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Classical and quantum dynamics of twisted (vortex) electron beams in electric and magnetic fields

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OUTLINE

- Twisted (vortex) particles
- From a pointlike Dirac particle to a centroid
- Twisted electrons in external electric and magnetic fields
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- Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field
- Summary

Twisted electrons and their interactions with external fields and matter are considered in detail in the following recent reviews:

K. Y. Bliokh, I. P. Ivanov, G. Guzzinati, L. Clark, R. Van Boxem, A. Beche, R. Juchtmans, M. A. Alonso, P. Schattschneider, F. Nori, and J. Verbeeck, **Theory and applications of free-electron vortex states, Phys. Rep. 690,** 1 (2017).

S. M. Lloyd,M. Babiker, G. Thirunavukkarasu, and J. Yuan, Electron vortices: Beams with orbital angular momentum, Rev. Mod. Phys. 89, 035004 (2017).

H. Larocque, I. Kaminer, V. Grillo, G. Leuchs, M. J. Padgett, R. W. Boyd, M. Segev, E. Karimi, \`Twisted' electrons, Contemp. Phys. 59, 126 (2018).

We base our explanations on our recent publications:

A. J. Silenko, Pengming Zhang and Liping Zou, Manipulating Twisted Electron Beams, Phys. Rev. Lett. 119, 243903 (2017);
A. J. Silenko, Pengming Zhang and Liping Zou, Relativistic Quantum Dynamics of Twisted Electron Beams in Arbitrary Electric and Magnetic Fields, Phys. Rev. Lett. 121, 043202 (2018).

Twisted (vortex) particles

Twisted particles are free particles which possess an intrinsic orbital angular momentum (OAM)

cylindrical coordinates $\mathbf{r}(\rho, \phi, z)$:

$$\psi_l(\mathbf{r}, t) = u(\rho, z)\exp(il\phi)\exp(ik_z z)\exp(-i\omega t).$$

$u(\rho, z)$:
- a Laguerre-Gaussian function;
- a Bessel function of the first kind;
- a wave function of a bandwidth-limited vortex beam

Laguerre-Gaussian beam:

$$\psi_{\ell,n}^{LG} \propto \left(\frac{r}{w(z)}\right)^{|\ell|} L_n^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)} + ik\frac{r^2}{2R(z)}\right)e^{i(\ell\varphi+kz)}e^{-i(2n+|\ell|+1)\zeta(z)},$$

where $L_n^{|\ell|}$ are the generalized Laguerre polynomials, $n = 0, 1, 2, \ldots$ is the radial quantum number, $w(z) = w_0\sqrt{1 + z^2/z_R^2}$ is the beam width, which slowly varies with z due to diffraction, $R(z) = z\left(1 + z_R^2/z^2\right)$ is the radius of curvature of the wavefronts, and $\zeta(z) = \arctan(z/z_R)$. Here, the characteristic transverse and longitudinal scales of the beam are the waist w_0 (the width in the focal plane $z = 0$) and the Rayleigh diffraction length z_R

$$w_0 \gg 2\pi/k, \qquad z_R = kw_0^2/2 \gg w_0.$$

Bessel beam:

$$\psi_\ell^B \propto J_{|\ell|}(\kappa r)\exp[i(\ell\varphi + k_z z)],$$

where J_ℓ is the Bessel function of the first kind, $\ell = 0, \pm 1, \pm 2, \ldots$ is an integer number (azimuthal quantum number), $k_z = p_z/\hbar$ is the longitudinal wave number, and $\kappa = p_\perp/\hbar$ is the transverse (radial) wave number.

The Bessel beams represent the simplest theoretical example of vortex beams. Despite the probability density of Bessel modes decaying as $|\psi_\ell^B| \sim 1/r$ when $r \to \infty$, these solutions are not properly localized in the transverse dimensions. Indeed, the integral $\int_0^\infty \left|\psi_\ell^B\right|^2 r\,dr$ diverges, and the function cannot be normalized with respect to the transverse dimensions. The delocalized nature of Bessel beams is reflected in the absence of diffraction and a single transverse quantum number ℓ (instead of two transverse quantum indices in the properly-localized modes).

