Monte Carlo Simulation of QGP $\rightarrow$ Hadron First-Order Phase Transitions in Heavy-Ion Collisions

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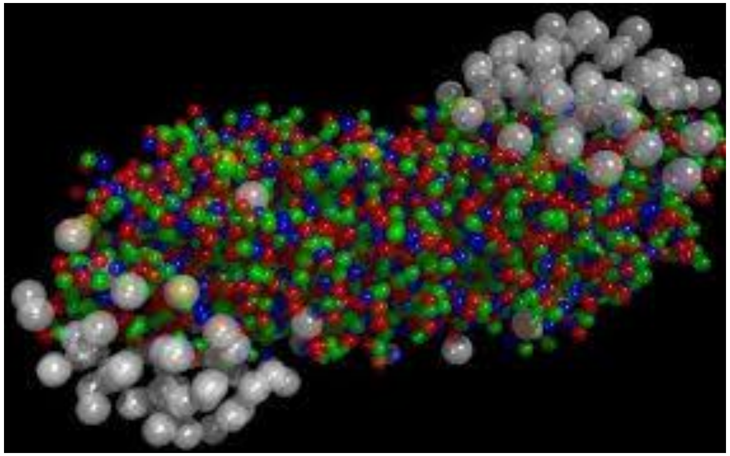
The XIV-th International School-Conference
The Actual Problems of Microworld Physics

In Memory of Professor Nikolai Shumeiko

Grodno, Belarus, August 12-24, 2018

Quark-Gluon Plasma



QGP ≡ *a (locally) thermally equilibrated state of matter in which quarks and gluons are deconfined from hadrons, so that color degrees of freedom become manifest over nuclear, rather than merely nucleonic, volumes.*

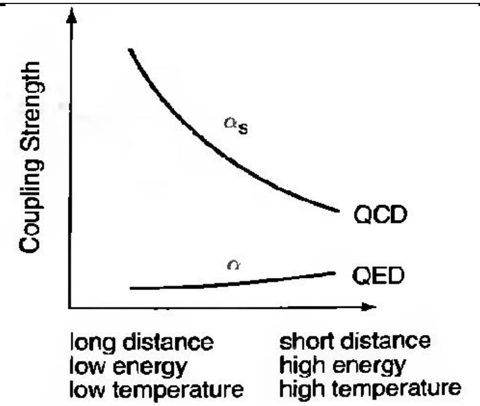
Quantum Chromo Dynamics

- Quantum Chromo Dynamics is the theory describing the interactions of quarks and gluons, the building blocks of the nucleons (proton + neutron).

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right) \psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$
$$= \bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \ ,$$

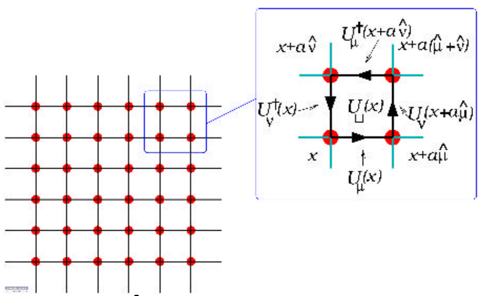
- Quarks are never observed as free particles, they turn in hadrons “confinement” (qq, qqq or qqq).
- All known hadron states are colors singlets (“white”)
- The quantities one wants to calculate in quantum theory are: probabilities and expectation values of a certain combinations (functionals) of fields.
- From these numbers one can subsequently extract information about particles mass, life time decay amplitudes …etc

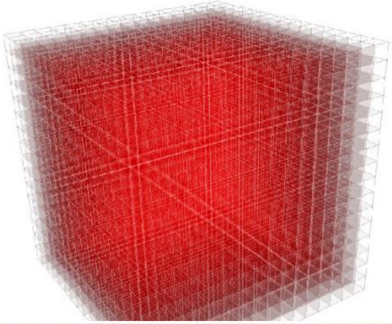
Lattice QCD



At high energies α_s is small. In this regime we treat quarks as free particles and observables can be calculated using perturbation theory.

- At low energies α_s is high. Perturbation theory fails we need a different approach.





Lattice QCD → Path integral method

→ Discretization of space-time …

Systematic errors :

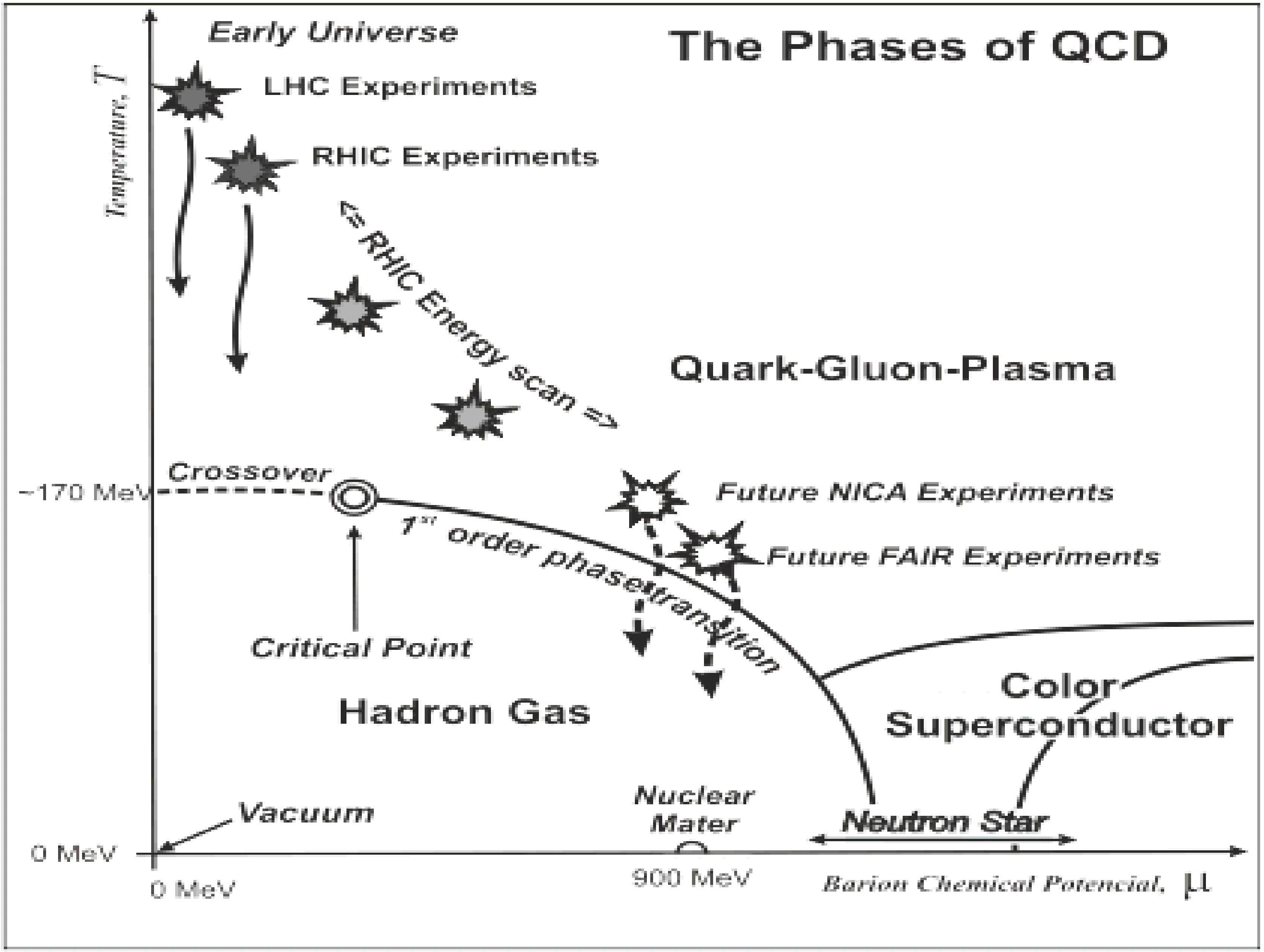
the lattice discretization from the lattice spacing *a.*

the number of point in the lattice universe (finite volume)

