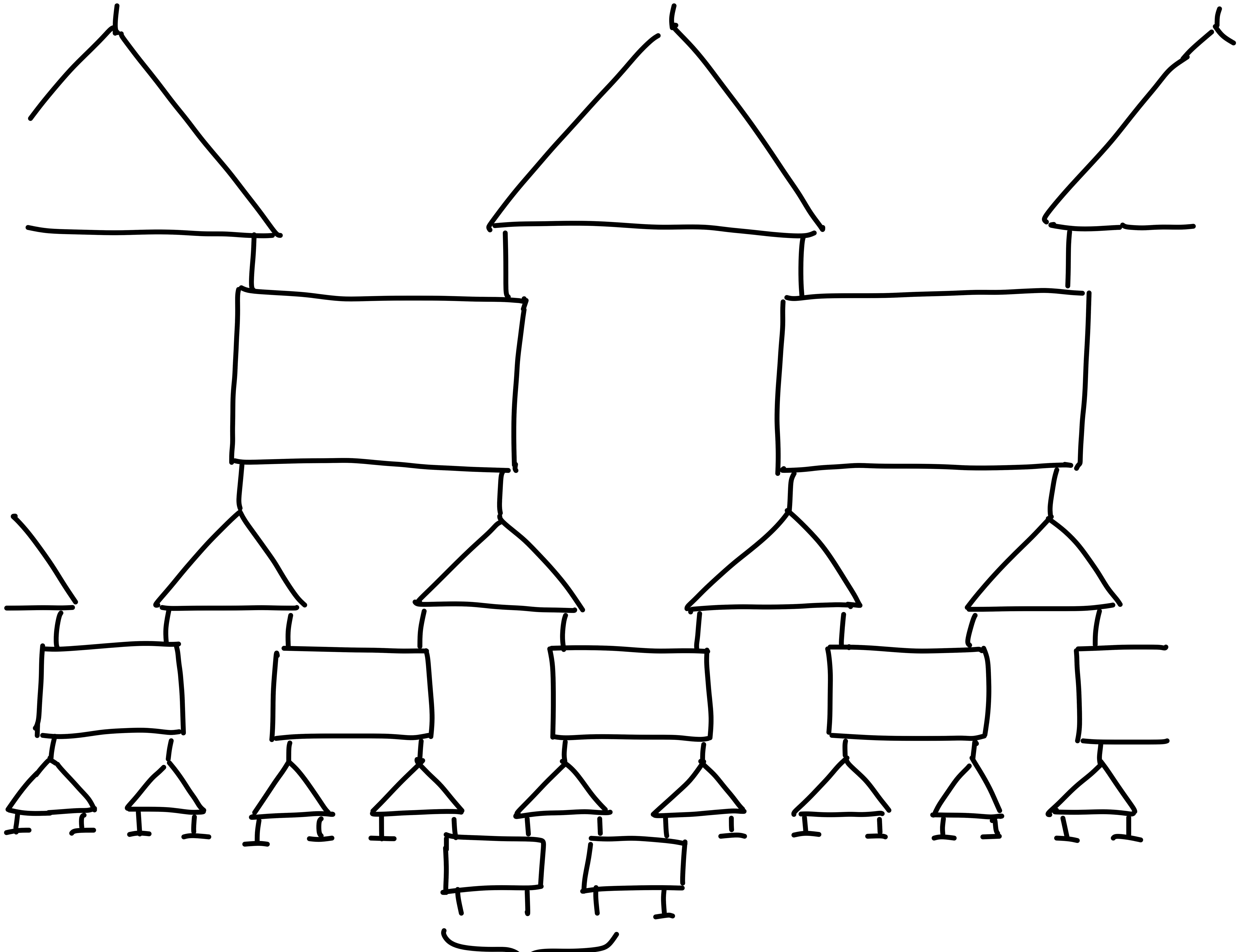

V3
V3
U3
U3
V2
V2
V2
V2
U2
U2
U2
U2
U2
V1
V1
U1
U1
U1

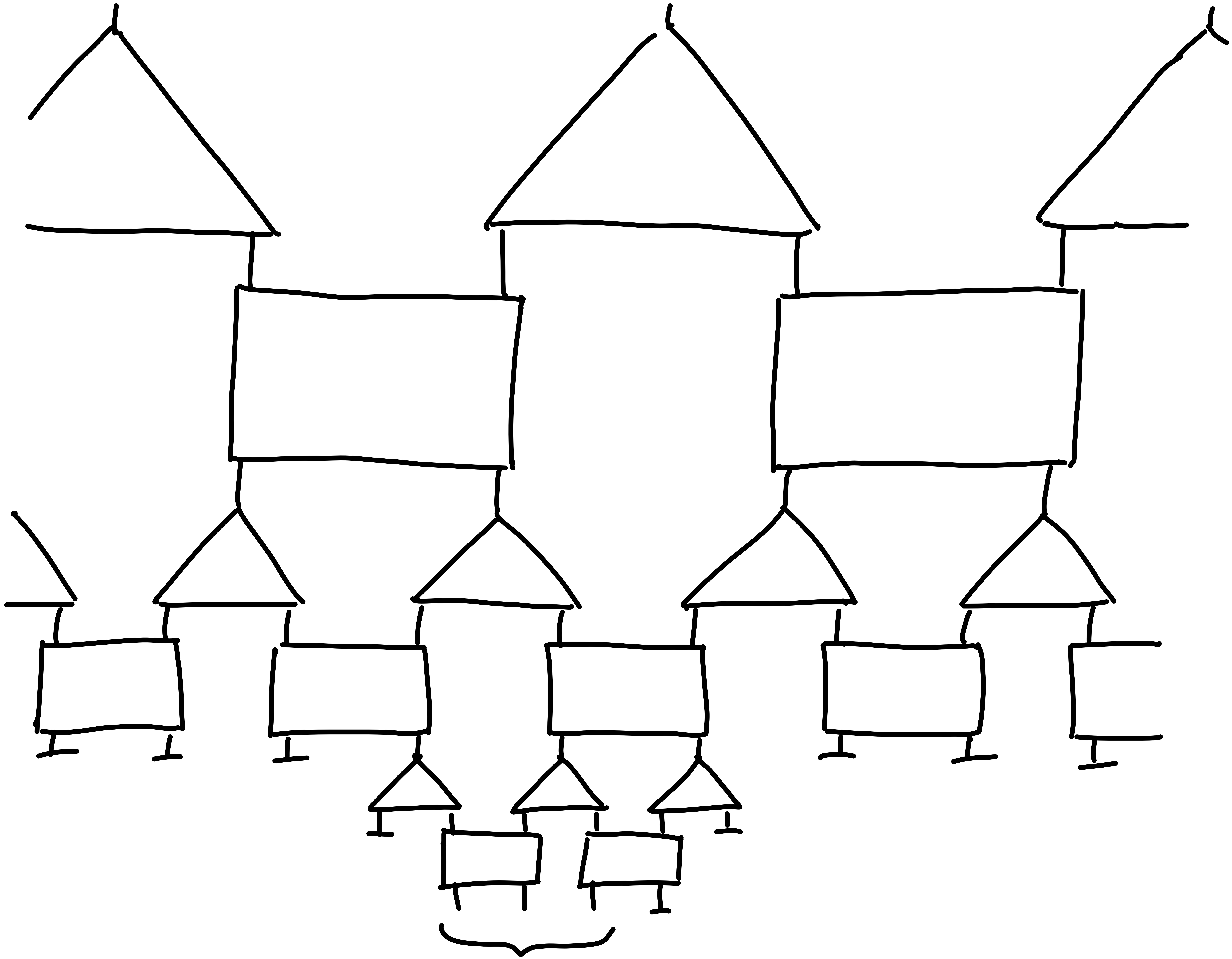


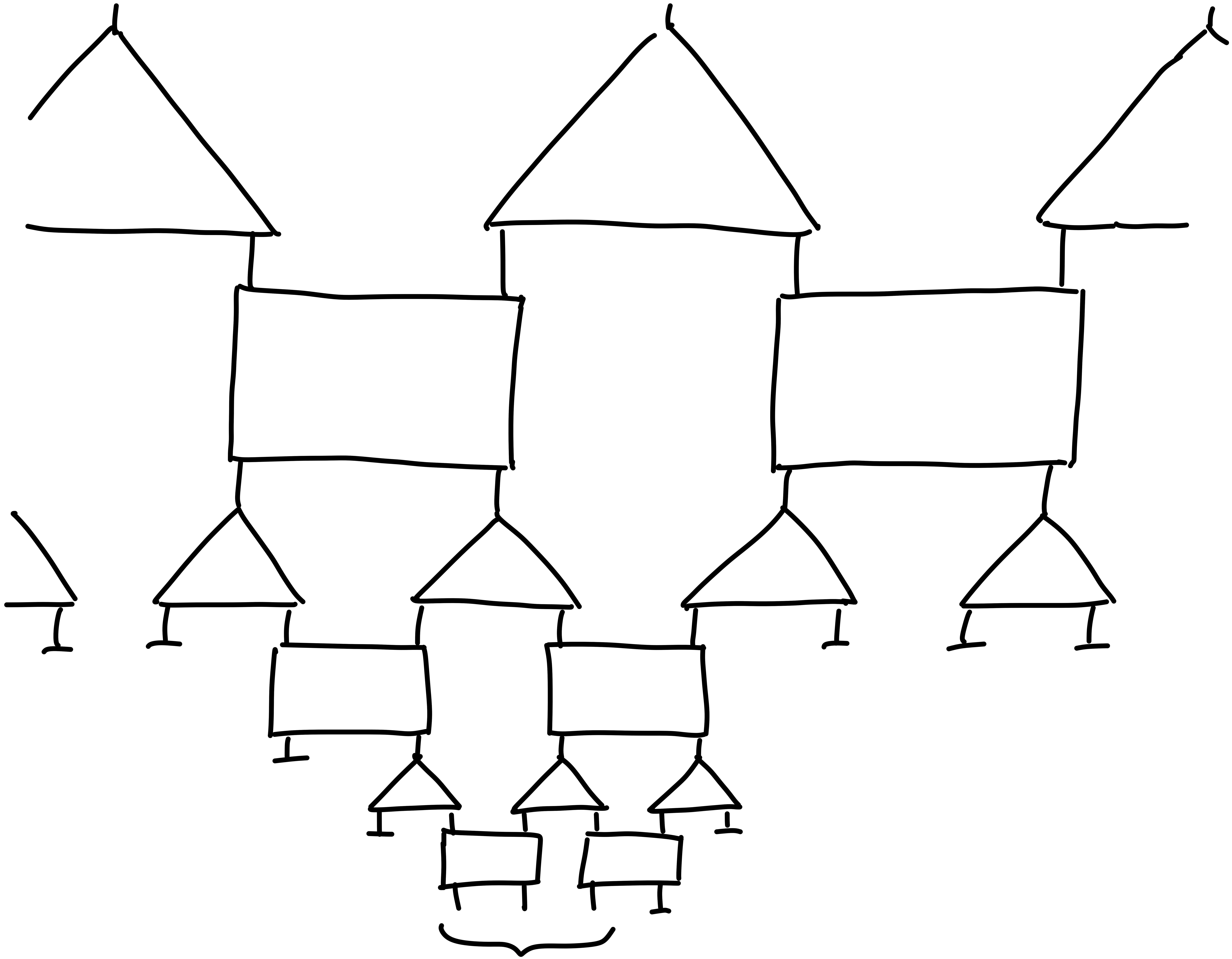


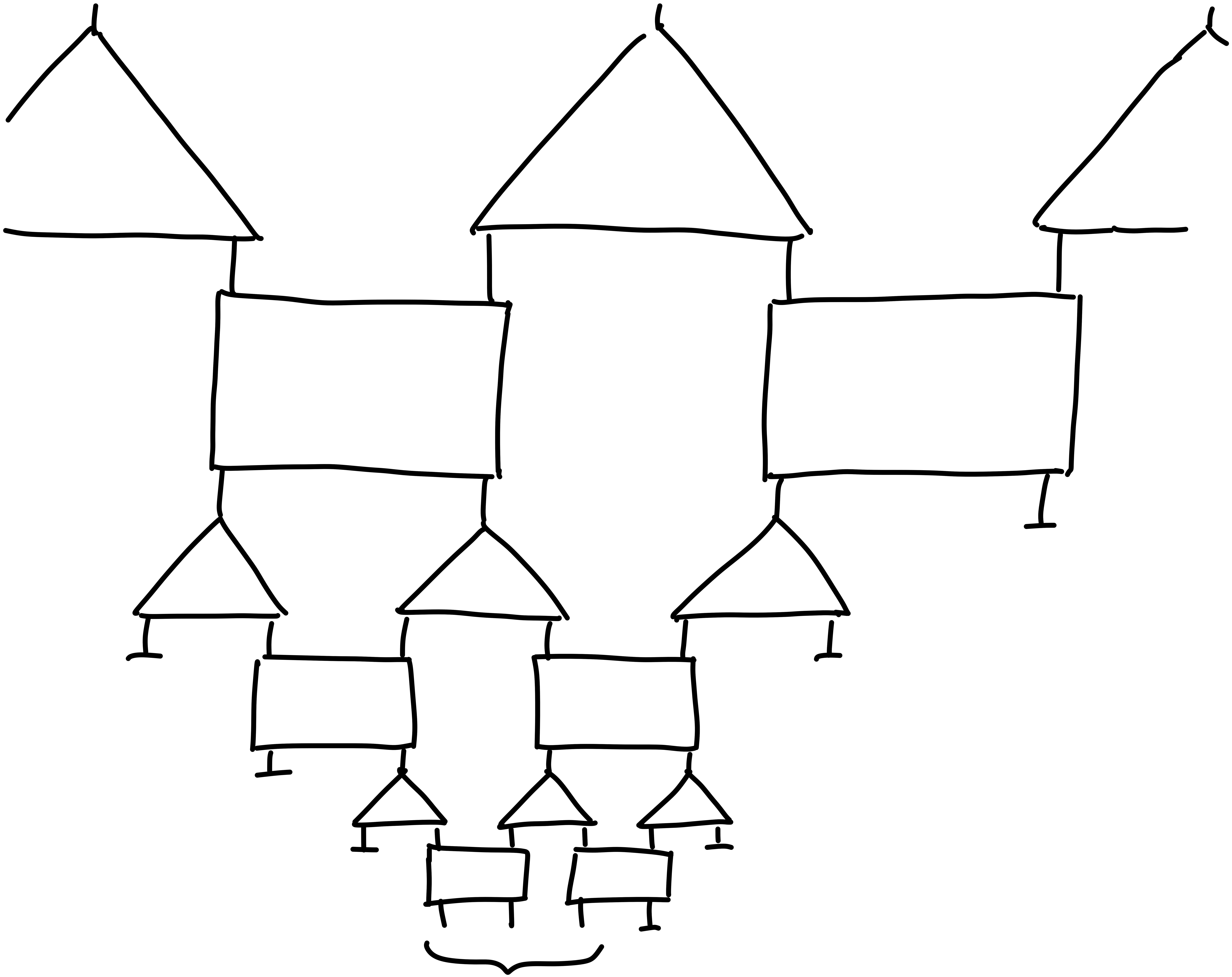


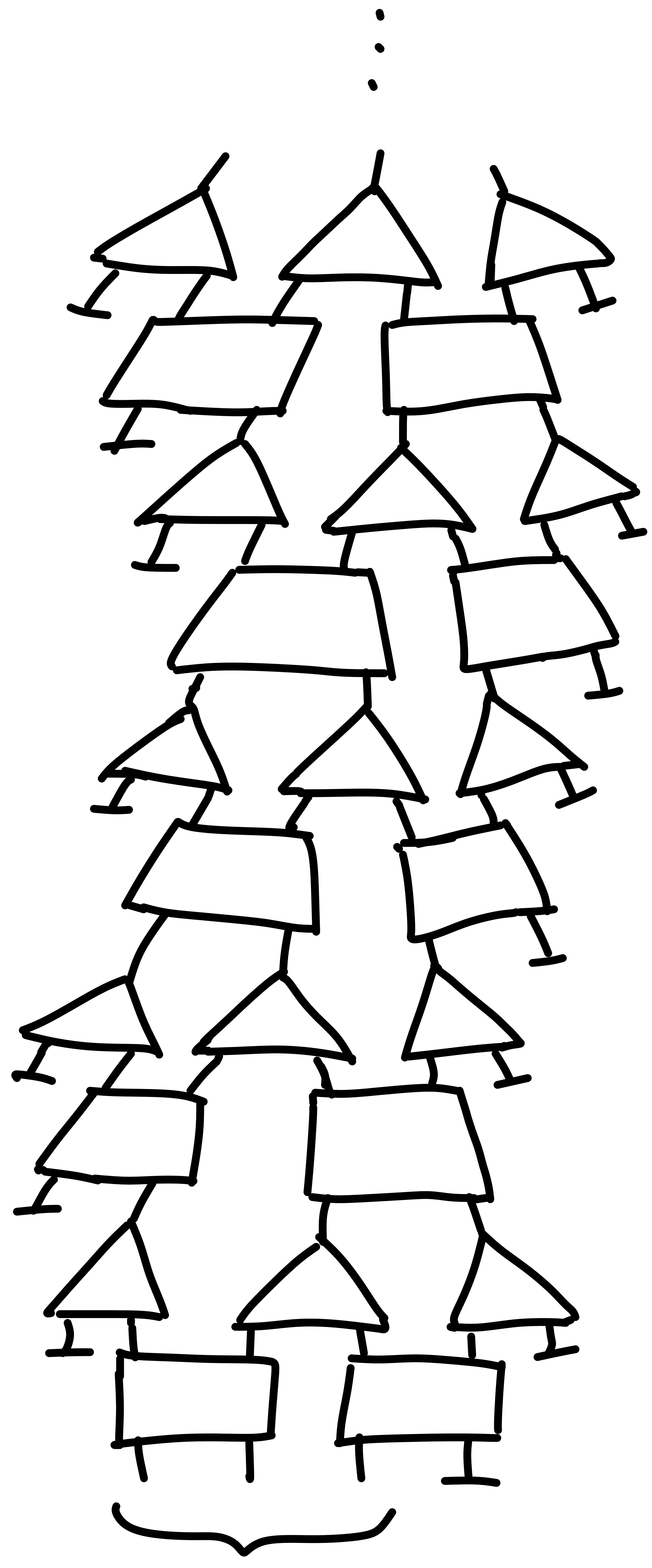


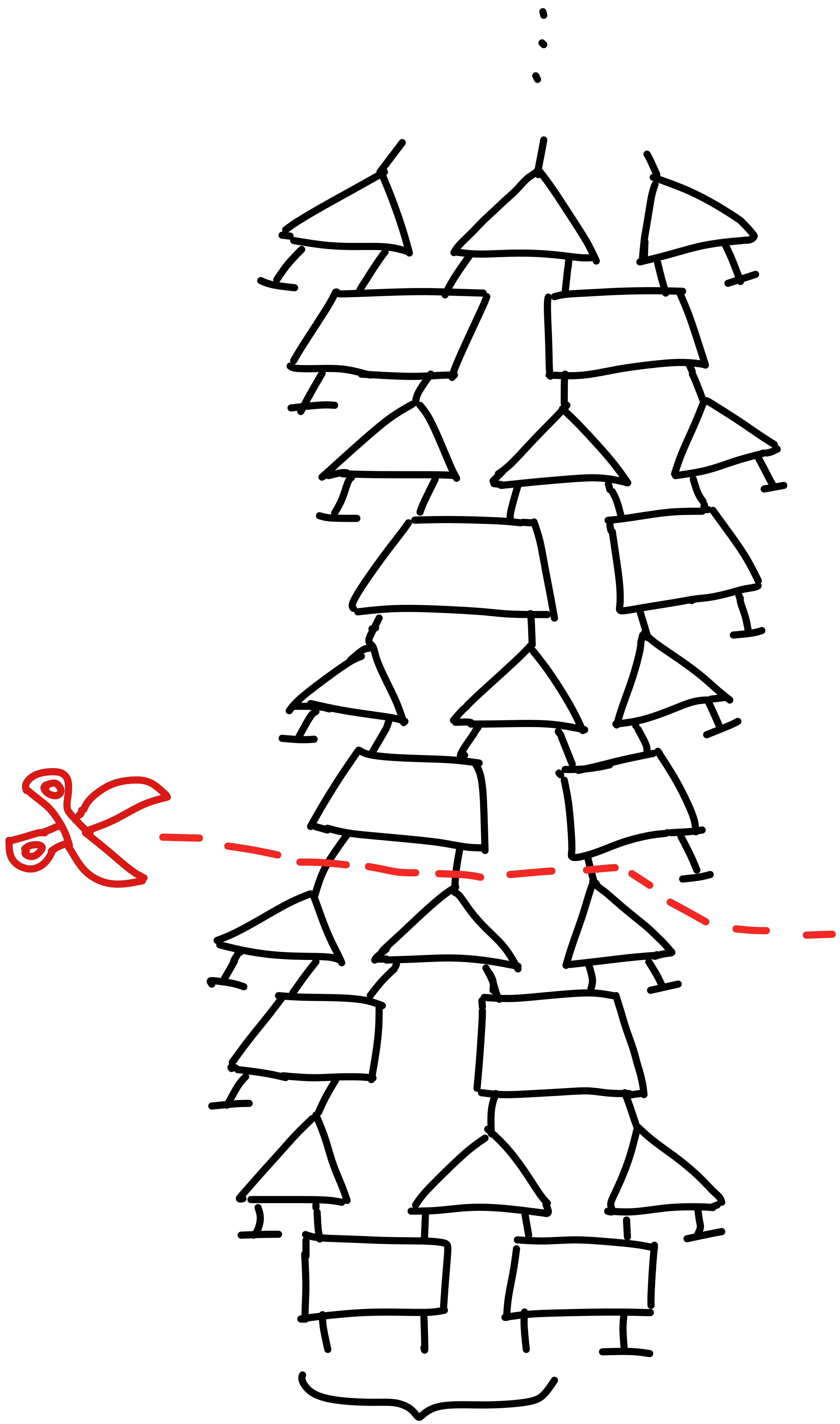


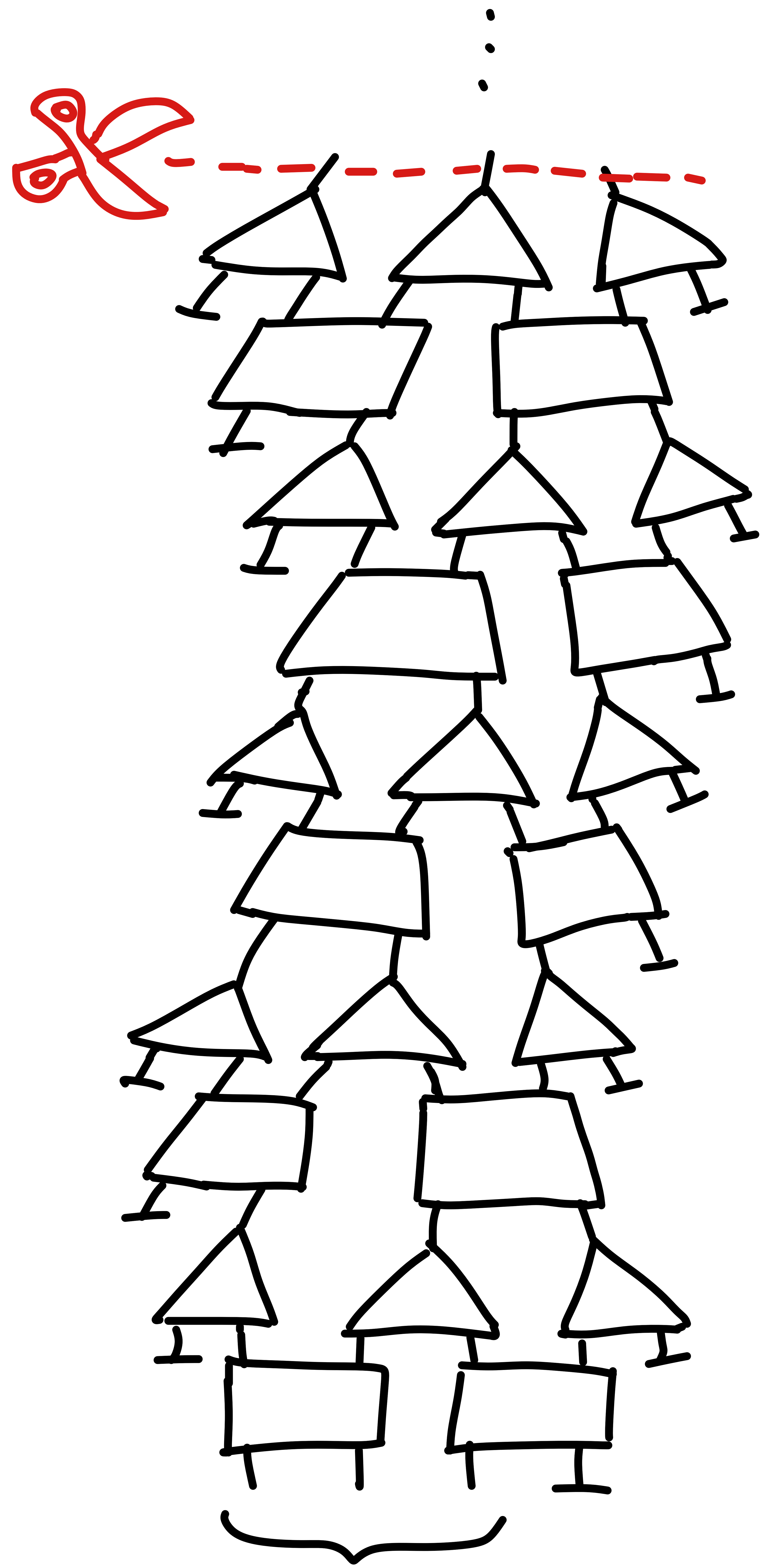


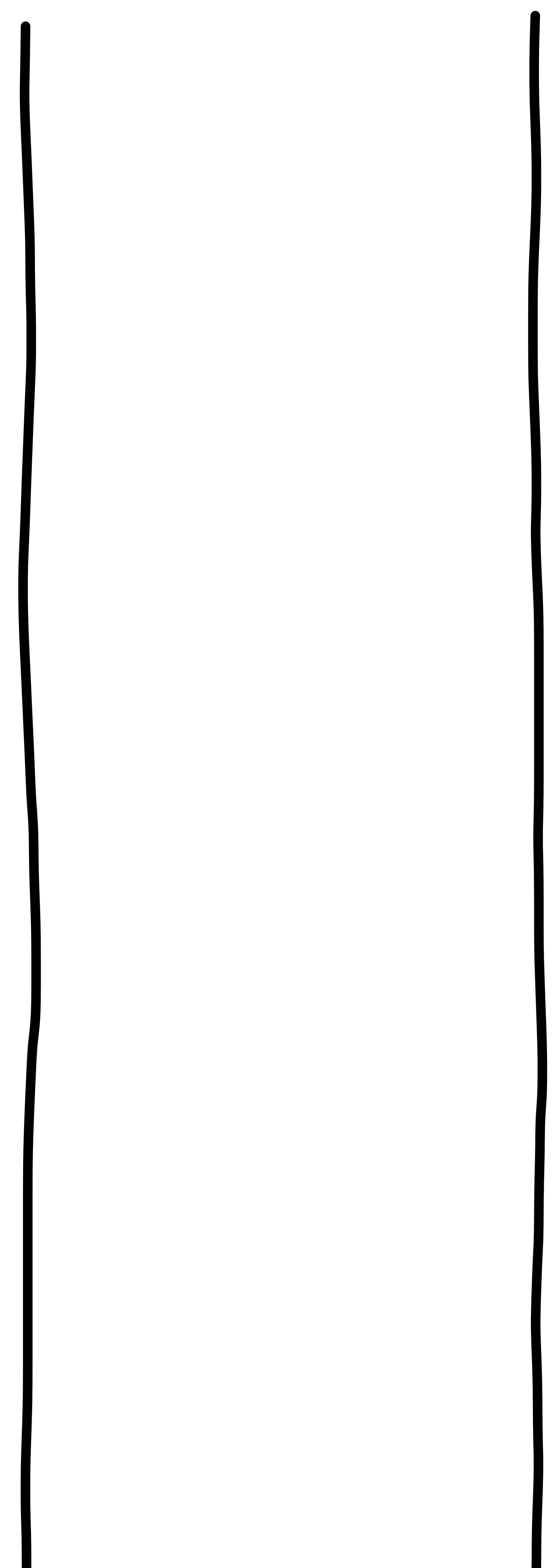


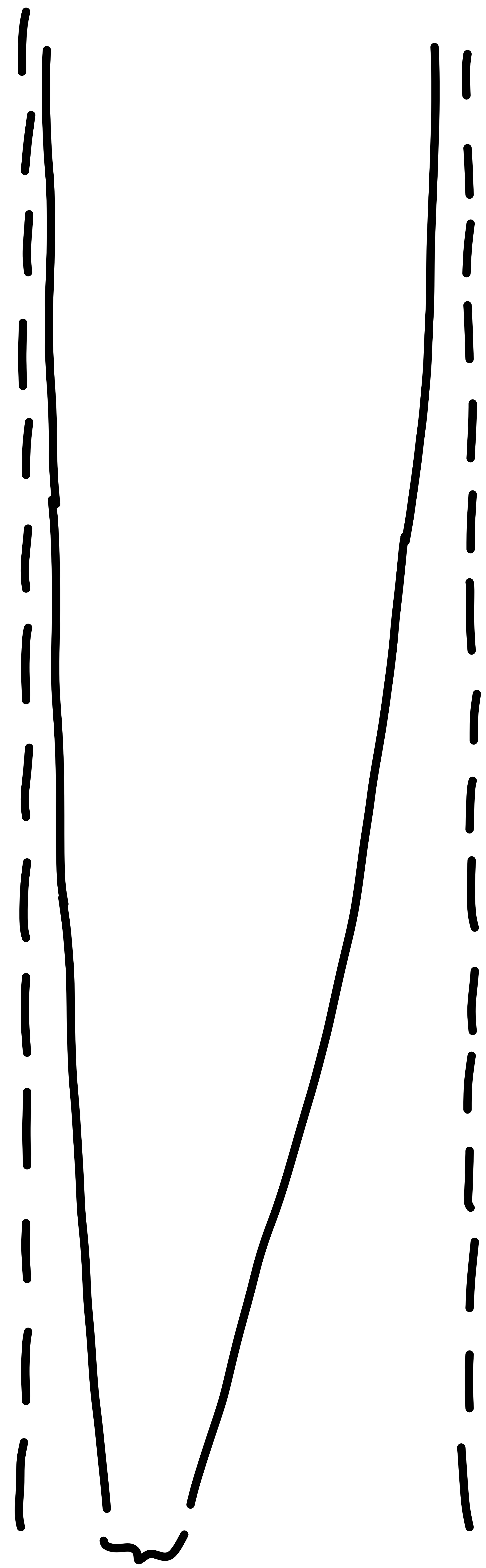


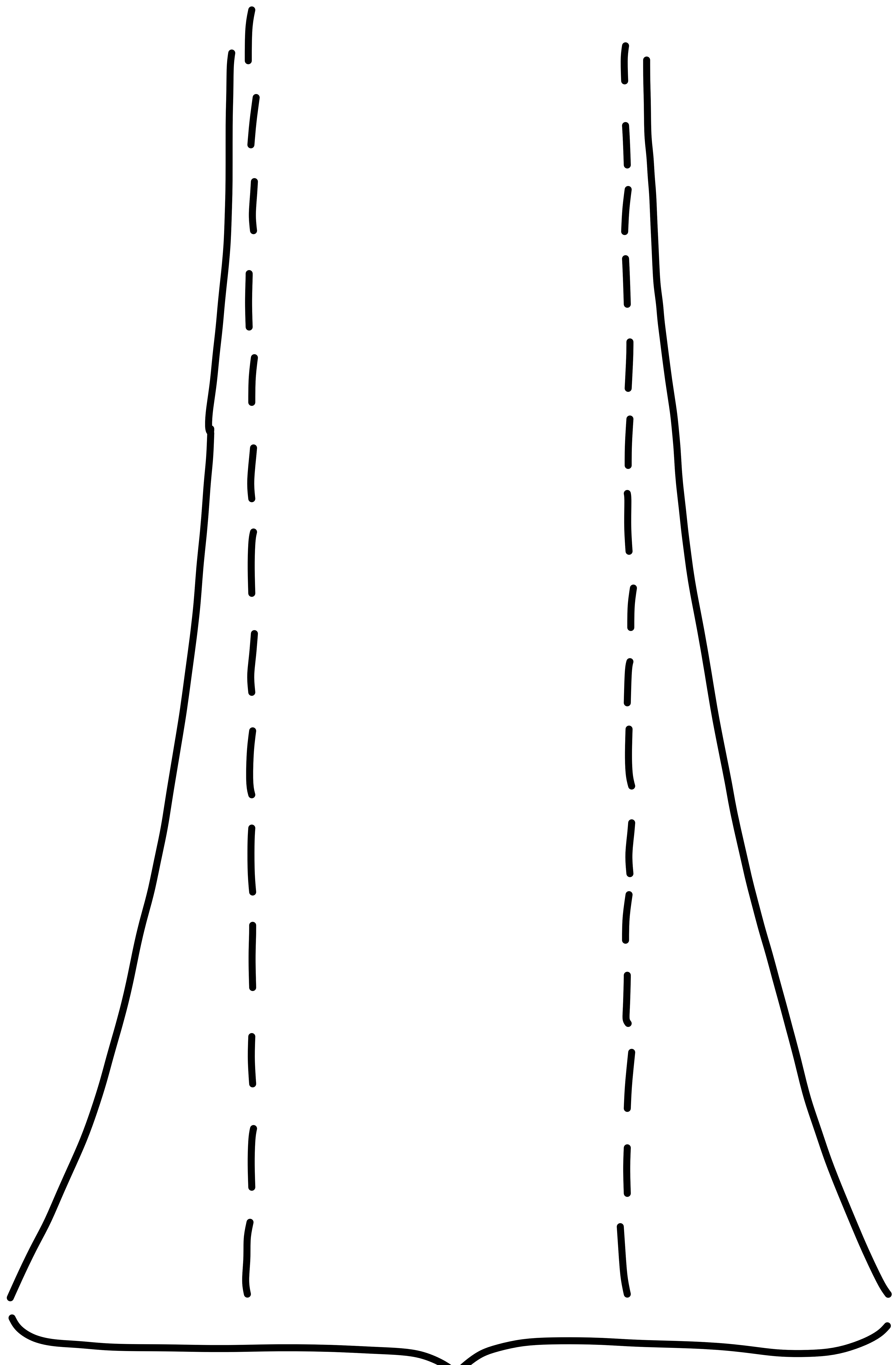


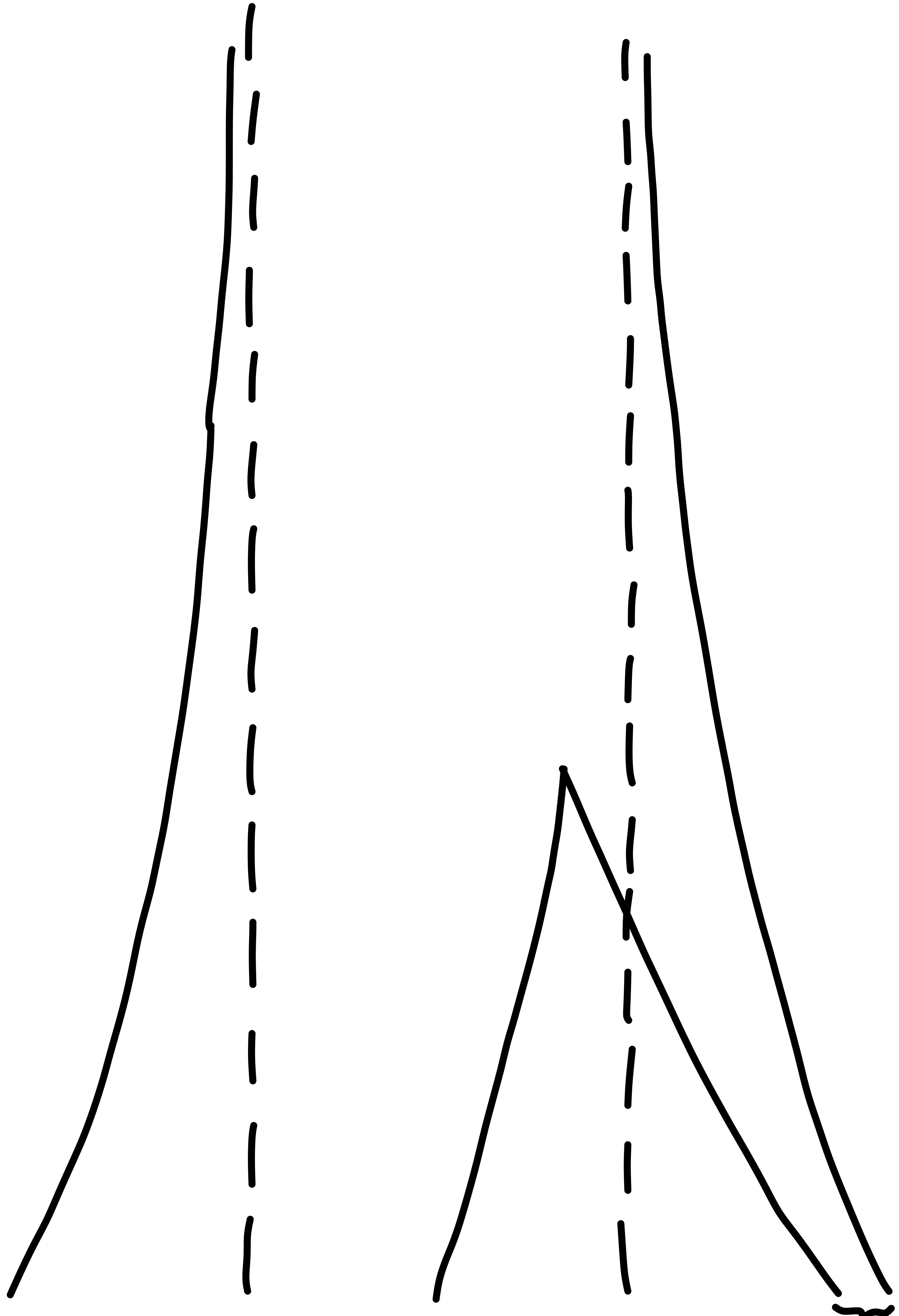


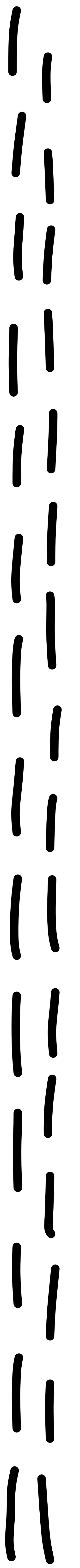


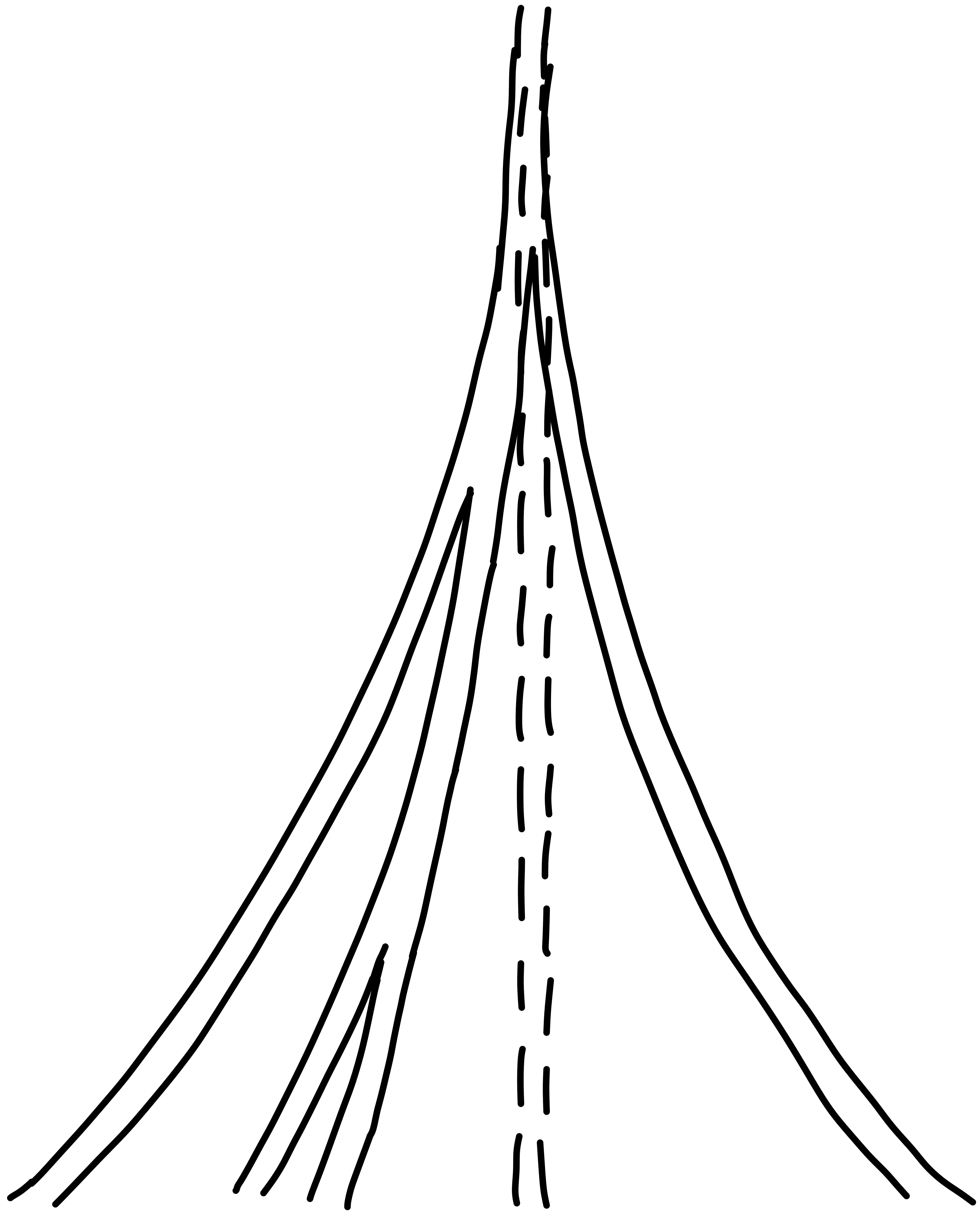


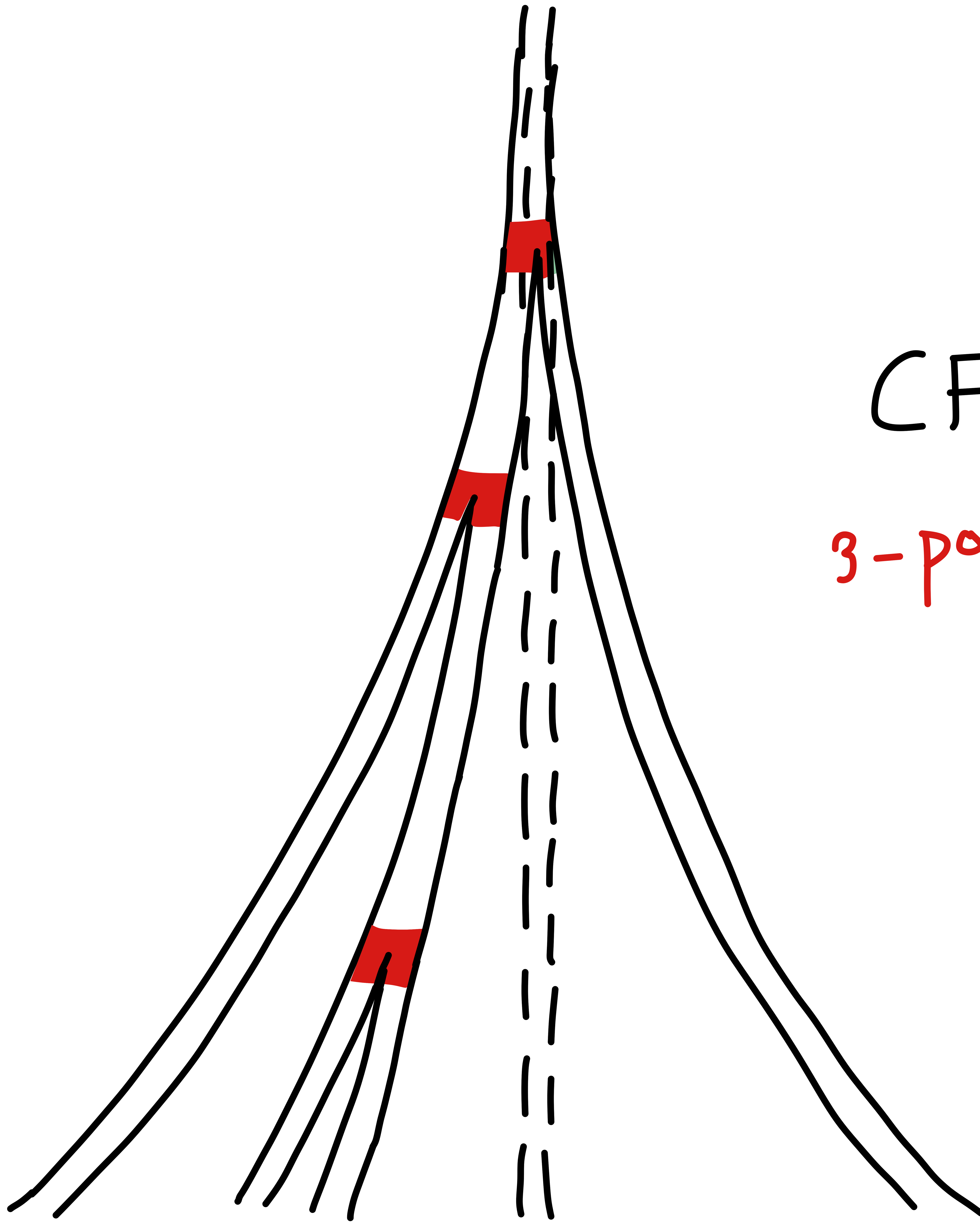






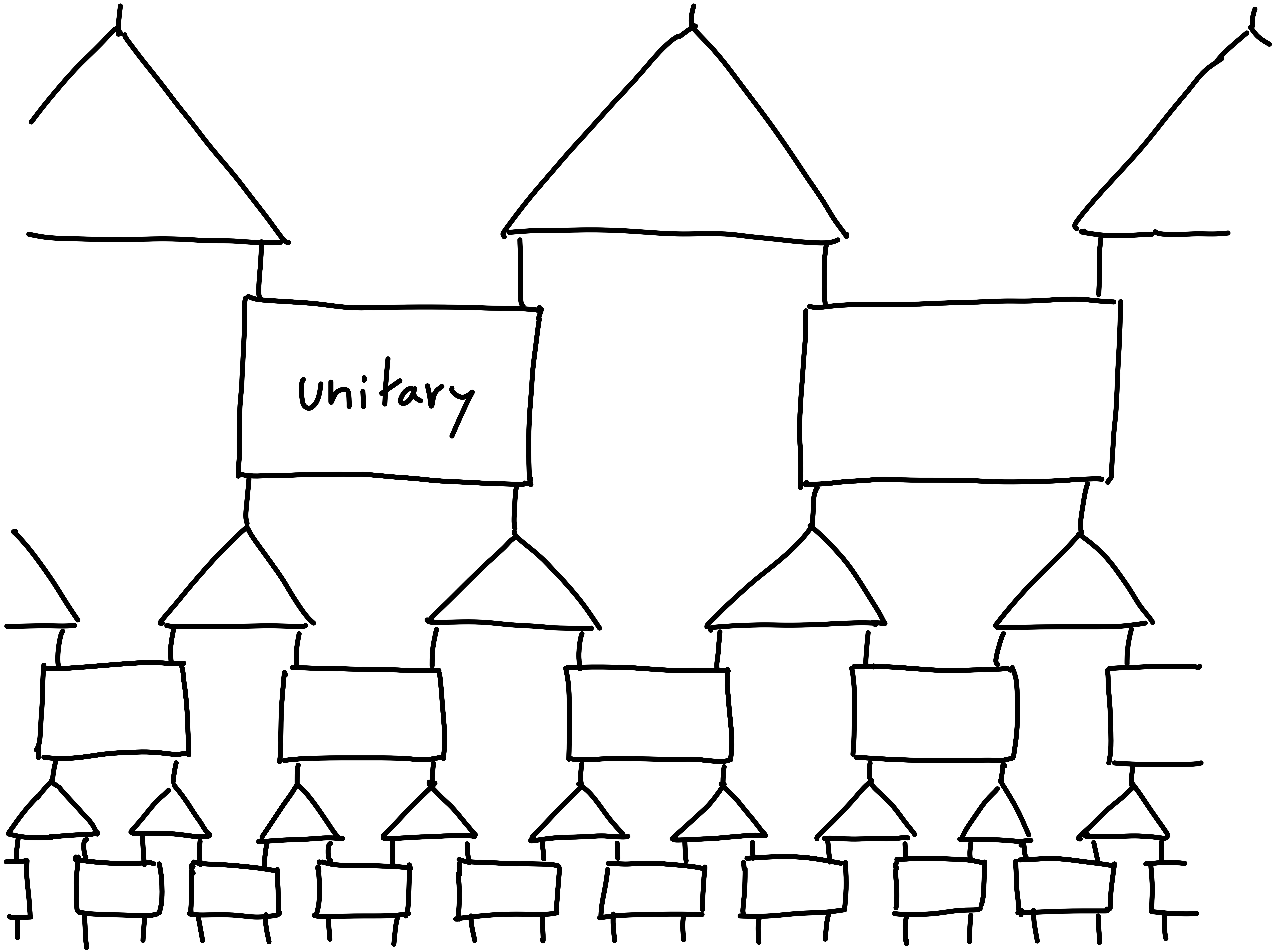


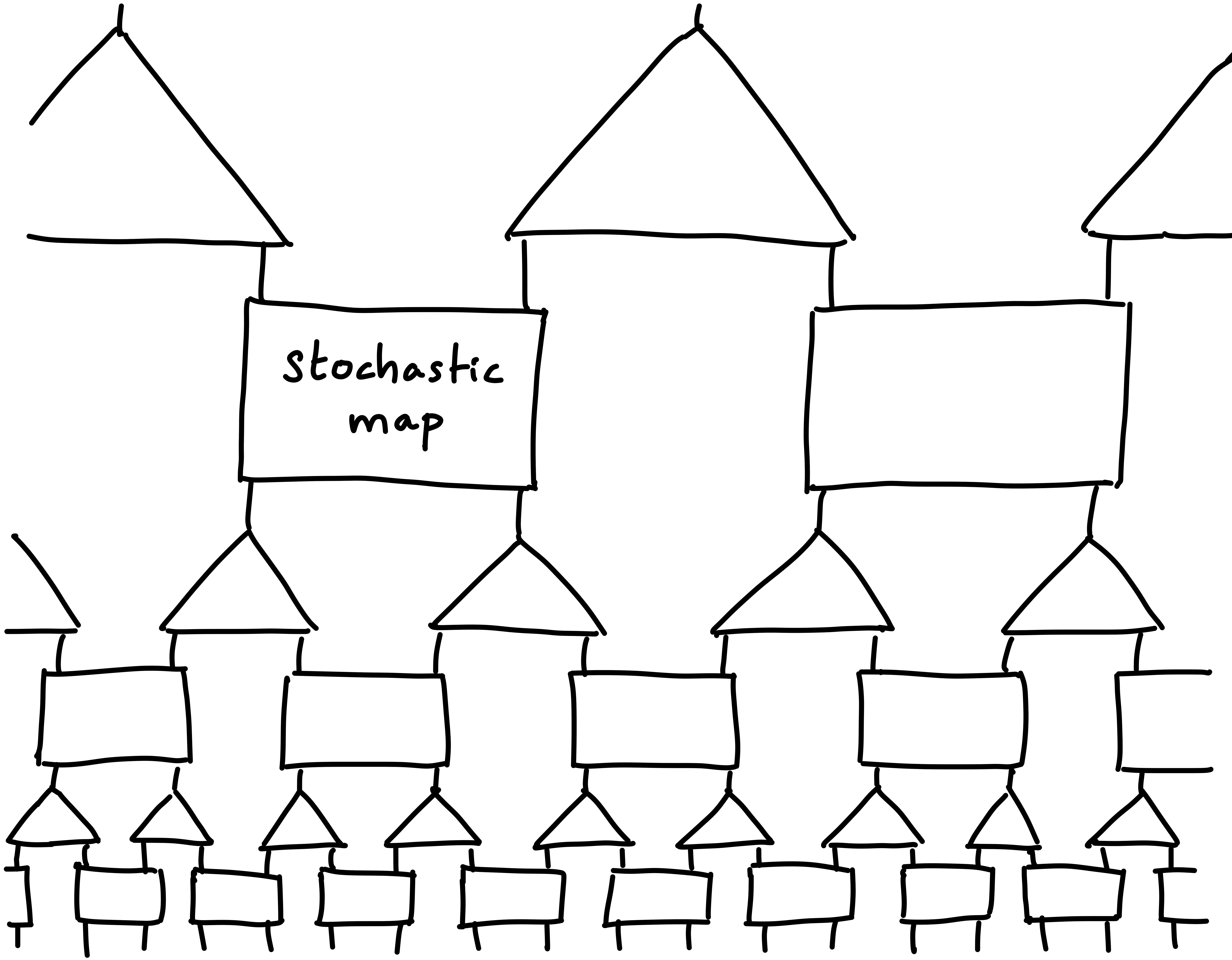


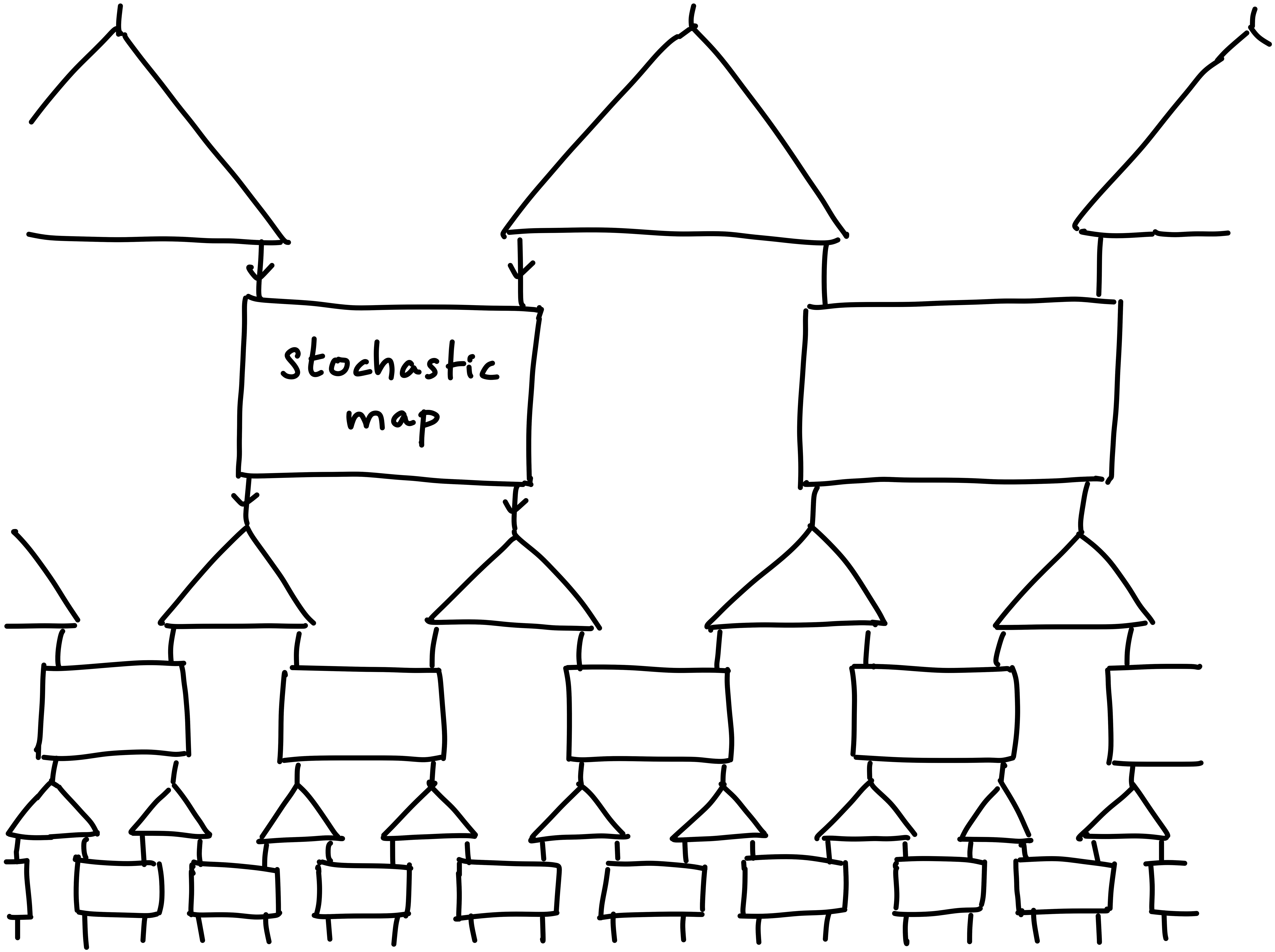


CFT

3-point function


unitary


Stochastic map

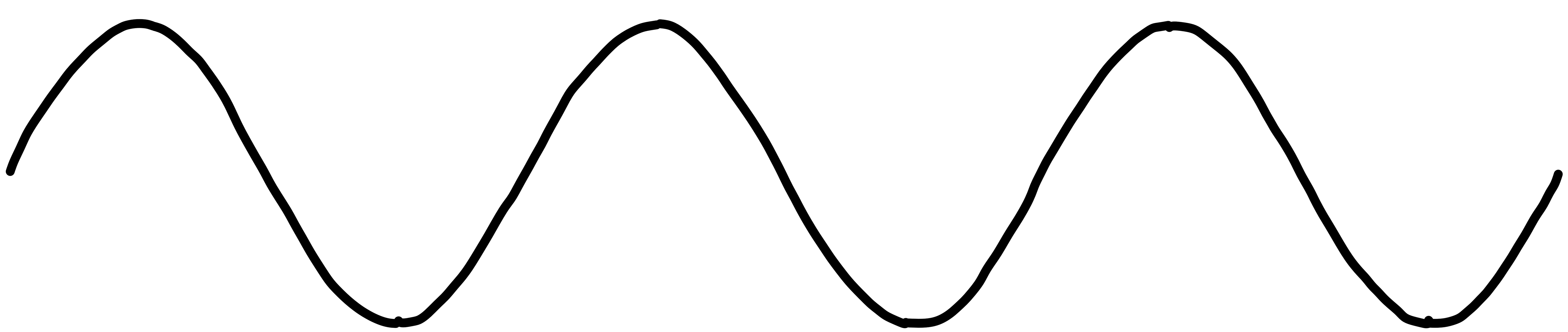


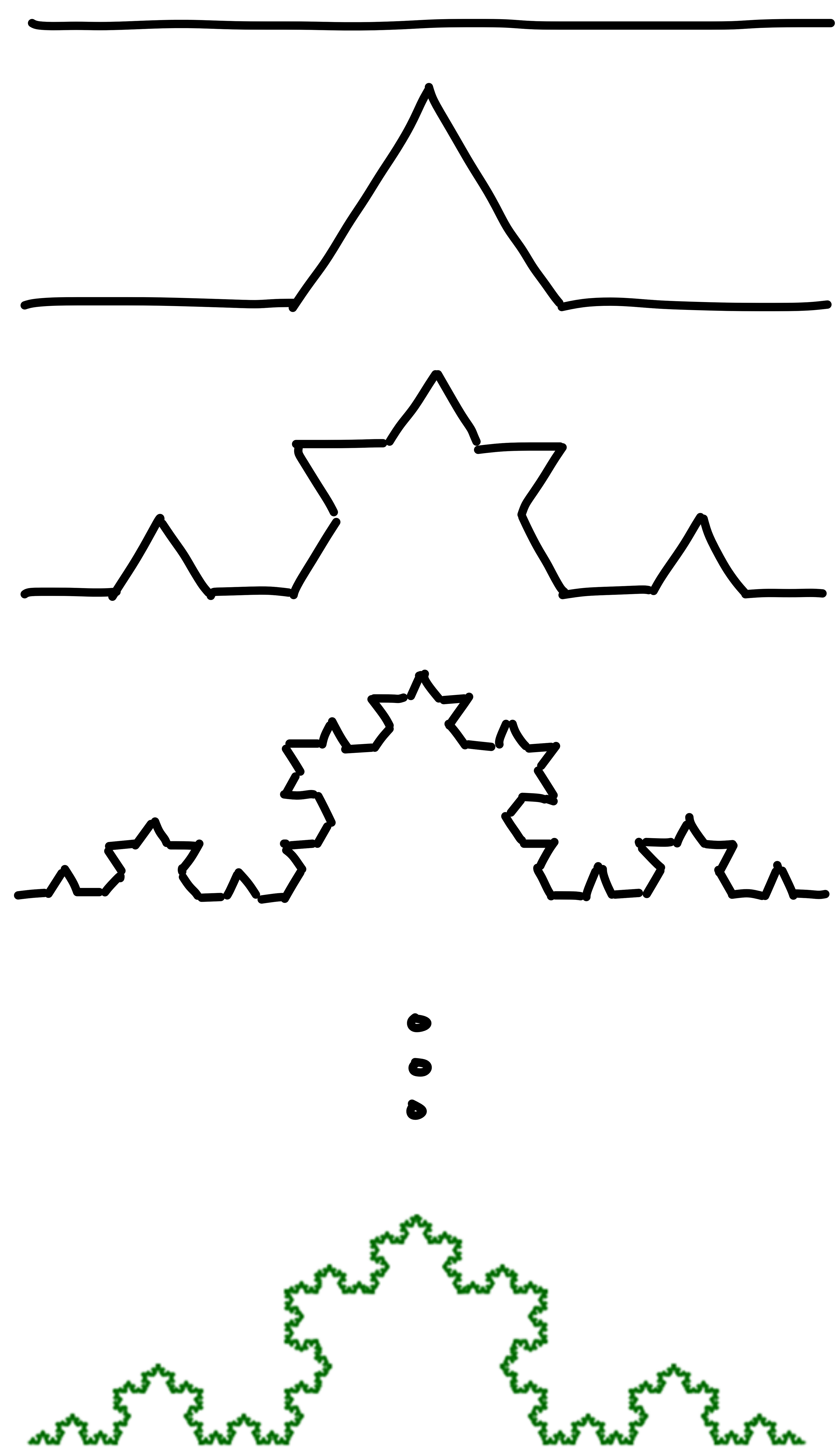
arXiv: 1301.3124

