Application of Bardin-Shumeiko approach for estimating radiative effects in lepton-nucleon scattering

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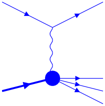
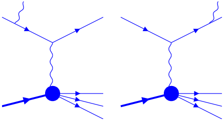
Model-Independent RC to DIS $ep \to eX$

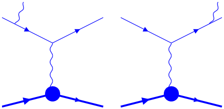
- The lowest order contribution.



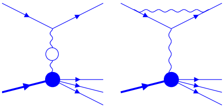
- Real photon emission from lepton line with hadronic inelastic tail. Contains the infrared divergence.



- Real photon emission from lepton line with hadronic elastic tail. Infrared free.



- Additional virtual particle contribution Last graph contains the infrared divergence.



Advantages of Model-Independent RC

- The task can be solved exactly.
- Model-Independent RC is rather large because of including so-called leading-order term $\log(Q^2/m^2)$.
- Uncertainties of the model-independent RC come only from fits and models used for structure functions.
- The calculation of model-dependent correction (box-type graphs, real photon emission from hadronic line) requires additional assumptions about hadron interaction, so it has additional pure theoretical uncertainties, which are hard to control.

Mo-Tsai Method (SLAC-PUB-848, 1971)

- Mo-Tsai firstly elaborated a systemic approach to calculate radiative corrections in elastic and inelastic electron scattering.
- For inelastic and deep inelastic processes they showed that actual Q^2 and W^2 going to hadronic part cover a wide kinematic region including the resonance region.
- Also they proved that elastic processes with the radiated photon (so-called radiative tail from elastic peak or simply elastic radiative tail) has to be added as a contribution to the total RC.
- One assumption (and limitation) in their calculations was the approximate way to consider the soft-photon contribution. Specifically, they introduced a parameter Δ such that $\Delta \ll m_e, E, E'$. Then they considered the region over photon energy E_γ and kept only the leading term $1/E_\gamma$. This allowed them to calculate the term with the soft photons analytically (even with a photon mass λ) and extract infrared divergence in the form of $\log(m_e/\lambda)$. The infrared divergence is canceled with respective term obtained when calculating loop diagrams (i.e., the vertex function). The correction from the region above Δ is evaluated numerically.

Covariant Approach of Bardin-Shumeiko (Nucl.Phys. B127 (1977) 242)

Bardin and Shumeiko improved the calculation approach in 5 aspects:

- They developed an approach for extraction and cancellation of the infrared divergence which is free of the artificial parameter Δ.
- They presented all results in the covariant form, so the formulas can be directly applied in any coordinate system.
- They developed a code TERAD that calculates RC for unpolarized target including nuclear targets; in this case radiative tail from quasielastic peak have to be considered and added.
- They suggested the radiative correction procedure of experimental data or unfolding procedure.
- With collaboration with Kukhto they obtained the formulas for RC on polarized protons.

Difference Between Mo-Tsai and Bardin-Shumeiko Methods

$$\frac{d\sigma_R}{d\Omega dE_\gamma} = f_0(E_\gamma) + \frac{f_1(E_\gamma)}{E_\gamma}, \text{ where } \frac{d\sigma_R}{d\Omega} = \int_0^{E_\gamma^{max}} dE_\gamma \frac{d\sigma_R}{d\Omega dE_\gamma} = \infty$$

Mo-Tsai: $\frac{d\sigma_R}{d\Omega} \to \frac{d\sigma_R^{soft}(\Delta)}{d\Omega} + \frac{d\sigma_R^{hard}(\Delta)}{d\Omega}$,

Direct integration $\frac{d\sigma_R^{hard}(\Delta)}{d\Omega} = \int_\Delta^{E_\gamma^{max}} dE_\gamma \frac{d\sigma_R}{d\Omega dE_\gamma}$

Integration with regularization: $\frac{d\sigma_R^{soft}(\Delta)}{d\Omega} = \int_\lambda^{\Delta} dE_\gamma \frac{f_1(0)}{E_\gamma}$

Bardin-Shumeiko: $\frac{d\sigma_R}{d\Omega} = \frac{d\sigma_{IR}}{d\Omega} + \frac{d\sigma_R}{d\Omega} - \frac{d\sigma_{IR}}{d\Omega} = \frac{d\sigma_{IR}}{d\Omega} + \frac{d\sigma_F}{d\Omega}$

Infrared part $\frac{d\sigma_{IR}}{d\Omega} = \frac{d\sigma_R^{soft}(\Delta)}{d\Omega} + \frac{d\sigma_{IR}^{hard}(\Delta)}{d\Omega}$,

Direct integration $\frac{d\sigma_F}{d\Omega}$ and $\frac{d\sigma_{IR}^{hard}(\Delta)}{d\Omega} = \int_\Delta^{E_\gamma^{max}} dE_\gamma \frac{f_1(0)}{E_\gamma}$

RC to DIS of Polarized Particles: POLRAD 2.0 (Comput.Phys.Commun. 104 (1997) 201)

- Akushevich and Shumeiko essentially improved the calculation of RC to polarized targets.
  - The most essential improvement was the idea of using the basis in the four-dimensional space and of expansion polarization vectors over momenta such as momenta of initial and final electrons and initial proton. This allowed to avoid a tedious and intricate procedure of tensor integration used before.
- Akushevich, Ilyichev, Soroko, Shumeiko and Tolkachev created the code POLRAD 2.0 that allows to calculations for:
  - RC in DIS on polarized targets of spin of 1/2 and 1. All contributions including quasielastic radiative tail were implemented.
  - RC to quadruple asymmetry for spin-one targets.
  - RC to semi-inclusive DIS (including polarized targets) in the simple quark-parton model i.e., for the three-dimensional cross section $d\sigma/dxdydz$.
  - Approximate contribution of double bremsstrahlung
  - Electroweak effects
  - The iterative procedure of RC of experimental data

Monte Carlo Generators RADGEN

Akushevich, Boettcher and Ryckbosch constructed the Monte Carlo generator RADGEN using POLRAD 2.0 (hep-ph/9906408).