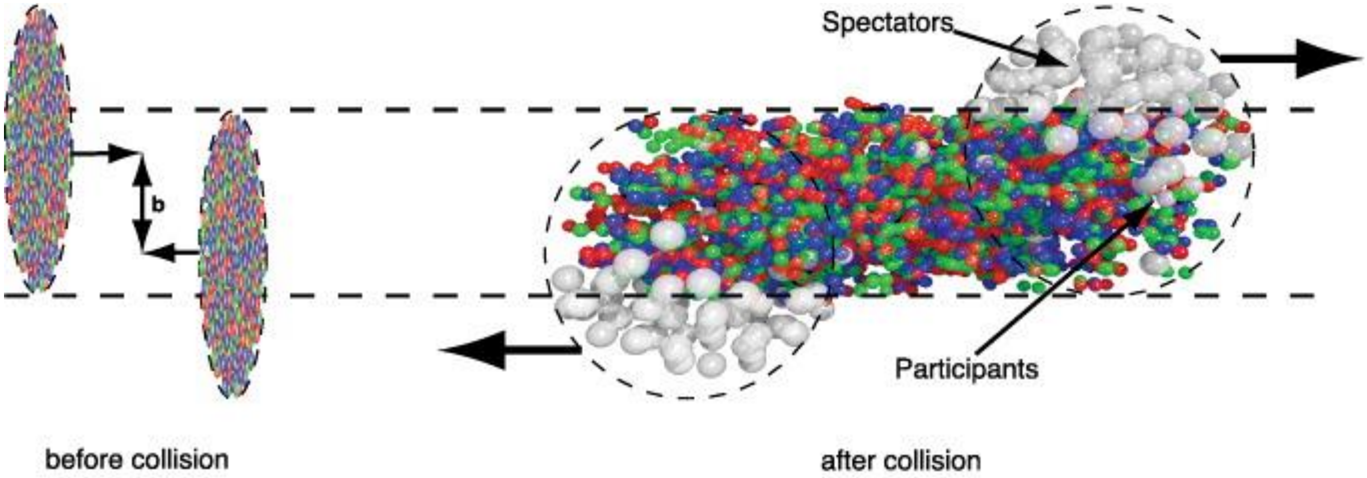
Statistical errors :

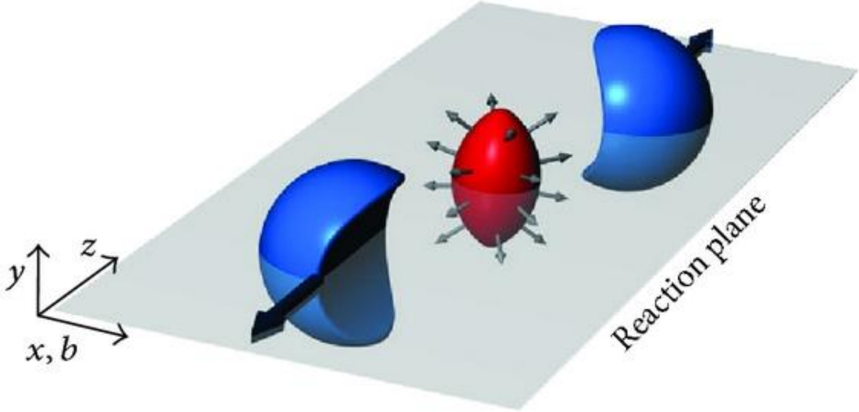
- the finite number of configurations used to compute the value of path integrals.

QGP phase diagramm



Heavy-Ion Collisions

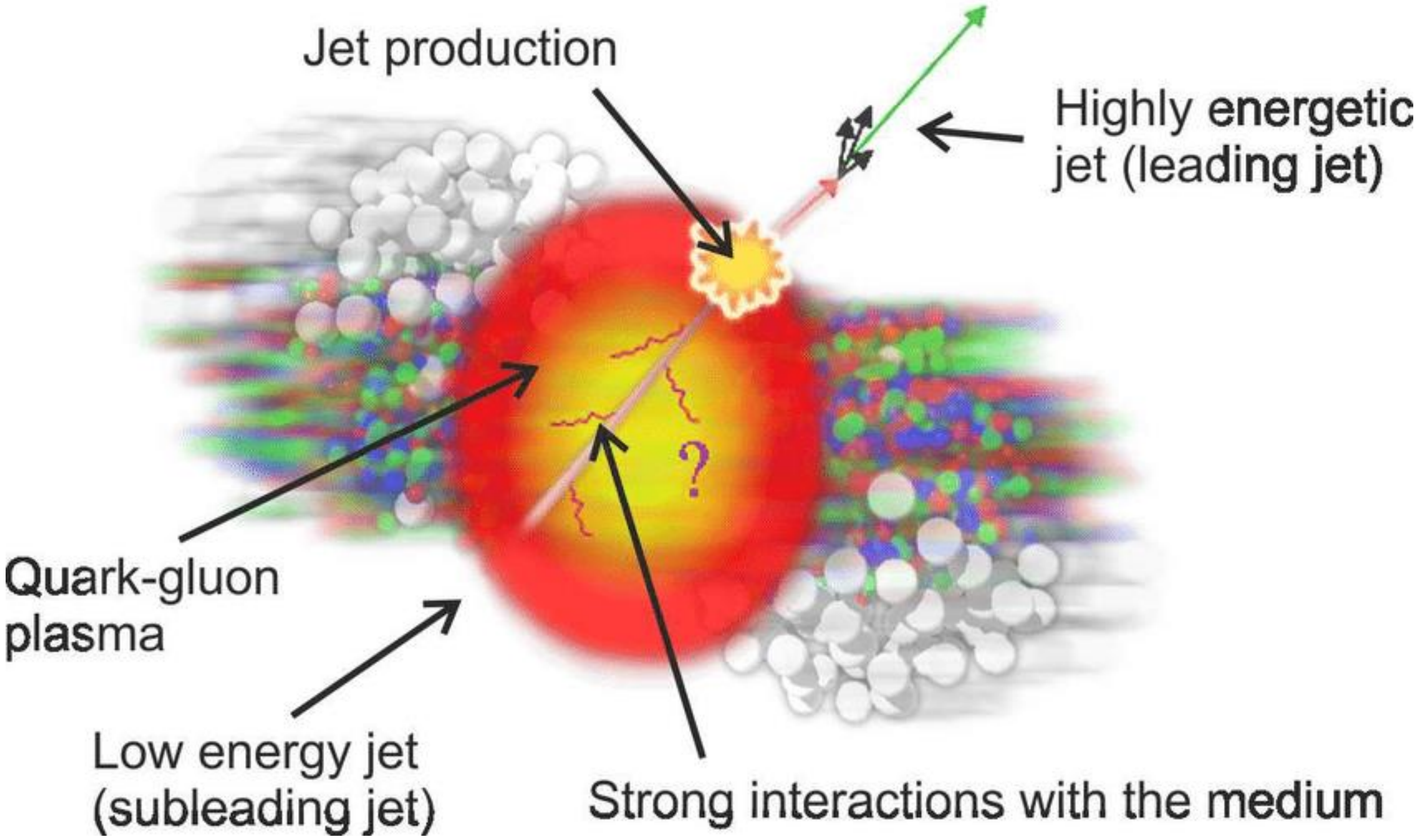




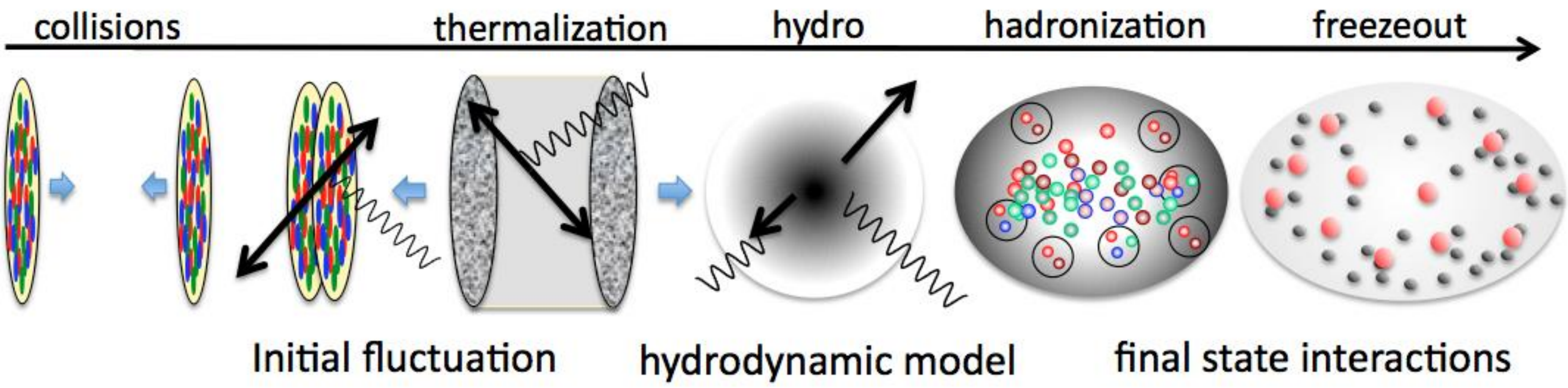
QGP signatures

- Single particle spectra (photons and dileptons)
- Strangeness production
- Photon and muon rates (and J/ψ melting)
- Elliptic flow
- Jet quenching
- Fluctuations