Renormalization in QFT

$$\frac{\partial^2}{\partial x^2}\phi(x) + m^2\phi(x) = 0$$

$$\frac{\partial^2}{\partial x^2}\phi(x) + m^2\phi(x) = 0$$





$$\pi = \{3,\ 3.1,\ 3.14,\ 3.145,\ \ldots\} \subset \mathbb{Q}$$

$\{x_1, x_2, \ldots\}$ is Cauchy if

$\forall \epsilon > 0 \ \exists N, \quad \forall i,j > N$

$\|x_i - x_j\| < \epsilon$

$\Lambda \longmapsto \psi_\Lambda$ is "Cauchy" if

$\forall\, \sigma, \epsilon \;\; \exists\, \Lambda_0 \,, \;\; \forall\, \Lambda, \Lambda' > \Lambda_0$

$d_\sigma(\psi_\Lambda, \psi_{\Lambda'}) < \epsilon$

$\Lambda \mapsto \psi_\Lambda$ is "Cauchy" if

$$\forall \sigma, \epsilon \;\; \exists \Lambda_0 , \;\; \forall \Lambda, \Lambda' > \Lambda_0$$

$$d_\sigma(\psi_\Lambda, \psi_{\Lambda'}) < \epsilon$$

observation scale

regulator

$\Lambda \longmapsto \psi_\Lambda$ is "Cauchy" if

$\forall \sigma, \epsilon \;\; \exists \Lambda_0 , \;\; \forall \Lambda, \Lambda' > \Lambda_0$

observation scale

