Description of Scalar Mesons as Two and Four Quark States.

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The XIV INTERNATIONAL SCHOOL-CONFERENCE

“THE ACTUAL PROBLEMS OF MICROWORLD PHYSICS”

In Memory of Professor Nikolai Shumeiko (1942-2016)

Grodno, August 24, 2018

"Where and what are the scalar mesons?"[*]

[*] P.Estabrooks, Phys. Rev. **D** 19, 2678 (1979)

Theoretical approaches

- Two quark models

Шабалин Е.П. ЯФ,т.40 (1984),

F.E.Close, A. Kirk, EPJ C21 (2001)

- Four quark models

R.L. Jaffe, Phys. Rev. D 15 (1977),

N.N. Achasov, Nucl.Phys. A728 (2003)…

- Gluebol , hybrids.

V. Vento EPJ A24 (2005),

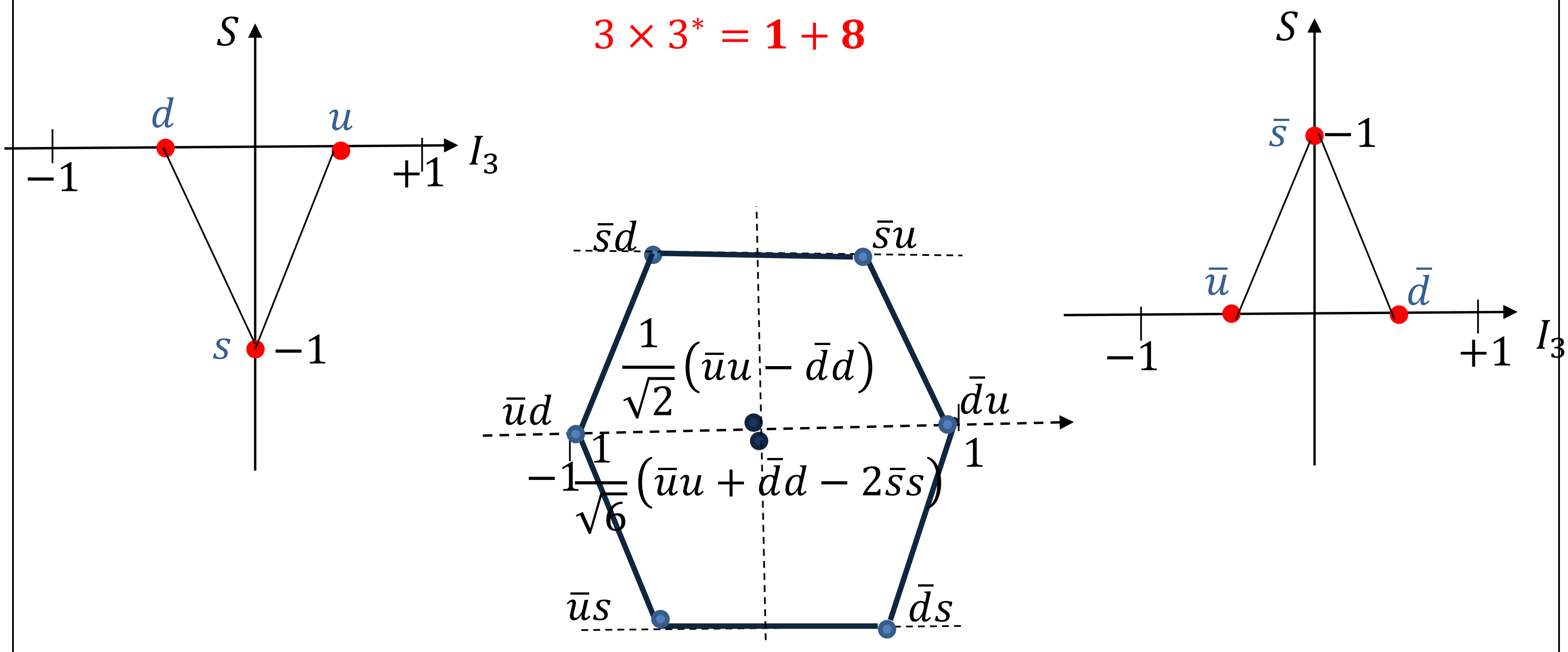
E.S. Swanson Phys.Rep.429 (2006)

NAIV QUARK MODEL

Gell-Mann, Zweig (1964):

Mesons- $q\bar{q}$-states, come from the product of 3 and 3^*representations of SU(3) group

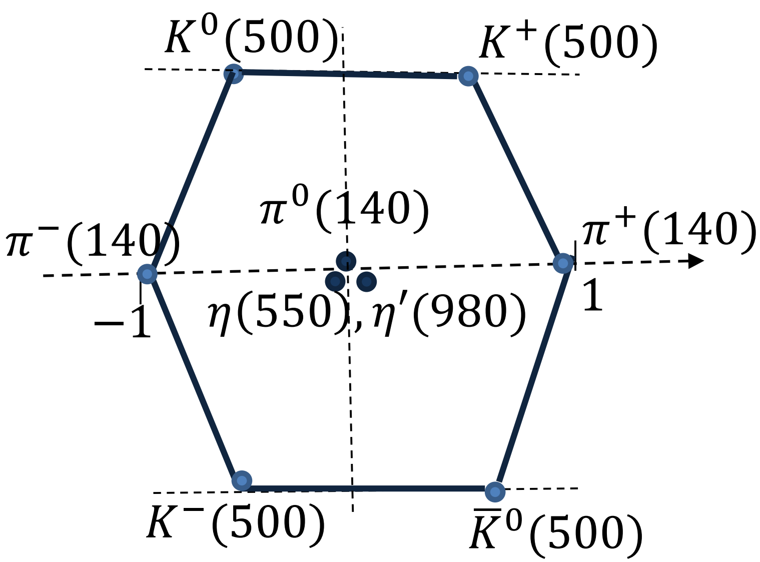
$$3 \times 3^* = \mathbf{1} + \mathbf{8}$$

