Femtoscopy of Heavy Ion Collisions

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- History
- QS correlations
- FSI correlations → femtoscopy with nonidentical particles
- Correlation asymmetries
- Correlation study of strong interaction
- Summary

History of Correlation femtoscopy

measurement of space-time characteristics R, $c\tau \sim$ fm
of particle production using particle correlations

Fermi'34, GGLP'60, Dubna (GKPLL..'71-) …

β-decay: Coulomb FSI between $e^{\pm}$ and Nucleus in β-decay modifies the relative momentum (**k**) distribution → Fermi (correlation) function

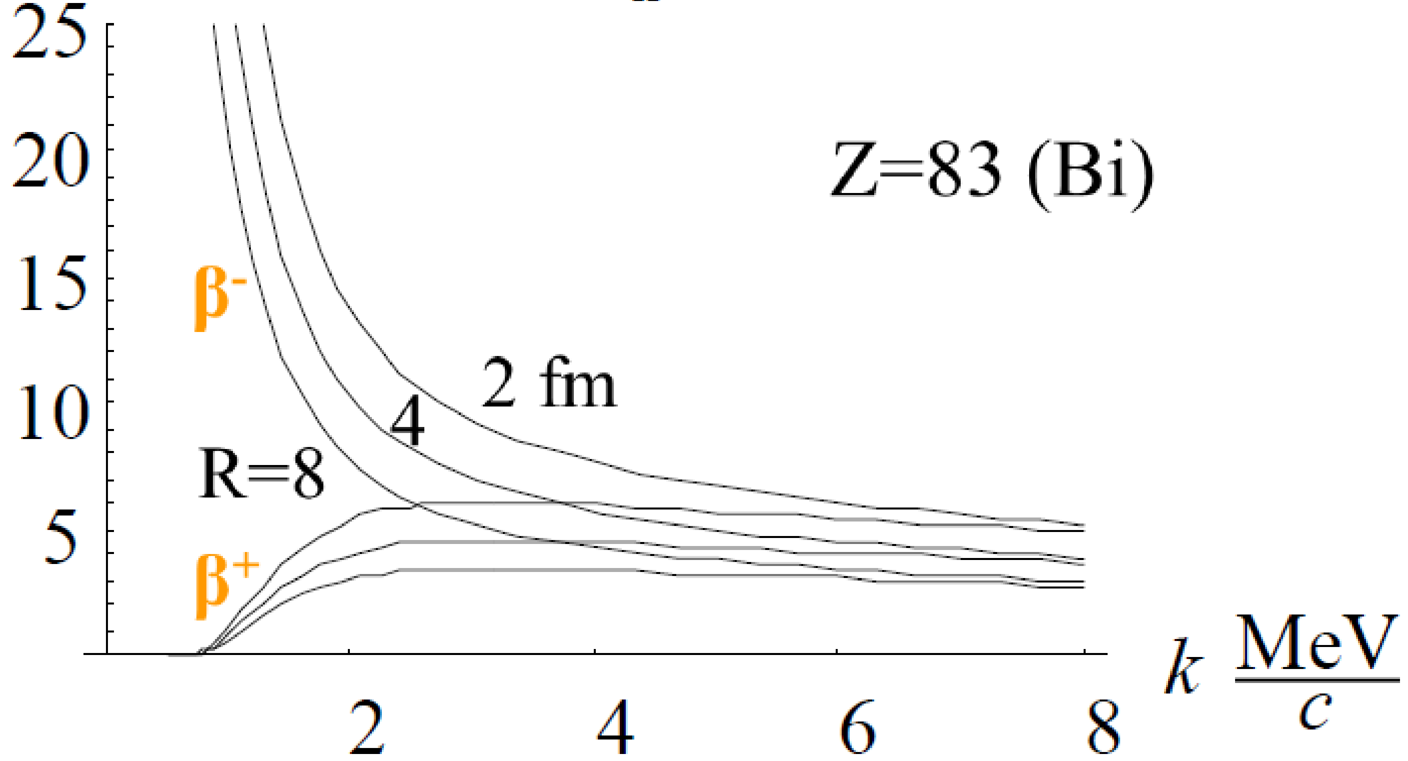
$$F(k,Z,R) = \langle |\psi_{-\mathbf{k}}(\mathbf{r})|^2 \rangle$$

is sensitive to Nucleus radius R if charge Z » 1

$\psi_{-\mathbf{k}}(\mathbf{r})$ = electron – residual Nucleus WF (Δt=0)

Fermi function in β-decay

$$F(k,Z,R)\ ) = \langle |\psi_{-\mathbf{k}}(\mathbf{r})|^2 \rangle \sim (kR)^{-(Z/137)^2}$$



Goldhaber, Goldhaber, Lee & Pais

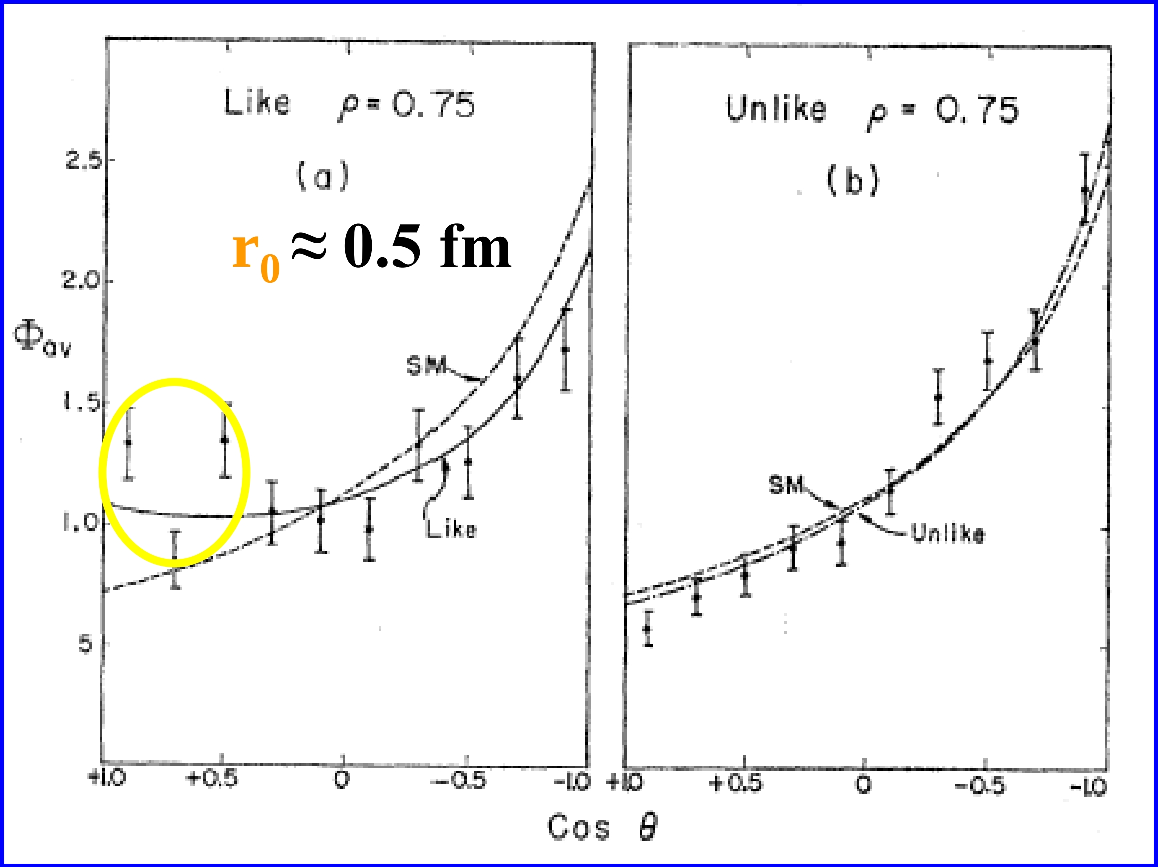
GGLP'60: enhanced $\pi^+\pi^+$, $\pi^-\pi^-$ vs $\pi^+\pi^-$ at small opening angles

– interpreted within SM as BE enhancement depending on fireball Gaussian radius r_0

$$\bar{p}\,p \rightarrow 2\pi^+\,2\pi^-\,n\pi^0$$

- SM multiplicity requires radius r_0 by a factor 3 larger !
- Later femtoscopy correlation analysis lead to $r_0 > 1$ fm

→ GGLP effect is likely dominated by dynamics (resonances)



Modern correlation femtoscopy formulated by Kopylov & Podgoretsky

KP'71-75: settled basics of correlation femtoscopy in > 20 papers (for non-interacting identical particles)

- **proposed CF= N^{corr}/N^{uncorr} & mixing techniques to construct N^{uncorr} & two-body approximation to calculate theor. CF**
- **showed that sufficiently smooth momentum spectrum allows one to neglect space-time coherence at small q***

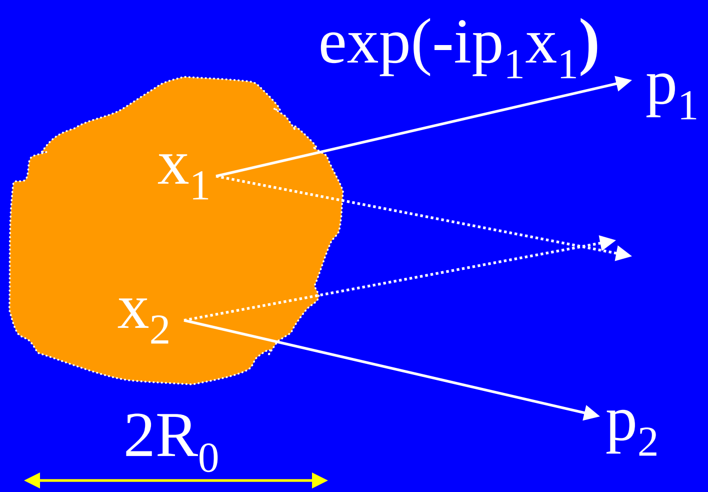
  smoothness approximation:

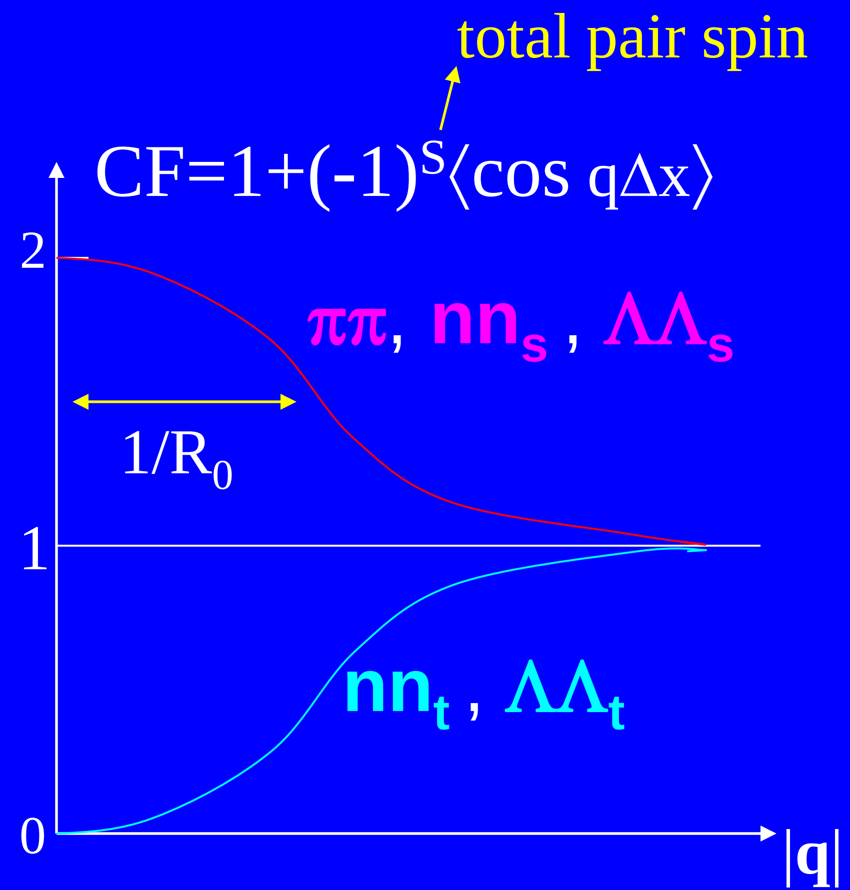
  $$|\int d^4x_1 d^4x_2 \psi_{p1p2}(x_1,x_2)\ldots|^2 \rightarrow \int d^4x_1 d^4x_2 |\psi_{p1p2}(x_1,x_2)|^2 \ldots$$
- **clarified role of space-time production characteristics: shape & time source picture from various q-projections**

QS symmetrization of production amplitude

→ *momentum correlations of identical particles are sensitive to space-time structure of the source*

KP'71-75





$q = p_1 - p_2 \xrightarrow{\textbf{PRF}} \{0, 2\mathbf{k}^*\}$

$\Delta x = x_1 - x_2 \rightarrow \{t^*, \mathbf{r}^*\}$

$$CF \rightarrow \langle\, |\psi^{S(sym)}_{-k^*}(r^*)|^2 \,\rangle = \langle\, |\, [\, e^{-ik^*r^*} + (-1)^S e^{ik^*r^*}]/\sqrt{2}\, |^2 \,\rangle$$

! CF of noninteracting identical particles is independent of t* in PRF

KP model of single-particle emitters

Probability amplitude to observe a particle with 4-coordinate x from emitter A at x_A can depend on $x-x_A$ only and so can be written as:

$$\langle x|\psi_A\rangle = (2\pi)^{-4}\int d^4\kappa\, u_A(\kappa)\exp[i\kappa(x-x_A)].$$

Transfering to 4-momentum representation: $\langle p|x\rangle = \exp(-ipx)$ $\Rightarrow$

$$\langle p|\psi_A\rangle = \int d^4x\, \langle p|x\rangle\langle x|\psi_A\rangle = u_A(p)\exp(-ipx_A)$$

and probability amplitude to observe two spin-0 bosons:

$$T_{AB}^{sym}(p_1,p_2) = [\langle p_1|\psi_A\rangle\langle p_2|\psi_B\rangle + \langle p_2|\psi_A\rangle\langle p_1|\psi_B\rangle]/\sqrt{2}.$$

Corresponding **momentum correlation function**:

$\Delta x = x_A - x_B$

$$R(p_1,p_2) = 1 + \frac{\Re\sum_{AB} u_A(p_1)u_B(p_2)u_A^*(p_2)u_B^*(p_1)\exp(-iq\Delta x)}{\sum_{AB}|u_A(p_1)u_B(p_2)|^2} \doteq 1 + \langle\cos(q\Delta x)\rangle$$

if $u_A(p_1) \approx u_A(p_2)$: "smoothness assumption"

Assumptions to derive KP formula

$$CF - 1 = \langle \cos q\Delta x \rangle$$

- two-particle approximation (small freeze-out PS density f)
~ OK, $\langle f \rangle \ll 1$? low p_t fig.

- smoothness approximation: $R_{emitter} \ll R_{source} \Leftrightarrow \langle|\Delta p|\rangle \gg \langle|q|\rangle_{peak}$
~ OK in HIC, $R_{source}^2 \gg 0.1\ fm^2 \approx p_t^2$-slope of direct particles

- neglect of FSI
OK for photons, ~ OK for pions up to Coulomb repulsion

- incoherent or independent emission
2π and 3π CF data consistent with KP formulae:

$$CF_3(123) = 1+|F(12)|^2+|F(23)|^2+|F(31)|^2+2\mathrm{Re}[F(12)F(23)F(31)]$$

$$CF_2(12) = 1+|F(12)|^2, \quad F(q) = \langle e^{iqx} \rangle$$

Phase space density from CFs and spectra

$$\bar{f}(p_t) \equiv \frac{\int d^3r\, f(p,r)\cdot f(p,r)}{\int d^3r\, f(p,r)}$$

$$\sim \frac{1}{2\pi m_\pi}\frac{dN}{p_t dp_t dy}\int d^3Q_{inv} C(p_t, Q_{inv})$$

$$f_{max}(p_t, r) \approx 2\sqrt{2}\bar{f}(p_t)$$

Bertsch'94

Lisa ..'05

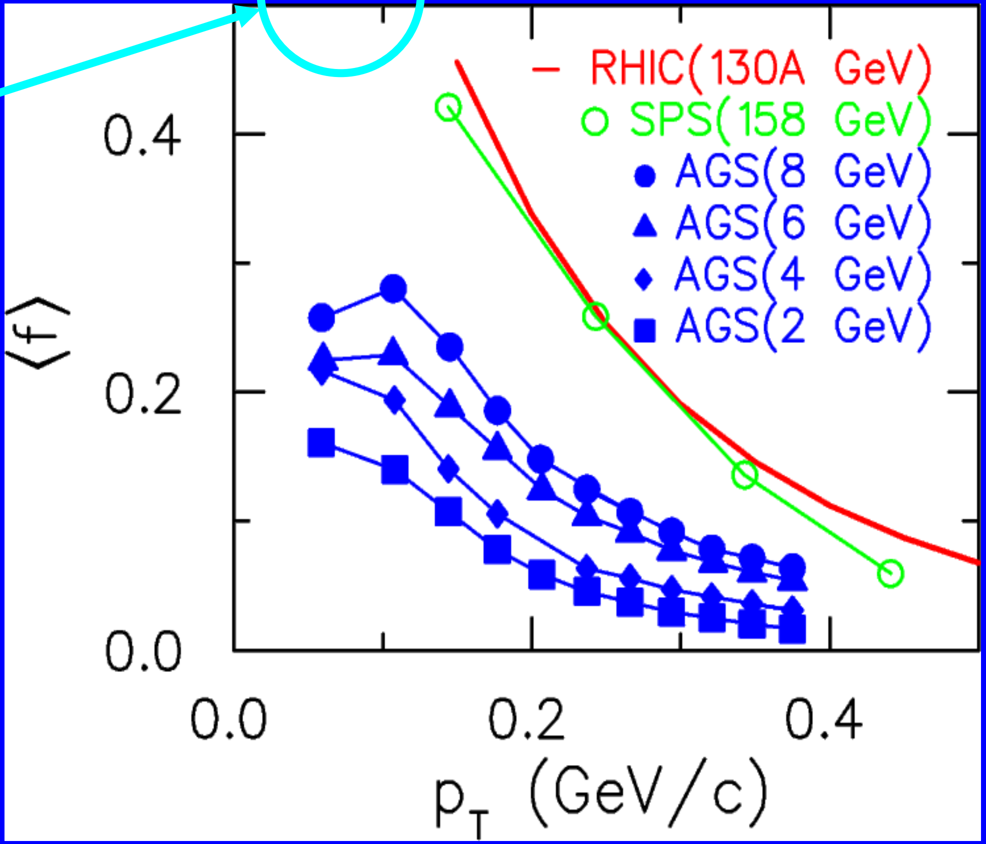
<f> rises up to SPS

May be high phase space density at low p_t ?

↓

? Pion condensate or laser

? Multiboson effects on CFs spectra & multiplicities



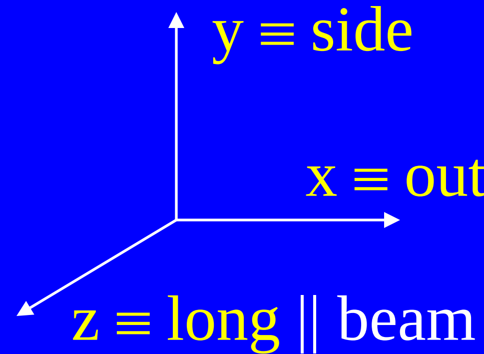
"General" parameterization at $|\mathbf{q}| \to 0$

Particles on mass shell & azimuthal symmetry $\Rightarrow$ 5 variables:

$\mathbf{q} = \{q_x, q_y, q_z\} \equiv \{q_{out}, q_{side}, q_{long}\}$, pair velocity $\mathbf{v} = \{v_x, 0, v_z\}$

$q_0 = \mathbf{qp}/p_0 \equiv \mathbf{qv} = q_x v_x + q_z v_z$

Grassberger'77
RL'78



$$\langle \cos q\Delta x \rangle = 1 - ½\langle (q\Delta x)^2 \rangle + .. \approx \exp(-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2 - 2R_{xz}^2 q_x q_z)$$

Femtoscopy or Interferometry radii:

$$R_x^2 = ½ \langle (\Delta x - v_x \Delta t)^2 \rangle, \; R_y^2 = ½ \langle (\Delta y)^2 \rangle, \; R_z^2 = ½ \langle (\Delta z - v_z \Delta t)^2 \rangle$$

Podgoretsky'83, Bertsch, Pratt'95; so called out-side-long parameterization

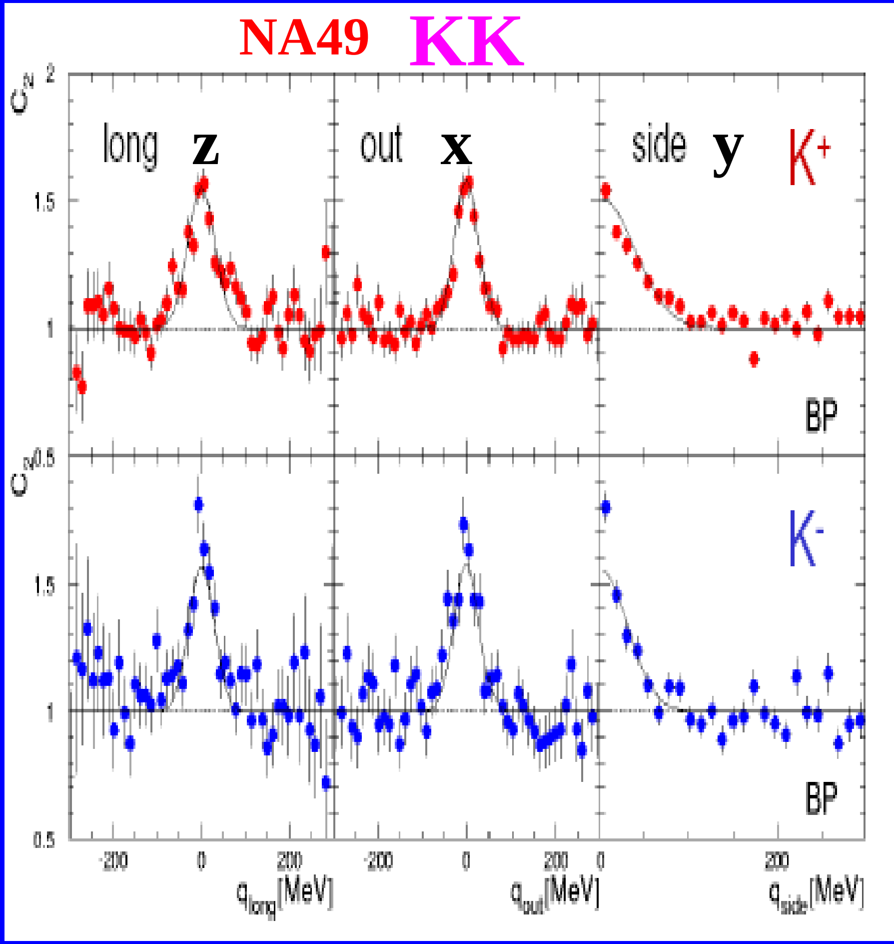
Csorgo, Pratt'91: LCMS $v_z = 0$

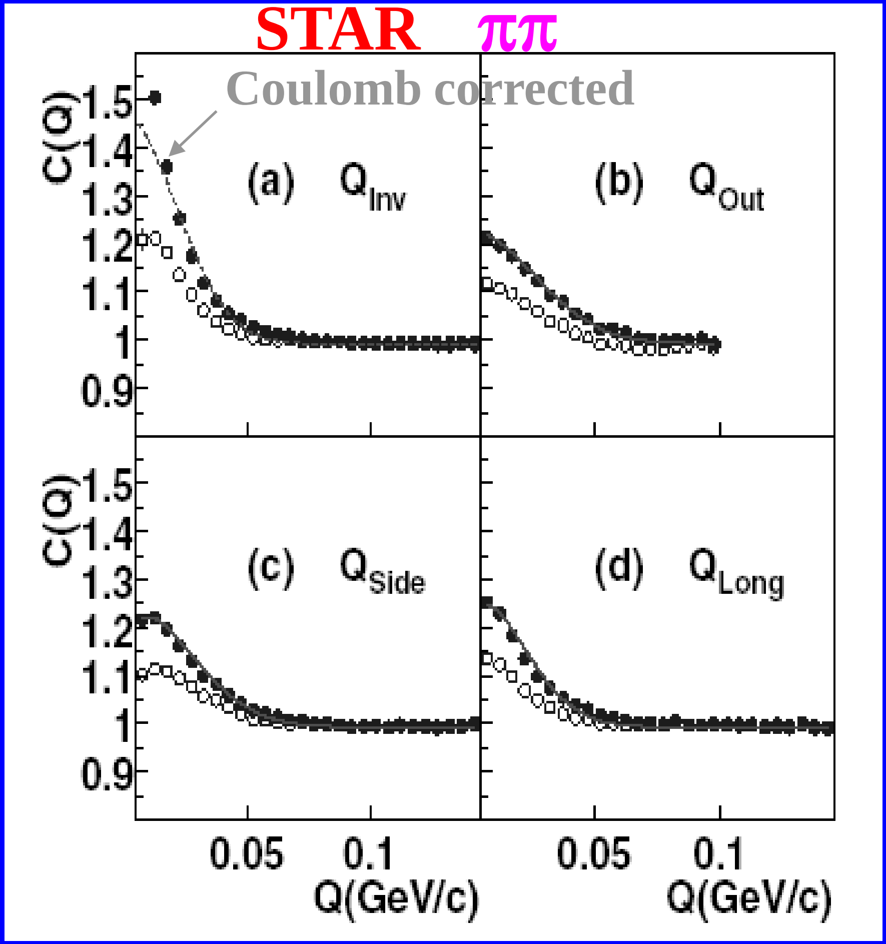
3-dim fit: $CF = 1 + \lambda \exp(-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2 - 2R_{xz}^2 q_x q_z)$

