Measurability Conception for Quantum Theory, Gravity and Thermodynamics. Some Basic Results and Application

Alexander Shalyt-Margolin
Research Institute for Nuclear Problems,Belarusian State University

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1. Preamble or Main target

Main target is investigation of quantum theory and gravity in terms of the **measurability** notion (**definition below**), with the aim to form the above-mentioned theories proceeding from the variations (increments) dependent on the **existent energies**. Then these theories should not involve the ***abstract*** infinitesimal variations $\boldsymbol{dt, dx_i, dp_i, dE, i = 1,\ldots,3}$

The main motive is the problem of divergences in the quantum theory and the gravity correct quantization

2. Necessary Preliminary Information

It is assumed that there is a minimal (universal) unit for measurement of the length $\boldsymbol{\ell}$ corresponding to some maximal energy $\boldsymbol{E_\ell = \frac{\hbar c}{\ell}}$ and a universal unit for measurement of time $\boldsymbol{\tau = \ell/c}$. Without loss of generality, $\boldsymbol{\ell}$ and $\boldsymbol{\tau}$ at Plank's level, i.e. $\boldsymbol{\ell = \kappa l_p, \tau = \kappa t_p,}$ where constant $\boldsymbol{\kappa}$ is on the order of $\mathbf{1}$.

I. $\boldsymbol{E_\ell \propto E_p}$ with the corresponding proportionality factor.

Then we consider a set of all nonzero momenta

$$\boldsymbol{P} = \{\boldsymbol{p}_{\boldsymbol{x}_i}\}, \boldsymbol{i} = \mathbf{1}, .., \mathbf{3}; |\boldsymbol{p}_{\boldsymbol{x}_i}| \neq \mathbf{0}. \qquad (2.1)$$

Primarily Measurable momenta

$$\boldsymbol{p}_{\boldsymbol{x}_i} \equiv \boldsymbol{p}_{\boldsymbol{N}_i} = \frac{\hbar}{\boldsymbol{N}_i \boldsymbol{\ell}}, \qquad (2.2)$$

where $\boldsymbol{N}_{\boldsymbol{i}}$ is an integer and $\boldsymbol{p}_{\boldsymbol{x}_i}$ is the momentum corresponding to the coordinate $\boldsymbol{x}_{\boldsymbol{i}}$.

Definition 1. ***Primary Measurability***

1.1. *Any variation in* $\boldsymbol{\Delta x}_{\boldsymbol{i}}$ *for the coordinates* $\boldsymbol{x}_{\boldsymbol{i}}$ *and* $\boldsymbol{\Delta t}$ *of the time* $\boldsymbol{t}$ *is considered* ***primarily measurable*** *if*

$$\boldsymbol{\Delta x}_{\boldsymbol{i}} = \boldsymbol{N}_{\boldsymbol{\Delta x}_i} \boldsymbol{\ell}, \boldsymbol{\Delta t} = \boldsymbol{N}_{\boldsymbol{\Delta t}} \boldsymbol{\tau}, \qquad (2.3)$$

where $\boldsymbol{N}_{\boldsymbol{\Delta x}_i} \neq \mathbf{0}$ *and* $\boldsymbol{N}_{\boldsymbol{\Delta t}} \neq \mathbf{0}$ *are integer numbers.*

1.2. *Let us define any physical quantity as* ***primary or elementary measurable*** *when its value is consistent with point* ***1.1*** *and formula* **(2.2).**

Then we consider formula *(2.3)* and **Definition 1.** with the addition of the momenta

$\boldsymbol{p_{x_0}} \equiv \boldsymbol{p_{N_0}} = \frac{\hbar}{\boldsymbol{N_0}\boldsymbol{\ell}}$, where $\boldsymbol{N_0}$ is an integer corresponding to the time coordinate ($\boldsymbol{N_{\Delta t}}$ in *(2.3)*).

For convenience, we denote **Primarily Measurable Quantities -- PMQ**. It is clear that **PMQ** is inadequate for studies of the physical processes.

$$\frac{\boldsymbol{\tau}}{\boldsymbol{N_t}} = \boldsymbol{p_{N_t c}} \frac{\boldsymbol{\ell^2}}{\boldsymbol{c}\hbar}$$
$$\frac{\boldsymbol{\ell}}{\boldsymbol{N_i}} = \boldsymbol{p_{N_i}} \frac{\boldsymbol{\ell^2}}{\hbar}, \mathbf{1} = \mathbf{1}, \ldots, \mathbf{3}, \qquad (2.4)$$

$\boldsymbol{p_{N_i}}, \boldsymbol{p_{N_t c}}$ are **Primarily Measurable** momenta, up to the fundamental constants are coincident with $\boldsymbol{p_{N_i}}, \boldsymbol{p_{N_t c}}$ and they may be involved at any stage of the calculations but, evidently, they are not **PMQ** in the general case.

Note: $\boldsymbol{\ell} = \boldsymbol{\kappa l_p}; \boldsymbol{l_p}^2 = \boldsymbol{G}\frac{\hbar}{\mathbf{c}^3}; \frac{\boldsymbol{\ell^2}}{\hbar} = \frac{\boldsymbol{\kappa}^2 \mathbf{G}}{\mathbf{c}^3}$

Definition 2. Generalized Measurability

We define any physical quantity at all energy scales as ***generalized measurable*** *or, for simplicity,* ***measurable*** *if any of its values may be obtained in terms of* ***PMQ*** *specified by points* ***1.1***, ***1.2*** *of* ***Definition 1***.

The main target of the author is to form a quantum theory and gravity only in terms of **measurable** quantities (or of **PMQ**).

A) **Low Energies,** $\boldsymbol{E \ll E_p}$.

Domain $P_{LE} \subset P$ (LE is abbreviation of "Low Energies") defined by the conditions

$$\boldsymbol{P_{LE} = \{p_{x_i}\}, i = 1,..,3; P_\ell \gg |p_{x_i}| \neq 0,} \qquad (2.5)$$

where $\boldsymbol{P_\ell = E_\ell / c}$--maximal momentum.

In this case ***Primarily Measurable Momenta (PMM)*** takes the form

$$\begin{aligned} &\boldsymbol{N_i = \frac{\hbar}{p_{x_i}\ell}, or} \\ &\boldsymbol{p_{x_i} \equiv p_{N_i} = \frac{\hbar}{N_i \ell}} \qquad (2.6) \\ &\boldsymbol{|N_i| \gg 1,} \end{aligned}$$

For $\boldsymbol{E \ll E_\ell}$, i.e. ($\boldsymbol{|N_i| \gg 1}$), **primary measurable** momenta are **sufficient** to specify the whole domain of the momenta to a high accuracy $\boldsymbol{P_{LE}}$.

Of course, all the calculations of point A) also comply with the **primary measurable** momenta $\boldsymbol{p}_{N_t c} \equiv \boldsymbol{p}_{N_0}$. Because of this, in what follows we understand P_{LE} as a set of the **primary measurable** momenta $p_{x_\mu} = p_{N_\mu}, (\mu = 0, \ldots, 3)$ with $|N_\mu| \gg 1$.

