RARE DECAYS OF STRANGE MESONS.

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QUARK MODEL

- **The hadronic interactions will be described in the** *Quark Confinement Model (QCM).*

[G.V.Efimov and M.A. Ivanov, *"The Quark Confinement Model of Hadron"*, IOP Publishing, 1993]. **This model is based on the following assumptions**

Lagrangian of the interaction between hadrons and quarks is obtaind

$$L_M = \frac{g_M}{\sqrt{2}} M^i \overline{q}^a_m \Gamma_\mu \lambda^{mn} q^a_n$$

$q^a_j = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$ **-are the quark fields,**

M^i- Euclidean fields connected with the fields of physical particles, λ and Γ are the Gell-Mann and Dirac matrixes, g_M are quark-meson coupling.

QUARK MODEL

- **The quark confinement is provided by nontrivial gluon vacuum background. The confinement ansatz in the case of one-loop quark diagrams:**

$$\int d\sigma_{VAC}\, Tr|M(x_1)S(x_1,x_2|B_{VAC})\ldots M(x_n)S(x_n,x_1|B_{VAC})| \rightarrow$$

$$\int d\sigma_{VAC}\, Tr|M(x_1)S_v(x_1-x_2)\ldots M(x_n)S_v(x_n-x_1)|,$$

- **where**

$$S_v(x_1-x_2) = \int \frac{d^4p}{i(2\pi)^4} e^{-ip(x_1-x_2)} \frac{1}{\sigma\Lambda_q - \hat{p}}$$

QUARK MODEL

The measure $d\sigma_v$ is defined by $\int \frac{d\sigma_v}{v-\hat{z}} = G(\hat{z}) = a(-z^2) + \hat{z}\, b(-z^2)$

- The confinement function G(z) is independent of quark color and flavor and represents an entire function decreasing in the Euclidean region faster then any power of z.
- The choice of G(z) or $a(-z^2)$ and $b(-z^2)$ is one of the model assumptions. We use $a(-z^2)$ and $b(-z^2)$ in a form

$$a(u) = 2e^{-u^2-0.5u},$$

$$b(u) = 2e^{-u^2+0.2u}.$$

- The parameters Λ are

$$\Lambda_u = \Lambda_d = \Lambda_n = 0,460\ GeV$$

$$\Lambda_s = 0,508\ GeV$$

QUARK MODEL

g_M -The coupling constants for meson-quark interaction are defined from so-called compositeness condition

$$Z_M = 1 + \frac{3g_M^2}{4\pi^2}\widetilde{\Pi}'_M(m_M) = 0$$

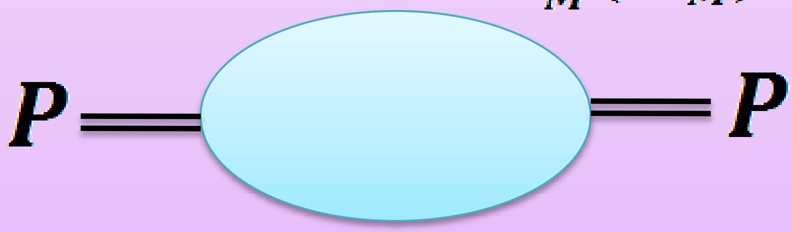
It is convenient to use

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\widetilde{\Pi}'_M(m_M)}$$

MESON-QUARK INTERACTION CONSTANTS

Pseudo scalar meson constant

$$h_M = \frac{3g_M^2}{4\pi^2} = \frac{1}{\widetilde{\Pi}'_M(m_M)}$$



$$\Pi_M(p^2) = \int \frac{d^4k}{4\pi^2 i} \int d\sigma_\lambda \, Sp\left\{\Gamma_\mu S_\lambda(\hat{k}+\hat{p})\Gamma_\mu S_\lambda(\hat{k})\right\}$$

$$\widetilde{\Pi}'_M(p^2) \sim F_{PP}(x,\Lambda)$$

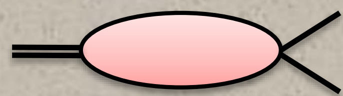
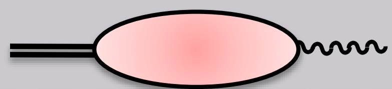
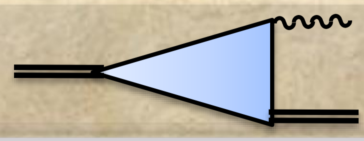
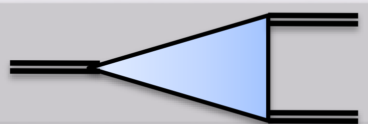
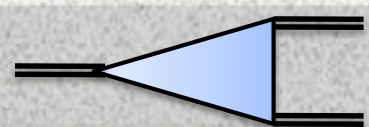
$$F_{PP}(x,\Lambda) = \int_0^\infty du\, b(u) + \frac{x}{4\Lambda^2}\int_0^1 du\, b\left(-\frac{x}{4\Lambda^2}u\right)\frac{1-\frac{u}{2}}{\sqrt{1-u}}$$