Correlation strength or chaoticity (λ)

Femtoscopy / Interferometry radii (R_x, R_y, R_z, R_{xz})

Examples CF data: NA49 & STAR





Probing source shape and emission duration

KP (71-75) …

Static Gaussian model with space and time dispersions $R_\perp^2, R_\parallel^2, \Delta\tau^2$

$$\rightarrow \begin{array}{l} R_x^2 = R_\perp^2 + v_\perp^2 \Delta\tau^2 \\ R_y^2 = R_\perp^2 \\ R_z^2 = R_\parallel^2 + v_\parallel^2 \Delta\tau^2 \end{array}$$

$\rightarrow$ **Emission duration**

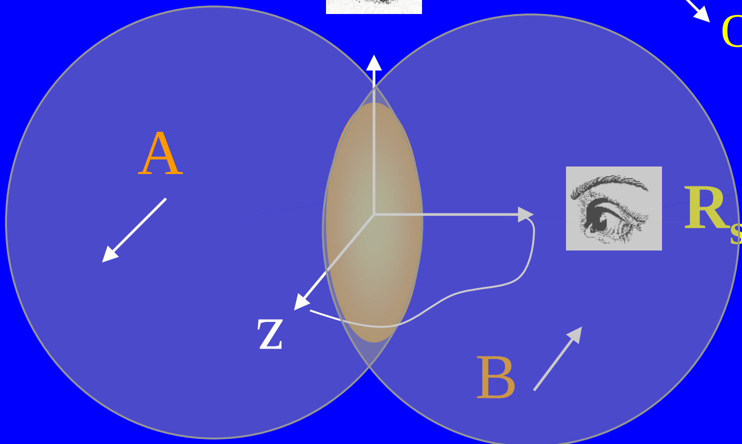
$$\boldsymbol{\Delta\tau^2 = (R_x^2 - R_y^2)/v_\perp^2}$$

If elliptic shape also in transverse plane

$\Rightarrow$ $\mathbf{R_y \equiv R_{side}}$ oscillates with pair **azimuth ϕ**

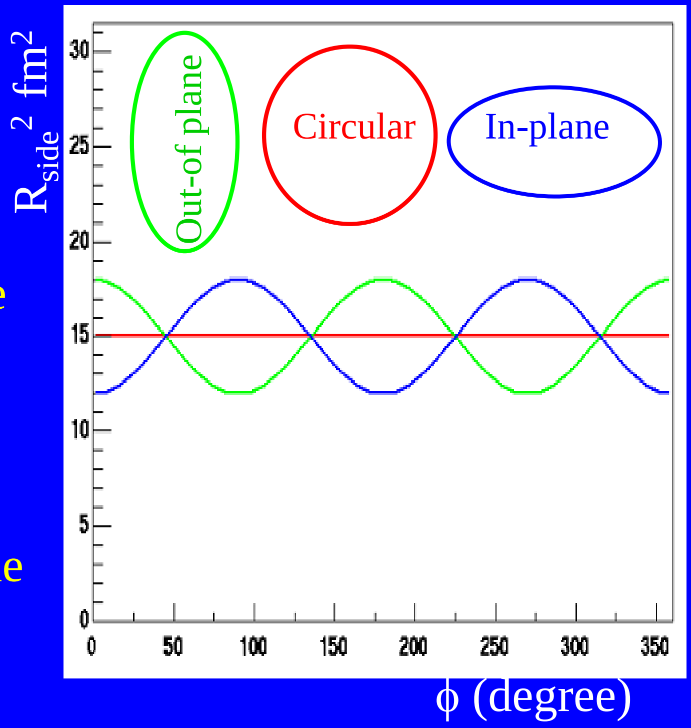
$\mathbf{R_{side}}$ **(ϕ=90°)** small

↘ Out-of reaction plane

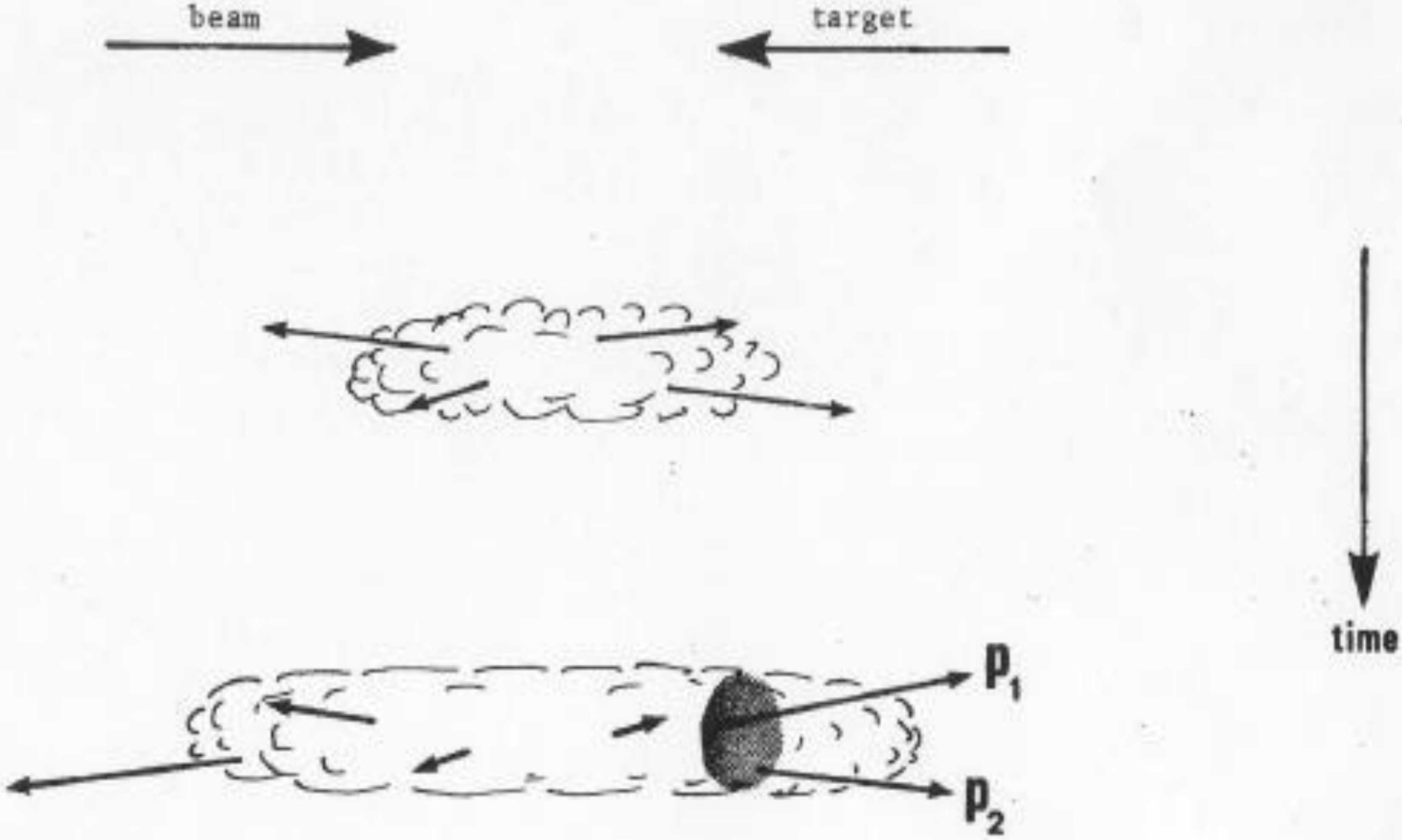


$\mathbf{R_{side}}$ **(ϕ=0°)** large

↘ In reaction plane



Grassberger'77: fire sausage



Probing source dynamics - expansion

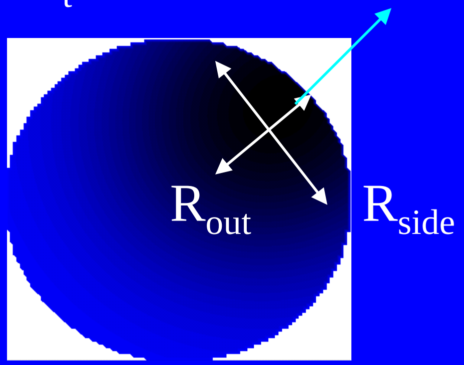
Dispersion of emitter velocities & limited emission momenta (**T**) ⇒
x-p correlation: interference dominated by pions from nearby emitters

→ **Interference probes only a part of the source**

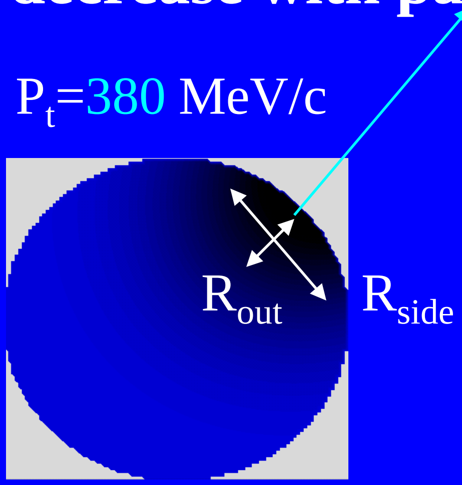
→ **Interferometry radii decrease with pair velocity**

Resonances GKP'71 ..
Strings Bowler'85 ..
Hydro Pratt'84,86
Kolehmainen, Gyulassy'86
Makhlin-Sinyukov'87
Bertch, Gong, Tohyama'88
Hama, Padula'88
Pratt, Csörgö, Zimanyi'90
Mayer, Schnedermann, Heinz'92
.....

P_t=160 MeV/c



P_t=380 MeV/c



Collective transverse flow β^F → $\mathbf{R_{side} \approx R/(1+m_t\,\beta^{F2}/T)^{1/2}}$

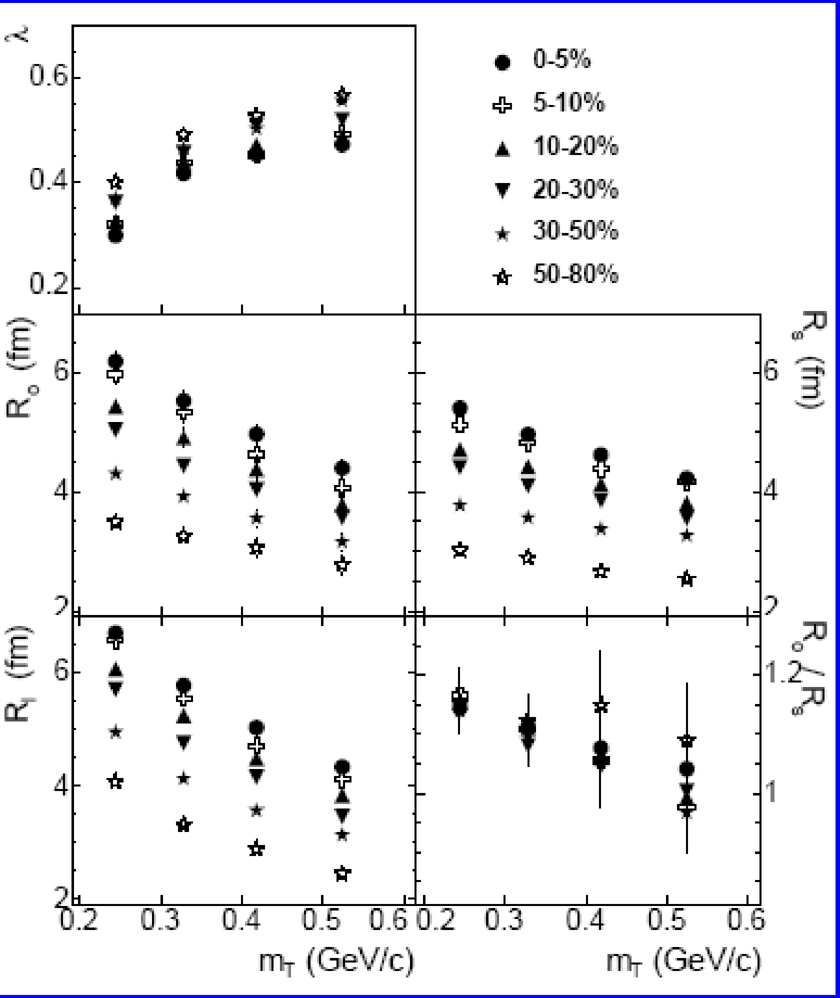
in LCMS: **1**

Longitudinal boost invariant expansion
during proper freeze-out (evolution) time τ → $\mathbf{R_{long} \approx (T/m_t)^{1/2}\,\tau/\underbrace{\cosh y}}$

AGS→SPS→RHIC: ππ radii

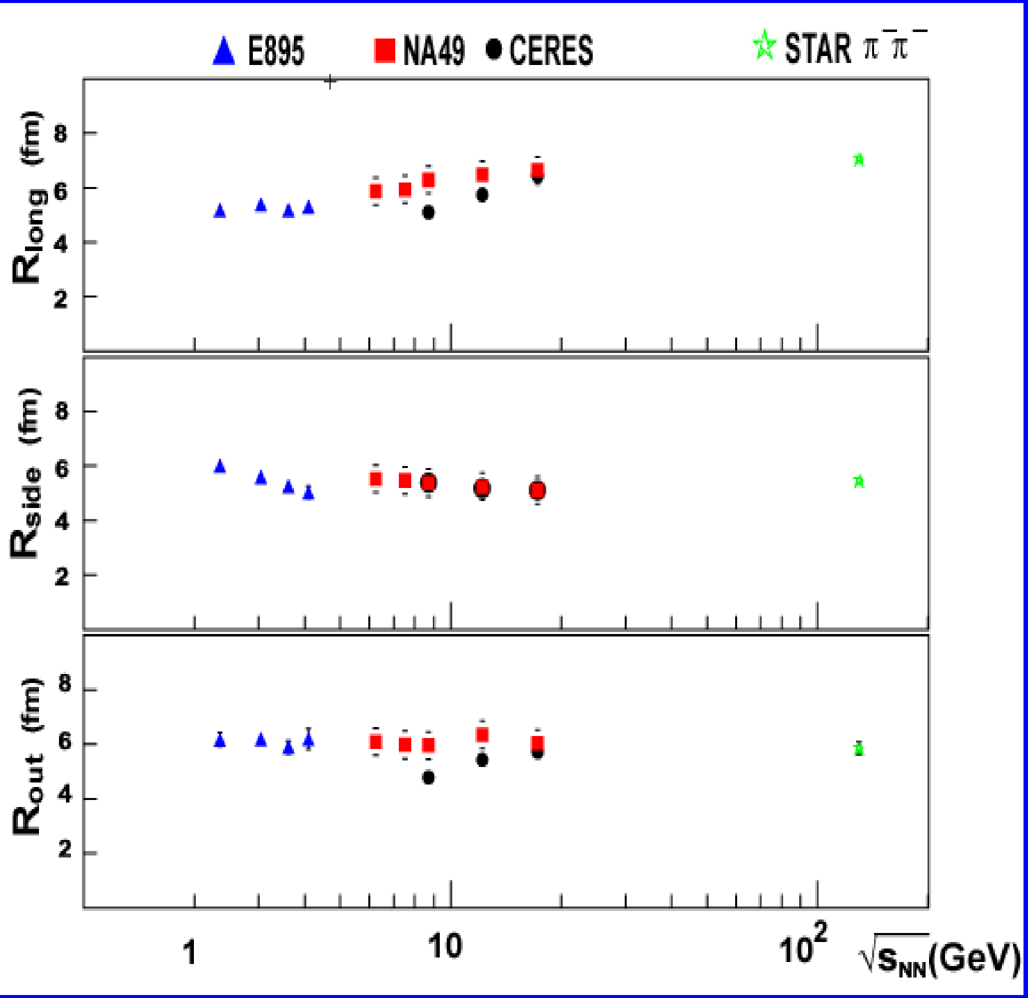
Clear centrality dependence

STAR Au+Au at 200 AGeV



Weak energy dependence

0-5% central Pb+Pb or Au+Au

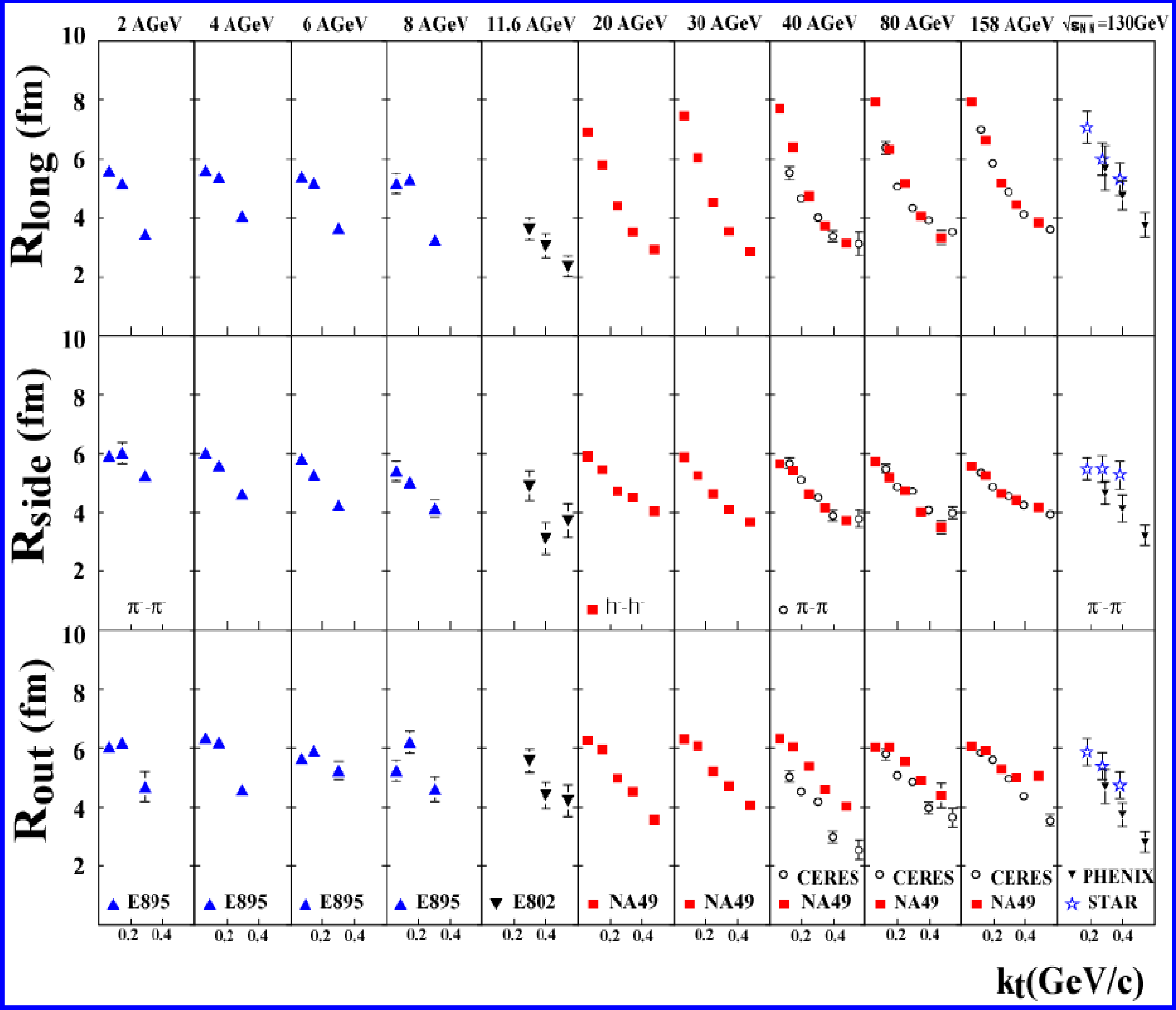


AGS→SPS→RHIC: ππ radii vs s_{NN} & p_t

***R*$_{long}$:**
increases smoothly & points to short evolution time
τ ~ 8-10 fm/c

***R*$_{side}$, *R*$_{out}$:**
change little & point to strong transverse flow
β_0^F ~ 0.4-0.6 & short emission duration
$\Delta\tau$ ~ 2 fm/c

