Complete one-loop electroweak corrections to polarized $e + e-$ scattering in SANC project

Yahor Dydyshka

on behalf of the SANC group

INP BSU/JINR

LHC days in Belarus, 26.02.2018

Outline

- Future e^+e^- colliders: linear vs. circular
- Polarized beams at future collider
- SANC branch for processes with polarized e^+e^- beams
- Polarized Bhabha scattering ($e^+e^- \to e^+e^-$) at NLO EW
- Preliminary results for polarized ($e^+e^- \to \mu^+\mu^-$) at NLO EW
- Numerical results and cross-checks
- Conclusion and plans

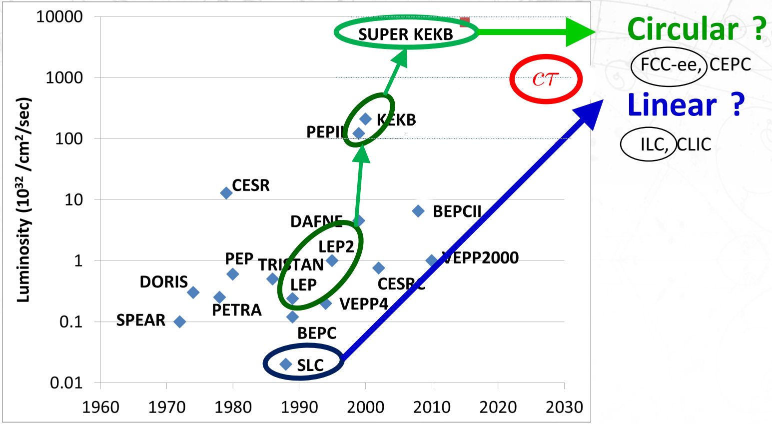
Future lepton collider projects

Mid-term perspectives (2030-2050):
The quest for precision: Linear or Circular

FCC (100 km)
First step: FCC-ee (88-400 GeV)
[Use the tunnel ultimately aimed at FCC-hh]

Future lepton collider projects

- Historically, e^+e^- colliders have been used for precision measurements
  - The accuracy of e^+e^- colliders led to predictions at higher scales (m_{top}, m_H, limits on NP)
    - And to [unexpected] discoveries (e.g., c quark, gluon, tau lepton, neutrino tau …)



The project of the Super Charm-Tau factory

Institute Nuclear Physics G.I. Budker of the SB RAS (Novosibirsk)

Installation colliding electron-positron beams will work in the region of total energies from 2 to 5 GeV with unprecedented high luminosity $10^{35} cm^{-2} c^{-1}$ and the longitudinal polarization of the electrons.

The main goal of the experiments at the Super Charm-Tau factory is to study the processes of birth-charmed quarks and tau leptons, using a data set that is 2 orders of magnitude more in volume than the one typed in the experiment BESIII.

Future lepton collider projects

Linear collider (e+e-)

- ILC; CLIC
- ILC: technology at hand, realization in Japan??

E_{cm}

- 250GeV – 1TeV, 91GeV (ILC)
- 500GeV – 3TeV (CLIC)

L ≈ $2\times10^{34}cm^{-2}s^{-1}$ (~$500fb^{-1}$/year)

→ Stat. uncertainty ~ $10^{-3}…10^{-2}$

Beam polarization

e- beam P = 80-90%

e+ beam

ILC: P = 30% baseline;

60% upgrade

CLIC: P ≥ 60% upgrade

Circular collider

- FCC-ee, TLEP
- CEPC

Projects under study

μ Collider

E_{cm}

91 GeV, 160GeV, 240GeV, 350GeV

L ≈ $10^{36}cm^{-2}s^{-1}$ (4 experiments)

→ Stat. uncertainty ≤10^{-3}

Beam polarization

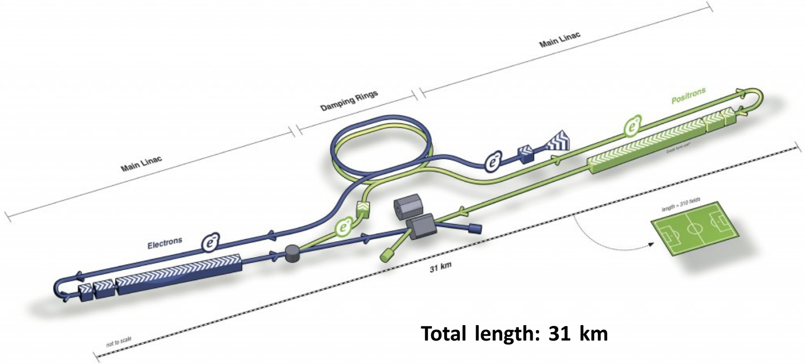
- Desired (?)