Bandwidth-limited vortex beams:

Fourier transformed truncated Bessel beams (FT-TBB):

$$\psi_{p,l}^{\text{FT-TBB}}(k_\rho, \phi, z) = i^l \lambda_{p,l} J_l'(\lambda_{p,l}) e^{il\phi} \frac{J_l(k_\rho R_{\max})}{k_l^{p,l\,2} - k_\rho^2},$$

where k_q is the transverse wave vector of the diffracted beams. At the focal plane of a lens of power $1/f$, the corresponding radial displacement (ρ) is given by fk_q/k_z $(\sim fk_q/k_0)$.

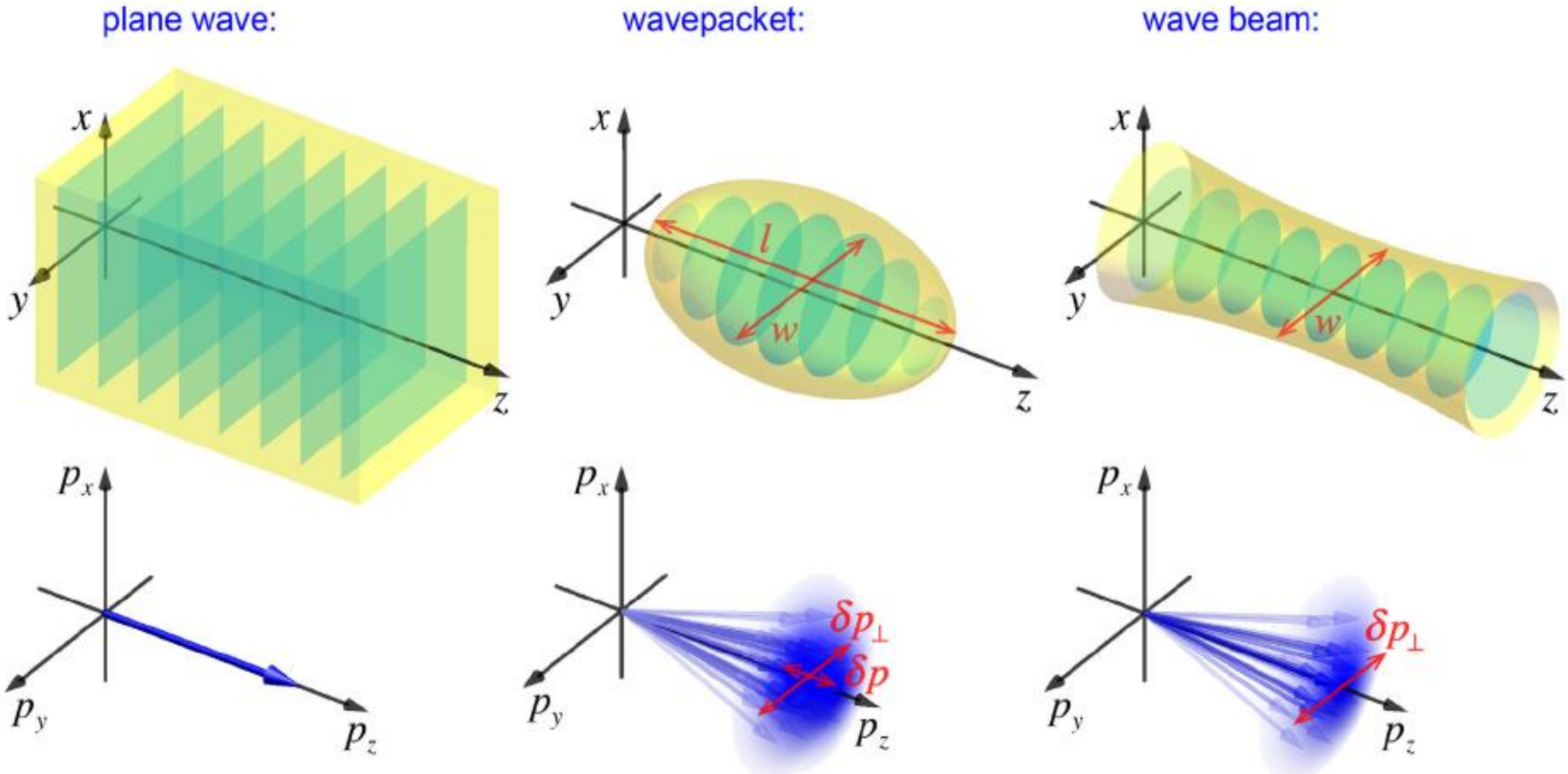
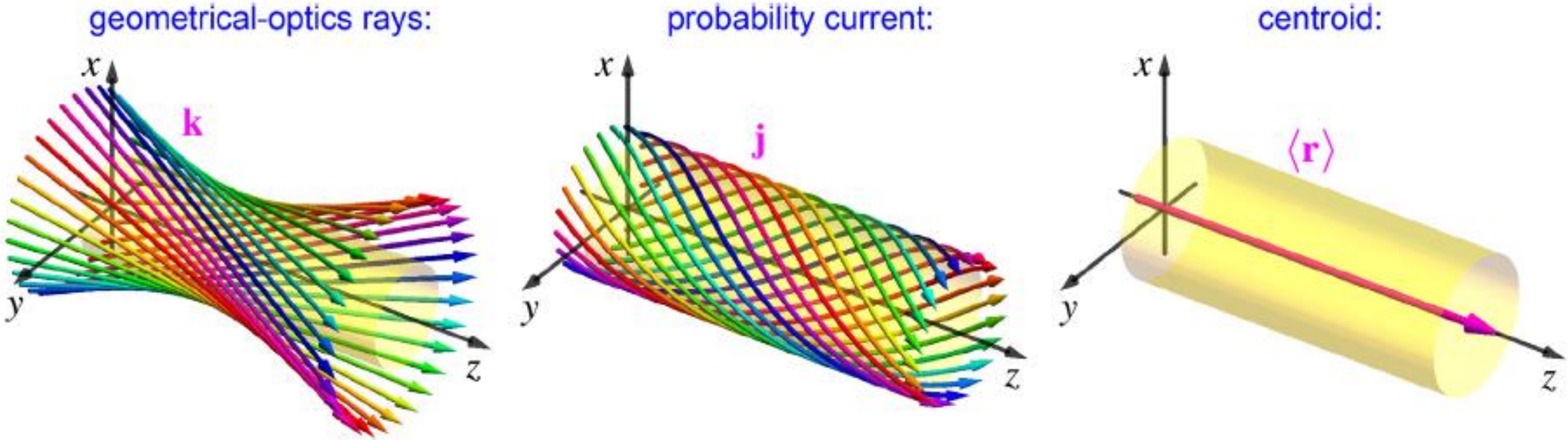