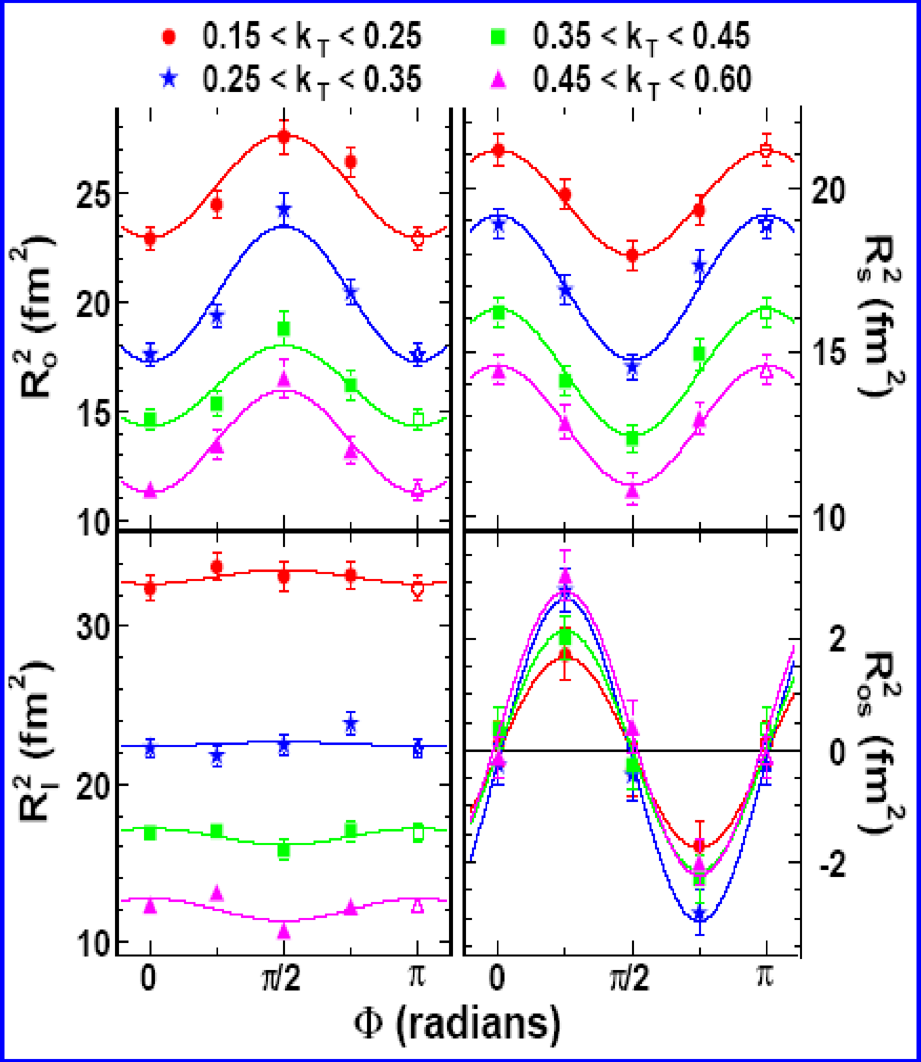
Central Au+Au or Pb+Pb



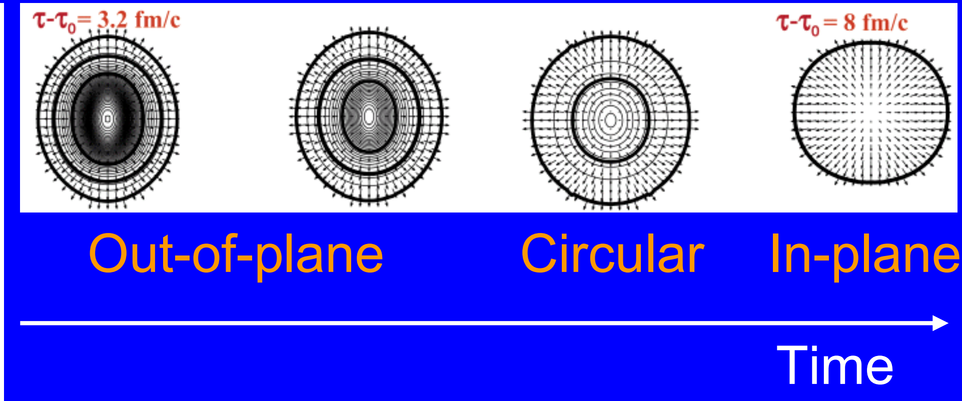
Interferometry wrt reaction plane

STAR'04 Au+Au 200 GeV 20-30%

$\pi^+\pi^+$ & $\pi^-\pi^-$



Typical hydro evolution

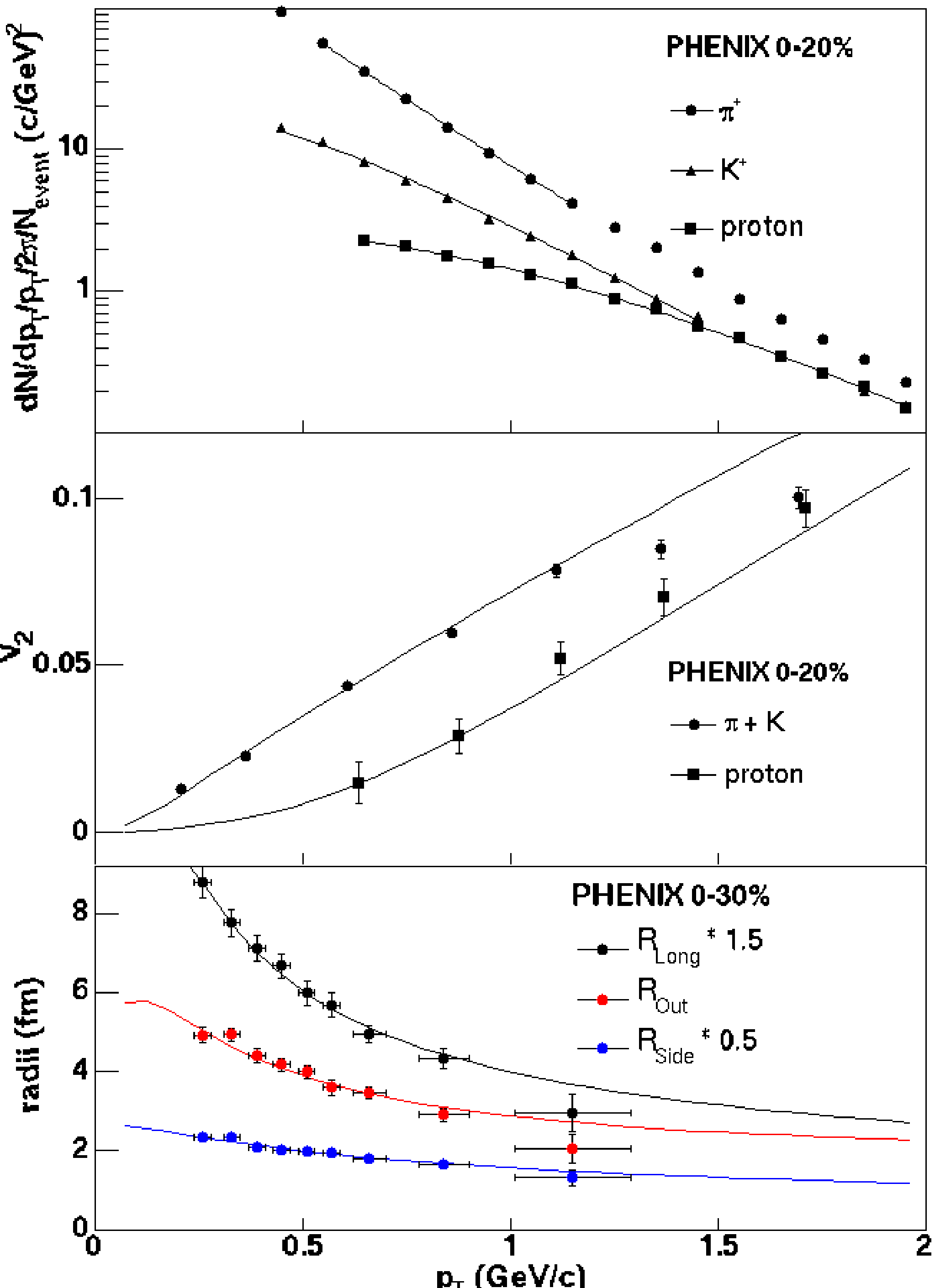


STAR data:

ϕ oscillations like for a static out-of-plane source stronger then Hydro & RQMD

⇓

Short evolution time



Hadro motivated BW fit of Au-Au 200 GeV

Retiere@LBL'05

T=106 ± 1 MeV

$<\beta_{InPlane}> = 0.571 \pm 0.004$ c

$<\beta_{OutOfPlane}> = 0.540 \pm 0.004$ c

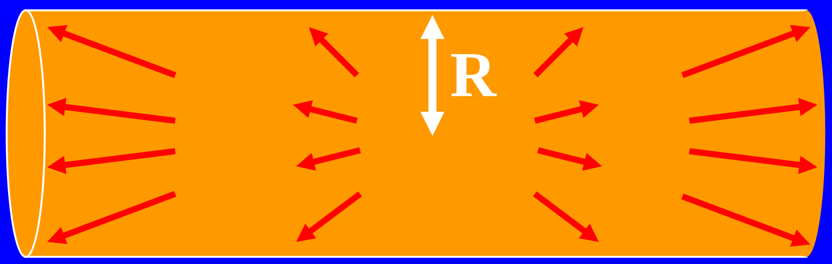
$R_{InPlane} = 11.1 \pm 0.2$ fm

$R_{OutOfPlane} = 12.1 \pm 0.2$ fm

Life time (τ) = 8.4 ± 0.2 fm/c

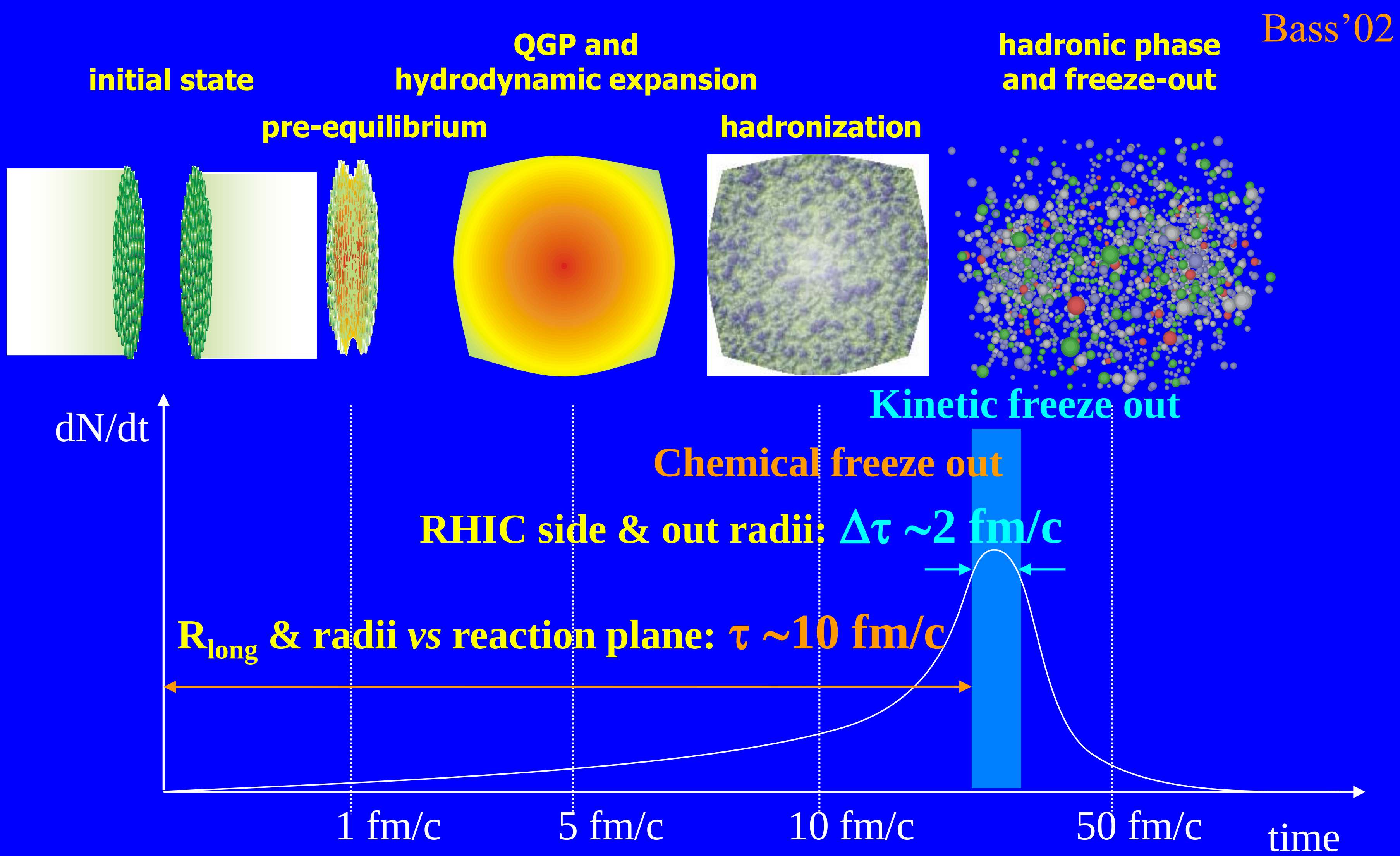
Emission duration = 1.9 ± 0.2 fm/c

χ^2/dof = 120 / 86



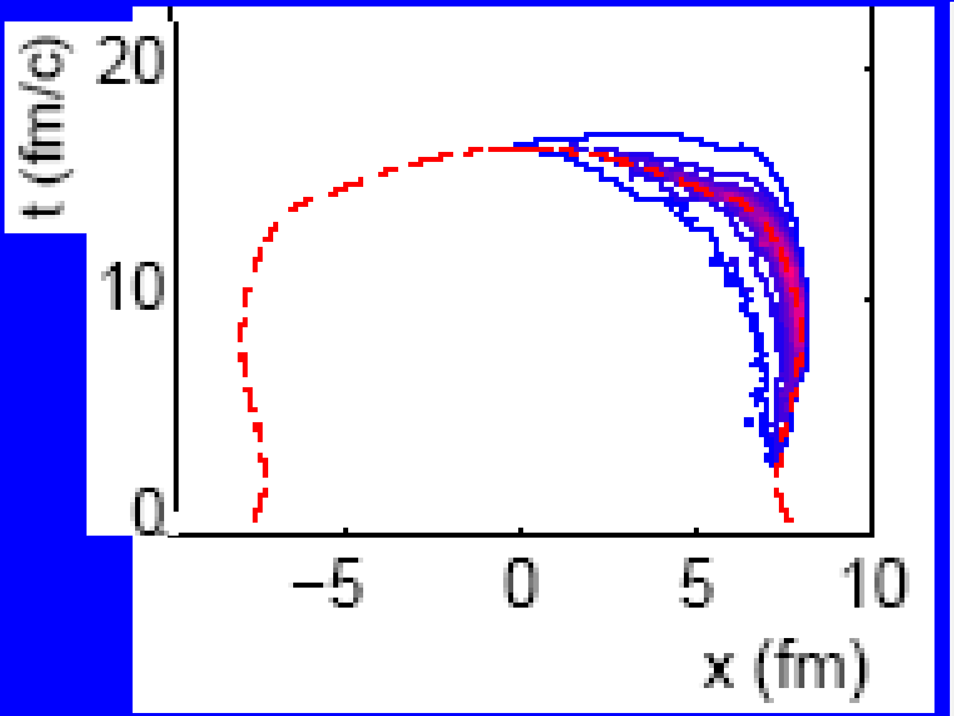
$\beta_x \approx \beta_0 (r/R)$ $\beta_z \approx z/\tau$

Expected evolution of HI collision vs RHIC data

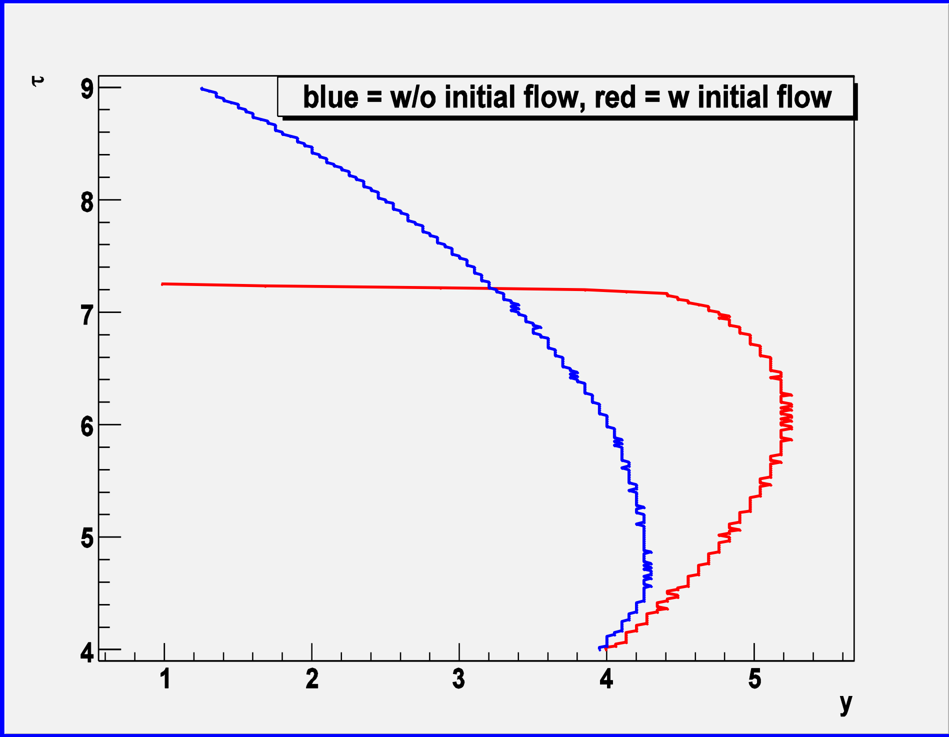


Femtoscopy puzzle: simple Hydro overestimates τ and $\Delta\tau$

2+1D Hydro
no initial flow
Kolb, Heinz'03



3+1D Hydro
w & w/o initial flow
Sinyukov, Karpenko'05



$$R_x^2 = ½ \langle (\Delta x - v_x \Delta t)^2 \rangle$$

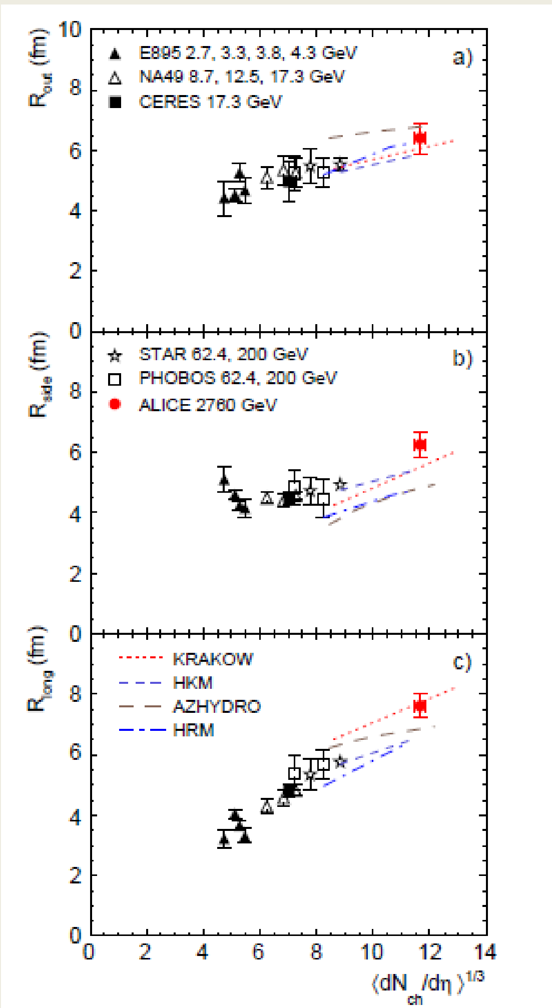
Initial flow → positive x-t correlation → smaller R_{out}

AMPT → positive x-t correlation → describes R_i Lin, Ko, Pal'02

Femtoscopy of Pb+Pb at LHC

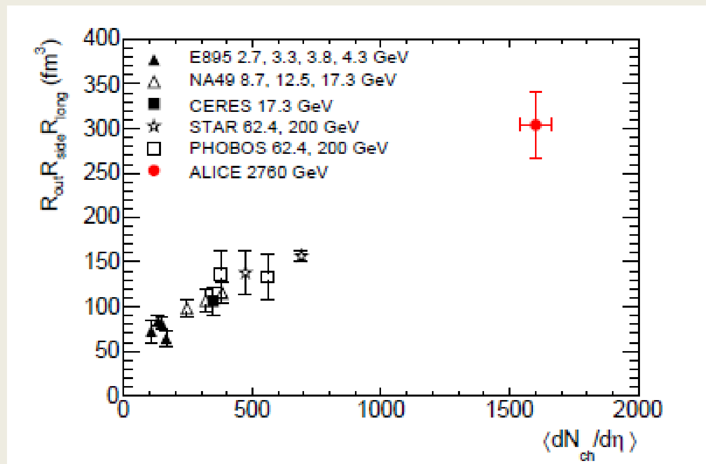
arXiv:1012.4035

All radii increase with N_{ch} from RHIC to LHC



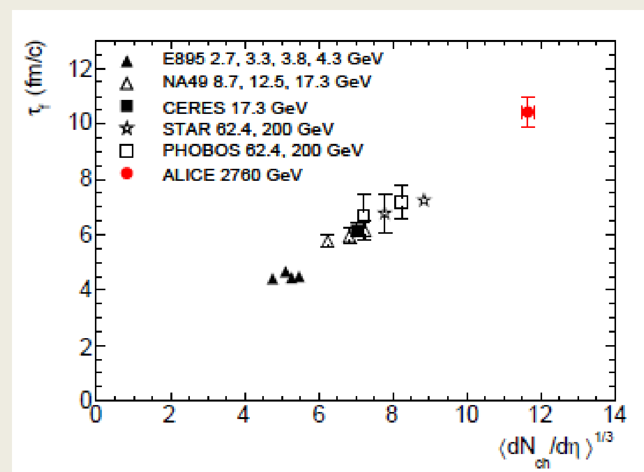
multiplicity scaling of the correlation volume

→ universal freeze-out density



The LHC fireball:
- **hotter**
- **lives longer &**
- **expands to a larger size**

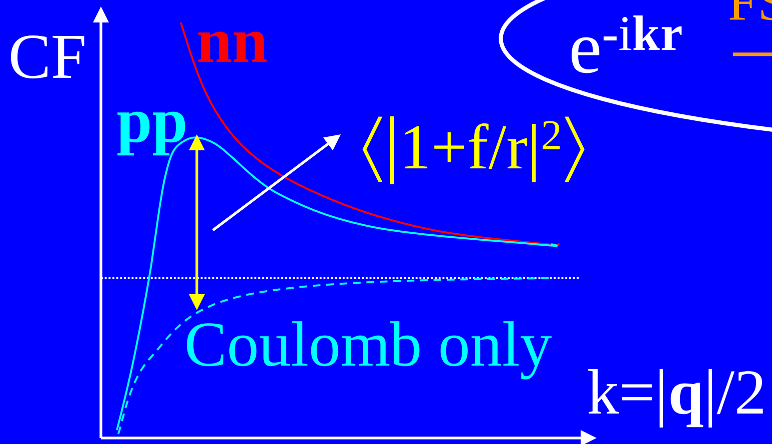
Freeze-out time τ_f from $R_{long}=\tau_f\,(T/m_t)^{1/2}$



Final State Interaction

Similar to Coulomb distortion of β-decay Fermi'34: $\langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle$

Migdal, Watson, Sakharov, … Koonin, GKW, LL, ...



$$e^{-i\mathbf{kr}} \xrightarrow{\text{FSI}} \psi_{-\mathbf{k}}(\mathbf{r}) \xrightarrow{\text{strong FSI}} [\; e^{-i\mathbf{kr}} + f(k)e^{ikr}/r \;]$$

s-wave: $f_c\sqrt{A_c}\cdot(G_0+iF_0)$

Coulomb: $e^{i\delta_c}\sqrt{A_c}$ — Point-like Coulomb factor

$$F = 1 + \frac{\mathbf{kr}+kr}{ka} + \ldots$$

a — Bohr radius

⇒ FSI is sensitive to source size r and scattering amplitude f

It **complicates CF analysis** but makes possible

→ **Femtoscopy with nonidentical particles** πK, πp, ..

including relative space-time asymmetries delays, flow

→ **Femtoscopy using Coalescence** **deuterons, ..**

→ **Study "exotic" scattering** ππ, πK, KK, πΛ, pΛ, ΛΛ, ..

"Fermi-like" CF formula

$$CF = \langle\ |\psi_{-k^*}(r^*)|^2\ \rangle$$

Koonin'77: nonrelativistic & unpolarized protons

RL, Lyuboshitz'82: generalization to relativistic & polarized & nonidentical particles & calculated the effect of nonequal times

Assumptions:

- same as for KP formula in case of pure QS &
- equal time approximation in PRF

RL, Lyuboshitz'82 → eq. time conditions: $|t^*| << m_{1,2}r^{*2}$
$|k^* t^*| << m_{1,2}r^*$

OK (usually, to several % even for pions) fig.

- $t_{FSI} = d\delta/dE >> t_{prod}$

t_{FSI} (s-wave) $= \mu f_0/k^* \rightarrow |k^*| = ½|q^*| <<$ hundreds MeV/c ≈ typical momentum transfer in production

RL, Lyuboshitz ..'98

& account for coupled channels within the same isomultiplet only: $\pi^+\pi^- \leftrightarrow \pi^0\pi^0$, $\pi^- p \leftrightarrow \pi^0 n$, $K^+K^- \leftrightarrow K^0\bar{K}^0$, ...

