Emitters decay for thermal reactor within particles birth and death model

M. Kravchenko, T. Korbut, A. Kuzmin, Ed. Rudak, A. Petrovski

m.kravch@sosn.bas-net.by

GRODNO - 2018

Birth and death model

Operation of thermal reactor is considered through *instantaneous birth and death rates*:

$$I(t) = \exp \int_{t_1}^{t_2} [\lambda(s) - \mu(s)] ds$$

First was presented in [1,2] in form:

$$I(t) = \int_0^t \frac{ds}{a - b \cdot e^{-s/c}}$$

$$c = \tau_{delayed}$$
$$b = c \cdot \beta$$
$$a = b + \tau_{promt}$$

Definition of neutron generation

Neutron generation – physical amount of neutrons in the breeding medium in a certain moment of time.

$$t_i \rightarrow \text{arbitrary}$$
$$t_{i+1} \rightarrow t_i + \tau_i$$

Defining a function form

Through the analytical investigation the function under integral was obtained:

$$I(t) = \int_0^t \frac{e^{-s/c}}{a - b \cdot e^{-s/c}} ds$$

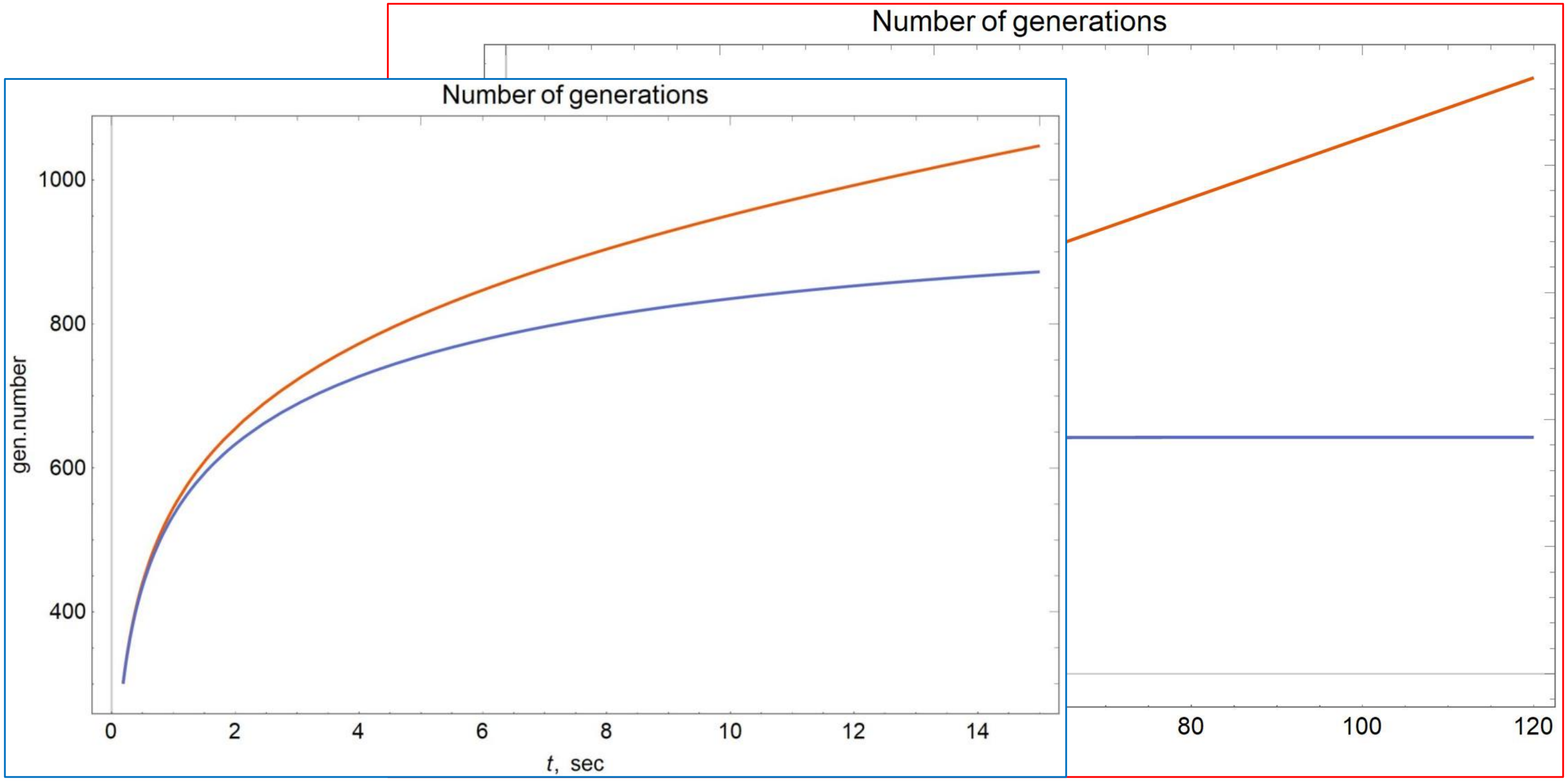
Previous function

$$I(t) = \int_0^t \frac{ds}{(a - b \cdot e^{-s/c})} = (c/a) \cdot \ln[(a \cdot e^{(t/c)} - b)/(a - b)]$$

Current function

$$I(t) = \int_0^t \frac{e^{-s/c}}{(a - b \cdot e^{-s/c})} ds = (c/b) \cdot \ln[(a - b \cdot e^{(-t/c)})/(a - b)]$$

