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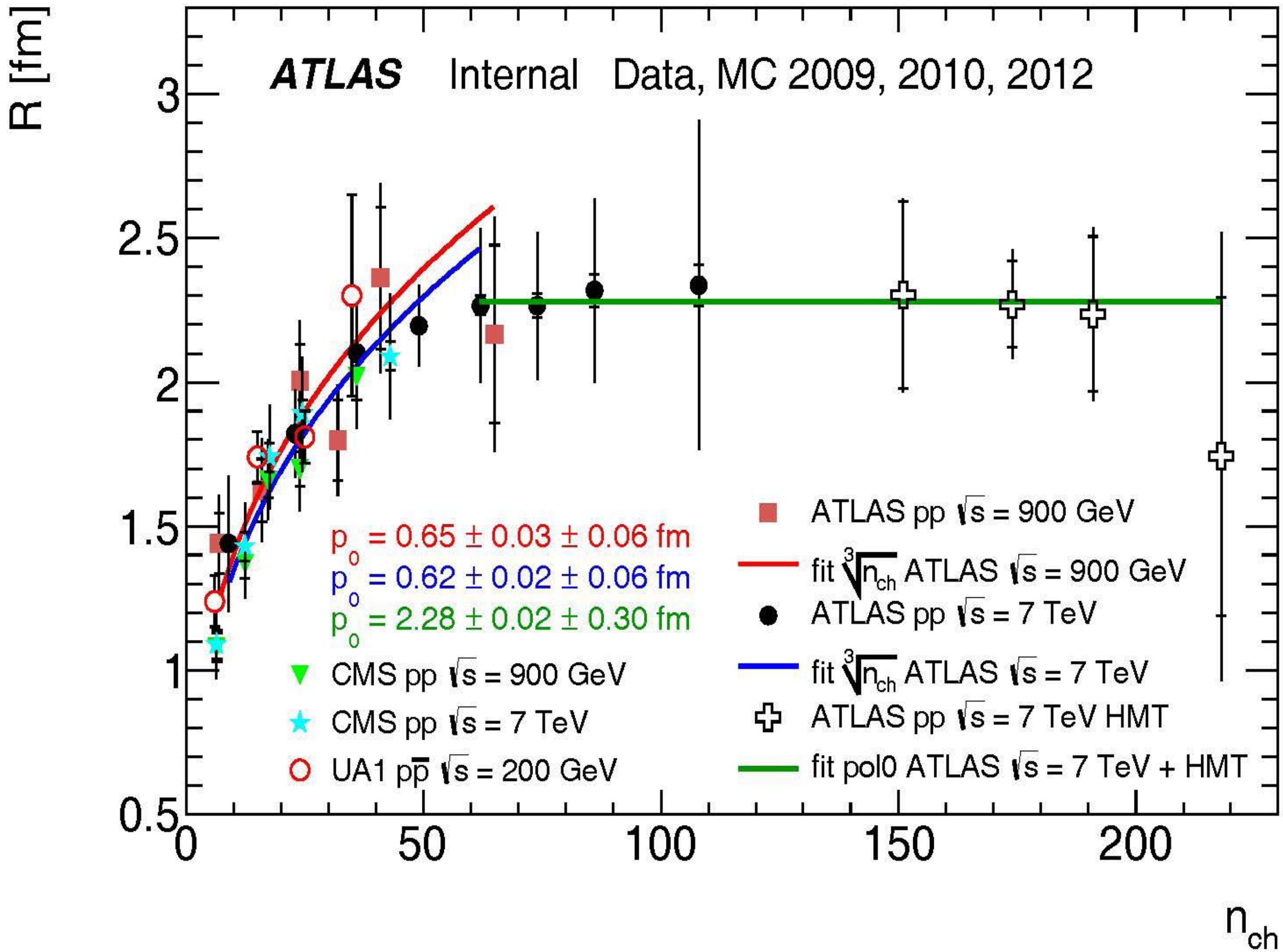
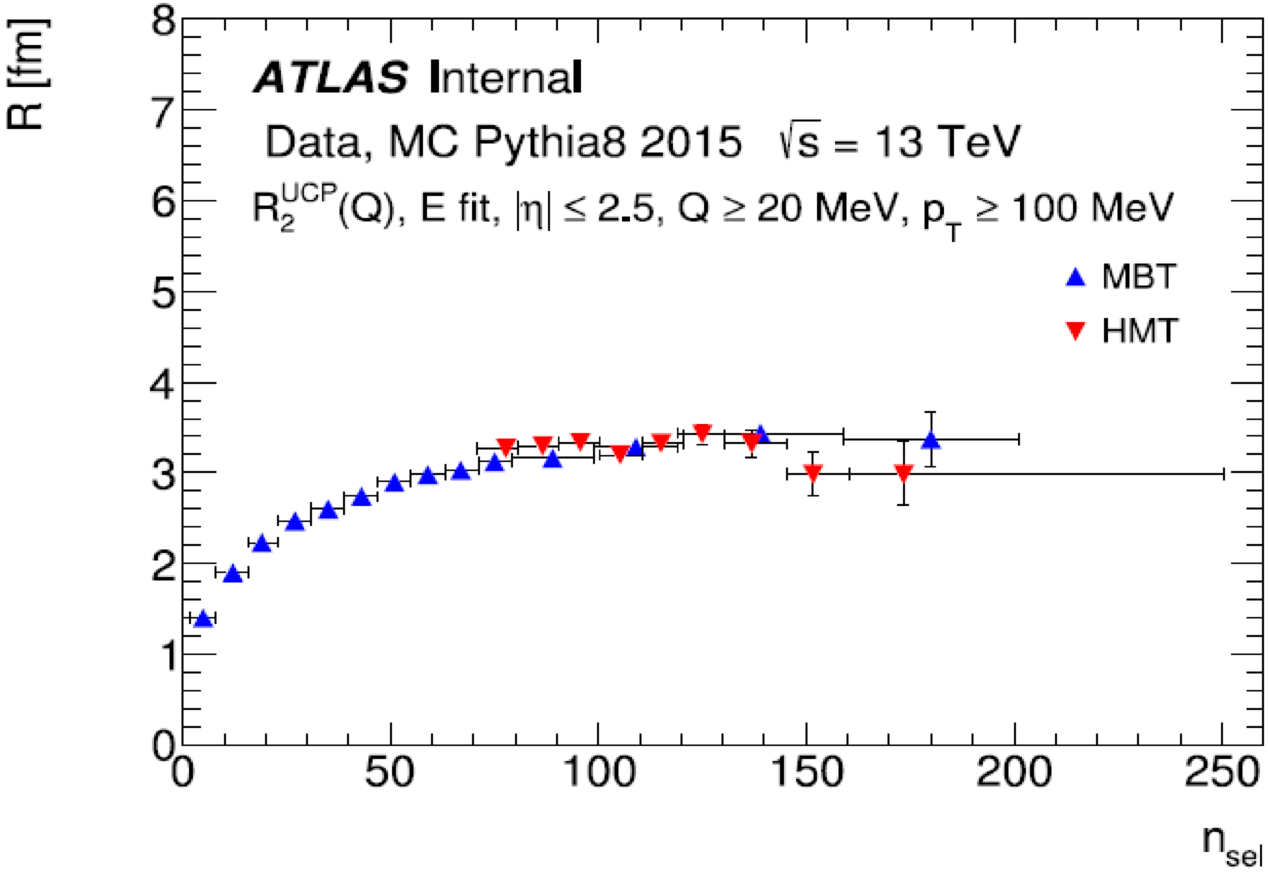
The model of hadrons and problem of the multi-particles production