Remark 2.2. ***It should be noted that, as all the experimentally involved energies E are low, they meet the condition $E \ll E_\ell$, specifically for LHC the maximal energies are $\approx 10TeV = 10^4 GeV$, that is by 15 orders of magnitude lower than the Planck energy $\approx 10^{19} GeV$. But since the energy E_ℓ is on the order of the Planck energy $E_\ell \propto E_p$, in this case all the numbers N_i for the corresponding momenta will meet the condition $min|N_i| \approx 10^{15}$. So, all the experimentally involved momenta are considered to be primary measurable momenta,i.e. P_{LE} at low energies $E \ll E_\ell$.***

So, in the proposed paradigm at low energies $\boldsymbol{E} \ll \boldsymbol{E_p}$ a set of the **primarily measurable** $\boldsymbol{P_{LE}}$ is discrete, and in every measurement of $\mu = 0, \ldots, 3$ there is the discrete subset $\boldsymbol{P}_{x_\mu} \subset \boldsymbol{P}_{LE}$:

$$\boldsymbol{P}_{x_\mu} \doteq \{\ldots, \boldsymbol{p}_{N_{x_\mu}-1}, \boldsymbol{p}_{N_{x_\mu}}, \boldsymbol{p}_{N_{x_\mu}+1}, \ldots\}. \qquad \textbf{(2.9)}$$

In this case, as compared to the canonical quantum theory, in continuous space-time we have the following substitution:

$$dp_\mu \mapsto \mathbf{\Delta p_{N_{x_\mu}}} = \mathbf{p_{N_{x_\mu}}} - \mathbf{p_{N_{x_\mu}+1}} = \mathbf{p_{N_{x_\mu}(N_{x_\mu}+1)}};$$

$$\frac{\partial}{\partial p_\mu} \mapsto \frac{\mathbf{\Delta}}{\mathbf{\Delta p_\mu}}, \frac{\partial \mathbf{F}}{\partial \mathbf{p_\mu}} \mapsto \frac{\mathbf{\Delta F}\left(\mathbf{p_{N_{x_\mu}}}\right)}{\mathbf{\Delta p_\mu}} = \frac{\mathbf{F}\left(\mathbf{p_{N_{x_\mu}}}\right) - \mathbf{F}\left(\mathbf{p_{N_{x_\mu}+1}}\right)}{\mathbf{p_{N_{x_\mu}}} - \mathbf{p_{N_{x_\mu}+1}}} = \frac{\mathbf{F}(\mathbf{p_{N_{x_\mu}}}) - \mathbf{F}(\mathbf{p_{N_{x_\mu}+1}})}{\mathbf{p_{N_{x_\mu}(N_{x_\mu}+1)}}}. \quad (2.10)$$

And for sufficiently high integer values of $|N_{x_\mu}|, \mu = 0, \ldots, 3$,

$\boldsymbol{\tau/N_t, \ell/N_{x_i}}$ (which are the **primarily measurable** momenta $\boldsymbol{P_{x_\mu}}$ up to fundamental constant) represent a **measurable** analog of small (and infinitesimal) space-time *increments* in the space-time variety $\mathcal{M} \subset \mathbf{R^4}$.

Because of this, for sufficiently high integer values of $|N_{x_\mu}|$, we have :

the following correspondence

$$\boldsymbol{dx_\mu} \mapsto \frac{\boldsymbol{\ell}}{\mathbf{N_{x_\mu}}};$$

$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial x_\mu}} \mapsto \frac{\mathbf{\Delta}}{\mathbf{\Delta_{N_{x_\mu}}}}, \frac{\boldsymbol{\partial F}}{\boldsymbol{\partial x_\mu}} \mapsto \frac{\mathbf{\Delta F(x_\mu)}}{\mathbf{\Delta_{N_{x_\mu}}}} = \frac{\mathbf{F}\left(\mathbf{x_\mu} + \frac{\boldsymbol{\ell}}{\mathbf{N_{x_\mu}}}\right) - \mathbf{F(x_\mu)}}{\boldsymbol{\ell}/\mathbf{N_{x_\mu}}}. \quad (2.11)$$

Now we formulate the ***principle of correspondence to a continuous theory.***

Correspondence to Continuous Theory (CCT).

At low energies $\boldsymbol{E \ll E_p}$ (or same $\boldsymbol{E \ll E_\ell}$) the infinitesimal space-time quantities

$\boldsymbol{dx_\mu; \mu = 0, \ldots, 3}$ and also infinitesimal values of the momenta $\boldsymbol{dp_i, i = 1, 2, 3}$ and of the energies $\boldsymbol{dE}$ form the basic instruments ("construction materials") for any theory in continuous space-time. Because of this, to construct the **measurable** variant of such a theory, (discrete case) (2.10) and (2.11) give the adequate substitutes for these quantities.

B) **High Energies,** $\boldsymbol{E \approx E_p}$.

In this case **primary measurable** momenta are

$$\boldsymbol{N_i = \frac{\hbar}{p_{x_i}\ell}, or}$$
$$\boldsymbol{p_{x_i} \equiv p_{N_i} = \frac{\hbar}{N_i\ell}} \qquad (2.12)$$
$$\boldsymbol{|N_i| \approx 1}.$$

where $\boldsymbol{N_i}$ is an integer number and $\boldsymbol{p_{x_i}}$ is the momentum corresponding to the coordinate $\boldsymbol{x_i}$.

The main difference of the case B) **High Energies** from the case A)**Low Energies** is in the fact that at **High Energies** the **primary measurable** momenta are *inadequate* for theoretical studies at the energy scales $\boldsymbol{E \approx E_p}$.

This is easily seen when we consider, e.g., the ***Generalized Uncertainty Principle (GUP)***, that is an extension of **Heisenberg's Uncertainty Principle (HUP)** , to (Planck) high energies

$$\boldsymbol{\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\Delta p}{\hbar}} \qquad (2.13)$$

where α' is a constant on the order of **1**.
Obviously, (2.13) leads to the minimal length ℓ on the order of the Planck length l_p

$$\boldsymbol{\Delta x_{min} = 2\sqrt{\alpha'} l_p \equiv \ell}. \qquad (2.14)$$

In his earlier works the author, using simple calculations, has demonstrated that for the equality in (2.13) at high energies $\boldsymbol{E \approx E_p, (E \approx E_\ell)}$ the **primary measurable** space quantity $\boldsymbol{\Delta x = N_{\Delta x}\ell}$, where $\boldsymbol{N_{\Delta x} \approx 1}$ is an integer number, results in the momentum $\boldsymbol{p(N_{\Delta x}, GUP)}$**:**

$$\boldsymbol{\Delta p \equiv p(N_{\Delta x}, GUP) = \frac{\hbar}{1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell}}. \qquad (2.15)$$

It is clear that for $\boldsymbol{N_{\Delta x} \approx 1}$ the momentum $\boldsymbol{\Delta p(N_{\Delta x}, GUP)}$ is not a **primary measurable** momentum.
On the contrary, at low energies $\boldsymbol{E \ll E_p, (E \ll E_\ell)}$ the **primary measurable** space quantity $\boldsymbol{\Delta x = N_{\Delta x}\ell}$, where $\boldsymbol{N_{\Delta x} \gg 1}$ is an integer number, due to the validity of the limit

$$\lim_{N_{\Delta x}\to\infty}\sqrt{N_{\Delta x}^2-1}=N_{\Delta x}, \qquad (2.16)$$

leads to the momentum $\Delta p(N_{\Delta x}, HUP)$:

$$\Delta p \equiv \Delta p(N_{\Delta x}, HUP) = \frac{\hbar}{1/2(N_{\Delta x}+\sqrt{N_{\Delta x}^2-1})\ell} \approx \frac{\hbar}{N_{\Delta x}\ell} = \frac{\hbar}{\Delta x}. \qquad (2.17)$$

It is inferred that, for sufficiently high integer values of $N_{\Delta x}$ the momentum $\Delta p(N_{\Delta x}, HUP)$ within any high accuracy may be considered to be the **primary measurable** momentum.

