Alternative way to understand the unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio

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Preface

The basis of my talk are the results of our joint with professor E.A. Kuraev papers, which were published in [1-4].

[1]. M.V. Galynskii, E.A. Kuraev, Alternative way to understand the unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio, Phys. Rev. D, **89**, 054005 (2014).

[2]. M.V. Galynskii, E.A. Kuraev, On the Physical Meaning of Sachs Form Factors and on the Violation of the Dipole Dependence of G_E and G_M on Q^2, JETP Lett. **96**, 6 (2012).

[3]. M. V. Galynskii, E. A. Kuraev, and Yu. M. Bystritskiy, Possible method for measuring the proton form factors in processes with and without proton spin flip, JETP Lett. **88**, 481 (2008).

[4]. M.V. Galynskii, E.A. Kuraev, arXiv: 1210.0634 [nucl-th].

Матричные элементы и сечение процесса $e\vec{p} \to e\vec{p}$

$$e(p_1) + p(q_1, s_1) \to e(p_2) + p(q_2, s_2) \tag{1}$$

$$M_{ep\to ep} = \frac{4\pi\alpha\, T}{q^2}\,,\ T = (J_e)_\mu (J_p)^\mu . \tag{2}$$

Матричные элементы электронного и протонного токов $(J_e)_\mu$ и $(J_p)^\mu$

$$(J_e)_\mu = \overline{u}(p_2)\gamma^\mu u(p_1)\,, \tag{3}$$
$$(J_p)^\mu = \overline{u}(q_2)\Gamma_\mu(q^2)u(q_1)\,, \tag{4}$$
$$\Gamma_\mu(q^2) = F_1\gamma_\mu + \frac{F_2}{4M}(\,\hat{q}\gamma_\mu - \gamma_\mu\hat{q}\,)\,, \tag{5}$$

$\overline{u}(p_i)u(p_i) = 2m,\ \overline{u}(q_i)u(q_i) = 2M,\ p_i^2 = m^2,\ q_i^2 = M^2 (i = 1, 2)$;
F_1 и F_2 – дираковский и паулиевский формфакторы протона,
$q = q_- = q_2 - q_1$ – 4-импульс, переданный протону;
s_1 и s_2 – 4-векторы поляризации начального и конечного протонов.

Дифференциальное сечение процесса (10) имеет вид:

$$\frac{d\sigma}{dQ^2} = \frac{\pi\alpha^2}{4I^2}\,\frac{|T|^2}{q^4}, \quad I^2 = (p_1 q_1)^2 - m^2 M^2\,. \tag{6}$$

The Rosenbluth formula in the arbitrary reference frame

The Rosenbluth formula in the arbitrary reference frame read as:

$$d\sigma = \frac{\alpha^2 do}{4I^2}\frac{1}{1+\tau_p}\left(\, G_E^2\, Y_I + \tau\, G_M^2\, Y_{II}\,\right)\frac{1}{q^4}\,, \tag{7}$$

$$Y_I = (p_+q_+)^2 + q_+^2 q^2, \quad Y_{II} = (p_+q_+)^2 - q_+^2(q^2+4m^2)\,,$$

$$p_+ = p_1 + p_2\,,\; q_+ = q_1 + q_2\,,\; I^2 = (p_1q_1)^2 - m^2M^2.$$

The Rosenbluth formulas in an arbitrary reference frame (7) as well as in the laboratory reference frame are expressed only through the squares of the form factors (FFs) Sachs G_E^2 and G_M^2.

It is the question arises: whether there is any physical meaning in the decomposition of G_E^2 and G_M^2 in Rosenbluth's cross section?
[A.I. Akhiezer and V.B. Berestetsky, Quantum Electrodynamics, Nauka, Moscow, 1969, in Russian, eq.(34.3.3), page 475]

1. Rosenbluth Method or Rosenbluth Technique

In elastic electron proton scattering $e(p_1) + p(q_1) \to e(p_2) + p(q_2)$ there are primarily two methods used to extract the proton form factors. The first method is the Rosenbluth separation method, which uses measurements of the unpolarized cross section and in the laboratory reference frame when $q_1 = (M, \vec{0})$ and $m_e = 0$ in one-photon exchange approximation read as [1]:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_1^3 \sin^4(\theta_e/2)} \frac{1}{1+\tau} \left(G_E^2 + \frac{\tau}{\varepsilon} G_M^2 \right) . \qquad (8)$$

$$G_E = F_1 - \tau F_2 \,, \; G_M = F_1 + F_2 \,. \qquad (9)$$

Here $\tau = Q^2/4M^2$, $Q^2 = -q^2 = 4E_1 E_2 \sin^2(\theta_e/2)$, $q = q_2 - q_1$, $\alpha = 1/137$ - fine structure constant, $\varepsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta_e/2)$, **$\varepsilon$ is the degree of the linear polarization of the virtual photon** [2-4]!

[1]. M. Rosenbluth, Phys. Rev. **79**, 615 (1950)
[2]. N. Dombey, Rev. Mod. Phys. **41**, 236 (1969).
[3]. A. Akhiezer, M. Rekalo, Fiz.Elem.Chast.Atom.Yadra **4**, 662 (1973).
[4]. M. Galynskii and M. Levchuk, Yad. Fiz. **60**, 2028 (1997).