$$d_\sigma(\psi_\Lambda, \psi_{\Lambda'}) < \epsilon$$

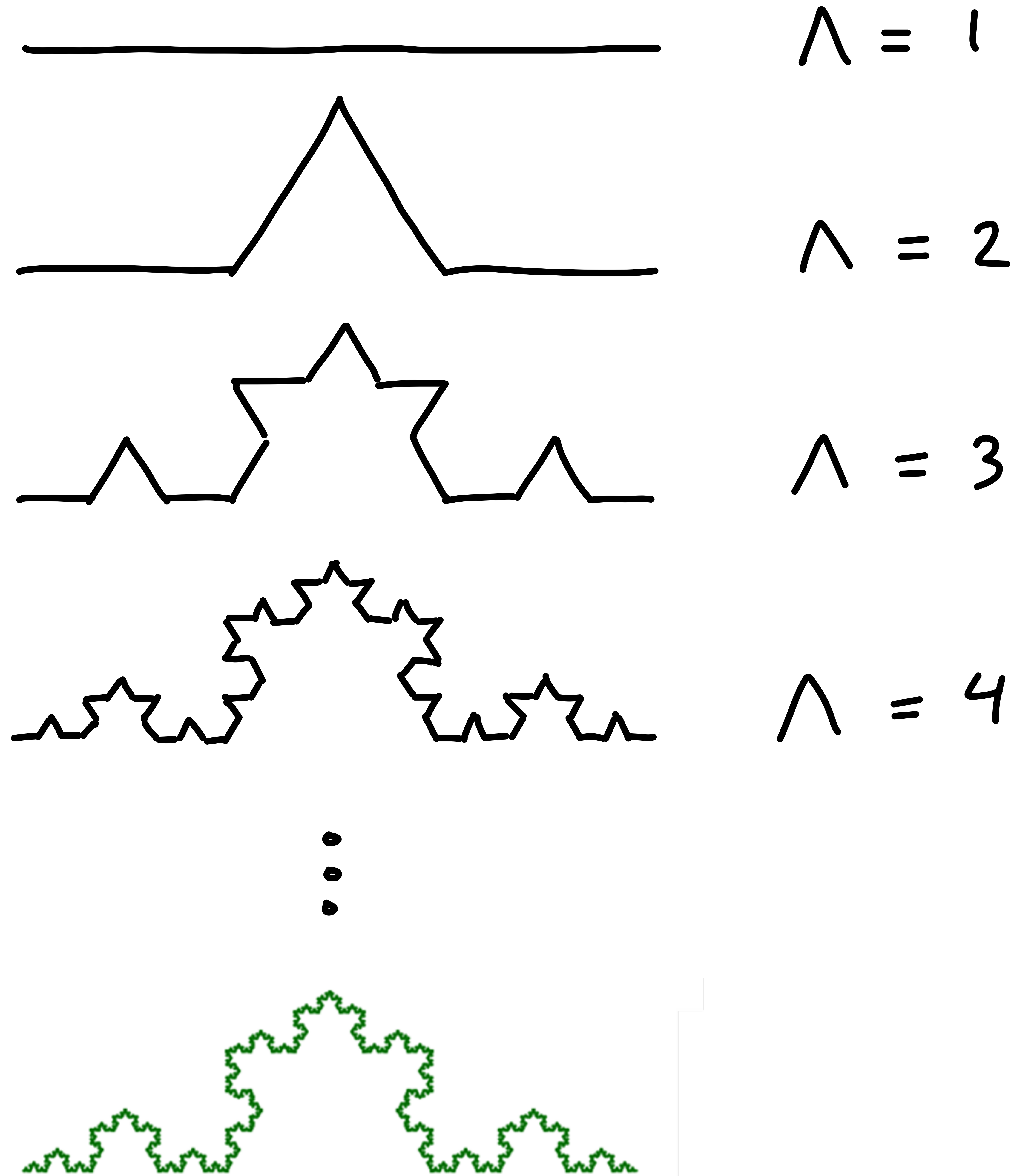
regulator

$\Lambda \longmapsto \psi_\Lambda$ is "Cauchy" if

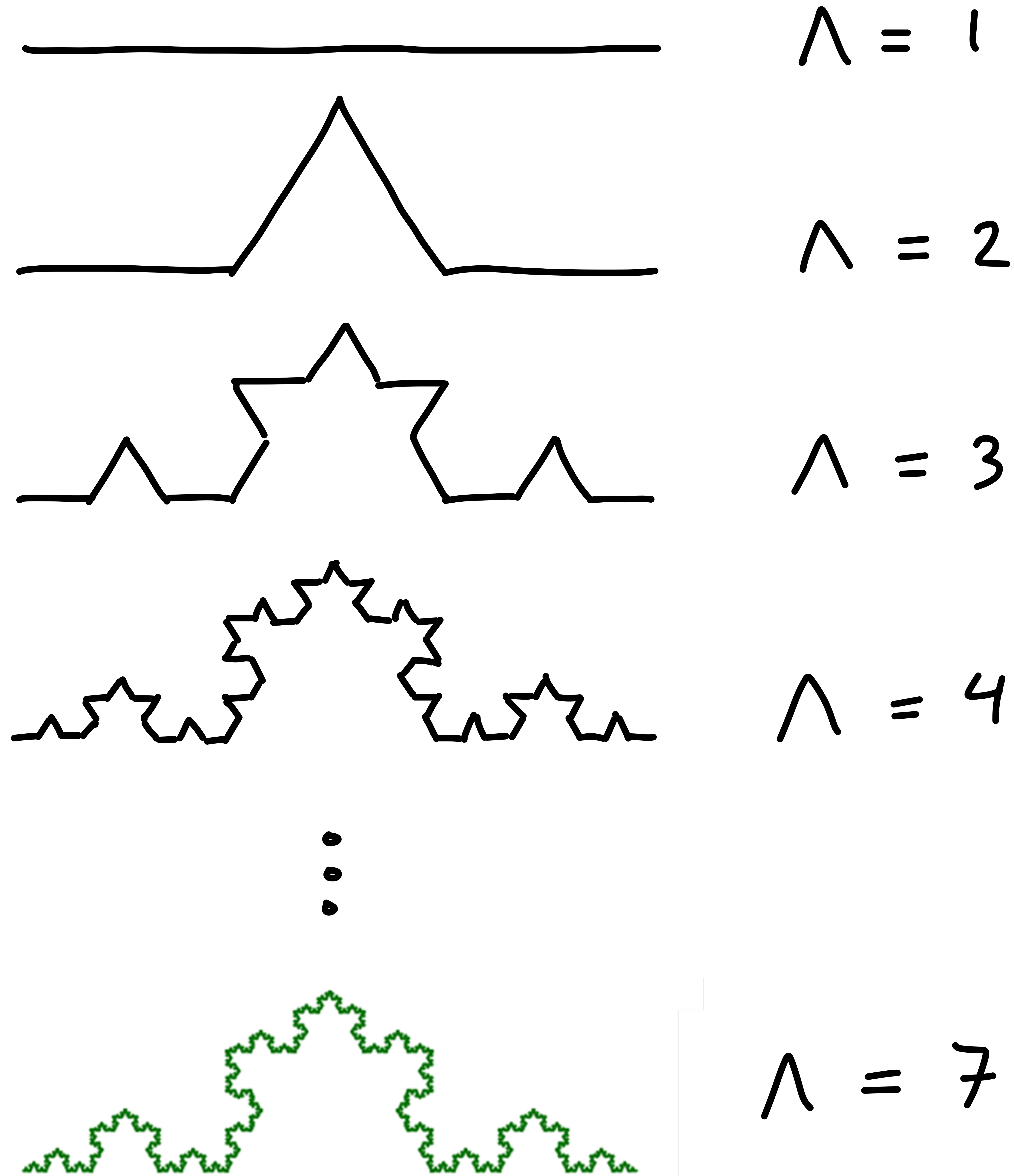
$\forall\, \sigma, \epsilon \;\; \exists\, \Lambda_0(\sigma, \epsilon) \;\; \forall\, \Lambda, \Lambda' > \Lambda_0$

observation scale

$$d_\sigma(\psi_\Lambda, \psi_{\Lambda'}) < \epsilon$$



Λ = 1
Λ = 2
Λ = 3
Λ = 4



$\sigma = 1$ pixel

take $\Lambda_0 = 7$

$$d_\sigma(\psi_\Lambda, \psi_{\Lambda'})$$

$$d_\sigma(\rho, \rho')$$

$$d_\sigma(\rho, \rho')$$

$$= d(N_\sigma(\rho), N_\sigma(\rho'))$$

$$d_\sigma(\rho, \rho')$$

$$= d(N_\sigma(\rho), N_\sigma(\rho'))$$

↑ distinguishability

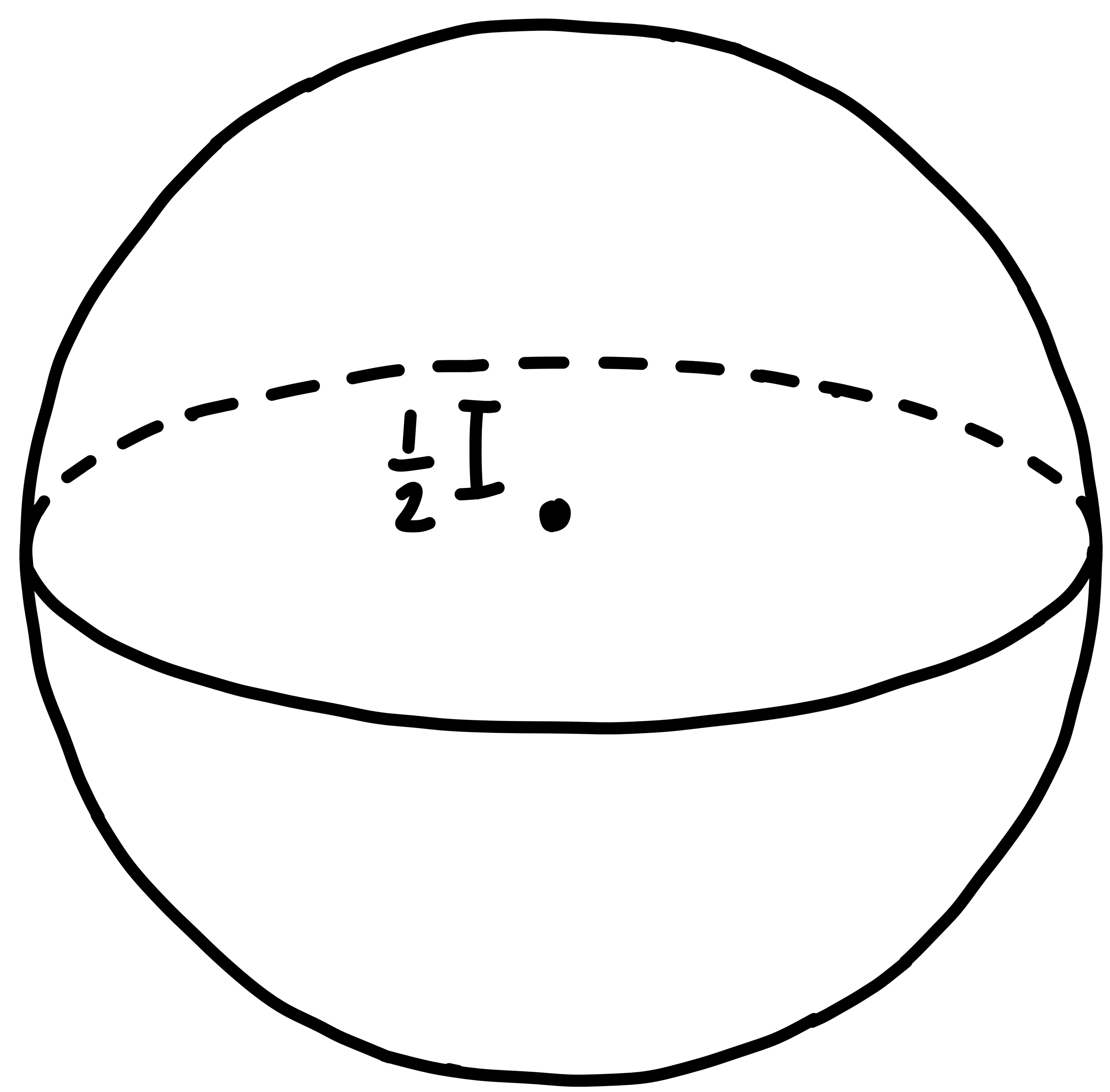
$$d_\sigma(\rho, \rho')$$

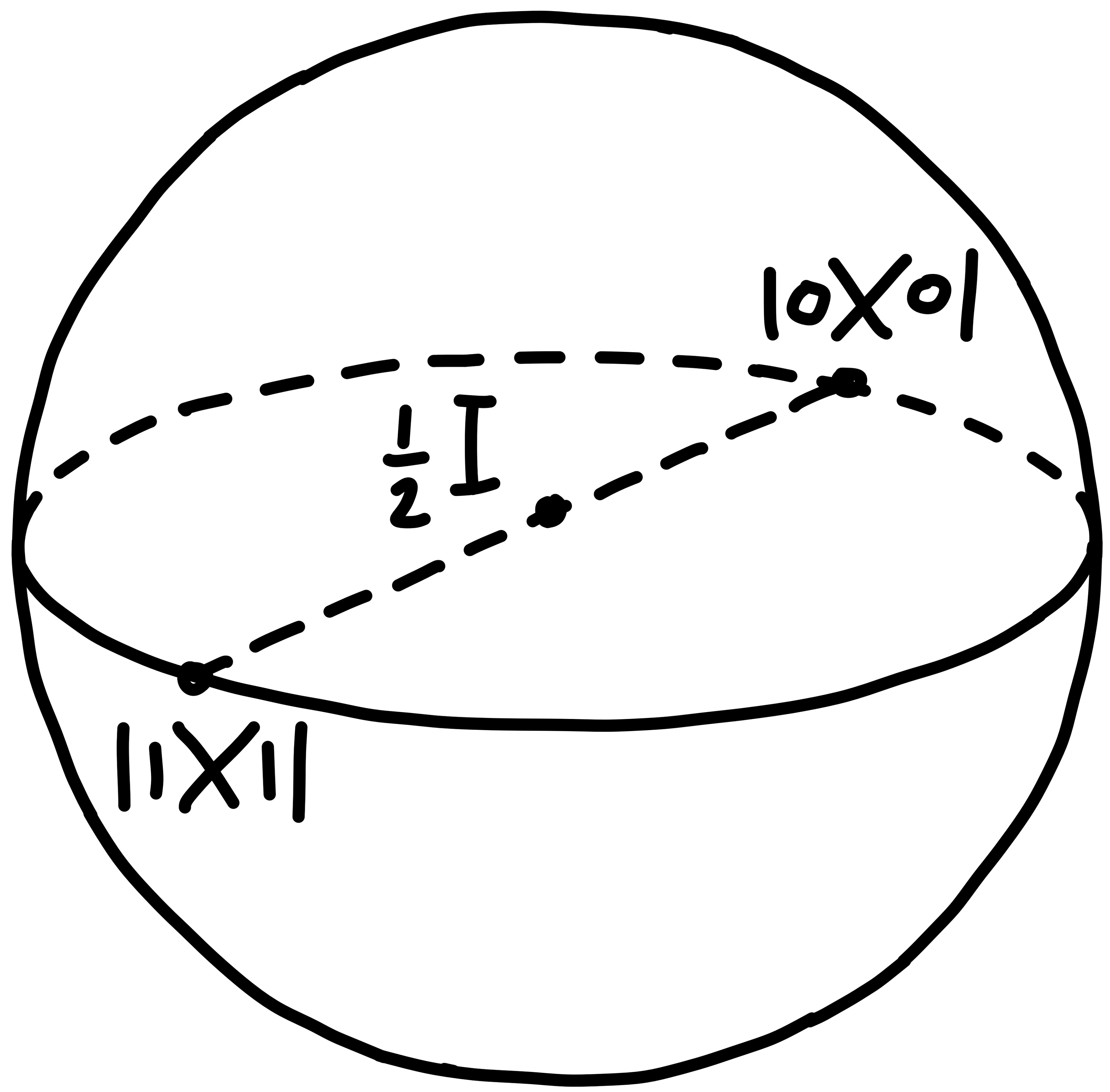
$$= d(N_\sigma(\rho), N_\sigma(\rho'))$$

distinguishability

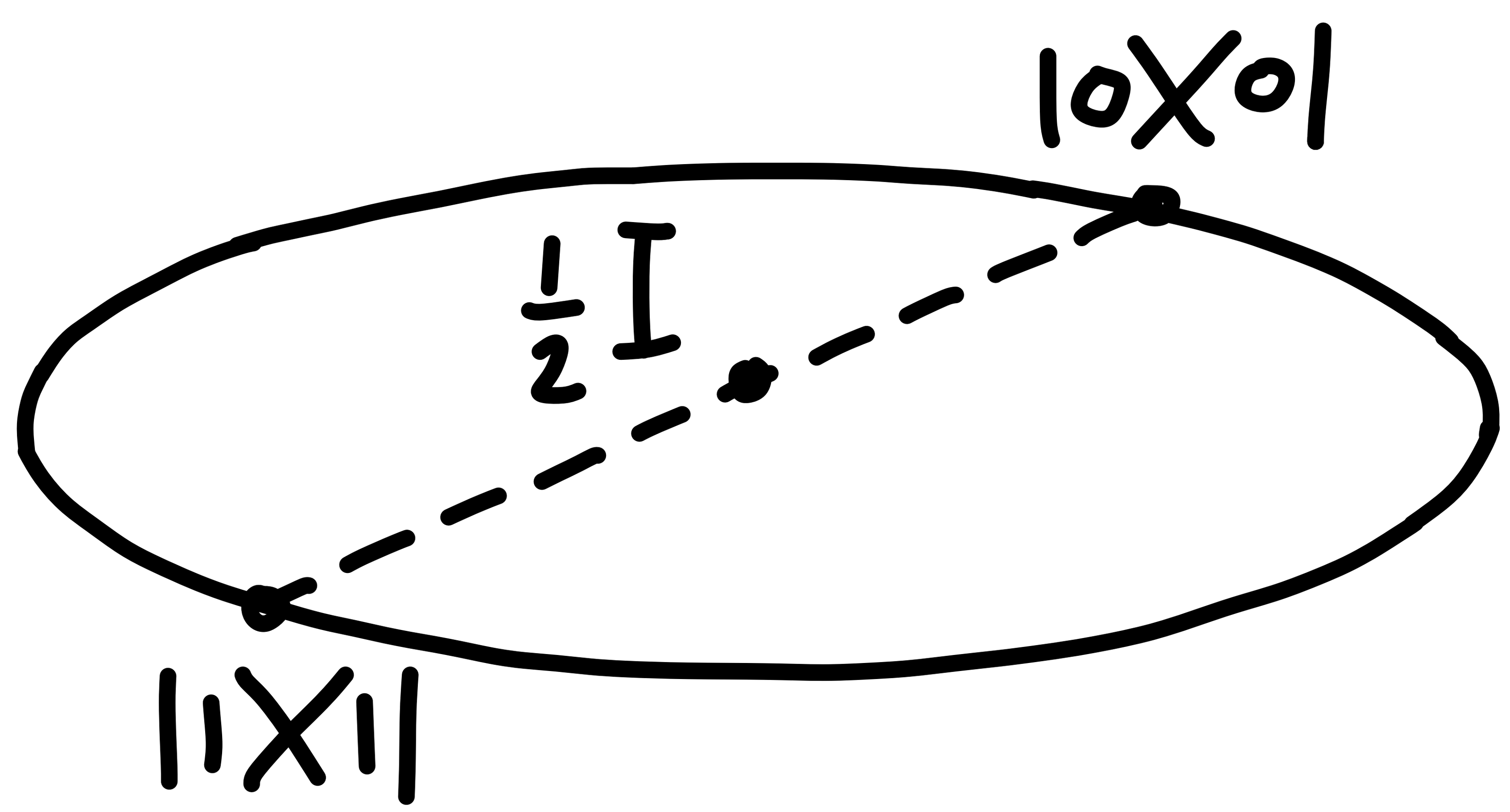
coarse-graining (CP map)

qubit (spin-$\frac{1}{2}$)

