
Computation of the propagator of a multiphoton vertex in the form of an algebraic sum of mean values

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 - 1. Пасько А.И. Скалозуб В.В. Ефективна електромагнітна взаємодія у щільному ферміонному середовищі
 - 2. Reznikov E.V., Skalozub V.V. Effective Three-photon Vertex in a Dense Fermionic Medium
 - 3. V. V. Skalozub and A. Yu. Tishchenko The polarization operator and the three-photon vertex in QED₂₊₁, in a dense medium
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Introduction

The study of nonlinear processes in dense media often encounters the problem of the "purity" of the medium: when creating a medium, in addition to the chemical potential, powerful magnetic fields arise, up to 10^{15} T, and possibly a spontaneous appearance of a chromomagnetic field.

To consider such cases, it is required to calculate the vertex function taking into account the presence of strong fields in a dense medium.

However, it is the combination of factors acting in the environment that can give the most clear "signals" for the formation of the environment, as well as allow one to explore its properties.

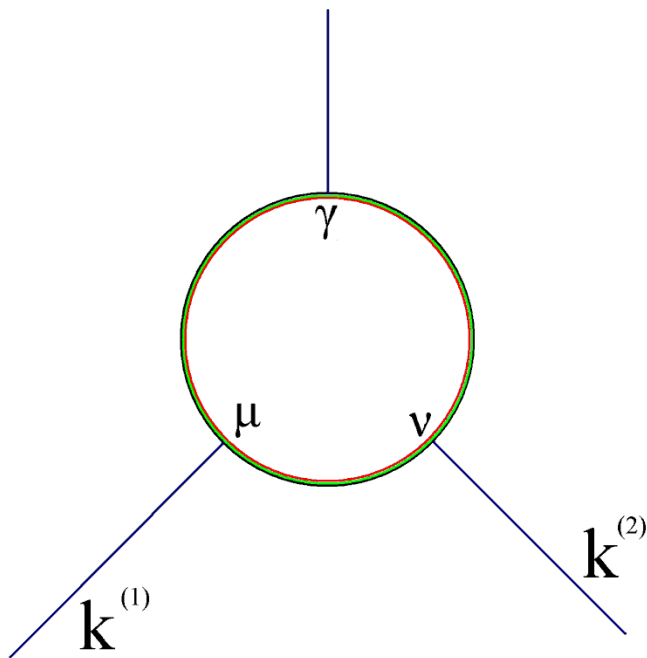
At the same time, a direct analytic solution of a vertex function in the general case is an extremely laborious task, coupled with a significant error probability. To eliminate this problem, we should use the replacement of integration over the fermion line by an algebraic procedure for finding the mean value.

The form of a three-photon diagram and the exact fermion propagator in a dense medium and in the presence of a chromomagnetic field.

The tensor of the vertex function has the following form

$$\Pi_{\mu\nu\gamma}(k, k', k'') = \delta(k + k' + k'') \frac{e^3}{(2\pi)^3} \int dp^4 (\gamma_\mu G(p+k) \gamma_\nu G(p-k') \gamma_\gamma G(p) + \gamma_\mu G(p) \gamma_\nu G(p+k') \gamma_\gamma G(p-k''))$$

Diagram in the medium

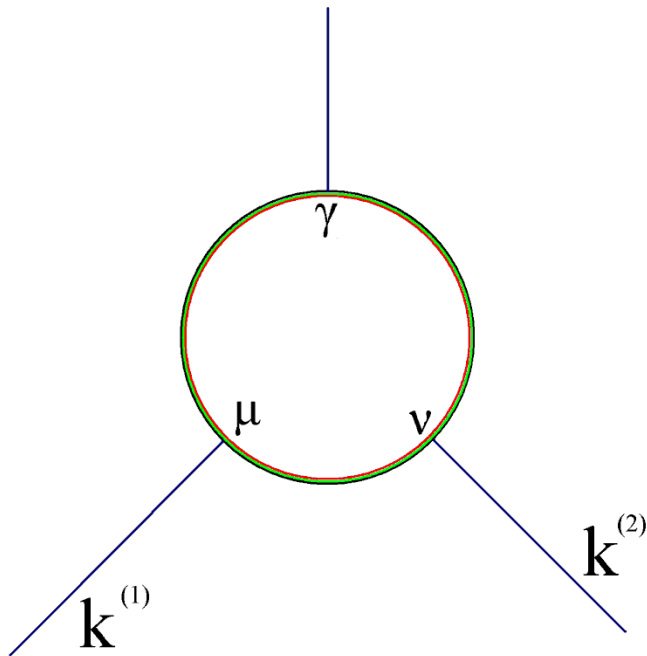


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The momentum of fermion in the Green functions, for a dense medium with a chromomagnetic field, is given by