Fig. 4. Schematic pictures of a plane wave, a wavepacket, and a wave beam in both real- and momentum-space representations. The real-space probability density distributions are schematically shown in yellow, while the phase fronts are shown in cyan, and azimuthal symmetry about the propagation z-axis is implied. Assuming the paraxial approximation $p_z \simeq p$, $p_\perp \ll p$, the characteristic dimensions of the real- and momentum-space distributions satisfy the uncertainty relations $l \sim \hbar/\delta p$ and $w \sim \hbar/\delta p_\perp$

In many problems, only the *transverse* localization of the electron is important. Then, one can consider states delocalized in the longitudinal dimension, $l = \infty$, and hence *monoenergetic*: $\delta p = \delta E = 0$. Such states are called *wave beams*, Fig. 4

Wave beams are localized with respect to two dimensions and are described by two discrete quantum numbers related to the transverse modal structure of the beam



Geometrical-optics rays, streamlines of the probability current, and centroid trajectory in a Bessel beam with ℓ = 2

The probability density and probability current density in quantum electron states are determined by

$$\rho = |\psi|^2, \qquad \mathbf{j} = \frac{1}{m_e}\left(\psi|\hat{\mathbf{p}}|\psi\right) = \frac{\hbar}{m_e}\mathrm{Im}\left(\psi^* \nabla \psi\right).$$

In the Bessel beams

$$\rho^B_{|\ell|}(r) \propto \left|J_{|\ell|}(\kappa r)\right|^2, \qquad \mathbf{j}^B_\ell(r,\varphi) = \frac{\hbar}{m_e}\left(\frac{\ell}{r}\bar{\boldsymbol{\varphi}} + k_z\bar{\mathbf{z}}\right)\rho^B_{|\ell|}(r),$$

The ℓ-dependent azimuthal component of the probability current, together with its longitudinal component, results in a spiraling current. This is a common feature of all vortex beams.