Graphical representation of total generation number



Parameters possible to calculate

$$t_k = -c \cdot \ln\left[\frac{a + (b-a)\exp[b \cdot k / c]}{b}\right]$$

Generation appearance time

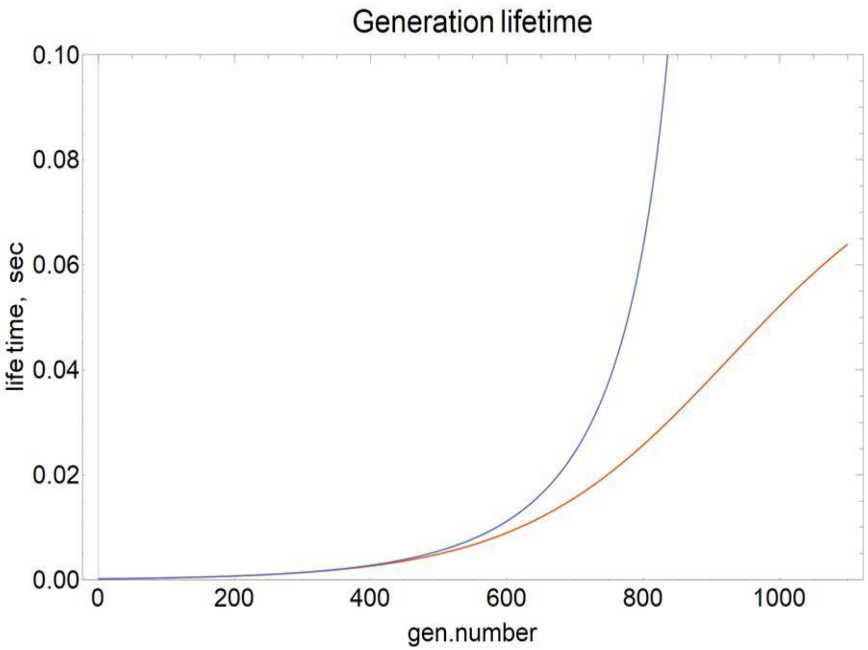
$$\tau_k = t_k - t_{k-1}$$

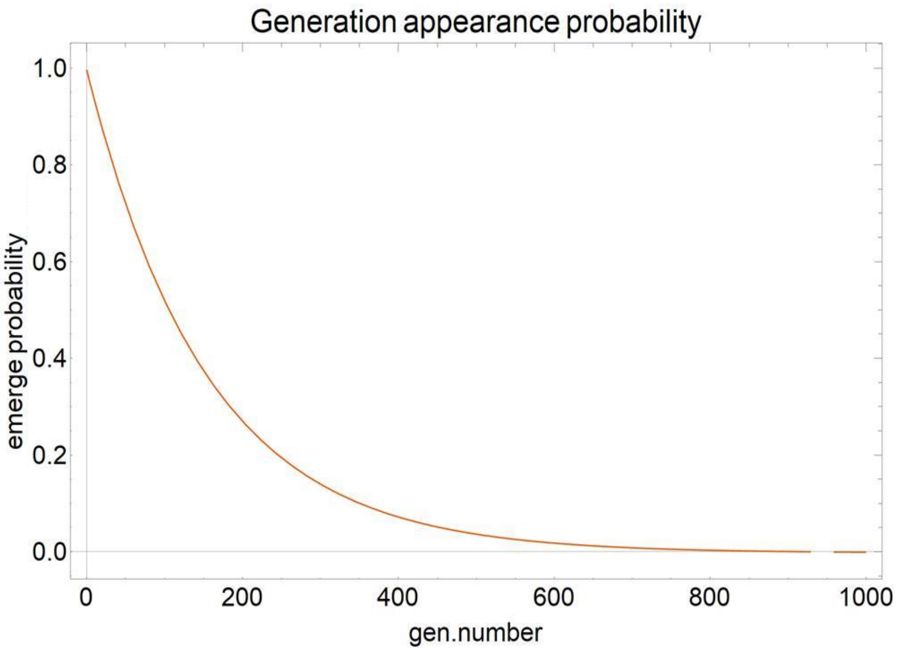
Generation lifetime

$$p_k = \tau_{мг} / \tau_k$$

Generation appearance probability

Graphical representation of life time and appearance probability