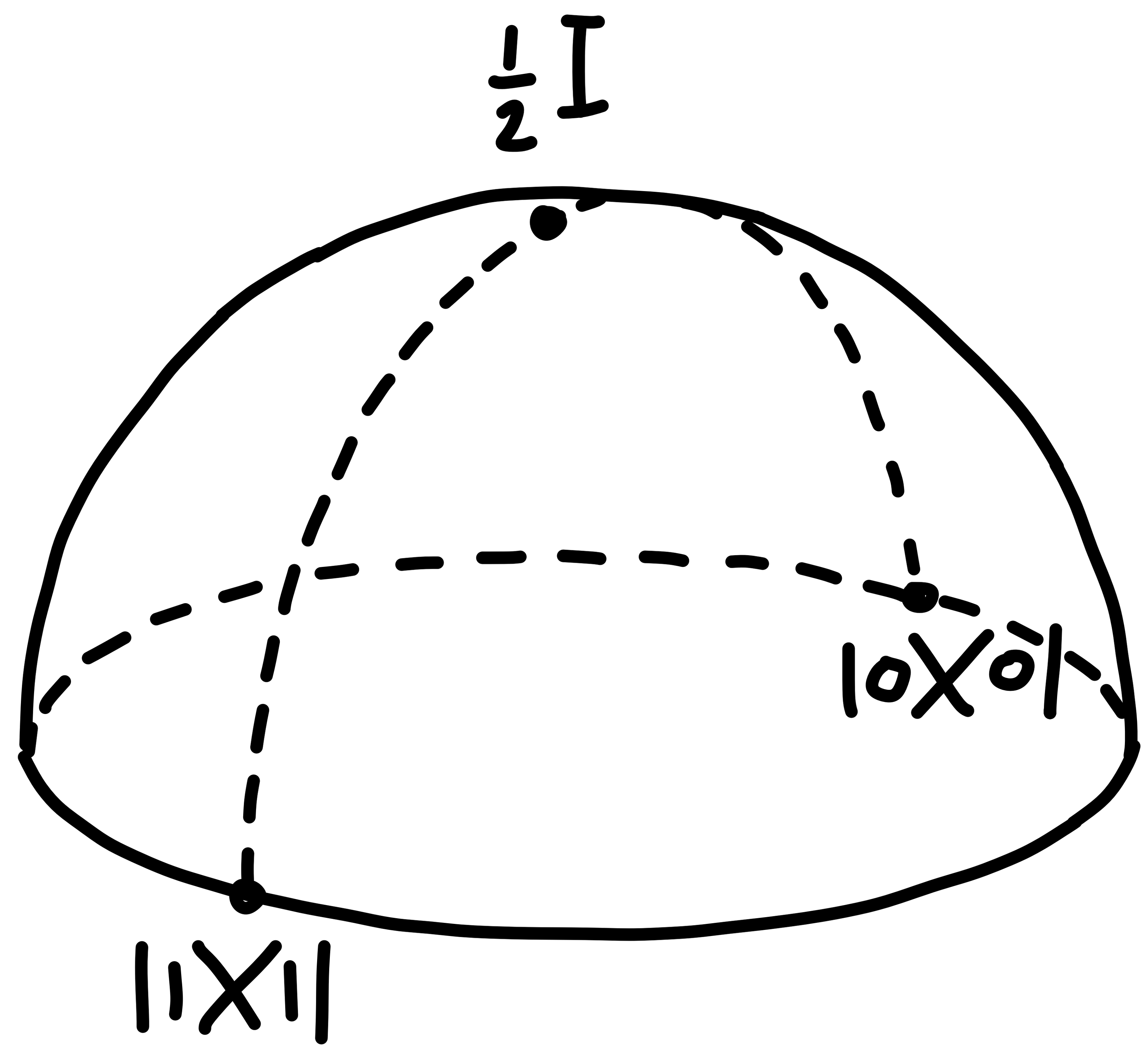


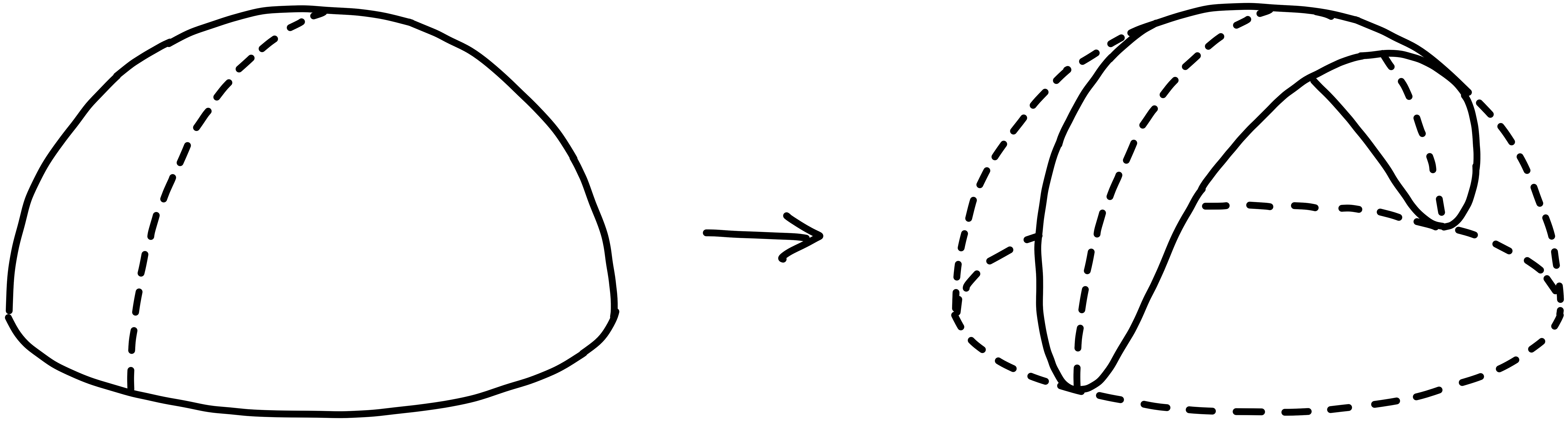


|0⟩⟨0|
$\frac{1}{2}I$
|1⟩⟨1|

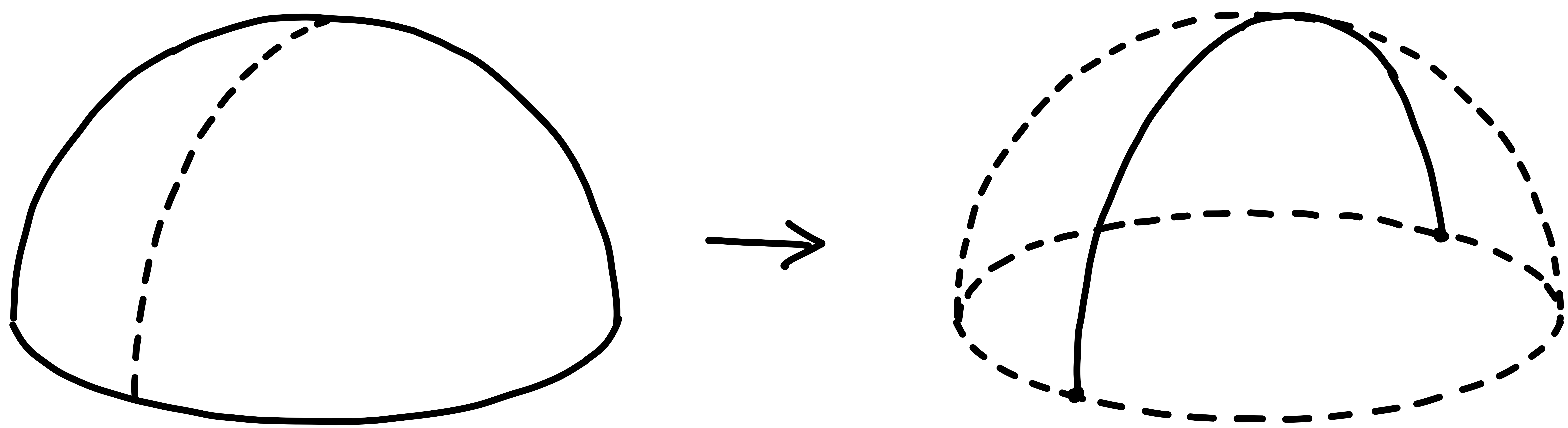

|0⟩⟨0|
½ I
|1⟩⟨1|

$$d(\rho, \rho') = \operatorname{acos} \operatorname{Tr} \sqrt{\sqrt{\rho}\,\rho'\sqrt{\rho}}$$

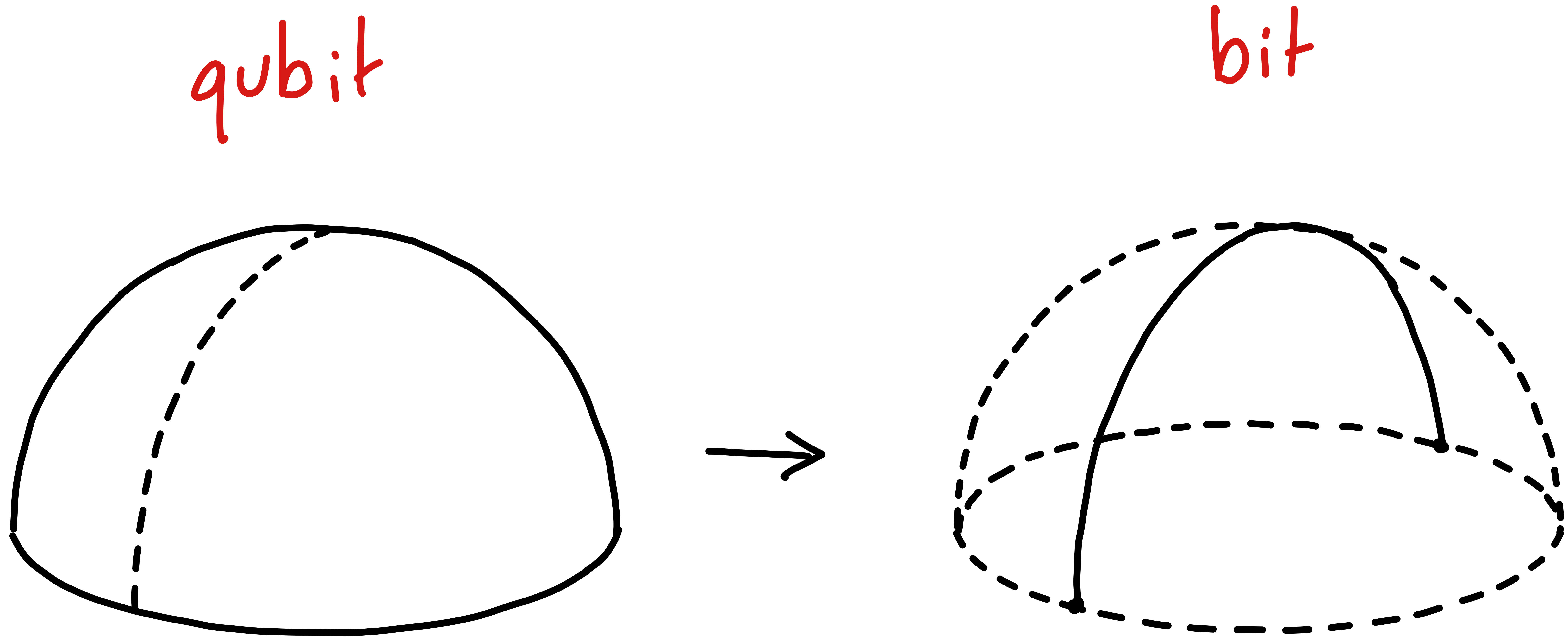




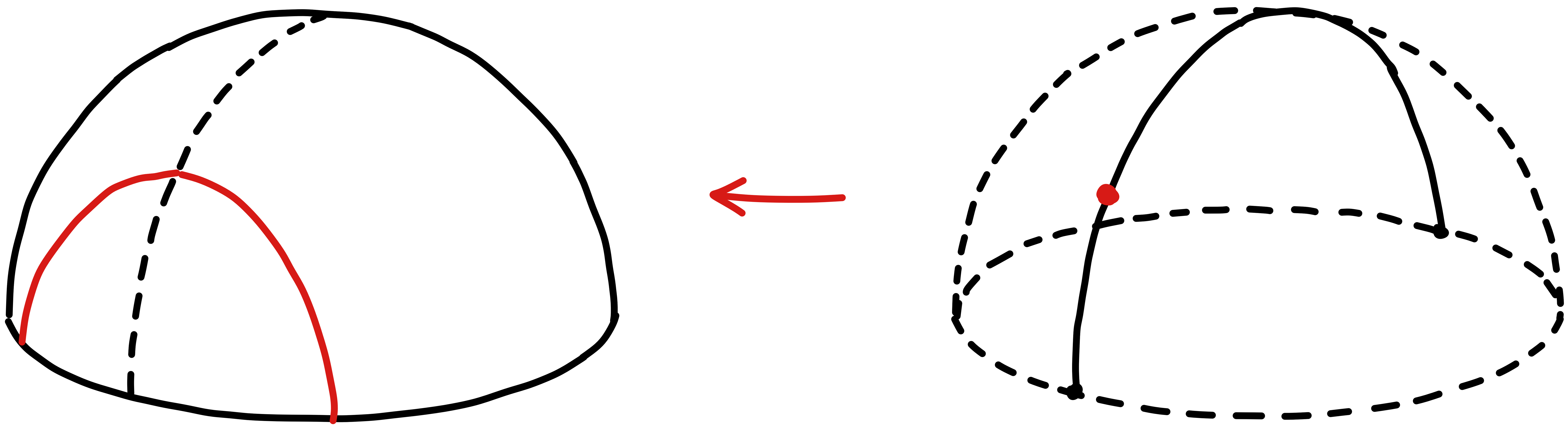
$$\begin{pmatrix} p & x \\ x & 1-p \end{pmatrix} \xmapsto{N_\lambda} \begin{pmatrix} p & x/\lambda \\ x/\lambda & 1-p \end{pmatrix}$$



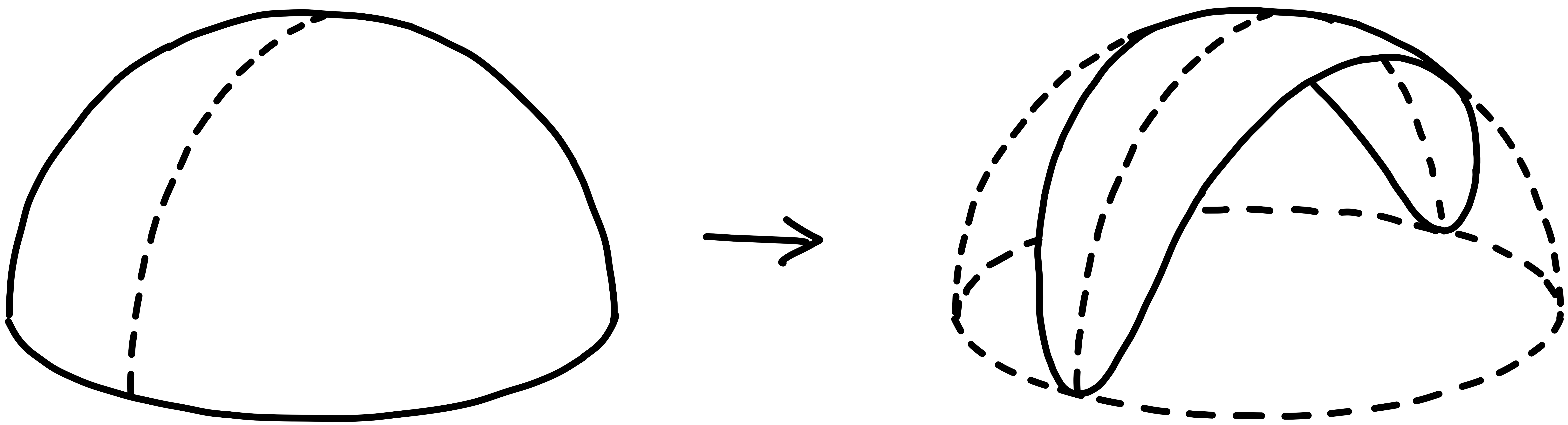
$$\begin{pmatrix} p & \times \\ \times & 1-p \end{pmatrix} \xmapsto{N_\infty} \begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}$$



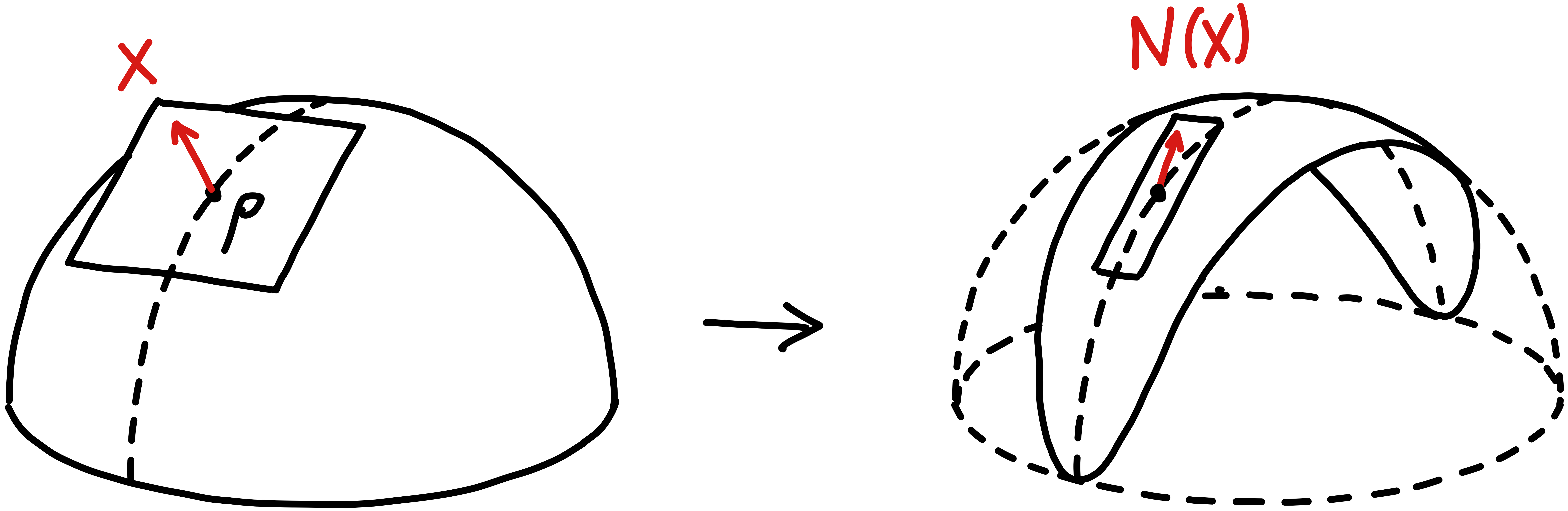
$$\begin{pmatrix} p & \times \\ \times & 1-p \end{pmatrix} \xmapsto{N_\infty} \begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}$$



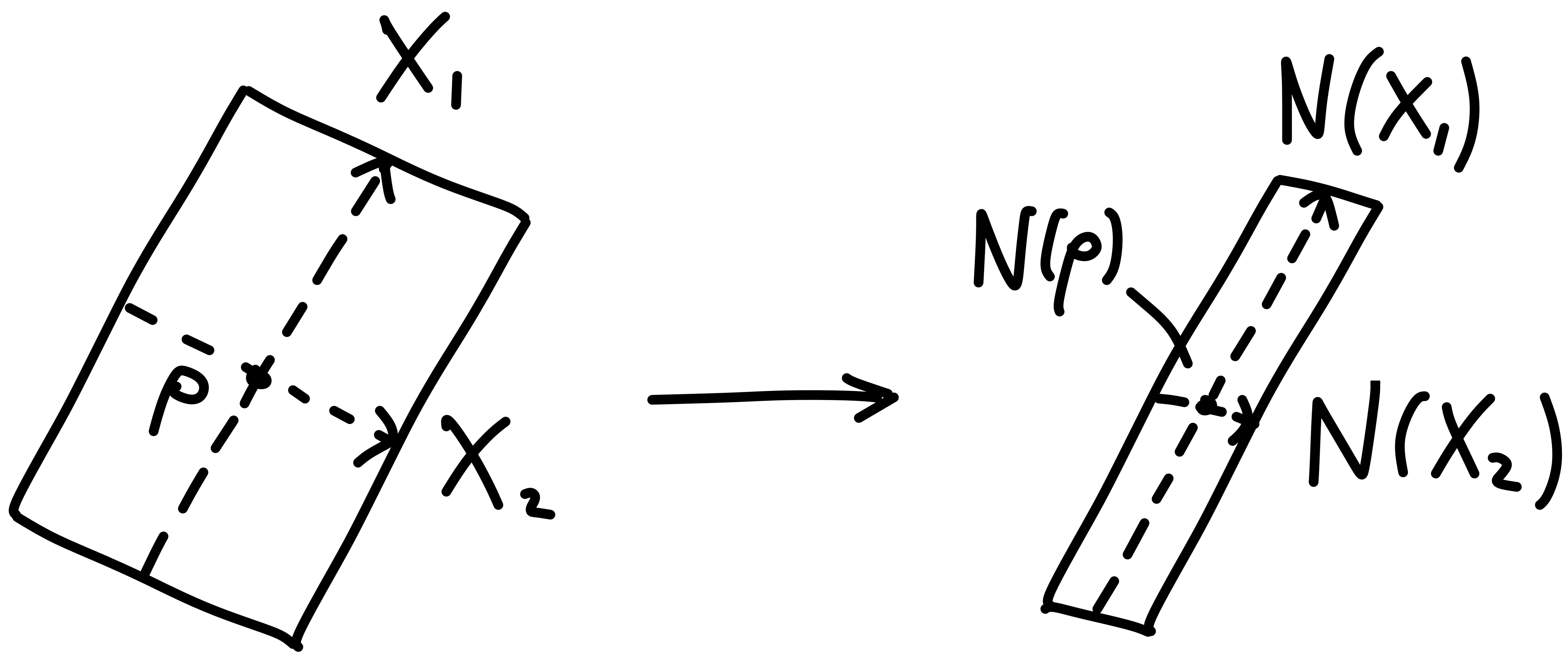
$$\begin{pmatrix} p & \times \\ \times & 1-p \end{pmatrix} \overset{N_\infty}{\longmapsto} \begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}$$



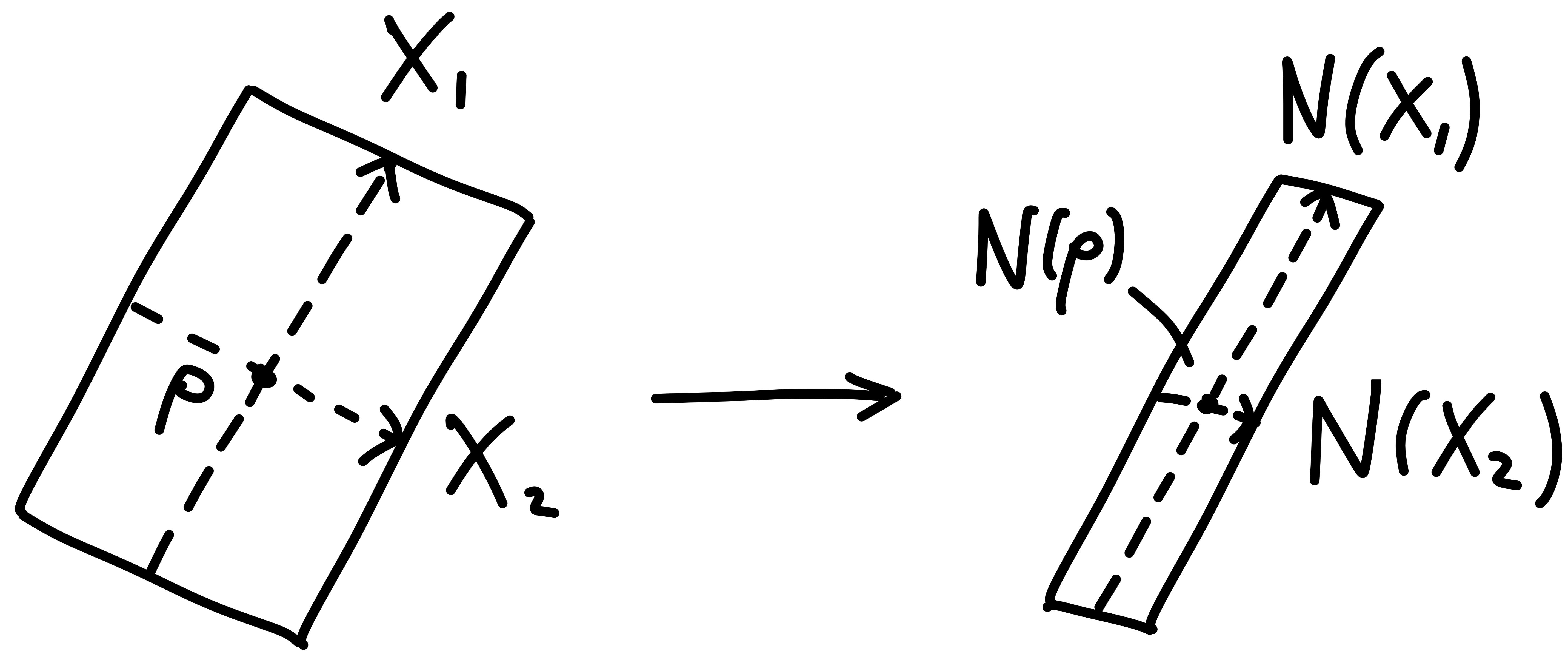
$$\begin{pmatrix} p & x \\ x & 1-p \end{pmatrix} \xmapsto{N} \begin{pmatrix} p & x/\lambda \\ x/\lambda & 1-p \end{pmatrix}$$



$$\langle X, Y \rangle_p \overset{N}{\longmapsto} \langle N(X), N(Y) \rangle_{N(p)}$$