EINTRODACTION

1. $K^+ \rightarrow \pi^0 l^+ \nu$

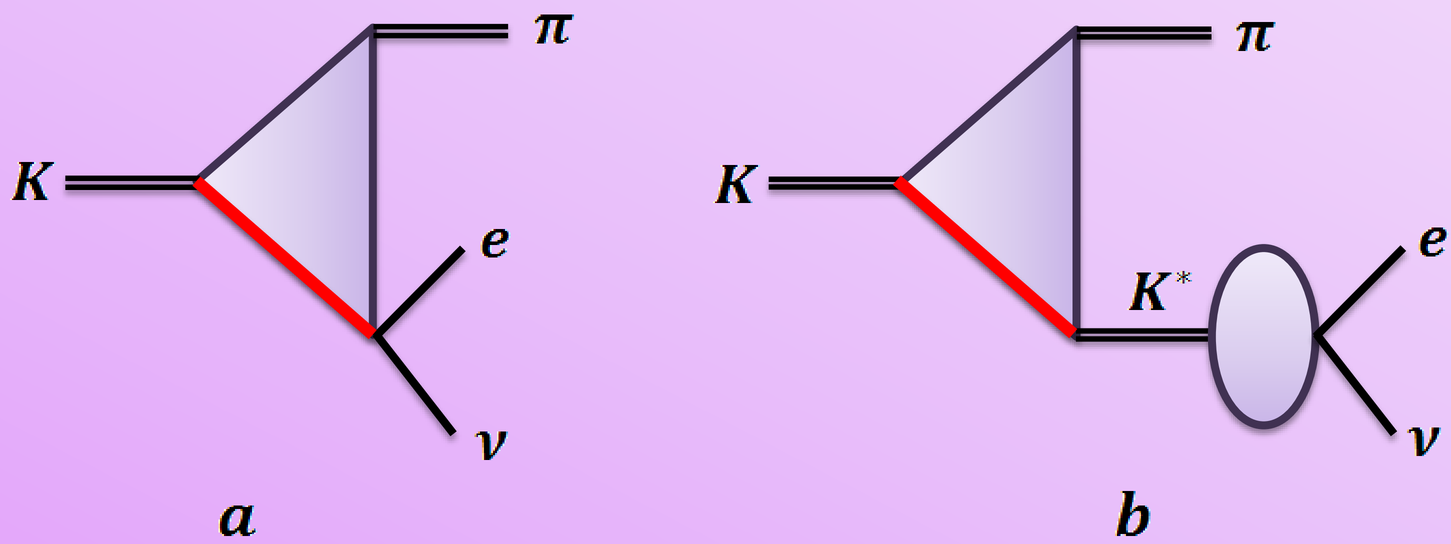
2. $K^+ \rightarrow \pi^+ l^+ l^-$

The main proccesses with S quarks

Proccesses	Vaiue	QCM	Exp.
$K \to \mu\, \nu$ 	f_K	$160\ MeV$	$157\ MeV$
$\varphi \to \gamma$ 	$g_{\varphi\gamma}$	$0,0901$	$0,0758$
$K^* \to K\gamma$ 	$g_{K^*K\gamma}$	$1,17\ GeV^{-1}$	$1,29\,GeV^{-1}$
$K^* \to K\pi$ 	$g_{K^*K\pi}$	$4,22$	$4,65$
$\varphi \to K\bar{K}$ 	$g_{\varphi K\bar{K}}$	$4,04$	$4,47$

K_{l_3} DECAY

$$M^{\mu}(p_1, p_2) = F_{+}(t)(p_1 + p_2)^{\mu} + F_{-}(t)(p_1 - p_2)^{\mu}$$



$$F_{+}(t) = F_{+}^{(a)}(t) + F_{+}^{(b)}(t) \qquad F_{-}(t) = F_{-}^{(a)}(t) + F_{-}^{(b)}(t)$$

$$t = (p_1 - p_2)^2$$

K_{l_3} DECAY

$$F_{+}^{(a)}(t) = \sqrt{2h_K h_\pi}\, F_{VPP}^{-}\left(t, m_K^2, m_\pi^2, \Lambda_s, \Lambda_u, \Lambda_u\right)$$

$$F_{-}^{(a)}(t) = \sqrt{2h_K h_\pi}\, F_{VPP}^{+}\left(t, m_K^2, m_\pi^2, \Lambda_s, \Lambda_u, \Lambda_u\right)$$

$$F_{+}^{(b)}(t) = -t\, F_{+}^{(a)}(t)\, h_{K^*}\, G_{K^*} F_{VV}(t)$$

$$F_{-}^{(b)}(t) = \left(m_K^2 + m_\pi^2\right) F_{+}^{(a)}(t)\, h_{K^*}\, G_{K^*} F_{VV}(t)$$

K_{l_3} DECAY

$$F^{+}_{VPP}\left(p^2,k_1^2,k_2^2,\Lambda_1,\Lambda_2,\Lambda_3\right)=\frac{\Delta p^2}{16\Lambda^2}\int_0^{u_\Delta} du\,u\,b\left(-\frac{p^2}{4\Lambda^2}u\right)\sqrt{1-u+\left(\frac{\Delta u}{2}\right)^2}+$$

$$+\frac{1}{2}\iiint_0^1 d\alpha_1 d\alpha_2\, d\alpha_3\;\delta(1-\alpha_1-\alpha_2-\alpha_3)\;b(-P)\times$$

$$\times\frac{P\cdot[(\alpha_1-\alpha_2)(\Lambda_1-\Lambda_3)(\Lambda_2-\Lambda_3)+\Lambda_3(\Lambda_1-\Lambda_2)]+\alpha_1 k_1^2+\alpha_2 k_2^2}{\alpha_1\Lambda_1^2+\alpha_2\Lambda_2^2+\alpha_3\Lambda_3^2}$$