Jet quenching, disappearing away jet



Heavy-Ion Collisions. Hydrodynamic model



Monte Carlo simulations

•**HIJING (Heavy Ion Jet INteraction Generator)**) is Monte Carlo generator with special emphasis on the role of minijets in pp, pA and AA reactions at collider energies. Based on a pQCD-inspired model, multiple minijet production is combined together with Lund-type model for soft interactions. Binary approximation and Glauber geometry for multiple interaction are used to simulate pA and AA collisions.

HYDJET++ (HYDrodynamics plus JETs is a Monte-Carlo event generator for simulation of relativistic heavy-ion AA collisions considered as a superposition of the soft, hydro-type state and the hard state resulting from multi-parton fragmentation.. HYDJET++ is designed for studying the multi-particle production in a wide energy range of heavy-ion experimental facilities: from FAIR and NICA to RHIC and LHC.

Ginzburg-Landau Model
Second order

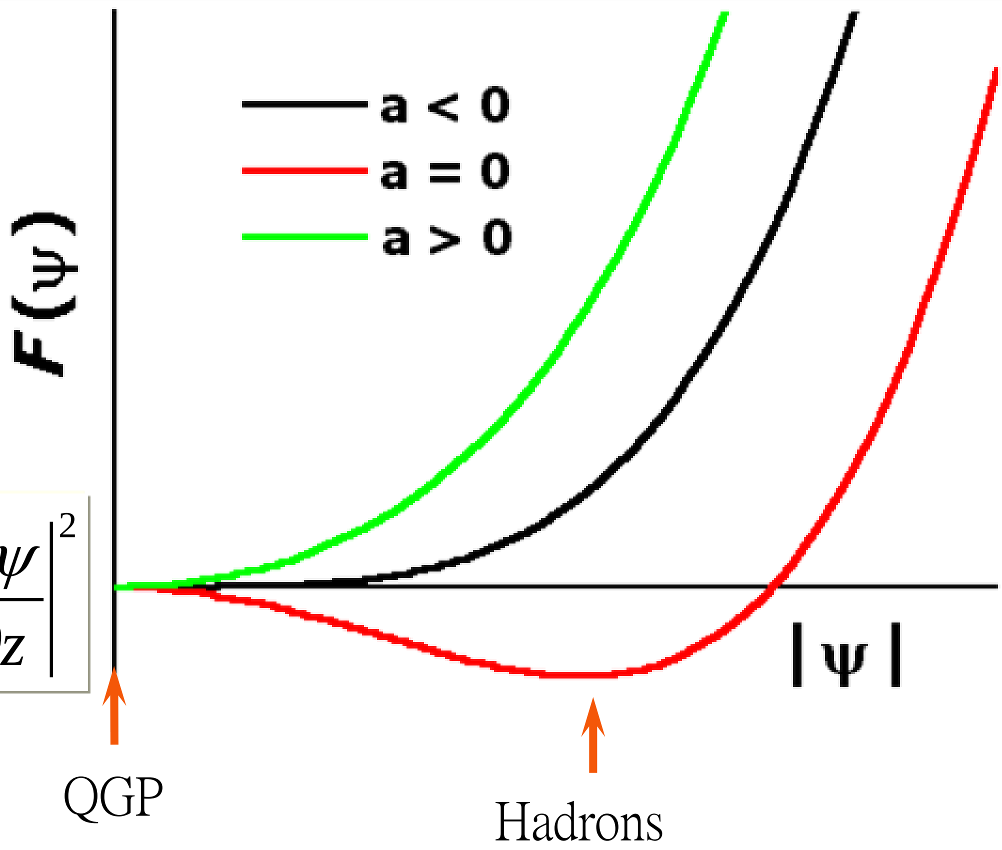
Hwa R. C., Phys. Rev. **D47**, (1993), 2773.

$\psi(z)$ is the order parameter.

In original GL model

free energy density is

$$F(\psi) = \underbrace{a|\psi|^2 + b|\psi|^4}+ c\left|\frac{\partial\psi}{\partial z}\right|^2$$



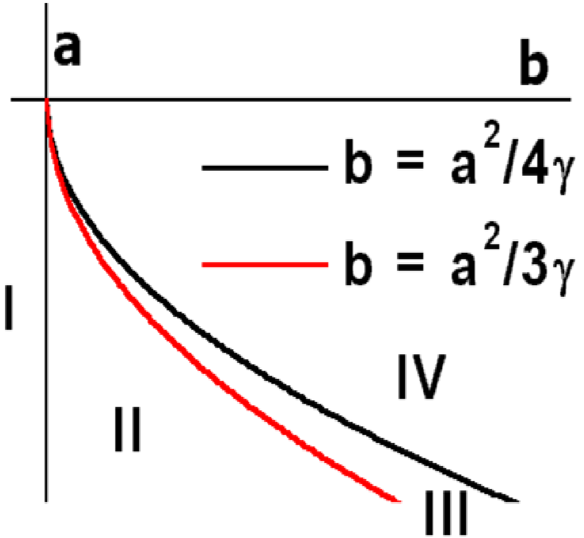
Generalized Ginzburg-Landau Model of phase transitions

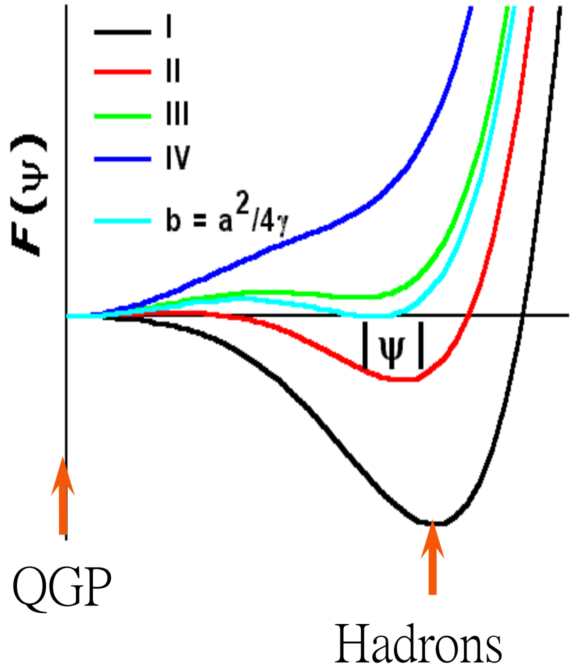
First order phase transition

Babichev L. F., Klenitsky D. V., Kuvshinov V. I., Phys. Lett. **B345**, (1995), 269.