3. Space-Time Metrics and Einstein Equations in Measurable Format

Low energies $E \ll E_p$,

According to the above-mentioned results, the **measurable** variant of gravity should be formulated in terms of the small **measurable** space-time quantities $\ell/N_{\Delta x_\mu}$ or same **primary measurable** momenta $p_{N_{\Delta x_\mu}}$.

Let us consider the case of the random metric $g_{\mu\nu} = g_{\mu\nu}(x)$ where $x \in R^4$ is some point of the four-dimensional space-time manifold $M \subseteq R^4$ defined in **measurable** terms.

Now, any such point $\boldsymbol{x} \in \{\boldsymbol{x}^{\chi}\} \in \boldsymbol{M}$ and any set of integer numbers $\{\boldsymbol{N}_{\Delta x^{\chi}}\}$ dependent on the point $\{\boldsymbol{x}^{\chi}\}$ with the property $|\boldsymbol{N}_{\Delta x^{\chi}}| \gg \mathbf{1}$ may be correlated to the **bundle** with the base R^4 as follows:

$$\mathbf{B}_{N_{x^{\chi}}} \doteq \{\boldsymbol{x}^{\chi}, \frac{\boldsymbol{\ell}}{N_{\Delta x^{\chi}}}\} \mapsto \{\boldsymbol{x}^{\chi}\}. \qquad (3.1)$$

It is clear that $\lim\limits_{|N_{\Delta x^{\chi}}| \to \infty} \mathbf{B}_{N_{\Delta x^{\chi}}} = \boldsymbol{R}^{4}$.

Then as a ***canonically measurable pre-image*** of the infinitesimal space-time interval square

$$\boldsymbol{ds^2}(\boldsymbol{x}) = \boldsymbol{g}_{\mu\nu}(\boldsymbol{x})\boldsymbol{dx^{\mu}dx^{\nu}} \qquad (3.2)$$

we take the expression

$$\boldsymbol{\Delta s}^{2}_{\{N_{\Delta x^{\chi}}\}}(\boldsymbol{x}) = \boldsymbol{g}_{\mu\nu}(\boldsymbol{x}, \boldsymbol{N}_{\Delta x^{\chi}}) \frac{\boldsymbol{\ell}^{2}}{N_{\Delta x^{\mu}} N_{\Delta x^{\nu}}}. \qquad (3.3)$$

Here $\boldsymbol{g}_{\mu\nu}(\boldsymbol{x}, \boldsymbol{N}_{\Delta x^{\chi}})$ -- metric with the property that minimal **measurable** variation of metric $\boldsymbol{g}_{\mu\nu}(\boldsymbol{x}, \boldsymbol{N}_{\Delta x^{\chi}})$ in point (3.3) for coordinate $\boldsymbol{\chi} - \boldsymbol{th}$ has form

$$\boldsymbol{\Delta g_{\mu\nu}(x, N_{\Delta x^\chi})_\chi = g_{\mu\nu}(x + \ell/N_{\Delta x^\chi}, N_{\Delta x^\chi}) - g_{\mu\nu}(x, N_{\Delta x^\chi})}, \quad (3.4)$$

and $\boldsymbol{\Delta_\chi g_{\mu\nu}(x, N_{\Delta x^\chi})}$; $\boldsymbol{\Delta_\chi g_{\mu\nu}(x, N_{\Delta x^\chi}) = \frac{\Delta g_{\mu\nu}(x, N_{\Delta x^\chi})_\chi}{\ell/N_{\Delta x^\chi}}}$.(3.5)

The $\boldsymbol{\Delta g_{\mu\nu}(x, N_{\Delta x^\chi})_\chi}$ is a **measurable** analog for the $\boldsymbol{dg_{\mu\nu}(x)}$ of the χ-th component $(\boldsymbol{dg_{\mu\nu}(x)})_\chi$ in a continuous theory, whereas $\boldsymbol{\Delta_\chi g_{\mu\nu}(x, N_{\Delta x^\chi})}$ is a **measurable** analog of the partial derivative $\boldsymbol{\partial_\chi g_{\mu\nu}(x)}$**.**

In this manner we obtain the (3.1)-formula induced bundle over the metric manifold $\boldsymbol{g_{\mu\nu}(x)}$:

$$\mathbf{B}_{g, N_{\Delta x^\chi}} \doteq \boldsymbol{g_{\mu\nu}(x, N_{\Delta x^\chi}) \mapsto g_{\mu\nu}(x)}. \quad (3.6)$$

The formula (3.3) may be written in terms of the **primary measurable** momenta $\left(\boldsymbol{p_{N_{\Delta x^i}}, p_{N_{\Delta x^0}}}\right) = \boldsymbol{p_{N_{\Delta x^\chi}}}$ as follows:

$$\boldsymbol{\Delta s^2_{N_{\Delta x^\chi}}(x) = \frac{\ell^4}{\hbar^2} g_{\mu\nu}(x, N_{\Delta x^\chi}) p_{N_{\Delta x^\mu}} p_{N_{\Delta x^\nu}}}. \quad (3.7)$$

Considering that $\boldsymbol{\ell \propto l_P}$ **(i.e.,** $\boldsymbol{\ell = \kappa l_P}$**),** where $\boldsymbol{\kappa = const}$ is on the order of **1**, in the general case to within the constant $\boldsymbol{\ell^4}/\hbar^2$, we have

$$\boldsymbol{\Delta s^2_{N_{\Delta x^\chi}}(x) = g_{\mu\nu}(x, N_{\Delta x^\chi}) p_{N_{\Delta x^\mu}} p_{N_{\Delta x^\nu}}}. \qquad (3.8)$$

Thus, we can obtain **measurable** (discrete) analogs all component **infinitesimal of General Relativity:**

In particular, the Christoffel symbols

$$\boldsymbol{\Gamma^\alpha_{\mu\nu}(x) = \frac{1}{2}\, g^{\alpha\beta}(x)\left(\partial_\nu\, g_{\beta\mu}(x) + \partial_\mu\, g_{\nu\beta}(x) - \partial_\beta\, g_{\mu\nu}(x)\right)} \qquad (4.2)$$

have the **measurable** analog

$$\boldsymbol{\Gamma^\alpha_{\mu\nu}(x, N_{x_\chi}) = \frac{1}{2}\, g^{\alpha\beta}(x, N_{x_\chi})\,(\Delta_\nu g_{\beta\mu}(x, N_{x_\chi}) + \Delta_\mu g_{\nu\beta}(x, N_{x_\chi}) - -\Delta_\beta g_{\mu\nu}(x, N_{x_\chi}))}. \qquad (4.3)$$