- **Twisted electron beams with large intrinsic OAMs (up to 1000*ħ*) have been recently obtained. At present, much attention is also devoted to interactions of such beams with nuclei and a laser field. A dynamics of the intrinsic OAM in external magnetic and electric fields is also studied. We can disregard the anomalous magnetic moment of the electron because its *g* factor is close to 2.**

The Schrödinger form of the relativistic quantum mechanics is provided by the relativistic Foldy-Wouthuysen transformation. The relativistic Foldy-Wouthuysen Hamiltonians generalize the nonrelativistic Schrödinger ones and are similar to the corresponding classical Hamiltonians.

A.J. Silenko, Foldy-Wouthuysen transformation for relativistic particles in external fields, J. Math. Phys. 44, 2952 (2003); **Foldy-Wouthyusen transformation and semiclassical limit for relativistic particles in strong external fields, Phys. Rev. A 77,** 012116 (2008); **Energy expectation values of a particle in nonstationary fields, Phys. Rev. A 91,** 012111 (2015); **General method of the relativistic Foldy-Wouthuysen transformation and proof of validity of the Foldy-Wouthuysen Hamiltonian, Phys. Rev. A 91,** 022103 (2015).

From a pointlike Dirac particle to a centroid

The exact relativistic Hamiltonian in the FW representation (the FW Hamiltonian) for a Dirac particle in a magnetic field is given by (Case, 1954; Tsai, 1973)

$$\mathcal{H}_{FW} = \beta\sqrt{m^2 + \boldsymbol{\pi}^2 - e\boldsymbol{\Sigma}\cdot\boldsymbol{B}},$$

where $\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}$ is the kinetic momentum, $\boldsymbol{B} = \nabla\times\boldsymbol{A}$ is the magnetic induction, and β and $\boldsymbol{\Sigma}$ are the Dirac matrices. This Hamiltonian is valid for a twisted and a untwisted particle. The spin angular momentum operator is equal to $\boldsymbol{s} = \hbar\boldsymbol{\Sigma}/2$. The magnetic field is, in general, nonuniform.

It is necessary to take into account that a twisted electron is a charged centroid (Bliokh et al., 2007; 2017). To describe observable quantum-mechanical effects, we need to present the Hamiltonian in terms of the centroid parameters. The centroid as a

whole is characterized by the center-of-charge radius vector $\boldsymbol{R}$ and by the kinetic momentum $\boldsymbol{\pi}' = \boldsymbol{P} - e\boldsymbol{A}(\boldsymbol{R})$, where $\boldsymbol{P} = -i\hbar\partial/(\partial\boldsymbol{R})$. The intrinsic motion is defined by the kinetic momentum $\boldsymbol{\pi}'' = \mathbf{p} - e[\boldsymbol{A}(\boldsymbol{r}) - \boldsymbol{A}(\boldsymbol{R})]$. Here $\mathbf{p} = -i\hbar\partial/(\partial\mathbf{r})$, $\mathbf{r} = \boldsymbol{r} - \boldsymbol{R}$, $\boldsymbol{\pi}' + \boldsymbol{\pi}'' = \boldsymbol{\pi}$, $\boldsymbol{P} + \mathbf{p} = \boldsymbol{p}$. Since

$$\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{A}(\boldsymbol{R}) + \frac{1}{2}\boldsymbol{B}(\boldsymbol{R}) \times \mathbf{r},$$

the operator $\boldsymbol{\pi}^2$ takes the form

$$\boldsymbol{\pi}^2 = \boldsymbol{\pi}'^2 + \mathbf{p}^2 - \frac{e}{2}[\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}] + \boldsymbol{\pi}'\cdot\boldsymbol{\pi}'' + \boldsymbol{\pi}''\cdot\boldsymbol{\pi}'.$$

After summing over partial waves with different momentum directions, $\langle\boldsymbol{\pi}'\cdot\boldsymbol{\pi}'' + \boldsymbol{\pi}''\cdot\boldsymbol{\pi}'\rangle = 0$. More precisely, the operator $\boldsymbol{\pi}'\cdot\boldsymbol{\pi}'' + \boldsymbol{\pi}''\cdot\boldsymbol{\pi}'$ has zero expectation values for any eigenstates of the operator $\boldsymbol{\pi}^2$ and, therefore, it can be omitted. It can be added that this summing can be performed for the squared Hamiltonian $\mathcal{H}_{FW}^2$.