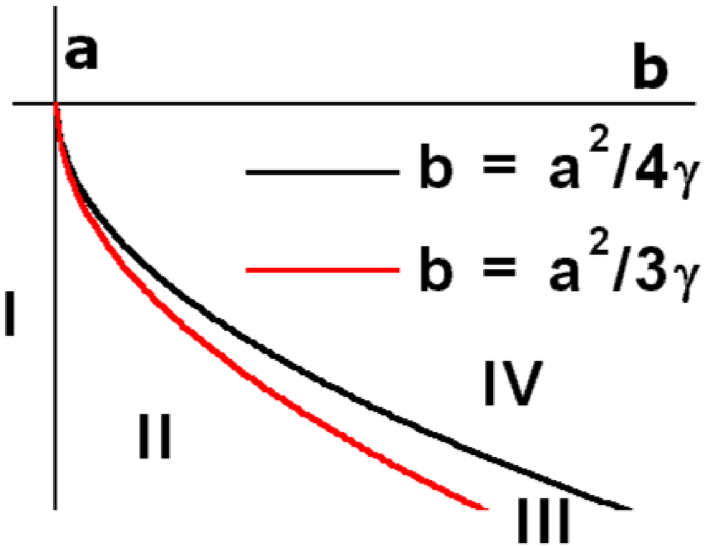
Generalized Ginzburg-Landau free energy density is

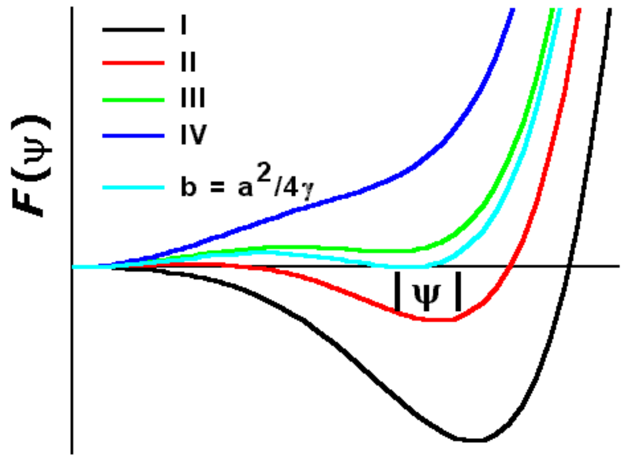
$$F(\psi) = b|\psi|^2 + a|\psi|^4 + \gamma|\psi|^6 + c\left|\frac{\partial\psi}{\partial z}\right|^2$$

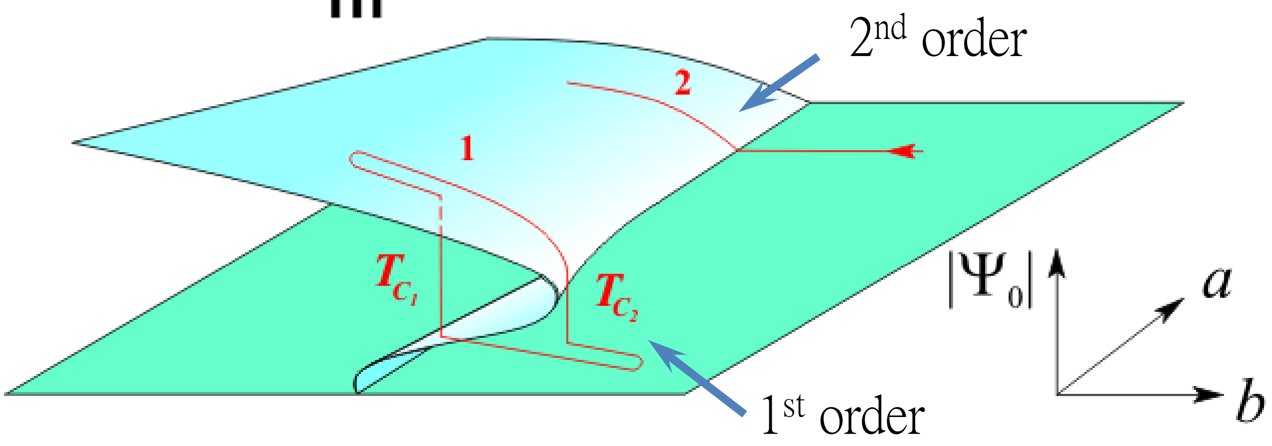




Ginzburg-Landau model of first order phase transitioins







Local multiplicity fluctuations as a signature of phase transitions in heavy-ion collisions

The *factorial moment* of order *q* is defined as average

$$F_q = \frac{\langle n(n-1)...(n-q+1) \rangle}{\langle n \rangle^q}$$

$$F_q = \frac{\sum_n n(n-1)...(n-q+1)P_n}{\left(\sum_n nP_n\right)^q}$$

n=n(δ) is a multiplisity of particle in the bin of the size δ
q is order factorial moment
$P_n(\delta)$ is probability of finding *n* particles in bin δ .

The *normalized factorial moments* (NFMs)

$$F_q = \frac{f_q}{f_1^q}$$

Local Multiplicity Fluctuations as a Signature of QGP → Hadrons Phase Transition

The phase transitions and scaling behavior of factorial moments

The *factorial moment* of order *q* is defined as average

$$F_q = \frac{\langle n(n-1)\ldots(n-q+1)\rangle}{\langle n\rangle^q}$$

$$F_q = \frac{\sum_n n(n-1)\ldots(n-q+1)P_n}{\left(\sum_n nP_n\right)^q}$$

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$P_n(\delta)$ is probability of finding *n* particles in bin δ .

The *normalized factorial moments* (NFMs)

$$F_q = \frac{f_q}{f_1^{\,q}}$$

Intermittency and scaling

Intermittency is defined as power-law dependence:

in the region of small phase bin size.

$$F_q \underset{\delta \to 0}{\propto} \delta^{-\phi_q}$$

intermittency indices

$$\beta_q = \phi_q / \phi_2$$

$$F_q \propto F_2^{\beta_q}$$

Scaling is defined as dependence

$$\beta_q = (q-1)^{\nu}$$

scaling exponent

In heavy-ion experiments.

$$\nu = 1.459 \pm 0.021 \qquad \nu = 1.550 \pm 0.120$$

Multiplicity distribution and fluctuations

P_n^0 is a probability to find n hadrons in a state with fixed $\psi(z)$

Taking fluctuations into account result in

$$P_n = Z^{-1} \int D\psi P_n^0 e^{-\mathrm{F}[\psi]}$$

Normalizing factor

Free energy

$$Z = \int D\psi e^{-\mathrm{F}[\psi]}$$

$$\mathrm{F}[\psi] = \int F(\psi(z))dz$$

Coherent States

Coherent State

$$|\psi\rangle_{CS} = \hat{D}[\psi]|0\rangle$$

$\hat{D}$ - displacement operator

Coherent states multiplicity distribution

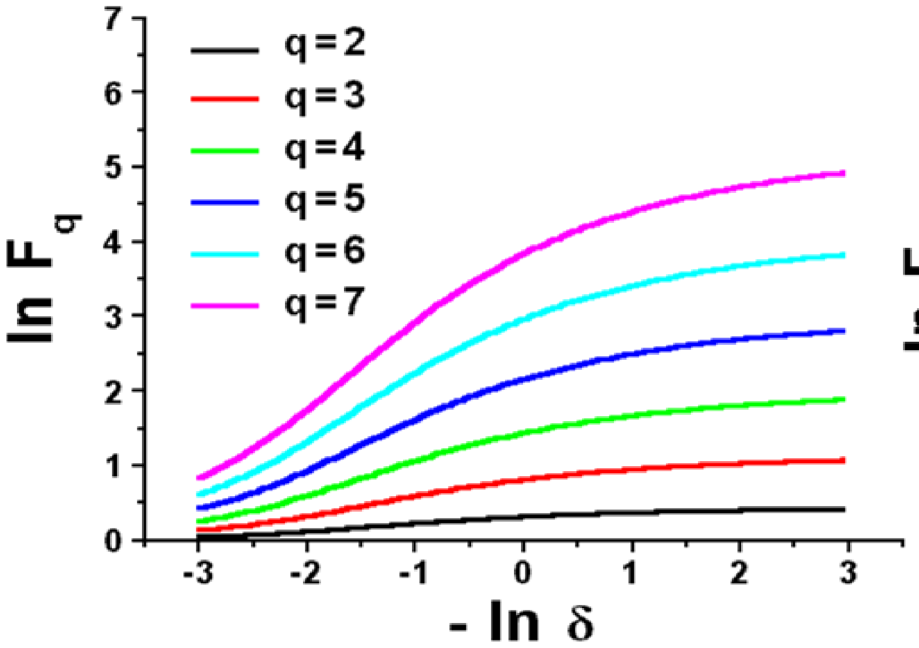
$$P_n^0 = \frac{1}{n!}|\psi|^{2n} e^{-|\psi|^2}$$