The cross section is represented in the sum of two positively definite contributions

$$\sigma_{obs} = \sigma_{non-rad} + \sigma_{rad}$$

where $\sigma_{non-rad}$ contains Born contribution, loop diagrams and soft photon emission and σ_{rad} is the contribution of additional hard photon emission with energy larger than a minimal photon energy ϵ_{min} associated with resolution in calorimeter.

In spite of introducing the artificial parameter ϵ_{min} there is no loosing an accuracy and no acquired dependence of the cross section of this parameter

Two modes for generator operation

- integrals are calculated for each event and grid for a simulation is stored
- look-up table calculated in advance is used for interpolation of the grid

Explicit Expression for the lowest order RC

The complete RC of the lowest order (and multiple soft photon contributions) calculated using the covariant technique (i.e., that used to create POLRAD, DIFFRAD, EXCLURAD, and other codes) is represented in the form

$$\sigma_{RC} = \sigma_0 \exp(\delta_{inf})(\delta_{VR} + \delta_{vac}) + \sigma_F$$

Here the corrections δ_{inf} and δ_{vac} come from the radiation of soft photons and the effects of vacuum polarization, the correction δ_{VR} is infrared-free sum of factorized parts of real and virtual photon radiation, and σ_F is an infrared free contribution from the process of emission of an additional real photon.

The contribution of hard photons σ_F is represented in the form of three-dimensional integral over kinematic variables of an unobserved photon.

$$\sigma_F = \alpha^3 C_{kin} \int d\Omega_k \int_0^{v_m} dv \sum_n \left[\frac{v f_{kin}}{\tilde{Q}^4} L^{(n)}_{\mu\nu,\mu'} T^{(n)}_{\mu\nu,\mu'} - \frac{f^0_{kin}}{v Q^4} L^{0(n)}_{\mu\nu,\mu'} T^{0(n)}_{\mu\nu,\mu'} \right]$$

The integrals need to be calculated numerically. This integral is finite for $v \to 0$ and not positively definite.

Other Codes for RC in *ep*-scattering

POLRAD 2.0 *FORTRAN code for the RC procedure of experimental data in polarized inclusive and semi-inclusive DIS. The iteration procedure based on MINUIT fitting the data is included. Estimation of higher order and electroweak corrections is done.*

RADGEN *Monte Carlo generator of radiative events in the DIS on polarized and unpolarized targets. Can be applied for RC generation in inclusive, semi-inclusive and exclusive DIS processes. This version uses a look-up table for photonic angles which provides for fast event generation.*

DIFFRAD *FORTRAN code for RC calculation in the processes of electroproduction of vector mesons. Versions with Monte Carlo and numerical integrations are available. Monte Carlo code allows to estimate RC to the quasi-real photoproduction case (i.e., the final electron is not detected)*

HAPRAD 2.0 *FORTRAN code for RC calculation in the processes of semi-inclusive hadron electroproduction. The contribution of the exclusive radiative tail is included.*

ELARADGEN 2.0 Monte Carlo generator of radiative events in the kinematics of polarized elastic ep-scattering measurements.

MASCARAD *FORTRAN code for RC calculation in elastic electron-nucleon scattering with a polarized target and/or recoil polarization. The experimental acceptances are accounted for.*

EXCLURAD *FORTRAN code for RC calculation in the process of exclusive pi electroproduction on a nucleon.*

Other research groups dealing with RC

Bohm, Hollik, Spiesberger: *One-loop correction in electroweak physics; general theory of renormalization in electroweak theory. We compared POLRAD 2.0 with code HERACLES produced by Hubert Spiesberger.*

Bardin et al.: *We worked in parallel using the same approach of covariant calculation of RC. We compared POLRAD 2.0 with the code HECTOR produced by Dima Bardin with collaborators.*

Eduard Kuraev: *Produced multiple brilliant results in quantum electrodynamics.*

- *Asymptotic expressions for loop integrals in non–collinear kinematics, JINR E2-98-53, hep-ph/0703048*
- *Approach for the calculation in leading log approximation, shifted kinematics, Phys.Rev. C77, 055206 (2008)*

Marc Vanderhaeghen: One-loop correction and soft photon emission in DVCS, Phys.Rev. C62(2000)025501.

Maximon and Tjon: Phys.Rev. C62 (2000) 054320; Contributions of the box and crossed-box (two-photon exchange) diagrams.

- *In the end of 60th Mo-Tsai firstly developed a systemic approach to calculate radiative corrections in elastic and inelastic electron scattering.*
- *One limitation in their calculations was the dependence of the final expressions on small parameter Δ.*
- *Bardin and Shumeiko improved the calculation approach in many way. Particularly they removed the dependence on Δ.*
- *Basing of Bardin-Shumeiko method there are a lot of codes both for the numerical estimation of radiative effect and the hard photon simulation have been constructed.*