Erroneous terminology (red color)

Citation 1: In electron scattering there are primarily two methods used to extract the proton form factors. The first method is the Rosenbluth or Longitudinal-Transverse (LT) separation method [1], which uses measurements of the unpolarized cross section, and the second is the polarization transfer or polarized target (PT) method, which requires measurement of the spin-dependent cross section.

Citation 1a: ε is the virtual photon longitudinal polarization parameter...

[1]. I. A. Qattan, J. Arrington, A. Alsaad, PRC 91 (2015) no.6, 065203.

Citation 2: ε is the longitudinal polarization of the virtual photon....

[2]. A. J. R. Puckett, E. J. Brash, M. K. Jones [et al],
Polarization Transfer Observables in Elastic Electron-Proton Scattering at $\mathbf{Q^2 = 2.5}$, 5.2, 6.8 and 8.5 GeV$^{\mathbf{2}}$, Phys. Rev. C 96, 055203 (2017),
last revised 10 Aug 2018 (this version, v3)

The correct terminology (red color)

In 2015 I found only one work [1] , where the written words about the physical meaning of the variable ε are absolutely correct....

Citation from [1]: "Let us introduce another set of kinematical variables: Q^2, and the degree of the linear polarization of the virtual photon, ε.

Citation from [2]: ε is the virtual photon transverse polarization

Citation from [3]: ε_T is the virtual photon transverse polarization

[1] G.I. Gakh, E. Tomasi-Gustafsson, Model independent analysis of polarization effects in elastic electron-deuteron scattering in presence of two-photon exchange. Nuclear Physics A 799 (2008), pp. 127–150.

[2] S. Riordan, Weak Neutral Current Studies with Positrons, https://arxiv.org/abs/1712.05314

[3] G.V. Fedotov, Iu.A. Skorodumina, V.D. Burkert, R.W. Gothe, K. Hicks [ea al] (CLAS Collaboration); https://arxiv.org/abs/1804.05136

Формула Розенблюта в ЛСО

$$\sigma_R = \frac{d\sigma}{d\Omega_e} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_1^3 \sin^4(\theta_e/2)} \frac{1}{1+\tau_p} \left(G_E^2 + \frac{\tau_p}{\varepsilon} G_M^2 \right). \tag{10}$$

В учебных пособиях по физике элементарных частиц говорится, что использование формфакторов Сакса (ФФС) является просто удобным, поскольку позволяет записать формулу Розенблюта (10) в простом и компактном виде. Поскольку такие соображения содержатся в том числе и в известных монографиях:
[1]. Ахиезер А.И., Берестецкий В.Б. КЭД (1969),
[2]. В.Б. Берестецкий, Е.М. Лифшиц, Л.П. Питаевский. КЭД (1989),
то они не подвергаются сомнениям и воспроизводятся в литературе вплоть до настоящего времени, например, в диссертации:

http://arxiv.org/abs/1508.01456 (2015),
A.J.R. Puckett, Recoil Polarization Measurements of the Proton Electromagnetic Form Factor Ratio to High Momentum Transfer.
MIT Ph.D. Thesis, accepted by MIT on Oct. 13 (2009) 313 pages.

Спиновые векторы начального и конечного протонов

Спиновые 4-векторы s_1 и s_2 начального и конечного протонов с 4-импульсами q_1 и q_2 в произвольной системе отсчета (ПСО). Условия ортогональности $(s_i q_i = 0)$ и нормировки $(s_i^2 = -1)$ позволяют однозначно определить выражения для их временных (s_{i0}) и пространственных $(\boldsymbol{s}_i)$ компонент $s_i = (s_{i0}, \boldsymbol{s}_i)$ через их 4-скорости $v_i = (v_{i0}, \boldsymbol{v}_i) = q_i/M$ $(i = 1, 2)$ следующим образом:

$$s_i = (s_{i0}, \boldsymbol{s}_i),\ s_{0i} = \boldsymbol{v}_i\, \boldsymbol{c}_i,\ \boldsymbol{s}_i = \boldsymbol{c}_i + \frac{(\boldsymbol{c}_i \boldsymbol{v}_i)\, \boldsymbol{v}_i}{1 + v_{i0}}\ , \tag{11}$$

где $\boldsymbol{c}_i$ $(\boldsymbol{c}_i^2 = 1)$ называются осями спиновых проекций.

В ЛСО, где $q_1 = (M, \boldsymbol{0})$, $q_2 = (q_{20}, \boldsymbol{q}_2)$, выберем оси спиновых проекций $\boldsymbol{c}_1$ и $\boldsymbol{c}_2$ так, чтобы они совпадали с направлением движения конечного протона:

$$\boldsymbol{c} = \boldsymbol{c}_1 = \boldsymbol{c}_2 = \boldsymbol{n}_2 = \boldsymbol{q_2}/|\boldsymbol{q}_2|\,. \tag{12}$$

Тогда спиновые 4-векторы протонов s_1 и s_2 в ЛСО принимают вид:

$$s_1 = (0, \boldsymbol{n}_2)\,,\ s_2 = (|\boldsymbol{v}_2|, v_{20}\, \boldsymbol{n_2})\,,\ \boldsymbol{n}_2 = \boldsymbol{q_2}/|\boldsymbol{q}_2|\,. \tag{13}$$

Дифференциальное сечения процесса $e\vec{p} \to e\vec{p}$ в ЛСО

Дифференциальное сечение процесса $e\vec{p} \to e\vec{p}$ в ЛСО для случая, когда $\boldsymbol{c} = \boldsymbol{c}_1 = \boldsymbol{c}_2 = \boldsymbol{n}_2 = \boldsymbol{q_2}/|\boldsymbol{q}_2|$:

$$\frac{d\sigma_{\delta_1,\delta_2}}{d\Omega_e} = \frac{\alpha^2 E_2 \cos^2(\theta_e/2)}{4E_1^3 \sin^4(\theta_e/2)} \frac{1}{1+\tau_p} \left(\frac{1+\delta_1\delta_2}{2}\, G_E^2 + \frac{1-\delta_1\delta_2}{2}\, \frac{\tau_p}{\varepsilon}\, G_M^2 \right), \quad (14)$$

где $\delta_{1,2}$ – удвоенные значения проекций спина начального и конечного протона на общую ось спиновых проекций (12) для обоих протонов; $-1 \leqslant \delta_{1,2} \leqslant 1$.