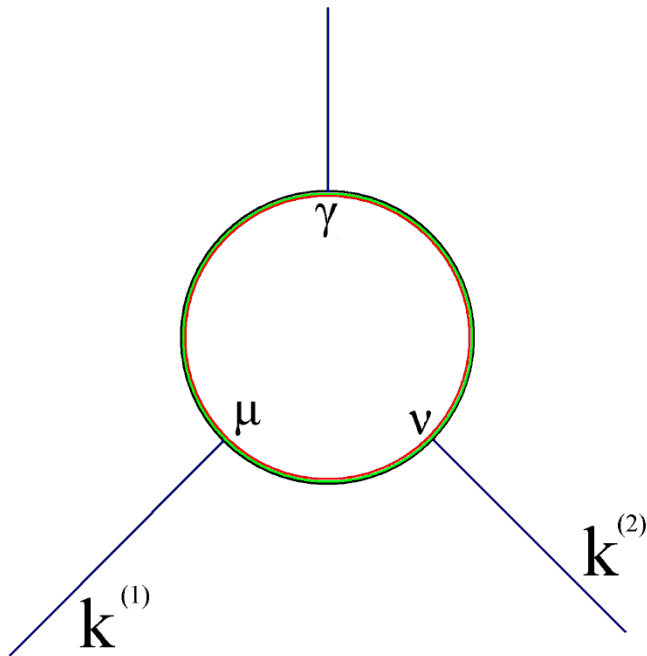
$$p_\mu = \begin{cases} p_i + ieA_i, i = 1..3 \\ p_4 + ieA_4 + i\mu \end{cases}, A_\mu = \bar{A}_\mu + \frac{\lambda^a}{2} \bar{A}_\mu^a$$

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then the Green's function takes the form

$$G(p) = \frac{-i\hat{p} + m}{\left((i\partial_4 + ieA_4 + i\mu)^2 + (i\partial_j + ieA_j)^2 + m^2 \right)}$$

The tensor of a three-photon vertex in a magnetized quark-gluon plasma.

General form of the vertex tensor

$$\Pi_{\mu\nu\gamma} = \frac{ie^3}{2\pi^3\beta} \left(\int \frac{d^3p}{[p^2+m^2]} \left(\frac{f_{\mu\nu\gamma}}{[(p+k^{(1)})^2+m^2][(p-k^{(2)})^2+m^2]} + \frac{\tilde{f}_{\mu\nu\gamma}}{[(p-k^{(1)})^2+m^2][(p+k^{(2)})^2+m^2]} \right) \right)$$

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function $f_{\mu\nu\gamma}$ depends on the variables $(k^{(1)}, k^{(2)}, p, m)$ and has the form

$$f^{(1)}_{\mu\nu\gamma}(k^{(1)}, k^{(2)}, p, m) = \int dp_4 \left(\begin{aligned} & \left[(\mathbf{p} - \mathbf{k}^{(2)}) \mathbf{p} + m^2 \right] \left[(p_\gamma + k_\gamma^{(1)}) \delta_{\mu\nu} + (p_\mu + k_\mu^{(1)}) \delta_{\nu\gamma} + (p_\nu + k_\nu^{(1)}) \delta_{\mu\gamma} \right] - \\ & \left[(\mathbf{p} + \mathbf{k}^{(1)}) \mathbf{p} + m^2 \right] \left[(p_\gamma - k_\gamma^{(1)}) \delta_{\mu\nu} - (p_\mu - k_\mu^{(2)}) \delta_{\nu\gamma} + (p_\nu - k_\nu^{(2)}) \delta_{\mu\gamma} \right] + \\ & \left[(\mathbf{p} + \mathbf{k}^{(1)}) (\mathbf{p} - \mathbf{k}^{(2)}) + m^2 \right] \left[p_\gamma \delta_{\mu\nu} - p_\mu \delta_{\nu\gamma} + p_\nu \delta_{\mu\gamma} \right] + \\ & \left[p_\nu + k_\nu^{(1)} \right] \left[(p_\gamma - k_\gamma^{(2)}) p_\mu - (p_\mu - k_\mu^{(2)}) p_\gamma \right] + \\ & \left[p_\gamma + k_\gamma^{(1)} \right] \left[(p_\nu - k_\nu^{(2)}) p_\mu - (p_\mu - k_\mu^{(2)}) p_\nu \right] + \\ & \left[p_\mu + k_\mu^{(1)} \right] \left[(p_\gamma - k_\gamma^{(2)}) p_\nu - (p_\nu - k_\nu^{(2)}) p_\gamma \right] \end{aligned} \right)$$