Future lepton collider projects

There are several proposals for future high-luminosity e^+e^- colliders, which are expected to measure electroweak precision observables, in particular Z-pole observables and the W mass, to significantly higher precision. The first proposal, the International Linear Collider (ILC), is planned to be a linear e^+e^- machine with adjustable center-of-mass energy in the range $\sqrt{s} \sim 90...500$ GeV, extendible to 1 TeV. It can accommodate polarized e^- and e^+ beams and is expected to collect more than 50 fb^{-1} of data near the Z pole and 100 fb^{-1} near the WW production threshold. An alternative proposal, the Future Circular Collider (FCC-ee), is based on a 80–100 km circumference accelerator ring with $\sqrt{s} \sim 90...350$ GeV. It has the potential to generate several ab^{-1} of data near the Z pole and a comparable amount at the WW threshold. Finally, there is the Circular Electron-Positron Collider (CEPC) proposal, which is also a ring collider with 50–70 km circumference and $\sqrt{s} \sim 90...250$ GeV. Its target luminosities are 150 fb^{-1} at the Z pole and 100 fb^{-1} near the WW threshold.

International Linear Collider

- For 20 years, there was only one such project on the market
  - A 500 GeV e^+e^- linear collider, now called "ILC", proposed in the early 1990's

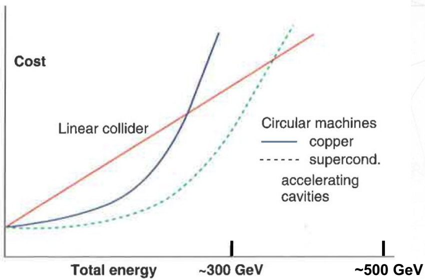


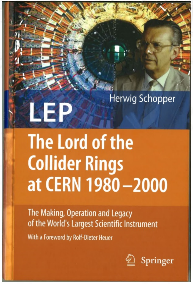
Total length: 31 km

- Why not a 500 GeV circular collider ?

Linear vs. Circular

- **Why not a 500 GeV circular collider ?**
  - **Synchrotron radiation in circular machines**
    - **Energy lost per turn grows like** $\Delta E \propto \frac{1}{R}\left(\frac{E}{m}\right)^4$ **, e.g., 3.5 GeV/turn at LEP2**
      - **Must compensate with R and accelerating cavities ⟶ Cost grows like E^4 too**





Author: Herwig Schopper (Former CERN DG)

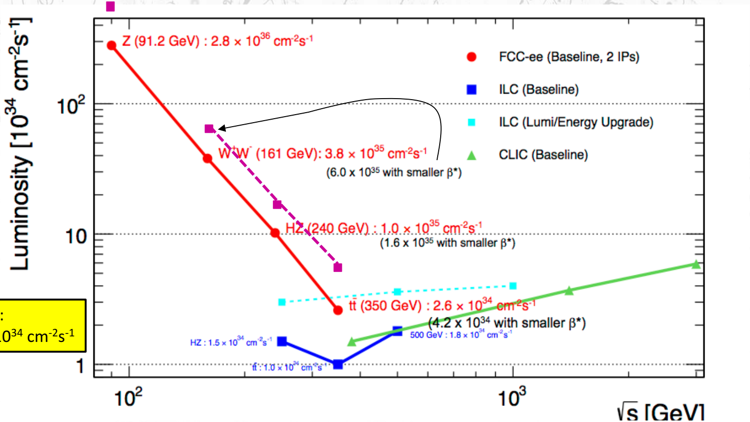
- **A 500+ GeV e^+e^- collider can only be linear. Cost of a circular collider is prohibitive**
  - **"Up to a centre of mass energy of 350 GeV at least, a circular collider with superconducting accelerating cavities is the cheapest option"**

H. Schopper, 2014

Linear vs. Circular

- **Performance target for e^+e^- colliders**

D. Schulte



LEP@Z-pole:
$L = 0.01 \times 10^{34}\ cm^{-2}s^{-1}$

- **Complementarity**
  - **Ultimate precision measurements with circular colliders (FCC-ee)**
  - **Ultimate e^+e^- energies with linear colliders (CLIC)**

Linear vs. Circular

- Performance target for e^+e^- colliders
  - Number of events per year for the FCC-ee