GRODNO 2018

Plan

- I ***Coherent states on the horosphere of the Lobachevsky momentum space as partons***
- Introduction. Some results of multi-particle production on LHC
- Kinematic of the of the $pp-$ collisions in the quasi Cartesian coordinates on the horosphere of Lobachevsky momentum space
- Conventional coherent states on horosphere of Lobachevsky momentum space.
- Space picture in the laboratory system and main hypothesis.
- Multiplicity distribution
- Coordinate and momentum representation of the coherent states and multiplicity distribution
- II ***Coherent states of particles moving in constant homogeneous non-Abelian gauge fields and dynamical nature of characteristic size hadrons.***
- III ***The model of the scalar hadron on the base of classical field theory***


ATLAS Internal Data, MC 2009, 2010, 2012
R [fm]
n_ch
p_0 = 0.65 ± 0.03 ± 0.06 fm
p_0 = 0.62 ± 0.02 ± 0.06 fm
p_0 = 2.28 ± 0.02 ± 0.30 fm
CMS pp √s = 900 GeV
CMS pp √s = 7 TeV
UA1 pp̄ √s = 200 GeV
ATLAS pp √s = 900 GeV
fit ∛n_ch ATLAS √s = 900 GeV
ATLAS pp √s = 7 TeV
fit ∛n_ch ATLAS √s = 7 TeV
ATLAS pp √s = 7 TeV HMT
fit pol0 ATLAS √s = 7 TeV + HMT
0.5
1
1.5
2
2.5
3
0
50
100
150
200



ATLAS Internal
Data, MC Pythia8 2015 √s = 13 TeV
R₂^UCP(Q), E fit, |η| ≤ 2.5, Q ≥ 20 MeV, p_T ≥ 100 MeV
MBT
HMT
R [fm]
n_sel
0
1
2
3
4
5
6
7
8
0
50
100
150
200
250

Typical sizes

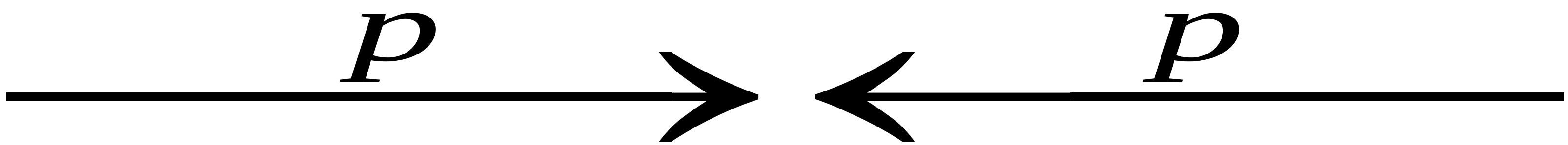
$$r_{0s} = 1,46 \cdot 10^{-15}, r_0 \square\, 2,3 \cdot 10^{-15}, r = \frac{1}{\sqrt{S}} \square\, 10^{-18}, \sqrt{S} = 7TeV$$

The ATLAS Collaboration. Two-particle Bose–Einstein correlations in *pp* collisions at $\sqrt{S}$ 0.9 and 7 TeV measured with the ATLAS detector. CERN-PH-EP-2014-264. Submitted to: EPJC.