Multiplication factor

Calculating multiplication factor from average number of neutrons in a generation:

$M(t_k) = M_0 e^{(\rho+\beta(t_k))t_k/\tau_k}$ – Average number of neutrons

$$K_{k,k-1} = M(t_k)/M(t_{k-1}) = \exp\int_{t_{k-1}}^{t_k}[\lambda(s)-\mu(s)]ds =$$

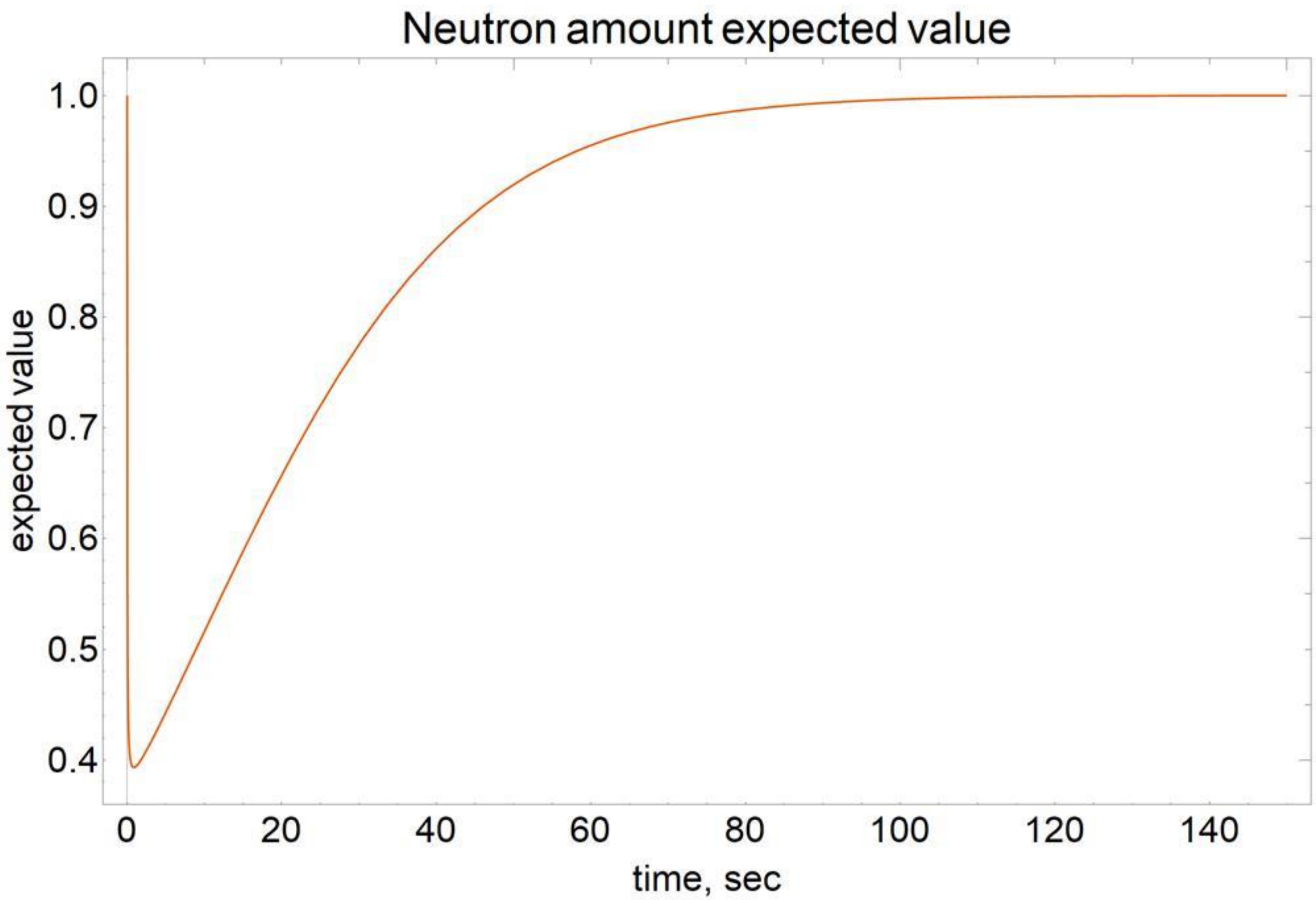
$$= \exp[(\rho+\beta(t_k))\cdot t_k/\tau_k - (\rho+\beta(t_{k-1}))\cdot t_{k-1}/\tau_{k-1}]$$