$$F^{-}_{VPP}\left(p^2,k_1^2,k_2^2,\Lambda_1,\Lambda_2,\Lambda_3\right)=\frac{1}{2}\int_0^{\infty} du\,b(u)+\frac{p^2}{8\Lambda^2}\int_0^{u_\Delta} du\,b\left(-\frac{p^2}{4\Lambda^2}u\right)\sqrt{1-u+\left(\frac{\Delta u}{2}\right)^2}+$$

$$+\frac{1}{2}\iiint_0^1 d\alpha_1 d\alpha_2\, d\alpha_3\;\delta(1-\alpha_1-\alpha_2-\alpha_3)\;b(-P)\times$$

$$\times\frac{P\cdot[(\alpha_1+\alpha_2)(\Lambda_1-\Lambda_3)(\Lambda_2-\Lambda_3)+\Lambda_3(\Lambda_1+\Lambda_2-\Lambda_3)]+\alpha_1 k_1^2+\alpha_2 k_2^2}{\alpha_1\Lambda_1^2+\alpha_2\Lambda_2^2+\alpha_3\Lambda_3^2}$$

$$\Lambda^2=\frac{1}{2}\left(\Lambda_1^2+\Lambda_2^2\right);\quad \Delta=\frac{\Lambda_2^2-\Lambda_1^2}{\Lambda_1^2+\Lambda_2^2}\;;\quad u_\Delta=\frac{2}{1+\sqrt{1-\Delta^2}}$$

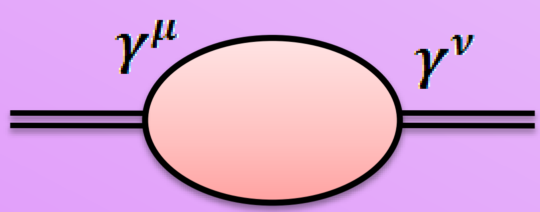
$$P=\frac{\alpha_1\alpha_2 p^2+\alpha_1\alpha_3 k_1^2+\alpha_2\alpha_3 k_2^2}{\alpha_1\Lambda_1^2+\alpha_2\Lambda_2^2+\alpha_3\Lambda_3^2}$$

K_{l_3} DECAY

$$F_{VV} = g^{\mu\nu}\left(\left[\sqrt{1-\Delta^2}-1\right]\cdot\left[b_1 - s^2\int_0^{u_\Delta} du\, u\, b(-us)\sqrt{1-u+\left(\frac{\Delta u}{2}\right)^2}\right] - \frac{\Delta^2 s^2}{2}\int_0^{u_\Delta} du\, u^2\, b(-us)\sqrt{1-u+\left(\frac{\Delta u}{2}\right)^2}\right) +$$

$$+\frac{1}{3\Lambda^2}\left[p^\mu p^\nu - g^{\mu\nu}\right]\left[b_0 + s\int_0^{u_\Delta} du\, u\, b(-us)\sqrt{1-u+\left(\frac{\Delta u}{2}\right)^2}\sqrt{1-u+\left(\frac{\Delta u}{2}\right)^2}\left(1-u+\left(\frac{\Delta u}{2}\right)^2\right)\right]$$

$$h_V G^{\mu\nu}(p^2) = \frac{1}{\Pi_1(p^2)-\Pi_1(m_V^2)}\left\{-g^{\mu\nu} + \frac{p^\mu p^\nu \Pi_2(p^2)}{\Pi_1(p^2)-\Pi_1(m_V^2)+p^2\Pi_2(p^2)}\right\}$$



$$g^{\mu\nu}\Pi_1(p^2) + p^\mu p^\nu \Pi_2(p^2)$$

K_{l_3} DECAY

$$F_\pm(t) = F_\pm(0)\left[1 + \lambda_\pm \frac{t}{m_\pi^2}\right]$$

$$\lambda_\pm = m_\pi^2 \frac{F'_\pm(0)}{F_\pm(0)}$$

$$\xi(0) = \frac{F_-(0)}{F_+(0)}$$

$$\lambda_0 = \lambda_+ + \frac{m_\pi^2}{m_K^2 - m_\pi^2}\xi(0)$$

K_{l_3} DECAY

Value	QCM	Exp[PDG]
λ_+	$0,037 \pm 0,004$	$0,0298 \pm 0,0005$
λ_-	$0,030 \pm 0,0036$	0
$\xi(0)$	$-0,39 \pm 0,047$	$-0,35 \pm 0,14$

$$F_+(m_K^2) + F_-(m_K^2) = 0,9\frac{f_K}{f_\pi}$$

INTERACTION LAGRANGIAN

$$K^+ \rightarrow \pi^+ l^+ l^-$$

$$\mathcal{L}_I = \mathcal{L}_M + \mathcal{L}_w^{eff} + \mathcal{L}_{em}$$

$$\mathcal{L}_{em} = eA_\mu\left(\bar{q}_i^a Q_{ij}\, \gamma^\mu q_j^a + \bar{l}\gamma^\mu l\right),\ \ Q = diag(2/3\,, -1/3\,, -1/3)$$

$$\mathcal{L}_M = \frac{g_M}{\sqrt{2}} M_i^\mu \bar{q}_m^a \Gamma_\mu \lambda_i^{mn} q_m^a$$

$$\mathcal{L}_w^{eff} = \frac{G_F}{2\sqrt{2}}\, V_{ud}\, V_{us} \sum_{i=1}^{6} c_i O_i$$