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Summation by p_4 in the function $f_{\mu\nu\gamma}$ in case of the presence of temperature occurs according to the formula

$$\frac{1}{\beta} \sum_{p_4} F(\dots p_4) = \frac{1}{\beta} \sum_n F\left(\dots \frac{(2n+1)\pi}{\beta}\right) = -\frac{1}{\beta} \sum_k f(z_k) \operatorname{Re} s(F(\dots z_k))$$

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Where $p_4 = (2n+1)\pi/\beta$ and $p_4 = z_k$ is the position of the poles of the function F , the function $f(z)$ is chosen in such a way that it is analytic everywhere except for points $p_4 = z_k = (2n+1)\pi/\beta$ at which poles with residues equal to one exist and tend to zero at infinity. These conditions are satisfied by a function of the form

$$f(z) = \frac{-i\beta}{1 + e^{i\beta z}}$$

The tensor of a three-photon vertex in a magnetized quark-gluon plasma.

Calculating the residues of the function, we obtain three groups of terms, combined by common factors

$$H_1 = \frac{\frac{1}{1 + e^{\beta(\varepsilon_0 - eA_4 - \mu)}} - \frac{1}{1 + e^{\beta(\varepsilon_0 + eA_4 + \mu)}}}{2i\varepsilon_0 \left[2((\mathbf{p}_j + eA_j)\mathbf{k}^{(1)}_j + i\varepsilon_0\mathbf{k}^{(1)}_4) + (\mathbf{k}^{(1)}_j)^2 + (\mathbf{k}^{(1)}_4)^2 \right] \left[-2((\mathbf{p}_j + eA_j)\mathbf{k}^{(2)}_j + i\varepsilon_0\mathbf{k}^{(2)}_4) + (\mathbf{k}^{(2)}_j)^2 + (\mathbf{k}^{(2)}_4)^2 \right]}$$

$$H_2 = \frac{\frac{1}{1 + e^{\beta(\varepsilon_1 - eA_4 - \mu)}} - \frac{1}{1 + e^{\beta(\varepsilon_1 + eA_4 + \mu)}}}{2i\varepsilon_1 \left[2((\mathbf{p}_j + eA_j)\mathbf{k}^{(1)}_j + i\varepsilon_1\mathbf{k}^{(1)}_4) - (\mathbf{k}^{(1)}_j)^2 + (\mathbf{k}^{(1)}_4)^2 \right] \left[-2((\mathbf{p}_j + eA_j)(\mathbf{k}^{(1)}_j + \mathbf{k}^{(2)}_j) + i\varepsilon_1(\mathbf{k}^{(1)}_4 + \mathbf{k}^{(2)}_4)) + (\mathbf{k}^{(2)}_j)^2 - (\mathbf{k}^{(1)}_j)^2 + (\mathbf{k}^{(1)}_4 + \mathbf{k}^{(2)}_4)^2 \right]}$$

$$H_3 = \frac{\frac{1}{1 + e^{\beta(\varepsilon_2 - eA_4 - \mu)}} - \frac{1}{1 + e^{\beta(\varepsilon_2 + eA_4 + \mu)}}}{2i\varepsilon_2 \left[2((\mathbf{p}_j + eA_j)(\mathbf{k}^{(1)}_j + \mathbf{k}^{(2)}_j) + i\varepsilon_2(\mathbf{k}^{(1)}_4 + \mathbf{k}^{(2)}_4)) + (\mathbf{k}^{(1)}_j)^2 - (\mathbf{k}^{(2)}_j)^2 + (\mathbf{k}^{(1)}_4 + \mathbf{k}^{(2)}_4)^2 \right] \left[2((\mathbf{p}_j + eA_j)\mathbf{k}^{(2)}_j + i\varepsilon_2\mathbf{k}^{(2)}_4) - (\mathbf{k}^{(2)}_j)^2 + (\mathbf{k}^{(2)}_4)^2 \right]}$$

где

$$\varepsilon_0 = \left((\mathbf{p}_j + eA_j)^2 + m^2 \right)^{1/2}, \varepsilon_1 = \left((\mathbf{p}_j + eA_j + \mathbf{k}^{(1)}_j)^2 + m^2 \right)^{1/2}, \varepsilon_2 = \left((\mathbf{p}_j + eA_j - \mathbf{k}^{(2)}_j)^2 + m^2 \right)^{1/2}$$

$j = 1..3$

The tensor of a three-photon vertex in a magnetized quark-gluon plasma.