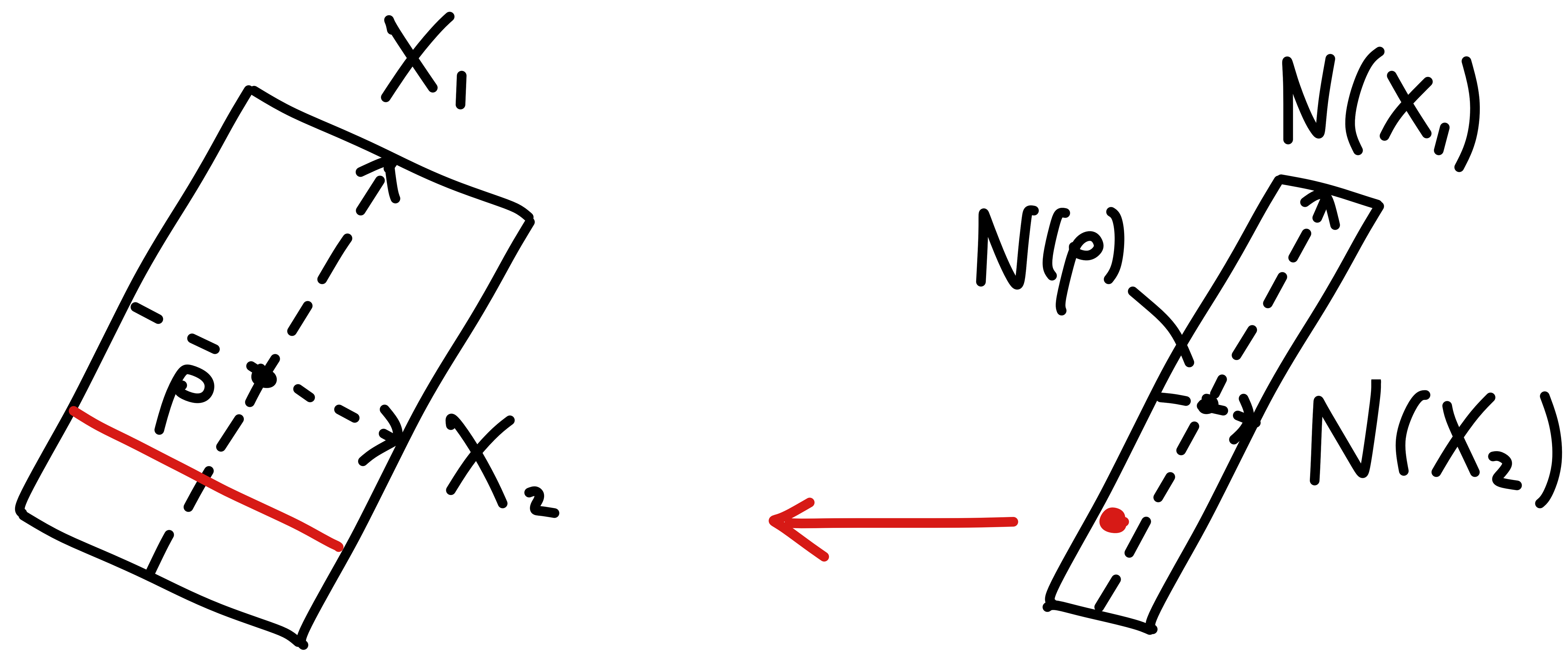


$$N^{*p} \cdot N(X_i) = \eta_i X_i$$



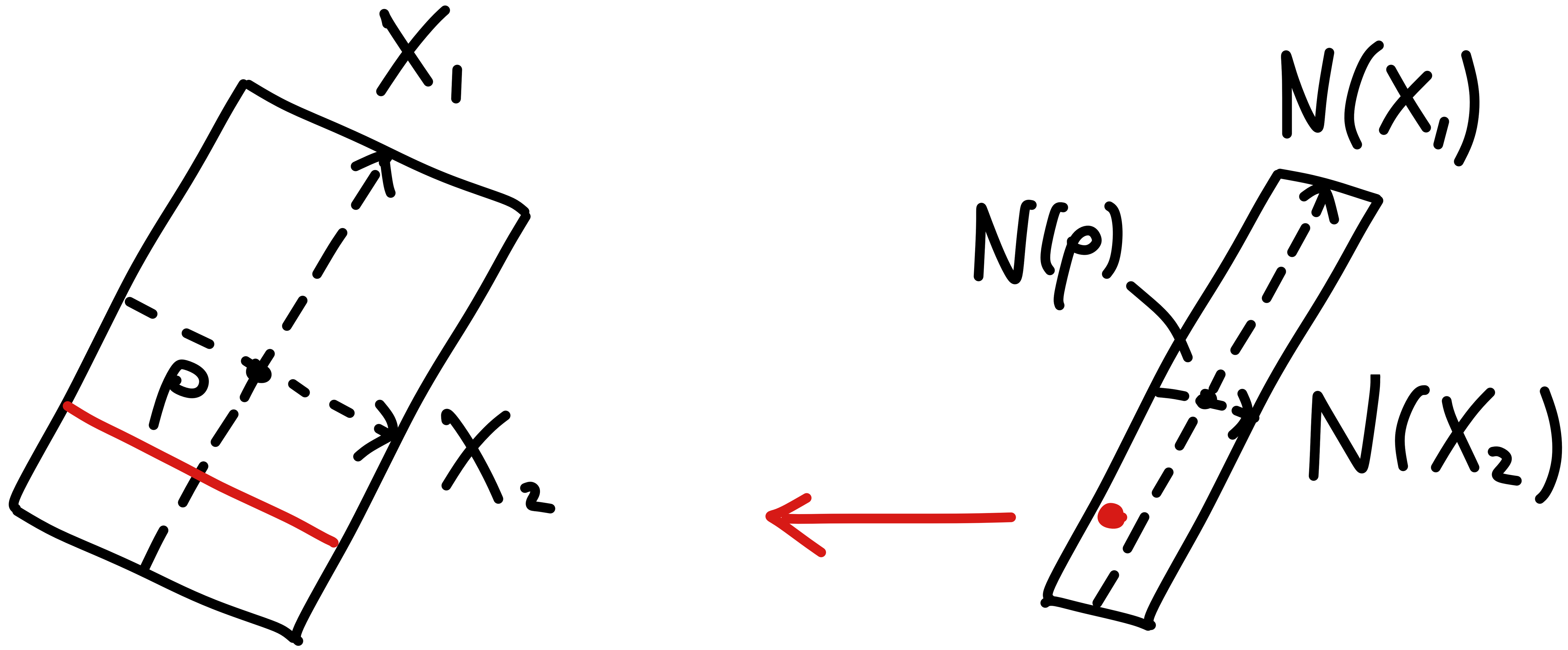
$$N^{*\rho} \cdot N(X_i) = \eta_i X_i$$

↑ Bayesian inference



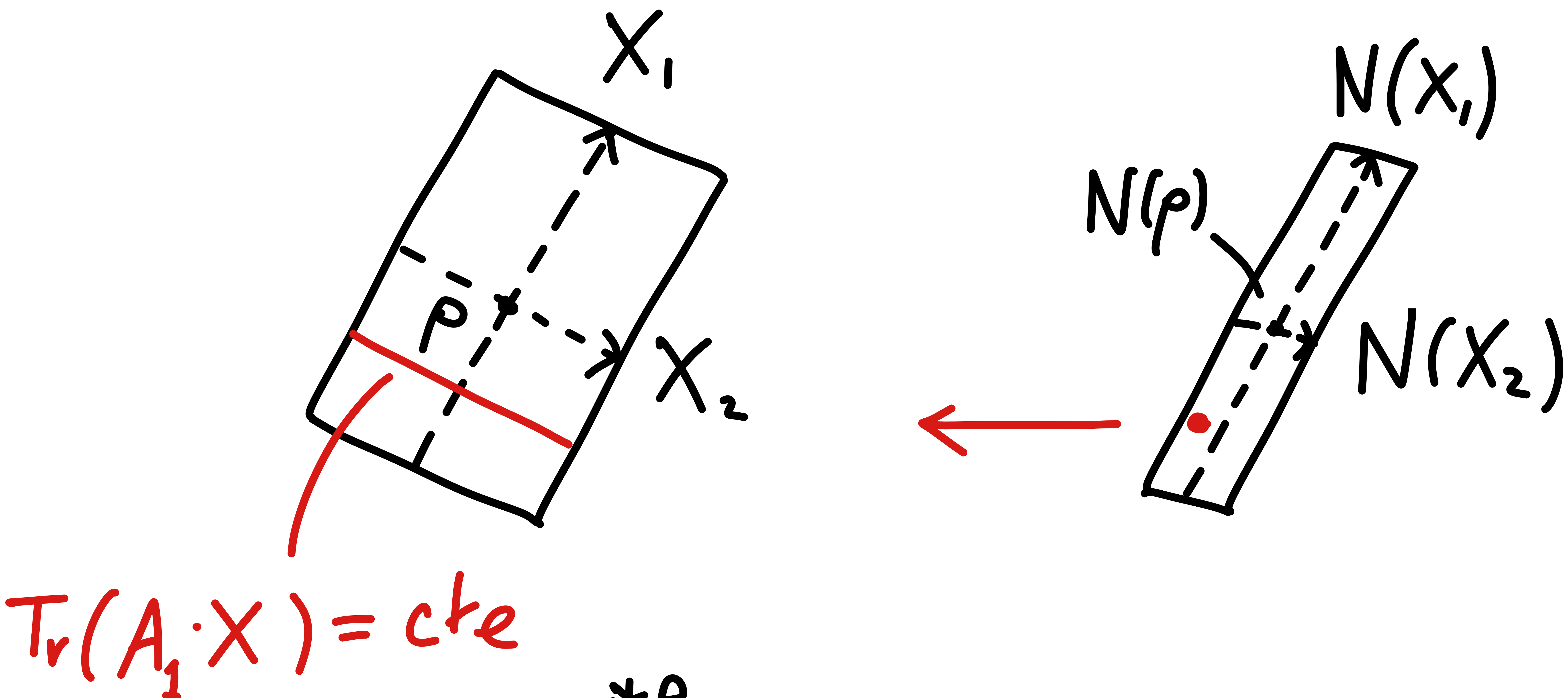
$$N^{*p} \cdot N(X_i) = \eta_i X_i$$

↑ Bayesian inference



$$N^{*\rho} \circ N(X_i) = \eta_i X_i$$

$$\left(N^{*\rho} \circ N\right)^{\dagger}(A_i) = \eta_i A_i$$



$$N^{*\rho} \circ N (X_i) = \eta_i X_i$$

$$(N^{*\rho} \circ N)^{\dagger}(A_i) = \eta_i A_i$$

$\rho \equiv$ quasi-free field vacuum

$N_{\sigma,h}$ gaussian coarse-graining

$\rho \equiv$ quasi-free field vacuum

$N_{\sigma, h}$ gaussian coarse-graining

$$A \longmapsto X_\sigma^T A X_\sigma + hI$$

$\rho \equiv$ quasi-free field vacuum

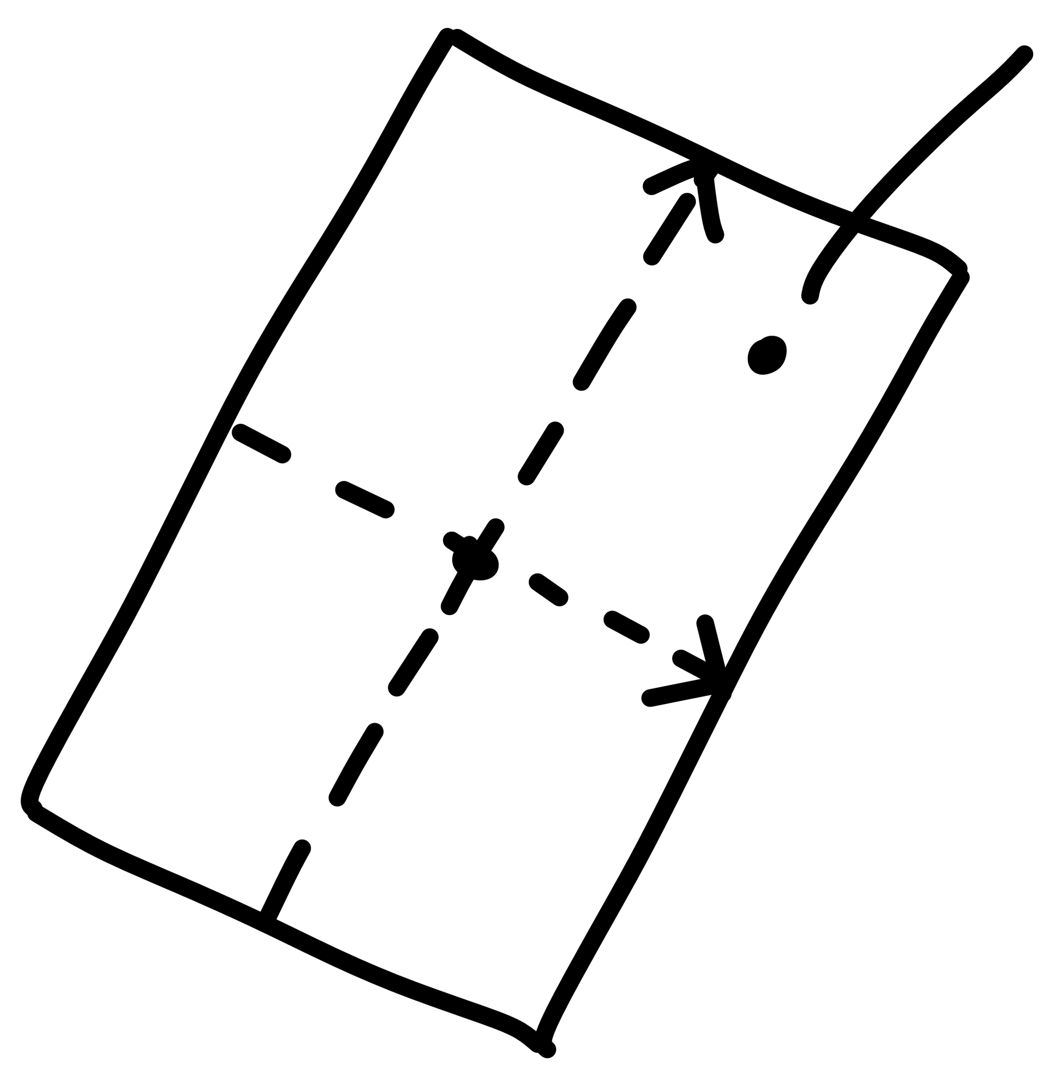
$N_{\sigma, h}$ gaussian coarse-graining

$$(N \circ N^{*,\rho})^{\dagger}(\phi_{k_1} \cdots \phi_{k_n}) \simeq h^{-n} \cdot e^{-\sigma^2 \sum_i |k_i|^2} \phi_{k_1} \cdots \phi_{k_n}$$

$\rho \equiv$ quasi-free field vacuum

$N_{\sigma, h}$ gaussian coarse-graining

$$(N \circ N^{*,\rho})^{\dagger}(\phi_{k_1} \cdots \phi_{k_n}) \simeq h^{-n} \cdot e^{-\sigma^2 \sum_i |k_i|^2} \phi_{k_1} \cdots \phi_{k_n}$$

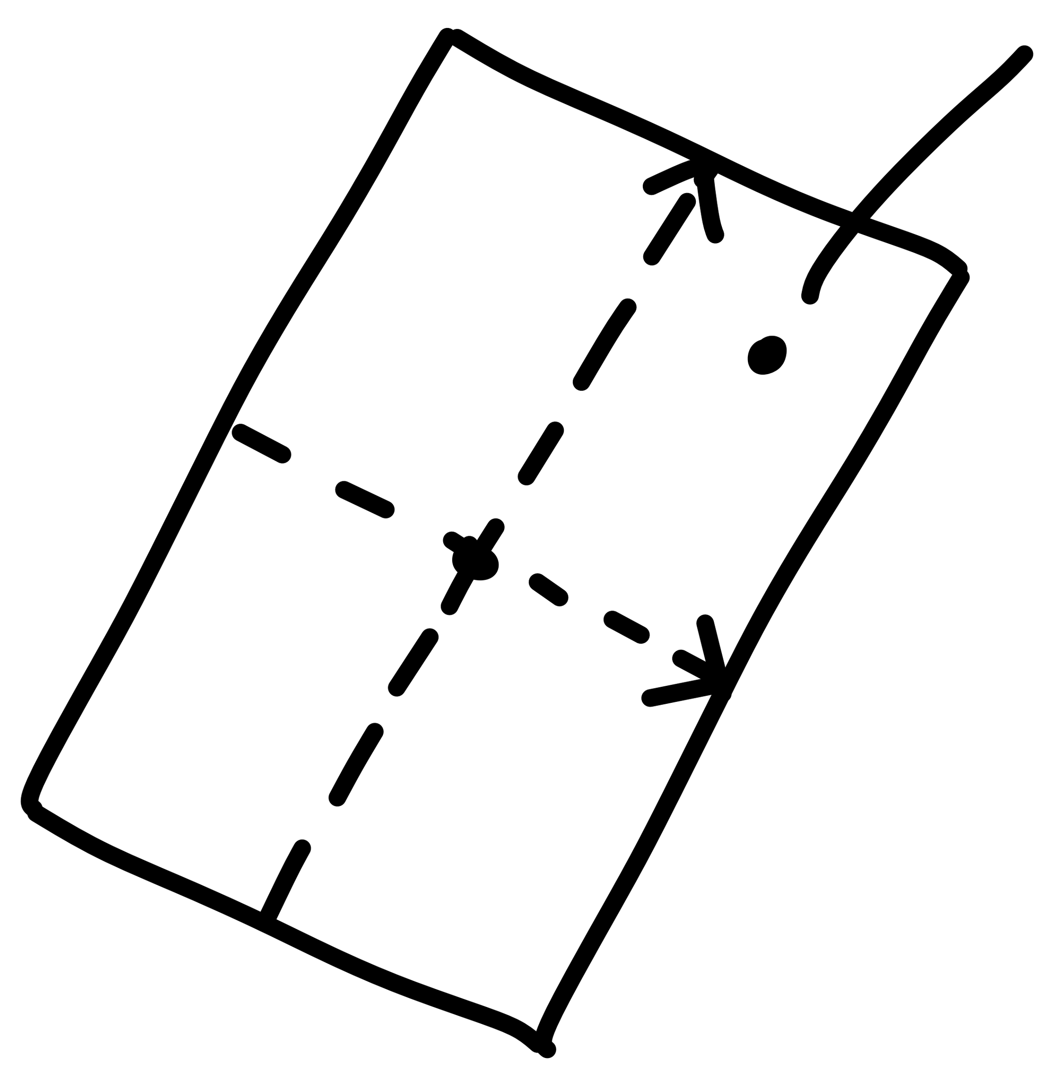


$\rho \equiv$ perturbative field vacuum

$N_{\sigma,h}$ gaussian coarse-graining

$$(N \circ N^{*,\rho})^{\dagger}(\phi_{k_1} \cdots \phi_{k_n}) \simeq h^{-m} \cdot e^{-\sigma^2 \sum_i |k_i|^2} \phi_{k_1} \cdots \phi_{k_n}$$

$m < n$



$\rho = |0\rangle\langle 0|$

$\Lambda \longmapsto \rho_\Lambda$ is "Cauchy" if

$\Lambda \longmapsto \rho_\Lambda$ is "Cauchy" if

$\forall \sigma$

$$\left(N_\sigma^{*,\rho_\Lambda} \circ N\right)^{+}\left(A_k^{\rho_\Lambda}\right) = \eta_k A_k^{\rho_\Lambda}$$

$\Lambda \longmapsto \rho_\Lambda$ is "Cauchy" if

$\forall \sigma, \epsilon \quad \exists \Lambda_0, \quad \forall \Lambda > \Lambda_0$

$\mathrm{Tr}\left(A_k^\Lambda \frac{d}{d\Lambda} \rho_\Lambda\right) = 0 \quad \forall k \quad \eta_k^\Lambda > \epsilon$

where $\left(N_\sigma^{*,\rho_\Lambda} \circ N\right)^+ \left(A_k^\Lambda\right) = \eta_k^\Lambda A_k^\Lambda$

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$\forall \sigma, \epsilon \quad \exists \Lambda_0, \quad \forall \Lambda > \Lambda_0$

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where $\left(N_\sigma^{*,\rho_\Lambda} \circ N\right)^{\dagger}\left(A_k^{\Lambda}\right) = \eta_k^{\Lambda} A_k^{\Lambda}$

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$$\mathrm{Tr}\left(A_k \frac{d}{d\Lambda}\rho_\Lambda\right) = 0 \quad \forall k \quad \eta_k > \epsilon$$

where $\left(N_\sigma^{*,\rho_\Lambda} \circ N\right)^{+}(A_k) = \eta_k A_k$

$\Lambda \longmapsto \rho_\Lambda$ is "Cauchy" if

$\forall \sigma, \epsilon \quad \exists \Lambda_0 , \quad \forall \Lambda > \Lambda_0$

$$\frac{d}{d\Lambda} Tr(A_k \rho_\Lambda) = 0 \quad \forall k \quad \eta_k > \epsilon$$

where $\left(N_\sigma^{*,\rho_\Lambda} \circ N\right)^+ (A_k) = \eta_k A_k$

Renormalization Equations

$$\forall \sigma \quad \exists \Lambda_0 , \quad \forall \Lambda > \Lambda_0$$

$$\frac{d}{d\Lambda} \langle \phi_{k_1} \cdots \phi_{k_n} \rangle_\Lambda = 0 \quad \forall n \quad \forall \sum_i |k_i|^2 < \frac{1}{\sigma^2}$$