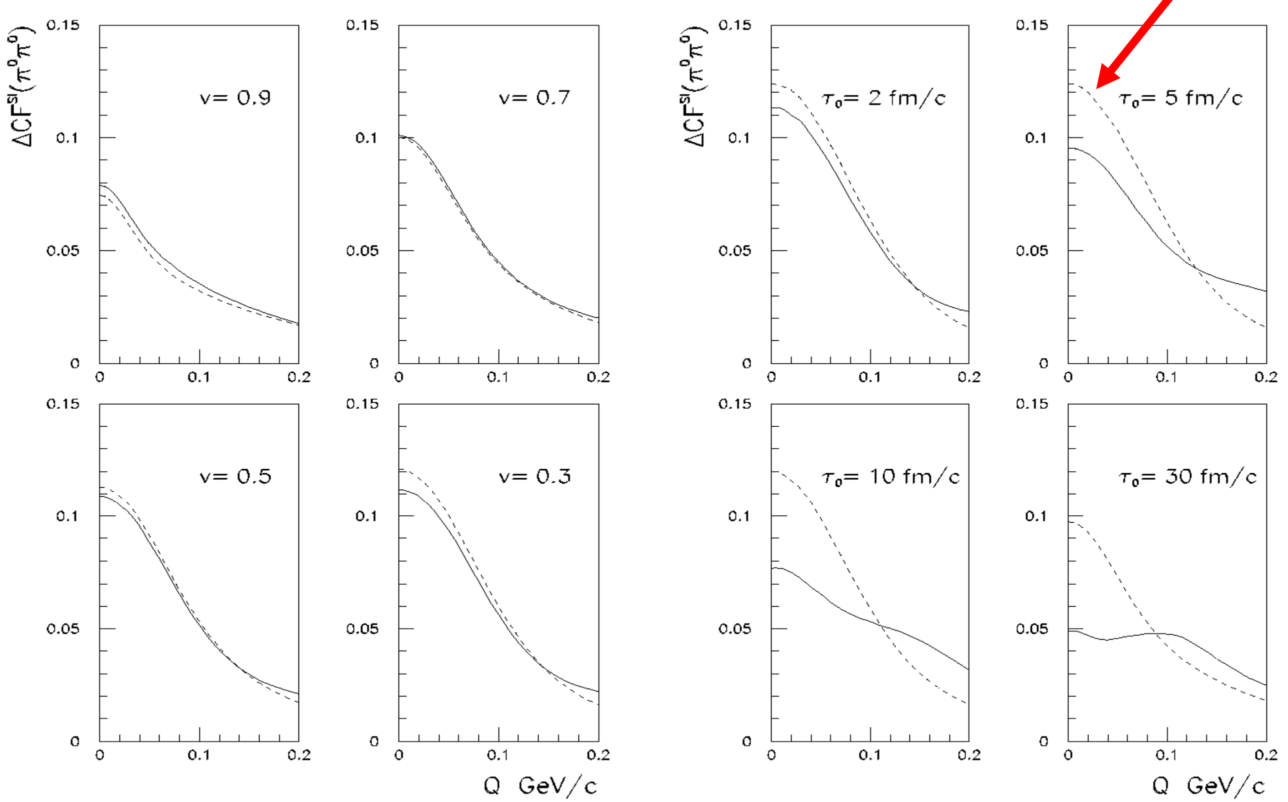
Effect of nonequal times in pair cms

RL, Lyuboshitz SJNP 35 (82) 770; RL nucl-th/0501065 $\Psi^{S(+)}_{p_1,p_2}(x_1,x_2) \rightarrow e^{iPX}\psi^{S}_{-\mathbf{k}^*}(\mathbf{r}^*)$

Applicability condition of equal-time approximation: $|t^*| \ll m_{1,2}r^{*2}$

$|k^*t^*| \ll m_{1,2}r^*$

r_0=2 fm τ_0=2 fm/c r_0=2 fm v=0.1



↓

OK for heavy particles & small k*

→ OK within 5% even for pions if $\Delta\tau=\tau_0 \sim r_0$ or lower

Correlation asymmetries

CF of **identical particles** sensitive to terms **even** in **k*r***
(e.g. through ⟨cos 2**k*r***⟩) → measures only
dispersion of the components of relative separation
$\mathbf{r}^* = \mathbf{r}_1{}^* - \mathbf{r}_2{}^*$ in pair cms

CF of **nonidentical particles** sensitive also to terms **odd** in **k*r***
→ measures also relative **space-time asymmetries - shifts** $\langle \mathbf{r}^* \rangle$

RL, Lyuboshitz, Erazmus, Nouais PLB 373 (1996) 30

→ Construct $\mathbf{CF}_{+x}$ and $\mathbf{CF}_{-x}$ with **positive** and **negative** **k*-projection** k^*_x on a given **direction x** and study CF-ratio $\mathbf{CF}_{+x}/\mathbf{CF}_{-x}$

CF-asymmetry for charged particles

Asymmetry arises mainly from Coulomb FSI

$CF \approx A_c(\eta)\ \langle|F(-i\eta,1,i\zeta)|^2\rangle$ $\eta=(k^*a)^{-1}$, $\zeta=\mathbf{k^*r^*}+k^*r^*$

$$F \xrightarrow[k^*<1/r^*]{r^*<<|a|} 1+\eta\,\zeta = 1+r^*/\underbrace{a}_{\text{Bohr radius}}+\mathbf{k^*r^*}/(k^*a)$$

Bohr radius ±226 fm for $\pi^\pm p$

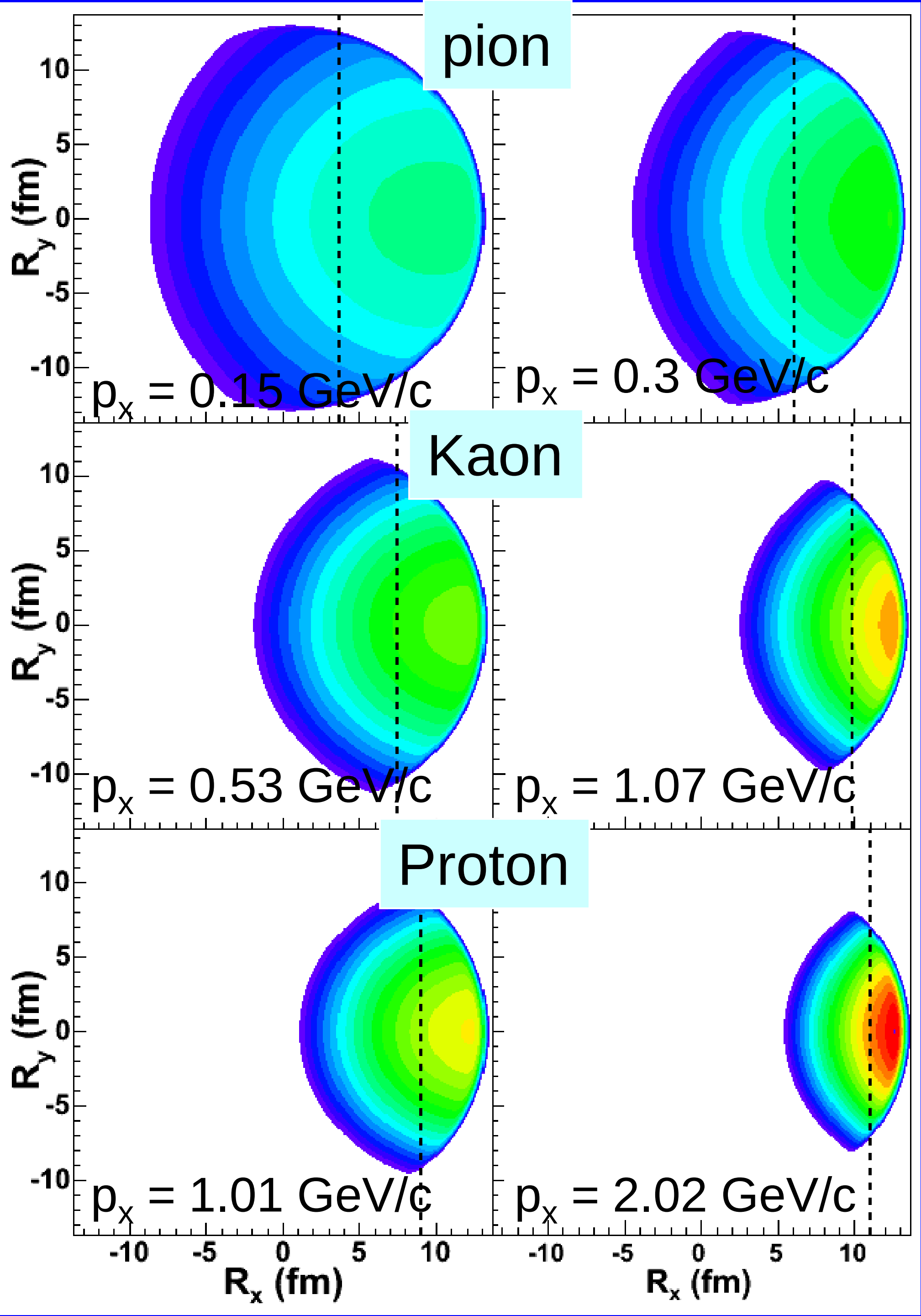
±388 fm for $\pi^+\pi^\pm$

$$\Rightarrow \boxed{CF_{+x}/CF_{-x} \xrightarrow{k^*\to 0} 1+2\,\langle\Delta x^*\rangle\,/a}$$

$\Delta x^* = x_1^* - x_2^* \equiv r_x^*$ → Projection of the relative separation **r*** in pair cms on the direction **x**

In LCMS ($v_z=0$) or **x** || **v**: $\boxed{\Delta x^* = \gamma_t(\Delta x - v_t\Delta t)}$

⇒ **CF asymmetry is determined by space and time asymmetries**



BW Retiere@LBL'05

Distribution of emission points at a given equal velocity:
- Left, $\beta_x = 0.73c$, $\beta_y = 0$
- Right, $\beta_x = 0.91c$, $\beta_y = 0$

Dash lines: average emission R_x
$\Rightarrow \langle R_x(\pi)\rangle < \langle R_x(K)\rangle < \langle R_x(p)\rangle$

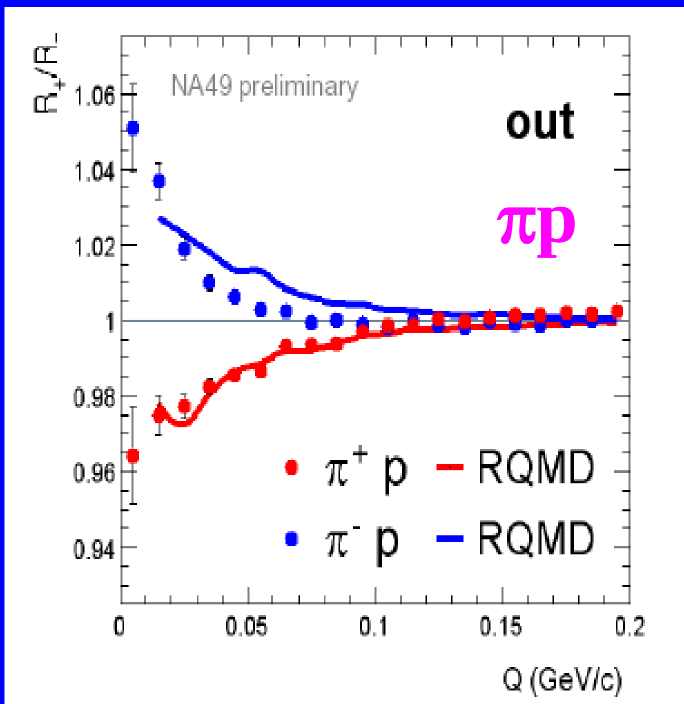
For a Gaussian density profile with a radius R_G and flow velocity profile $\beta^F(r) = \beta_0 r/R_G$
RL'04, Akkelin-Sinyukov'96 :

$$\langle x\rangle = R_G \beta_x \beta_0 / [\beta_0^2 + T/m_t]$$

NA49 & STAR out-asymmetries

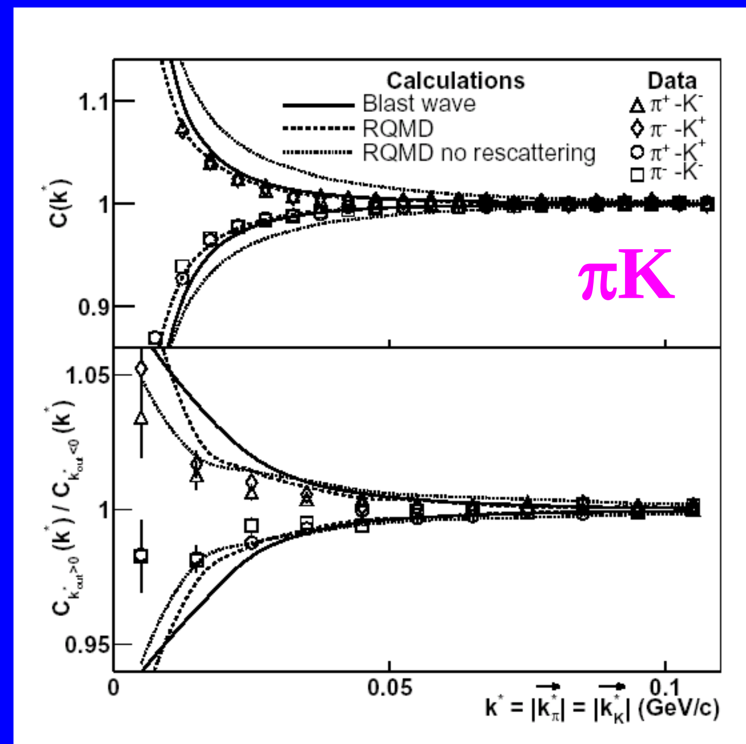
Pb+Pb central 158 AGeV

not corrected for ~ 25% impurity
r* RQMD scaled by 0.8



Au+Au central $\sqrt{s_{NN}}$=130 GeV

corrected for impurity



→ **Mirror symmetry** (~ same mechanism for + and – mesons)

→ **RQMD, BW ~ OK ⇒ points to strong transverse flow**

(⟨Δt⟩ gives only ~ ¼ of CF asymmetry)

Analytical dependence of CF on s-wave scatt. amplitudes f(k) and source radius r_0 LL'81

using spherical wave in the outer region ($r>\varepsilon$) & inner region ($r<\varepsilon$) correction:

$\Rightarrow$ FSI contribution to the CF of nonidentical particles, assuming Gaussian separation distribution $W(r)=\exp(-r^2/4r_0^2)/(2\sqrt{\pi}\, r_0)^3$

single channel & no Coulomb

at $kr_0 \ll 1$: $\Delta CF^{FSI} = \frac{1}{2}|f_0/r_0|^2[1-d_0/(2r_0\sqrt{\pi})]+2\mathrm{Re}f_0/(r_0\sqrt{\pi})$

f_0 & d_0 are the s-wave scatt. length and eff. radius determining the scattering amplitude in the effective range approximation:

$$f(k) = \sin\delta_0 \exp(i\delta_0)/k \approx (1/f_0+\tfrac{1}{2}d_0k^2 - ik)^{-1}$$

f_0 and d_0 : characterizing the nuclear force

$u(r) = e^{i\delta} r\psi(r)$

$f_0 = -a$

$d_0 \approx r_0$

at $k \rightarrow 0$

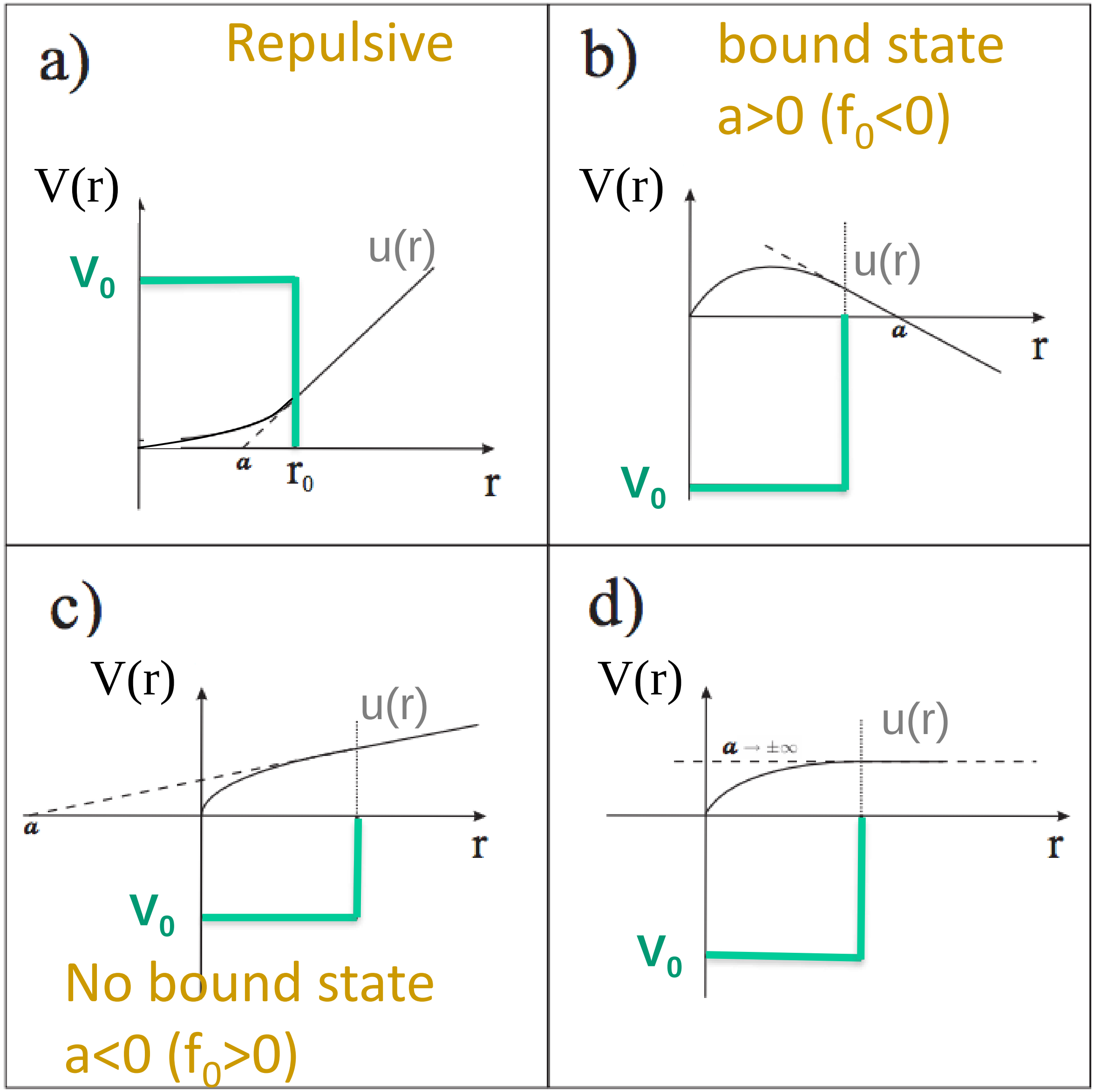
$r > r_0$

$u(r) \sim (r - a)$

Resonance:

$f_0 > 0$

$d_0 < 0$



f_0 and d_0 : How to measure them in scattering experiments
(not always possible with reasonable statistics)

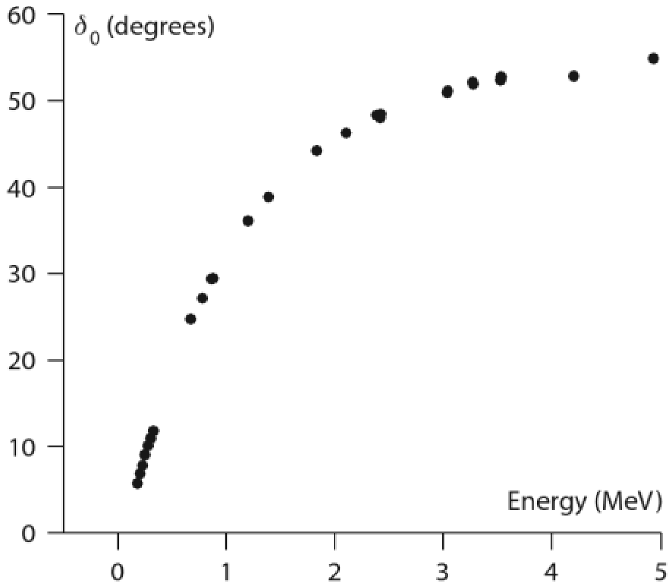


Figure 2.11 Phase shift variation as a function of the incident proton energy for proton-proton collision. The experimental points are from reference [JB50].

$$\frac{d\sigma}{d\Omega} = \left[\left(\frac{d\sigma}{d\Omega}\right)_c + \left(\frac{d\sigma}{d\Omega}\right)_n + \left(\frac{d\sigma}{d\Omega}\right)_{cn}\right]$$

Coulomb   Nuclear   Crossterm

$$\left(\frac{d\sigma}{d\Omega}\right)_c = \left(\frac{e^2}{2E_p}\right)^2 \left\{\frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} - \frac{\cos\left\{\eta \ln\left[\tan^2(\theta/2)\right]\right\}}{\sin^2(\theta/2)\cos^2(\theta/2)}\right\}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_n = \frac{\sin^2\delta_0}{k^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cn} = -\frac{1}{2}\left(\frac{e^2}{E_p}\right)^2 \frac{\sin\delta_0}{\eta}\left\{\frac{\cos\left[\delta_0 + \eta\ln\sin^2(\theta/2)\right]}{\sin^2(\theta/2)} + \frac{\cos\left[\delta_0 + \eta\ln\cos^2(\theta/2)\right]}{\cos^2(\theta/2)}\right\}$$

"*Nuclear physics in a nutshell*",
Carlos A. Bertulani. Princeton U Press (2007).

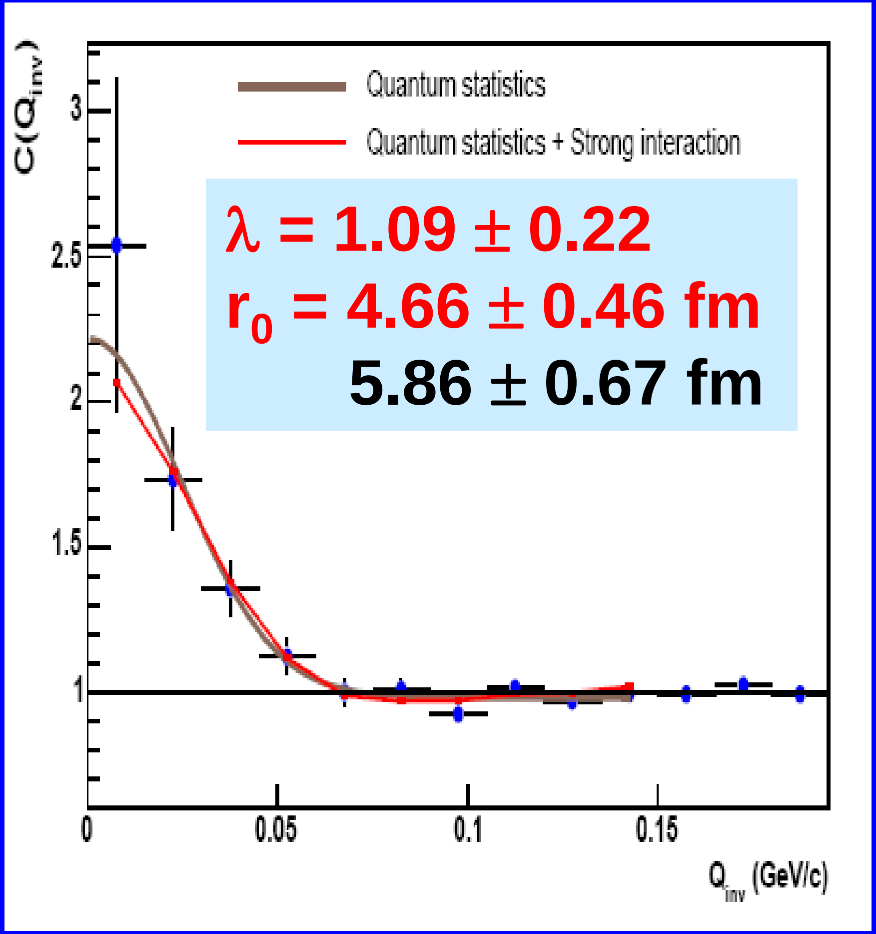
$$k\cot(d_0) » \frac{1}{f_0} + \frac{1}{2} d_0 k^2$$

f_0 and d_0 can be extracted by studying the phase shift vs. energy.

FSI effect on CF of neutral kaons

Lyuboshitz-Podgoretsky'79:
K_sK_s from $K\bar{K}$ also show BE enhancement

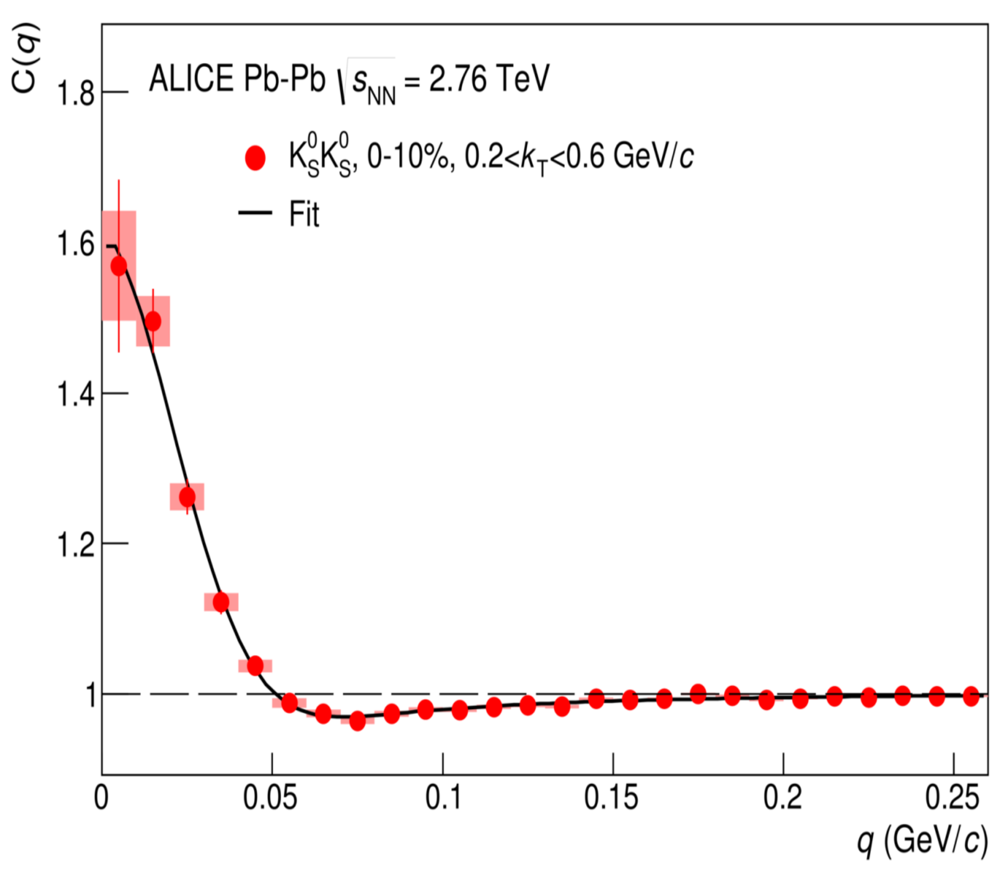
STAR data on CF(K_sK_s)



Goal: no Coulomb. But R may go up by ~1 fm if neglecting FSI in $K\bar{K}$ (~50% K_sK_s)↔ $f_0(980)$ & $a_0(980)$

RL-Lyuboshitz'82

ALICE data on CF(K_sK_s)



Even stronger effect of KK-bar FSI on $K_s K_s$ correlations in pp-collisions at LHC

ALICE: PLB 717 (2012) 151

e.g. for $k_t < 0.85$ GeV/c, N_{ch}=1-11 the neglect of FSI increases λ by ~100% and R_{inv} by ~40%

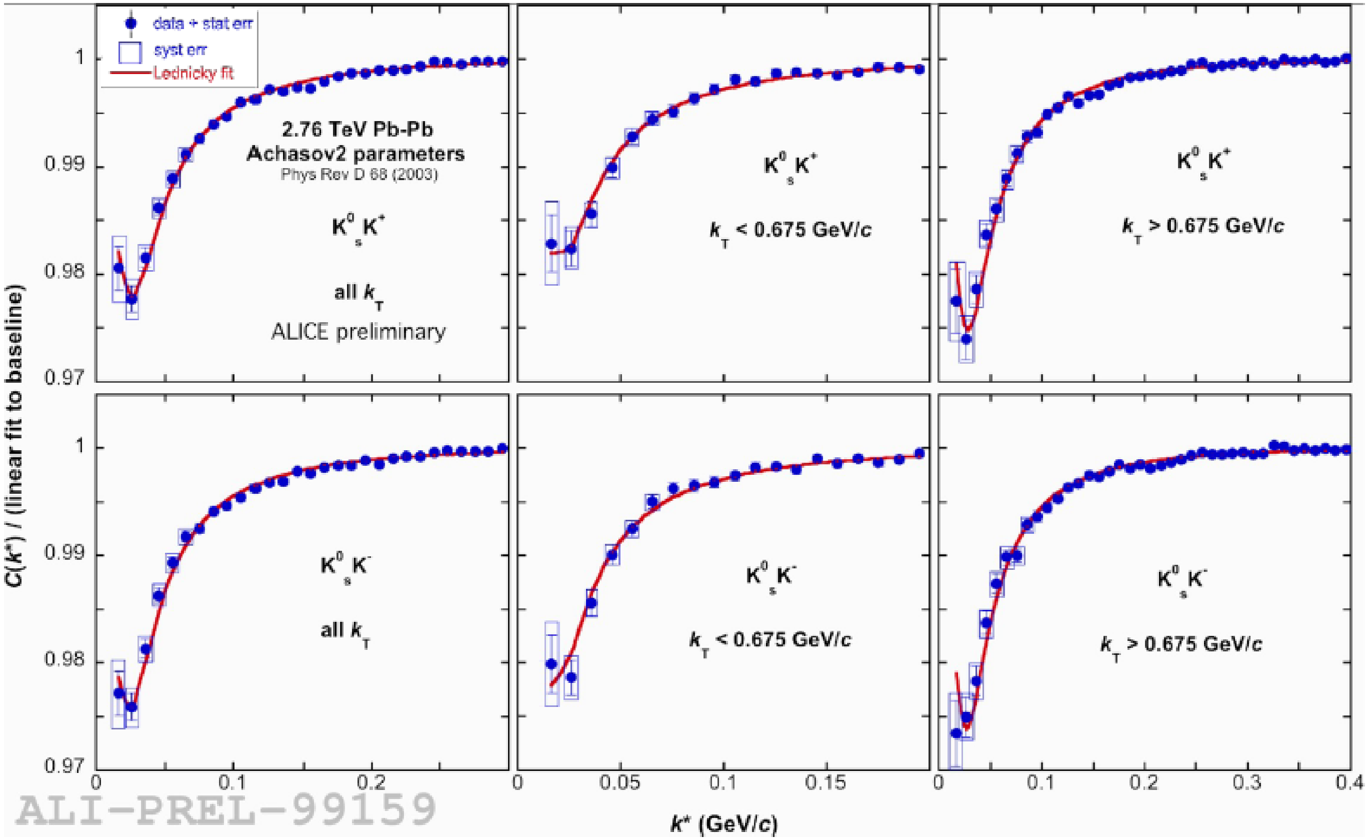
$\lambda = 0.64 \pm 0.07 \rightarrow 1.36 \pm 0.15 > 1$!