The temporal part of the vertex tensor then takes the form

$$\Pi_{444} = \frac{ie^3}{2\pi^3} \int d^3 p$$

$$\begin{aligned}
 & H_1 \left(4(i\varepsilon_0 - eA_4)^3 + 4(\hat{k}^{(1)}_4 - \hat{k}^{(2)}_4)(i\varepsilon_0 - eA_4)^2 - (i\varepsilon_0 - eA_4)(\hat{k}^{(2)}_4 \hat{k}^{(1)}_4 - k^{(2)}_j k^{(1)}_j) + ((p_j + eA_j)(k^{(2)}_j \hat{k}^{(1)}_4 + k^{(1)}_j \hat{k}^{(2)}_4)) \right) + \\
 & \left. \begin{aligned}
 & + H_2 \left(\begin{aligned}
 & \left(4(i\varepsilon_1 - eA_4 - k^{(1)}_4)^3 + 4(\hat{k}^{(1)}_4 - \hat{k}^{(2)}_4)(i\varepsilon_1 - eA_4 - k^{(1)}_4)^2 \right) - \\
 & - (i\varepsilon_1 - eA_4 - k^{(1)}_4) \left(4(\hat{k}^{(2)}_4 \hat{k}^{(1)}_4) - (3(\mathbf{k}^{(1)})^2 + 2((p_j + eA_j)(2k^{(1)}_j + k^{(2)}_j) - (\varepsilon_0 - k^{(1)}_4)(2k^{(1)}_4 + k^{(2)}_4)) + \mathbf{k}^{(2)} \mathbf{k}^{(1)}) \right) + \\
 & + \left((p_j + eA_j)(k^{(1)}_j + k^{(2)}_j) - (\varepsilon_0 - k^{(1)}_4)(k^{(1)}_4 + k^{(2)}_4) \right) \hat{k}^{(1)}_4 + \left((\mathbf{k}^{(1)})^2 + (p_j + eA_j)k^{(1)}_j - (\varepsilon_0 - k^{(1)}_4)k^{(1)}_4 \right) (k^{(1)}_4 - k^{(2)}_4) \right) \right) + \\
 & + H_3 \left(\begin{aligned}
 & \left(4(i\varepsilon_2 - eA_4 + k^{(2)}_4)^3 + 4(\hat{k}^{(1)}_4 - \hat{k}^{(2)}_4)(i\varepsilon_2 - eA_4 + k^{(2)}_4)^2 \right) - \\
 & - (i\varepsilon_2 - eA_4 + k^{(2)}_4) \left(4\hat{k}^{(2)}_4 \hat{k}^{(1)}_4 - (3(\mathbf{k}^{(1)})^2 + 2((p_j + eA_j)(2k^{(2)}_j + k^{(1)}_j) - (\varepsilon_0 + k^{(2)}_4)(2k^{(2)}_4 + k^{(1)}_4)) + \mathbf{k}^{(2)} \mathbf{k}^{(1)}) \right) - \\
 & - \left((p_j + eA_j)k^{(2)}_j - (\varepsilon_0 + k^{(2)}_4)k^{(2)}_4 \right) - (\mathbf{k}^{(2)})^2 \hat{k}^{(1)}_4 - \left((p_j + eA_j)(2k^{(2)}_j + k^{(1)}_j) - (\varepsilon_0 + k^{(2)}_4)(2k^{(2)}_4 + k^{(1)}_4) \right) - (\mathbf{k}^{(2)})^2 \hat{k}^{(2)}_4 \right) \right)
 \end{aligned} \right)
 \end{aligned}
 \right)
 \end{aligned}$$

The tensor of a three-photon vertex in a magnetized quark-gluon plasma.