√s (GeV)	90 (Z)	160 (WW)	240 (HZ)	350 (tt)	350+ (WW→H)
Lumi (ab^{-1}/yr)	30	4	1	0.3	0.3
Events/year	1.5×10^{12}	1.5×10^{7}	2.0×10^{5}	2.0×10^{5}	2.0×10^{4}
# years	6	2	5	5	
Events@FCCee	10^{13}	3×10^{7}	10^{6}	10^{6}	10^{5}

- Total running time of ~18 years (~10 years with recent more optimistic lumi numbers)

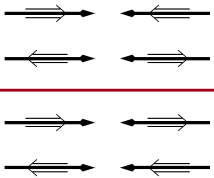
  - Scenario for the ILC precision physics programme (first 10-15 years)
    - with ±80% / ±30% polarization for e^-/e^+ beams

# years	3 ? (*)	3 ? (*)	3	1	4
Tot lumi (ab^{-1})	0.1	0.5	0.5	0.2	0.5
Events@ILC	3×10^{9} (*)	2×10^{6} (*)	1.4×10^{5}	10^{5}	3.5×10^{4}

(*) No design available at the Z pole and the WW threshold: non-trivial to achieve with a linear collider

Beam polarization

Consider s-channel processes (ee→ff)



		e^- e^+	Contibution due to polarization
$J_Z = 0$	σ_{RR}		$\frac{1+P_{e^-}}{2}\frac{1+P_{e^+}}{2}$
	σ_{LL}		$\frac{1-P_{e^-}}{2}\frac{1-P_{e^+}}{2}$
$J_Z = 1$	σ_{RL}		$\frac{1+P_{e^-}}{2}\frac{1-P_{e^+}}{2}$
	σ_{LR}		$\frac{1-P_{e^-}}{2}\frac{1+P_{e^+}}{2}$

P_{e-} = -1:
100% left-polarized e-
P_{e+} = -1:
100% right-polarized e+

$$\sigma_{ij}^{meas} = \sigma_0 (1 - P_{e-} P_{e+})(1 + A_{LR} P_{eff})$$

$$P_{eff} = \frac{P_{e-} - P_{e+}}{1 - P_{e-} P_{e+}}$$

σ_0 - unpolarized cross section

- Measurement with equal number of (+ -) and (- +) helicity pattern only increases statistics if both beams are polarized

→ Enhancement of effective luminosity with e+ polarization:

$L_{eff} = (1-P_{e+}P_{e-})$ → for (P_{e+};P_{e-}) = (∓80%; ±60%): L is factor ~1.5 higher

A_{LR} measurement at Zpeak: Blondel Scheme

- Most sensitive to weak mixing angle: A_{LR}

$$A_{LR} = \frac{A_{LR}^{meas}}{P} = A_e = \frac{2 v_e a_e}{v_e^2 + a_e^2}$$ (independent of the final state) $$\frac{v_e}{a_e} = 1 - 2\sin^2\theta_{eff}^{lept}$$

- Perform 4 independent measurements with different helicity combinations

$$\sigma_{\pm\pm} = \tfrac{1}{4}\sigma_0\left[1 + P_{e^+}P_{e-} + A_{LR}\left(\pm P_{e^+} \pm P_{e-}\right)\right]$$ =0 (SM) if both beams 100% polarized

$$\sigma_{\mp\pm} = \tfrac{1}{4}\sigma_0\left[1 - P_{e^+}P_{e-} + A_{LR}\left(\mp P_{e^+} \pm P_{e-}\right)\right]$$

- determination of P_{e+} and P_{e-}, and A_{LR} simultaneously ($A_{LR}\neq 0$) (equal polarization for + and – helicity):

$$A_{LR} = \left[\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})}{(\sigma_{+-} + \sigma_{-+} + \sigma_{++} + \sigma_{--})} \cdot \frac{(-\sigma_{+-} + \sigma_{-+} - \sigma_{++} + \sigma_{--})}{(-\sigma_{+-} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}\right]^{1/2}$$

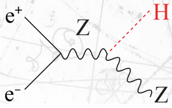
$$P_{e^\pm} = \left[\frac{(\sigma_{+-} + \sigma_{-+} - \sigma_{++} - \sigma_{--})}{(\sigma_{+-} + \sigma_{-+} + \sigma_{++} + \sigma_{--})} \cdot \frac{(\mp\sigma_{+-} \pm \sigma_{-+} - \sigma_{++} + \sigma_{--})}{(\mp\sigma_{+-} \pm \sigma_{-+} + \sigma_{++} - \sigma_{--})}\right]^{1/2}$$

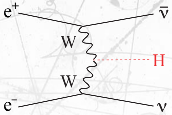
Polarisations P_{e+}, P_{e-} can be measured independently from polarimeters.