$$r_{эф} \leq \frac{\ln P(S)}{m_\pi}$$

A.A.Logunov, M.A.Mestvirishvili, V.A. Petrov. In book General principles of the quantum fields theory and hadrons interactions under high energy M. : Nauka 1977

Kinematics of the collisions of hadrons



$$p_1 = (ip_{01}, \vec{p}_1) \qquad p_2 = (ip_{02}, \vec{p}_2)$$

$$p_1^{\,2} = \vec{p}_1^{\,2} - p_{01}^{\,2} = \vec{p}_2^{\,2} - p_{02}^{\,2} = -m_p^{\,2} \qquad (1)$$

- Here m_p - proton mass, we use the system of the physical units with $\hbar = c = 1$, $\sqrt{s}$ - system mass energy of protons.

Kinematics of the collisions of hadrons

$$S = -(p_1 + p_2)^2 = -P^2 = -(p_{x1} + p_{x2})^2 - (p_{y1} + p_{y2})^2 - (p_{z1} + p_{z2})^2 + (p_{01} + p_{02})^2, (2)$$

$$P = (\vec{P}, iP_0) = [p_{x1} + p_{x2}, p_{y1} + p_{y2}, p_{z1} + p_{z2}, +i(p_{01} + p_{02})] \quad (3)$$

In the laboratory systems, where second proton is in the rest

$$P = (\vec{P}, iP_0) = [p_x, p_y, p_z, i(p_0 + m_p)] \quad (4)$$

and four dimensional momentum of the falling proton denote as $p = (p_x, p_y, p_z, ip_0) = (\vec{p}, ip_0)$

Main hypothesis

As you know, in the laboratory frame the incident particle (hadron) is attened in the direction of the movement due to the Lorentz contraction. In this case, transverse degrees of freedom (x and y) are important since at high energies the kinetic energy of the hadron constituents (partons) is much larger than the energy of their interaction. Therefore hadron moving at a speed close to the speed of light can be seen as a set of almost free partons. Since components of hadron move in unison before collision therefore this state of a hadron can be considered as a coherent state of its transverse excitations i.e. partons.

The main hypothesis is that the incident particle is a coherent state of partons i.e. transverse excitations on horosphere of the relativistic Lobachevsky momentum space.

Yu.A. Kurochkin, Yu.A. Kulchitsky, S.N. Harkusha, N.A. Russakovich Hadron as coherent state jn horosphere of the Lobachevsky momentum space. Письма в ЭЧАЯ 2016 Т.13, №3 (201)б 454-460. // Phys. Part. Nuclei Lett., 2016, V. 13, № 3, P. 285-288.

Quasi Cartesian coordinates on the horosphere of Lobachevsky momentum space

$$P_z = \frac{\sqrt{S}}{2}[e^{q_z/\sqrt{S}} + (\frac{q_x{}^2 + q_y{}^2}{S} + 1)e^{-q_z/\sqrt{S}}], \qquad P_x = q_x e^{-q_z/\sqrt{S}}, \qquad (5)$$

$$P_y = q_y e^{-q_z/\sqrt{S}}, \qquad P_0 = \frac{\sqrt{S}}{2}[e^{q_z/\sqrt{S}} + (\frac{q_x{}^2 + q_y{}^2}{S} - 1)e^{-q_z/\sqrt{S}}] \qquad (6)$$

Inverse formulas

$$q_x = \frac{P_x\sqrt{S}}{P_z - P_0}, q_y = \frac{P_y\sqrt{S}}{P_z - P_0}, q_z = \sqrt{S}\ln\frac{\sqrt{S}}{P_z - P_0} \qquad (7)$$

Quasi Cartesian coordinates on the horosphere of Lobachevsky momentum space

Metric element in these coordinates expressed as follows

$$dl_m{}^2 = e^{-2q_z/\sqrt{S}}(dq_x{}^2 + dq_y{}^2) + dq_z{}^2 \qquad (8)$$

The element of the volume in the momentum space in the horospherical (quasi Cortesian) coordinates

$$dV_m = \sqrt{g}\,dq_x dq_y dq_z = e^{-2q_z/\sqrt{S}}\,dq_x dq_y dq_z \qquad (9)$$

Separation coordinates

$$\Psi_1(x,y)\Psi_2(z) \leftrightarrow F\{\varphi_1(q_x,q_y)\varphi_2(q_z) \qquad (10)$$

Quantum mechanical variables (momenta, coordinates) on horosphere

$$q_x, q_y, \quad x = -i\hbar \frac{\partial}{\partial q_x}, \quad y = -i\hbar \frac{\partial}{\partial q_y}. \tag{11}$$

Heisenberg-Weyl algebra is the base for definition of the coherent states

$$\left[x, q_x\right] = \left[y, q_y\right] = -i\hbar I, \quad \left[x, y\right] = \left[q_x, q_y\right] = 0,$$
$$\left[x, I\right] = \left[I, y\right] = \left[q_x, I\right] = \left[q_y, I,\right] = 0, \tag{12}$$

Creation and annihilation operators connected with our problem

$$a_x = \frac{Rq_x + i\frac{x}{R}}{\sqrt{2}}, a_x^+ = \frac{Rq_x - i\frac{x}{R}}{\sqrt{2}}, a_y = \frac{Rq_y + i\frac{y}{R}}{\sqrt{2}}, a_y = \frac{Rq_y - i\frac{y}{R}}{\sqrt{2}}; \tag{13}$$