- **The FW Hamiltonian summed over the partial waves takes the form**

$$\mathcal{H}_{\mathrm{FW}} = \beta\epsilon - \beta\frac{e}{4}\left[\frac{1}{\epsilon}\Lambda\cdot \boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{\Lambda}\frac{1}{\epsilon}\right],$$

$$\epsilon = \sqrt{m^2 + \boldsymbol{\pi}'^2 + \mathbf{p}^2}, \qquad \boldsymbol{\Lambda} = \boldsymbol{L} + \boldsymbol{\Sigma}.$$

kinetic momentum of centroid (→ $\boldsymbol{\pi}'^2$) internal momentum (→ $\mathbf{p}^2$)

The momentum and the intrinsic OAM can have different mutual orientations in different Lorentz frames.

As a rule, the intrinsic OAM and the momentum of the twisted electron are collinear in the lab frame. However, it does not take place in other frames. The Lorentz transformation of the OAM from the lab frame ($\boldsymbol{L}=L_z\mathbf{e}_z$) to the rest frame results in $\boldsymbol{L}^{(0)}=\boldsymbol{L}$. The OAM in the frame moving with the arbitrary velocity V relative to the particle rest frame is given by

$$\boldsymbol{L} = \frac{\epsilon}{mc^2}\boldsymbol{L}^{(0)} - \frac{(\boldsymbol{L}^{(0)}\cdot\boldsymbol{p})\boldsymbol{p}}{m(\epsilon+mc^2)}, \qquad \epsilon = \frac{mc^2}{\sqrt{1-\frac{V^2}{c^2}}}$$

Twisted electrons in external electric and magnetic fields

There is a significant difference between the orbital angular momentum (OAM) and the spin. The OAM is formed by the spatial components of the antisymmetric tensor $L^{\mu\nu} \equiv x^\mu p^\nu - x^\nu p^\mu$. Unlike the OAM, the conventional spin ζ is defined by the spatial part of the four-component spin pseudovector a^μ in the particle rest frame.

The spatial components of the spin tensor $\mathfrak{S}^{\mu\nu}$ form the three-component spatial pseudovector $\mathfrak{S}$, which is not equivalent to ζ.

An interaction of the electric and magnetic dipole moments, $\boldsymbol{d}$ and $\boldsymbol{\mu}$, with the external fields is defined by the general Hamiltonian

$$H = -\boldsymbol{d} \cdot \boldsymbol{E} - \boldsymbol{\mu} \cdot \boldsymbol{B},$$

where all quantities are defined in the lab frame.

Now we can take into account that the quantities $\boldsymbol{L}^{(0)}$ and $\boldsymbol{\mu}^{(0)}$ are connected with the nonrotating instantaneous inertial frame and can perform the relativistic transformation of the dipole moments to the lab frame:

$$\mathbf{d} = \boldsymbol{\beta} \times \boldsymbol{\mu}^{(0)} = \boldsymbol{\beta} \times \boldsymbol{\mu}, \qquad \boldsymbol{\mu} = \boldsymbol{\mu}^{(0)} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{\mu}^{(0)}),$$

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}, \qquad \gamma = (1-\beta^2)^{-1/2},$$

where we have used the fact $\boldsymbol{d}^{(0)}=0$.

A.J. Silenko, Spin precession of a particle with an electric dipole moment: Contributions from classical electrodynamics and from the Thomas effect, Phys. Scripta 90, 065303 (2015).

As a result, the Hamiltonian is given by

$$H = -\frac{e}{2mc}\left[\boldsymbol{B}\cdot\boldsymbol{L}^{(0)} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{B})(\boldsymbol{\beta}\cdot\boldsymbol{L}^{(0)}) - (\boldsymbol{\beta}\times\boldsymbol{E})\cdot\boldsymbol{L}^{(0)}\right]$$
$$= -\frac{e}{2mc\gamma}[\boldsymbol{B}\cdot\boldsymbol{L} - (\boldsymbol{\beta}\times\boldsymbol{E})\cdot\boldsymbol{L}].$$

The corresponding relativistic FW Hamiltonian is similar:

$$\mathcal{H}_{\rm FW} = \beta\epsilon + e\Phi - \beta\frac{e}{4}\left[\frac{1}{\epsilon}\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R}) + \boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}\frac{1}{\epsilon}\right]$$

$$+\frac{e}{4}\left\{\frac{1}{\epsilon^2}\boldsymbol{L}\cdot[\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R})] - [\boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}']\cdot\boldsymbol{L}\frac{1}{\epsilon^2}\right\}.$$

In this equation, spin effects are disregarded because they can be neglected on the condition that $L \gg 1$. The term $e\Phi$ does not include the interaction of the intrinsic OAM with the electric field.