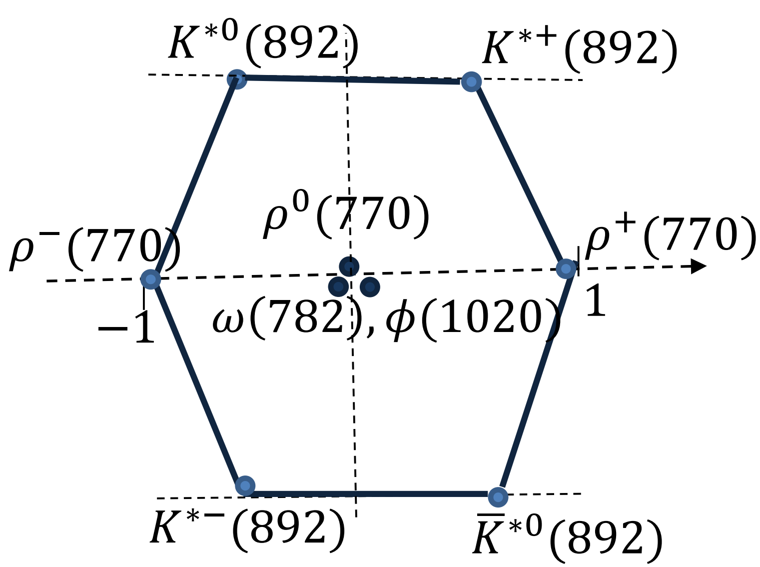


NAIV QUARK MODEL

Pseudoscalars 0^{-+}

Vectors 1^{--}

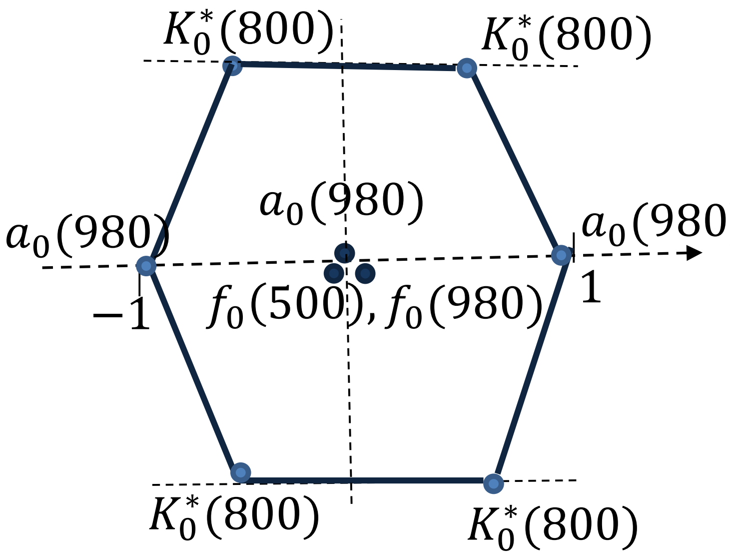


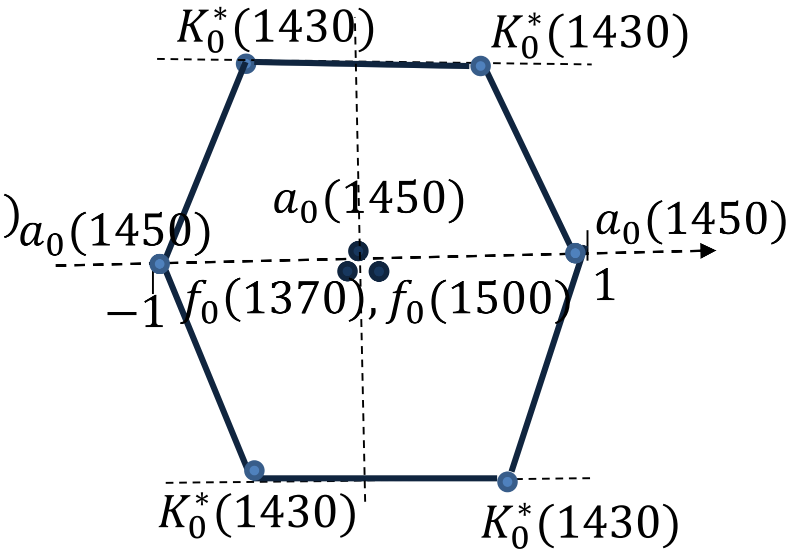


SCALAR MESONS

MESON	MASS (MeV)	WIDTH(MeV)	ISOSPIN, STRANGENESS
$f_0(500), \sigma$	$400 - 550$	$400 - 700$	$I = 0, S = 0$
$K_0^*(800), \kappa$	682 ± 29	547 ± 24	$I = \frac{1}{2}, S = \pm 1$
$f_0(980)$	990 ± 20	$40 - 100$	$I = 0, S = 0$
$a_0(980)$	980 ± 20	$50 - 100$	$I = 1, S = 0$
$f_0(1370)$	$1200 - 1500$	$200 - 500$	$I = 0, S = 0$
$K_0^*(1430)$	1425 ± 50	270 ± 80	$I = \frac{1}{2}, S = \pm 1$
$a_0(1450)$	1474 ± 19	265 ± 13	$I = 1, S = 0$
$f_0(1500)$	1505 ± 6	109 ± 7	$I = 0, S = 0$
$f_0(1700)$	1720 ± 6	135 ± 8	$I = 0, S = 0$

SCALAR MESONS





TWO QUARK MODEL (QCM)

- *Quark Confinement Model (QCM).*

[G.V.Efimov and M.A. Ivanov, *"The Quark Confinement Model of Hadron"*, IOP Publishing, 1993]. This model is based on the following assumptions

Lagrangian of the interaction between hadrons and quarks is obtained

$$L_M = \frac{g_M}{\sqrt{2}} M^i \bar{q}_m^a \Gamma_\mu \lambda^{mn} q_n^a$$

$q_j^a = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$ -are the quark fields,

M^i - Euclidean fields connected with the fields of physical particles, λ and Γ are the Gell-Mann and Dirac matrixes, g_M are quark-meson coupling.

TWO QUARK MODEL (QCM)

$$\mathcal{L}_I^s = \frac{g_s}{\sqrt{2}} s(x)\bar{q}(x)\left(I - i\frac{\mathrm{H}}{\Lambda}\overleftrightarrow{\partial}\right)\lambda_s q(x)$$

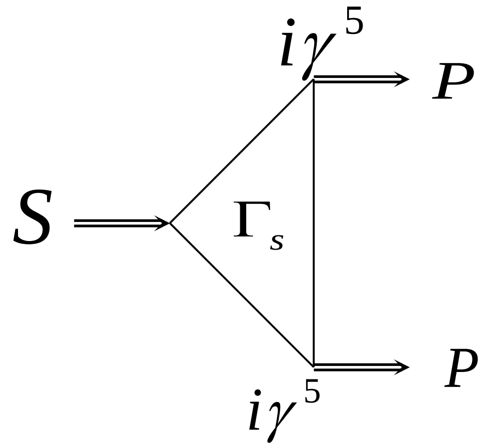
$$\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$$

$$\lambda_s = \begin{cases} diag(1, -1, 0) \Rightarrow a_0 \\ diag\left(\cos\delta_s, \cos\delta_s, -\sqrt{2}\sin\delta_s\right) \Rightarrow \varepsilon \\ diag\left(-\sin\delta_s, -\sin\delta_s, -\sqrt{2}\cos\delta_s\right) \Rightarrow f_0 \end{cases}$$

parameters: H, δ_s

Evaluation of one-loop diagrams

G.V.Efimov and M.A. Ivanov, "*The Quark Confinement Model of Hadron*", IOP Publishing, 1993.

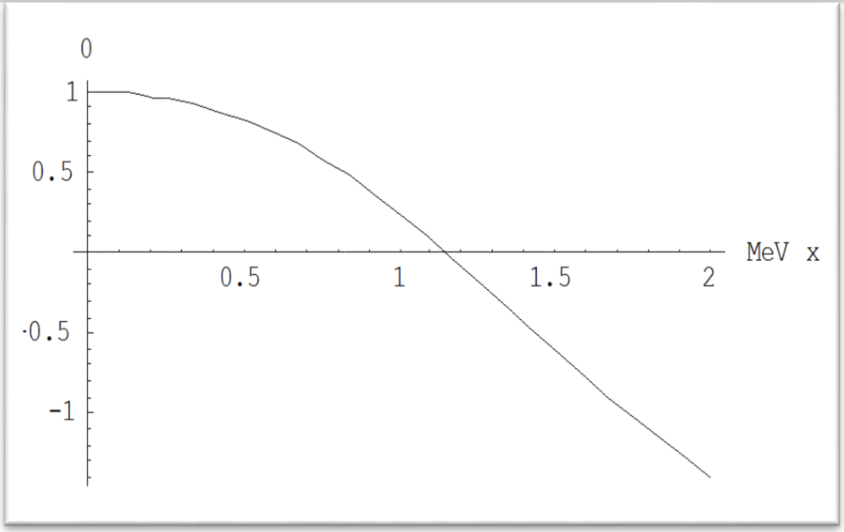


$$I_{SPP}=\int d\sigma_\lambda \int \frac{d^4k}{4\pi^2 i} Tr\left\{\Gamma_s \frac{1}{\Lambda v_\lambda-\hat{k}}\gamma^5 \frac{1}{\Lambda v_\lambda-(\hat{k}+\hat{q}_1)}\gamma^5 \frac{1}{\Lambda v_\lambda-(\hat{k}+\hat{p})}\right\}$$

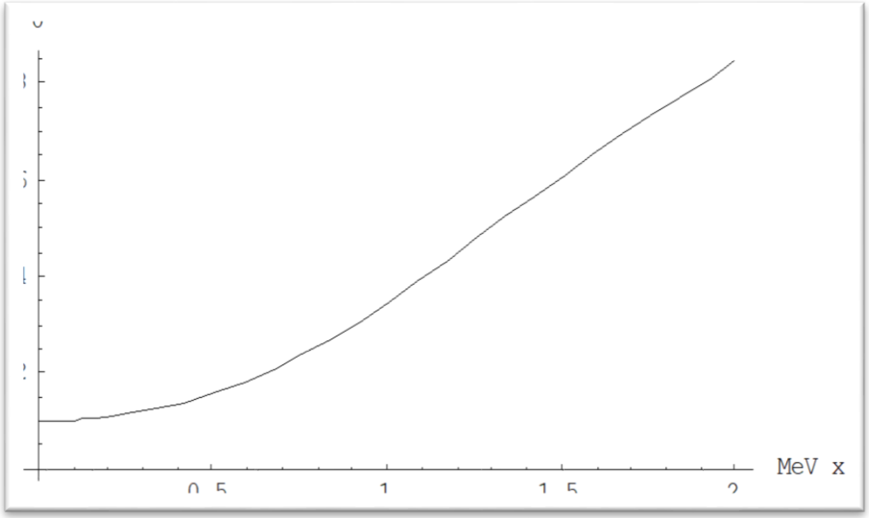
$$\int d\sigma_\lambda \frac{v_\lambda}{v_\lambda^2+X}=a(X), \qquad \int d\sigma_\lambda \frac{1}{v_\lambda^2+X}=b(X).$$