EFFECTIVE LAGRANGIAN

$$\mathcal{L}_w^{eff} = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{6} c_i O_i$$

$$O_1 = (\bar{d} O_L^{\mu} s)(\bar{u} O_L^{\mu} u) - (\bar{d} O_L^{\mu} u)(\bar{u} O_L^{\mu} s) \qquad \Delta I = 1/2$$

$$O_2 = (\bar{d} O_L^{\mu} u)(\bar{u} O_L^{\mu} s) + (\bar{d} O_L^{\mu} s)(\bar{u} O_L^{\mu} u) + 2(\bar{d} O_L^{\mu} s)(\bar{d} O_L^{\mu} d) + 2(\bar{d} O_L^{\mu} s)(\bar{s} O_L^{\mu} s) \qquad \Delta I = 1/2$$

$$O_3 = (\bar{d} O_L^{\mu} u)(\bar{u} O_L^{\mu} s) + (\bar{d} O_L^{\mu} s)(\bar{u} O_L^{\mu} u) - (\bar{d} O_L^{\mu} s)(\bar{s} O_L^{\mu} s) \qquad \Delta I = 1/2$$

$$O_4 = (\bar{d} O_L^{\mu} u)(\bar{u} O_L^{\mu} s) + (\bar{d} O_L^{\mu} s)(\bar{u} O_L^{\mu} u) - (\bar{d} O_L^{\mu} s)(\bar{d} O_L^{\mu} d) \qquad \Delta I = 3/2$$

$$O_5 = (\bar{d} O_L^{\mu} \lambda^a s) \sum_{q=u,d,s} (\bar{q} O_R^{\mu} \lambda^a q) \qquad \Delta I = 1/2$$

$$O_6 = (\bar{d} O_L^{\mu} s) \sum_{q=u,d,s} (\bar{q} O_R^{\mu} q) \qquad \Delta I = 1/2$$

$$O_L^{\mu} = \gamma^{\mu}(1 - \gamma^5)\,; \quad O_R^{\mu} = \gamma^{\mu}(1 + \gamma^5)$$

EFFECTIVE LAGRANGIAN

$$c_1 = -\chi_1^{4/b}\left(0,98\cdot\chi_2^{0,42} + 0,01\cdot\chi_2^{0,80}\right) + 0,04\chi_1^{-2/b}\left(\chi_2^{0,42} - \chi_2^{-0,30}\right)$$

$$c_2 = 0,2\chi_1^{-2/b}\left(0,96\cdot\chi_2^{-0,30} + 0,03\cdot\chi_2^{0,12}\right) - 0,02\chi_1^{4/b}\left(\chi_2^{0,42} - \chi_2^{-0,30}\right)$$

$$c_5 = 10^{-2}\chi_1^{4/b}\left(3,3\chi_2^{0,42} + 0,3\chi_2^{-0,3} - 3,9\chi_2^{0,80} + 0,3\chi_2^{-0,12}\right) + \\ +0,01\chi_1^{-2/b}\left(-0,1\chi_2^{0,42} - 2,9\chi_2^{-0,30} - 1,4\chi_2^{0,80} - 1,4\chi_2^{-0,12}\right)$$

$$c_6 = 10^{-2}\chi_1^{4/b}\left(4,8\chi_2^{0,42} - 0,6\chi_2^{-0,3} - 2,9\chi_2^{0,80} - 1,3\chi_2^{-0,12}\right) + \\ +0,01\chi_1^{-2/b}\left(-0,2\chi_2^{0,42} - 5,8\chi_2^{-0,30} - 1,0\chi_2^{0,80} + 7,0\chi_2^{-0,12}\right)$$

$$\begin{pmatrix} c_3 \\ c_4 \end{pmatrix} = \chi_2^{-2/9}\chi_1^{-2/b}\begin{pmatrix} 2/15 \\ 2/3 \end{pmatrix}$$

where

$$\chi_1 = 1 + b\cdot\frac{\bar{g}^2(m_c)}{16\pi^2}\cdot ln\frac{m_w^2}{m_c^2} \qquad \chi_2 = 1 + 9\cdot\frac{\bar{g}^2(m)}{16\pi^2}\cdot ln\frac{m_c^2}{m^2}$$

$$b = 11 - \frac{2}{3}N$$

EFFECTIVE LAGRANGIAN COEFFICIENTS

	Input values	c_1	c_2	c_3	c_4	c_5
Donogue J., et al	$\alpha_s = 1$ $m_t = 40\ GeV$	-2,38	0,1	0,084	0,42	-0,047
Minkowski P.	$\alpha_s(m_W) = 0,1$ $\mu = 3GeV$	-3,04	0,32	0,22	1,08	-0,13
Guberina B., et al	$\alpha_s = 1$ $\Lambda = 0,1GeV$ $\lvert\varepsilon\rvert = 1,1$	-5,11	0,03	0,04	0,2	-0,17
QCM	$\mu = 0,25GeV$ $\alpha_s = 0,45$	-1,97	0,12	0,093	0,47	-0,036