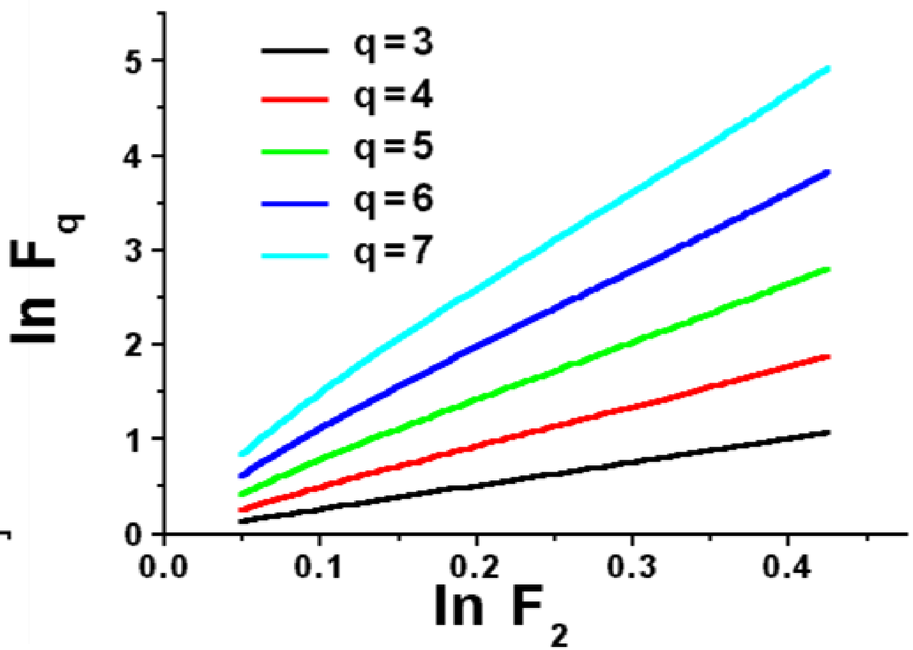
Coherent States (GL)

Second ***order potential***

$$\ln F_q \underset{\delta \to 0}{\propto} -\phi_q \ln \delta$$

$$\ln F_q \propto \beta_q \ln F_2$$





Hwa R. C., Phys. Rev. **D47**, (1993), 2773.

Coherent States

First order potential

$$\ln F_q \underset{\delta \to 0}{\propto} -\phi_q \ln \delta$$

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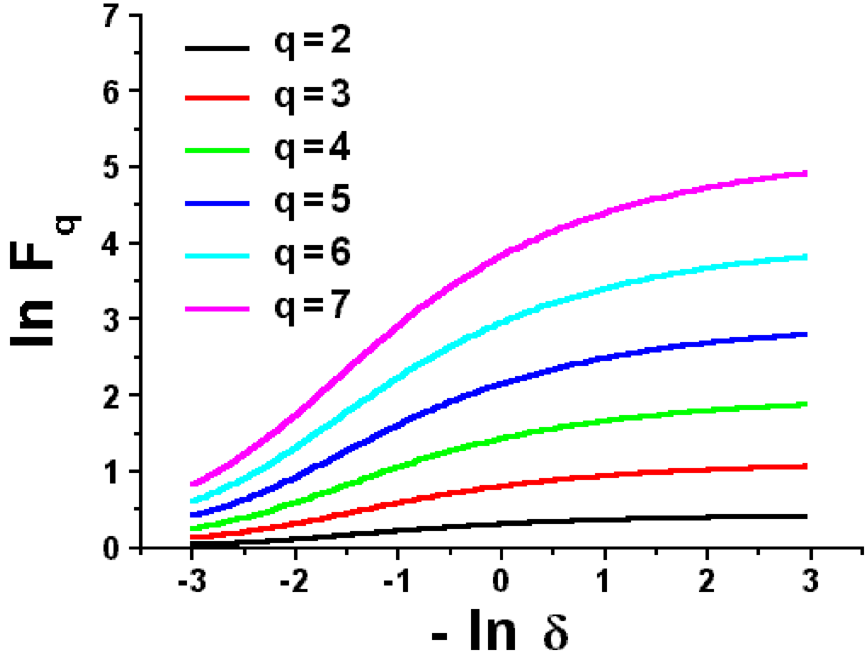
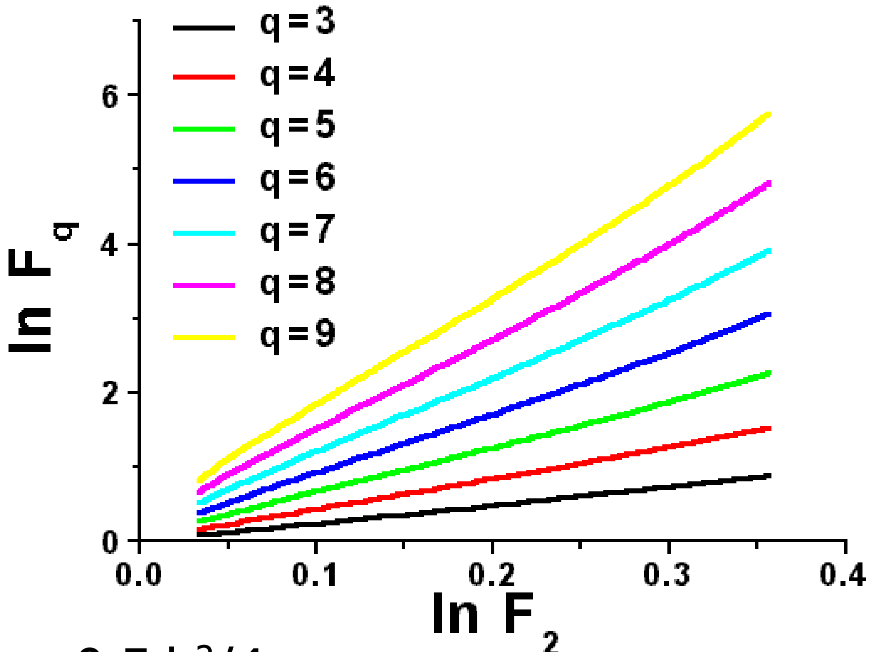




b = -3; a = 0.5 $b^2/4$

Babichev L. F., Klenitsky D. V., Kuvshinov V. I.,
Phys. Lett. **B345**, (1995), 269.

Scaling Exponent. Experimental data

Coherent states, *second* order potential (GL)

$$\nu = 1.305$$

Coherent states, *first* order potential (GL)

$$1.32 < \nu < 1.33$$

Heavy ion collisions

$$\nu = 1.459 \pm 0.021$$

Jain P. L. et al., Phys. Lett. **B 236**, (1990), 219;
Phys. Rev.**C44**, (1991), 854; Phys. Rev. **C48**, (1993), 517.

$$\nu = 1.550 \pm 0.120$$

Hwa R. C., Nazirov M. T., Phys. Rev. Lett. {\bf 69}, (1992), 741.

Squeezed States

Л.Ф. Бабичев, А.А. Букач, В.И. Кувшинов, В.А. Шапоров
Ядерная физика. – 2004. - Т. 67, № 3. - С. 593-600.

Coherent Squeezed State

$$|\psi,\eta\rangle_{CSS} = \hat{D}[\psi]\hat{S}[\eta]|0\rangle$$

Scaling Squeezed State

$$|\psi,\eta\rangle_{CSS} = \hat{S}[\eta]\hat{D}[\psi]|0\rangle$$

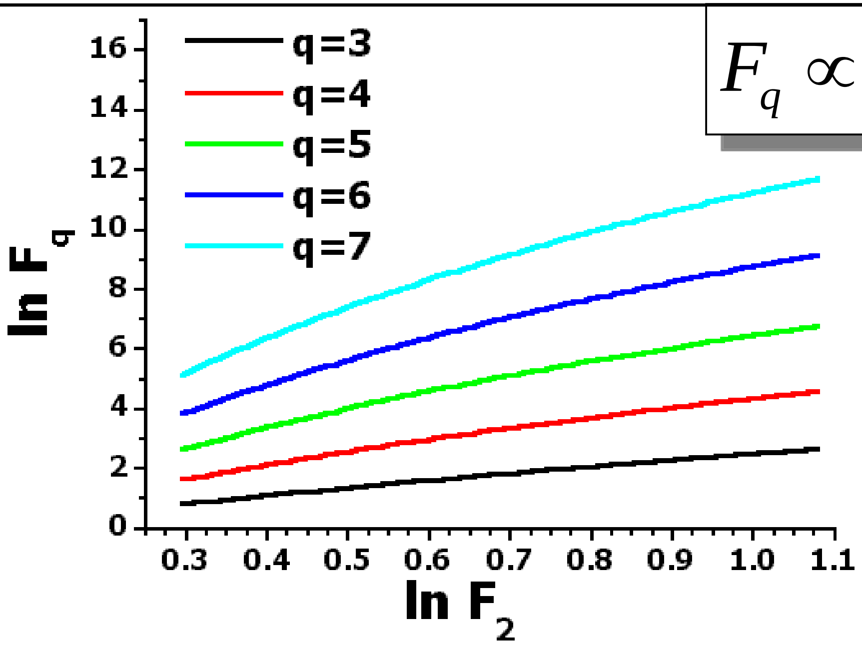
$\hat{S}$ - squeezing operator; $\hat{D}$ - displacement operator