Mass dependence of $S \longrightarrow PP$ in QCM

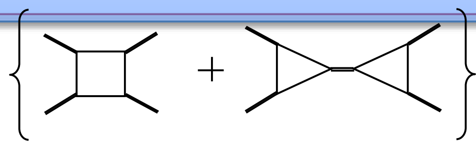
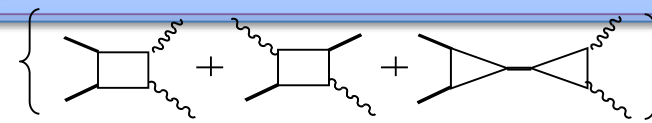
$$\Gamma_s = I$$



$$\Gamma_s = I - \frac{H}{\Lambda}\hat{p}$$



Determination of additional parameters in QCM

1.  $= 0 \; \big|_{m_\pi \to 0}$  $= 0 \; \big|_{m_\pi \to 0}$

$$\begin{cases} \int_0^\infty du\, b(u) = 2\Lambda^2 [\int_0^\infty du\, a(u) - 4H \int_0^\infty du\, u\, b(u)] h_\varepsilon(H) D_\varepsilon(0) \\ 5b(0) = -2\Lambda_s^2 \cos\delta_s (5\cos\delta_s - \sqrt{2}\sin\delta_s)[\int_0^\infty du\, a(u) - 4H \int_0^\infty du\, u\, b(u)] a(0) h_\varepsilon(H) D_\varepsilon(0) \end{cases}$$

$$R = -\frac{5\cos\delta_s [\int_0^\infty du\, a(u) - 4H \int_0^\infty du\, u\, b(u)]}{(5\cos\delta_s - \sqrt{2}\sin\delta_s) a(0)} = 1$$

$$\Gamma_{\exp}\left(f_0 \to \pi\pi\right)$$

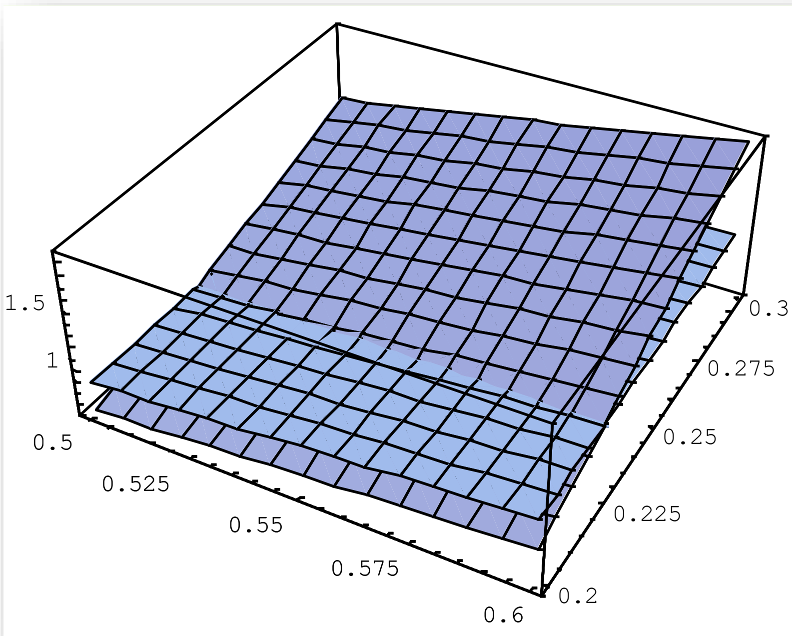
2.

$$\Gamma(f_0 \to \pi\pi) = \frac{3}{16\pi}\sqrt{1 - \frac{4m_\pi^2}{m_{f_0}^2}}\, g_{f_0\pi\pi}^2 (m_{f_0}^2, 0, 0) \frac{1}{m_{f_0}}$$

$$g_{SP_1P_2}(m_s^2, m_{P_1}^2, m_{P_2}^2) = Sp\lambda_s \{\lambda_{P_1}, \lambda_{P_2}\} \Lambda \frac{\sqrt{h_{P_1} h_{P_2} h_S(H)}}{6} I_{SPP}(m_s, m_{P_1}, m_{P_2})$$

Determination of additional parameters in QCM

$$g = \frac{\Gamma_{QCM}(f_0 \to \pi\pi)}{\Gamma_{\exp}(f_0 \to \pi\pi)} \qquad R = -\frac{5\cos\delta_s[\int_0^\infty du\,a(u) - 4H\int_0^\infty du\,u\,b(u)]}{(5\cos\delta_s - \sqrt{2}\sin\delta_s)a(0)} = 1$$



$$H = 0.54, \qquad \delta_s = 17^\circ$$

MASS OF LIHGTEST SCALAR

$$M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t)$$

a, b, c, d - *isotopic indexes*

$$A(s,t,u) = A(s,u,t)$$

$$A(s,t,u) = I_{box}^{\pi\pi}(s,t,u) + S^{\pi\pi}(s,t,u) + V^{\pi\pi}(s,t,u)$$

$$I_{box}^{\pi\pi}(s,t,u) = I_{box}(s,t,u,m_\pi^2,m_\pi^2,m_\pi^2,m_\pi^2,\Lambda_n,\Lambda_n,\Lambda_n,\Lambda_n)$$

$$S^{\pi\pi}(s,t,u) = F_{S\pi\pi}^2(s)\left(\frac{\cos^2\delta_s}{\Pi_s(s)-\Pi_s(m_1^2)} + \frac{\sin^2\delta_s}{\Pi_s(s)-\Pi_s(m_2^2)}\right) +$$
$$+F_{S\pi\pi}^2(t)\left(\frac{\cos^2\delta_s}{\Pi_s(t)-\Pi_s(m_1^2)} + \frac{\sin^2\delta_s}{\Pi_s(t)-\Pi_s(m_2^2)}\right)$$

$$F_{S\pi\pi}(x) = F_{S\pi\pi}(x, m_\pi^2, m_\pi^2)$$

m_1 *-mass of* $f_0(500)$, m_2 *-mass of* $f_0(980)$

MASS OF LIHGTEST SCALAR

$$V^{\pi\pi}(s,t,u) = \frac{1}{\Pi_1(m_\rho^2)}\left[\left(F^-(s)\right)^2(t-u) + \left(F^-(t)\right)^2(s-u)\right]$$

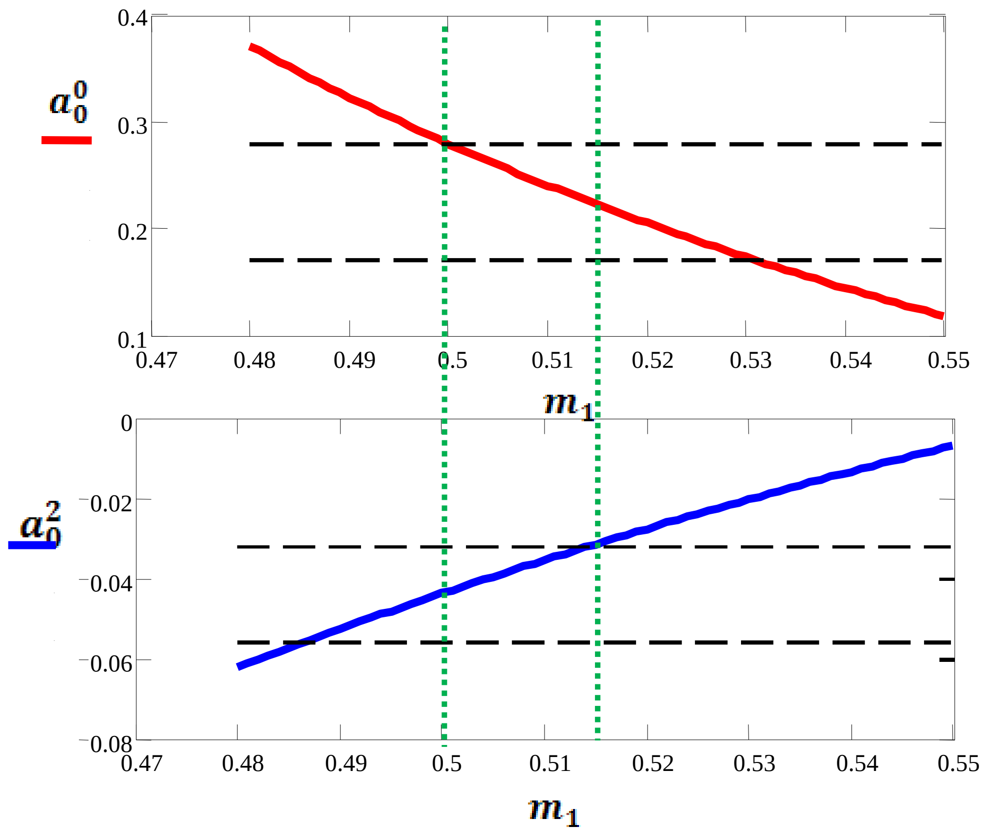
The amplitudes for the three possible channels $(I = 0, 1, 2)$