Сечение (14), есть сумма двух слагаемых, которые отвечают за вклад переходов без переворота ($\sim G_E^2$) и с переворотом спина протона ($\sim G_M^2$), что обеспечивают поляризационные множители ω_+ и ω_- при G_E^2 и G_M^2:

$$\omega_+ = (1+\delta_1\delta_2)/2,\ \omega_- = (1-\delta_1\delta_2)/2\,. \quad (15)$$

Из выражения (19) следует, что если $\delta_1 = 1, \delta_2 = 1$, то сечение процесса определяется только G_E^2, поскольку ω_+ и ω_- при G_E^2 и G_M^2 равны: $\omega_+ = 1$, $\omega_- = 0$. Если $\delta_1 = 1, \delta_2 = -1$, то сечение процесса определяется только G_M^2, при этом поляризационные множители при G_E^2 и G_M^2 равны: $\omega_+ = 0$, $\omega_- = 1$.

Физический смысл разбиения формулы Розенблюта

Выше сказанное позволяет переписать выражение (19) для сечения процесса $e\vec{p} \to e\vec{p}$, выделив явным образом вклады переходов без переворота ($\sigma^{\uparrow\uparrow}$) и с переворотом ($\sigma^{\downarrow\uparrow}$) спина протона:

$$\frac{d\sigma_{\delta_1,\delta_2}}{d\Omega_e} = \omega_+\sigma^{\uparrow\uparrow} + \omega_-\sigma^{\downarrow\uparrow}\,, \tag{16}$$

$$\sigma^{\uparrow\uparrow} = \sigma_M\, G_E^2,\;\; \sigma^{\downarrow\uparrow} = \sigma_M \frac{\tau_p}{\varepsilon}\, G_M^2\;, \tag{17}$$

где σ_M – выражение, стоящее перед скобками в (8) и в (14). Усредняя и суммируя сечение (16) по поляризациям начального и конечного протонов, получаем выражение для формулы Розенблюта (10) в другом представлении:

$$\sigma_R = \sigma^{\uparrow\uparrow} + \sigma^{\downarrow\uparrow}\,. \tag{18}$$

Следовательно, физический смысл разбиения формулы Розенблюта (10) на сумму двух слагаемых, содержащих только $G_E^{\,2}$ и $G_M^{\,2}$, заключается в том, что оно представляет собой сумму сечений без переворота и с переворотом спина протона (18) в случае, когда начальный покоящийся протон полностью поляризован вдоль направления движения конечного протона.

2. Polarization transfer method of Akhiezer and Rekalo

A.I. Akhiezer and M.P. Rekalo proposed a method for measuring the ratio of the Sachs form factors in the reaction $\vec{e}p \to e\vec{p}$ [1,2]. Their method relies on the phenomenon of polarization transfer from the longitudinally polarized initial electron to the final proton and requires measurement of the spin-dependent cross section. This method is called by the polarization transfer or polarized target (PT) method. In papers [1,2] was shown that the ratio of the degrees of longitudinal (P_l) and transverse (P_t) polarizations of the scattered proton has the form

$$\frac{P_l}{P_t} = -\frac{G_M}{G_E}\frac{E_1 + E_2}{2M}\tan\frac{\theta_e}{2}\,. \tag{19}$$

[1] A. Akhiezer, M. Rekalo, DAN SSSR **13**, 572 (1968),
[2] A. Akhiezer, M. Rekalo, Fiz.Elem.Chast.Atom.Yadra **4**, 662 (1973).

The discrepancy between the RT and JLab experiments

With the aid of Rosenbluth's technique, it was found that the experimental dependences of G_E and G_M on Q^2 are well described up to $5-6$ GeV^2 by the dipole-approximation expression

$$G_E = G_M/\mu_p = G_D(Q^2) \equiv \frac{1}{(1+Q^2/0.71)^2} \sim \frac{1}{Q^4},\ \ \mu_p \frac{G_E}{G_M} \approx 1\,, \quad (20)$$

where μ_p is the proton magnetic moment ($\mu_p = 2.79$).