arXiv: 1402.4949 1509.03249
1310.3188 1511.05090

TL;DR

TL;DR

given ρ : prior

TL;DR

given ρ : prior
N_σ : experimental limitations

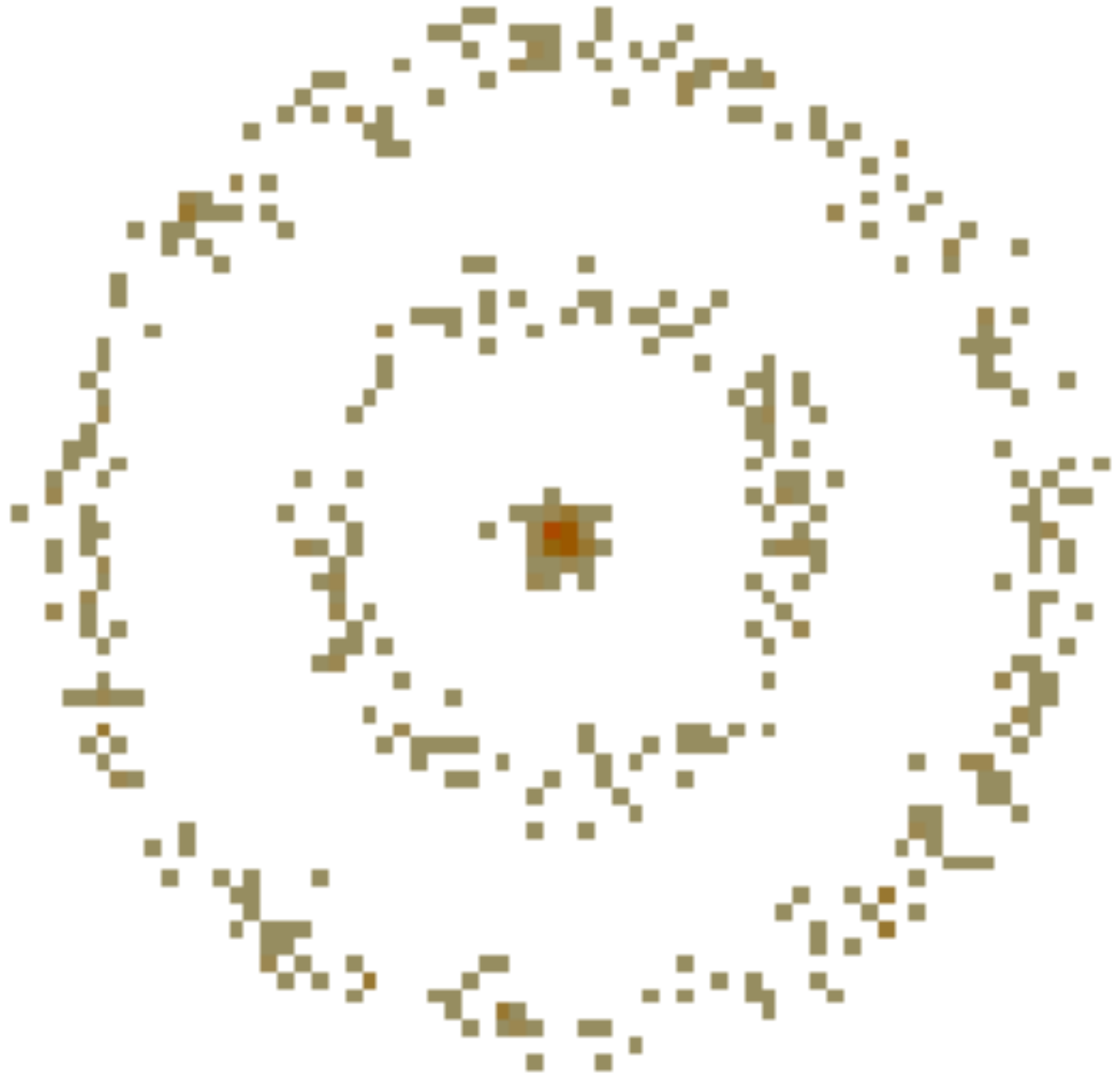
TL;DR

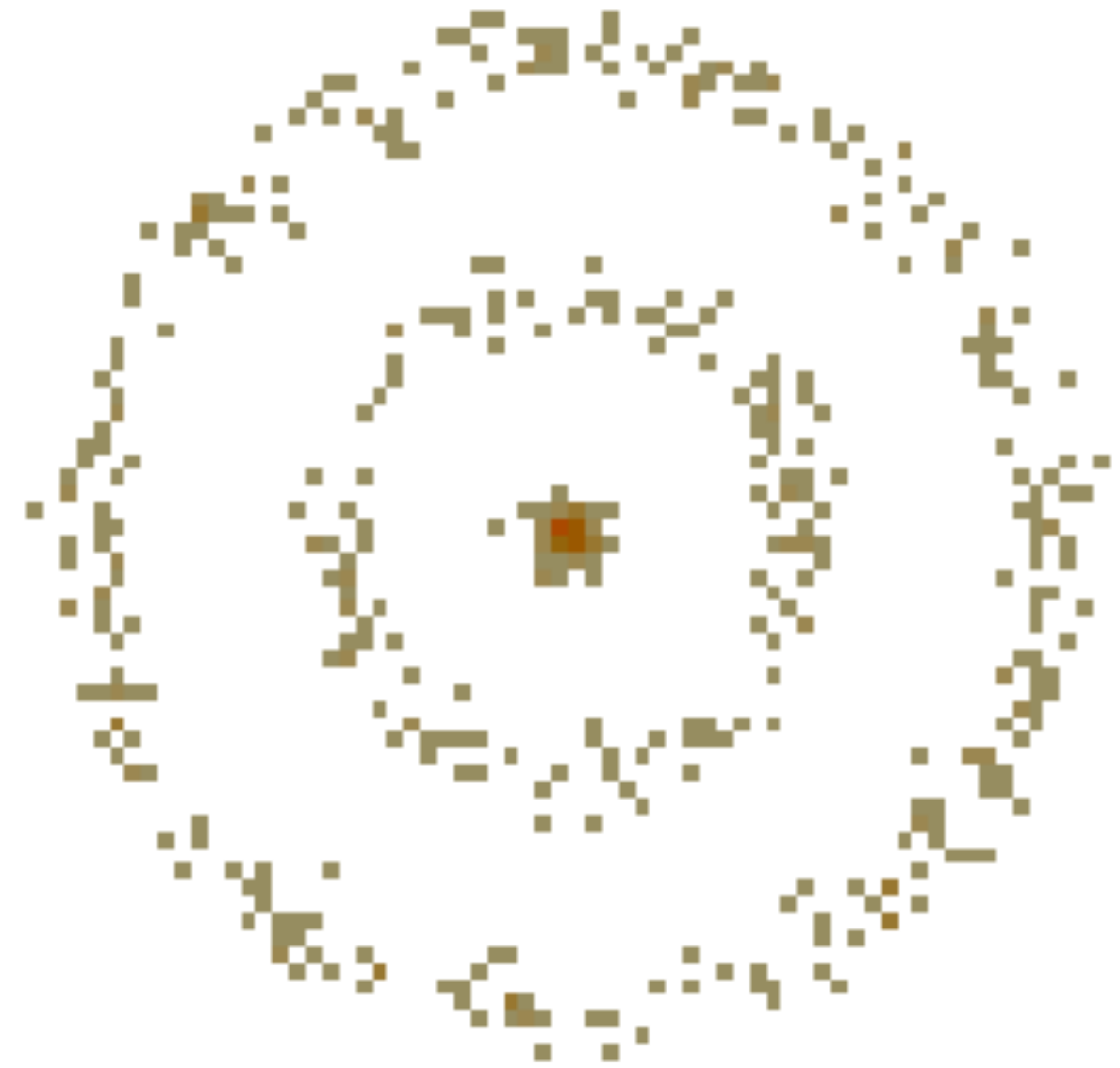
given ρ : prior

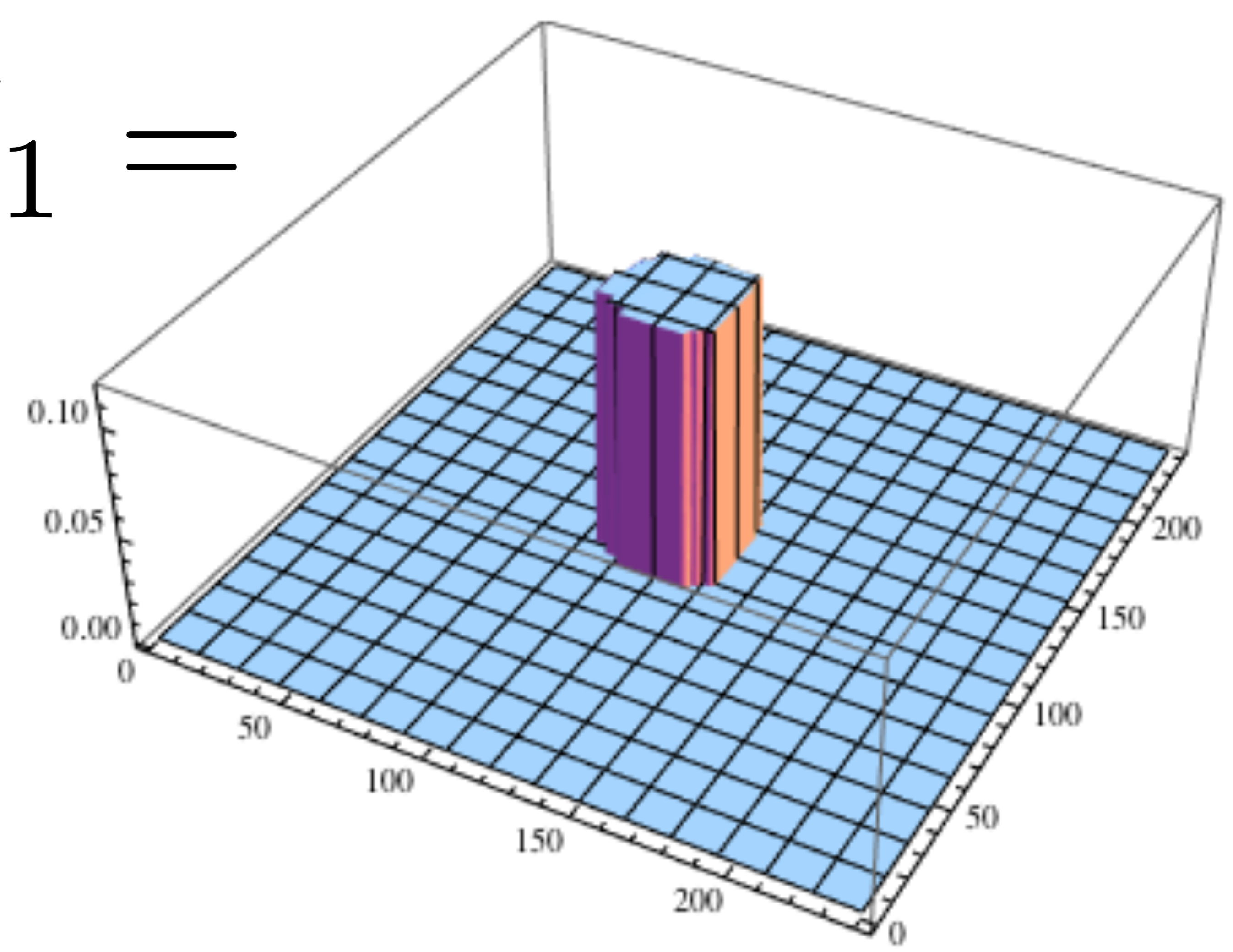
N_σ : experimental limitations

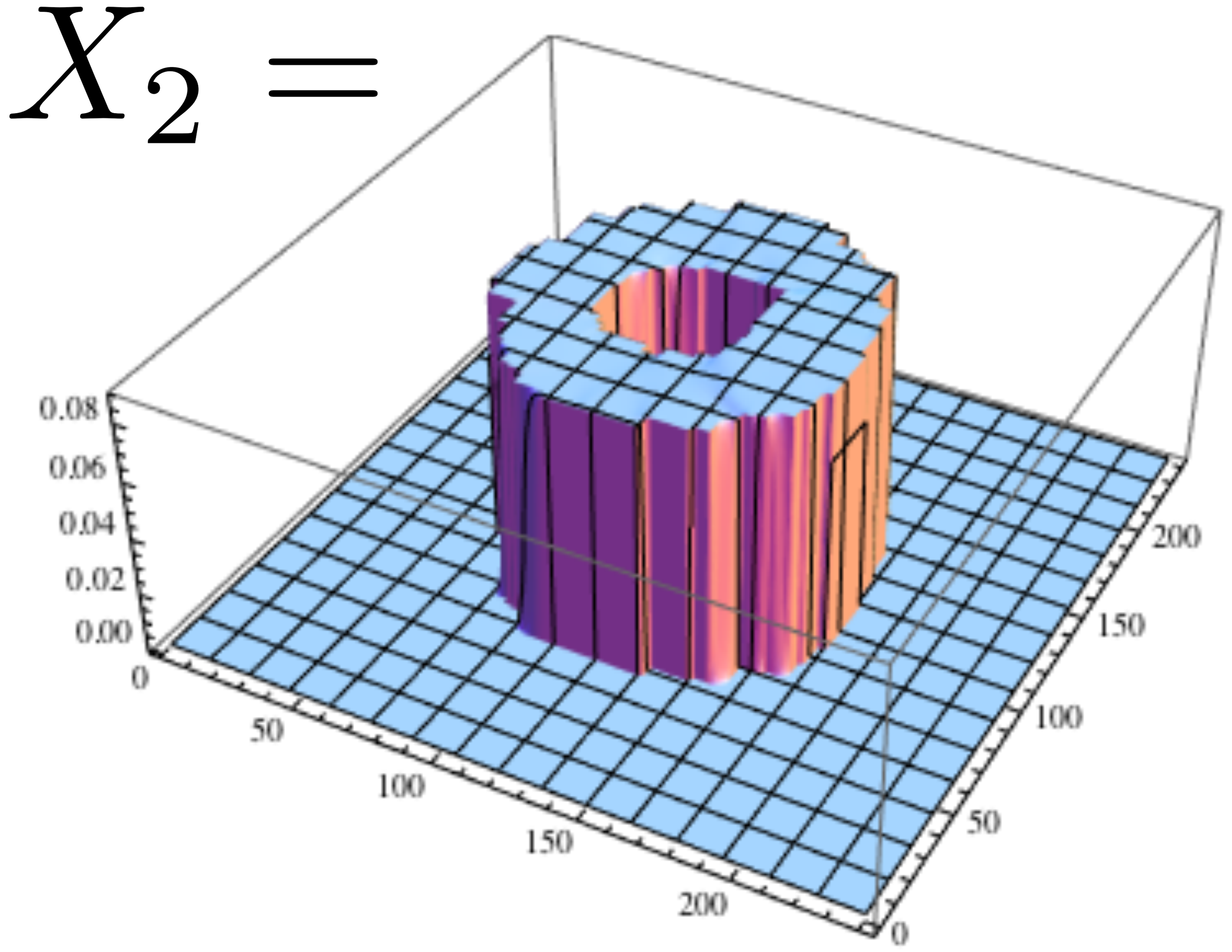
eigenvectors of $N_\sigma^{*\rho} \cdot N_\sigma$ are relevant data $x_1 \ldots x_N$

$$\rho(x) = \frac{1}{N} \sum_{i=1}^{N} \delta(x_i - x)$$

$\rho =$ 

$\rho =$ 

$X_1 =$ 
0.10
0.05
0.00
0
50
100
150
200

$X_2 =$ 
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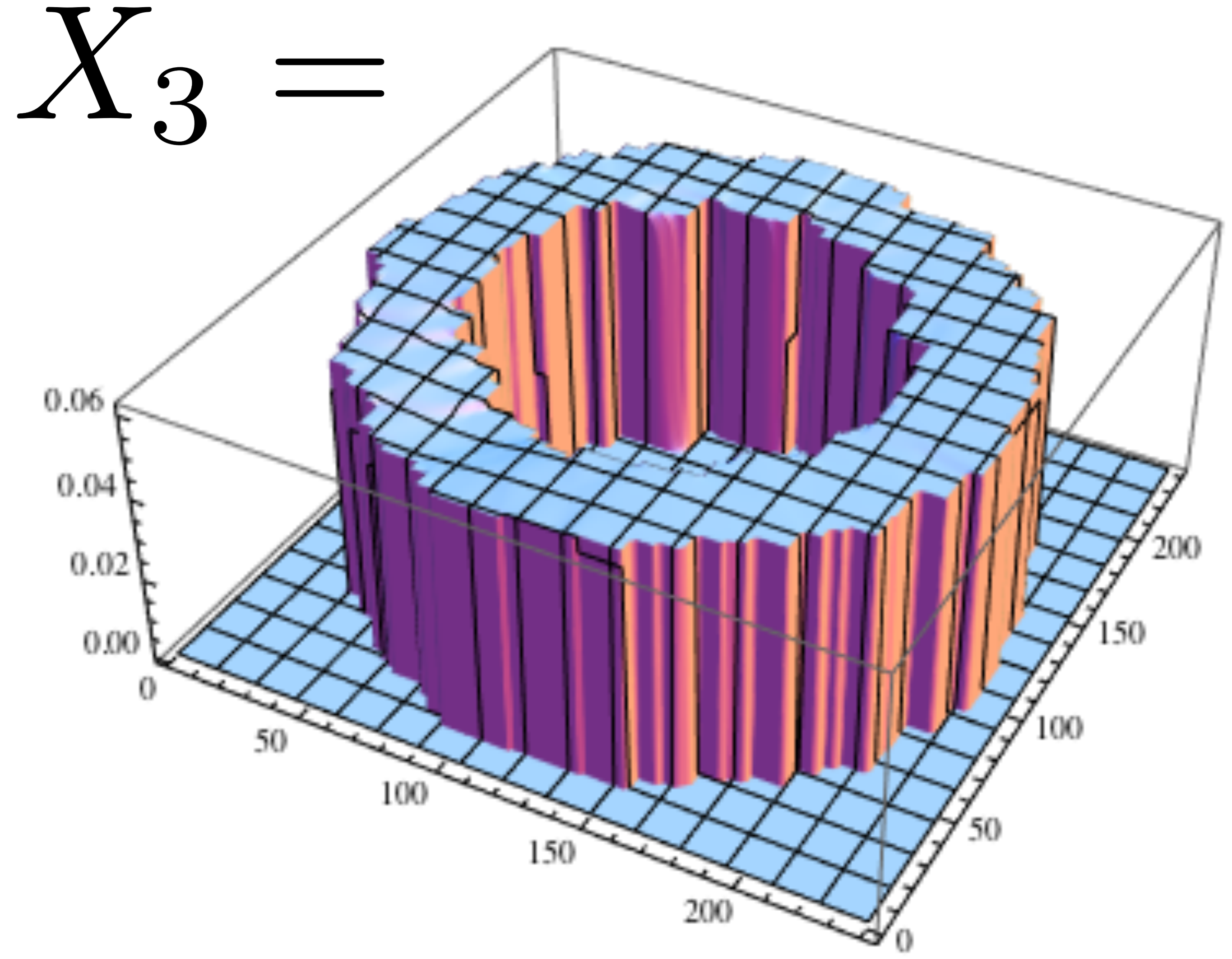
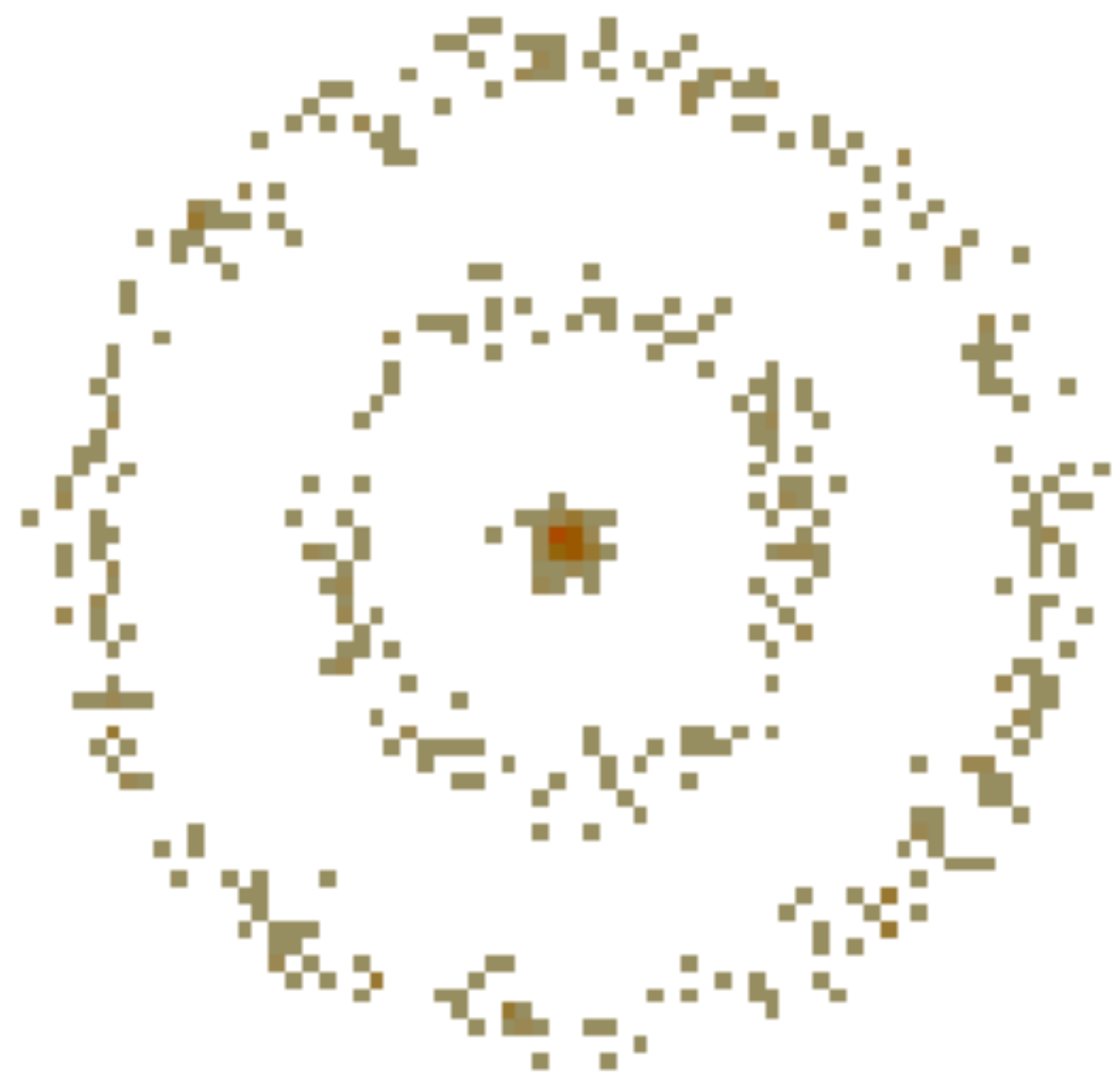
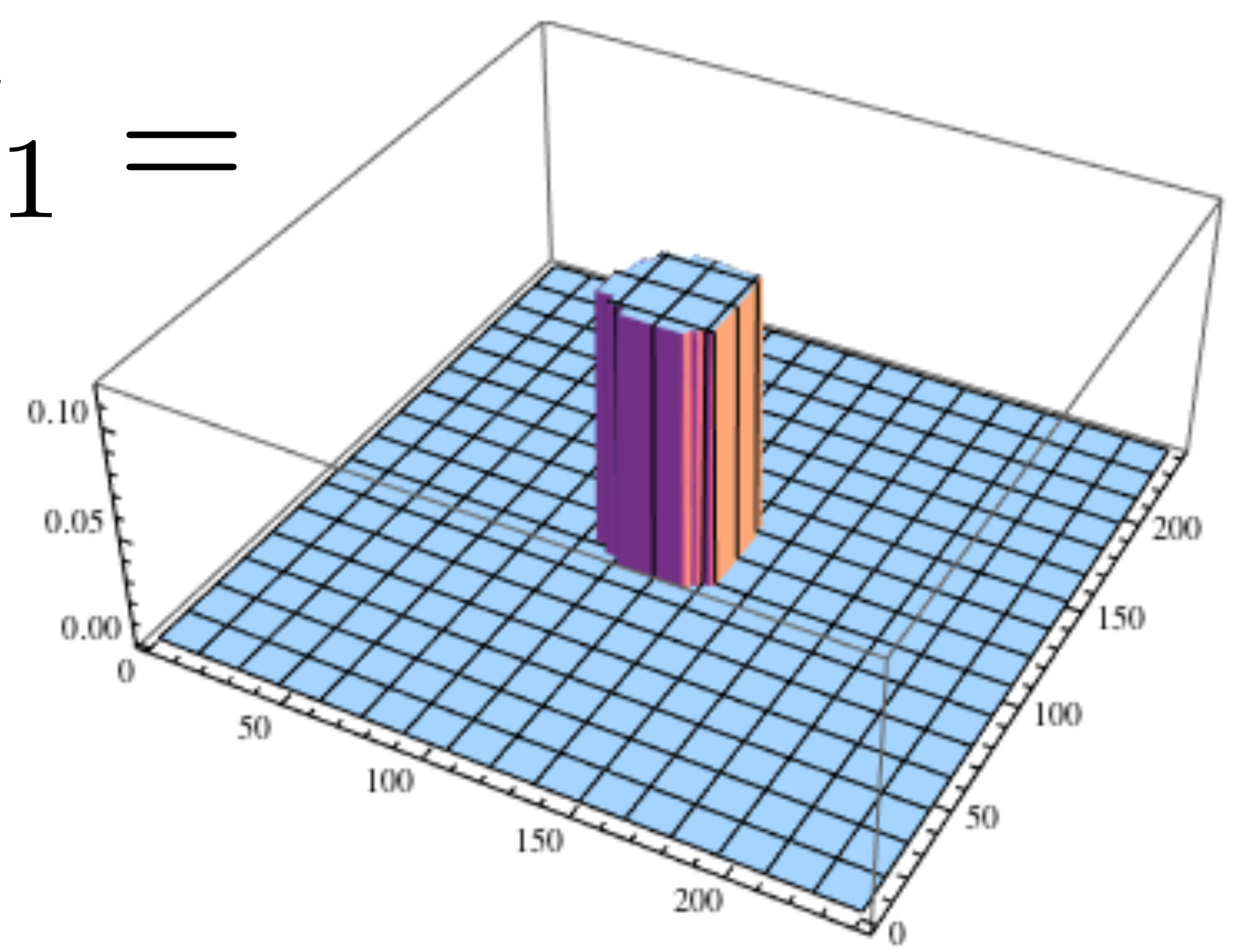
0.06
0.04
0.02
0.00
0
50
100
150
200