Yu. A. Kurochkin, I. Rybak, Dz. V. Shoukovy *Coherent states on the horosphere of the three dimensional Lobachevsky space. Doklady NAS Belarus 2014*

Heisenberg-Weyl algebra in terms of the creation and annihilation operators

(14)

$$\left[a_k, a_l^+\right] = \delta_{kl} I, \left[a_k^+, a_l^+\right] = \left[a_k, a_l\right] = \left[a_k, I\right] = \left[a_k^+, I\right] = 0.$$

- where $k, l = 1, 2$ corresponds to x and y
- Definition of the coherent states and some
- properties

$$a_x \left|z_1\right\rangle = z_1 \left|z_1\right\rangle, a_y \left|z_2\right\rangle = z_2 \left|z_2\right\rangle$$

(15)

$$\left\langle z_1 \middle| z_1 \right\rangle = \exp\left|z_1\right|^2, \qquad \left\langle z_2 \middle| z_2 \right\rangle = \exp\left|z_2\right|^2$$

Переломов А.М. Обобщенные когерентные состояния и их применения. М.: Наука. 1987.

Definition of the coherent states and some properties

$$|z_1, z_2\rangle = \exp\left(z_1 a_x^+\right)\exp\left(z_2 a_y^+\right)|0,0\rangle \quad (16)$$

- $|0,0\rangle$ -vacuum state, $a_x|0,0\rangle = a_y|0,0\rangle = 0$ (17)

$$\int |z_1, z_2\rangle\langle z_1, z_2| d\mu(z_1, z_2) = \int |z_1\rangle\langle z_1| d\mu(z_1)\int |z_2\rangle\langle z_2| d\mu(z_2) = I \quad (18)$$

$$\Delta x \Delta q_x = \frac{\hbar}{2}, \quad \Delta y \Delta q_y = \frac{\hbar}{2} \quad (19)$$

Average number of the excited quanta

$$\bar{n}_1 = \exp(-|z_1|^2)\langle z_1 | a^+{}_x a_x | z_1 \rangle = |z_1|^2, \quad \bar{n}_2 = \exp(-|z_2|^2)\langle z_2 | a^+{}_y a_y | z_2 \rangle = |z_2|^2 \quad (20)$$

$$\bar{n} = \bar{n}_1 + \bar{n}_2 = |z_1|^2 + |z_2|^2 \quad (21)$$

Multiplicity distribution

$$P(n) = \frac{\exp(-\bar{n})\bar{n}^n}{n!} \quad (22)$$

Coordinate representation of coherent states

$$\langle x, y | z_1, z_2 \rangle \propto e^{i\frac{\sqrt{2}}{R}(\beta_1 x+\beta_2 y)} \times e^{-\frac{1}{2R^2}[(x-\sqrt{2}R\alpha_1)^2+(y-\sqrt{2}R\alpha_2)^2]} \qquad (23)$$

$$\left|\langle x, y | z_1, z_2 \rangle\right|^2 \propto e^{-\frac{1}{2R^2}[(x-\sqrt{2}R\alpha_1)^2+(y-\sqrt{2}R\alpha_2)^2]} \qquad (24)$$

$$z_1=\alpha_1+i\beta_1, \alpha_1=|z_1|\cos\vartheta_1, \beta_1=|z_1|\sin\vartheta_1,\; z_2=\alpha_2+i\beta_2, \alpha_2=|z_2|\cos\vartheta_2, \beta_2=|z_2|\sin\vartheta_2$$

$$\sqrt{2}R\alpha_1=\sqrt{2}R|z_1|\cos\vartheta_1=\sqrt{2n}R\cos\vartheta_1=r_0, R=0{,}84\times10^{-15} \qquad (25)$$