$K^+ \to \pi^+ l^+ l^-$

$$\mathcal{M}(K^+ \to \pi^+ l^+ l^-) = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us} \cdot e \cdot F_+(q^2, m_K^2, m_\pi^2) \cdot p^\mu \frac{-i g^{\mu\nu}}{q^2 + i\epsilon} (-ie) \bar{l}(k) \gamma^\nu l(k')$$

where

$$F_+(q^2, m_K^2, m_\pi^2) = \sqrt{h_K h_\pi} \frac{3\Lambda^2}{8\sqrt{2\pi}} \Phi\left(\frac{q^2}{\Lambda^2}, \frac{m_K^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2}\right)$$

$$\Gamma(K^+ \to \pi^+ e^+ e^-) = \frac{{G_F}^2 \alpha^2}{128\pi^3 m_K^3} V_{ud}^2 V_{us}^2 \frac{9 h_K h_\pi}{8\pi} \times$$

$$\times \int_{4m_e^2}^{(m_K - m_\pi)^2} dq^2\, q^2 \left(1 + \frac{2m_e^2}{q^2}\right) \lambda^{3/2}\left(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2}\right) \lambda^{1/2}\left(1, \frac{m_e^2}{q^2}, \frac{m_e^2}{q^2}\right) \left|\Lambda^2 \Phi\left(\frac{q^2}{\Lambda^2}, \frac{m_K^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2}\right)\right|^2$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$$

$K^+ \to \pi^+ l^+ l^-$

$$\Phi\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right) = \Phi_1\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right) + \Phi_2\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right)$$

$$\Phi_2 = \Phi_A + \Phi_P$$

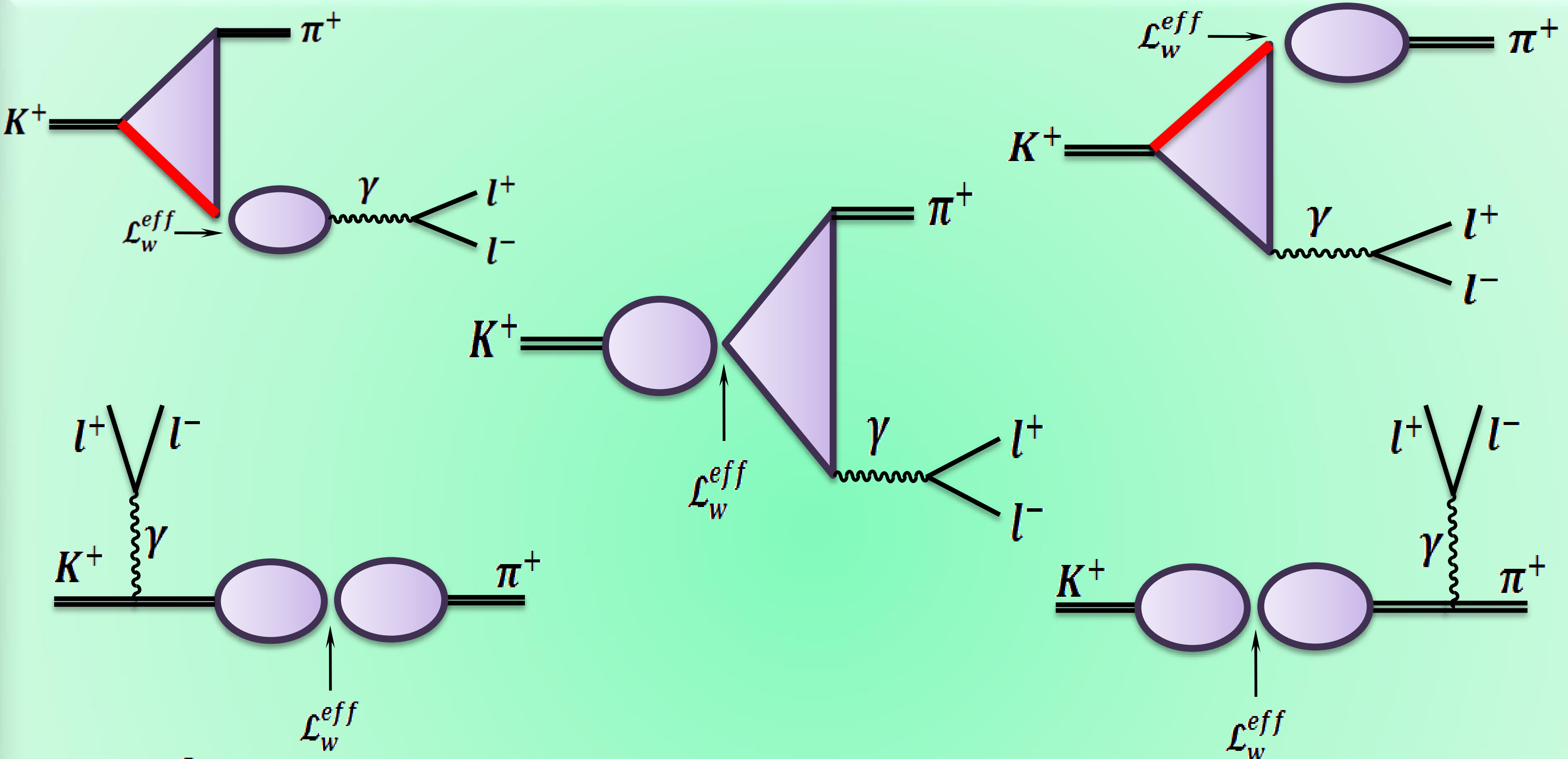
Where Φ_A are the contribution of the intermediate axial-vector meson, Φ_P- pseudo scalar meson

$$\Phi_1\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right) = \frac{2}{3}(-c_1 - 2c_2 - 2c_3 - 2c_4)G_1 + \frac{4}{9}(-c_1 - 2c_2 + 3c_3 + 3c_4)G_2 - \frac{2}{9}c_5 G_3$$

$$\Phi_A\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right) = \frac{2}{3}(c_1 + 2c_2 + 2c_3 + 2c_4)G_{A1} + \\ + \frac{4}{9}(-c_1 - 2c_2 + 3c_3 + 3c_4)G_{A2} - \frac{2}{9}c_5 G_{A3}$$