$X_3 =$ 
0.06
0.04
0.02
0.00
0
50
100
150
200

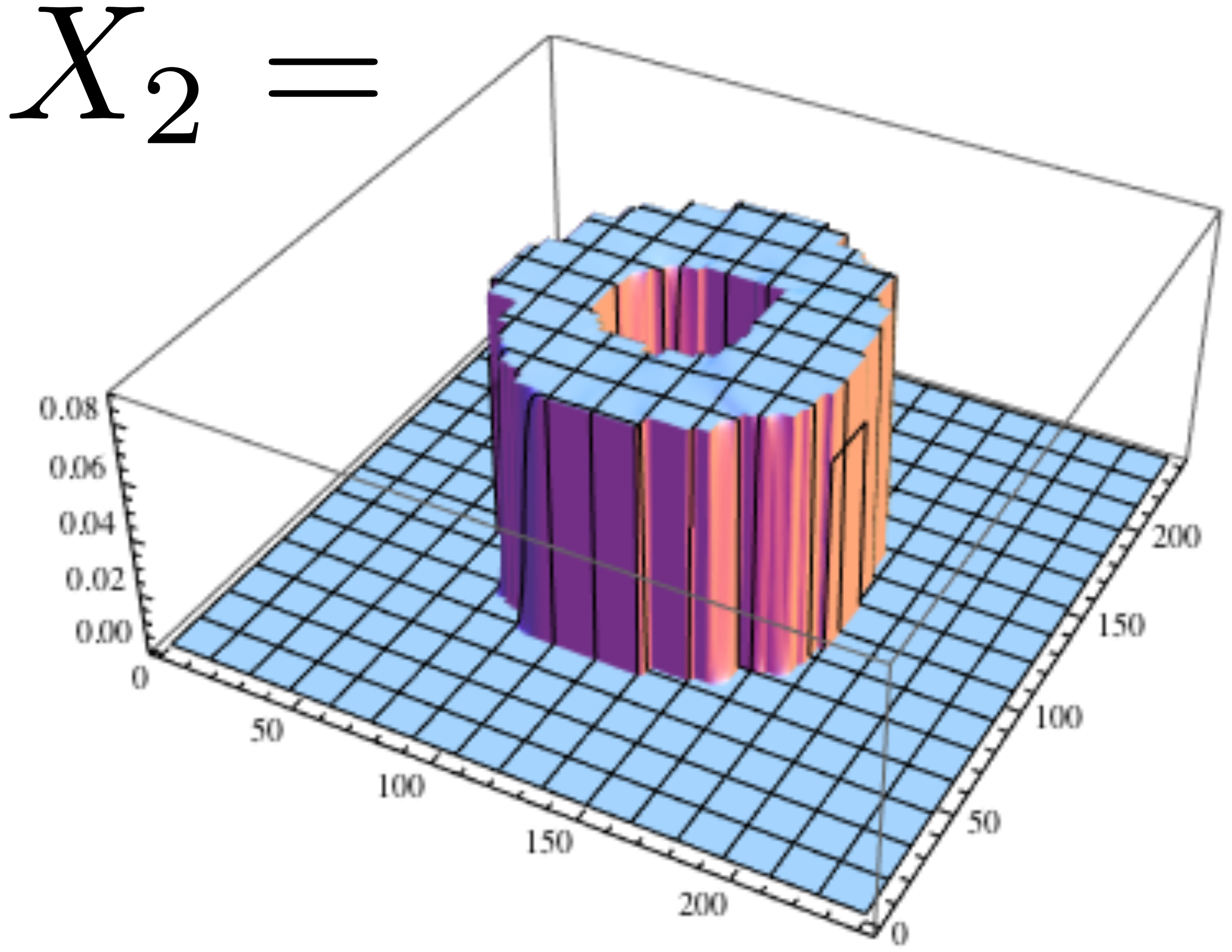
$\rho =$



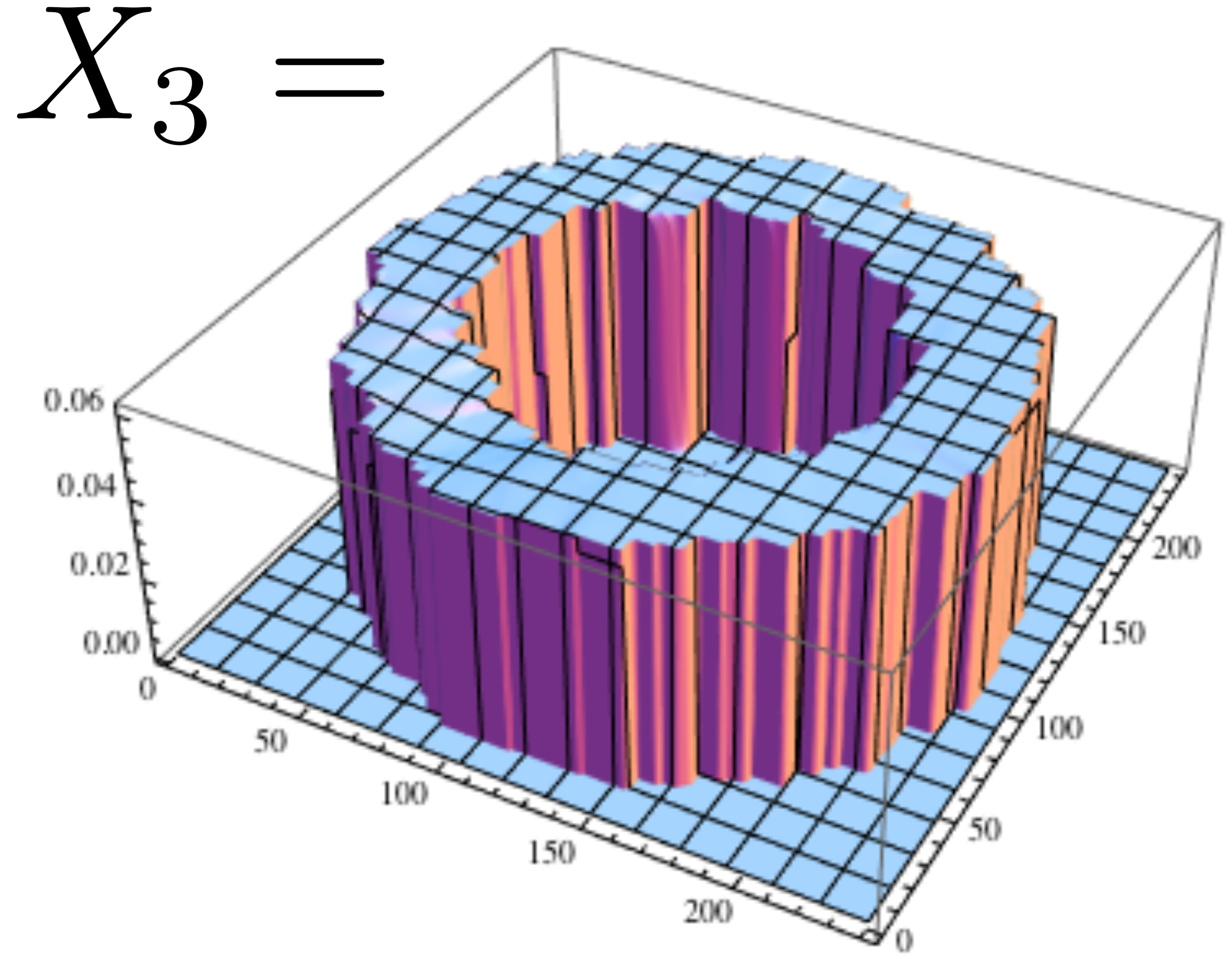
$X_1 =$



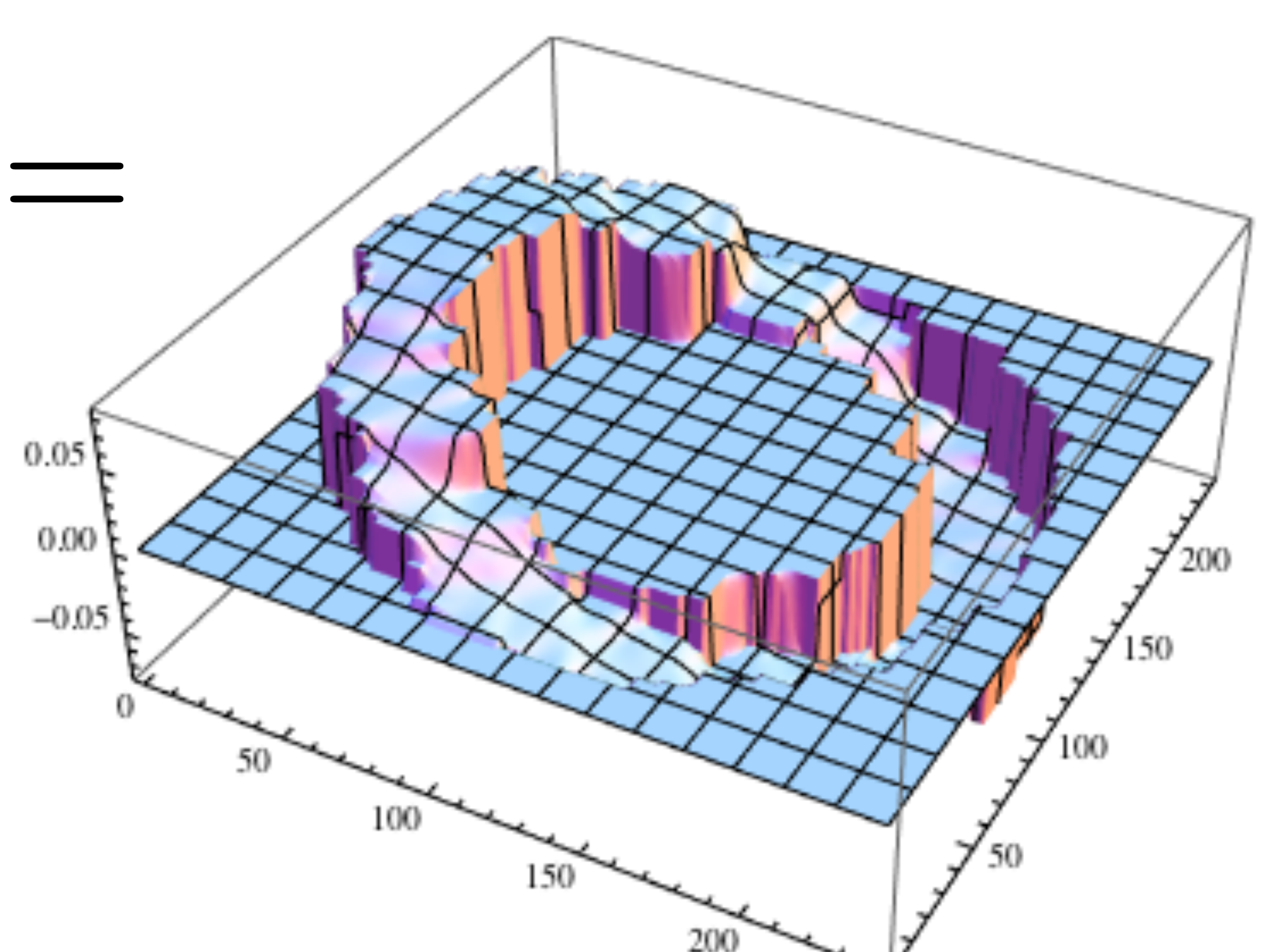
$X_2 =$



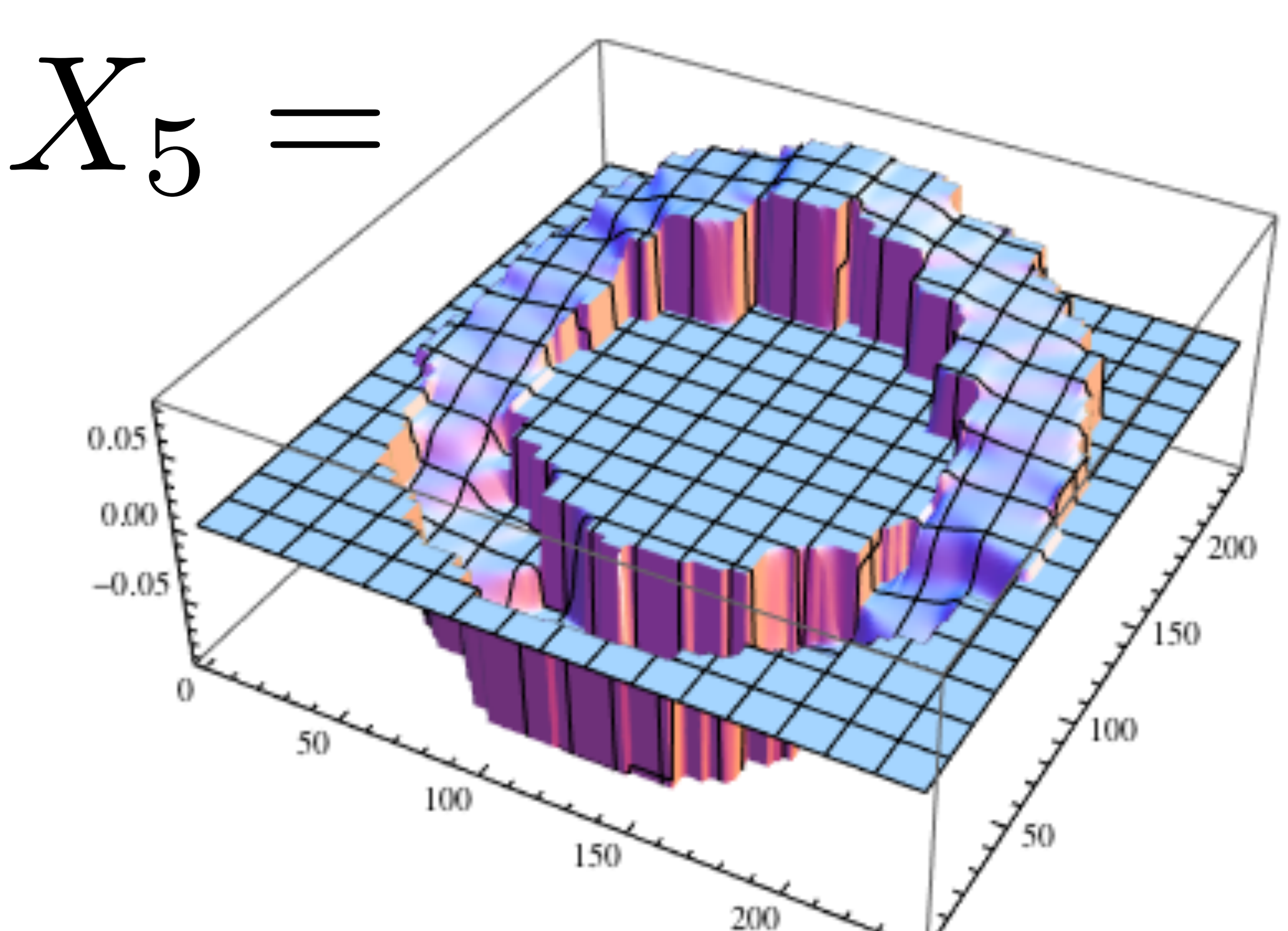
$X_3 =$

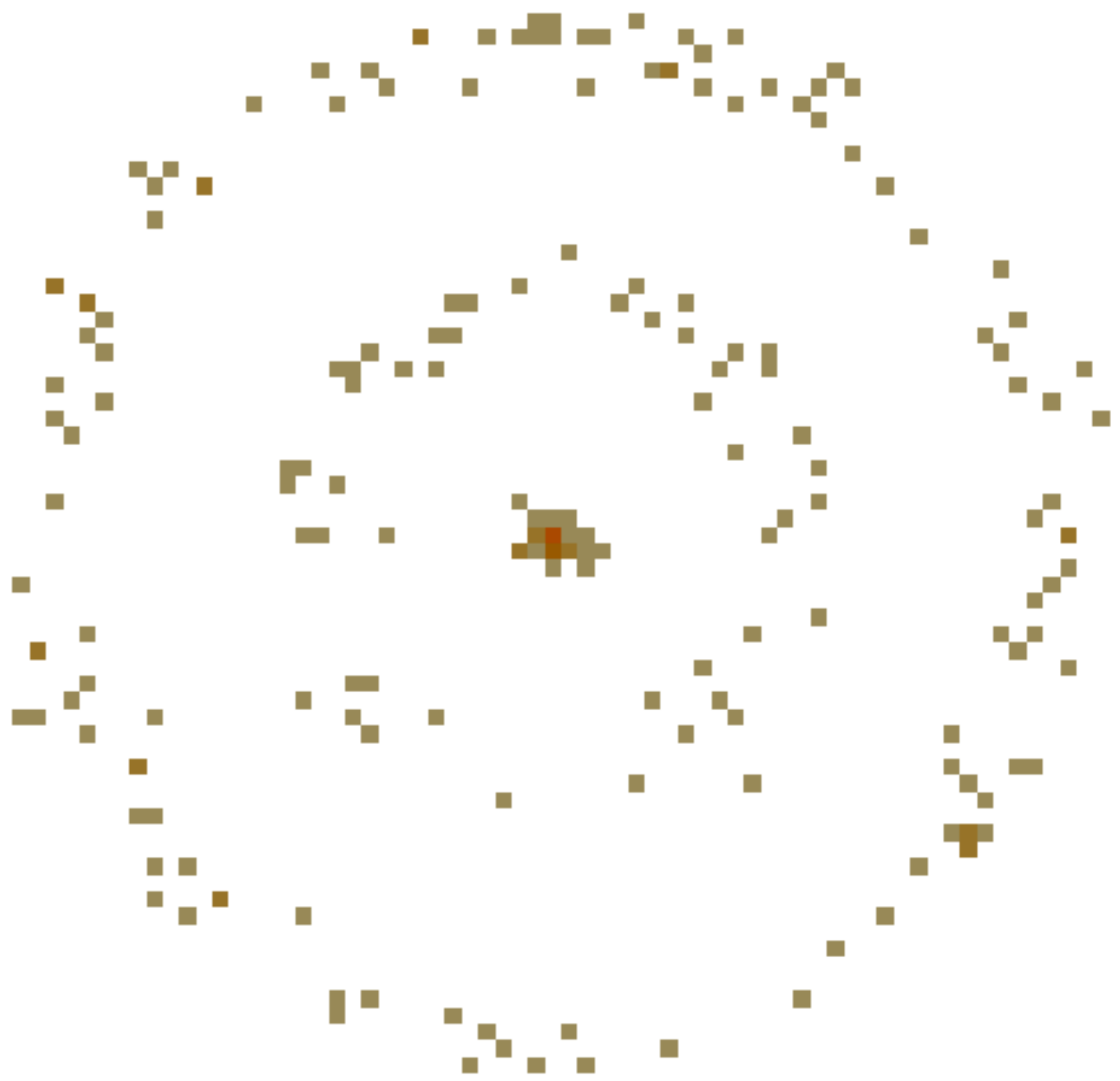


$X_4 =$

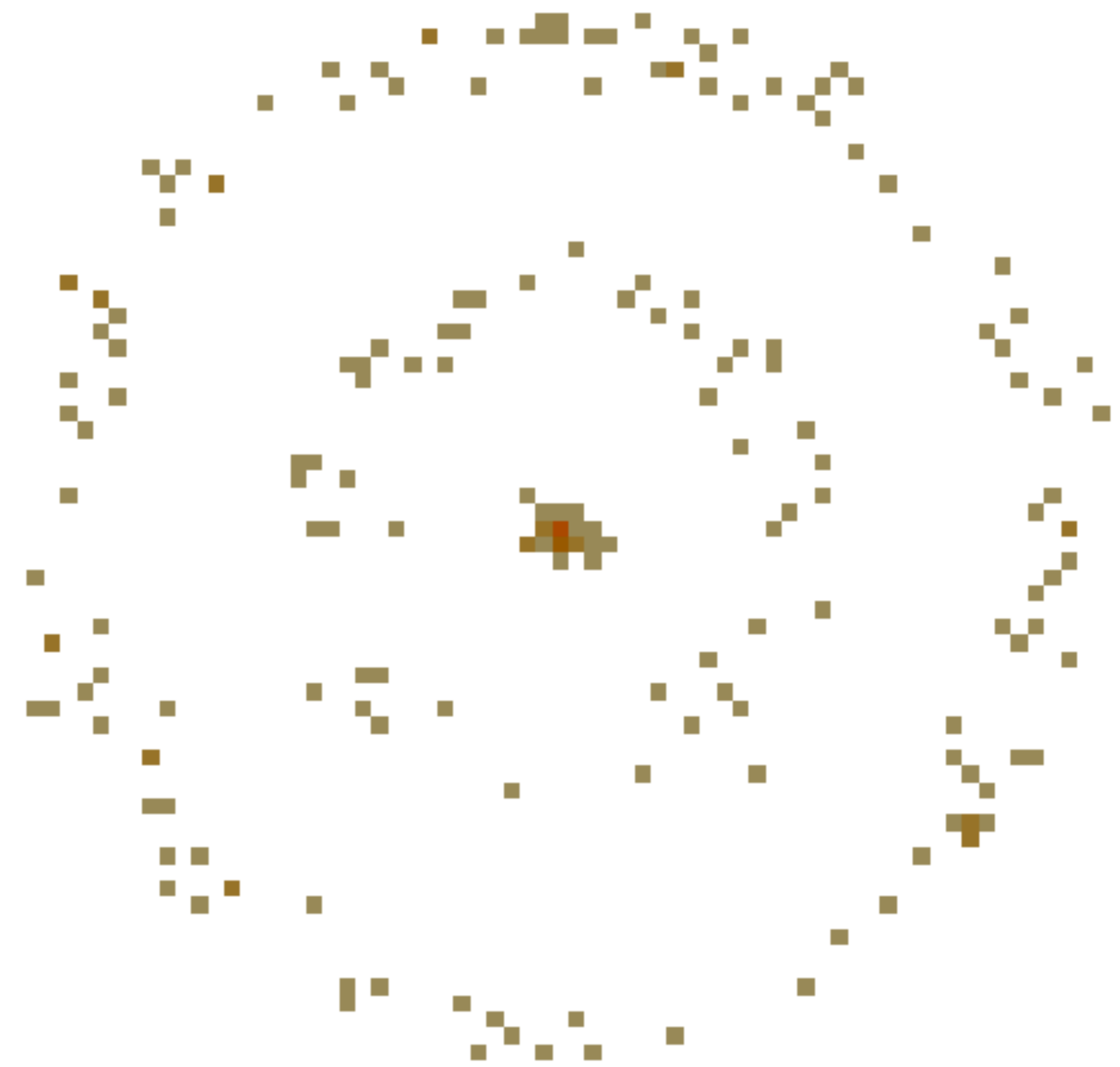


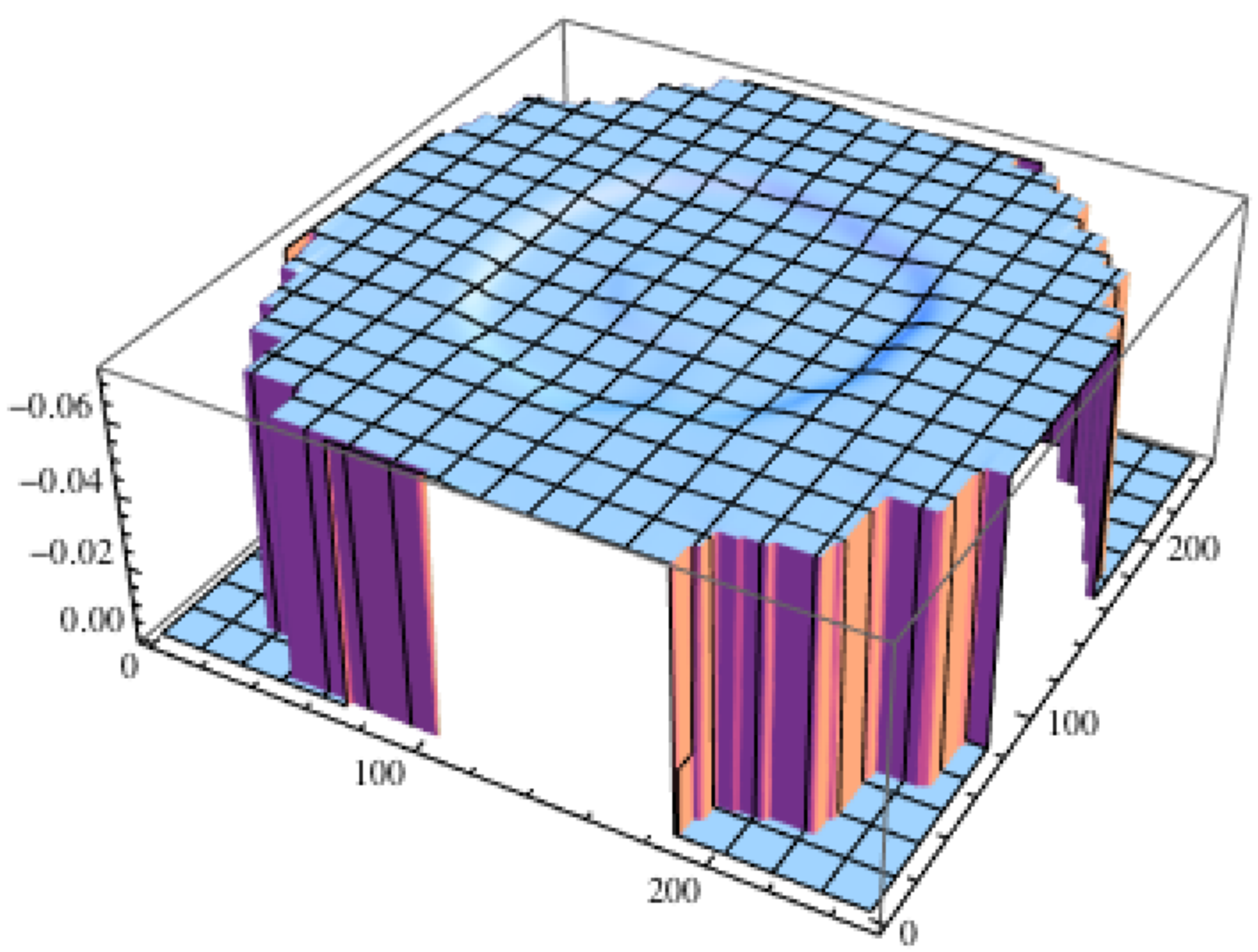
$X_5 =$

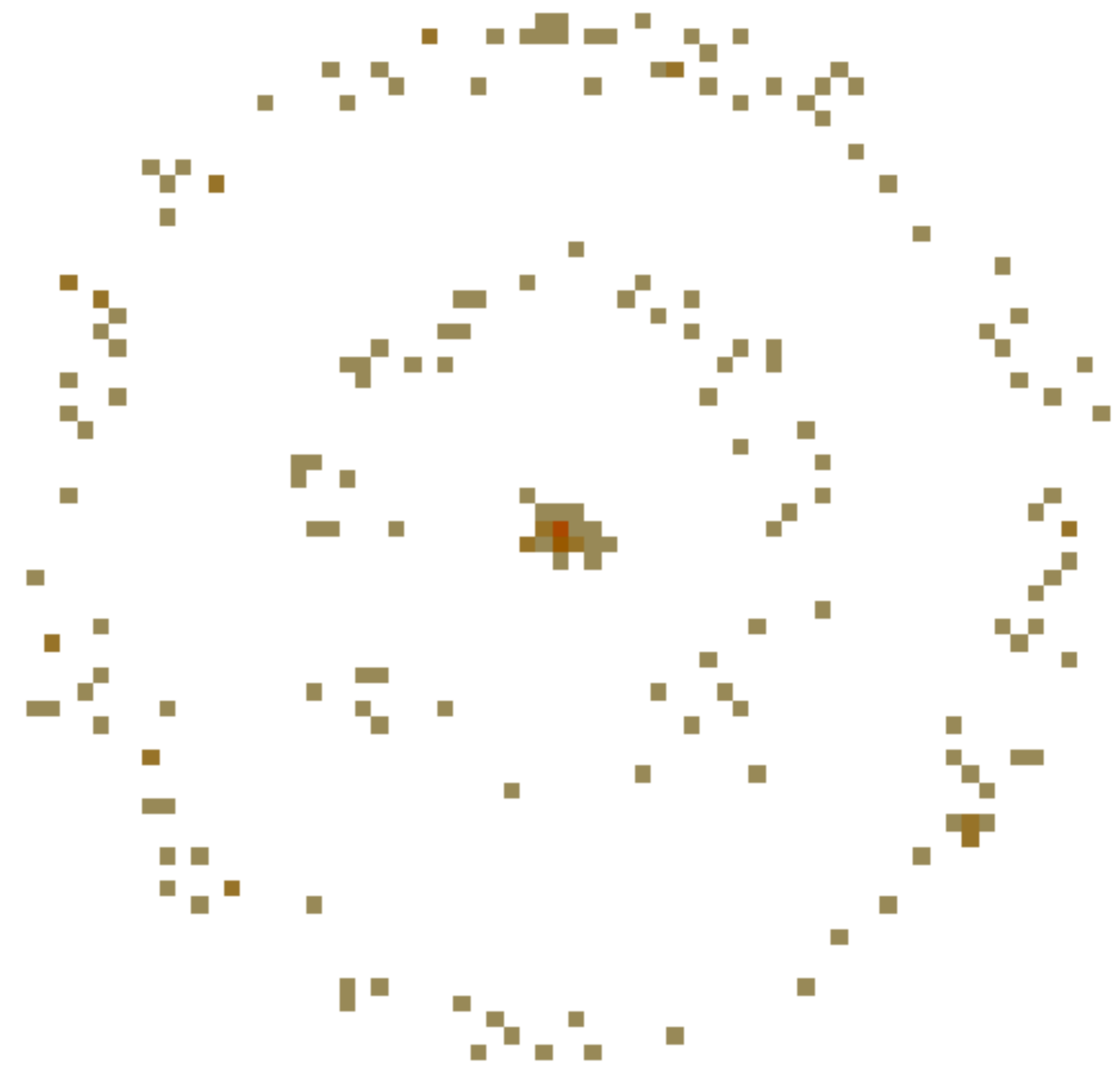


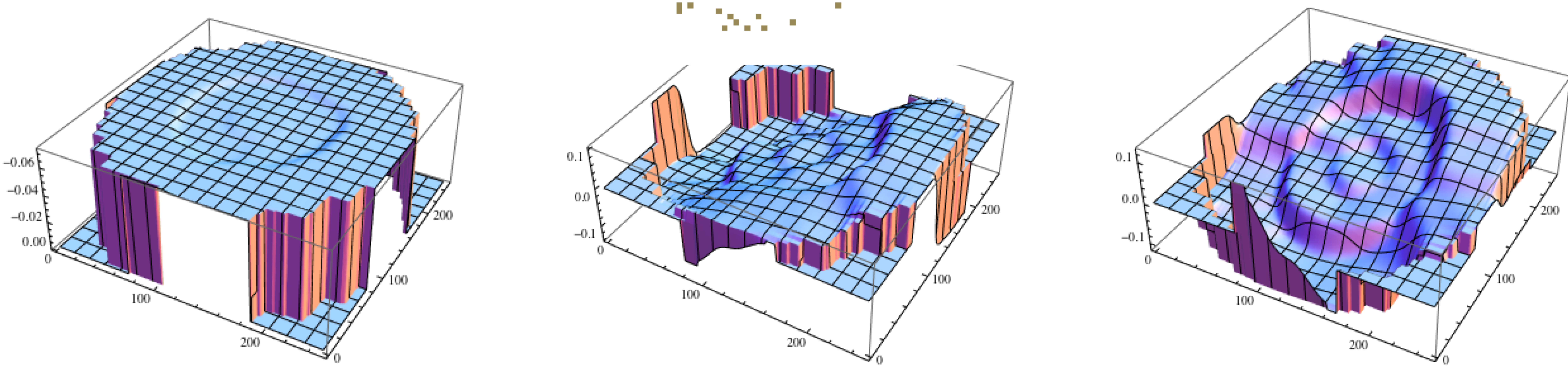
$\rho =$ 

$$\rho =$$



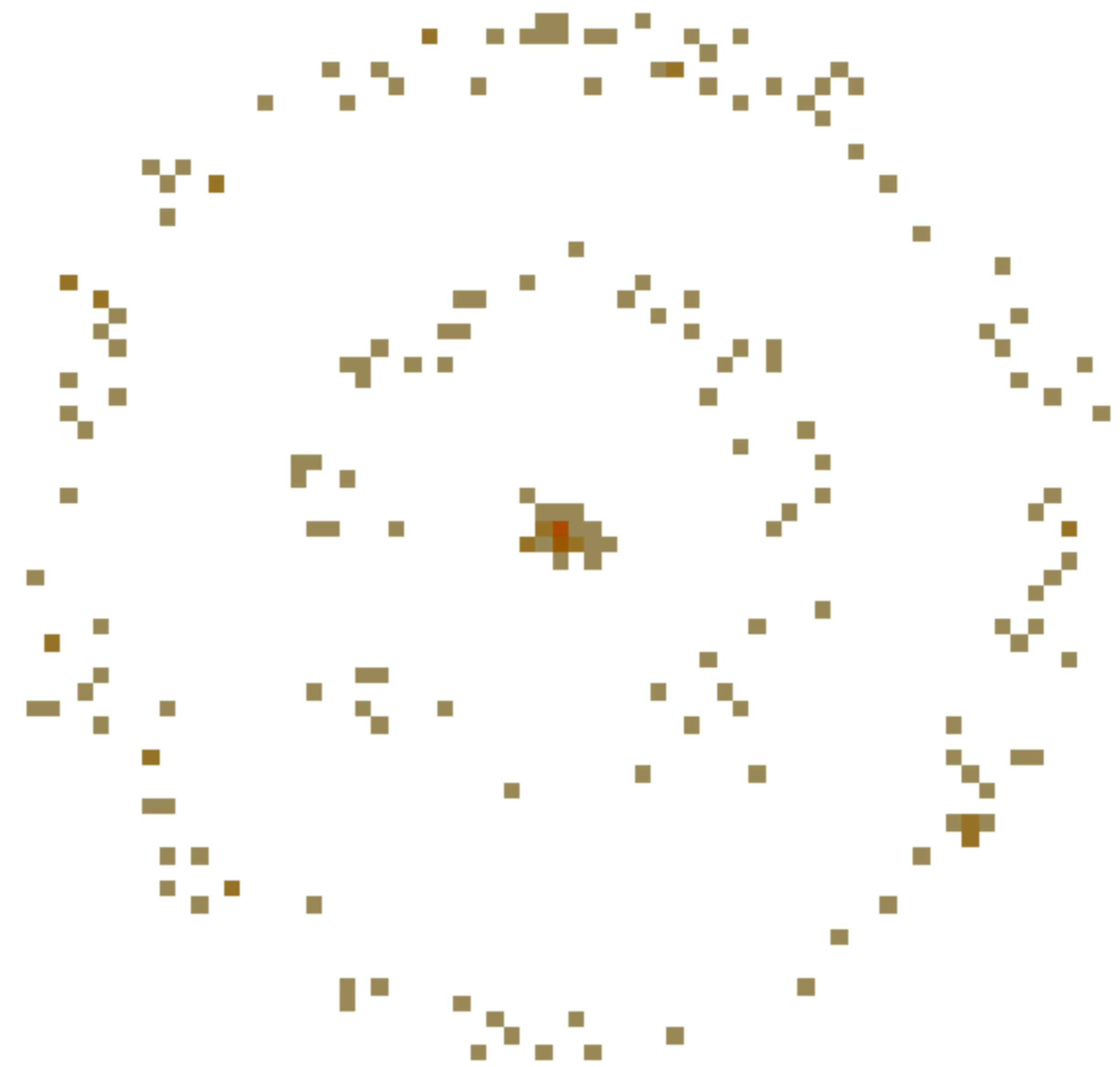


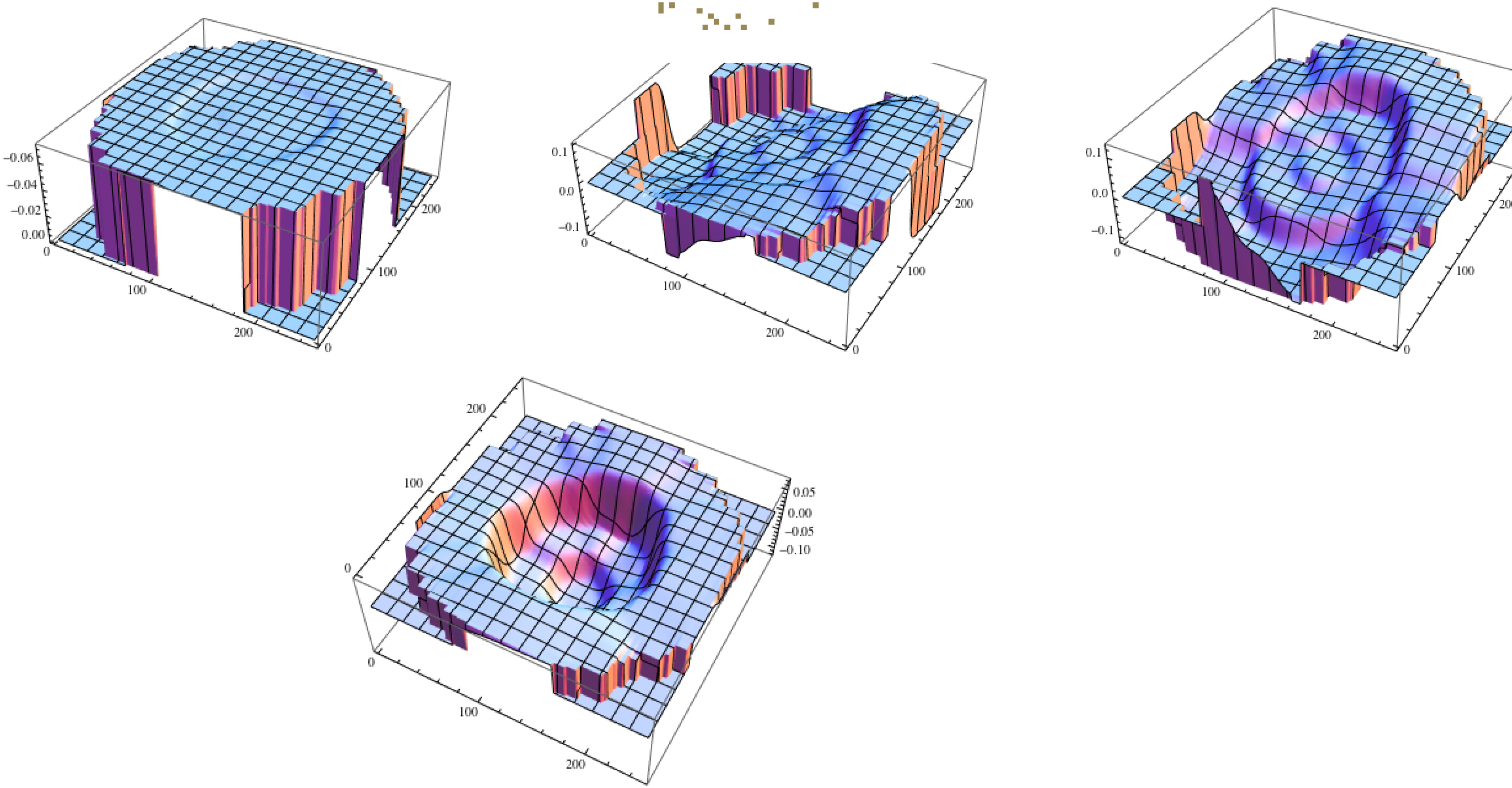
$\rho =$ 



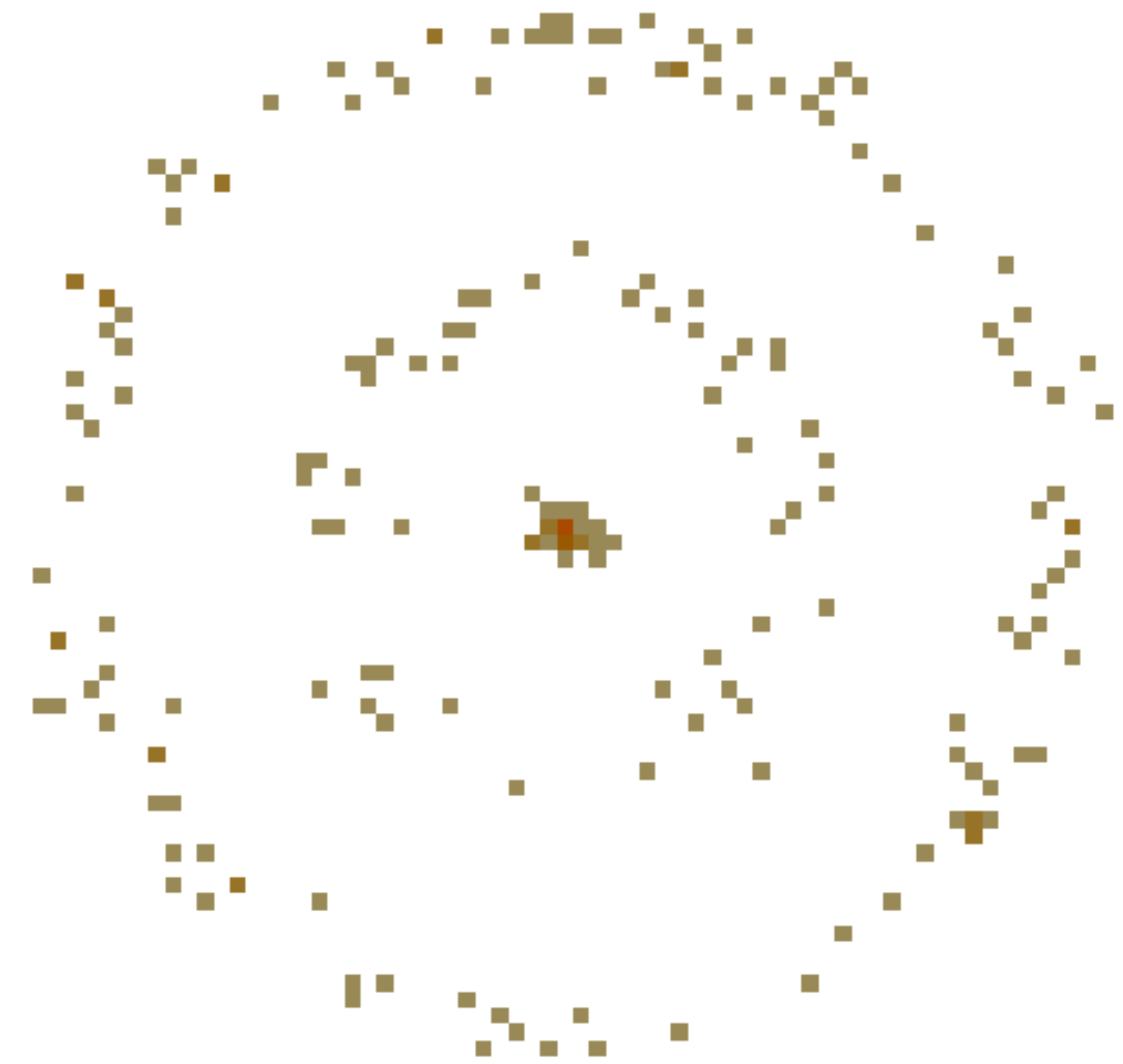
−0.06
−0.04
−0.02
0.00
0
100
200
0
100
200
0.1
0.0
−0.1
0
100
200
0
100
200
0.1
0.0
−0.1
0
100
200
0
100
200

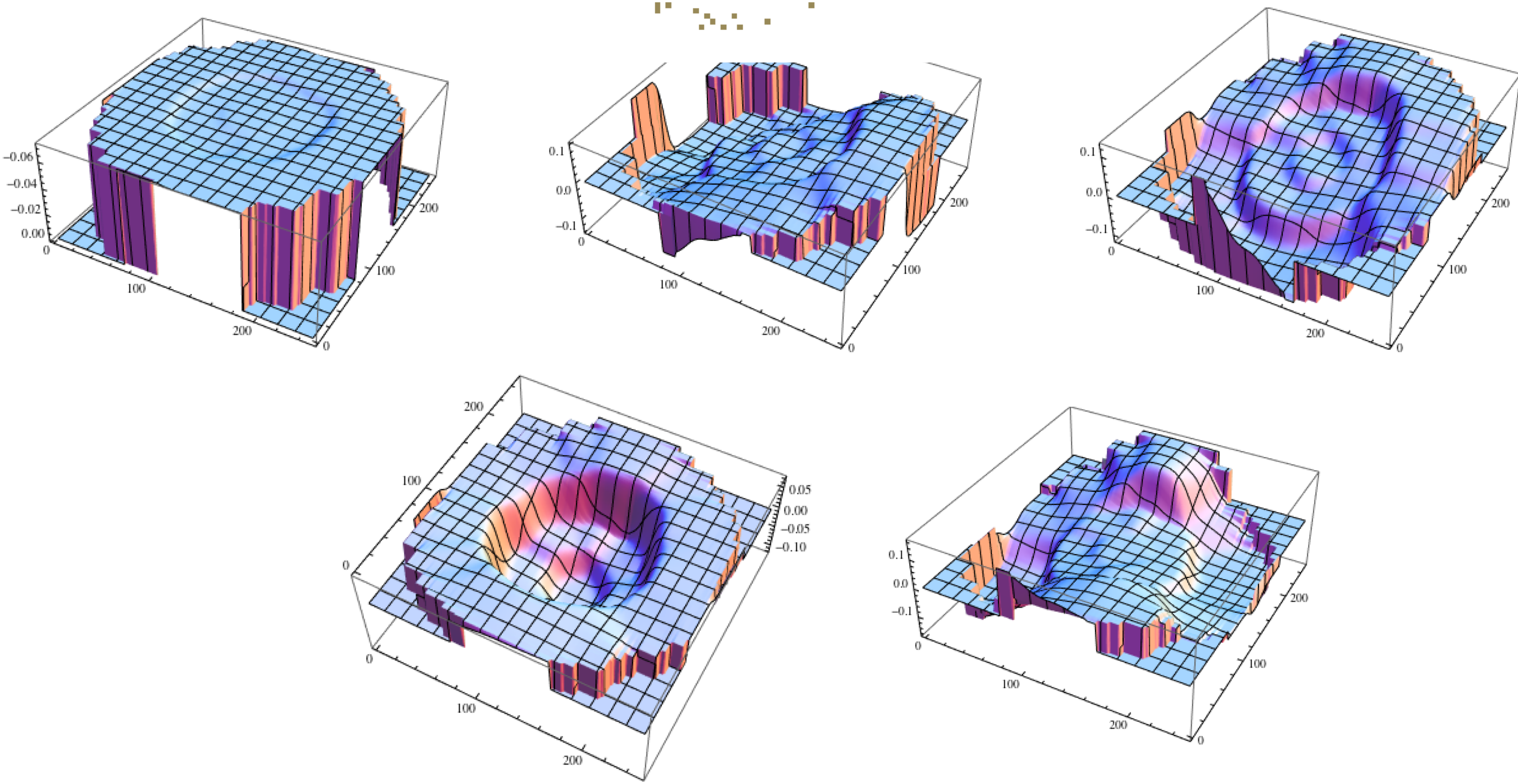
$\rho =$





$$\rho =$$





−0.06
−0.04
−0.02
0.00
0
100
200
100
200
0
0.1
0.0
−0.1