Similarly, for the ***Riemann tensor*** in a continuous theory we have:

$$\boldsymbol{R^\mu{}_{\nu\alpha\beta}(x) \equiv \partial_\alpha \Gamma^\mu_{\nu\beta}(x) - \partial_\beta \Gamma^\mu_{\nu\alpha}(x) + \Gamma^\mu_{\gamma\alpha}(x)\, \Gamma^\gamma_{\nu\beta}(x) - \Gamma^\mu_{\gamma\beta}(x)\, \Gamma^\gamma_{\nu\alpha}(x)}. \qquad (4.4)$$

With the use of formula (4.3), we can get the corresponding **measurable** analog, i.e. the quantity $\boldsymbol{R^\mu{}_{\nu\alpha\beta}(x, N_{x_\chi})}$**.** In a similar way we can obtain the **measurable** variant of *Ricci* *tensor*, $\boldsymbol{R_{\mu\nu}(x, N_{x_\chi}) \equiv R^{\alpha}{}_{\mu\alpha\nu}(x, N_{x_\chi})}$, and the **measurable** variant of *Ricci scalar*:

$\boldsymbol{R(x, N_{x_\chi}) \equiv R_{\mu\nu}(x, N_{x_\chi})\, g^{\mu\nu}(x, N_{x_\chi})}$.

So, for the ***Einstein Equations*** **(EE)** in a continuous theory

$$\boldsymbol{R_{\mu\nu} - \frac{1}{2}\, R\, g_{\mu\nu} - \frac{1}{2}\, \Lambda\, g_{\mu\nu} = 8\,\pi\, G\, T_{\mu\nu}} \qquad (4.5)$$

we can derive their **measurable** analog, for short denoted as **(EEM)** ***Einstein Equations Measurable--lattice approximation of*** **(EE):**

$$\boldsymbol{R_{\mu\nu}(x, N_{x_\chi}) - \frac{1}{2}\, R(x, N_{x_\chi})\, g_{\mu\nu}(x, N_{x_\chi}) - \frac{1}{2}\, \Lambda(x, N_{x_\chi})\, g_{\mu\nu}(x, N_{x_\chi}) = \\ = 8\,\pi\, G\, T_{\mu\nu}(x, N_{x_\chi}),} \qquad (4.6)$$

where $\boldsymbol{G}$ **--** Newton's gravitational constant.

(**EEM**) given by formula represents **deformation** of the Einstein equations (**EE**) in the sense of the Definition given **Ludwig Faddeev in 1989** with the deformation parameter N_{x_χ}, and we have

$$\lim_{|N_{x_\chi}| \to \infty} (\boldsymbol{EEM}) = (\boldsymbol{EE}). \qquad (4.9)$$

We denote this deformation as $(\boldsymbol{EEM})[\boldsymbol{N_{x_\chi}}]$. Since at low energies $\boldsymbol{E \ll E_P}$ and to within the known constants we have $\boldsymbol{\ell}/\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}} = \boldsymbol{p}_{\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}}$, the following deformations of (EU) are equivalent to

$$(\boldsymbol{EEM})[\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}] \equiv (\boldsymbol{EEM})[\boldsymbol{p}_{\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}}]. \qquad (4.10)$$

4. Some Important Comments and Problems

What are the advantages of this approach?

4.1. At low energies far from the Planck energies $\boldsymbol{E} \ll \boldsymbol{E}_{\boldsymbol{p}}$ we replace the space-time manifold space-time manifold $\boldsymbol{M} \subseteq \boldsymbol{R}^{\boldsymbol{4}}$ by the lattice model $\boldsymbol{Latt}^{\boldsymbol{le}}_{\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}}\boldsymbol{M}$ where the upper index $\boldsymbol{le}$ is the abbreviation for "low energies"), with the nodes taken at the points $\{\boldsymbol{x}_{\boldsymbol{\chi}}\} \in \boldsymbol{M}$ that all the edges belonging to $\{\boldsymbol{x}_{\boldsymbol{\chi}}\}$ have the size $\boldsymbol{\ell}/\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}$,where $\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}$ - integers having the property $|\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}| \gg \boldsymbol{1}$.
The lattice model $\boldsymbol{Latt}^{\boldsymbol{le}}_{\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}}\boldsymbol{M}$ is dynamic and dependent on the existing energies. In this case all the main attributes of a gravitational theory in the manifold $\boldsymbol{M}$,including Einstein Equations, have their adequate analogs on $\boldsymbol{Latt}^{\boldsymbol{le}}_{\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}}\boldsymbol{M}$.

Correspondence to Continuous Theory (CCT).

4.2. Possibly $(\boldsymbol{EEM})[\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}]$ on $\boldsymbol{Latt}^{\boldsymbol{le}}_{\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}}\boldsymbol{M}$ under some conditions for $\{\boldsymbol{N}_{\boldsymbol{x}_{\boldsymbol{\chi}}}\}$ have not pathological solutions in the form of the ***Closed Time-like Curves (CTC),*** involved in some models of General Relativity.

Example: K.Godel's solution of Einstein Equations:

$$ds^2 = \frac{1}{2\omega^2}\left(-(dt + e^x dz)^2 + dx^2 + dy^2 + \frac{1}{2}e^{2x}dz^2\right).$$
$$-\infty < t, x, y, z < \infty, \omega = const \neq 0$$

Gödel, K. (1949). «An example of a new type of cosmological solution of Einstein's field equations of gravitation», Rev. Mod. Phys. 21: 447–450.

4.3. As in the well-known works by S.Hawking all the results have been obtained within the scope of the semiclassical approximation, seeking for a solution of the above-mentioned problem is of primary importance. More precisely, we must find, *how to describe* *thermodynamics and quantum mechanics using the "language" of the* **measurable** *variant of gravity and what is the difference (if any) from the continuous treatment in this case*.