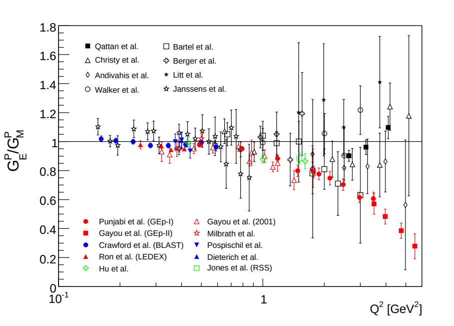
Precision experiments based on employing of the method of Akhiezer and Rekalo were performed at JLab. They showed that, in the range of $0.5 < Q^2 < 5.5$ GeV^2, there was a linear decrease in the ratio $R = \mu_p G_E/G_M$ with increasing Q^2:

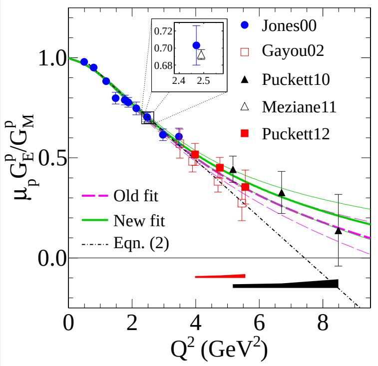
$$R \equiv \mu_p G_E/G_M \approx 1 - 0.13\,(Q^2 - 0.04)\, \approx 1 - \frac{1}{8}\,Q^2\,, \qquad (21)$$

which indicates that G_E falls faster than G_M. In the non-relativistic limit, this fact could be interpreted as indicating that the spatial distributions of charge and magnetization currents in the proton are definitely different.

Polarization transfer experiments JLab data for G_E^p/G_M^p

A. Puckett *et al.*, PRC, **85** (2012) 045203 →





World data (left figure) of the ratio $\mu_p G_{E_p}/G_{M_p}$ using the Rosenbluth method (black symbols) and from polarization experiments by Akhiezer and Rekalo method (colored symbols).

$$R \equiv \mu_p G_E/G_M \approx 1 - 0.13\,(Q^2 - 0.04) \approx 1 - \frac{1}{8}\,Q^2\,, \qquad (22)$$

Present status of the question

In order to resolve this contradiction, it was assumed that the discrepancy in question may be caused by disregarding, in the respective analysis, the contribution of two-photon exchange (TPE):

[P. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003)].
[A.V. Afanasev, S. J. Brodsky [et al], PRD **72**, 013008 (2005).]
[C. Perdrisat *et al.*, Prog. Part. Nucl. Phys. **59**, 694 (2007)] (review).

In works [GK1,GK2] we discuss questions related to the interpretation of unexpected results of measurements of the proton form factors ratio G_E/G_M in the high-precision polarization experiments done in JLab.

[GK1]. M.V. Galynskii, E.A. Kuraev, Alternative way to understand the unexpected results of the JLab polarization experiments to measure the Sachs form factors ratio, Phys. Rev. D, **89**, 054005 (2014).

[GK2]. M.V. Galynskii, E.A. Kuraev, On the Physical Meaning of Sachs Form Factors and on the Violation of the Dipole Dependence of G_E and G_M on Q^2, JETP Lett. **96**, 6 (2012).

Present status of the question

At the present time, three experiments aimed at studying the contribution of TPE are known:

1) experiment at the VEPP-3 storage ring in Novosibirsk,
2) the EG5 CLAS experiment at JLab,
3) the OLYMPUS experiment at the DORIS accelerator at DESY.

[exp1] I. A. Rachek, *et al.*, Phys. Rev. Lett. 114 (2015) 062005.
[exp2] D. Adikaram *et al.*, Phys. Rev. Lett., 114 (2015) 062003.
[exp3] B.S. Henderson *et al.* Phys. Rev. Lett. 118 (2017) 092501.

Предварительные результаты работ [exp1] - [exp3] показали, что учет вклада двухфотонного обмена, как и следовало ожидать, может устранить противоречия до значений Q^2 не более 2–3 ГэВ^2.

Jan C. Bernauer: https://arxiv.org/abs/1804.06665 (18 Apr 2018)

In the results, a small two-photon exchange effect is visible, significantly different from theoretical calculation. This paper discusses the possibilities for future measurements at larger momentum transfer.

Where does the pQCD behavior begin?

It is, in general, admitted that the onset of the asymptotic regime of pQCD starts around the J/Ψ mass squared, i.e. at $Q^2 \approx 9.0$ GeV^2. It was first observed in work [R. Arnold *et al.*, PRL **57**, 174 (1986)] that the proton magnetic FF, G_M, follows the asymptotic pQCD predictions of [Lepage and Brodsky, PRD **22**, 2157 (1980)], and $Q^4 G_M$ becomes nearly constant starting at $Q^2 \approx 9$ GeV^2. The answer to the question what is in general admitted at present on the onset of pQCD can be found in [1,2]:
[1] A. Courtoy and S. Liuti, Phys.Lett.B **726**, 320 (2013).
[2] S. Brodsky *et al.*, Phys.Rev.D **81**, 096010 (2010).
In this works based on using completely different approaches, it is shown that the point of transition from non-perturbative QCD to pQCD correspond to a momentum scale $Q_0 \sim 1$ GeV. For this reason we will below assume that HSM of pQCD starts at the lower boundary of the considered region, i.e. around $Q_0 \sim 1$ GeV.

Where does the pQCD behavior begin?

[1] A. Courtoy and S. Liuti, Phys.Lett.B **726**, 320 (2013).