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100
200
100
200
0
0.1
0.0
−0.1
0
100
200
100
200
0
200
100
0
0.05
0.00
−0.05
−0.10
0
100
200
0.1
0.0
−0.1
0
100
200
100
200
0

data

$x_i \in \{$

$\dots \}$

"kernel PCA"

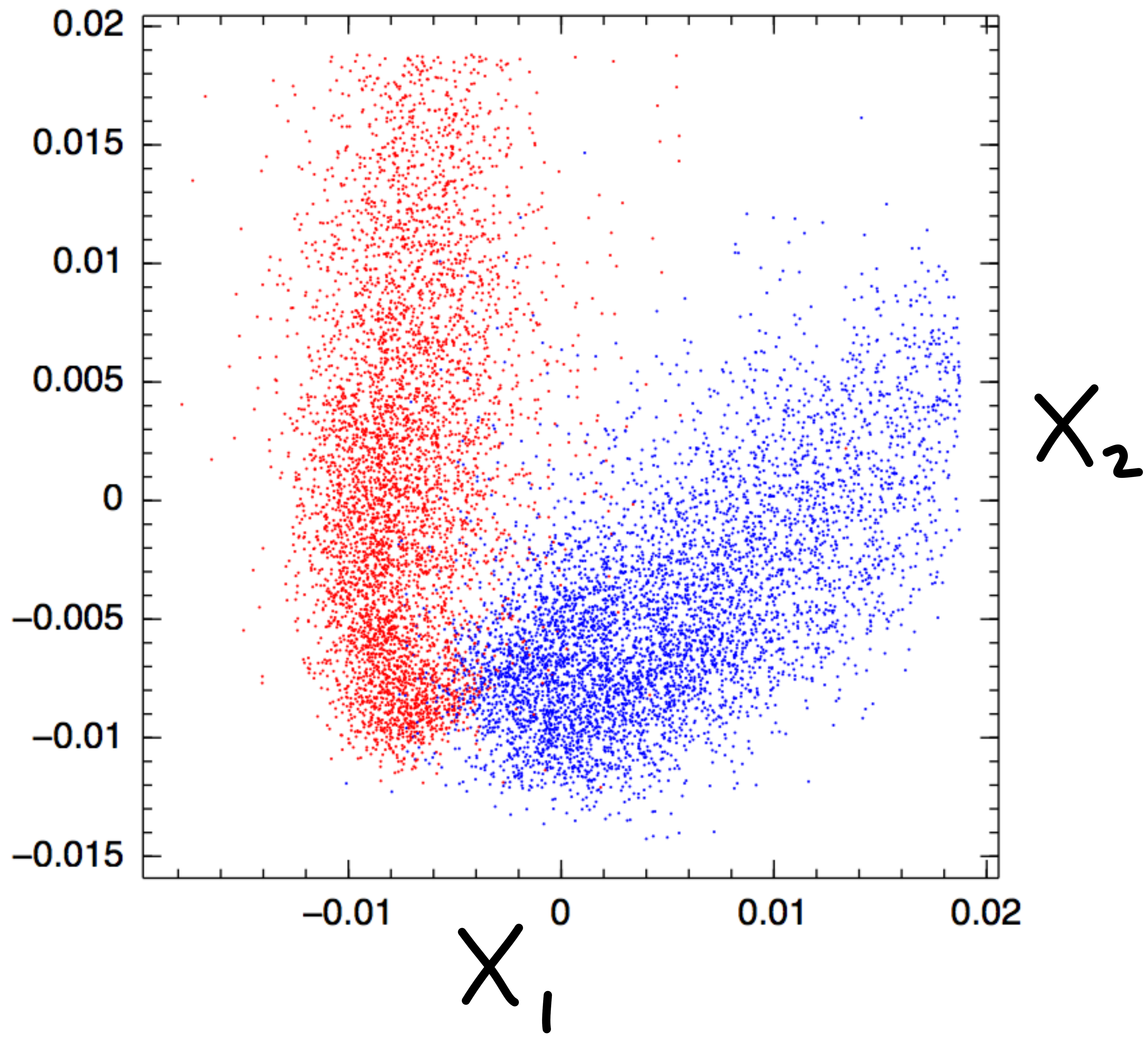
$$K_{ij} = \frac{1}{N} \langle \delta_{x_i} , N^{*\rho} \circ N(\delta_{x_j}) \rangle_\rho$$

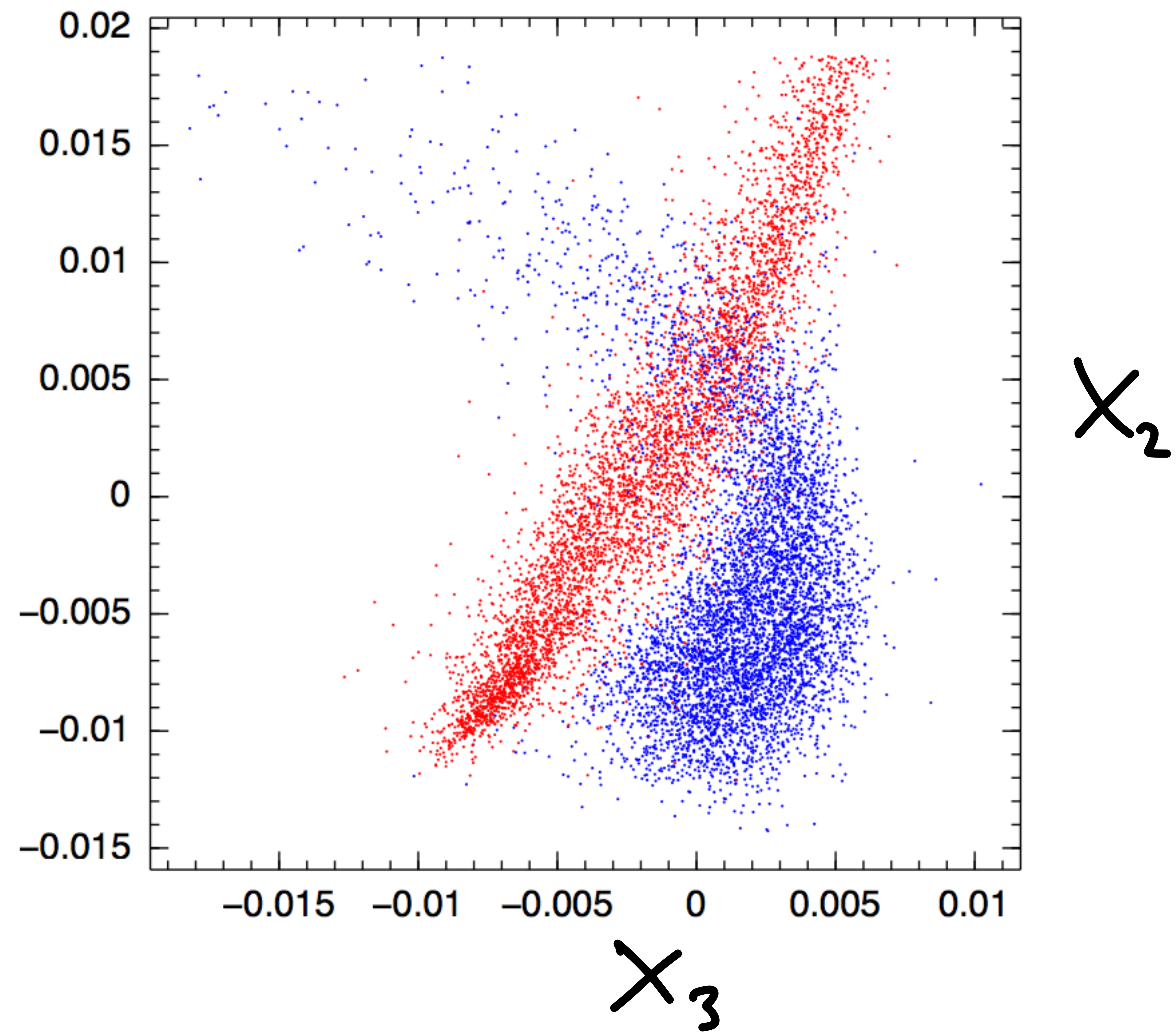
"kernel PCA"

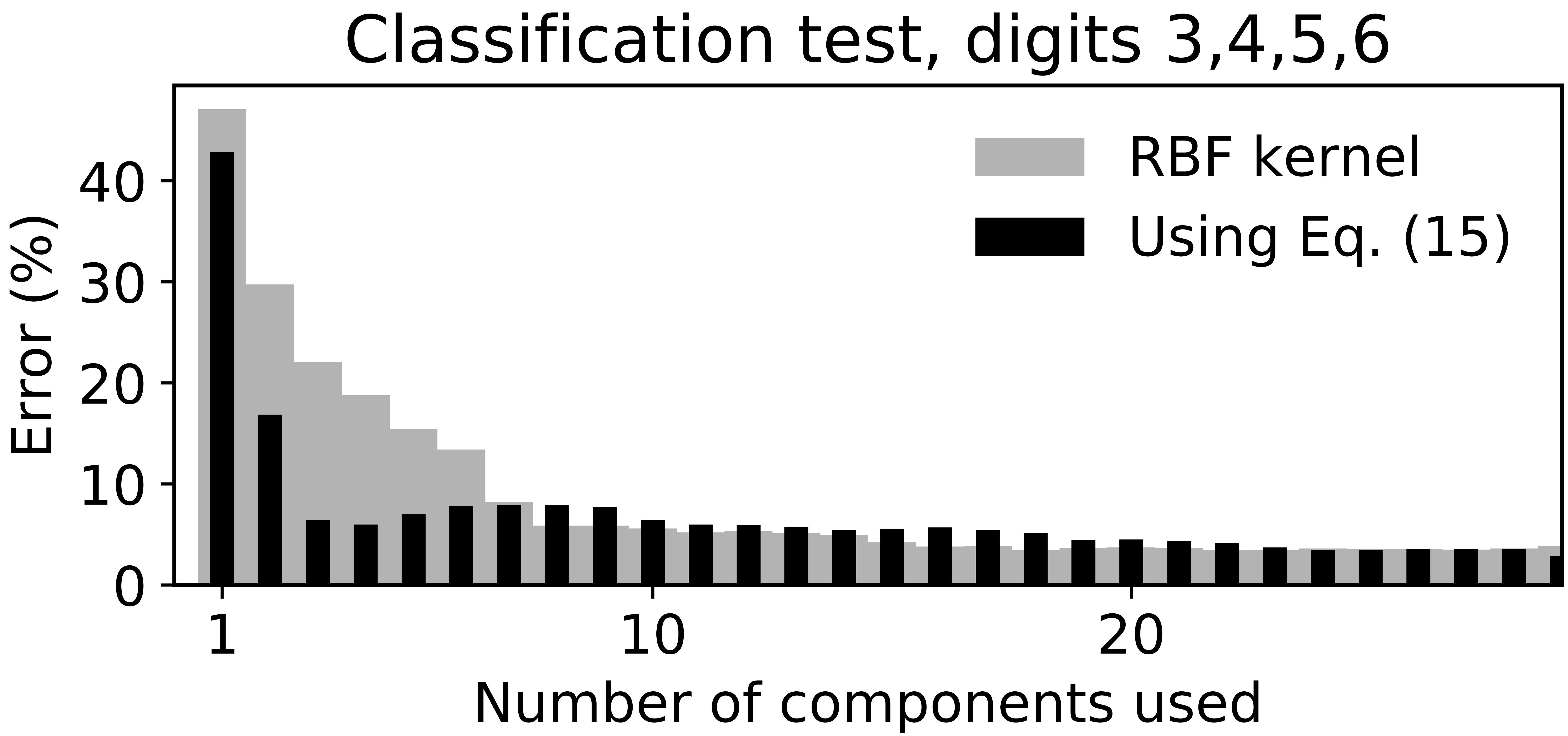
$$K_{ij} = \frac{1}{N} \left\langle \delta_{x_i}, N^{*\rho} \circ N(\delta_{x_j}) \right\rangle_\rho$$

$$= \int dx \, \frac{p(x|x_i)\, p(x|x_j)}{\sum_k p(x|x_k)} = \left\langle \frac{p(x|x_j)}{\sum_k p(x|x_k)} \right\rangle_{p(x|x_i)}$$

Digits 5, 6


0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.01
0
0.01
0.02
X_2
X_1


0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.015
−0.01
−0.005
0
0.005
0.01
X_2
X_3



arXiv: 1802.05756

Questions?