$R_{inv} = 0.96 \pm 0.04 \rightarrow 1.35 \pm 0.07$ fm

ArXiv.org:1506.07884

Correlation femtoscopy with nonidentical particles

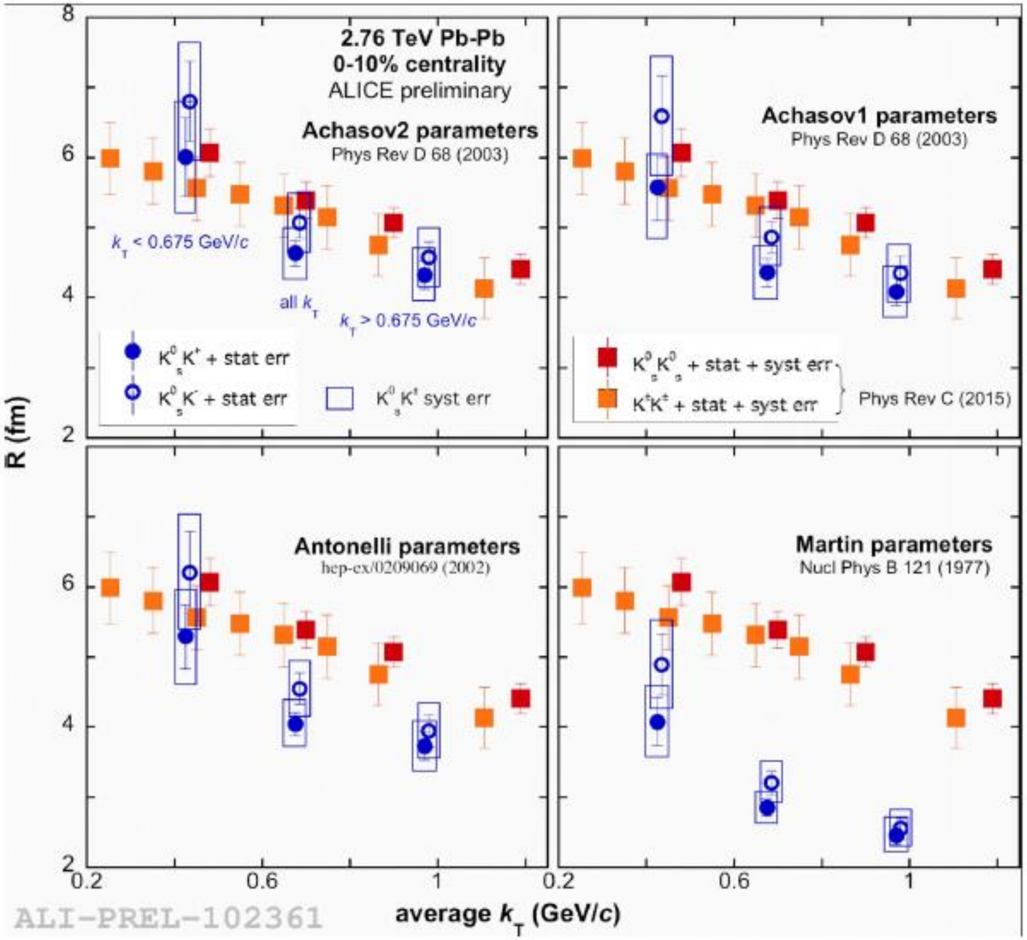
Correlations for K^0_S-K^{ch}



- Correlation function from strong interaction well described by theoretical formula, dominated by $a_0(980)$ resonance, sensitive to the exact values of resonance parameters

Correlation femtoscopy with nonidentical particles

Radii for K^0_S-K^{ch} correlations



- Radii for K^0_S-K^{ch} expected same as in K^0_S-K^0_S and K^{ch}-K^{ch}
- ALICE data favors Achasov a_0 resonance parameters

NA49 central Pb+Pb 158 AGeV vs RQMD: FSI theory OK

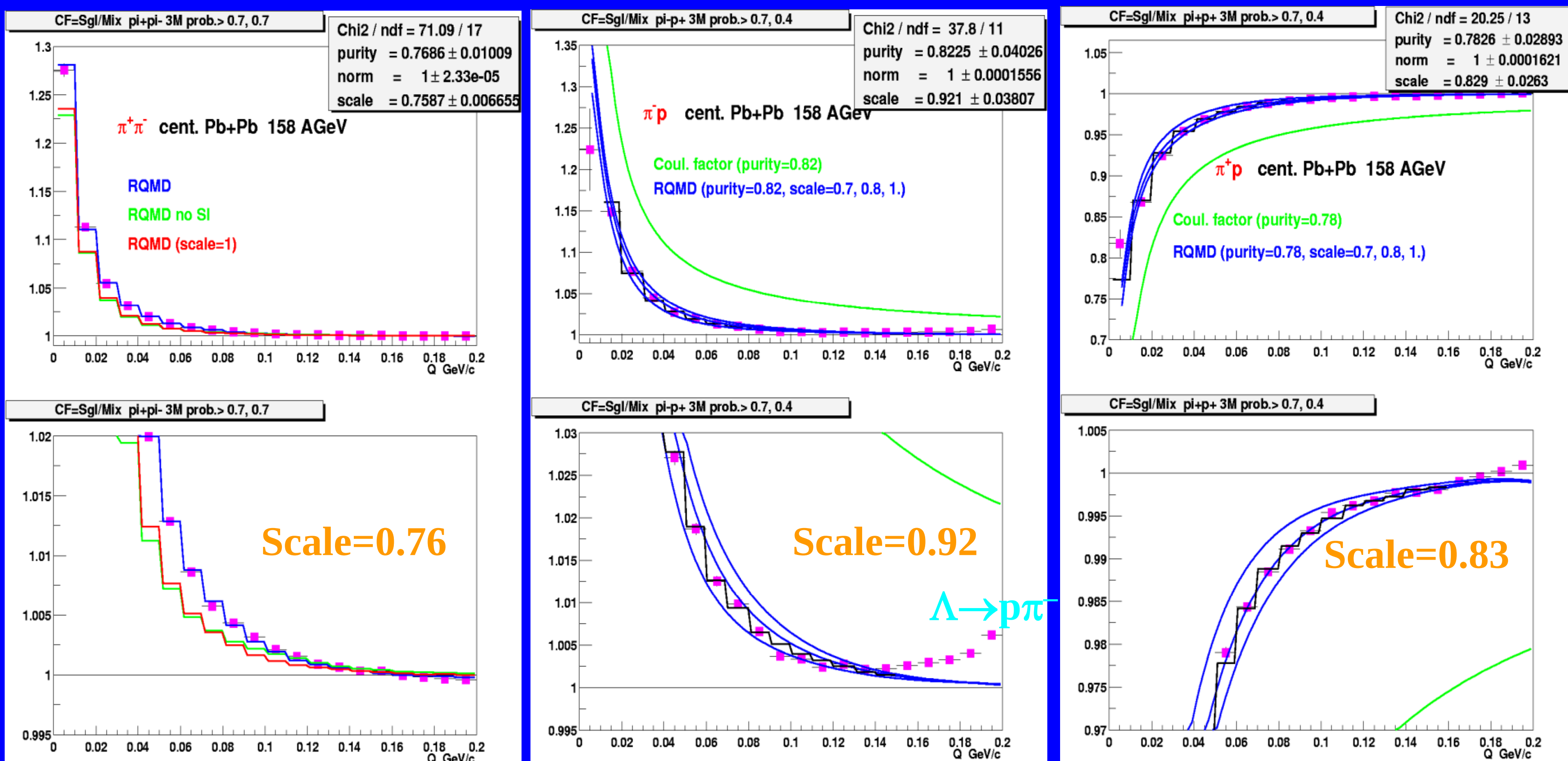
Long tails in RQMD: $\langle r^* \rangle$ = **21** fm for r^* < **50** fm

29 fm for r^* < **500** fm

Fit **CF=Norm [Purity RQMD($r^* \rightarrow$ Scale•r^*)+1-Purity]**

⇒ **RQMD overestimates r^* by 10-20% at SPS** cf **~ OK at AGS**

worse at RHIC



Correlation femtoscopy with nonid. particles

pΛ CFs at AGS & SPS & STAR

Goal: No Coulomb suppression as in pp CF &

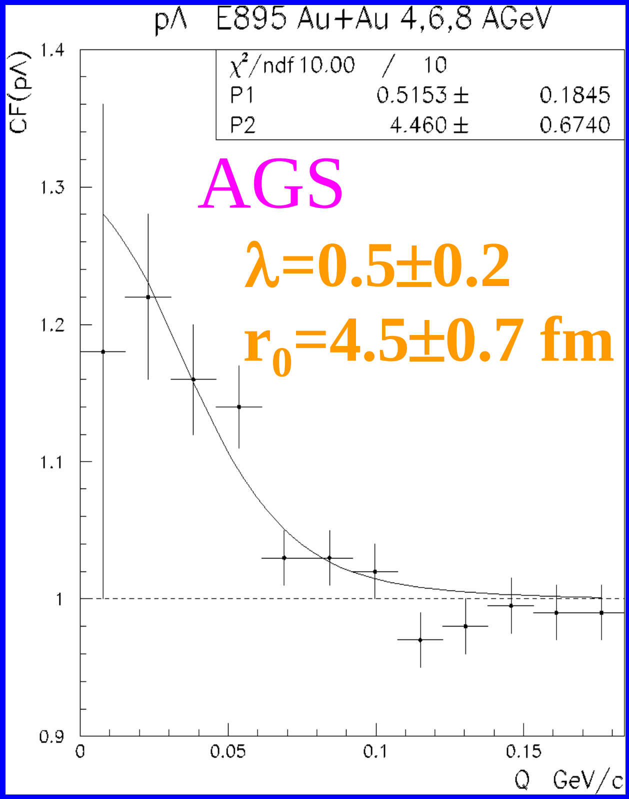
Wang-Pratt'99 Stronger sensitivity to r_0

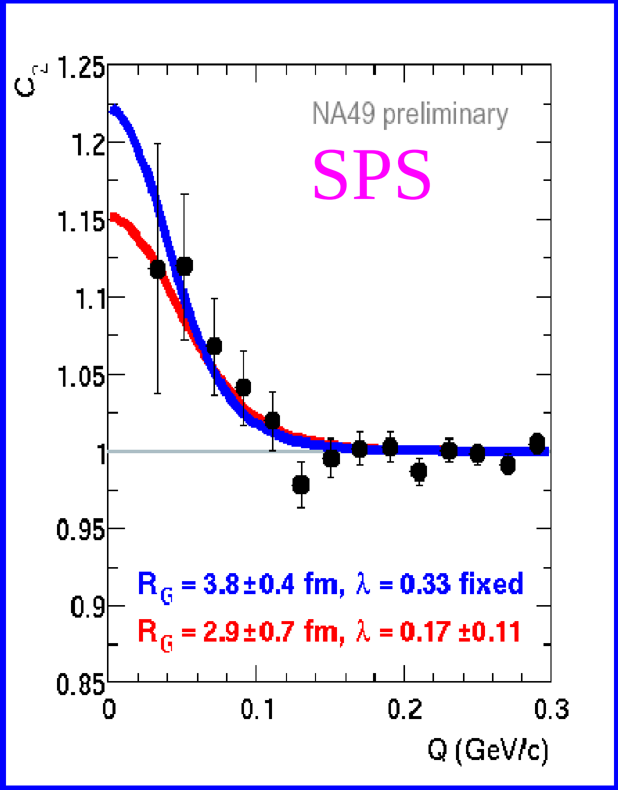
Fit using RL-Lyuboshitz'82 with

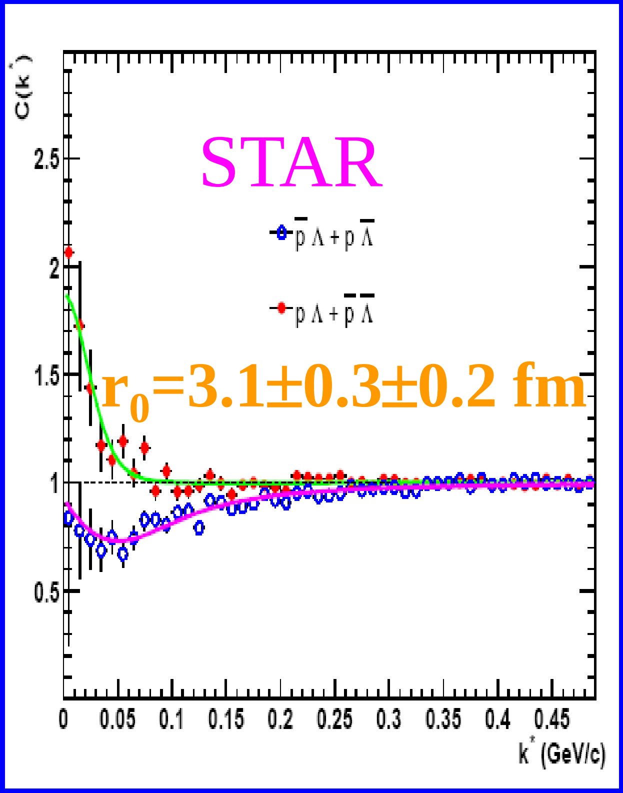
	singlet	triplet
Scattering lengths, fm:	2.31	1.78
Effective radii, fm:	3.04	3.22

λ consistent with estimated impurity

r_0~ 3-4 fm consistent with the radius from pp CF & m_t scaling







Pair purity problem for pΛ CF @ STAR

Particle	Identification	Fraction Primary
p	$76 \pm 7\%$	$52 \pm 4\%$
$\overline{p}$	$74 \pm 7\%$	$48 \pm 4\%$
Λ	$86 \pm 6\%$	$45 \pm 4\%$
$\overline{\Lambda}$	$86 \pm 6\%$	$45 \pm 4\%$

$\Rightarrow$ **PairPurity ~ 15%**

Assuming no correlation for misidentified particles and particles from weak decays

$\rightarrow$ $$C^{corr}_{measured}(k^*) = \frac{C_{measured}(k^*) - 1}{\text{PairPurity}} + 1$$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0} \right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi} r_0} F_1(Q r_0) - \frac{\Im f^S(k^*)}{r_0} F_2(Q r_0) \right],$$

where $F_1(z) = \int_0^z dx e^{x^2 - z^2}/z$ and $F_2(z) = (1 - e^{-z^2})/z$.

$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - i k^* \right)^{-1}$$

$\leftarrow$ Fit using RL-Lyuboshitz'82 (for np)

Pairs	Fractions (%)
p_{prim}-Λ_{prim}	15
p_{Λ}-Λ_{prim}	10
p_{Σ^+}-Λ_{prim}	3
p_{prim}-Λ_{Σ^0}	11
p_{Λ}-Λ_{Σ^0}	7
p_{Σ^+}-Λ_{Σ^0}	2
p_{prim}-Λ_{Ξ}	9
p_{Λ}-Λ_{Ξ}	5
p_{Σ^+}-Λ_{Ξ}	2

$\leftarrow$ but, there can be residual correlations for particles from weak decays requiring knowledge of $\Lambda\Lambda$, pΣ, $\Lambda\Sigma$, $\Sigma\Sigma$, pΞ, $\Lambda\Xi$, $\Sigma\Xi$ correlations

Correlation study of strong interaction

$\pi^+\pi^-$ & $\Lambda\Lambda$ & $\bar{p}\Lambda$ & $\bar{p}\bar{p}$ s-wave scattering parameters from NA49 and STAR

Fits using RL-Lyuboshitz'82

$\bar{p}\Lambda$: STAR data accounting for residual correlations
- Kisiel et al, PRC 89 (2014) : assuming a universal Imf_0
- Shapoval et al PRC 92 (2015): Gauss. parametr. of res. CF

Ref_0 ≈ 0.5 fm, Imf_0 ≈ 1 fm, r_0 ≈ 3 fm

$\Lambda\Lambda$: NA49: $|f_0(\Lambda\Lambda)| << f_0(NN)$ ~ 20 fm

STAR, PRL 114 (2015): $f_0(\Lambda\Lambda)$ ≈ -1 fm, $d_0(\Lambda\Lambda)$ ≈ 8 fm

$\pi^+\pi^-$: NA49 vs RQMD with SI scale: $f_0 \rightarrow$ sisca f_0 (=0.232fm)

sisca = 0.6±0.1 compare with

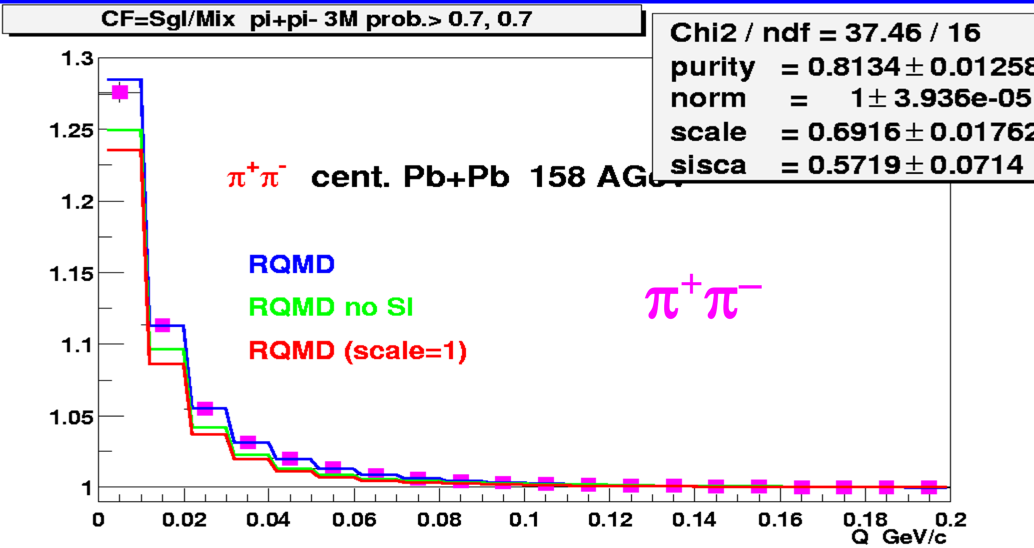
~0.8 from SχPT & BNL data E765 $K \rightarrow e\nu\pi\pi$

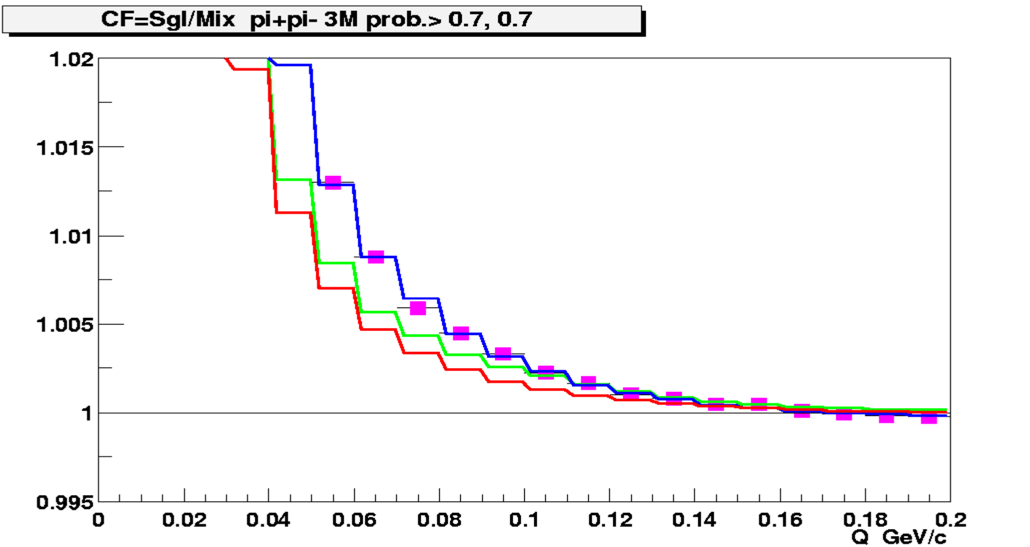
Here a suppression can be due to eq. time approx.