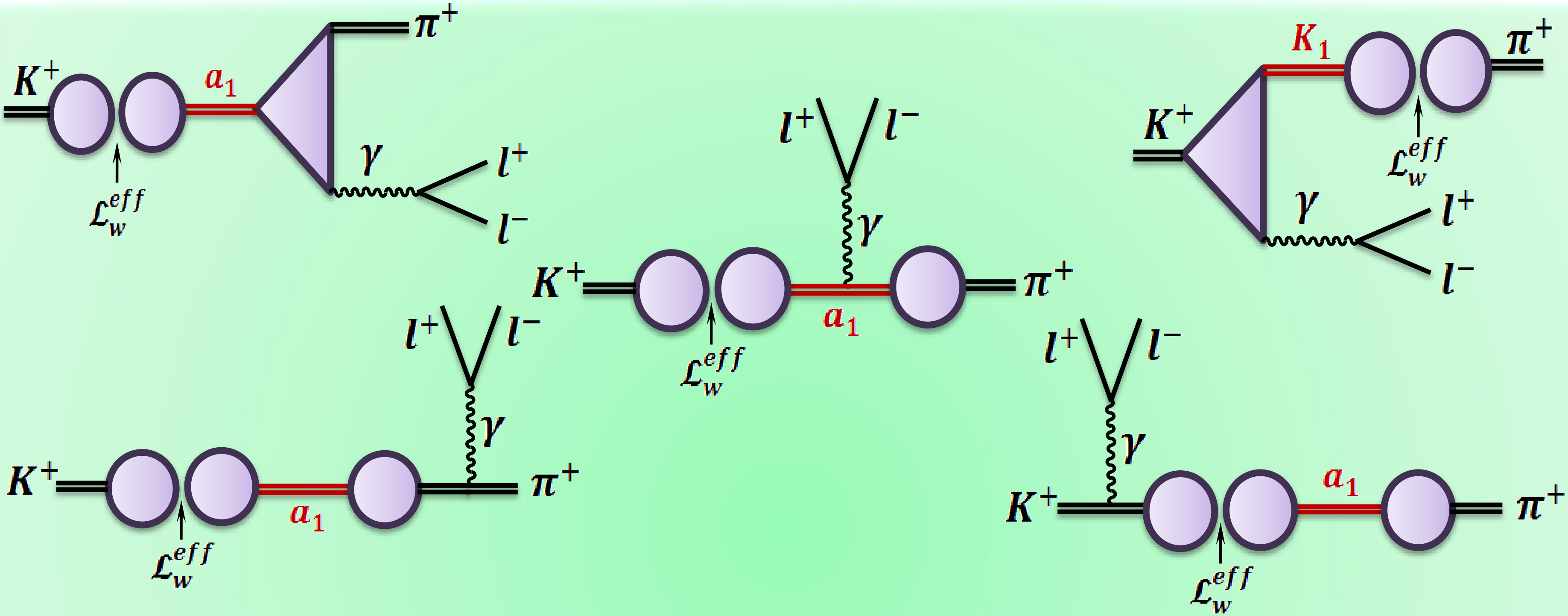
$$\Phi_P\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right) = \frac{2}{3}(c_1 + 2c_2 + 2c_3 + 2c_4)G_{P1} + \frac{2}{3}c_5 G_{P3}$$

$K^+ \to \pi^+ l^+ l^-$



$$\Phi_1\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right)=\frac{2}{3}(-c_1-2c_2-2c_3-2c_4)G_1+\frac{4}{9}(-c_1-2c_2+3c_3+3c_4)G_2-\frac{2}{9}c_5G_3$$

$K^+ \to \pi^+ l^+ l^-$



$$\Phi_A\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right)=\frac{2}{3}(c_1+2c_2+2c_3+2c_4)G_{A1}+$$
$$+\frac{4}{9}(-c_1-2c_2+3c_3+3c_4)G_{A2}-\frac{2}{9}c_5G_{A3}$$

$$\Phi_P\left(\frac{q^2}{\Lambda^2},\frac{m_K^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2}\right)=\frac{2}{3}(c_1+2c_2+2c_3+2c_4)G_{P1}+\frac{2}{3}c_5G_{P3}$$

$K^+ \to \pi^+ l^+ l^-$

$$G_1 = D_{AP}(m_K^2) T_{APV}(m_K^2, m_\pi^2, q^2) + D_{AP}(m_\pi^2) T_{APV}(m_\pi^2, m_K^2, q^2)$$

$$G_2 = q^2 D_{VV}(q^2) T_{PPV}(m_K^2, m_\pi^2, q^2)$$

$$G_3 = \left(D_{PP}(m_K^2) + D_{PP}(m_\pi^2) \right) T_{PPV}(m_K^2, m_\pi^2, q^2)$$

$$G_{A1} = -2 D_{AP}(m_K^2) T_{APV}(m_K^2, m_\pi^2, q^2) \frac{\Pi_1(m_K^2) + m_K^2 \Pi_2(m_K^2)}{\Pi_1(m_K^2) - \Pi_1(m_A^2) + m_K^2 \Pi_2(m_K^2)} -$$