The equation of motion of the intrinsic OAM has the form

$$\frac{d\boldsymbol{L}}{dt} = i[\mathcal{H}_{\rm FW}, \boldsymbol{L}] = \frac{1}{2}(\boldsymbol{\Omega}\times\boldsymbol{L} - \boldsymbol{L}\times\boldsymbol{\Omega}),$$

$$\boldsymbol{\Omega} = -\beta\frac{e}{4}\left\{\frac{1}{\epsilon}, \boldsymbol{B}(\boldsymbol{R})\right\} + \frac{e}{4}\left[\frac{1}{\epsilon^2}\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R}) - \boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}'\frac{1}{\epsilon^2}\right].$$

In the classical limit,

$$\boldsymbol{\Omega} = -\frac{e}{2mc\gamma}[\boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{E}].$$

The corresponding equation of the spin precession is very different.

Stern-Gerlach-like force

The operator of the total force is given by

$$\begin{aligned} \boldsymbol{F} &= \frac{d\boldsymbol{\pi}'}{dt} = \frac{\partial \boldsymbol{\pi}'}{\partial t} + i[\mathcal{H}_{\mathrm{FW}}, \boldsymbol{\pi}'] \\ &= e\boldsymbol{E}(\boldsymbol{R}) + \beta \frac{e}{4}\left\{\frac{1}{\epsilon}, (\boldsymbol{\pi}' \times \boldsymbol{B}(\boldsymbol{R}) - \boldsymbol{B}(\boldsymbol{R}) \times \boldsymbol{\pi}')\right\} + \boldsymbol{F}_{SGl}. \end{aligned}$$

Lorentz force

Beam splitting in nonuniform electric and magnetic fields is conditioned by the Stern-Gerlach-like force operator

$$\boldsymbol{F}_{\mathrm{SGI}} = \beta\frac{e}{4}\left\{\frac{1}{\epsilon}\nabla[\boldsymbol{L}\cdot\boldsymbol{B}(\boldsymbol{R})] + \nabla[\boldsymbol{B}(\boldsymbol{R})\cdot\boldsymbol{L}]\frac{1}{\epsilon}\right\}$$

$$-\frac{e}{4}\left\{\frac{1}{\epsilon^2}\nabla(\boldsymbol{L}\cdot[\boldsymbol{\pi}'\times\boldsymbol{E}(\boldsymbol{R})]) - \nabla([\boldsymbol{E}(\boldsymbol{R})\times\boldsymbol{\pi}']\cdot\boldsymbol{L})\frac{1}{\epsilon^2}\right\}.$$

The force is exerted to the center of charge of the centroid.

Manipulations of twisted electron beams

The results obtained permit us to develop methods for the manipulation of electron vortex beams

A. J. Silenko, Pengming Zhang and Liping Zou, Manipulating Twisted Electron Beams, Phys. Rev. Lett. 119, 243903 (2017).

Separations of beams with opposite directions of the OAM

The beam separation can be achieved in a longitudinal magnetic field. The nonuniform longitudinal magnetic field leads to a force acting on the OAM. The direction of this force depends on that of the OAM. As a result, accelerations of particles with oppositely directed OAMs have different signs. Therefore, the beam with a given OAM direction can be extracted (e.g., with the Wien filter). We should mention that a transversal magnetic field can destroy a beam coherence as a result of Larmor precession.

Freezing the intrinsic OAM in electromagnetic fields

As in spin physics, it is important to consider a condition which allows one to freeze the intrinsic OAM (i.e., to keep the orbital helicity constant) in electromagnetic fields. These fields deflect the beam. We consider a potential for a beam deflection without a change of the orbital helicity h_{orb}. In this case, the angular velocity of the relativistic Larmor precession should be equal to the angular velocity of the rotation of the momentum direction $\mathbf{N}=\mathbf{p}/p\approx\mathbf{V}/V$:

$$\frac{dN}{dt} = \omega \times N, \qquad \omega = -\frac{e}{mc\gamma}\left(B - \frac{N \times E}{\beta}\right).$$

The standard geometry is $E\perp B\perp V$. The condition $\Omega_L=\omega$ is satisfied when

$$\boldsymbol{B} = \left(\frac{2}{\beta^2} - 1\right)\boldsymbol{\beta} \times \boldsymbol{E}.$$

The device defined this equation is a deflector of the twisted electron beams which freezes the OAM relative to the momentum direction. It rotates the beam direction with the angular velocity

$$\boldsymbol{\Omega}_L = \boldsymbol{\omega} = -\frac{e\boldsymbol{B}}{mc\gamma(\gamma^2+1)}.$$

For standard beams with energy ~10^2 keV, the deflection is rather effective.