$\bar{p}\bar{p}$: STAR, Nature (2015): f_0 and d_0 coincide with table pp-values

Correlation study of strong interaction

CF=Norm [Purity RQMD(r* → Scale•r*)+1-Purity]





$\pi^+\pi^-$ scattering length
f_0 from NA49 CF

Fit CF($\pi^+\pi^-$) by RQMD
with SI scale:
$f_0 \rightarrow$ sisca f_0^{input}
f_0^{input} = 0.232 fm

sisca = 0.6±0.1
Compare with
~0.8 from SχPT
& BNL E765
$K \rightarrow e\nu\pi\pi$

Correlation study of strong interaction

ΛΛ scattering lengths f_0 from STAR correlation data

Fit using RL-Lyuboshitz (82): **$\lambda \approx 0.18$, $r_0 \approx 3$ fm, $a_{res} \approx -0.04$, $r_{res} \approx 0.4$ fm**

$f_0 \approx -1$ fm, $d_0 \approx 8$ fm $\Rightarrow$ - no s-wave resonance

- bound state possible

$$CF=1+\lambda[\Delta CF^{FSI}+\sum_S \rho_S (-1)^S \exp(-r_0^2 Q^2)]$$

$$+ a_{res} \exp(-r_{res}^2 Q^2)$$

$\rho_0 = \frac{1}{4}(1-P^2)$ $\rho_1 = \frac{1}{4}(3+P^2)$ P=Polar.=0

$$\Delta CF^{FSI} = 2\rho_0[\tfrac{1}{2}|f^0(k)/r_0|^2(1-d_0^0/(2r_0\sqrt{\pi}))$$

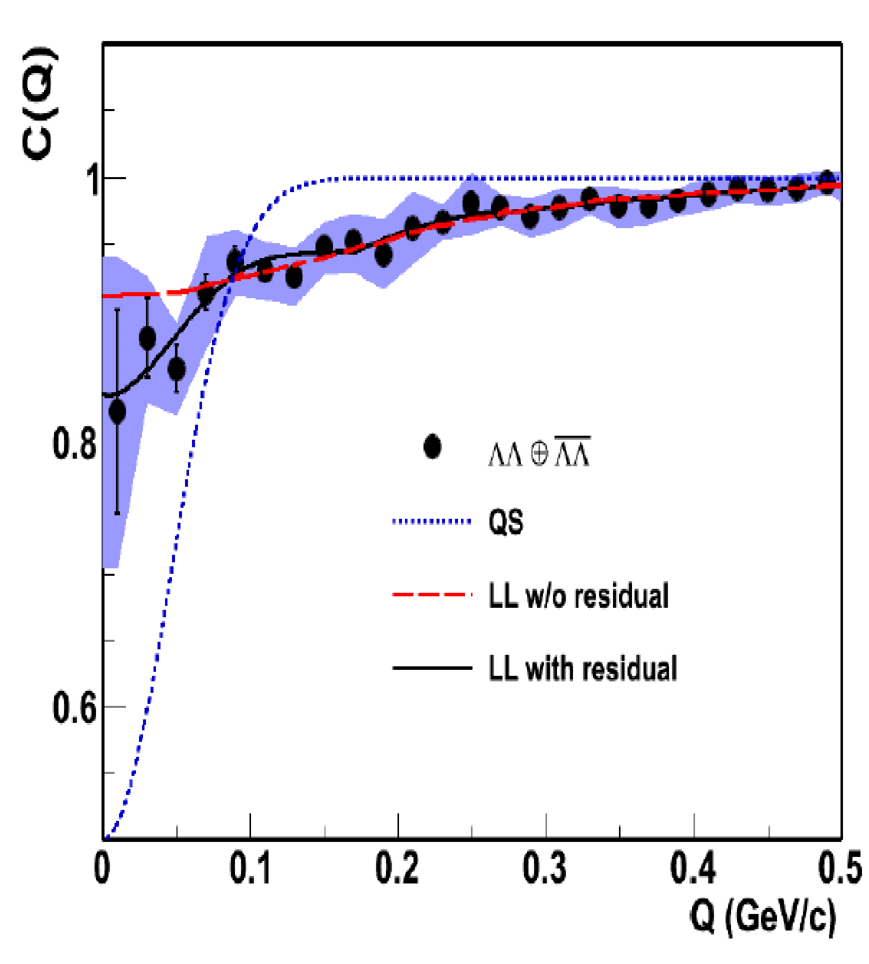
$$+2\mathrm{Re}(f^0(k)/(r_0\sqrt{\pi}))F_1(r_0 Q)$$

$$-\mathrm{Im}(f^0(k)/r_0)F_2(r_0 Q)]$$

$f^S(k)=(1/f_0^S+\tfrac{1}{2}d_0^S k^2 - ik)^{-1}$ $k=Q/2$

$F_1(z)=\int_0^z dx \exp(x^2-z^2)/z$

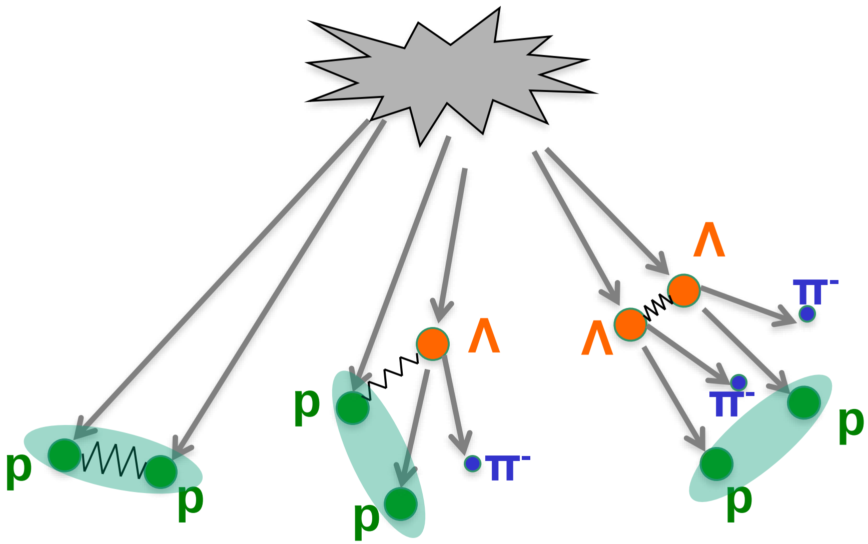
$F_2(z)=[1-\exp(-z^2)]/z$



Correlation study of strong pp & $\bar{p}\bar{p}$ interaction at STAR

***Nature* 527, 345 (2015)**

Residual pp correlation



The observed (anti)protons can come from weak decays of already correlated primary particles, hence introducing residual correlations which contaminate the CF
(generally cannot be treated as a constant impurity).

Residual correlation

Taking dominant contributions due to residual correlation, the measured correlation function can be expressed as :

$$C_{measured}(k^*) = 1 + x_{pp}\left[C_{pp}(k^*;R_{pp}) - 1\right] + x_{p\Lambda}\left[\tilde{C}_{p\Lambda}(k^*;R_{p\Lambda}) - 1\right] + x_{\Lambda\Lambda}\left[\tilde{C}_{\Lambda\Lambda}(k^*;R_{\Lambda\Lambda}) - 1\right]$$

where

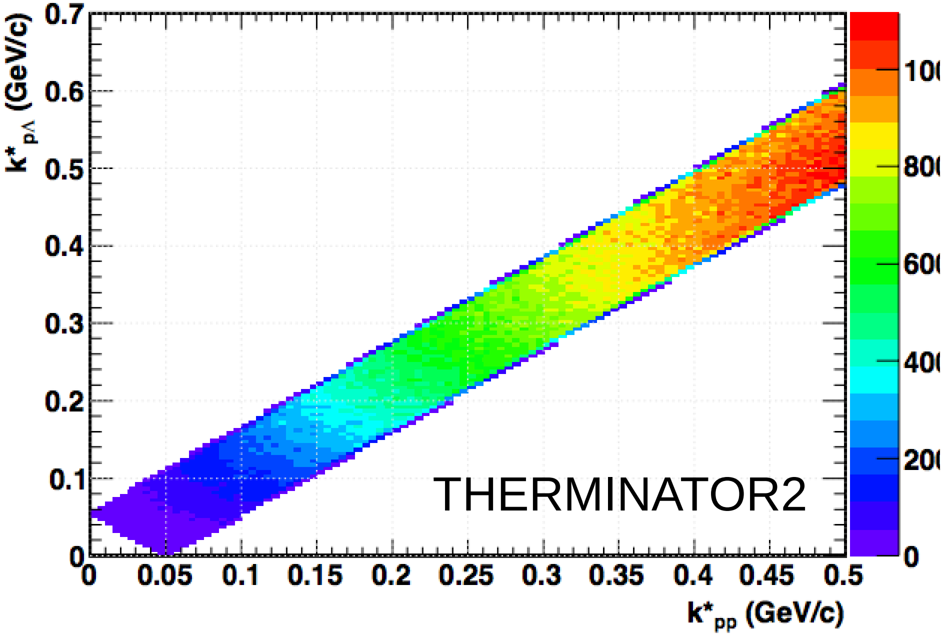
$$\tilde{C}_{p\Lambda}(k^*_{pp}) = \int C_{p\Lambda}(k^*_{p\Lambda})T(k^*_{p\Lambda}, k^*_{pp})dk^*_{p\Lambda}$$

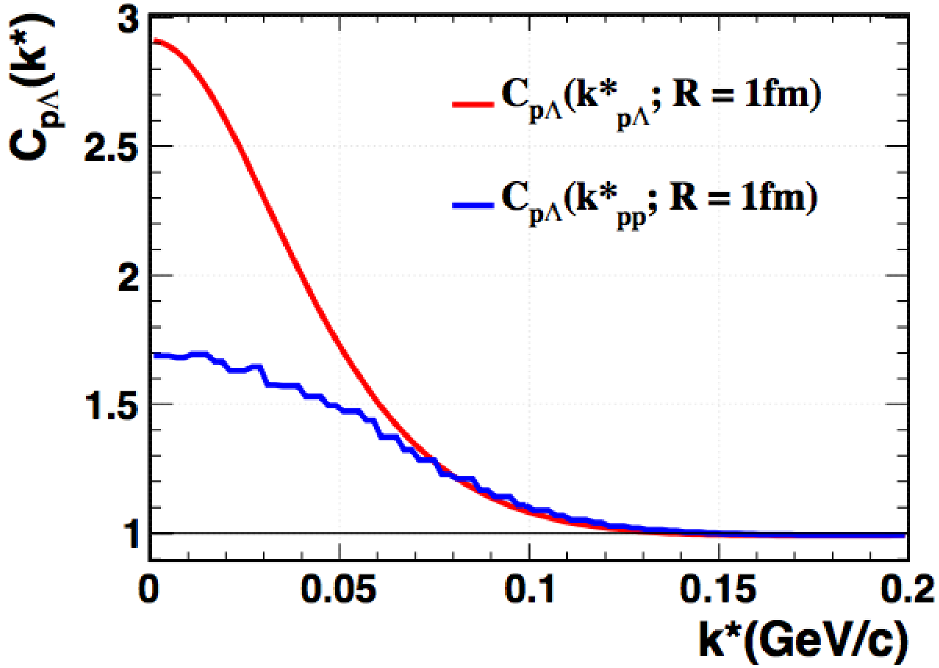
$$\tilde{C}_{\Lambda\Lambda}(k^*_{pp}) = \int C_{\Lambda\Lambda}(k^*_{\Lambda\Lambda})T(k^*_{\Lambda\Lambda}, k^*_{pp})dk^*_{\Lambda\Lambda}$$

	DCA	x_{pp}	$x_{p\Lambda}$	$x_{\Lambda\Lambda}$
proton-proton	$2cm$	0.45	0.375	0.077
proton-proton	$1cm$	0.51	0.335	0.055
pbar-pbar	$2cm$	0.42	0.385	0.092
pbar-pbar	$1cm$	0.485	0.35	0.063

- $C_{pp}(k^*)$ and $C_{p\Lambda}(k^*_{p\Lambda})$ are calculated by the Lednicky and Lyuboshitz model.
- $C_{\Lambda\Lambda}(k^*_{\Lambda\Lambda})$ is from STAR publication (PRL 114 22301 (2015)).
- Regard $R_{p\Lambda}$ and $R_{\Lambda\Lambda}$ are equal to R_{pp}.
- T is the corresponding tranform matirces, generated by THERMINATOR2, to transform $k^*_{p\Lambda}$ to k^*_{pp}, as well as $k^*_{\Lambda\Lambda}$ to k^*_{pp}.

Transformation Matrix





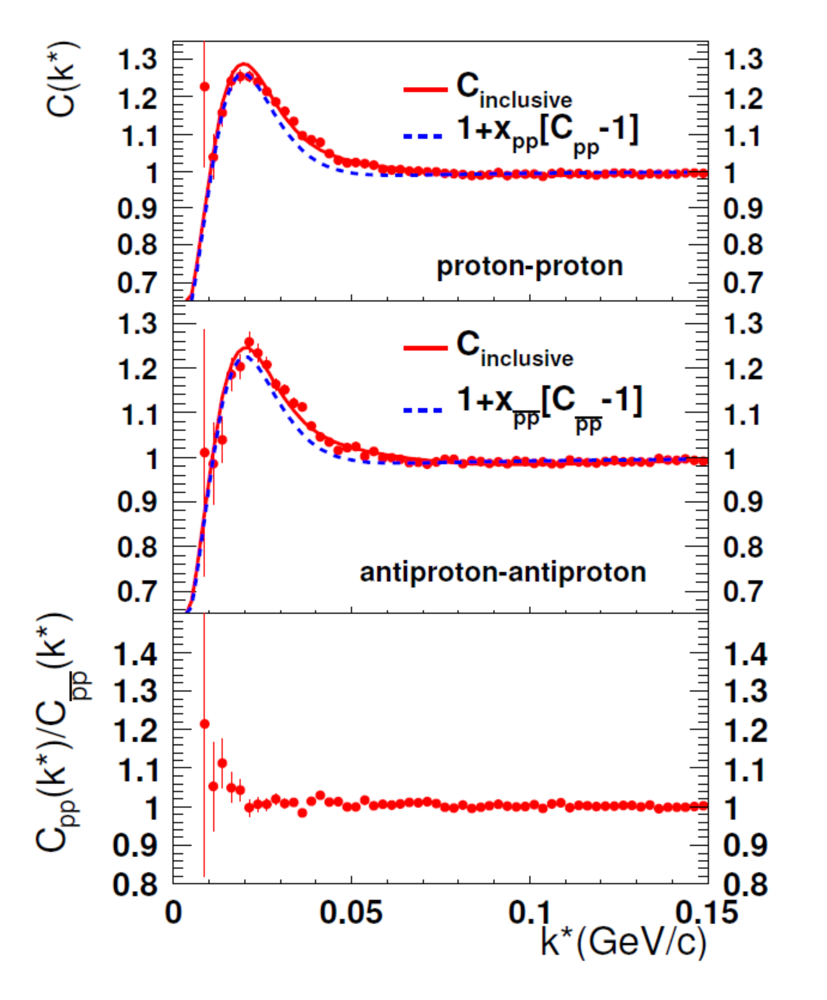
$$\tilde{C}_{p\Lambda}(k^*_{pp}) = \int C_{p\Lambda}(k^*_{p\Lambda})T(k^*_{p\Lambda}, k^*_{pp})dk^*_{p\Lambda}$$

$$\tilde{C}_{\Lambda\Lambda}(k^*_{pp}) = \int C_{\Lambda\Lambda}(k^*_{\Lambda\Lambda})T(k^*_{\Lambda\Lambda}, k^*_{pp})dk^*_{\Lambda\Lambda}$$

Correlation study of strong interaction

$\bar{p}\bar{p}$ s-wave scattering parameters from STAR correlation data

$$C_{\text{inclusive}}(k^*) = 1 + x_{pp}[C_{pp}(k^*; R_{pp}) - 1] + x_{p\Lambda}[\tilde{C}_{p\Lambda}(k^*; R_{p\Lambda}) - 1] + x_{\Lambda\Lambda}[\tilde{C}_{\Lambda\Lambda}(k^*) - 1]$$



	DCA	x_{pp}	$x_{p\Lambda}$	$x_{\Lambda\Lambda}$
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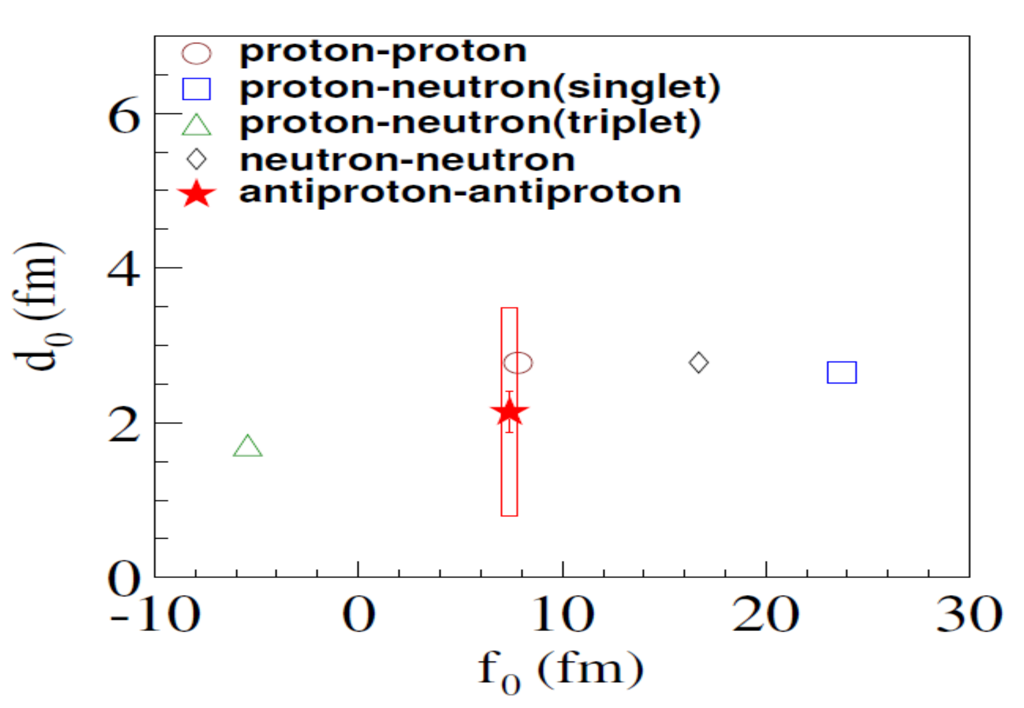


"This paper announces an important discovery! … offers important original contribution to the forces in antimatter!" – *Nature* Referee A

"… significance of the results can be considered high since this is really the first and only result available on the interaction between the antiprotons ever." – *Nature* Referee B

"… are of fundamental interest for the whole nuclear physics community and possible even beyond for atomic physics applications or condensed matter physicists. … I think that this paper is most likely one of the five most significant papers published in the discipline this year" – *Nature* Referee C

Summary

- Assumptions behind femtoscopy theory in HIC OK at $k \to 0$.
- Wealth of data on correlations of various particle species ($\pi^{\pm}, K^{\pm 0}, p^{\pm}, \Lambda, \Xi$) is available & gives unique space-time info on production characteristics including collective flows.
- Rather direct evidence for strong transverse flow in HIC at SPS & RHIC comes from nonidentical particle correlations.
- Original hydro calculations strongly overestimated out & long radii at RHIC. Solved by 3D hydro with crossover EoS + initial flow + hadronic transport.
- Info on two-particle strong interaction: $\pi\pi$ & $\Lambda\Lambda$ & $\bar{p}\Lambda$ & $\bar{p}\bar{p}$ scattering lengths & effective radii from correlation HIC data (on a way to solving the problem of residual correlations). A good perspective: high statistics RHIC & LHC data.

Apologize for skipping

- Multiboson effects Podgoretsky, Zajc, Pratt, RL …
- Coalescence (d, $\bar{d}$ data from NA49)
- Beyond Gaussian form RL, Podgoretsky, ..Csörgö .. Chung ..
- Imaging technique Brown, Danielewicz, ..
- Multiple FSI effects Wong, Zhang, ..; Kapusta, Li; Cramer, ..
- Spin correlations Alexander, Lipkin; RL, Lyuboshitz
- ……

Multiboson effects

- **Coherent emission: pion laser, DCC …**

→ **Correlation strength $\lambda < 1$ due to coherence Fowler-Weiner'77**

But: impurity, Long-Lived Sources (LLS), .. Deutschman'78
RL-Podgoretsky'79

→ **3π CF normalized to 2π CFs: get rid of LLS effect Heinz-Zhang'97**

But: problem with 3π Coulomb & extrapolation to Q_3=0

→ **Coherence modification of FSI effect on 2π CFs Akkelin ..'00**

But: requires precise measurement at low Q

- **Chaotic emission: Podgoretsky'85, Zajc'87, Pratt'93 ..**

See **RL et al.** **PRC 61 (00) 034901** & refs therein & **Heinz .. AP 288 (01) 325**

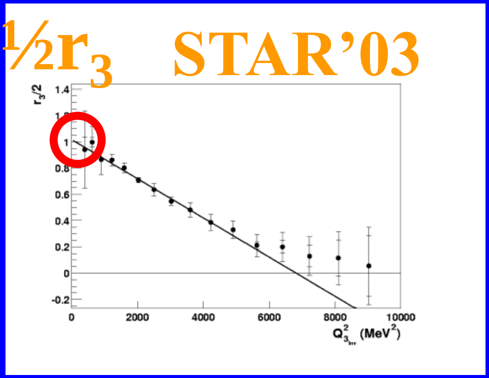
Increasing PSD:	**rare gas**	**→**	**BE condensate**
Widening of n_π distribution:	**Poisson**		**BE**
Narrowing of spectrum width:	**Δ**		**$\Delta/(2r_0\Delta) < \Delta$**
Widening of CF width:	**$1/r_0$**		**$\to \infty$**
Decreasing CF strength at fixed n:	**$\lambda = 1$**		**$\to 0$**

3π data on chaotic fraction ε

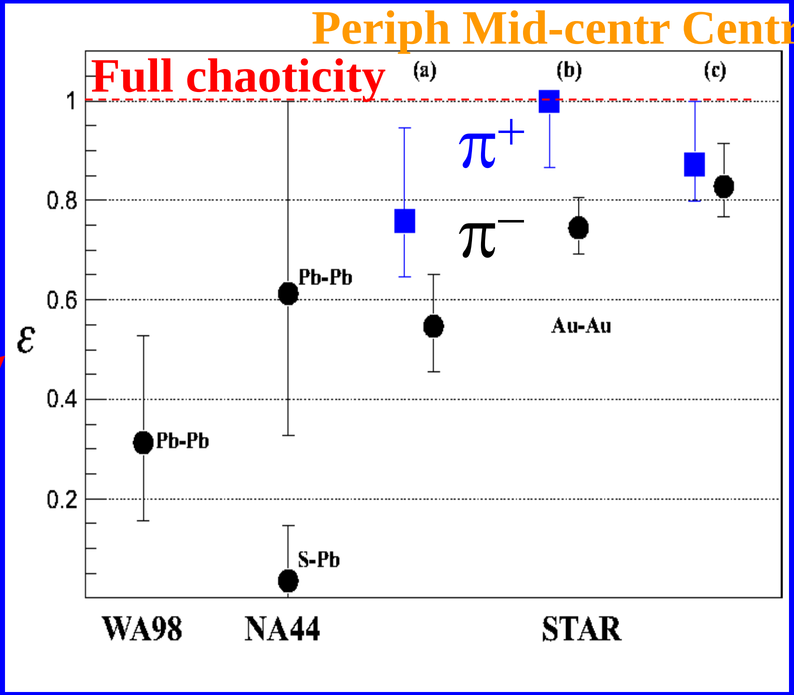
Construct ratio r_3 in which LLS contributions to $C_3 = CF_3 - 1$ and $C_2 = CF_2 - 1$ cancel out   Heinz-Zhang'97

$$r_3 = [C_3(123) - C_2(12) - C_2(23) - C_2(31)]/[C_2(12)\, C_2(23)\, C_2(31)]^{1/2}$$

Interpolate to $r_3(Q_3=0)$, $Q_3 = (Q_{12}^2 + Q_{23}^2 + Q_{31}^2)^{1/2}$

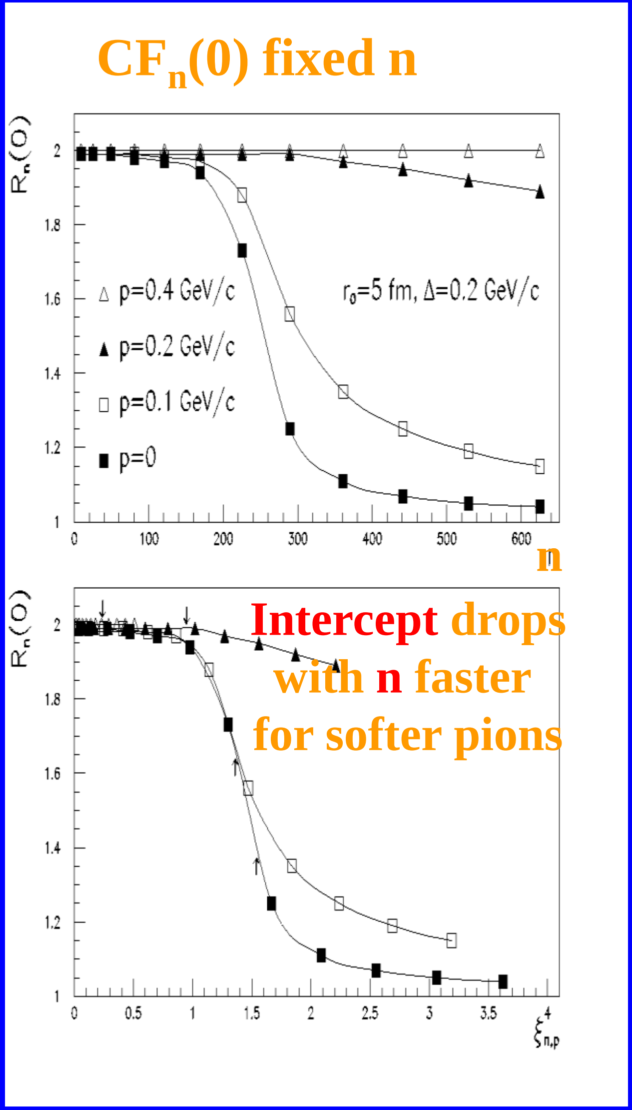


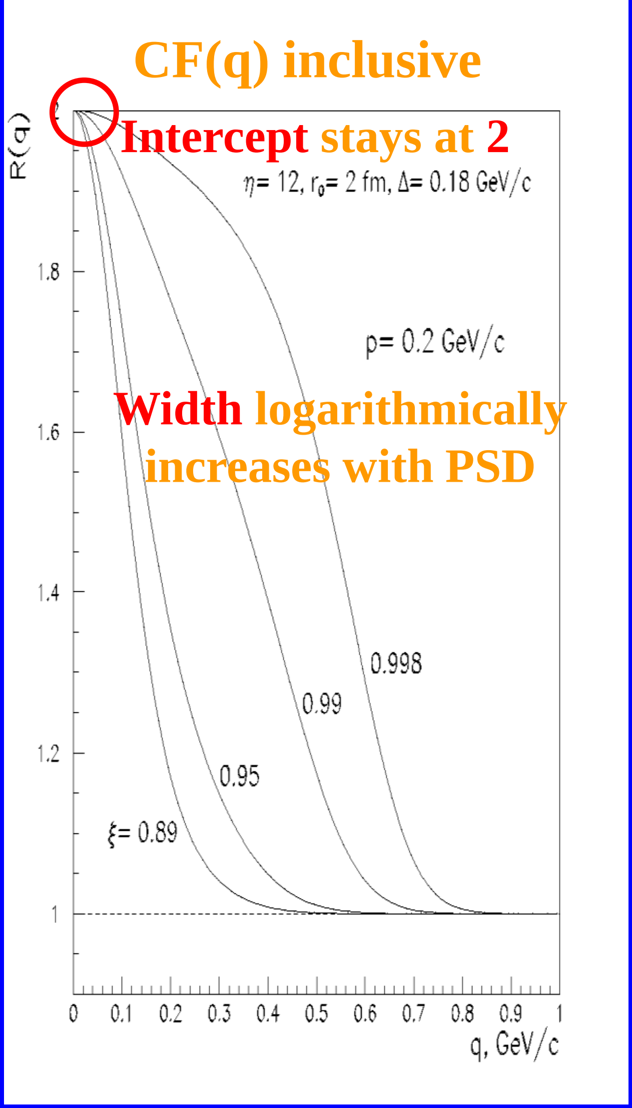
$$\tfrac{1}{2} r_3(0) = \varepsilon^{1/2}(3-2\varepsilon)/(2-\varepsilon)^{3/2}$$