$$-2 D_{AP}(m_\pi^2) T_{APV}(m_\pi^2, m_K^2, q^2) \frac{\Pi_1(m_\pi^2) + m_\pi^2 \Pi_2(m_\pi^2)}{\Pi_1(m_\pi^2) - \Pi_1(m_A^2) + m_\pi^2 \Pi_2(m_\pi^2)}$$

$$G_{A2} = q^2 D_{VV}(q^2) \times$$

$$\times \left[\frac{D_{AP}(m_\pi^2) T_{PPV}(m_\pi^2, m_K^2, q^2)}{\Pi_1(m_\pi^2) - \Pi_1(m_A^2) + m_\pi^2 \Pi_2(m_\pi^2)} - \frac{D_{AP}(m_K^2) T_{PPV}(m_K^2, m_\pi^2, q^2)}{\Pi_1(m_K^2) - \Pi_1(m_A^2) + m_K^2 \Pi_2(m_K^2)} \right]$$

$$G_{A3} = \left(D_{PP}(m_K^2) + D_{PP}(m_\pi^2) \right) \times$$

$$\times \left[\frac{D_{AP}(m_K^2) T_{APV}(m_K^2, m_\pi^2, q^2)}{\Pi_1(m_K^2) - \Pi_1(m_A^2) + m_K^2 \Pi_2(m_K^2)} - \frac{D_{AP}(m_\pi^2) T_{APV}(m_\pi^2, m_K^2, q^2)}{\Pi_1(m_\pi^2) - \Pi_1(m_A^2) + m_\pi^2 \Pi_2(m_\pi^2)} \right]$$

$K^+ \to \pi^+ l^+ l^-$

$$G_{P1} = \left(m_K^2 D_{AA}^2(m_K^2) - m_\pi^2 D_{AP}^2(m_\pi^2)\right) \frac{T_{PPV}(m_K^2, m_\pi^2, q^2)}{2\left(D_{PP}(m_K^2) - D_{PP}(m_\pi^2)\right)}$$

$$G_{P3} = \left(D_{PP}^2(m_K^2) - D_{PP}^2(m_\pi^2)\right) \frac{T_{PPV}(m_K^2, m_\pi^2, q^2)}{2\left(D_{PP}(m_K^2) - D_{PP}(m_\pi^2)\right)}$$

$$A_0 \equiv \int_0^\infty du\, a(u) \qquad B_0 \equiv \int_0^\infty du\, b(u)$$

$$A_1 \equiv \int_0^\infty du\, u\, a(u) \qquad B_1 \equiv \int_0^\infty du\, u\, b(u)$$

$K^+ \to \pi^+ l^+ l^-$

$$D_{AP}(p^2) = -\Lambda\left(A_0 + \mu^2 \int_0^1 du\, a(-u\mu^2)\sqrt{1-u}\right)$$

$$D_{VV}(p^2) = \frac{1}{3}\left(B_0 + \mu^2 \int_0^1 du\, b(-u\mu^2)\sqrt{1-u}\left(1+\frac{u}{2}\right)\right)$$

$$D_{PP}(p^2) = -\Lambda^2\left(B_1 + 2\mu B_0 + 2\mu^2 \int_0^1 du\, b(-u\mu^2)\sqrt{1-u}\right)$$

$$D_{AA}(p^2) = -\Lambda^2\left(2B_1 + \frac{4}{3}\mu B_0 + \frac{4}{3}\mu^2 \int_0^1 du\, b(-u\mu^2)\sqrt{(1-u)^3}\right)$$

$K^+ \to \pi^+ l^+ l^-$

$$T_{APV}(p_1^2, p_2^2, q^2) = -D_{AP}(p_1^2) + \frac{1}{\Lambda}\int_0^1 \{d^3\alpha\}\left(p_1^2\alpha_3 + p_2^2\alpha_3 + q^2(1-\alpha_3)\right) a(Q)$$

$$T_{PPV}(p_1^2, p_2^2, q^2) = B_0 + \frac{q^2}{4\Lambda^2}\int_0^1 du\, b\left(-u\frac{q^2}{4\Lambda^2}\right)\sqrt{1-u} +$$

$$+\frac{1}{\Lambda^2}\int_0^1 \{d^3\alpha\}(p_1^2\alpha_1 + p_2^2\alpha_2) b(Q)$$