$$T^0(s,t,u) = 3A(s,t,u) + A(t,s,u) + A(u,t,s)$$
$$T^1(s,t,u) = A(t,s,u) - A(u,s,t)$$
$$T^2(s,t,u) = A(t,s,u) + A(u,s,t)$$

The scattering length

$$a^I = \frac{1}{32\pi} T^I(4m_\pi^2, 0, 0)$$

MASS OF LIHGTEST SCALAR



$m_1 = 500MeV$

$a_0^0 = 0.28, a_0^2 = -0.044$

$m_1 = 515MeV$

$a_0^0 = 0.22, a_0^2 = -0.032$

TWO QUARK MODEL (CCQM)

- *Covariant Constintuent Quark Model (CCQM).*

T. Branz, A. Faessler, T. Gutsche, M.A. Ivanov, J.G. Körner, V. E. Lyubovitskij

Phys. Rev. D81, 034010 (2010)

$$L_{int}^{st}(x) = g_M M(x) \int dx_1 \int dx_2\, F_M(x, x_1, x_2) \bar{q}_1(x_1) \lambda_M \Gamma_M q_2(x_2)$$

$F_M(x, x_1, x_2)$-vertex function, characterizing the finite size of the meson

TWO QUARK MODEL (CCQM)

To satisfy translational invariance the vertex function has to obey the identity

$$\boldsymbol{F_M(x+a, x_1+a, x_2+a) = F_M(x, x_1, x_2)}$$

for any vector a.

$$\boldsymbol{F_M(x, x_1, x_2)} = \delta^4\left(x - \sum_{i=1}^{2} w_i x_i\right)\Phi_M\left((x_1 - x_2)^2\right)$$

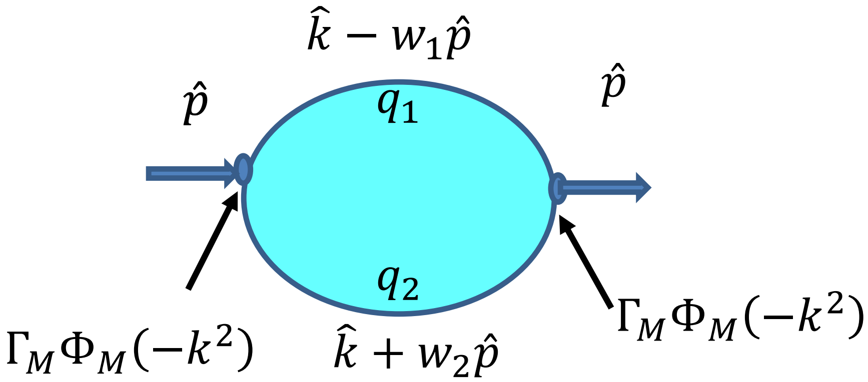
$$w_i = \frac{m_i}{m_1 + m_2}$$

m_1, m_2- masses of constituent quarks

The simplest choice: $\boldsymbol{\Phi_M(-l^2) = \exp\left(-\frac{l^2}{\Lambda_M^2}\right)}$

$\boldsymbol{\Lambda_M^2}$-characterizes the size of the meson

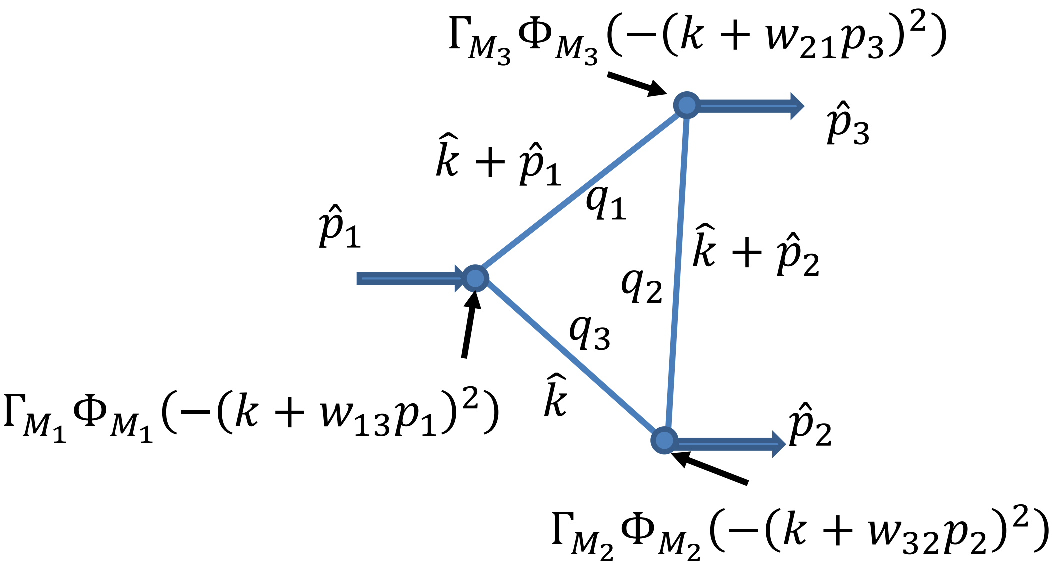
TWO QUARK MODEL (CCQM)



$$w_{1,2} = \frac{m_{q_{1,2}}}{m_{q_1}+m_{q_2}}$$

$$\Pi_M(p^2) = 3g_M^2 \int \frac{d^4k}{(2\pi)^4 i}\Phi_M^2(-k^2)Tr\{\Gamma_M\, S_{q_1}(\hat{k} - w_1\hat{p})\Gamma_M S_{q_2}(\hat{k} + w_2\hat{p})\}$$

TWO QUARK MODEL (CCQM)



$$w_{ij} = \frac{m_{q_i}}{m_{q_i+}m_{q_j}}$$

$$\boldsymbol{T_{M_1M_2M_3}(\hat{p}_{M_1},\hat{p}_{M_2},\hat{p}_{M_3}) = 3g_{M_1}g_{M_2}g_{M_3}} \times$$

$$\times \int \frac{d^4k}{(2\pi)^4 i} \Phi_{M_1}(-(k+w_{13}p_1)^2)\Phi_{M_2}(-(k+w_{32}p_3)^2)\Phi_{M_3}(-(k+w_{21}p_3)^2)\cdot$$

$$\cdot\, Tr\{\Gamma_{M_1}S_{q_1}(\hat{k}+\hat{p}_1)\Gamma_{M_2}S_{q_2}(\hat{k}+\hat{p}_2)\Gamma_{M_3}S_{q_3}(\hat{k})\}$$

TWO QUARK MODEL (CCQM)

$$S_q(\hat{k}) = \frac{1}{m_q - \hat{k} - i\epsilon}$$ *-free propagator of constituent quark*

Fock-Shwinger representation:

$$S_q(\hat{k}+\hat{p}) = \frac{1}{m_q - \hat{k} - \hat{p}} = \frac{m_q + \hat{k} + \hat{p}}{m_q^2 - (k+p)^2} =$$

$$= \left(m_q + \hat{k} + \hat{p}\right) \int_0^\infty d\alpha\, e^{-\alpha\left(m_q^2 - (k+p)^2\right)}$$