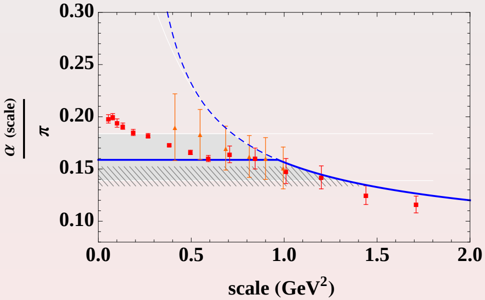


Рис. : Extraction of α_s. The blue dashed curve represents the exact NLO solution for the running coupling in $\overline{\text{MS}}$ scheme. The solid blue curve represents the running coupling obtained from our analysis using inclusive electron scattering data at large x. Owing to large x resummation, at lower values of the scale, $\alpha_s = \alpha_{s,\text{NLO}}$ (min) is frozen as explained in the text. The grey area represents the region where the freezing occurs for JLab data, while the hatched area corresponds the freezing region determined from SLAC data.

Where does the pQCD behavior begin?

In [3], within the analytic perturbation theory (APT) approach using the rules of the Gerasimov-Drell-Hearn, it is shown that the point of "crosslinking"of the perturbative and nonperturbative regimes in APT is significantly lower than that obtained in the framework of the standard pQCD, where $Q_0 \sim 1$ GeV. The main reason for such a significant forwarding down of Q within the APT approach is the disappearance of the nonphysical singularities of the perturbation theory series.

It should be noted that in the known work of Belitsky *et al.* [4] the authors have performed numerical calculations in the framework of pQCD in the region of $0.5 \leq Q^2 \leq 5.5$ GeV^2; therefore, they proceeded from the assumption that the onset of pQCD starts already at $Q^2 = 0.5$ GeV^2.

It is very likely that the results of Ref. [4] are an indirect proof of the correctness results of Ref. [3] obtained in the framework of the APT.

[3] R. Pasechnik, D. Shirkov and O. Teryaev, PRD **78**, 071902 (2008).

[4] A. Belitsky, X. Ji, and F. Yuan, PRL **91**, 092003 (2003).

What is the hard-scattering mechanism of pQCD?

G. Lepage and S. Brodsky, Phys. Rev. D **22**, 2157 (1980).

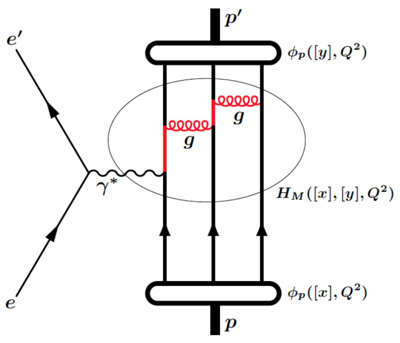


Рис. : A typical hard gluon-exchange process in elastic electron-proton scattering ($e + p \rightarrow e' + p'$). There are two hard quark propagators and two gluon ones which contribute to the counting rule in the elastic form factor.

Diagonal spin basis (DSB)

F.I. Fedorov, TMF **2**, № 3, 343 (1970):

$$\vec{a} = \vec{q}_2/q_{20} - \vec{q}_1/q_{10} \; . \tag{23}$$

The direction of $\vec{a}$ (23) have property that the projections of the spins of both particles on it simultaneously have definite values.

S. Sikach, Vesti AN BSSR, ser. fiz.-m.n, № 2, 84 (1984)

In the diagonal spin basis (DSB) spin 4-vectors s_1 and s_2 of protons with 4-momenta q_1 and q_2 $(s_1 q_1 = s_2 q_2 = 0, s_1^2 = s_2^2 = -1)$ have the form:

$$s_1 = -\frac{(v_1 v_2) v_1 - v_2}{\sqrt{(v_1 v_2)^2 - 1}} \; , \quad s_2 = \frac{(v_1 v_2) v_2 - v_1}{\sqrt{(v_1 v_2)^2 - 1}} \; , v_1 = \frac{q_1}{M}, \; v_2 = \frac{q_2}{M}, \tag{24}$$

The spin vectors (24) obviously do not change under transformations of the Lorentz little group (little Wigner group) common to particles with 4-momenta q_1 and q_2: $L_{q_1,q_2} q_1 = q_1$, $L_{q_1,q_2} q_2 = q_2$. Therefore, the DSB naturally makes it possible to describe the spin states of systems of any two particles by means of the spin projections on the common direction given by the 3-vector (23).

Diagonal spin basis (DSB)

Since vector $\vec{a}$ (23) is the difference of two vectors and the geometrical image of the difference of two vectors is the diagonal of a parallelogram, hence the name "diagonal spin basis"given by academician F.I. Fedorov.

Let us consider the realization DSB in the rest frame of the initial proton, where $q_1 = (M, \vec{0})$. Here $\vec{a}$ (23) equal $\vec{a} = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|$, i.e. common direction for spin projection is the direction of the motion of the final proton, thus this final proton polarization state is a helicity and spin 4-vectors s_1 и s_2 (24) have the form:

$$s_1 = (0, \vec{n}_2),\; s_2 = (|\vec{v}_2|, v_{20}\, \vec{n_2}), \vec{c}_1 = \vec{c}_2 = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,, \tag{25}$$

axis of spin projections $\vec{c}_1$ and $\vec{c}_2$ is coincide with the direction of the final proton.