5. Natural Transition to High Energies (Quantum Region).

5.1. ***However, minimal measurable increments for the energies $E \approx E_P$ are not of the form ℓ/N_{x_μ} because the corresponding momenta $\{p_{N_{x_\chi}}\}$ are no longer primary measurable, as indicated by the results in Section 2.***

So, in the proposed paradigm the problem of the ultraviolet generalization of the low-energy **measurable** gravity $(\boldsymbol{EEM})[\boldsymbol{N_{x_\chi}}]$ is actually reduced to the problem: what becomes with the **primary measurable** momenta $\{\boldsymbol{p}_{N_{x_\chi}}\}, |\boldsymbol{N}_{x_\chi}| \gg \mathbf{1}$ **at high (Planck's) energies?!** In a relatively simple case of **GUP** in Section 2 we have the answer.
In more general case **KMM (Kempf,Mangano,Mann,** ***Hilbert space representation of the minimal length uncertainty relation, Phys. Rev. D 1995, 52, 1108–1118).***

$$\boldsymbol{\Delta x \Delta p} \geq \hbar(\mathbf{1} + \boldsymbol{\beta}(\boldsymbol{\Delta p})^{\mathbf{2}}) \qquad (5.1)$$

$$\boldsymbol{\Delta x_0} = 2\hbar\sqrt{\boldsymbol{\beta}} \doteq \boldsymbol{\ell}$$

when (5.1) is equality, $\boldsymbol{\Delta p} = \boldsymbol{p}_{N_{\Delta x^\mu}}$ **- generalized measurable**

$$\boldsymbol{l_H(p_{N_{\Delta x^\mu}}) \doteq \frac{\ell^2}{\hbar} p_{N_{\Delta x^\mu}}, |N_{\Delta x^\mu}| \approx 1}$$

$$\boldsymbol{\Delta s^2_{N_{\Delta x^\chi}}(x,q) \doteq g_{\mu\nu}\left(x, N_{x_\chi}, \mathbf{q}\right) l_H(p_{N_{\Delta x^\mu}}) l_H(p_{N_{\Delta x^\nu}}) \doteq \frac{\ell^4}{\hbar^2} g_{\mu\nu}\left(x, N_{x_\chi}, \mathbf{q}\right) p_{N_{\Delta x^\mu}} p_{N_{\Delta x^\nu}};\ |N_{x_\chi}| \approx 1}$$

$$\boldsymbol{p_{N_{x_\chi}}, (|N_{x_\chi}| \approx 1) \overset{|N_{x_\chi}|\approx 1 \to |N_{x_\chi}| \gg 1}{\Rightarrow} p_{N_{x_\chi}}, (|N_{x_\chi}| \gg 1).} \qquad (5.2)$$

$$\boldsymbol{l_H(p_{N_{x_\chi}}), (|N_{x_\chi}| \approx 1) \overset{|N_{x_\chi}|\approx 1 \to |N_{x_\chi}| \gg 1}{\Rightarrow} \frac{\ell}{N_{x_\chi}}, (|N_{x_\chi}| \gg 1).}$$

$$\boldsymbol{\Delta_{\mathbf{q}} g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q})_\chi \doteq g_{\mu\nu}(x + l_H(p_{N_{x_\chi}}), N_{x_\chi}, \mathbf{q}) - g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q}),} \qquad (5.3)$$

$$\boldsymbol{\Delta_{\chi,q} g_{\mu\nu}\left(x, N_{x_\chi}, \mathbf{q}\right) \doteq \frac{\Delta_{\mathbf{q}} g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q})_\chi}{l_H(p_{N_{x_\chi}})}.}$$

$$\mathbf{EEM[q]} \doteq R_{\mu\nu}(x, N_{x_\chi}, \mathbf{q}) - \frac{1}{2} R(x, N_{x_\chi}, \mathbf{q})\, g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q}) - \frac{1}{2} \Lambda(x, N_{x_\chi}, \mathbf{q})\, g_{\mu\nu}(x, N_{x_\chi}, \mathbf{q}) =$$
$$= 8\,\pi\, G\, T_{\mu\nu}(x, N_{x_\chi}, \mathbf{q})$$

$$\lim_{\mathrm{E} \ll \mathrm{E_{max}}} \mathbf{EEM[q]} = \mathbf{EEM}, \qquad \mathbf{or} \quad \lim_{|N_{x_\chi}| \gg 1} \mathbf{EEM[q]} = \mathbf{EEM}$$

$$\mathbf{Finally} \quad \lim_{|N_{x_\chi}| \to \infty} \mathbf{EEM[q]} = \mathbf{Einstein\ Equations} = \lim_{\ell \to 0} \mathbf{EEM[q]}$$

6. Generalized Uncertainty Principles in Thermodynamics

Now we consider the thermodynamics uncertainty relations between the inverse temperature and interior energy of a macroscopic ensemble (***Bohr*,*Heisenberg*,*Lavenda*,...):**

$$\Delta \frac{1}{T} \geq \frac{k_B}{\Delta U}, \ (k_B \text{ is the Boltzmann constant}) \qquad \mathbf{(6.1)}$$

(Uncertainty Principle in Thermodynamics **UPT**)

At very high energies the capacity of the heat bath can no longer to be assumed infinite at the Planck scale. In this case an additional term should be introduced into **(6.1)**

$$\Delta \frac{1}{T} \geq \frac{k_B}{\Delta U} + \eta\, \Delta U, \ (\eta \text{ is a coefficient}) \qquad \mathbf{(6.2)}$$

(*Generalized Uncertainty Principle in Thermodynamics* -- GUPT)
Shalyt-Margolin,Tregubovich,Mod.Phys.Lett.A. Mod. Phys. Lett. A,19, 71 (2004); R. Carroll, Fluctuations, Information, Gravity and the Quantum Potential. *Fundam.Theor.Phys.148*, Springer, N.Y., 2006; A.Farmany, Acta Phys. Pol. B., 40, 1569(2009)

Dimension and symmetry reasons give

$$\eta = \frac{k_B}{E_p^2} \quad or \quad \eta = \alpha' \frac{k_B}{E_p^2} \qquad \textbf{(6.3)}$$

(6.2) leads to the fundamental (inverse) temperature.

$$T_{max} = \frac{\hbar}{2\sqrt{\alpha'}t_p k_B} = \frac{E_p}{2\sqrt{\alpha'}k_B} = \frac{T_p}{2\sqrt{\alpha'}} = \frac{\hbar}{t_{min}k_B},$$
$$\beta_{min} = \frac{1}{k_B T_{max}} = \frac{t_{min}}{\hbar} \qquad \textbf{(6.4)}$$

Thus, we obtain the system of generalized uncertainty relations in a symmetric form

$$\begin{cases} \boldsymbol{\Delta x} & \geq & \frac{\hbar}{\boldsymbol{\Delta p}} + \boldsymbol{\alpha}' \left(\frac{\boldsymbol{\Delta p}}{\boldsymbol{P_{pl}}}\right) \frac{\hbar}{\boldsymbol{P_{pl}}} \\ \boldsymbol{\Delta t} & \geq & \frac{\hbar}{\boldsymbol{\Delta E}} + \boldsymbol{\alpha}' \left(\frac{\boldsymbol{\Delta E}}{\boldsymbol{E_p}}\right) \frac{\hbar}{\boldsymbol{E_p}} \\ \boldsymbol{\Delta \frac{1}{T}} & \geq & \frac{\boldsymbol{k_B}}{\boldsymbol{\Delta U}} + \boldsymbol{\alpha}' \left(\frac{\boldsymbol{\Delta U}}{\boldsymbol{E_p}}\right) \frac{\boldsymbol{k_B}}{\boldsymbol{E_p}} \end{cases} \qquad \textbf{(6.5)}$$

or in the equivalent form

$$\begin{cases} \boldsymbol{\Delta x} & \geq & \frac{\hbar}{\boldsymbol{\Delta p}} + \boldsymbol{\alpha}' \boldsymbol{l_p^2} \frac{\boldsymbol{\Delta p}}{\hbar} \\ \boldsymbol{\Delta t} & \geq & \frac{\hbar}{\boldsymbol{\Delta E}} + \boldsymbol{\alpha}' \boldsymbol{t_p^2} \frac{\boldsymbol{\Delta E}}{\hbar} \\ \boldsymbol{\Delta \frac{1}{T}} & \geq & \frac{\boldsymbol{k_B}}{\boldsymbol{\Delta U}} + \boldsymbol{\alpha}' \frac{\boldsymbol{1}}{\boldsymbol{T_p^2}} \frac{\boldsymbol{\Delta U}}{\boldsymbol{k_B}} \end{cases} \qquad \textbf{(6.6)}$$

Here T_p is the Planck temperature: $\boldsymbol{T_p = E_p/k_B}$**.** In this case, without the loss of generality and for symmetry, it is assumed that a dimensionless constant in the right-hand side of **GUP** and in the right-hand side of **GUPT** is the same -- α'.