$$K_{tot} = M(t_k)/M_0 = \exp[(\rho+\beta(t_k)\cdot t_k/\tau_k] \rightarrow K_{tot} = \prod_{i=1}^{k} K_{i,i-1}$$

Relation between the model and the point reactor

$$\langle \lambda(t_k) - \mu(t_k) \rangle \tau_k = (\rho + \beta) \cdot (t_k / \tau_k - t_{k-1} / \tau_{k-1}) -$$
$$\beta \cdot (\exp[t_k / c] \cdot t_k / \tau_k - \exp[t_{k-1} / c] \cdot t_{k-1} / \tau_{k-1})$$

Neutron amount expected value



Conclusions

- The birth and death model is suitable for describing the evolution of an emitter nuclei ensemble in a breeding medium.
- The chosen function for describing the process of birth-death allows us to use the Poisson distribution formalism to obtain analytic expressions. Also the physical meaning of the binomial distribution is preserved in describing the generations of neutrons and their lifetime.
- The mathematical apparatus described allows one to obtain an analytical expression for the multiplication factor - one of the most significant parameters of the breeding medium.

References

1. Корбут, Т.Н. **Тепловой реактор как аналог ADS-систем с внутренним источником нейтронов** / Т.Н. Корбут, А.В. Кузьмин, Э.А. Рудак // Известия РАН. Серия физическая т. 79, №4. 2015. – с.503-511.
2. Korbut, T.N. **Thermal Nuclear Reactor as an Analog of ADS Systems with Internal Sources of Neutrons** / T.N. Korbut, A.V. Kuz'min, E.A. Rudak // Bulletin of the Russian Academy of Sciences. Physics, v. 79, №4. – 2015. – pp. 461-469. © Allerton Press, Inc., 2015.
3. Рудак, Э.А**. Формализм модели рождения и гибели частиц в приближении субпуассоновского распределения**: сб. материалов Междунар. конф. Молодых ученых Ч. 2. / Нац. акад. наук Беларуси, Совет молодых ученых; редкол.: В. Г. Гусаков [и др.]. – Минск: Беларуская навука, 2018. – с. 285-291.
4. Корбут, Т.Н. **Статитстическое описание распада ансамбля ядер эмиттеров в рамках субпуассоновского распределение** / Т.Н. Корбут, Э.А. Рудак, А.М. Петровский // Известия РАН. Серия физическая, т.2, № 10, 2018. – с. 80-86

Thank You for attention!