Breit system, where $\vec{q_2} = -\vec{q}_1$, is a special case of DSB. In the Breit system where $q_1 = (q_0, -\vec{q}), q_2 = (q_0, \vec{q})$, the spin states of the initial and final protons are helicity, so they spin 4-vectors s_1 и s_2 in DSB have the form:

$$s_1 = (-|\vec{v}|, v_0\vec{n}_2),\; s_2 = (|\vec{v}|, v_0\vec{n}_2)\,, \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,. \tag{26}$$

Spin operators in the DSB

In the DSB all spin operators for initial and final proton have the same form:

$$\sigma = \sigma_1 = \sigma_2 = \gamma^5 \hat{s_1} \hat{v_1} = \gamma^5 \hat{s_2} \hat{v_2} = \gamma^5 \hat{b}_0 \hat{b}_3 = i \hat{b}_1 \hat{b}_2 \ , \tag{27a}$$

$$\sigma^{\pm\delta} = \sigma_1^{\pm\delta} = \sigma_2^{\pm\delta} = -i/2 \gamma^5 \hat{b}_{\pm\delta}, \ b_{\pm\delta} = b_1 \pm i\delta b_2 \ , \ \delta = \pm 1 \ , \tag{27b}$$

$$\sigma u^\delta(q_i) = \delta u^\delta(q_i) \ , \ \sigma^{\pm\delta} u^{\mp\delta}(q_i) = u^{\pm\delta}(q_i). \tag{27c}$$

The set of unit 4-vectors b_0, b_1, b_2, b_3 is an orthonormal basis of 4-vectors b_A, $b_A b_B = g_{AB}$ $(A, B = 0, 1, 2, 3)$:

$$(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa b_2^\sigma, \ (b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa p_1^\sigma / \rho \, , b_3 = \frac{q_-}{\sqrt{-q_-^2}}, \ b_0 = \frac{q_+}{\sqrt{q_+^2}} \ , \tag{28}$$

where $q_- = q_2 - q_1$, $q_+ = q_2 + q_1$, $\varepsilon_{\mu\nu\kappa\sigma}$ is the Levi-Civita tensor ($\varepsilon_{0123} = -1$), ρ is determined from the normalization conditions $b_1^2 = b_2^2 = b_3^2 = -b_0^2 = -1$.

[M. Galynskii, S. Sikach, Phys.Part.Nucl. **29**, 469 (1998)]

The matrix elements of the proton current in the DSB

Matrix elements (amplitudes) for proton current defined as:

$$(J_p^{\pm\delta,\delta})_\mu = \overline{u}^{\pm\delta}(q_2)\Gamma_\mu(q^2)u^\delta(q_1)\,, \tag{29}$$

$$\Gamma_\mu(q^2) = F_1\,\gamma_\mu + \frac{F_2}{4M}(\,\hat{q}\gamma_\mu - \gamma_\mu\hat{q}\,)\,. \tag{30}$$

They were calculated in DSB by S.Sikach (1984):

$$(J_p^{\delta,\delta})_\mu = 2G_E M(b_0)_\mu\,, \tag{31}$$

$$(J_p^{-\delta,\delta})_\mu = -2\delta M\sqrt{\tau}G_M(b_\delta)_\mu\,. \tag{32}$$

For the point particles with mass m_q the amplitude have the form

$$(J_q^{\delta,\delta})_\mu = 2\,m_q\,(b_0)_\mu\,, \tag{33}$$

$$(J_q^{-\delta,\delta})_\mu = -2\,m_q\,\delta\sqrt{\tau_q}\,(b_\delta)_\mu\,. \tag{34}$$

where $\tau_q = Q_q^2/4m_q^2$.

Спиральные амплитуды в БСО

Упражнение (8.7) в [НМ], МЭ протонного тока в БСО
$q_1 = (q_0, -\boldsymbol{q}), q_2 = (q_0, \boldsymbol{q}),\ q = q_2 - q_1 = (0, 2\boldsymbol{q})$.

$$(J_p^{-\lambda,\lambda})_\mu = 2MG_E(b_0)_\mu, (J_p^{\lambda,\lambda})_\mu = -2\lambda|\boldsymbol{q}|G_M(b_\lambda)_\mu, \tag{35}$$

$(b_0)^\mu = (1,0,0,0),\ (b_1)^\mu = (0,1,0,0),\ (b_2)^\mu = (0,0,1,0)$,
$(b_3)^\mu = (0,0,0,1),\ b_\lambda = b_1 + i\lambda b_2,\ \lambda = \pm 1,\ 2|\boldsymbol{q}| = \sqrt{Q^2}$.

$$J_\mu^{\lambda,\lambda} = J_\mu^{\uparrow\downarrow} = J_\mu^{\delta,-\delta},\ J_\mu^{-\lambda,\lambda} = J_\mu^{\downarrow\downarrow} = J_\mu^{-\delta,\,-\delta}.$$

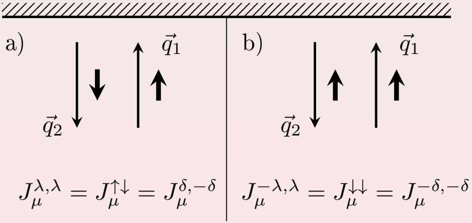


Рис. : 2. Система Брейта начального и конечного протонов: $\boldsymbol{q}_1 + \boldsymbol{q}_2 = 0$.

$$(J_p^{\delta,\delta})_\mu = 2MG_E(b_0)_\mu, (J_p^{-\delta,\delta})_\mu = -2\delta M\sqrt{\tau_p}\,G_M(b_\delta)_\mu. \tag{36}$$

[НМ] Ф. Хелзен, А. Мартин. – М.: Мир, 1987.– 456 с.