7. Minimal Inverse Temperature and Measurability. Duality

Now, let us return to the thermodynamic relation in the case of equality:

$$\boldsymbol{\Delta\frac{1}{T} = \frac{k_B}{\Delta U} + \eta\, \Delta U,} \qquad \textbf{(7.1)}$$

that is equivalent to the quadratic equation

$$\boldsymbol{\eta\,(\Delta U)^2 - \Delta\frac{1}{T}\Delta U + k_B = 0}. \qquad \textbf{(7.2)}$$

leading directly to $(\Delta\frac{1}{T})_{min}$

$$\boldsymbol{(\Delta\frac{1}{T})_{min} = 2\sqrt{\alpha'}\,\frac{k_B}{E_p}} \qquad \textbf{(7.4)}$$

It is clear that $(\Delta\frac{1}{T})_{min}$ corresponds to T_{max}

$$\boldsymbol{T_{max} \approx T_p \gg 0}. \qquad \textbf{(7.6)}$$

In this case $\Delta\frac{1}{T} \approx \frac{1}{T}$ and, of course, we can assume that

$$(\frac{\mathbf{1}}{\boldsymbol{T}})_{\boldsymbol{min}} = \tilde{\boldsymbol{\tau}} = \frac{\mathbf{1}}{\boldsymbol{T_{max}}}. \qquad \textbf{(7.7)}$$

Trying to find from formula **(7.7)** a minimal unit of measurability for the inverse temperature and introducing the ``Integrality Condition" (IC)

$$\frac{1}{T} = N_{1/T}\tilde{\tau},\ N_{1/T} > 0 \text{ is an integer number} \qquad \textbf{(7.8)}$$

analog of the **primary measurability** notion into thermodynamics.

Definition 3 (**Primary Thermodynamic Measurability**)

(1) *Let us define a quantity having the dimensions of inverse temperature as* **primarily measurable** *when it satisfies the relation* **(7.8)**.

(2) Let us define any physical quantity in thermodynamics as **primarily measurable** when its value is consistent with point (1) of this Definition.

Definition 3 in thermodynamics is analogous to the **Primary Measurability** in a quantum theory (**Definition 1**).

Now we consider the quadratic equation **(7.2)** in terms of **measurable quantities** in the sense of **Definition 3**. In accordance with this definition we can write

$\boldsymbol{\Delta \frac{1}{T} = N_{\Delta(1/T)}\tilde{\tau}}$, $\boldsymbol{N_{\Delta(1/T)} > 0}$ is an integer number.

This quadratic equation takes the following form:

$$\boldsymbol{\eta\,(\Delta U)^2 - N_{\Delta(1/T)}\tilde{\tau}\Delta U + k_B = 0}. \qquad \textbf{(7.9)}$$

we can find the " **measurable**" roots of this equation:

$$\boldsymbol{(\Delta U)_{meas,\pm} = \frac{[N_{\Delta(1/T)} \pm \sqrt{N^2_{\Delta(1/T)} - 1}]\tilde{\tau}}{2\eta} =}$$
$$\boldsymbol{= \frac{2k_B[N_{\Delta(1/T)} \pm \sqrt{N^2_{\Delta(1/T)} - 1}]\tilde{\tau}}{\tilde{\tau}^2} =} \qquad \textbf{(7.10)}$$
$$\boldsymbol{\frac{2k_B[N_{\Delta(1/T)} \pm \sqrt{N^2_{\Delta(1/T)} - 1}]}{\tilde{\tau}}}.$$

The last line in is associated with the obvious relation $2\eta = \frac{\tilde{\tau}^2}{2k_B}$.

In this way we derive a complete analog of the corresponding relation from a quantum theory by replacement

$$\boldsymbol{\Delta p_\pm \Rightarrow \Delta U_{meas,\pm}; N_{\Delta x} \Rightarrow N_{\Delta(1/T)}; \hbar \Rightarrow k_B}. \qquad \textbf{(7.11)}$$

As, for **low temperatures and energies**, $T \ll T_{max} \propto T_p$, we have $1/T \gg 1/T_p$ and hence $\Delta(1/T) \gg 1/T_p$ and $N_{\Delta(1/T)} \gg 1$.

Only the minus sign in **(7.10)** is consistent with high and low energies.

So, taking the root value in **(7.9)** corresponding to this sign and multiplying the nominator and denominator in **(7.9)** by $N_{\Delta(1/T)} + \sqrt{N^2_{\Delta(1/T)} - 1}$, we obtain

$$(\boldsymbol{\Delta U})_{\boldsymbol{meas}} = \frac{\boldsymbol{k_B}}{\frac{1}{2}(\boldsymbol{N_{\Delta(1/T)}} + \sqrt{\boldsymbol{N^2_{\Delta(1/T)}} - \mathbf{1}})\tilde{\boldsymbol{\tau}}} \qquad \textbf{(7.11)}$$

to have a complete analog of the corresponding relation from quantum theory by substitution.Then it is clear that, in analogy with QT, for low energies and temperatures $N_{\Delta(1/T)} \gg 1$ may be rewritten as

$$(\boldsymbol{\Delta U})_{\boldsymbol{meas}} = (\boldsymbol{\Delta U})_{\boldsymbol{meas}}(\boldsymbol{T} \ll \boldsymbol{T_{max}}) = \frac{\boldsymbol{k_B}}{\frac{1}{2}(\boldsymbol{N_{\Delta(1/T)}} + \sqrt{\boldsymbol{N^2_{\Delta(1/T)}} - \mathbf{1}})\tilde{\boldsymbol{\tau}}} \approx$$
$$\approx \frac{\boldsymbol{k_B}}{\boldsymbol{N_{\Delta(1/T)}}\tilde{\boldsymbol{\tau}}}, \boldsymbol{N_{\Delta(1/T)}} \gg \mathbf{1}, \qquad \textbf{(7.12)}$$

and , at high energies, for $T \approx T_{max}; N_{\Delta(1/T)} \approx 1$, we have:

$$(\boldsymbol{\Delta U})_{meas} = (\boldsymbol{\Delta U})_{meas}(T \approx T_{max}) = \frac{k_B}{1/2(N_{\Delta(1/T)} + \sqrt{N^2_{\Delta(1/T)} - 1})\tilde{\tau}}, \quad \textbf{(7.13)}$$

$N_{\Delta(1/T)} \approx 1$.