Momentum representation of coherent states

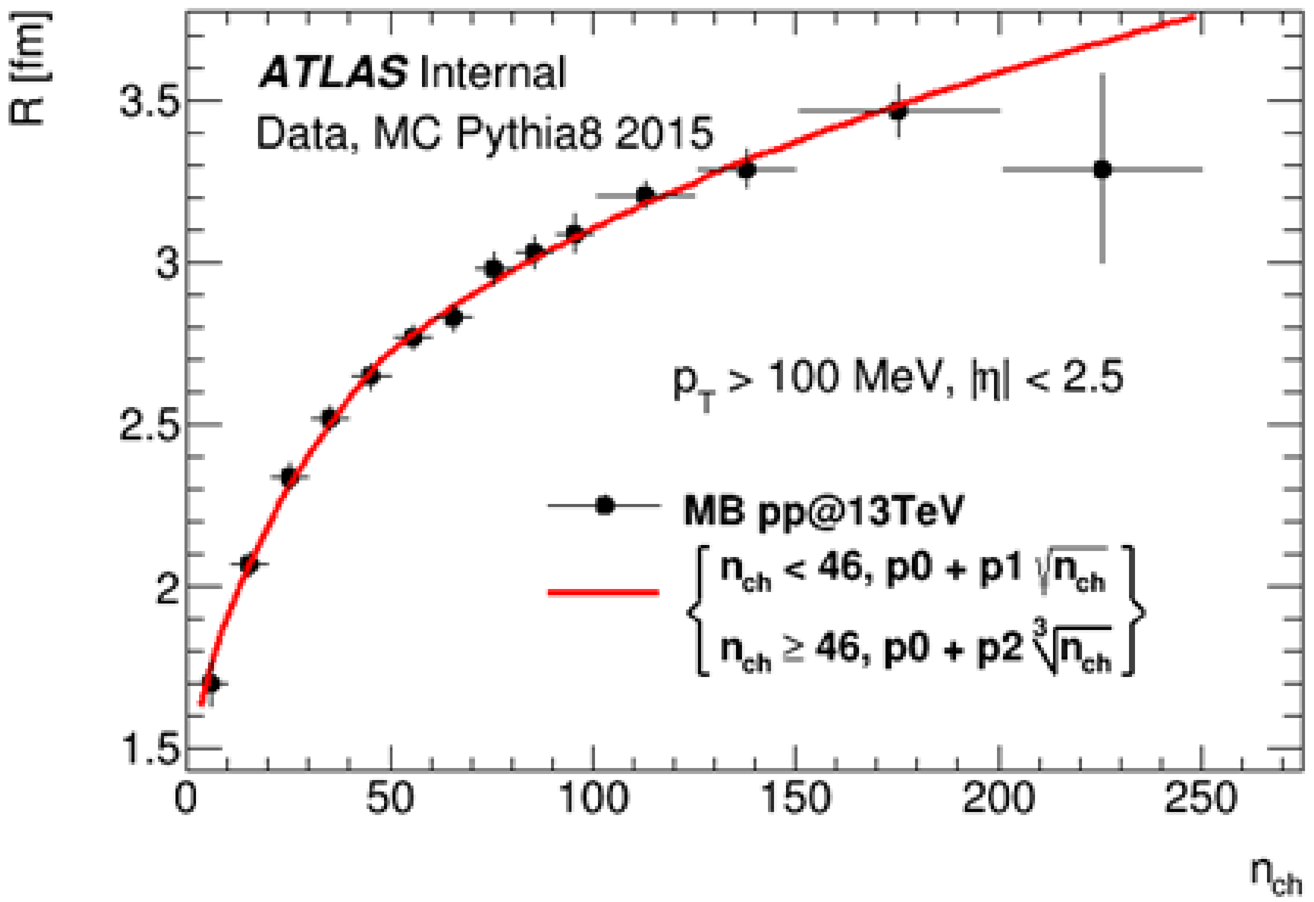
$$\left\langle q_x, q_y \middle| z_1, z_2 \right\rangle \square\ e^{iR\sqrt{2}(\alpha_1 q_x + \alpha_2 q_y)} \times e^{-R^2/_2[(q_x - \sqrt{2}/_R \beta_1)^2 + (q_y - \sqrt{2}/_R \beta_2)^2]} \qquad (26)$$

$$\left| \left\langle q_x, q_y \middle| z_1, z_2 \right\rangle \right|^2 \square\ e^{-R^2/_2[(q_x - \sqrt{2}/_R \beta_1)^2 + (q_y - \sqrt{2}/_R \beta_2)^2]} \qquad (27)$$

$$z_1 = \alpha_1 + i\beta_1, \alpha_1 = |z_1| \cos \vartheta_1, \beta_1 = |z_1| \sin \vartheta_1,\ z_2 = \alpha_2 + i\beta_2, \alpha_2 = |z_2| \cos \vartheta_2, \beta_2 = |z_2| \sin \vartheta_2$$

$$\sqrt{2} R \alpha_1 = \sqrt{2} R |z_1| \cos \vartheta_1 = \sqrt{2n} R \cos \vartheta_1 = r_0, R = 0{,}84 \times 10^{-15}$$

Fitting of the experimental data



Denotations to previous pictures

- p0 = 1.26 $\pm$ 0.15, p1 = 0.21 $\pm$ 0.03, (28)
  p2 = 0.40 $\pm$ 0.03.

Conclusion to part I

- Thus, in our approach, we distinguish two stages of the process.
- 1. At each stage, the spatial distribution of hadronic matter is different.

  2.In the first stage it is two-dimensional, on the second it is three-dimensional.

|| Coherent states of particles moving in constant homogeneous non-Abelian gauge fields and dynamical nature of characteristic size hadrons.

Quark in external gluon field of magnetic type

$$i\gamma_\mu (p_\mu + gT^a A^a{}_\mu)\psi = m\psi \qquad (29)$$

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad T^5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad T^6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad (30)$$

$$T^7 = \frac{1}{2}\begin{pmatrix} 0 & & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Kurochkin Yu.A. , Harkusha S.N. , Kulchitsky Yu.A. , Russakovich N.A. /On describing quark in external gluon field of magnetic type // Proceeding of the National Academy of Sciences of Belarus. Physics and Mathematics series - 2017, n.4, P. 39-43.