Squeezed states multiplicity distribution

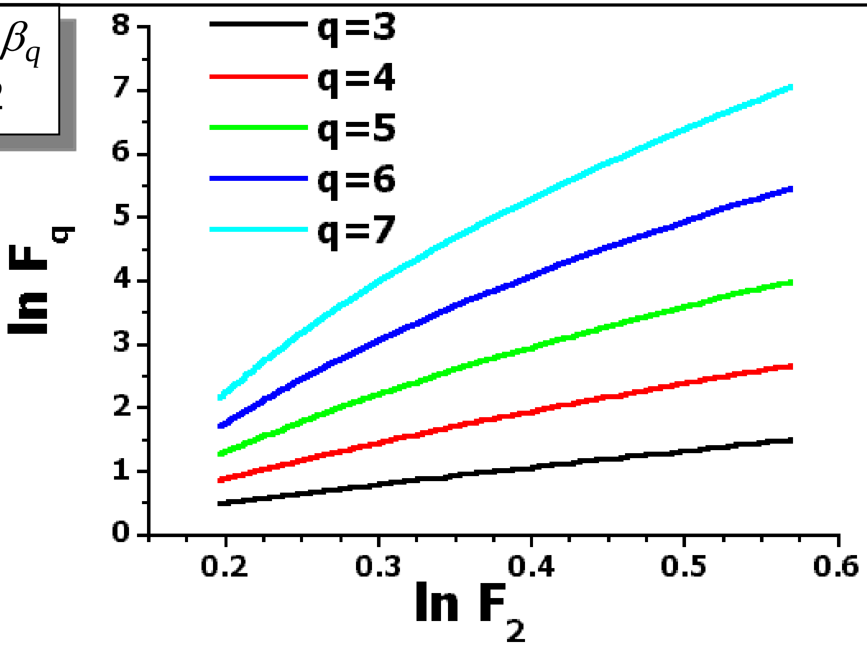
$$P_n^0 = \frac{1}{n!}\frac{1}{\cosh r}\left(\frac{\tanh r}{2}\right)^n |H_n(\xi_1)|^2 e^{\xi_2}$$

H_n - Hermite polynomials; $r = |\eta|$ - squeeze factor

Intermittency



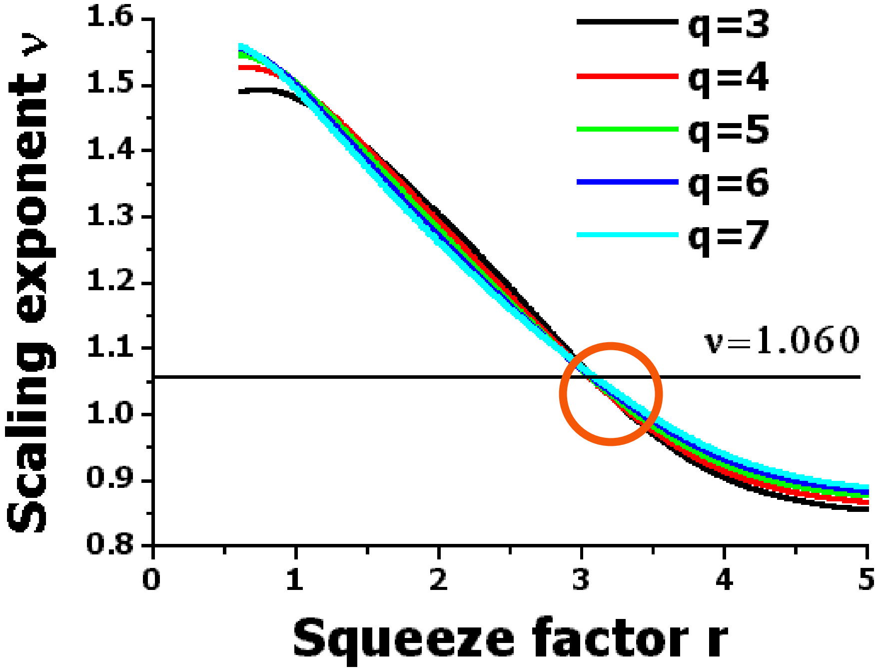
$$F_q \propto F_2^{\beta_q}$$



Coherent squeezed states
a = -1.0, b = 0.007,
r = 3.0

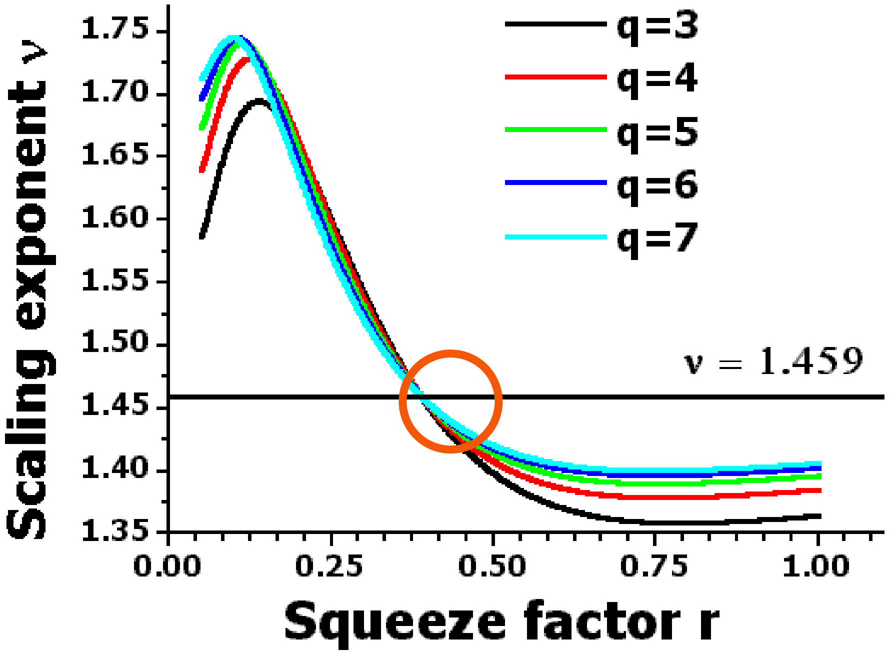
Scaling squeezed states
a = -10, b = 0.20055,
r = 0.3876