The spatial part of the vertex tensor takes the form

$$\Pi_{ijl} = \frac{ie^3}{\pi^3} \int d^3 p \left(\begin{aligned} & \left[H_1 \mathbf{pk}^{(2)} - H_2 (\mathbf{pk}^{(2)} + (\mathbf{k}^{(1)})^2 + 2\mathbf{pk}^{(1)}) - H_3 ((\mathbf{k}^{(2)})^2 - \mathbf{pk}^{(2)}) \right] \left[(p_l + eA_l + k_l^{(1)}) \delta_{ij} + (p_i + eA_i + k_i^{(1)}) \delta_{jl} + (p_j + eA_j + k_j^{(1)}) \delta_{il} \right] - \\ & - \left[H_1 \mathbf{pk}^{(1)} - H_2 ((\mathbf{k}^{(1)})^2 + \mathbf{pk}^{(1)}) - H_3 ((\mathbf{k}^{(2)})^2 - 2\mathbf{pk}^{(2)} - \mathbf{pk}^{(1)}) \right] \left[(p_l + eA_l - k_l^{(1)}) \delta_{ij} - (p_i + eA_i - k_i^{(2)}) \delta_{jl} + (p_j + eA_j - k_j^{(2)}) \delta_{il} \right] + \\ & + \left[\begin{aligned} & H_1 (\mathbf{pk}^{(1)} - \mathbf{pk}^{(2)} - \mathbf{k}^{(2)} \mathbf{k}^{(1)}) - \\ & - H_2 ((\mathbf{k}^{(1)})^2 + \mathbf{pk}^{(1)} + \mathbf{pk}^{(2)} + \mathbf{k}^{(1)} \mathbf{k}^{(2)}) - \\ & - H_3 ((\mathbf{k}^{(1)})^2 + \mathbf{pk}^{(1)} - \mathbf{pk}^{(2)} + \mathbf{k}^{(1)} \mathbf{k}^{(2)}) \end{aligned} \right] \left[(p_l + eA_l) \delta_{ij} - (p_i + eA_i) \delta_{jl} + (p_j + eA_j) \delta_{il} \right] + \\ & \left[H_1 + H_2 + H_3 \right] \left[\begin{aligned} & \left[p_j + eA_j + k_j^{(1)} \right] \left[(p_l + eA_l - k_l^{(1)}) (p_i + eA_i) - (p_i + eA_i - k_i^{(2)}) (p_l + eA_l) \right] + \\ & + \left[p_l + eA_l + k_l^{(1)} \right] \left[(p_j + eA_j - k_j^{(2)}) (p_i + eA_i) - (p_i + eA_i - k_i^{(2)}) (p_j + eA_j) \right] + \\ & + \left[p_i + eA_i + k_i^{(1)} \right] \left[(p_l + eA_l - k_l^{(1)}) (p_j + eA_j) - (p_j + eA_j - k_j^{(2)}) (p_l + eA_l) \right] \end{aligned} \right] \end{aligned} \right)$$

Replacement of integration over the fermion line by an algebraic procedure for finding the mean value

At first we use an exponential representation for the combination of three propagation functions:

$$\frac{1}{k_1^2} \frac{1}{(p-k_1)^2 + m^2 - eq\sigma F} \frac{1}{(p-k_1+k_2)^2 + m^2 - eq\sigma F} = -\int_0^\infty s ds \int_0^1 du e^{-is\chi(u)}$$
$$\chi(u) = (k_1 - k_2 - up)^2 + u(1-u)[m^2 - (\gamma p)^2] + u^2(m^2 - eq\sigma F)$$
$$\sigma F = \frac{1}{2} \sigma^{\mu\nu} F_{\mu\nu}$$

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At second - the transformation of the k integration into a matrix form by the use of a variable ξ that is complementary to k

$$[\xi_\mu, k_\nu] = ig_{\mu\nu} \text{ and by observing that } (\xi' \text{ is the eigenvalue of } \xi) \quad \int \frac{(dk)}{(2\pi)^4} f(k) = \langle \xi' = 0 | f(k) | \xi' = 0 \rangle$$

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For a fermion line, between two photon lines, the cross section will have the form

$$M^{(0)} = \frac{\alpha}{2\pi} m \left[-\frac{eH}{2m^2} + \left(\frac{eH}{m^2}\right)^2 \left(\frac{4}{3} \ln \frac{m^2}{2eH} + \frac{13}{18} \right) + \left(\frac{eH}{m^2}\right)^3 \left(\frac{14}{3} \ln \frac{m^2}{2eH} + \frac{32}{5} \ln 2 + \frac{83}{90} \right) \right]$$

Conclusions

- At creating a real dense medium, strong fields will be formed in it, presence of which can significantly distort the effects that occur when a chemical potential exists in a "clean" medium.
- In addition, the presence of a nonzero temperature of the medium will also affect the behavior of the vertex tensor
- At the same time, accounting for effects generated by a combination of strong fields, temperature and chemical potential, will help describe new processes taking place in real dense media.
- To obtain an analytical solution in the general case, we should use the replacement of integration over the fermion line by an algebraic procedure for finding the mean value.

■ Thank for your attention