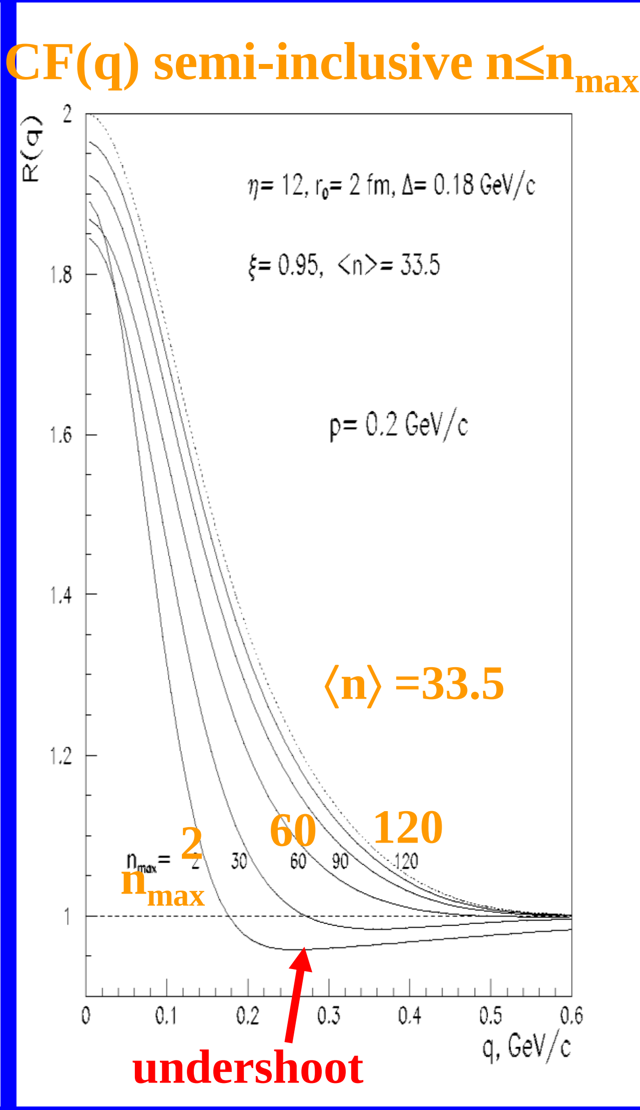


Within large (systematic) errors STAR data is consistent with full chaoticity

Multiboson effects on CFs

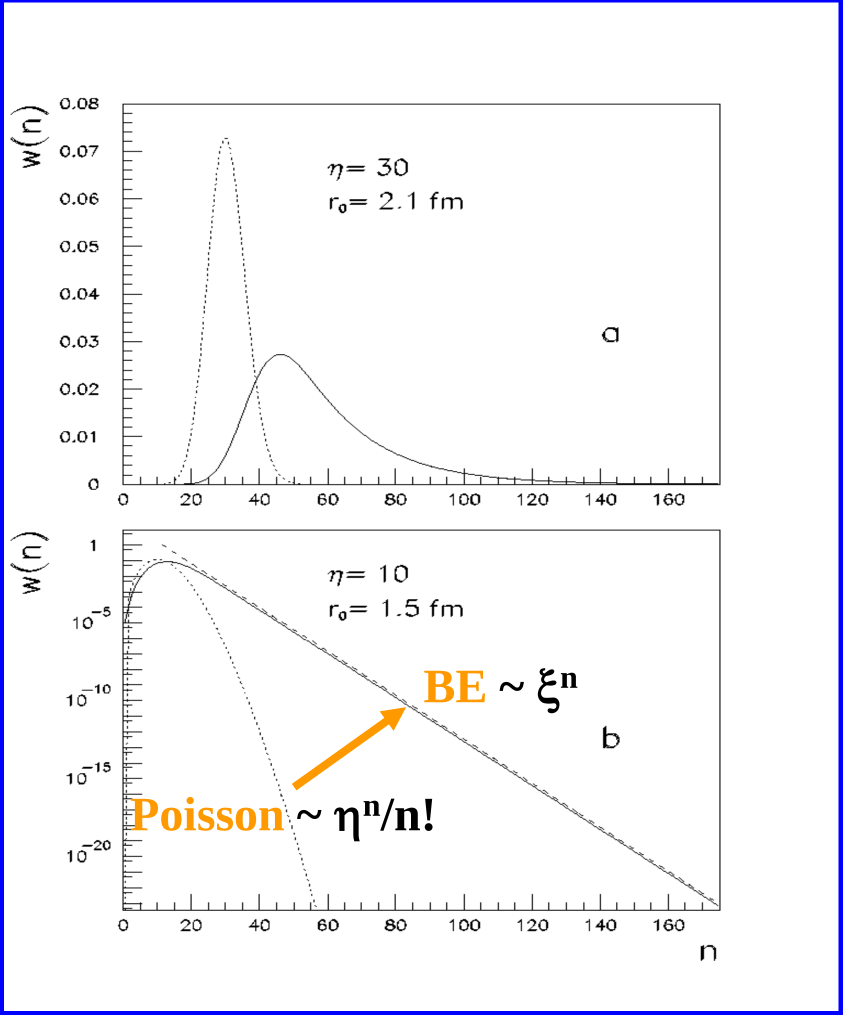


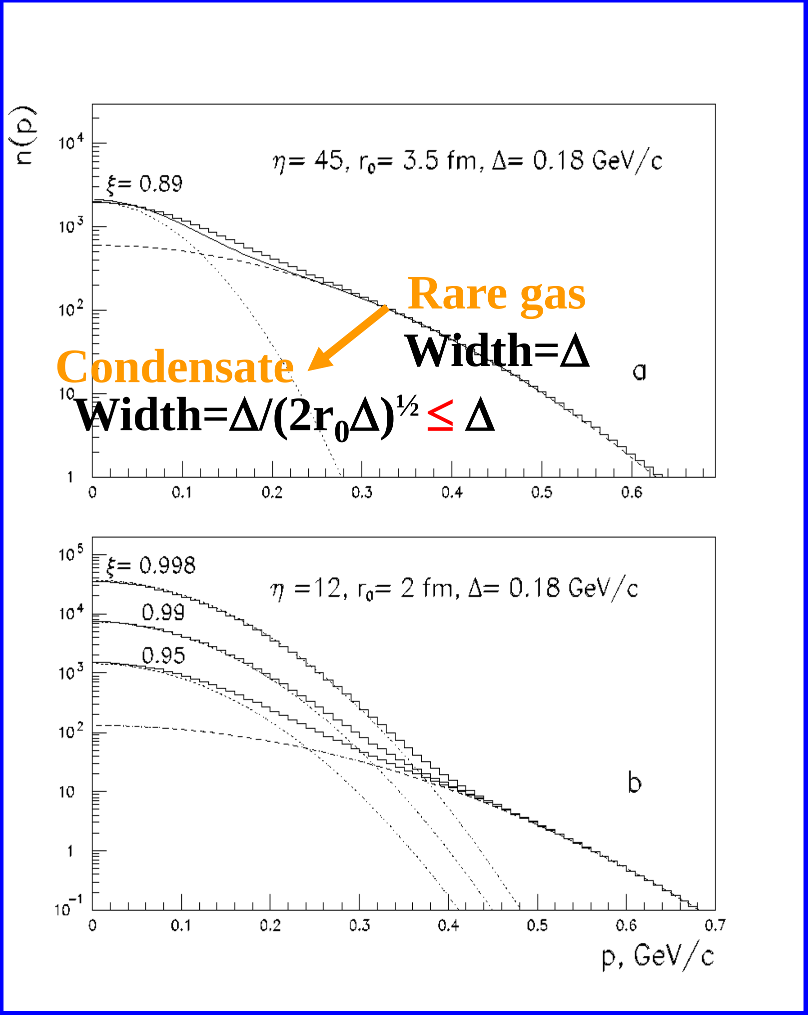




Multiboson effects on n_π & spectra

Measure of PSD: $\xi=\eta/(r_0\Delta+½)^3 \leq 1$





Longitudinal boost-invariant expansion

el. sources of lifetime τ produced at t=z=0 uniformly distr. in rapidity η and decaying according to thermal law $\exp(-E^*/T)$

$t = \tau \cosh(\eta) \quad z = \tau \sinh(\eta)$

$E = m_t \cosh(y) \quad p_z = m_t \sinh(y) \quad E^* = m_t \cosh(y - \eta)$

In LCMS: pair rapidity y=0 so

$$G \sim \exp(-E^*/T) = \exp(-m_t \cosh\eta / T) \approx \exp(-m_t/T) \exp[-\eta^2 / 2(T/m_t)]$$

$$\Rightarrow \quad \langle \eta^2 \rangle \approx (T/m_t)$$

$$R_z^2 = \langle (z - \langle z \rangle)^2 \rangle \equiv \langle z'^2 \rangle \quad R_y^2 = \langle y'^2 \rangle \quad R_x^2 = \langle (x' - v_x t')^2 \rangle$$

$$R_z^2 = \langle (\tau \sinh(\eta))^2 \rangle = \langle \tau^2 \rangle \langle (\sinh(\eta))^2 \rangle \approx \langle \tau^2 \rangle (T/m_t)$$

$$R_x^2 = \langle x'^2 \rangle - 2v_x \langle x't' \rangle + v_x^2 \langle t'^2 \rangle \qquad \langle t'^2 \rangle \approx \langle (\tau - \langle \tau \rangle)^2 \rangle \equiv (\Delta\tau)^2$$

$R_z \rightarrow \langle \tau \rangle$ = evolution time **$R_x \rightarrow \Delta\tau$ = emission duration if $\langle x't' \rangle = 0$ & $\langle x'^2 \rangle = \langle y'^2 \rangle$**

Transverse expansion

Thermal law & gaussian tr. density profile $\exp(-r^2/ 2r_0^2)$

& linear tr. flow velocity profile $\beta^F(r) = \beta_0^F r / r_0$

Nonrelativistic case: $\beta_t^{T2} = \beta^{F2} + \beta_t^2 - 2\,\beta^F \beta_t \cos\varphi$

$\vec{\beta}_t = \vec{\beta}^F + \vec{\beta}_t^T$ $\quad x = r\cos\varphi$ (**out**) $\quad y = r\sin\varphi$ (**side**)

β_t = **tr. velocity** $\quad \beta_t^T$ = **tr. thermal velocity**

$G \sim \exp(-\beta_t^{T2} m_t / 2T) \exp(-r^2/ 2r_0^2)$

$= \exp[- (\beta_0^{F2} r^2/r_0^2 + \beta_t^2 - 2\,\beta_0^F \beta_t\, x/r_0)\, m_t / 2T - r^2/ 2r_0^2]$

$\Rightarrow$ $\langle y\rangle = 0 \quad \langle x\rangle = r_0\, \beta_t\, \beta_0^F / [\beta_0^{F2} + T/m_t]$

$R_y^2 = \langle y'^2\rangle = \langle x'^2\rangle = r_0^2 / [1 + \beta_0^{F2}\, m_t /T]$

$x' = x - \langle x\rangle$

Note: for a box-like profile $(r < R) \rightarrow \langle x'^2\rangle < \langle y'^2\rangle$

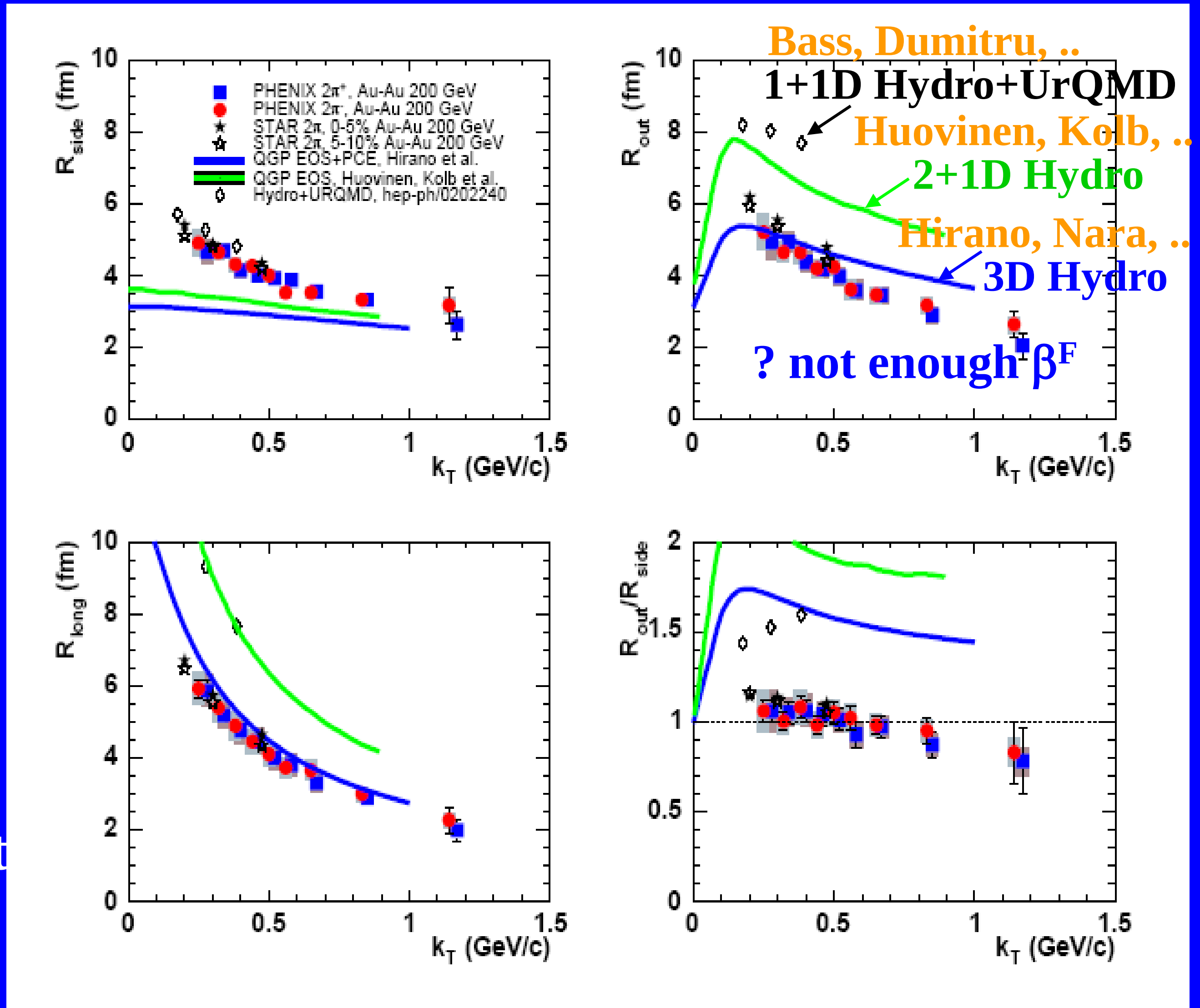
Femtoscopy Puzzle ?

Hydro assuming ideal fluid explains strong collective (π) flows at RHIC but not the femtoscopy results

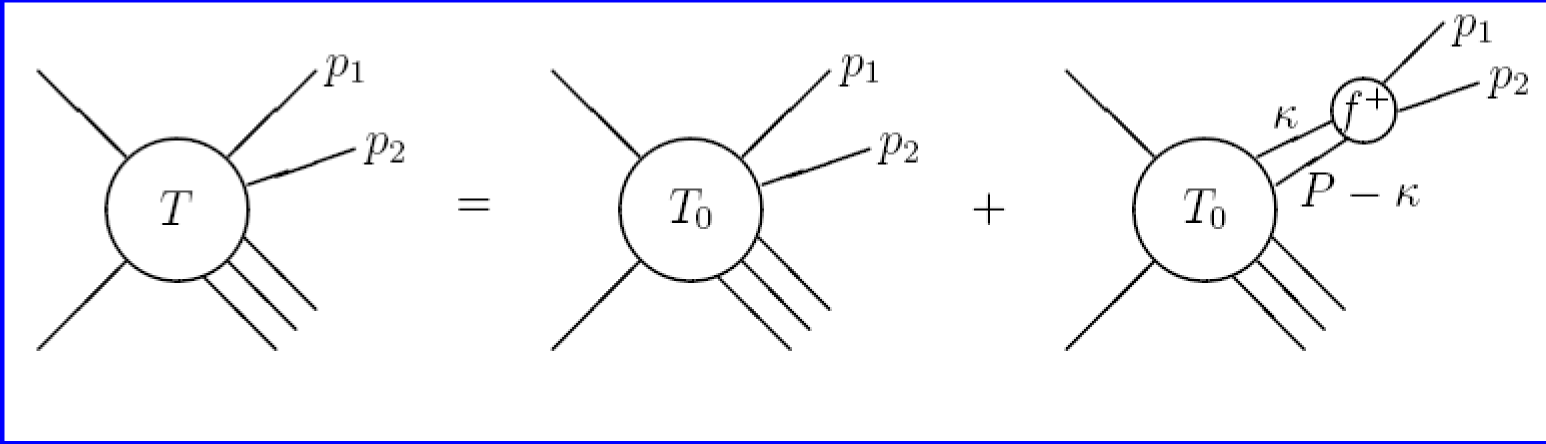
But comparing 1+1D H+UrQMD with 2+1D Hydro

→ kinetic evolution ~ conserves R_{out},R_{long} & increases R_{side} at small p_t (resonances ?)

⇒ Good prospect for 3D Hydro + hadron transport + ? initial β^F



BS-amplitude Ψ



$$T(p_1, p_2; \alpha) = T_0(p_1, p_2; \alpha) + \Delta T(p_1, p_2; \alpha)$$

$$\Delta T(p_1, p_2; \alpha) = \frac{\mathrm{i}\sqrt{P^2}}{2\pi^3} \int \mathrm{d}^4\kappa \frac{T_0(\kappa, P-\kappa; \alpha) f^{S*}(p_1, p_2; \kappa, P-\kappa)}{(\kappa^2 - {m_1}^2 - \mathrm{i}0)[(P-\kappa)^2 - {m_2}^2 - \mathrm{i}0]}$$

Inserting KP amplitude $T_0(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)\exp(-ip_1x_A - ip_2x_B)$ in ΔT and taking the amplitudes $u_A(\kappa)$ and $u_B(P-\kappa)$ out of the integral at $\kappa \approx p_1$ and $P-\kappa \approx p_2$ (again "smoothness assumption") ⇒

Plane waves $\exp(-ip_1x_A - ip_2x_B)$ → BS-amplitude $\Psi_{p_1p_2}(x_A, x_B)$:

$$T(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)\,\Psi_{p_1p_2}(x_A, x_B)$$

Note

- Formally (FSI) correlations in beta decay and multiparticle production are determined by the same (Fermi) function $\langle|\psi_{-\mathbf{k}}(\mathbf{x})|^2\rangle$

- But it appears for different reasons in
beta decay: a weak **r**-dependence of $\psi_{-\mathbf{k}}(\mathbf{r})$ within the nucleus volume + point like emission + equal times and in
multiparticle production in usual events of HIC:
a small space-time extent of the emitters compared to their separation + sufficiently small phase space density + usually a small effect of nonequal times

Coalescence: deuterons ..

WF in **continuous** **pn** spectrum $\psi_{-k^*}(r^*) \rightarrow$ WF in **discrete** **pn** spectrum $\psi_b(r^*)$

$$E_d d^3N/d^3p_d = B_2\, E_p d^3N/d^3p_p\, E_n d^3N/d^3p_n \qquad p_p \approx p_n \approx \tfrac{1}{2} p_d$$

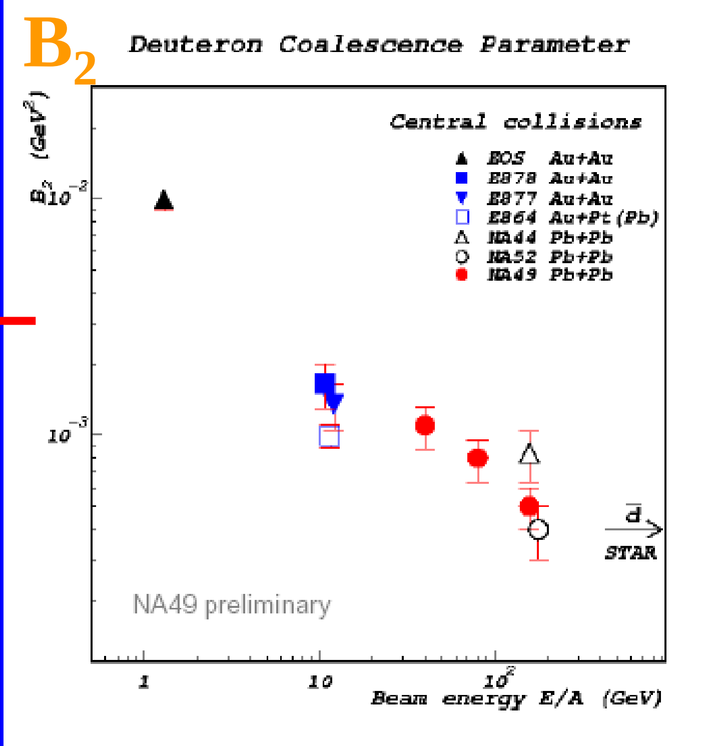
Coalescence factor: $B_2 = (2\pi)^3 (m_p m_n/m_d)^{-1} \rho_t \langle |\psi_b(r^*)|^2 \rangle \sim R^{-3}$

Lyuboshitz (88) ..