Flipping the intrinsic OAM

If an electron vortex beam with an upward or a downward orbital polarization is confined in a storage ring, the direction of the intrinsic OAM can be flipped. A flip of the OAM is similar to that of the spin and can be fulfilled by the method of the magnetic resonance. A significant difference between the flips of the OAM and the spin consists in different dependences of the resonance frequencies on the electric and magnetic fields. A spin flip frequency in a storage ring is defined by the Thomas-Bargmann-Mishel-Telegdi equation whose distinction from the corresponding equation for the OAM is evident. The OAM flip can be forced by a longitudinal (azimuthal) magnetic field oscillating with the resonance frequency. A Wien filter with a vertical electric field and a radial magnetic field (when the two fields oscillate with the resonance frequency) can also be used for the OAM flip.

Rotation of the intrinsic OAM in crossed electric and magnetic fields as a critical experiment

We propose a simple critical experiment for a verification of the results obtained. For this purpose, crossed electric and magnetic fields ($E \perp B \perp \beta$) satisfying the relation $E = -\beta \times B$ can be used. Such fields characterizing the Wien filter do not affect a beam trajectory. We suppose the fields E and B to be uniform. In the considered case, the classical limit of the relativistic equation for the angular velocity of precession of the intrinsic OAM is given by

$$\Omega^{(W)} = -\frac{e\left(m^2 + \mathfrak{p}^2\right)}{2\epsilon^3} B.$$

Here $\mathfrak{p}$ is the momentum characterizing the internal motion inside of the centroid, $\varepsilon = m\gamma$. It is necessary to use a single twisted electron beam possessing a standard orbital polarization collinear to the beam momentum. The intrinsic OAM rotates with the angular frequency $\Omega^{(W)}$ and reverses its direction with the angular frequency $2\Omega^{(W)}$. The device is the intrinsic-OAM rotator. As twisted electron beams are relativistic, a quantitative verification of the results obtained can be fulfilled.

Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field

The well-known effect is the radiative spin polarization of electron or positron beams in storage rings caused by the synchrotron radiation (Sokolov-Ternov effect). The radiative spin polarization acquired by unpolarized electrons is opposite to the direction of the main magnetic field. The reason for the effect is a dependence of spin-flip transitions from the initial particle polarization. It follows from the previously obtained results (Ivanov, 2012; Ivanov et al., 2016) that quantum-electrodynamics effects are rather similar for twisted and untwisted particles. We can note the evident similarity between interactions of the spin and the intrinsic OAM with the magnetic field. In particular, energies of stationary states depend on projections of the spin and the intrinsic OAM on the field direction. This similarity validates the existence of the effect of the radiative orbital polarization. As well as the radiative spin polarization, the corresponding orbital polarization acquired by unpolarized twisted electrons should be opposite to the direction of the main magnetic field.

The effect is conditioned by transitions with a change of a projection of the intrinsic OAM. The probability of such transitions is large enough if the electron energy is not too small. Similarly to the spin polarization, the orbital one is observable when electrons are accelerated up to the energy of the order of 1 GeV. The acceleration can depolarize twisted electrons but cannot vanish *L*. During the process of the radiative polarization, the average energy of the electrons should be kept unchanged.

A. J. Silenko, Pengming Zhang and Liping Zou, Relativistic Quantum Dynamics of Twisted Electron Beams in Arbitrary Electric and Magnetic Fields, Phys. Rev. Lett. 121, 043202 (2018).

Summary

- Main distinctive features of the twisted (vortex) particles have been discussed
- General equation defining dynamics of twisted electrons in external electric and magnetic fields has been derived
- Methods of manipulations of twisted electron beams have been considered
- Rotation of the intrinsic OAM in crossed electric and magnetic fields is proposed as a critical experiment for a verification of the results obtained
- Sokolov-Ternov-like effect of a radiative orbital polarization of twisted electron beams in a magnetic field is found


Thank you for your attention