Non –Abelian color field

$$A^a{}_\mu = (0, 0, H^a x_1, 0) \qquad (31)$$

$$\nabla_\mu F^a{}_{\mu\nu} = \partial_\mu F^a{}_{\mu\nu} + g f^{abc} F^b{}_{\mu\nu} A^c{}_\nu = 0 \qquad (32)$$

$$F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + g f^{abc} A^b{}_\mu A^c{}_\nu \qquad (33)$$

Quark in external gluon field of magnetic

$$i\gamma_{\mu}(p_{\mu} + gT^{a}H^{a}x_{1})\psi = m\psi \qquad (34)$$

$$i\gamma_{\mu}(p_{\mu} + g\Lambda_{k}\,\mathrm{I}\,x_{1})\psi = m\psi, \qquad (35)$$

$$T^{a}H^{a} = \frac{1}{2}\begin{pmatrix} H^{3} + \frac{1}{\sqrt{3}}H^{8} & H^{1} - iH^{2} & H^{4} - iH^{5} \\ H^{1} + iH^{2} & -H^{3} + \frac{1}{\sqrt{3}}H^{8} & H^{6} - iH^{7} \\ H^{4} + iH^{5} & H^{6} + iH^{7} & -\frac{2}{\sqrt{3}}H^{8} \end{pmatrix} \qquad (36)$$

$$\left|T^{a}H^{a} - \Lambda\right| = 0 \qquad (37)$$

Quark in external gluon field of magnetic type

,

$$(\vec{\sigma}\vec{\mathrm{p}} + g\Lambda_k \mathrm{I}\, x_1)\varphi = (\varepsilon + m)\chi \qquad (38)$$

,

$$(\vec{\sigma}\vec{\mathrm{p}} + g\Lambda_k \mathrm{I}\, x_1)\chi = (\varepsilon - m)\varphi$$

,

$$\{\vec{\mathrm{p}}^2 + g^2\Lambda_k{}^2 x^2 + g\Lambda_k(\sigma_3 + 2xp_2)\}\varphi = (\varepsilon^2 - m^2)\varphi \qquad (39)$$

,

,

$$\sigma_3\varphi = \mu\varphi \qquad \mu = \pm 1 \qquad \varphi \to \Phi_{\Lambda_k \mu}$$

,

$$\Phi_{\Lambda_k \mu}(x_1) = e^{i(p_2 x_2 + p_3 x_3)}\Upsilon_{\Lambda_k \mu}(x_1) \qquad (40)$$

,

Quark in external gluon field of magnetic type

,

$$\left(-\frac{d^2}{d\xi^2}+\xi^2\right)\Upsilon_{\Lambda_k\mu}(\xi)=\frac{\varepsilon^2-m^2-p_3{}^2+g\Lambda_k\mu}{g\Lambda_k}\Upsilon_{\Lambda_k\mu}(\xi) \qquad (41)$$

,

,

$$\xi=\sqrt{g\Lambda_k}\,(x_1-\frac{p_2}{g\Lambda_k}) \qquad (42)$$

,

,

,

,

Quark in external gluon field of magnetic type

,

$$\varepsilon^2 = m^2 + p_3{}^2 + g\Lambda_k(2n+1-\mu) \qquad (43)$$

,

,

$$R_k = \sqrt{\frac{\hbar c}{2g\Lambda_k}} \qquad (44)$$

,

,

,

,

Conclusion to part II

- Thus, we showed the principal possibility of introducing coherent states of particles moving in constant homogeneous non-Abelian gauge fields and interacting with these fields.
- An important feature of the considered model is that here each "color" degree of freedom corresponds to its characteristic size

Part III The model of scalar hadron on the base of classical field theory

- The task of this part of the talk the further development of field-theoretical models for describing hadrons as coherent states as transverse excitations, considered as partons, determining the limitations imposed by the field-theoretical model on the initial phenomenological model.
- In this part, we consider the case of the Klein-Fock equation describing a scalar relativistic particle, for example, π- meson.

Separation of transverse and longitudinal variables in the Klein-Fock equation and the Levy-Civita transformation

$$\Box\varphi(\underline{x}, x_0) = \partial_\mu{}^2\varphi(\underline{x}, x_0) = \frac{m^2c^2}{\hbar^2}\varphi(\underline{x}, x_0) \tag{45}$$

$$\Box = \partial_\mu{}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2} \tag{46}$$

Separation of transverse and longitudinal variables in the Klein-Fock equation and the Levy-Civita transformation

We will seek the solution of equation (45) by dividing the variables into "transverse" and "longitudinal" variables. Obviously, before the introduction of the interaction that separates the direction in space, the separation of variables into "transverse" and "longitudinal" parts is conventional. Then the solution of equation (45) can be represented in the form

$$\varphi(\underline{x}, x_0) = \varphi_1(x, y)\varphi_2(z, t) \qquad (47)$$

Separation of transverse and longitudinal variables in the Klein-Fock equation and the Levy-Civita transformation

$$\frac{\partial^2 \varphi_1(x,y)}{\partial x^2} + \frac{\partial^2 \varphi_1(x,y)}{\partial y^2} = -k_\perp^2 \varphi_1(x,y) \qquad (48)$$

$$\frac{\partial^2 \varphi_2(z,t)}{\partial z^2} - \frac{\partial^2 \varphi_2(z,t)}{\partial t^2} = -k_\parallel^2 \varphi_2(z,t) = (m^2 + k_\perp^2)\varphi_2(z,t) \qquad (49)$$

Levy-Civita transformation (complex functions).

we made next substitution in (48) :

$$\varphi_1(x, y) \longrightarrow \Phi_1(\eta, \xi) \qquad (50)$$

and introduce new variables

$$x + iy = (\eta + i\xi)^2 = \eta^2 - \xi^2 + 2i\eta\xi \qquad (51)$$

than this equation presents as follow

$$\frac{\partial^2 \Phi_1(\eta, \xi)}{\partial \eta^2} + \frac{\partial^2 \Phi_1(\eta, \xi)}{\partial \xi^2} = 4k_\perp^2(\eta^2 + \xi^2)\Phi_1(\eta, \xi) \qquad (52)$$

Levy-Civita transformation (functions of the double variable).