Triplet fraction = ¾ ↲ unpolarized Ns

Usually: **n** → **p**

Much **stronger energy dependence** of $B_2 \sim R^{-3}$ than expected from pion and proton interferometry radii

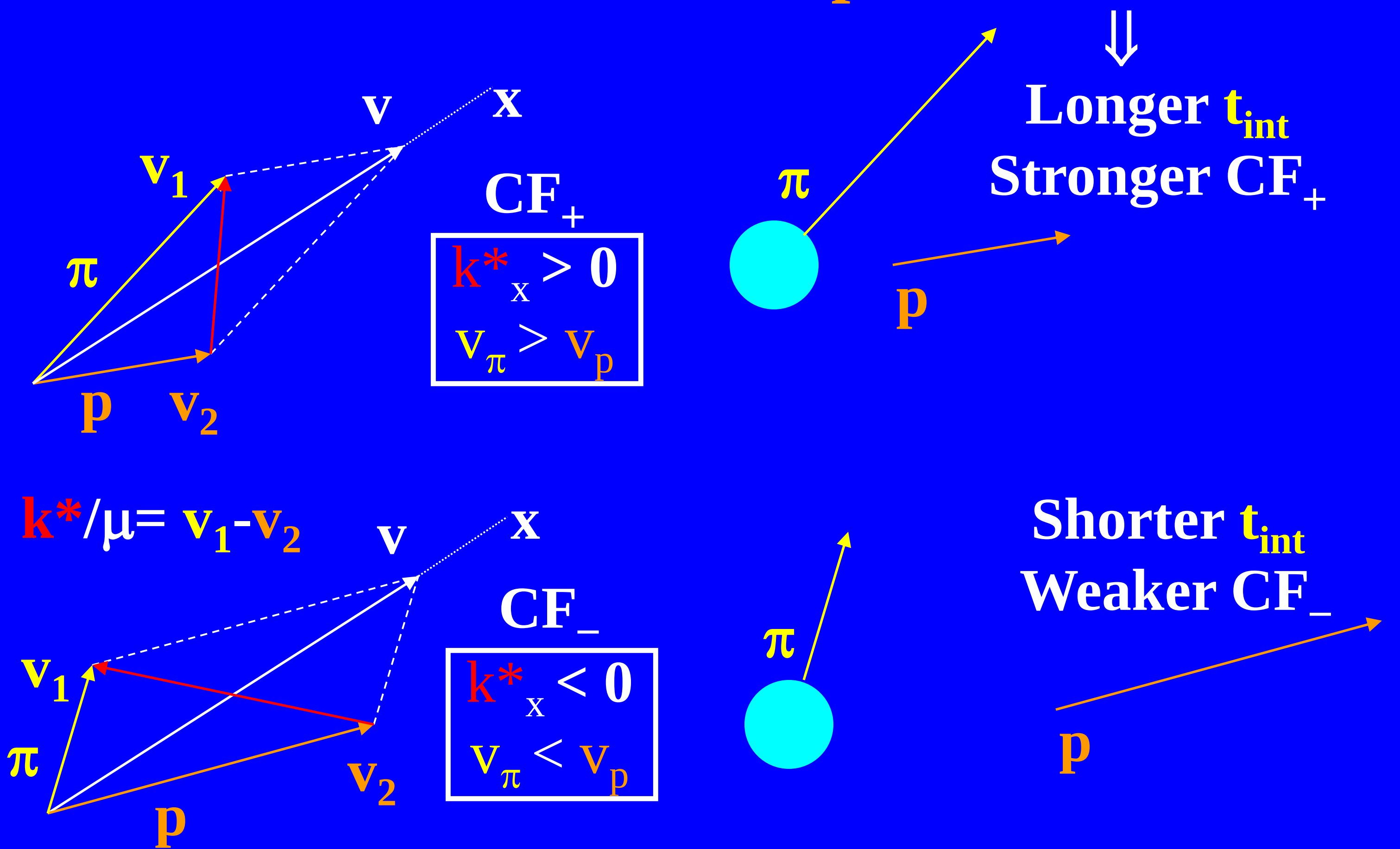


R(pp) ~ 4 fm from AGS to SPS

Simplified idea of CF asymmetry

(valid for Coulomb FSI)

Assume π emitted later than p or closer to the center

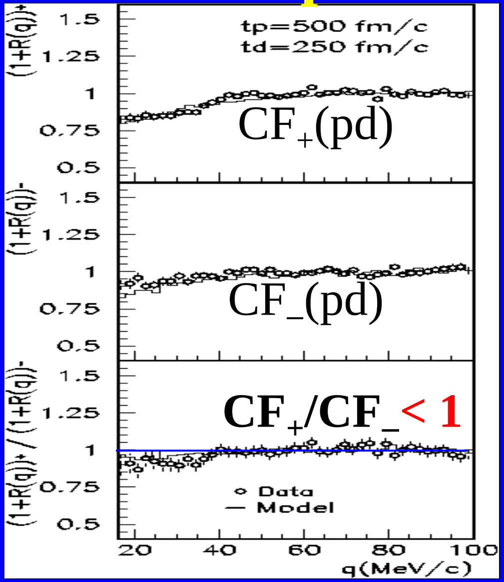


Large lifetimes evaporation or phase transition

x || v |Δx| << |Δt| → CF-asymmetry yields time delay

Ghisalberti (95) **GANIL**

Pb+Nb → **p+d**+X



Deuterons earlier than protons in agreement with coalescence

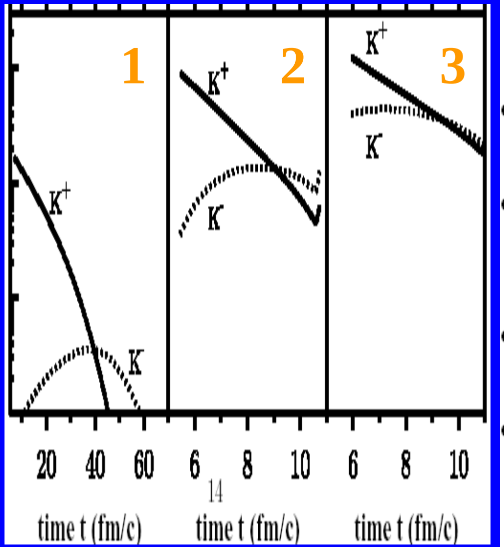
$e^{-tp/\tau}\, e^{-tn/\tau} \approx e^{-td/(\tau/2)}$ since $t_p \approx t_n \approx t_d$

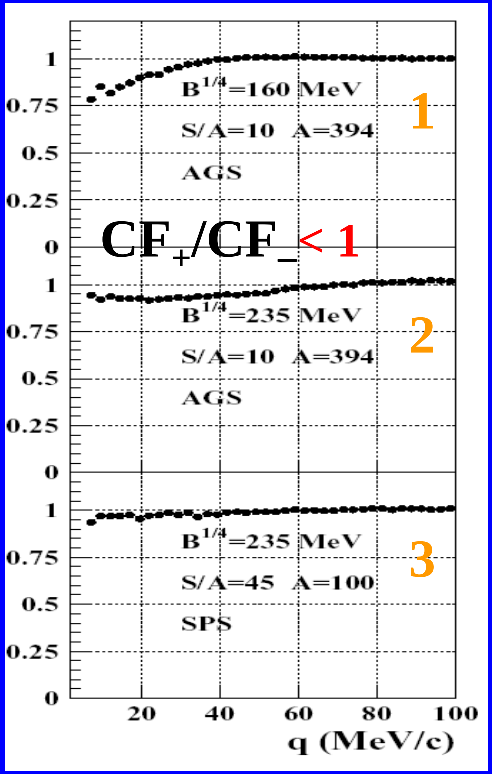
Strangeness distillation:

K+ earlier than **K−** in baryon rich QGP

Two-phase thermodynamic model

Ardouin et al. (99)





Usually: $\langle \Delta x \rangle$ and $\langle \Delta t \rangle$ comparable

RQMD Pb+Pb $\rightarrow$ **πp** +X central 158 AGeV : $\langle \Delta x \rangle$ = **-5.2** fm

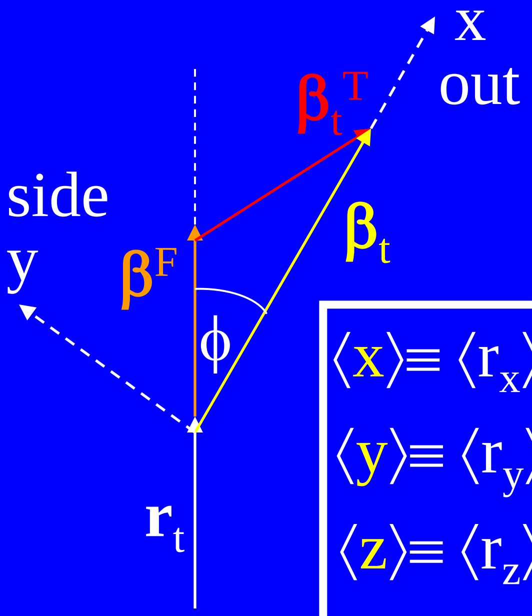
$\langle \Delta t \rangle$ = **2.9** fm/c

π^+p-asymmetry effect $2\langle \Delta x^* \rangle / a \approx$ **-8%** $\leftarrow \langle \Delta x^* \rangle$ = **-8.5** fm

Shift $\langle \Delta x \rangle$ in out direction is due to collective transverse flow & higher thermal velocity of lighter particles

$\langle x_p \rangle > \langle x_K \rangle > \langle x_\pi \rangle > 0$

RL'99-01



β^F = flow velocity β_t^T = transverse thermal velocity

$\beta_t = \beta^F + \beta_t^T$ = observed transverse velocity

$\langle x \rangle \equiv \langle r_x \rangle = \langle r_t \cos\phi \rangle = \langle r_t \, (\beta_t^2 + \beta^{F2} - \beta_t^{T2})/(2\beta_t \beta^F) \rangle$

mass dependence

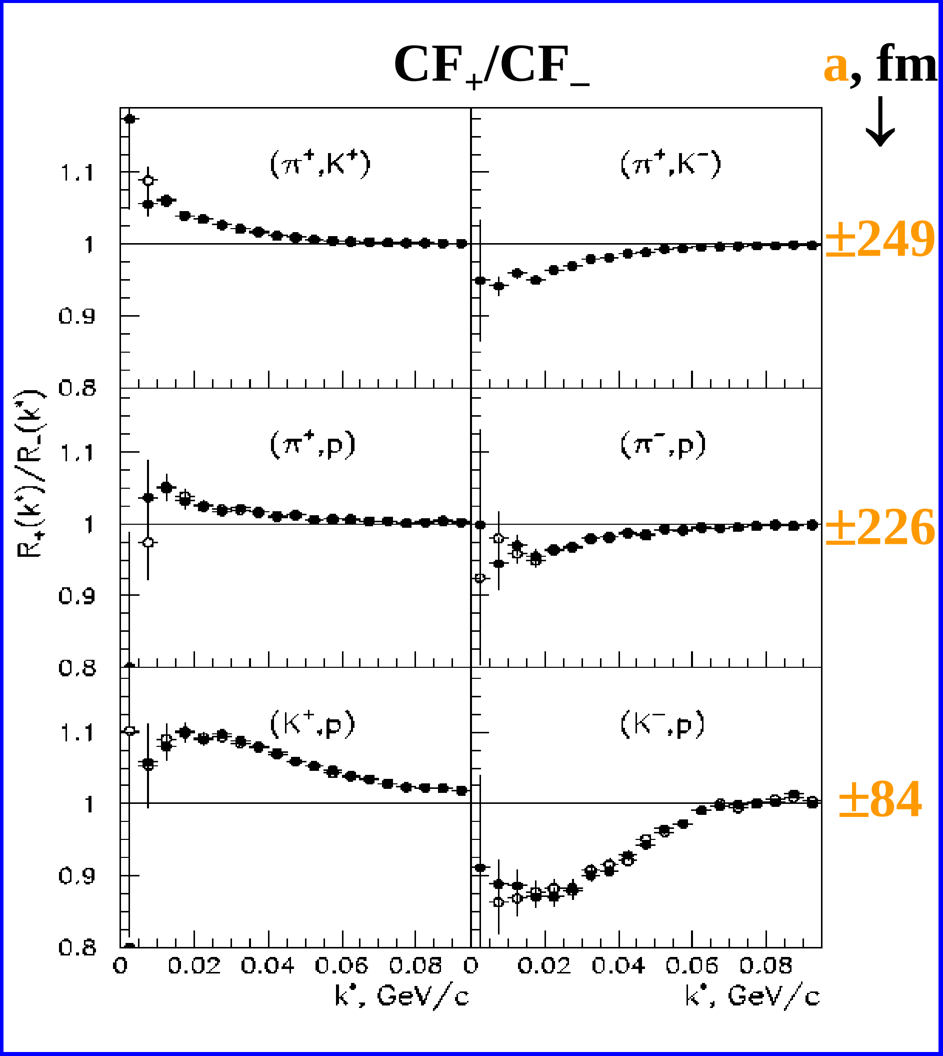
$\langle y \rangle \equiv \langle r_y \rangle = \langle r_t \sin\phi \rangle = 0$

$\langle z \rangle \equiv \langle r_z \rangle \rightarrow \langle \tau \sinh\eta \rangle = 0$ in LCMS & Bjorken long. exp.

measures edge effect at $y_{CMS} \neq 0$

ad hoc time shift $\Delta t = -10$ fm/c

Sensitivity test for ALICE Erazmus et al. (95)



$$CF_+/CF_- \xrightarrow{k^* \to 0} 1+2\,\langle \Delta x^* \rangle / a$$

Here $\langle \Delta x^* \rangle = -\langle \gamma v \Delta t \rangle$

↓

CF-asymmetry scales as

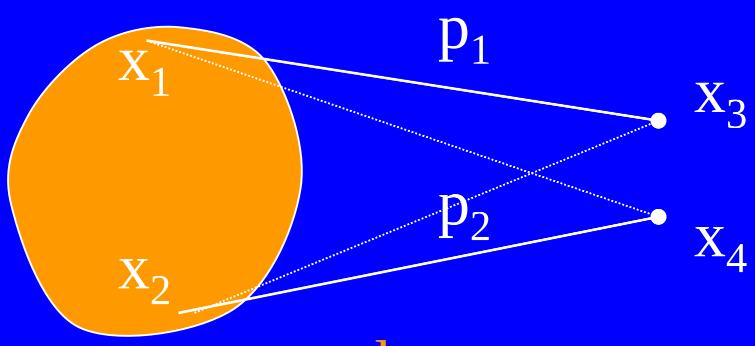
$-\langle \Delta t \rangle / a$

Delays of several fm/c can be easily detected

	collective flow	chaotic source motion
x^2-p correlation	yes	yes
$T_{eff} \uparrow$ with m	yes	yes
$R \downarrow$ with m_t	yes	yes
x-p correlation	yes	no
$\langle \Delta x \rangle \neq 0$	yes	no
CF asymmetry	yes	yes if $\langle \Delta t \rangle \neq 0$

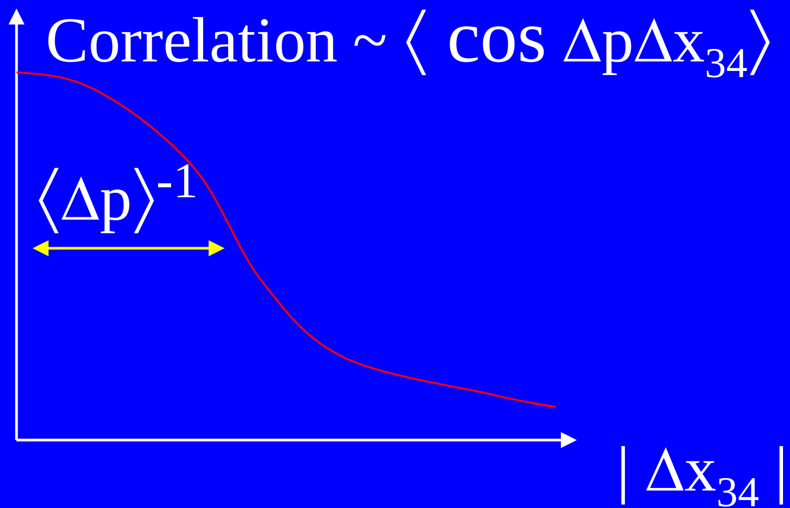
Intensity interferometry of classical electromagnetic fields in Astronomy HBT‘56 $\rightarrow$ product of single-detector currents

cf conceptual quanta measurement $\rightarrow$ two-photon counts



star

detectors-antennas tuned to mean frequency $\langle\omega\rangle$



Space-time correlation measurement in Astronomy $\rightarrow$

source **momentum picture** $\langle\Delta p\rangle=\langle\omega\Delta\theta\rangle \rightarrow$ star angular radius $\langle\Delta\theta\rangle$

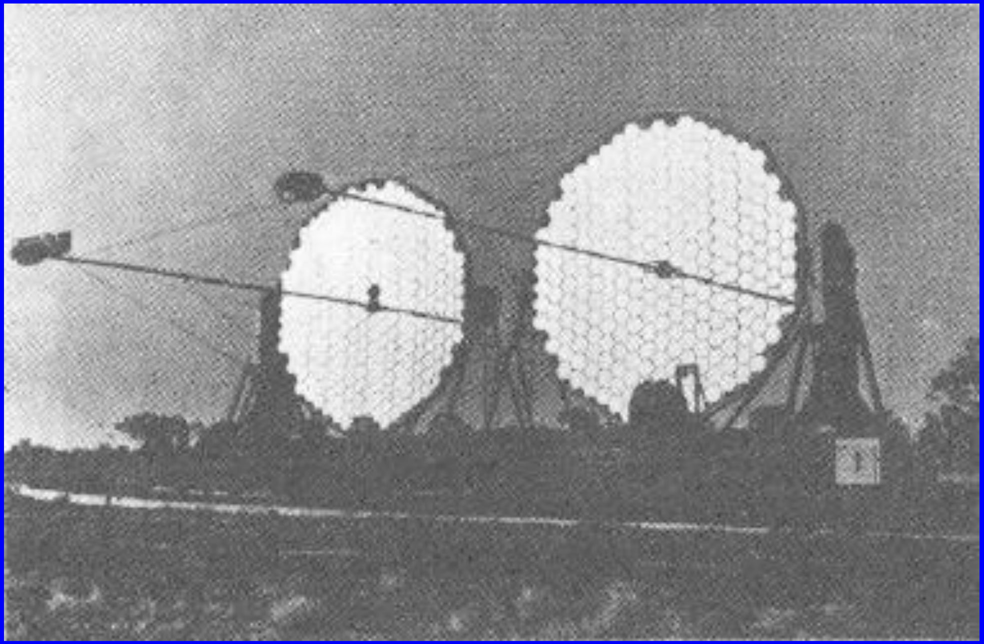
$\rightarrow$ **no info on star R,τ**

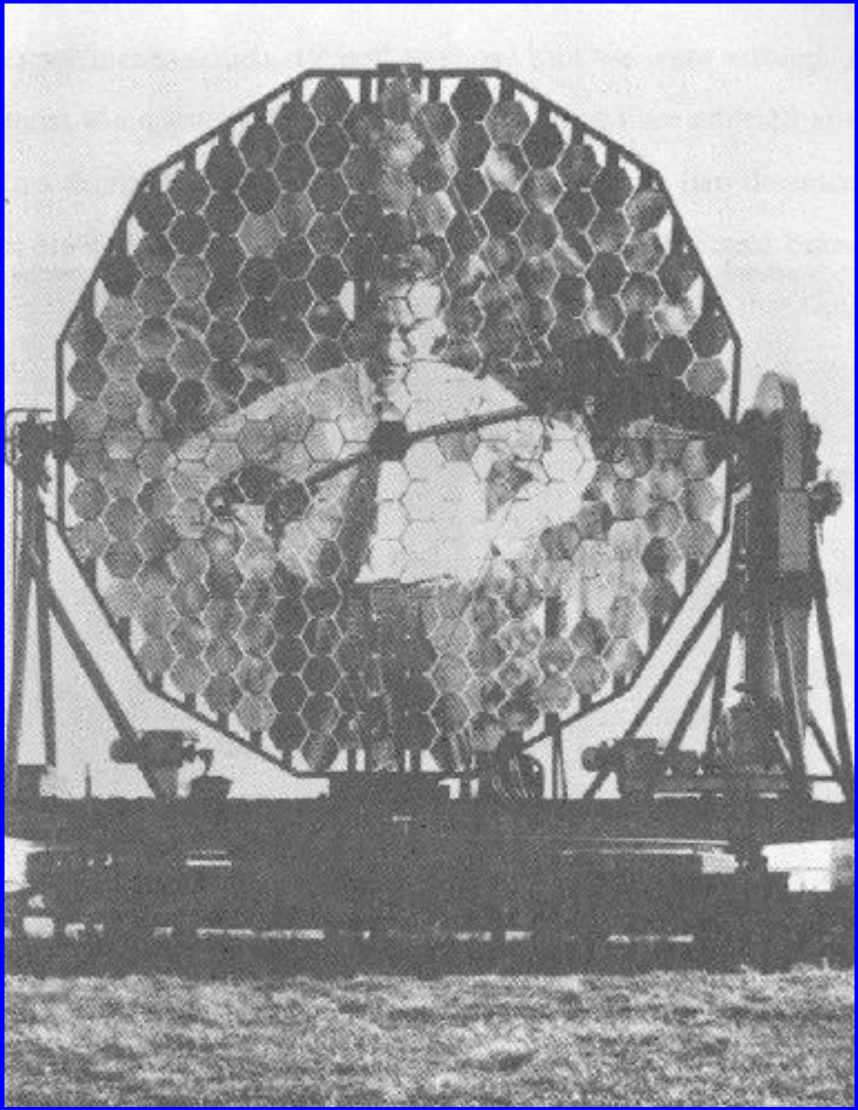
KP’75 orthogonal to

momentum correlation measurement in particle physics $\rightarrow$

source **space-time picture** $\langle\Delta x\rangle$

HBT paraboloid mirrors focusing the light from a star on photomultipliers





→ **momentum correlation** (GGLP, KP) measurements are impossible in Astronomy due to extremely large stellar space-time dimensions

while

→ **space-time correlation** (HBT) measurements can be realized also in Laboratory:

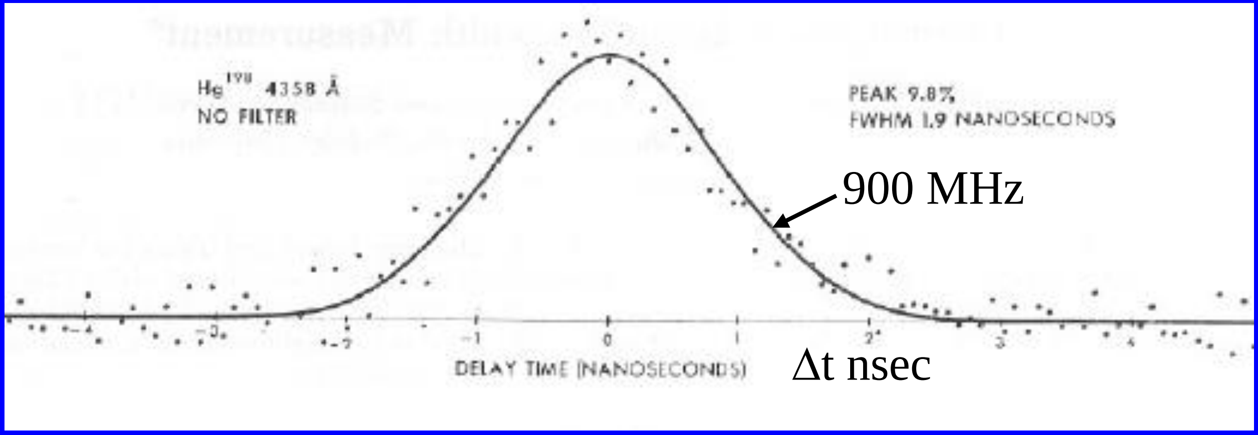
Intensity-**correlation spectroscopy** Goldberger,Lewis,Watson’63-66

Measuring phase of x-ray scattering amplitude Fetter’65

& spectral line shape and width Glauber’65

Phillips, Kleiman, Davis’67:

linewidth measurement from a mercurury discharge lamp



Formalism of independent one-particle sources

$\langle x|\psi_A\rangle = (2\pi)^{-4} \int d^4\kappa\, u_A(\kappa) \exp[-i\, \kappa(x-x_A)]$

$\langle \kappa|x\rangle = \exp(i\, \kappa x)$

$\langle \kappa|\psi_A\rangle = \int d^4x\, \langle \kappa|x\rangle\langle x|\psi_A\rangle = u_A(\kappa) \exp(i\, \kappa x_A)$

Momentum (femtoscopic) correlations:

$\mathrm{Ampl}(p) = \langle p|\psi_A\rangle = u_A(p) \exp(i\, p x_A)$

$\mathrm{Ampl}(p_1,p_2) = 2^{-1/2}\, [u_A(p_1)u_A(p_2) \exp(i\, p_1x_A + i\, p_2x_B) + 1 \longleftrightarrow 2]$

$\mathrm{Corr}(p_1,p_2) = 2\mathrm{Re}\{\exp(i\, q\Delta x)\, u_A(p_1)u_B(p_2)u_A^*(p_2)u_B^*(p_1)\, \times$
$\times\, [|u_A(p_1)u_B(p_2)|^2 + |u_A(p_2)u_B(p_1)|^2]^{-1}\} \rightarrow \cos[(p_1-p_2)(x_A-x_B)]$

Space-time (spectroscopic) correlations:

$\mathrm{Ampl}(x) = \langle x|\psi_A\rangle \sim \exp[i\, p_A(x_A-x)]$ for ~ monochrom. source

$\mathrm{Ampl}(x_3,x_4) \sim \exp\{i\, p_A(x_A-x_3) + i\, p_B(x_B-x_4)] + 3 \longleftrightarrow 4\}$

$\mathrm{Corr}(x_3,x_4) \sim \cos[(p_A-p_B)(x_3-x_4)]$! No explicit dependence on x_A, x_B

Femtoscopy through Emission function $G(p,x)$

One particle:

$$E\, d^3N/d^3p = \sum_\alpha |T_\alpha(p)|^2 = \int d^4x\, d^4x'\, \exp[-i\, p(x-x')]\, \sum_\alpha T_\alpha(x) T_\alpha^*(x')$$

$$= \int d^4x\, G(p,x) \qquad x,x' \rightarrow x = ½(x+x'),\ \varepsilon = x - x'$$

$G(p,x)$ = partial Fourier transform of space-time density matrix $\sum_\alpha T_\alpha(x) T_\alpha^*(x')$

Two id. pions:

$$E_1 E_2 d^6N/d^3p_1 d^3p_2 = \int d^4x_1 d^4x_2\, [G(p_1,x_1;p_2,x_2) + G(p,x_1;p,x_2)\cos(q\Delta x)]$$

$$p = ½(p_1+p_2) \quad q = p_1 - p_2 \quad \Delta x = x_1 - x_2$$

$$Corr(p_1,p_2) = \int d^4x_1 d^4x_2 G(p,x_1;p,x_2)\cos(q\Delta x) / \int d^4x_1 d^4x_2 G(p_1,x_1;p_2,x_2)$$

$$\approx \langle \cos(q\Delta x) \rangle \rightarrow \exp(-\textstyle\sum_i R_i^2 q_i^2 - \tau^2 q_0^2)$$

if $G(p_1,x_1;p_2,x_2) = G(p_1,x_1)G(p_2,x_2)$ $\quad G(p,x) \sim \exp(-\sum_i x_i^2/2R_i^2 - x_0^2/2\tau^2)$

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