TWO QUARK MODEL (CCQM)

$$\Pi_M(p^2) = 3\frac{g_M^2}{4\pi^2}\int\frac{d^4k}{(2\pi)^2 i}Tr\left\{\Gamma_M\left(m_{q_1}+\left(\hat{k}-w_1\hat{p}\right)\right)\,\Gamma_M\left(m_{q_2}+\left(\hat{k}+w_2\hat{p}\right)\right)\right\}\times$$

$$\times\int_0^\infty d\alpha_1\int_0^\infty d\alpha_2 e^{ak^2+2kr-z_0}$$

$$a=\frac{2}{\Lambda_M^2}+\alpha_1+\alpha_2 \qquad r=p(\alpha_1 w_1-\alpha_2 w_2)=pb$$

$$z_0=-m_1^2\alpha_1-m_2^2\alpha_2+p^2\left(w_1^2\alpha_1+w_2^2\alpha_2\right)$$

$$T_{M_1M_2M_3}\left(\hat{p}_{M_1},\hat{p}_{M_2},\hat{p}_{M_3}\right)=\frac{3g_{M_1}g_{M_2}g_{M_3}}{4\pi^2}\times$$

$$\int\frac{d^4k}{(2\pi)^4 i}Tr\{\Gamma_{M_1}(m_{q_1}+\hat{k}+\hat{p}_1)\Gamma_{M_2}(m_{q_2}+\hat{k}+\hat{p}_2)\Gamma_{M_3}(m_{q_3}+\hat{k})\}\times$$

$$\times\int_0^\infty d\alpha_1\int_0^\infty d\alpha_2\int_0^\infty d\alpha_3 e^{a(\alpha)k^2+2kr(\alpha,p)-z_0(\alpha,m_q,p)}$$

TWO QUARK MODEL (CCQM)

$$k^{\mu} e^{ak^2+2kr+z_0} = \frac{1}{2}\frac{\partial}{\partial r^{\mu}} e^{ak^2+2kr+z_0}$$

$$k^{\mu} k^{\nu} e^{ak^2+2kr+z_0} = \frac{1}{2}\frac{\partial}{\partial r^{\mu}}\frac{1}{2}\frac{\partial}{\partial r^{\nu}} e^{ak^2+2kr+z_0}$$

$$Tr\left\{\Gamma_M\left(m_{q_1}+(\hat{k}-w_1\hat{p})\right)\,\Gamma_M\left(m_{q_2}+(\hat{k}+w_2\hat{p})\right)\right\}\Longrightarrow$$

$$\Longrightarrow Tr\{\Gamma_M(m_{q_1}+\gamma^{\mu})\,\Gamma_M(m_{q_2}+\gamma^{\nu})\}\left(\frac{1}{2}\frac{\partial}{\partial r^{\mu}}-w_1p^{\mu}\right)\left(\frac{1}{2}\frac{\partial}{\partial r^{\nu}}+w_2p^{\nu}\right)$$

$$Tr\{\Gamma_{M_1}(m_{q_1}+\hat{k}+\hat{p}_1)\Gamma_{M_2}(m_{q_2}+\hat{k}+\hat{p}_2)\Gamma_{M_3}(m_{q_3}+\hat{k})\}\Longrightarrow$$

$$Tr\{\Gamma_{M_1}(m_{q_1}+\gamma^{\mu})\Gamma_{M_2}(m_{q_2}+\gamma^{\nu})\Gamma_{M_3}(m_{q_3}+\gamma^{\sigma})\}\times$$

$$\times\left(\frac{1}{2}\frac{\partial}{\partial r^{\mu}}+w_1p_1^{\mu}\right)\left(\frac{1}{2}\frac{\partial}{\partial r^{\nu}}+w_2p_2^{\nu}\right)\left(\frac{1}{2}\frac{\partial}{\partial r^{\sigma}}\right)$$

TWO QUARK MODEL (CCQM)

$$\int \frac{d^4k}{4\pi^2 i} e^{a(\alpha)k^2+2kr(\alpha,p)-z_0(\alpha,m_q,p)} = \{k_0 = ik_4; k_E^2 \leq 0, p_E^2 \leq 0\} =$$

$$= \frac{1}{a(\alpha)} e^{-\frac{r^2(\alpha,p)}{a(\alpha)}-z_0(\alpha,m_q,p)}$$

$$\frac{\partial}{\partial r^\mu} e^{-\frac{r^2}{a}} = e^{-\frac{r^2}{a}}\left(-\frac{2r^\mu}{a}+\frac{\partial}{\partial r^\mu}\right)$$

$$\frac{\partial}{\partial r^\mu}\frac{\partial}{\partial r^\nu} e^{-\frac{r^2}{a}} = e^{-\frac{r^2}{a}}\left(-\frac{2r^\mu}{a}+\frac{\partial}{\partial r^\mu}\right)\left(-\frac{2r^\nu}{a}+\frac{\partial}{\partial r^\nu}\right)$$

$$\left[\frac{\partial}{\partial r^\mu}, r^\nu\right] = g^{\mu\nu}$$

TWO QUARK MODEL (CCQM)

For any graph one ,obtains

$$G = \int_0^\infty d^n\alpha\, F(\alpha_1, \alpha_2, \cdots, \alpha_n)$$

The set of Schwinger parameters $\alpha_i$can be turned into a simplex by introducing an additional t–integration via the identity

$$1 = \int_0^\infty dt\, \delta\left(t - \sum_{i=1}^{n} \alpha_i\right)$$

$$G = \int_0^\infty d^n\alpha \int_0^\infty dt\, \delta\left(t - \sum_{i=1}^{n} \alpha_i\right) F(\alpha_1, \alpha_2, \cdots, \alpha_n) = \{\alpha_i = t\alpha_i\} =$$

$$= \int_0^\infty dt\, t^{n-1} \int_0^1 d^n\alpha\delta\left(1 - \sum_{i=1}^{n} \alpha_i\right) F(t\alpha_1, t\alpha_2, \cdots, t\alpha_n)$$

TWO QUARK MODEL (CCQM)

One can remove all possible thresholds present in the initial quark diagram
by cutting the scale integration at the upper limit corresponding
to the introduction of an infrared cutoff :

$$\int_0^\infty dt \to \int_0^{\frac{1}{\lambda^2}} dt$$

So

$$G^c = \int_0^{\frac{1}{\lambda^2}} dt\, t^{n-1} \int_0^1 d^n\alpha\, \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, t\alpha_2, \cdots, t\alpha_n)$$

TWO QUARK MODEL (CCQM)

Model parameters:

Universal cutoff parameter λ:

$$\lambda = 0,181\ GeV$$

Constituent quark masses

$$m_u = m_d = 0,241\ GeV$$

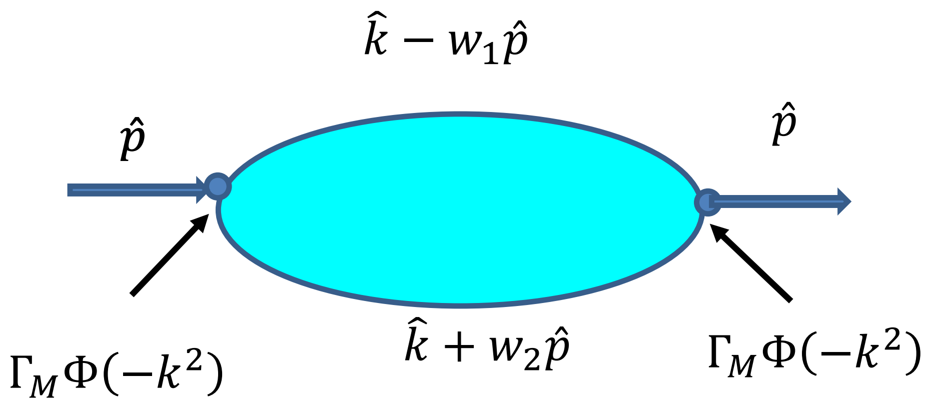
$$m_s = 0,428\ GeV$$

Size parameters of hadrons Λ_M

$$\Lambda_\pi = 0,711 GeV$$

$$\Lambda_S - have\ to\ be\ determined$$

TWO QUARK MODEL (CCQM)



Compositeness condition:

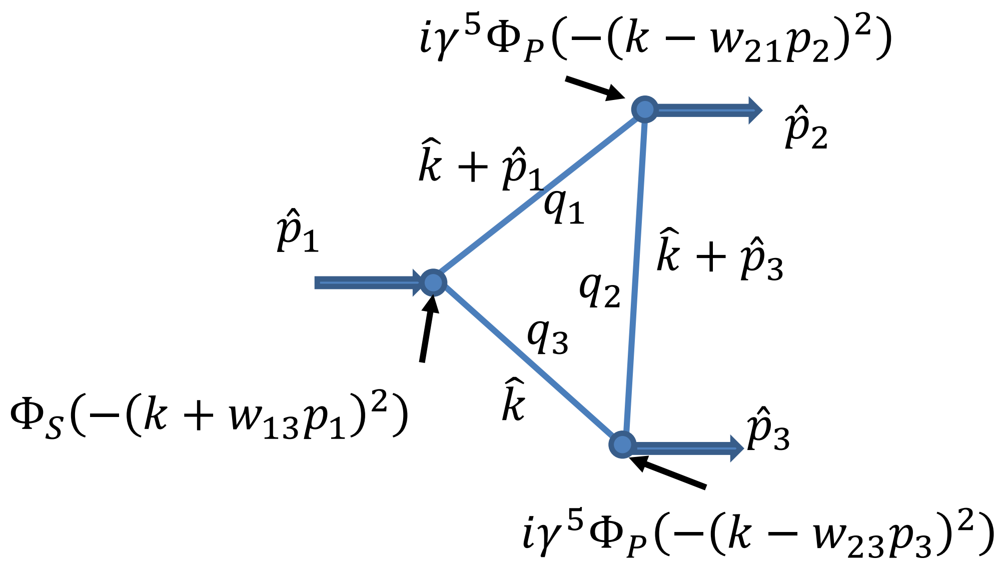
$$Z_M = 1 - \Pi'_M(p^2)\Big|_{p_M^2 = m_M^2} = 0$$

$$\Pi_S(p^2) = 3g_S^2 \int \frac{d^4k}{(2\pi)^4 i} \Phi^2(-k^2) Tr\{I\, S_1(\hat{k} - w_1\hat{p}) I\, S_2(\hat{k} + w_2\hat{p})\}$$

$$\Pi'_S(p^2) = \frac{1}{2p^2} 3g_S^2 \int \frac{d^4k}{(2\pi)^4 i} \Phi^2(-k^2)$$

$$Tr\{S_1(\hat{k} - w_1\hat{p}) w_1\hat{p} S_1(\hat{k} - w_1\hat{p}) S_2(\hat{k} + w_2\hat{p})\} + m_1 \leftrightarrows m_2$$

TWO QUARK MODEL (CCQM)

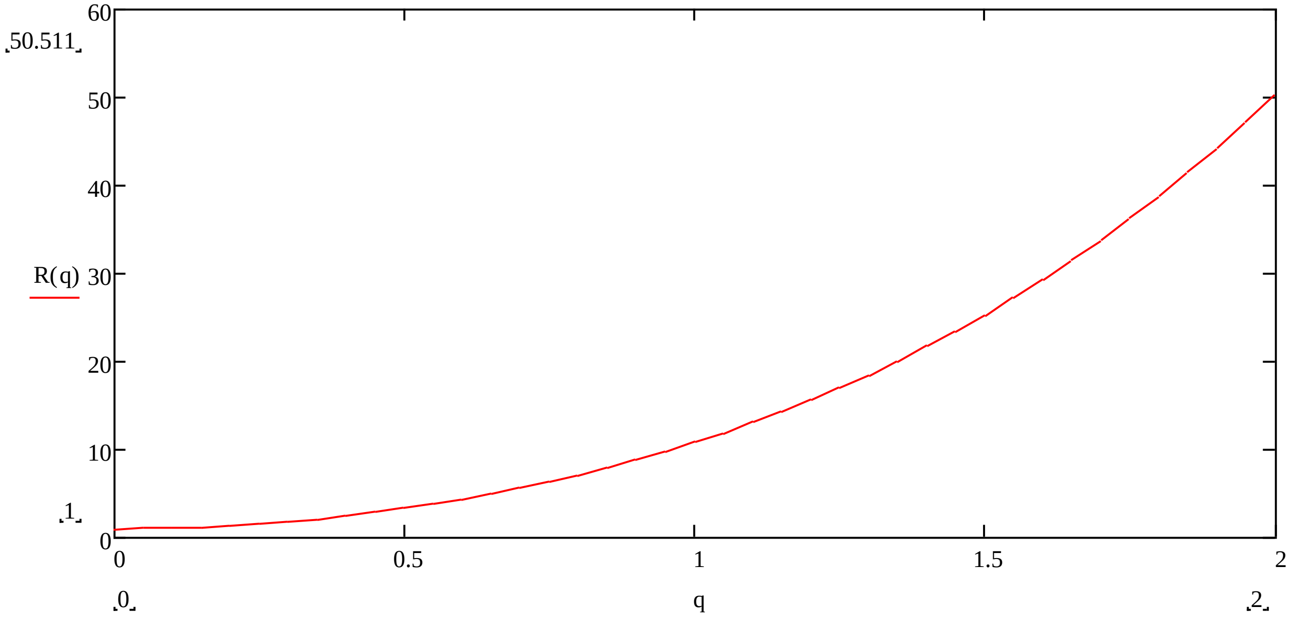


$$T_{SPP}(\hat{p}_{M_1},\hat{p}_{M_2},\hat{p}_{M_3}) = 3g_S g_{P_1} g_{P_2} \times$$

$$\times \int \frac{d^4k}{(2\pi)^4 i} \Phi_S(-(k+w_{13}p_1)^2)\Phi_P(-(k-w_{23}p_3)^2)\Phi_P(-(k-w_{21}p_2)^2)\cdot$$

$$\cdot Tr\{IS_{q_1}(\hat{k}+\hat{p}_1)i\gamma^5 S_{q_2}(\hat{k}+\hat{p}_3)i\gamma^5 S_{q_3}(\hat{k})\}$$

Mass dependence of S ⟶ PP in QCCM
(two quark model)



FOUR QUARK MODEL

- Scalars as diquark- antidiquark states

$$f_0(600)(\sigma) \Rightarrow (ud)(\bar{u}\bar{d})$$

$$K_0^*(800), (\kappa) \Rightarrow (su)(\bar{u}\bar{d}); (sd)(\bar{u}\bar{d}) + c.c$$

$$f_0(980) \Rightarrow \frac{(su)(\bar{s}\bar{u}) + (sd)(\bar{s}\bar{d})}{\sqrt{2}}$$

$$a_0(980) \Rightarrow (su)(\bar{s}\bar{d}); \frac{(su)(\bar{s}\bar{u}) - (sd)(\bar{s}\bar{d})}{\sqrt{2}}; (sd)(\bar{s}\bar{u})$$

FOUR QUARK MODEL

❑For four-quark states

$$J_M(x) = \int dx_1 \int dx_2 \int dx_3 \int dx_4\, \delta\left(x - \sum_{i=1}^{4} \varpi_i x_i\right) \Phi_M\left(\sum_{i<j} (x_i - x_j)^2\right) \cdot$$

$$\cdot \frac{1}{\sqrt{2}} \varepsilon_{abc}\varepsilon_{dec}\{[q_a(x_4)C\Gamma_{M_1} q_b(x_1)][\bar{q}_d(x_3)\Gamma_{M_2} C\bar{q}_d(x_2)] + \Gamma_{M_1} \leftrightarrow \Gamma_{M_2}\}$$

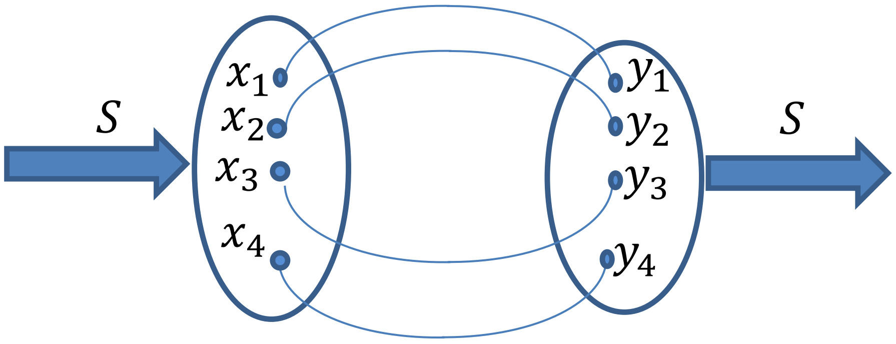
$$\varpi_i = \frac{m_{q_i}}{\sum m_{q_i}} \qquad C = \gamma^0\gamma^2 \quad C = C^\dagger = C^{-1} = -C^T \quad C\Gamma^T C^{-1} = \pm\Gamma; \begin{array}{l} + \text{for } \mathrm{S}, \mathrm{P}, A \\ - \text{for } \mathrm{V}, T \end{array}$$