- In equation (49) we made transformation of variables

$$(z + jt) = (\chi + j\zeta)^2 = \chi^2 + \zeta^2 + 2j\chi\zeta \quad (53)$$

$$\varphi_2(z,t) \rightarrow \Phi_2(\chi,\zeta).$$

- and
- Here analytical **functions over the double variable are used** $j^2 = 1$ (54)
- Then instead (49) we have:

$$\frac{\partial^2\Phi_2(\chi,\zeta)}{\partial\chi^2} - \frac{\partial^2\Phi_2(\chi,\zeta)}{\partial\zeta^2} = 4(m^2 + k_\perp{}^2)(\chi^2 - \zeta^2)\Phi_2(\chi,\zeta) \quad (55)$$

Introduction of the coherent states

$$L_1\Phi_1(\eta',\xi') = \frac{\partial^2\Phi_1(\eta',\xi')}{\partial\eta'^2} + \frac{\partial^2\Phi_1(\eta',\xi')}{\partial\xi'^2} - (\eta'^2 + \xi'^2)\Phi_1(\eta',\xi') = 0 \tag{56}$$

$$\eta' = 2\sqrt{k_\perp}\,\eta \qquad\qquad \xi' = 2\sqrt{k_\perp}\,\xi \tag{57}$$

$$a_1^{\pm} = \frac{1}{\sqrt{2}}\left(\xi' \mp \frac{\partial}{\partial\xi'}\right) \qquad\qquad a_2^{\pm} = \frac{1}{\sqrt{2}}\left(\eta' \mp \frac{\partial}{\partial\eta'}\right) \tag{58}$$

$$L_1 = -2(a_1^{+}a_1^{-} + a_2^{+}a_2^{-} + 2) \tag{59}$$

Introduction of the coherent states

$$L_2\Phi_2(\chi',\zeta') = \frac{\partial^2\Phi_2(\chi',\zeta')}{\partial\chi'^2} - \frac{\partial^2\Phi_2(\chi',\zeta')}{\partial\zeta'^2} - (\chi'^2 - \zeta'^2)\Phi_2(\chi',\zeta') = 0. \tag{60}$$

$$\chi' = 2\sqrt[4]{m^2 + k_\perp{}^2}\,\chi \qquad \zeta' = 2\sqrt[4]{m^2 + k_\perp{}^2}\,\zeta \tag{61}$$

$$b_1^\pm = \frac{1}{\sqrt{2}}\left(\chi' \mp \frac{\partial}{\partial\chi'}\right), \qquad b_2^\pm = \frac{1}{\sqrt{2}}\left(\zeta' \mp \frac{\partial}{\partial\zeta'}\right). \tag{62}$$

$$L_1 = -2(b_1^+ b_1^- - b_2^+ b_2^-). \tag{63}$$

Heisenberg-Weyl algebra

(64)

$$\left[a_k^-, a_l^+\right] = \delta_{kl} I, \left[a_k^+, a_l^+\right] = \left[a_k^-, a_l^-\right] = \left[a_k^-, I\right] = \left[a_k^+, I\right] = 0.$$

$$\left[b_k^-, b_l^+\right] = \delta_{kl} I, \left[b_k^+, b_l^+\right] = \left[b_k^-, b_l^-\right] = \left[b_k^-, I\right] = \left[b_k^+, I\right] = 0.$$

Coherent states

$$a^{-}{}_{1}\left|\alpha_1\right\rangle = \alpha_1\left|\alpha_1\right\rangle, \; a^{-}{}_{2}\left|\alpha_2\right\rangle = \alpha_2\left|\alpha_2\right\rangle. \qquad (65)$$

$$b^{-}{}_{1}\left|\beta_1\right\rangle = \beta_1\left|\beta_1\right\rangle, \; b^{-}{}_{2}\left|\beta_2\right\rangle = \beta_2\left|\beta_2\right\rangle. \qquad (66)$$

$$-2(n_1 + n_2 + 2) = 0 \qquad (67)$$

$$-2(n_3 - n_4) = 0 \qquad (68)$$

Reciprocaly invariant equation

$$(L_1 + L_2)\Phi = \frac{\partial^2 \Phi}{\partial \eta'^2} + \frac{\partial^2 \Phi}{\partial \xi'^2} + \frac{\partial^2 \Phi}{\partial \chi'^2} - \frac{\partial^2 \Phi}{\partial \zeta'^2} - (\eta'^2 + \xi'^2 + \chi'^2 - \zeta'^2)\Phi = 0, \quad (69)$$

$$a_1^+ a_1^- + a_2^+ a_2^- + b_1^+ b_1^- - b_2^+ b_2^- + 2 = n_1 + n_2 + n_3 - n_4 + 2 = 0 \quad (70)$$