On the dependence G_E and G_M on Q^2

Since $|b_0| = 1$ and $|b_\delta| = \sqrt{2}$ and they are does not depend on Q^2, then from the (31), (32), (33), (34) we can easily obtain the dependence on Q^2 for (absolute) values of the matrix elements of proton currents $J_p^{\pm\delta,\delta}$ and point particles $J_q^{\pm\delta,\delta}$:

$$J_p^{\delta,\delta} = 2\,M\,G_E,\; J_p^{-\delta,\delta} = 2\,M\,\sqrt{\tau}\,G_M\,, \tag{37}$$

$$J_q^{\delta,\delta} = 2\,m_q\,,\; J_q^{-\delta,\delta} = 2\,m_q\,\sqrt{\tau_q}\,. \tag{38}$$

Note that the factorization of $2M$ and $2m_q$ in the expressions (37), (38) is caused by the normalization bispinors $\bar{u}_i u_i = 2m_i$. Below during the computation is more convenient to use the normalization of $\bar{u}_i u_i = 1$, and instead of (37), (38) we will use the expressions:

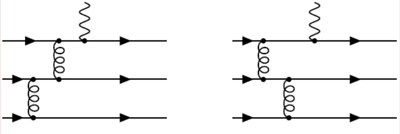
$$J_p^{\delta,\delta} = \,G_E, J_p^{-\delta,\delta} = \,\sqrt{\tau}\,G_M\,, \tag{39}$$

$$J_q^{\delta,\delta} = 1\,,\;\; J_q^{-\delta,\delta} = \sqrt{\tau_q}\,. \tag{40}$$

[M.Galynskii, E.Kuraev, Phys. Rev D **89**, 054005 (2014)]

On the dependence G_E and G_M on Q^2

Let us consider the HSM of pQCD in the process $ep \to ep$ that is realized as we believe at $Q^2 \geq 1$ GeV^2. In this case the leading contribution to the proton current $J_p^{\pm\delta,\delta}$ can be presented as a sum of the hard gluon exchange processes, where the proton is replaced by a set of three almost on mass shell quarks as illustrated in figure below



We suppose the masses of quarks m_q to be equal to $1/3$ of the proton mass M and the fraction of their transfer momenta to be equal, we have

$$\tau_q = \tau \,. \tag{41}$$

Under such simplifying assumptions it can easily be verified that the matrix element corresponding to the sum of two gauge-invariant diagrams, shown in this figure, has the form

$$(J_{p_{1,2}}^{\pm\delta,\delta})^\mu \sim (J_q^{\pm\delta,\delta})^\nu (J_q^{\pm\delta,\delta})_\nu (J_q^{\pm\delta,\delta})^\mu / Q^6 \,. \tag{42}$$

On the dependence G_E and G_M on Q^2

Therefore, the absolute magnitudes of the proton current matrix elements $J_p^{\pm\delta,\delta}$ that correspond to the contribution of the full set of possible Feynman diagrams can be written as the product of three point-quark current amplitudes $J_q^{\pm\delta,\delta}$ (40) divided by Q^6,

$$J_p^{\pm\delta,\delta} \sim J_q^{\pm\delta,\delta} J_q^{\pm\delta,\delta} J_q^{\pm\delta,\delta} / Q^6 \,. \tag{43}$$

There are two possibilities for a proton non-spin-flip transition: (i) none of the three quarks undergoes a spin-flip transition and (ii) two quarks undergo a spin-flip transition, while the third does not. We denote the number of such ways as $n_{qE}^{-\delta,\delta} = [0,2]$.
Proton spin-flip can also proceed in two ways: (i) one quark undergoes a spin-flip transition, while the other two do not, and (ii) all three quarks undergo a spin-flip transition. We denote the number of such ways by $n_{qM}^{-\delta,\delta} = [1,3]$. Thus, there are in all four combinations to be considered:

$$n_{qE}^{-\delta,\delta} \times n_{qM}^{-\delta,\delta} = (0,1) \oplus (0,3) \oplus (2,1) \oplus (2,3) \,. \tag{44}$$

Note due to Eqs. (40), (41) at $\tau \ll 1$ ($\tau \gg 1$) the quark transition without (with) spin-flip dominates. Therefore, the sets (0,1) and (2,3) are realized at $\tau \ll 1$ and $\tau \gg 1$, respectively.

The set (0,1), $G_E, G_M \sim 1/Q^6$, $G_E/G_M \sim 1$

Let us consider the first (0,1) set. We use for the amplitudes of protons and point-like quarks currents expressions (39):

$$J_p^{\delta,\delta} = G_E, J_p^{-\delta,\delta} = \sqrt{\tau}\, G_M\,,$$
$$J_q^{\delta,\delta} = 1\,, \quad J_q^{-\delta,\delta} = \sqrt{\tau}\,.$$

The matrix elements of the proton current $J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$ must be proportional to G_E and G_M, respectively; as a result, we have

$$J_p^{\delta,\delta} = G_E \sim 1 \times 1 \times 1\,/\,Q^6\,, \tag{45}$$
$$J_p^{-\delta,\delta} = \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times 1 \times 1/\,Q^6\,, \tag{46}$$

where the factors of unity and $\sqrt{\tau}$ on the right-hand side of Eqs. (45) and (46) correspond to non-spin-flip transitions for three pointlike quarks and to the spin-flip transition for one quark. As a result, we have

$$G_E \sim \frac{1}{Q^6}, G_M \sim \frac{1}{Q^6}\,, \frac{G_E}{G_M} \sim 1\,. \tag{47}$$

Therefore, for the set (0,1) the FFs ratio G_E/G_M behaves in just the same way as in the dipole case. However, the dependencies $G_E \sim 1/Q^6$, $G_M \sim 1/Q^6$ are not dipole ones.