Scalar current:

$$J_{4q}(x_1, x_2, x_3, x_4) = \varepsilon_{abc}[q_a^T(x_3)C\gamma^5 q_b(x_1)] \cdot \varepsilon_{dec}[\bar{q}_d(x_4)\gamma^5 C\bar{q}_e^T(x_2)]$$

FOUR QUARK MODEL

- Mass operator:



FOUR QUARK MODEL

Mass operator

$$\Pi(x-y) = i\int dx_1 \cdots \int dx_4\, \delta\left(x - \sum x_i \varpi_i\right) \Phi\left(\sum_{i<j} \left(x_i - x_j\right)^2\right)$$

$$\int dy_1 \cdots \int dx_y\, \delta\left(y - \sum y_i \varpi_i\right) \Phi\left(\sum_{i<j} \left(y_i - y_j\right)^2\right) \langle 0 | T\{J_q(x_1, \cdots, x_4) J_q(y_1, \cdots, y_4)\} | 0 \rangle$$

Jucobi coordinates:

$$x_i = x + \sum_{j=1}^{3} w_{ij} \rho_j^x \qquad y_i = y + \sum_{j=1}^{3} w_{ij} \rho_j^y$$

$$\Pi(x-y) = i\int d^3\vec{\rho}_x \Phi(\vec{\rho}_x^{\,2}) \int d^3\vec{\rho}_y \Phi\left(\vec{\rho}_y^{\,2}\right) \langle 0 | T\{J_q(x_1, \cdots, x_4) J_q(y_1, \cdots, y_4)\} | 0 \rangle$$

FOUR QUARK MODEL

$$\widetilde{\Pi}(p,p') = \int dx e^{-ipx} \int dy\, e^{-p'y} \Pi(x-y)$$

$$\widetilde{\Pi}(p,p') = (2\pi)^4 \delta^{(4)}(p-p')\widetilde{\Pi}(p^2)$$

$$\widetilde{\Pi}(p^2) = 12 \prod_{i=1}^{3} \left[\frac{d^4 k}{(2\pi)^4 i}\right] \widetilde{\Phi}(-\vec{\omega}^2) \cdot$$

$$\cdot\, Tr\{\gamma^5 S_1(k_1 - w_1 p)\gamma^5 S_3(k_3 + w_3 p)\} \cdot Tr\{\gamma^5 S_2(k_2 - w_2 p)\gamma^5 S_4(k_1 + k_2 - k_3 + w_4 p)\}$$

$$\vec{\omega}^2 = \frac{1}{2}[k_1^2 + k_2^2 + k_3^2 + k_1 k_2 - k_1 k_3 - k_2 k_3]$$

FOUR QUARK MODEL

$$\frac{d}{dp^2}\widetilde{\Pi}(p,p') = \frac{1}{2p^2}p^\alpha\frac{\partial}{\partial p^\alpha}\widetilde{\Pi}(p,p')$$

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$$\frac{d}{dp^2}\widetilde{\Pi}(p,p') = \frac{1}{2p^2}12\prod_{i=1}^{3}\left[\frac{d^4k_i}{(2\pi)^4 i}\right]\breve{\Phi}^2(-\vec{\omega}^2)\cdot\{\cdots\}$$

$$\{\cdots\} = -w_1 Tr\{\gamma^5 S_1(k_1-w_1p)\hat{p}S_1(k_1-w_1p)\gamma^5 S_3(k_3+w_3p)\}\cdot Tr\{\cdots\}+$$

$$+w_3 Tr\{\gamma^5 S_1(k_1-w_1p)\gamma^5 S_3(k_3+w_3p)\hat{p}S_3(k_3+w_3p)\}\cdot Tr\{\cdots\}-$$

$$-w_2 Tr\{\cdots\}\cdot Tr\{\gamma^5 S_2(k_2-w_2p)\hat{p}S_2(k_2-w_2p)\gamma^5 S_4(k_1+k_2-k_3+w_4p)\}+$$

$$+w_4 Tr\{\cdots\}\cdot Tr\{\gamma^5 S_2(k_2-w_2p)\gamma^5 S_4(k_1+k_2-k_3+w_4p)\hat{p}S_4(k_1+k_2-k_3+w_4p)\}$$

$$S_1(k_1-w_1p)\hat{p}S_1(k_1-w_1p) = \frac{(m_1+k_1-w_1p)\hat{p}(m_1+k_1-w_1p)}{(m_1^2-(k_1-w_1p)^2)^2} =$$

$$= (m_1+k_1-w_1p)\hat{p}(m_1+k_1-w_1p)\int_0^\infty d\alpha_1\alpha_1 e^{-\alpha_1[m_1^2-(k_1-w_1p)^2]}$$

FOUR QUARK MODEL

$$\widetilde{\Pi}'(p^{2\prime}) = 12 \prod_{j=1}^{4} \int_{0}^{\infty} \alpha_j \prod_{i=1}^{3} \left[\frac{d^4 k_i}{(2\pi)^4 i}\right] num\{k_i, \alpha_j\} e^z$$

$$z = kak + 2kr + z_0$$

$$k = \{k_1, k_2, k_3\}$$

r-3-vector, $r_i = b_i(\alpha, w)p_i$

$a = a(\alpha)$ − 3 × 3 matrix

z= $z(p^2, w, m_q, \alpha)$

FOUR QUARK MODEL

$$\widetilde{\Pi}'(p^2) = 12 \prod_{j=1}^{4} \int_0^{\infty} d\alpha_j \prod_{i=1}^{3} \left[\frac{d^4 k_i}{(2\pi)^4 i} \right] num\{k_i, \alpha_j\} e^{z}$$

$$num\{k_i, \alpha_j\} e^{kak+2kr+z_0} = num \left\{ \frac{1}{2} \frac{\partial}{\partial r_i}, \alpha_j \right\} e^{kak+2kr+z_0}$$

$$\prod_{i=1}^{3} \left[\frac{d^4 k_i}{(2\pi)^4 i} \right] e^{kak+2kr+z_0} = \frac{1}{(4\pi)^6} \frac{1}{|a|^2} e^{-ra^{-1}r+z_0}$$

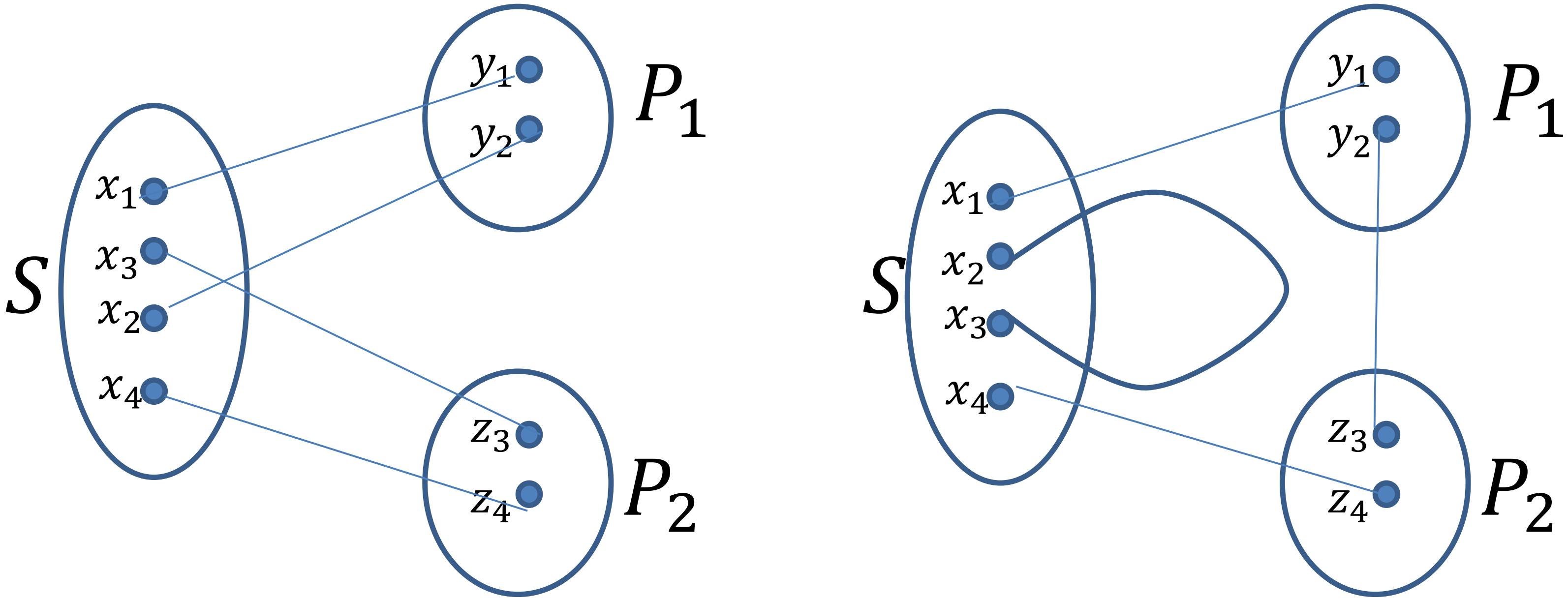
$$num \left\{ \frac{1}{2} \frac{\partial}{\partial r_i}, \alpha_j \right\} e^{-ra^{-1}r+z_0} = e^{-ra^{-1}r+z_0} \left\{ \frac{1}{2} \frac{\partial}{\partial r_i} - (a^{-1}r)_i, \alpha_j \right\}$$