Solutions. Coordinate represantation

$$\langle \eta', \xi', \chi', \zeta' \| \alpha_1, \alpha_2, \beta_1, \beta_2 \rangle = \Phi(\eta', \xi', \chi', \zeta') = \frac{1}{\pi} e^{i(\alpha'_1 \alpha''_1 + \alpha'_2 \alpha''_2 + \beta'_1 \beta''_1 - \beta'_2 \beta''_2)} \times$$

$$\times e^{i\sqrt{2}(\alpha''_1 \eta' + \alpha''_2 \xi' + \beta''_1 \chi' - \beta''_2 \zeta')} \times e^{-\frac{1}{2}[(\eta' - \sqrt{2}\alpha'_1)^2 + (\xi' - \sqrt{2}\alpha'_2)^2 + (\chi' - \sqrt{2}\beta'_1)^2 - (\zeta' - \sqrt{2}\beta'_2)^2}$$

$$\alpha_1 = \alpha'_1 + i\alpha''_1, \alpha_2 = \alpha'_2 + i\alpha''_2, \beta_1 = \beta'_1 + i\beta''_1, \beta_2 = \beta'_2 + i\beta''_2$$

Solutions. Momentum represantation

$$\left\langle p_{\eta'}, p_{\xi'}, p_{\chi'}, p_{\zeta'} \right\| \left| \alpha_1, \alpha_2, \beta_1, \beta_2 \right\rangle = \Phi(p_{\eta'}, p_{\xi'}, p_{\chi'}, p_{\zeta'}) = \frac{1}{\pi} e^{-i(\alpha'_1\alpha''_1+\alpha'_2\alpha''_2+\beta'_1\beta''_1-\beta'_2\beta''_2)} \times$$

$$\times e^{-i\sqrt{2}(\alpha'_1 p_{\eta'}+\alpha'_2 p_{\xi'}+\beta'_1 p_{\chi'}-\beta'_2 p_{\zeta'})} \times e^{-\frac{1}{2}[(p_{\eta'}-\sqrt{2}\alpha''_1)^2+(p_{\xi'}-\sqrt{2}\alpha''_2)^2+(p_{\chi'}-\sqrt{2}\beta''_1)^2-(p_{\zeta'}-\sqrt{2}\beta''_2)^2}$$

$$\alpha_1 = \alpha'_1 + i\alpha''_1, \alpha_2 = \alpha'_2 + i\alpha''_2, \beta_1 = \beta'_1 + i\beta''_1, \beta_2 = \beta'_2 + i\beta''_2$$

Conclusions

- In this part it is shown that by performing Lévi-Civita transformations in the Klein-Fock equation and seperating the variables into "transverse" and "longitudinal“ it is possible to define coherent states in a standard way. Solutions of the Klein-Fock equation in the form of coordinate and impulse representations of the data of coherent states are found. The "transverse" variables are interpreted as the parton degrees of freedom of the hadron (-meson) described by the scalar Klein-Fock equation. It is established that in this approach the number of partons depends on the energy and longitudinal degrees of freedom.

References

- Kurochkin Yu.A. , Harkusha S.N. , Kulchitsky Yu.A. , Russakovich N.A. /On describing quark in external gluon field of magnetic type // Vestsi Natsyianal'nai akademii navuk Belarusi. Seria fizika-matematychnykh navuk = Proceeding of the National Academy of Sciences of Belarus. Physics and Mathematics series -2017, n.4, P. 39-43.
- Kurochkin, Y. Hadron as Coherent State on the Horosphere of the Lobachevsky Momentum Space. Kurochkin Y., Kulchitsky Y., Harkusha S., and Russakovich N. / Письма в ЭЧАЯ 2016, Т.13, № 3(201), С.454-460, Physics of the Particles and Nuclei Letters, (2016), Vol. 13, No 3, pp. 285-288.
- Переломов А.М. /Обобщенные когерентные состояния и их применения - М.:Наука. -1987- 268 с.
- Точные решения релятивистских волновых уравнений/ В.Г. Богров, Д.М. Гитман, И.М. Тернов, В.Р. Халилов, В.Н. Шаповалов //Точные решения релятивистских волновых уравнений -Новосибирск, Наука- 1982-143 с.
- Kalnins E.G. / Coulomb-Oscillator duality in space of constant curvature// E.G. Kalnins, W. Miller, Jr., G.S. Pogosyan J. Math. Phys. V. 41, №5, 2000, 2629-

References

- М.Н. Олевский Триортогональные системы в пространствах постоянной кривизны, в которых уравнение допускает полное разделение переменных // Мат. Сб. 1950, Т. 27, С. 379–426.
- Ю.П. Никитин, И.Л Розенталь Теория множественных процессов, м.: Атомиздат, (1976), 230 с.
- Л.Д. Ландау и С.З. Беленький. Гидродинамическая теория множественного образования частиц УФН, (1955), т.56, 309.
- В.А. Матвеев Глубоконеупругие лептон-адронные процессы при высоких энергиях.Международная школа молодых ученых по физике высоких энергий. Сборник статей. Гомель 25.08.-5.09. 1973. Дубна.1973 С.81-172.
- Р.М. Мурадян Автомодельность в инклюзивных реакциях/ Препринт ОИЯИ. Р2-6762, Дубна ОИЯИ, 1972.
- В.П. Шелест, Г.М. Зиновьев, В.А. Миранский. Модели сильновзаимодействующих частиц II, М.: Атомиздат, (1976), 247 с.
- R. Rohl et all /The size of proton//Nature 09250, Vol. 466, (2010), doi:10.1038
- Ю.А. Курочкин, Л.М. Томильчик// О параметризации преобразований комплексной группы Лоренца для пространств с вещественной метрикой/ Докл. нац. акад. Наук Беларуси.- 2018-Т.62, № 2. С. 159-163.

- Thank you for your attention