Scaling



Coherent Squeezed States

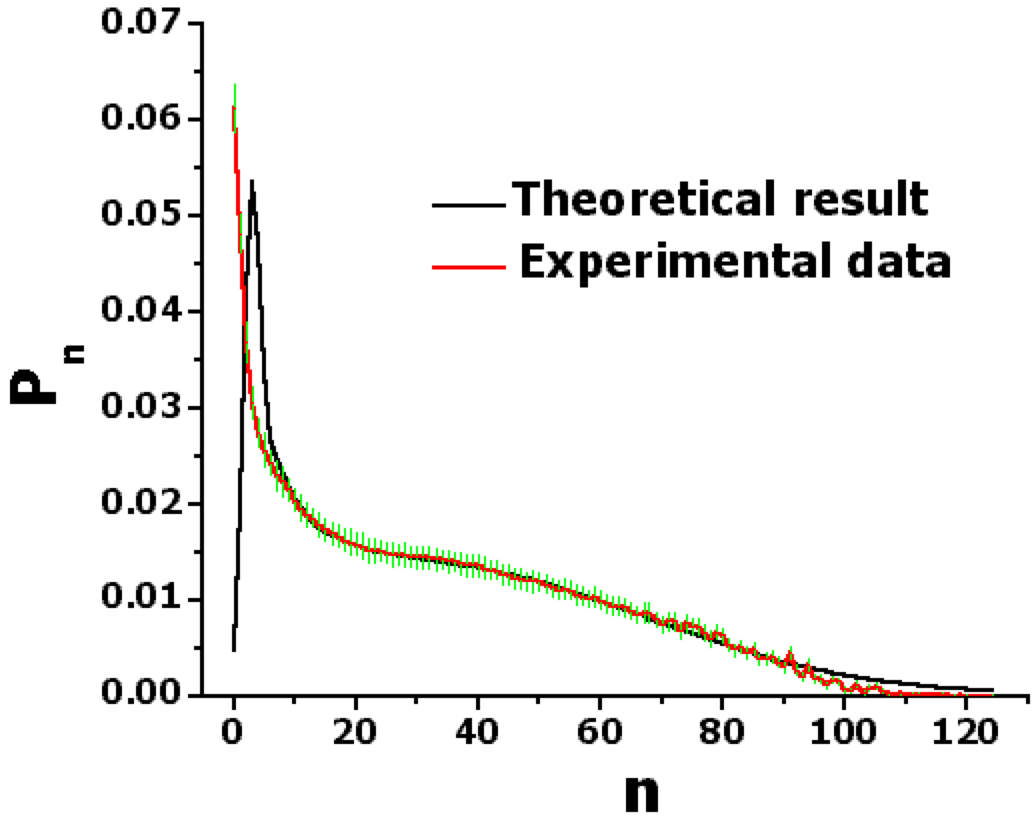
a = -1.0, b = 0.007



Scaling Squeezed States

a = -10, b = 0.20054

Distribution fit



Experimental data are taken from NA36 experiment

Theoretical results corresponds to parameters

a = -0.99871

b = 0.39610

r = 0.81358

-ln δ = -2.3485

$\chi^2/(N-4) = 0.450$

(n = 6...99)

Data Center of JIPNR-Sosny

Simulation of the heavy ions interaction processes at high energies with Monte Carlo generator HIJING

We simulated N = 10^5 Pb+Pb collisions for center of mass energies in range $\sqrt{S_{NN}}$ from 500 to 1400 GeV.

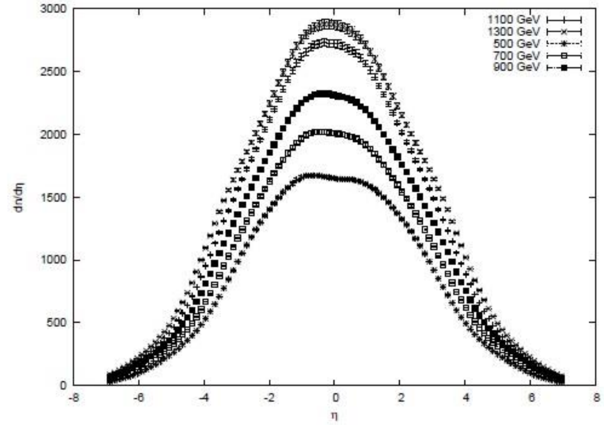


FIG. 1: Distribution of the multiplicity of secondary particles for the pseudorapidity η for $\sqrt{s} = 500, 700, 900, 1100, 1300$ GeV

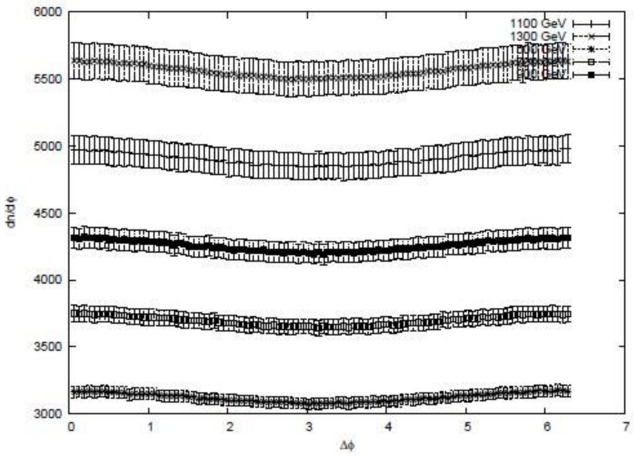


FIG. 3: Distribution azimuthal angle of emission particle with respect to the reaction plane $\Delta\phi$ for energy 500, 700, 900, 1100,1300 GeV

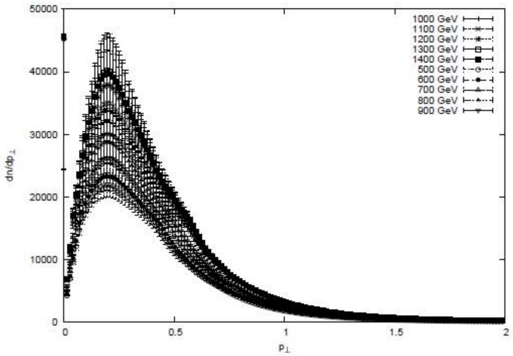


FIG. 4: Transverse momentum distribution for different energy (from 500 to 1400 GeV).

Simulation of the heavy ions interaction processes at high energies with Monte Carlo generator HIJING

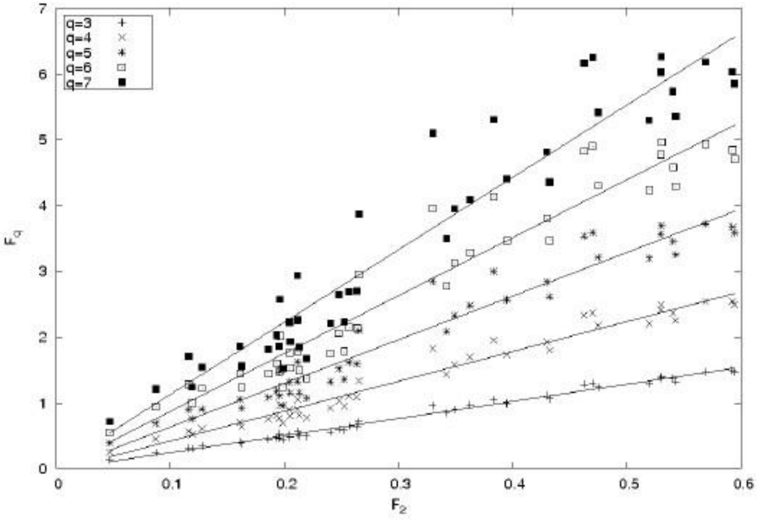


FIG. 8: Example of dependence $\ln(\mathcal{F}_q)$ of $\ln(\mathcal{F}_2)$, $q = 1 \ldots 7$

TABLE I: The value of the difference of the scaling exponents for different energies of interaction (Pb + Pb)

$\sqrt{s}$, GeV	γ_η on [0; 5,0], $\Delta = 0,42$	γ_ϕ on [0; 3,0], $\Delta = 0,19$	$\gamma_{p_\perp}$ on [0; 2,0], $\Delta = 0,21$
500	1,21 ± 0,03	0,86 ± 0,03	1,22 ± 0,16
600	1,22 ± 0,03	0,86 ± 0,06	1,22 ± 0,16
700	1,23 ± 0,03	0,86 ± 0,07	1,22 ± 0,17
800	1,24 ± 0,03	0,86 ± 0,08	1,21 ± 0,20
900	1,25 ± 0,04	0,85 ± 0,08	1,21 ± 0,20
1000	1,28 ± 0,05	0,84 ± 0,11	1,20 ± 0,32
1100	1,26 ± 0,04	0,85 ± 0,10	1,21 ± 0,25
1200	1,27 ± 0,04	0,84 ± 0.10	1.24 ± 0.29
1300	1,28 ± 0,05	0.85 ± 0.11	1.20 ± 0.33
1400	1,29 ± 0,05	0.85 ± 0.10	1.20 ± 0.32

Simulation of the heavy ions interaction processes at high energies with Monte Carlo generator **HYDJET++**

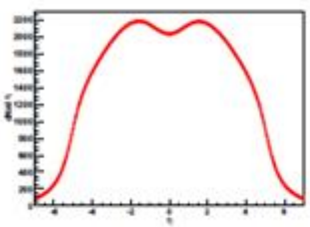


Рисунок 8 Распределение множественности по псевдобыстроте η для энергии $\sqrt{s}$=3000 GeV