Precision Higgs physics at the FCC-ee

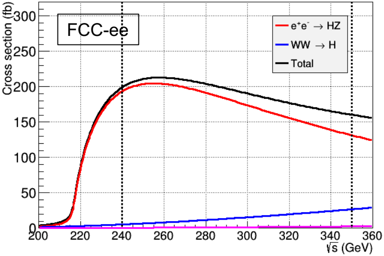
- **Dominant production processes for √s ≤ 500 GeV**

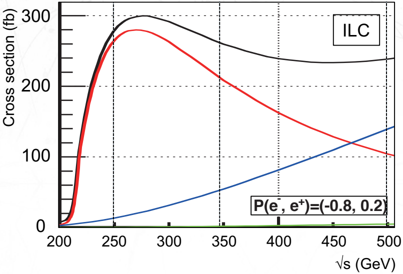
Higgs-strahlung





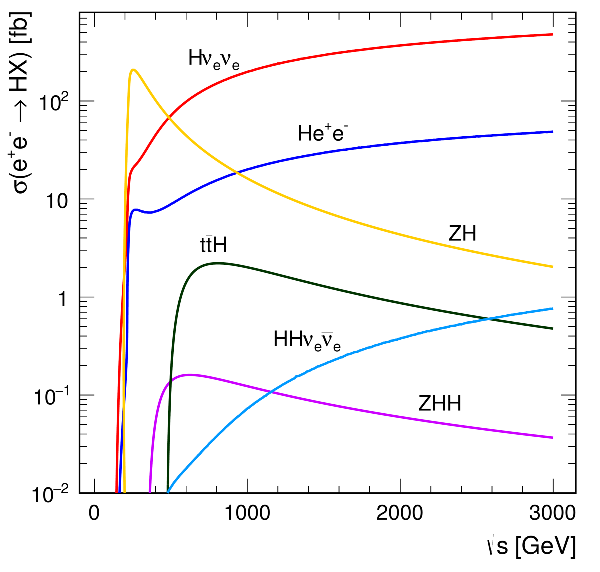
Boson fusion





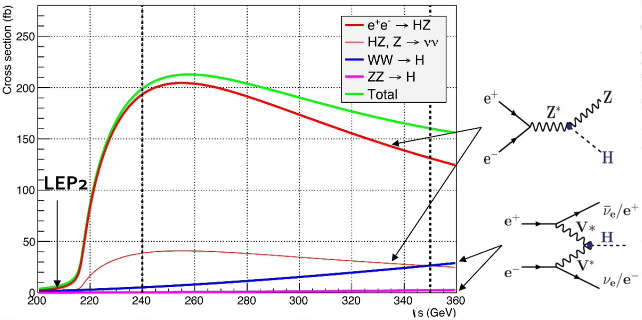
- **Effect of beam polarization (exercise)**
  - **Higgs-strahlung cross section multiplied by $1 - P_- P_+ - A_e \times (P_- - P_+)$**
  - **Boson fusion cross section multiplied by $(1 - P_-) \times (1 + P_+)$**

$e+e-$ Higgs factory



$e+e-$ Higgs factory

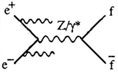
- Interest for circular collider projects grew up again after first LHC results
  - The Higgs boson is light – LEP2 almost made it: only moderate √s increase needed

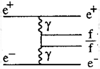


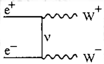
  - Need to go up to the top-pair threshold (350+ GeV) anyway to study the top quark
  - There seems to be no heavy new physics below 500 GeV
    - The interest of √s = 500 GeV (and even 1 TeV) is now very much debated
  - Way out: study with unprecedented precision the Z, W, H bosons and the top quark
    - Highest possible luminosities at 91, 160, 240 and 350 GeV are needed

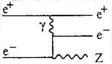
Precision Higgs physics at the FCC-ee

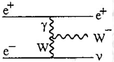
- **Physics backgrounds are "small"**
  - For example, at $\sqrt{s}$ = 240 GeV