The set (2,3), $G_E, G_M \sim 1/Q^4, G_E/G_M \sim 1$

For the set (2,3) we have

$$J_p^{\delta,\delta} = G_E \sim \sqrt{\tau} \times \sqrt{\tau} \times 1\,/\,Q^6\ , \qquad (48)$$

$$J_p^{-\delta,\delta} = \sqrt{\tau}\,G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau}\,/\,Q^6\ . \qquad (49)$$

Hence, we obtain

$$G_E \sim \frac{1}{Q^4},\ G_M \sim \frac{1}{Q^4},\ \frac{G_E}{G_M} \sim 1\ . \qquad (50)$$

Therefore, the dipole dependence in the behavior of the FFs G_E and G_M on Q^2 occurs in the set (2,3) at $\tau \gg 1$ in the case when a number of quark transitions with spin-flip saturation takes place.
Thus, our approach is in fact a generalization of constituent-counting rules for the massive quarks. Note, in Ref. [5] to estimate the leading contribution of the HSM in the proton magnetic FF within the standard pQCD with massless quarks, a method similar to our approach was used. At the same time, formulas (16), (17) in Ref. [5] and our formulas (49) are the same and reproduce the well-known result obtained in the works of Brodsky within the framework of the constituent-counting rules before the development of QCD.

[5] H. Kawamura *et al.*, Phys. Rev. D **88**, 034010 (2013)

Spin Parametrization for $\boldsymbol{G_E/G_M}$

The non-spin-flip and spin-flip proton-current amplitudes ($J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$) can be represented as the linear combinations

$$J_p^{\delta,\delta} = \alpha_0\, J_q^{\delta,\delta} J_q^{-\delta,-\delta} J_q^{\delta,\delta} + \alpha_2\, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{\delta,\delta}, \tag{51}$$

$$J_p^{-\delta,\delta} = \beta_1 J_q^{-\delta,\delta} J_q^{\delta,\delta} J_q^{-\delta,-\delta} + \beta_3\, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{-\delta,\delta}, \tag{52}$$

where the coefficients α_0, α_2, β_1, and β_3 have a clear physical meaning that is determined by their indices. From Eqs. (51) and (52), we have

$$\frac{G_E}{G_M} = \frac{\alpha_0 + \alpha_2\,\tau}{\beta_1 + \beta_3\,\tau}\,. \tag{53}$$

This expression may serve as a basis for constructing spin parametrization and fits experimental data obtained by measuring the ratio G_E/G_M. We showed above that at $\tau \ll 1$ the quark transition without spin-flip dominates; the set (0,1) with the minimal number of spin-flip quarks, where $G_E/G_M \sim 1$, must occur. In this case the coefficients α_0 and β_1 in Eq. (53) must have the values close to unity. With allowance for this comment, we expand the right-hand side of (53) in a power series for τ. As a result, we get the law of a linear decrease in the ratio $R = G_E/G_M$ as Q^2 increases,

$$R \approx 1 - (\beta_3 - \alpha_2)\,\tau\,. \tag{54}$$

Conclusion

We have discussed in the one-photon exchange approximation the questions related to the interpretation of the JLab polarization experiment's unexpected results to measure the Sachs FFs ratio G_E/G_M in the region $1.0 \leq Q^2 \leq 8.5$ GeV^2. For this purpose, in the case of the HSM of the pQCD, we calculated the hard kernel of the proton current matrix elements $J_p^{\pm\delta,\delta}$ for the full set of spin combinations corresponding to a number of the spin-flipped quarks, which contribute to the proton transition without spin-flip ($J_p^{\delta,\delta}$) and with the spin-flip ($J_p^{-\delta,\delta}$).
This allows us to state that
(i) around the lower boundary of the considered region the leading scaling behavior of the Sachs FFs has the form $G_E, G_M \sim 1/Q^6, G_E/G_M \sim 1$, but it is not dipole dependence,
(ii) since for quarks $J_q^{\delta,\delta} \sim 1$ and $J_q^{-\delta,\delta} \sim \sqrt{\tau}$, then the dipole dependence ($G_E, G_M \sim 1/Q^4$) is realized in the asymptotic regime of pQCD when $\tau \gg 1$ in the case when the quark transitions with spin-flip dominate,
(iii) the asymptotic regime of pQCD in the JLab experiments has not yet been achieved, and it is likely that the asymptotic regime for G_E occurs at higher values Q^2 than for G_M,

Conclusion

(iv) the linear decrease of the ratio G_E/G_M at $\tau < 1$ is due to additional contributions to $J_p^{\delta,\delta}$ by spin-flip transitions of two quarks and an additional contribution to $J_p^{-\delta,\delta}$ by spin-flip transitions of three quarks,
(v) one of our predictions is the realization (restoration) of a dipole dependence of the Sachs form factors and the value $R = 1$ for higher values of Q^2 (at $\tau \gg 1$).

Thus, abandoning the massless quarks, we were able to explain in the one-photon exchange approximation the unexpected results of measurements of the proton Sachs FFs ratio and analytically derive the experimentally established formula of the linear decrease law for this ratio at $\tau < 1$.

Developed by us an approach is essentially a generalization of the constituent-counting rules of the perturbative QCD (pQCD) for the case of massive quarks.

We believe that the interpretation presented above can be considered as a possible way to solve the G_E/G_M problem.

THANK FOR YOUR ATTENTION