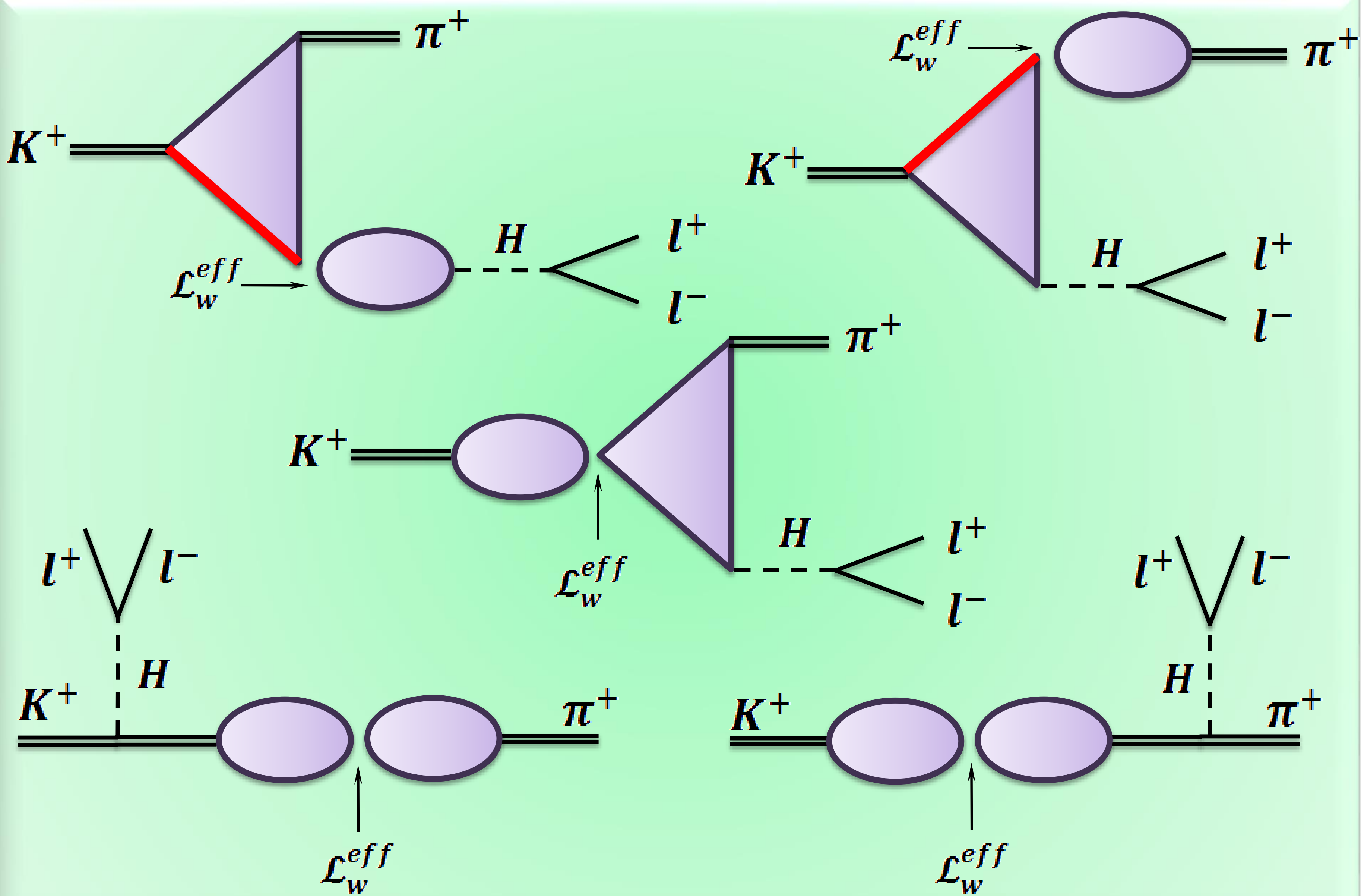
$$Q \equiv \frac{p_1^2\alpha_1\alpha_3 + p_2^2\alpha_2\alpha_3 + q^2\alpha_1\alpha_2}{\Lambda^2} \qquad \mu^2 \equiv \frac{p^2}{4\Lambda^2}$$

$K^+ \to \pi^+ l^+ l^-$

Decay	$Br_{exp} \times 10^{-7}$	$Br_1 \times 10^{-7}$	$Br_2 \times 10^{-7}$
$K^+ \to \pi^+ e^+ e^-$	$2,99 \pm 0,22$	$5,58 \pm 0,56$	$3,23 \pm 0,56$
$K^+ \to \pi^+ \mu^+ \mu^-$	$< 2,3$	$1,15 \pm 0,12$	$0,73 \pm 0,14$

$$\alpha_s = 0,05 - 1,0\,; \quad \mu = 0,9 - 1,9\ GeV$$

$K^+ \to \pi^+ l^+ l^-$



π+
K+
H
l+
l−
L_w^eff
π+
K+
H
l+
l−
L_w^eff
π+
K+
H
l+
l−
L_w^eff
l+
l−
H
K+
π+
L_w^eff
l+
l−
H
K+
π+
L_w^eff

$K^+ \to \pi^+ l^+ l^-$

$$\Gamma_{\rm H}(K^+ \to \pi^+ e^+ e^-) = \frac{{G_F}^2 \alpha^2}{128\pi^3 m_K^3} V_{ud}^2 V_{us}^2 \frac{9 h_K h_\pi}{32\pi^2} \Lambda^4 \times$$

$$\times \int_{4m_e^2}^{(m_K - m_\pi)^2} dq^2\, q^2 \lambda^{3/2}\left(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2}\right) \lambda^{1/2}\left(1, \frac{m_e^2}{q^2}, \frac{m_e^2}{q^2}\right) \frac{g_H^2}{\left(m_H^2 - q^2\right)^2} \left|\Lambda^2 \Phi_{\rm H}\left(\frac{q^2}{\Lambda^2}, \frac{m_K^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2}\right)\right|^2$$

$$\Phi_{\rm H}\left(\frac{q^2}{\Lambda^2}, \frac{m_K^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2}\right) = \frac{4}{3}(c_1 + 2c_2 + 2c_3 + 2c_4)H_1 + \frac{2}{3}(-2c_1 - c_2 - c_3 - c_4)H_2 +$$

$$+\frac{2}{3}(c_1 + 2c_2 + 2c_3 + 2c_4)H_3 - \frac{2}{3} c_5 (A_1 H_{\blacksquare} - 2H_4 + H_5 + H_6)$$

$$g_{Hf\bar{f}} = \frac{m_f}{v} \qquad v = \left(\sqrt{2} G_F\right)^{-1/2} \approx 246\ GeV \qquad m_H \cong 125\ GeV$$