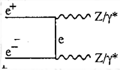


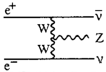










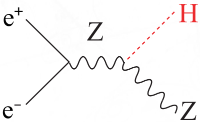


$e^+e^- \to q\bar{q}, l^+l^-$	$\gamma\gamma \to q\bar{q}, l^+l^-$, m > 30 GeV	$e^+e^- \to W^+W^-$	$e^+e^- \to Ze^+e^-$	$e^+e^- \to We\nu$	$e^+e^- \to ZZ$	$e^+e^- \to Z\nu\bar{\nu}$
60 pb	30 pb	16 pb	3.8 pb	1.4 pb	1.3 pb	32 fb

- "Blue" cross sections decrease like 1/s
- "Green" cross sections increase slowly with s

Add $e^+e^- \to t\bar{t}$ for $\sqrt{s}$ > 345 GeV — 0.6 pb

- To be compared to



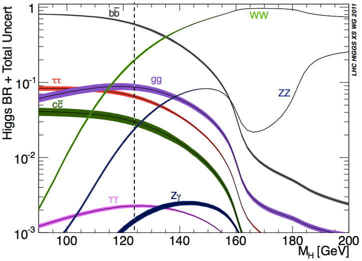
200 fb

- Only one to two orders of magnitude smaller
  - vs. 11 orders of magnitude in pp collisions
    - Trigger is 100% efficient (no need for trigger with ILC – all crossings are recorded)
    - All Higgs events are useful and exploitable
    - Signal purity is large

Precision Higgs physics at the FCC-ee

- **The plan is to run at √s = 240-250 GeV and 350-500 GeV in order to**
  - Determine all Higgs couplings in a model-independent way
  - Infer the Higgs total decay width
  - Evaluate (or set limits on) the Higgs invisible or exotic decays
    - Through the measurements of $\sigma(e^+e^- \rightarrow H+X) \times BR(H \rightarrow YY)$

with Y = b, c, g, W, Z, γ, τ, μ , invisible



m_H = 125 GeV

Decay	BR [%]	Unc. [%]
bb	57.7	3.3
ττ	6.32	5.7
cc	2.91	12.2
μμ	0.022	6.0
WW	21.5	4.3
gg	8.57	10.2
ZZ	2.64	4.3
γγ	0.23	5.0
Zγ	0.15	9.0
ΓH [MeV]	4.07	4.0

- m_H = 125 GeV is a very good place to be for precision measurements !
  - All decay channels open and measurable – can test new physics from many angles

Precision Higgs physics at the FCC-ee

- **Comparison with LHC**

Coupling	HL-LHC	ILC (+)	FCC-ee
κ_W	2-5%	0.8%	0.19%
κ_Z	2-4%	0.6%	0.15%
κ_b	4-7%	1.5%	0.42%
κ_c	–	2.7%	0.71%
κ_τ	2-5%	1.9%	0.54%
κ_μ	~10%	20%	6.2%
κ_γ	2-5%	7.8%	1.5%
κ_g	3-5%	2.3%	0.8%
$\kappa_{Z\gamma}$	~12%	?	?
BR_{invis}	~10-15%?	< 0.5%	< 0.1%
Γ_H	~50%?	3.8%	0.9%
κ_t	7-10%	18%	13% (*)
κ_H	30-50% ?	77%	80%(*)

(*) indirect

Model-independent results (ILC, FCC-ee)

κ_W – κ_μ: **Sensitive to new physics at tree level**
Expected effects < 5% / Λ^2_{NP}
1% precision needed for Λ_{NP}~1TeV
Sub-percent needed for Λ_{NP}>1TeV

κ_γ – $\kappa_{Z\gamma}$: **Sensitive to new physics in loops**

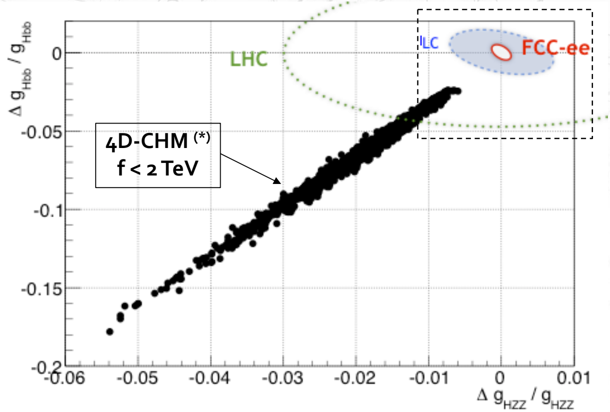
BR_{invis}, Γ_H: **Sensitive to light dark matter particles (sterile ν, χ, …) and to other exotic decays**