$(\boldsymbol{\Delta U})_{meas}$ is not **primarily measurable** thermodynamic quantity.Therefore

Definition 4. Generalized Measurability in Thermodynamics

Any physical quantity in thermodynamics may be referred to as **generalized-measurable** or, for simplicity, **measurable** if any of its values may be obtained in terms of the **Primary Thermodynamic Measurability** of **Definition 3**.

It is clear that we have the limiting transition

$$(\boldsymbol{\Delta U})_{meas}(T \approx T_{max}) \overset{(N_{\Delta(1/T)} \approx 1) \to (N_{\Delta(1/T)} \gg 1)}{\longrightarrow} (\boldsymbol{\Delta U})_{meas}(T \ll T_{max} \propto T_p),$$

that is analogous to the corresponding formula in a quantum theory.

Comment

Naturally, the problem of compatibility between the **measurability** definitions in quantum theory and in thermodynamics arises.On the basis of the previous formulae we can state:

measurability *in quantum theory and* ***thermodynamic measurability*** *are completely* *compatible and consistent as the minimal unit of inverse temperature* $\tilde{\tau}$ *is nothing else but the minimal time* $t_{min} = \tau$ *up to a constant factor. And hence* $N_{1/T}, (N_{\Delta(1/T)})$ *is nothing else but* $N_t, (N_{\Delta t})$. *Then it is clear that* $N_t = N_{a=tc}$.

8. Black Holes and Measurability

Now let us show the applicability this results to a quantum theory of black holes. Consider the case of Schwarzschild's black hole.

Naturally, it is important to study the transition from low to high energies in the indicated case.

We investigate in measurable format gravitational dynamics at low $E \ll E_p$ ***and at high*** $E \approx E_p$ ***energies in the case of the Schwarzschild black hole and in a more general case of the space with static spherically-symmetric horizon in space-time.***

More general cases have been thoroughly studied from the viewpoint of gravitational thermodynamics in remarkable works of professor T. Padmanbhan

The case of a static spherically-symmetric horizon in cont space-time is considered, the horizon being described by the metric

$$\boldsymbol{ds^2 = -f(r)c^2dt^2 + f^{-1}(r)dr^2 + r^2d\Omega^2}. \qquad \textbf{(8.1)}$$

The horizon location will be given by a simple zero of the function $f(r)$, at the radius $r = a$.

Then at the horizon $r = a$ Einstein's field equations

$$\boldsymbol{\frac{c^4}{G}\left[\frac{1}{2}f'(a)a - \frac{1}{2}\right] = 4\pi P a^2} \qquad \textbf{(8.2)}$$

where $P = T_r^r$ is the trace of the momentum-energy tensor and radial pressure. Therewith, the condition $f(a) = 0$ and $f'(a) \neq 0$ must be fulfilled.

It is known that for horizon spaces one can introduce the temperature that can be identified with an analytic continuation to imaginary time. In the case under consideration

$$\boldsymbol{k_B T = \frac{\hbar c f'(a)}{4\pi}}. \qquad \textbf{(8.3)}$$

It is shown that in the initial (continuous) theory the Einstein Equation for horizon spaces in the differential form may be written as a thermodynamic identity (the first principle of thermodynamics)

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_B T}\ \underbrace{\frac{c^3}{G\hbar} d\left(\frac{1}{4} 4\pi a^2\right)}_{dS}\ \underbrace{-\ \frac{1}{2}\frac{c^4 da}{G}}_{-dE} = \underbrace{P d\left(\frac{4\pi}{3} a^3\right)}_{P\,dV}. \qquad \textbf{(8.4)}$$

where, as noted above, T -- temperature of the horizon surface, S --corresponding entropy, E-- internal energy, V -- space volume.

It is impossible to use **(30b)** in the formalism under consideration because, as follows from the given results da, dS, dE, dV are not **measurable quantities**.

First, we assume that a value of the radius r at the point a is a **primarily measurable quantity** i.e. $a = a_{meas} = N_a \ell$, where $N_a > 0$ - integer, and the temperature T from the left-hand side of **(8.4)** is the **measurable** temperature $T = T_{meas}$ in the sense of **Definition 3.**

Then, in terms of **measurable** quantities, first we can rewrite **(8.2)** as

$$\frac{c^4}{G}\left[\frac{2\pi k_B T}{\hbar c}a_{meas}-\frac{1}{2}\right]=4\pi P a_{meas}^2. \qquad \textbf{(9.5)}$$

We express $a = a_{meas} = N_a \ell$ in terms of the deformation parameter $\alpha_a = \frac{1}{{N_a}^2}$

$$a = \ell \alpha_a^{-1/2}; \qquad \textbf{(8.6)}$$

the temperature T is expressed in terms of $T_{max} \propto T_p$.

Then, considering that $T_p = E_p/k_B$, equation **(8.5)** may be given as

$$\frac{c^4}{G}\left[\frac{\pi E_p}{\sqrt{\alpha'}N_{1/T}\hbar c}\ell\alpha_a^{1/2}-\frac{1}{2}\alpha_a\right]=4\pi P\ell^2. \qquad \textbf{(8.7)}$$

Because $\ell = 2\sqrt{\alpha'}l_p$ and $l_p = \frac{\hbar c}{E_p}$, we have

$$\frac{c^4}{G}\left[\frac{2\pi E_p}{N_{1/T}\hbar c}l_p\alpha_a^{1/2}-\frac{1}{2}\alpha_a\right]=\frac{c^4}{G}\left[\frac{2\pi}{N_{1/T}}\alpha_a^{1/2}-\frac{1}{2}\alpha_a\right]=4\pi P\ell^2. \quad \textbf{(8.8)}$$

Note that in its initial form this equation has been considered in a continuous theory, i.e. at low energies $E \ll E_p$. Consequently, in the present formalism it is implicitly meant that the "measurable counterpart" of this equation also initially considered at low energies, in particular, $N_{1/T} \gg 1$.

Let us consider two different cases.

8.1. *Measurable case for low energies:* $E \ll E_p$. Then $a = a_{meas} = N_a \ell$, where the integer number is $N_a \gg 1$ or similarly $N_{1/T} \gg 1$.

As this takes place, $\alpha_a \equiv \alpha_a(HUP)$ is a **primarily measurable** quantity (**Definition 1**), $\alpha_a \approx N_a^{-2}$, though taking a discrete series of values but varying smoothly, in fact *continuously*. **(8.8)** is a quadratic equation with respect to $\alpha_a^{1/2} \approx N_a^{-1}$ with the two parameters $N_{1/T}^{-1}$ and P. In this terms, the equation **(8.8)** may be rewritten as

$$\boldsymbol{\frac{c^4}{G}[\frac{2\pi}{N_{1/T}}\alpha_a^{1/2}(HUP) - \frac{1}{2}\alpha_a(HUP)] = 4\pi P\ell^2}. \qquad \textbf{(8.9)}$$

So, at low energies the equation **(8.9)** written for the discretely-varying $\boldsymbol{\alpha_a}$ may be considered in a continuous theory. As a result, in the case under study we can use the basic formulae from a continuous theory considering them valid to a high accuracy.