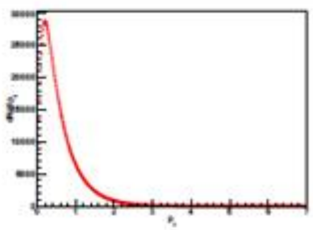


Рисунок 9 Распределение множественности по поперечному импульсу для для энергии $\sqrt{s}$=3000 GeV

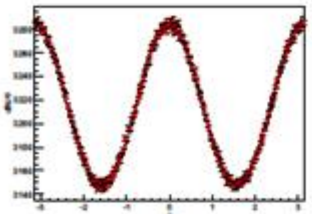


Рисунок 10 Распределение множественности по азимутальный углу частицы относительно плоскости реакции *Delta Phi* для энергии 3000 GeV

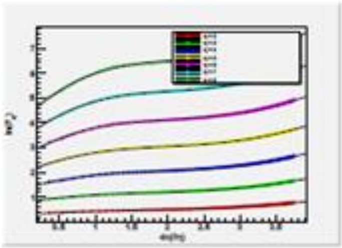


Рисунок 11 Зависимость $\ln(\mathcal{F}_q(\delta\eta))$ от $-\ln(\delta\eta)$ для $\sqrt{s}=1$ GeV, $q=2\ldots8$ for $\sqrt{s}=3000$ GeV

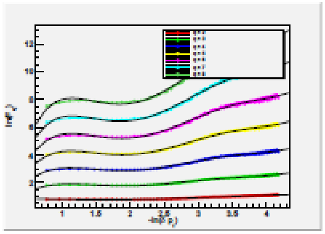


Рисунок 12 Зависимость $\ln(\mathcal{F}_q(\delta p_t))$ от $-\ln(\delta p_t)$ для $\sqrt{s}=1$ GeV, $q=2\ldots8$ для $\sqrt{s}=3000$ GeV

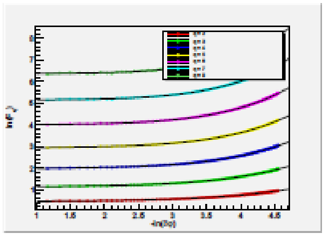


Рисунок 13 Зависимость $\ln(\mathcal{F}_q(\delta\varphi))$ от $-\ln(\delta\varphi)$ для $\sqrt{s}=1$ GeV, $q=2\ldots8$ для $\sqrt{s}=3000$ GeV

Таблица 3 Значение разностной скэйлинговой экспоненты γ для различных энергий в (Pb+Pb)столкновениях

$\sqrt{s}$, GeV	γ_η on [0; 3.5], $\Delta=0,070$	γ_φ on [0;2.5], $\Delta=0,035$	$\gamma_{p_\perp}$ on [0;0.2], $\Delta=0,030$
3000	0.696 ± 0.009	0.655 ± 0.003	1.484 ± 0.010
3500	0.691 ± 0.010	0.646 ± 0.004	1.518 ± 0.010
4500	0.695 ± 0.012	0.617 ± 0.004	1.574 ± 0.009
5500	0.712 ± 0.015	0.584 ± 0.004	1.621 ± 0.009

Event-by-Event Multiplicity Fluctuations for first order phase transition

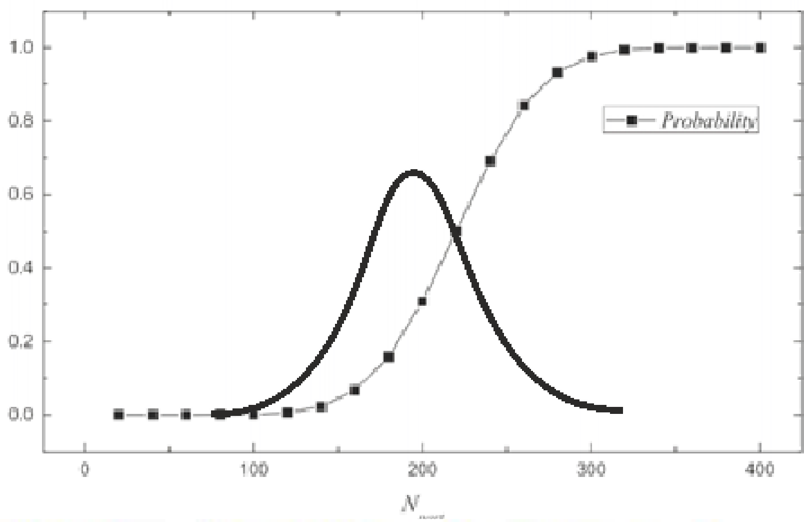
Let us assume that the beam energy required for the phase transition during heavy ion collisions is achieved and the probability of the 1^{st} order phase transition (ϖ_i) versus N_{part} has distribution

$$\varpi_i(x_i) = \frac{1}{\sqrt{2\pi} < x^2 >} \exp\left(-\frac{x_i^2}{2 < x^2 >}\right) \quad (1)$$

$$x_i = (N_{part})_i - < N_{part} > ,$$

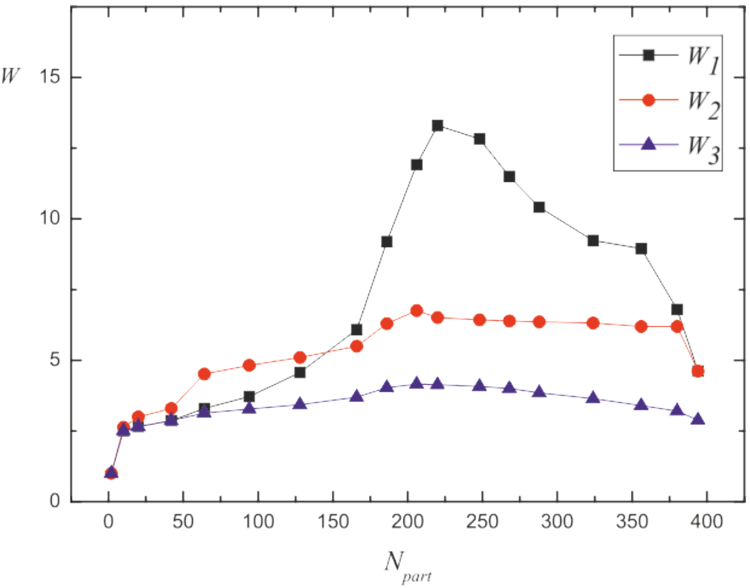
$$< x^2 >= \frac{\sum_{i=1}^{\mathcal{N}} ((N_{part})_i - < N_{part} >)}{\mathcal{N}}$$

where $(N_{part})_i$ is the number of participating nucleons in i-th event, $\mathcal{N}$ is the number of events, $< N_{part} >$ is the mean number of participating nucleons among all $\mathcal{N}$ events, and $< x^2 >$ characterizes the intensity of fluctuations.



Probability distribution function of the 1^{st} order phase transition QGP – hadron in even-by- event collisions versus N_{part}.

Event-by-Event Multiplicity Fluctuations



Multiplicity fluctuations of negatively charged particles calculated with Monte Carlo generator HIJING. The line with black squares draws the multiplicity fluctuations W_1 taking into account formula (1); the line with circles draws the multiplicity fluctuations W_2 if we do not take into account the growth of fluctuations in the vicinity of the 1^{st} order phase transition; the line with triangles draws the multiplicity fluctuations W_2 without QGP phase.

$$W = \frac{var(N)}{<N>} = \frac{<(N_i - <N>)^2>}{<N>}$$

$$= \frac{<(N^2> - <N>^2>}{<N>},$$

$$<(N_i - <N>)^2> = \frac{\sum_{i=1}^{\mathcal{N}} (N_i - <N>)^2}{\mathcal{N}}$$

Negatively charged particles multiplicity fluctuations (W) were calculated as scaled variance , where $<N>$ is mean number of finally produced negatively charged particles produced during one event, $var(N)$ is variance, $\mathcal{N}$ is number of events, N_i is number of negatively charged particles finally produced during *i-th* event $(i = 1, \ldots, \mathcal{N})$.

Conclusion

- With Monte Carlo generator **HIJING** we can simulate the effects of first-order phase transition of QGP $\rightarrow$ hadrons.

- The scaling exponents g calculated by Monte Carlo generator **HYDJET++** have large errors

- The developed method can be used to take in account the growth of fluctuations near the 1st order phase transition. This method can be used to get information on signatures of phase transitions in nuclear matter.

Thank you for your attention!