$$\frac{\partial}{\partial r_i^{\alpha}} r_j^{\beta} = \delta_{ij} g^{\alpha\beta} + r_j^{\beta} \frac{\partial}{\partial r_i^{\alpha}}$$

$$\boldsymbol{\widetilde{\Pi}'(p^2) = \frac{12}{(4\pi)^6} \prod_{j=1}^{4} \int_0^{\infty} d\alpha_j \frac{1}{|a|^2} e^{-ra^{-1}r+z_0} num \left\{ \frac{1}{2} \frac{\partial}{\partial r_i} - \left(a^{-1}r\right)_{i'} \alpha_j \right\}}$$

FOUR QUARK MODEL

$S \rightarrow PP$ DECAY



FOUR QUARK MODEL

$$\langle 0 | T\{J_S(x_1, x_2, x_3, x_4) J_{P_1}(y_1, y_2) J_{P_2}(z_1, z_2)\} | 0 \rangle =$$
$$= -6\, Tr\{\gamma_5 S_1(x_1 - y_1)\gamma_5 S_2(y_2 - x_2)\gamma_5 S_4(x_4 - y_4)\gamma_5 S_3(y_3 - x_3)\}$$

$$M(p, q_1, q_2) =$$
$$= -6 i g_S g_{P_1} g_{P_2} \prod_{i=1}^{3} \int \frac{d\omega_i}{(2\pi)^4} \breve{\Phi}_S(-\vec{\omega}^2) \int \frac{dl_1}{(2\pi)^4} \breve{\Phi}_{P_1}(-l_1^2) \int \frac{dl_2}{(2\pi)^4} \breve{\Phi}_{P_2}(-l_2^2) \cdot$$
$$\cdot \prod_{j=1}^{4} \int \frac{dk_j}{(2\pi)^4 i} Tr\{\gamma_5 S_1(k_1)\gamma_5 S_2(k_2)\gamma_5 S_4(k_4)\gamma_5 S_3(k_3)\}$$
$$\int d\rho_1 \int d\rho_2 \int d\rho_3 \int dy_1 \int dy_2 \int dz_1 \int dz_2\, \delta\left(y - \sum_{i=1}^{2} v_i y_i\right) \delta\left(z - \sum_{i=3}^{4} u_i z_i\right)$$
$$exp\{-ipx + iq_1 y + iq_2 z - i\vec{\rho}\vec{\omega} - il_1(y_1 - y_2) - il_2(z_3 - z_4) - ik_1(x_1 - y_1) -$$
$$-ik_2(y_2 - x_2) - ik_3(z_3 - x_3) - ik_4(x_4 - z_4)\}$$

FOUR QUARK MODEL

S	P_1	P_2
$w_i = \frac{m_i}{\sum_{i=1}^{4} m_i}$	$v_i = \frac{m_i}{m_1 + m_2}$	$u_i = \frac{m_i}{m_3 + m_4}$
$x_i = x + \sum_{j=1}^{3} w_{ij}\rho_j$ $w_{ij} = \frac{m_i}{m_i + mj}$	$y = v_1 y_1 + v_2 y_2$	z$= u_3 z_3 + u_4 z_4$
$x \to -p - k_1 + k_2 + k_3 - k_4 = 0$	$y_1 \to v_1 q_1 - l_1 + k_1 = 0$ $y_2 \to v_2 q_1 + l_1 - k_2 = 0$	$z_3 \to u_3 q_2 - l_2 - k_3 = 0$ $z_4 \to u_4 q_2 + l_2 + k_4 = 0$

FOUR QUARK MODEL

$$k_4 = k_3 - q_2$$
$$l_1 = k_1 + v_1 q_1$$
$$k_2 = k_1 + q_1$$
$$l_2 = -k_3 + u_3 q_3$$

$$\rho_1 \to -\omega_1 - k_1 w_{11} + k_2 w_{21} + k_3 w_{31} - k_4 w_{41} = 0$$
$$\rho_2 \to -\omega_2 - k_1 w_{12} + k_2 w_{22} + k_3 w_{32} - k_4 w_{42} = 0$$
$$\rho_3 \to -\omega_3 - k_1 w_{11} + k_2 w_{23} + k_3 w_{33} - k_4 w_{43} = 0$$

$$\Downarrow$$

$$\omega_1 = -\frac{1}{2\sqrt{2}}(2k_1 + (1 + w_1 - w_2)q_1 + (w_1 - w_2)q_2)$$
$$\omega_2 = \frac{1}{2\sqrt{2}}(2k_2 - (w_3 - w_4)q_1 + (1 - w_3 + w_4)q_2)$$
$$\omega_3 = \frac{1}{2}\left((w_3 + w_4)q_1 - (w_1 + w_2)q_2\right)$$

FOUR QUARK MODEL

$$M(p, q_1, q_2) = i(2\pi)^4 \delta(p - q_1 - q_2) T(p^2, q_1^2, q_2^2)$$

$$T(p^2, q_1^2, q_2^2) = -6 g_S g_{P_1} g_{P_2} \int \frac{d^4 k_1}{(2\pi)^4 i} \int \frac{d^4 k_2}{(2\pi)^4 i} \widetilde{\Phi}_S(-\vec{\omega}^2) \widetilde{\Phi}_{P_1}(-(k_1 + v_1 q_1)^2) \cdot$$

$$\cdot \widetilde{\Phi}_{P_2}(-(k_2 + u_4 q_2)^2) Tr\{\gamma_5 S_1(k_1) \gamma_5 S_2(k_1 + q_1) \gamma_5 S_4(k_2) \gamma_5 S_3(k_2 + q_2)\}$$

$$S_i(k) = \frac{m_i + \hat{k}}{m_i^2 - k^2} = \left(m_i + \hat{k}\right) \int_0^\infty d\alpha_i e^{-\alpha_i\left(m_i^2 - k^2\right)}$$

FOUR QUARK MODEL

$$T(p^2, q_1^2, q_2^2) = -6 g_S g_{P_1} g_{P_2} \prod_{i=1}^{4} \int_0^{\infty} d\alpha_i \prod_{j=1}^{2} \int \frac{d^4 k_j}{(2\pi)^4 i} num[k] e^z$$

$$num[k] = Tr\{\gamma_5 (m_1 + k_1) \gamma_5 (m_2 + k_1 + q_1) \gamma_5 (m_4 + k_2) \gamma_5 (m_3 + k_2 + q_2)\}$$

$z = kak + 2kr + z_0$ $\quad$ a - 2 × 2 matrix $\quad$ $r = (r_1, r_2)$, $\quad$ $r_1 = b_{11} q_1 + b_{12} q_2$

$r_2 = b_{21} q_1 + b_{22} q_2$

$$T(p^2, q_1^2, q_2^2) = -\frac{6 g_S g_{P_1} g_{P_2}}{(4\pi)^4} \prod_{j=1}^{4} \int_0^{\infty} d\alpha_j \frac{1}{|a|^2} e^{-ra^{-1}r + z_0} num\left\{\frac{1}{2} \frac{\partial}{\partial r_j} - \left(a^{-1} r\right)_j\right\}$$

Mass dependence of $S \to PP$ in QCCM (four quark model)