In particular, in the notation used for ***Schwarzschild's black hole***,
we have

$$\boldsymbol{r_s = N_a\ell = \frac{2GM}{c^2}; M = \frac{N_a\ell c^2}{2G}}. \qquad \textbf{(8.10)}$$

As its temperature is given by the formula

$$\boldsymbol{T_H = \frac{\hbar c^3}{8\pi G M k_B}}, \qquad \textbf{(8.11)}$$

at once we get

$$\boldsymbol{T_H = \frac{\hbar c}{2\pi k_B N_a \ell} = \frac{\hbar c \alpha_a^{1/2}}{2\pi k_B \ell}}. \qquad \textbf{(8.12)}$$

8.2. *Measurable case for high energies:* $\boldsymbol{E \approx E_p}$. Then, a is the **generalized measurable** quantity $\boldsymbol{a = a_{meas} = 1/2(N_a + \sqrt{N_a^2 - 1})\ell}$, with the integer $\boldsymbol{N_a \approx 1}$**.**

The quantity

$$\boldsymbol{\Delta a_{meas}(q) = 1/[2\left(N_a + \sqrt{N_a^2 - 1}\right)\ell] - N_a \ell = 1/[2\left(\sqrt{N_a^2 - 1} - N_a\right)\ell]} \quad \textbf{(8.14)}$$

may be considered as a **quantum correction** for the **measurable** radius $\boldsymbol{r = a_{meas}}$, that is infinitesimal at low energies $\boldsymbol{E \ll E_p}$ and not infinitesimal for high energies $\boldsymbol{E \approx E_p}$**.**

In this case there is no possibility to replace **GUP** by **HUP**. In equation **(8.8)** $\alpha_a \equiv \alpha_a(GUP) = 1/[\boldsymbol{1/2(N_a + \sqrt{N_a^2 - 1}})]^2$ is a **generalized measurable** quantity.

In this case the number $N_{1/T}$ is replaced by $1/2(N_{1/T} + \sqrt{N_{1/T}^2 - 1})$, i.e. the equation is of the form **(quantum gravity analog)**

$$\frac{c^4}{G}\left[\frac{2\pi}{1/2(N_{1/T}+\sqrt{N_{1/T}^2-1})}\alpha_a^{1/2}(GUP) - \frac{1}{2}\alpha_a(GUP)\right] = 4\pi P\ell^2. \quad \textbf{(8.15)}$$

In so doing the theory becomes really discrete, and the solutions of **(8.15)** take a discrete series of values for every N_a or ($\alpha_a(GUP)$**)** sufficiently close to 1.

In this formalism for a "quantum" Schwarzschild black hole (i.e. at high energies $E \approx E_p$**)** formula black hole temperature T_H is replaced by **quantum black hole** temperature in measurable format

$$T_H(Q) = \frac{\hbar c}{\pi k_B(N_a+\sqrt{N_a^2-1})\ell} = \frac{\hbar c\alpha_a^{1/2}(GUP)}{2\pi k_B\ell}. \qquad \textbf{(8.16)}$$

and its quantum correction $\Delta T_H(Q) = T_H(Q) - T_H$ (from formula (8.12)).

Similarly for for the mass M of a Schwarzschild black hole and other quantities.

Remark 8.1.

A minimal value of $N_a = 1$ is ***unattainable*** because in this case obtain a value of the length l that is below the minimum $l < \ell$ for the momenta and energies above the maximal ones, and that is impossible. **Thus, we always have** $N_a \geq 2$**.**

Remark 8.2. It is clear that we have the following transition:

$$\boldsymbol{Eq}.(\mathbf{8.15})(\boldsymbol{E}\approx\boldsymbol{E_p})\overset{(\boldsymbol{N_a}\approx\mathbf{1})\to(\boldsymbol{N_a}\gg\mathbf{1})}{\to}\boldsymbol{Eq}.(\mathbf{8.9})(\boldsymbol{E}\ll\boldsymbol{E_p})$$

.

Remark 8.3. So, all the members of the gravitational equation apart from $\boldsymbol{P}$, are expressed in terms of the measurable parameter $\boldsymbol{\alpha_a}$**.** From this it follows that P should be also expressed in terms of the measurable parameter α_a, i.e. $P=P(\alpha_a)$: $E\ll E_p$, $P=P[\alpha_a(HUP)]$ at low energies and $E\approx E_p$,$P=P[\alpha_a(GUP)]$ at high energies.

Finally, as $\boldsymbol{\alpha_a}(\boldsymbol{HUP})=\frac{\boldsymbol{\ell^2}}{\hbar^2}\boldsymbol{p_{N_a}}^{\mathbf{2}}$ ***and*** $\boldsymbol{p_{N_a}}$ ***are the primarily measuramle momenta*** $\boldsymbol{p_{N_a}}=\frac{\hbar}{\boldsymbol{N_a\ell}}$ ***and*** $\boldsymbol{\alpha_a}(\boldsymbol{GUP})=\frac{\boldsymbol{\ell^2}}{\hbar^2}\boldsymbol{p}(\boldsymbol{N_a},\boldsymbol{GUP})^{\mathbf{2}}$ ***and*** $\boldsymbol{p}(\boldsymbol{N_a},\boldsymbol{GUP})^{\mathbf{2}}$ ***are the generalized measuramle momenta*** $\boldsymbol{p}(\boldsymbol{N_a},\boldsymbol{GUP})=\frac{\hbar}{\frac{1}{2}\left(\boldsymbol{N_a}+\sqrt{\boldsymbol{N_a^2}-\mathbf{1}}\right)\boldsymbol{\ell}}$ ***then evidentlly, that Einstein Equations in measurable form at all energies scales for spherically symmetric space with horizon is special case of general consideration measurable form Einstein Equations Sections 4–6.***

Afterword

<u>Thanu Padmanabhan</u> showed, that ***"Gravity is an intrinsically***

quantum phenomenon" Mod. Phys. Lett. A17 (2002) 1147-1158 and many other papers,**for example** Mod.Phys.Lett.A25:1129-1136,2010, and so on

Key Publications

1) Minimal Length and the Existence of Some Infinitesimal Quantities in Quantum Theory and Gravity, ***Advances in High Energy Physics, Volume 2014 (2014), Article ID 195157, 8 pages;***

2) Minimal Length, Measurability and Gravity, ***Entropy*** **2016*, 18*(3)*, 80*;**

3) Minimal Length, Minimal Inverse Temperature, Measurability and Black Holes, ***EJTP 14, No.37 (2018) 35–54;***

4) The Measurability Notion in Quantum Theory, Gravity and Thermodynamics: Basic Facts and Implications, ***Chapter 8 in " Horizons in World Physics. Volume 292",pp.199--244, Nova Science Publishers,2017, NY,USA.***

5) Gravity in Measurable Format and Natural Transition to High Energies, ***NPCS,21(2) (2018),138 - 163, ...*** **and other papers.**