κ_t, κ_H: **Need higher energy to improve on LHC**

(+) Factor 2 smaller errors if lumi upgrade and an additional 10-15 years of running

Precision Higgs physics at the FCC-ee

- **Higgs couplings are affected by new physics**
  - **Example: Effect on κ_Z and κ_b for 4D-Higgs Composite Models**



Precision electroweak physics at the FCC-ee

- Reminder: The FCC-ee goals in numbers (after commissioning)

√s (GeV)	Running time	FCC-ee Statistics	ILC	LEP
91	5-6 years	10^{12} (10^{13}) Z decays (Tera-Z)	3×10^{9} (*)	2×10^{7}
161	2 years	3×10^{7} WW pairs (Oku-W)	2×10^{6} (*)	4×10^{4}
350	5 years	10^{6} top pairs (Mega-Top)	10^{5}	–

(*) Estimate: not in the core programme

- FCC-ee is the ultimate Z, W, Higgs and top factory
  - 10 to 3,000 times the ILC targeted statistics at the same energies
  - 10^{5} times more Zs and 10^{3} times more Ws than LEP1 and LEP2
    - Potential statistical accuracies are mind-boggling !

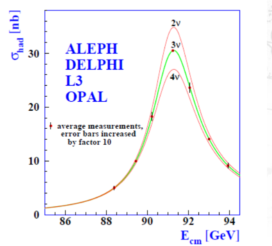
- Predicting accuracies with 200 times smaller statistical precision than at LEP is difficult
  - Conservatively, use LEP experience for systematics. This is just the start

- Example: The uncertainty on E_{BEAM} (2 MeV) was the dominant uncertainty on m_Z, Γ_Z
  - Can we do significantly better at FCC-ee ?

Precision electroweak physics at the FCC-ee

- EW precision measurements at FCC-ee (see arXiv:1308.6176)

Z reconance: TeraZ



Lineshape

- E_{beam} Exquisite (unique!) — Exquisite E_{beam} (unique!)
- m_Z, Γ_Z to < 100 keV (2.2 MeV)

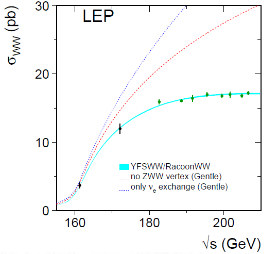
Asymmetries

- $\sin^2\theta_W$ to 6×10^{-6} (1.6 × 10^{-4})
- $\alpha_{QED}(m_Z)$ to 3×10^{-5} (1.5 × 10^{-4})

Branching ratios R_l, R_b

- $\alpha_S(m_Z)$ to 0.0002 (0.002)

WW threshold scan: OkuW



Threshold scan

- m_W to 0.5 MeV (15 MeV)

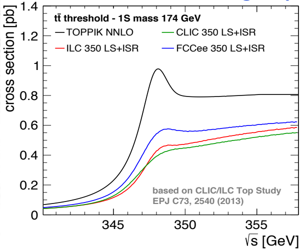
Branching ratios R_l, R_b

- $\alpha_S(m_Z)$ to 0.0002

Radiative return $e^+e^- \rightarrow Z\gamma$

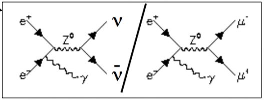
- N_ν to 0.0004 (0.008)

$t\bar{t}$ threshold scan: MegaTop



Threshold scan

- m_{top} to 10 MeV (500 MeV)
- λ_{top} to 10%
- EW couplings to 1%



Estimated experimental precision

Now:

Quantity	Theory error	Exp. error
M_W [MeV]	4	15
$\sin^2\theta^l_{eff}[10^{-5}]$	4.5	16
Γ_Z [MeV]	0.5	2.3
$R_b[10^{-5}]$	15	66

Quantity	ILC	FCC-ee	CEPC	Projected theory error
M_W [MeV]	3–4	1	3	1
$\sin^2\theta^l_{eff}[10^{-5}]$	1	0.6	2.3	1.5
Γ_Z [MeV]	0.8	0.1	0.5	0.2
$R_b[10^{-5}]$	14	6	17	5–10

The estimated error for the theoretical predictions of these quantities is given, under the assumption that $O(\alpha\alpha_s^2)$, fermionic $O(\alpha^2\alpha_s)$, fermionic $O(\alpha^3)$, and leading four-loop corrections entering through the ρ-parameter will become available.

We concentrated on taking into account *polarization effects*.

Precision electroweak physics at the FCC-ee

- The predictions of m_{top}, m_W, m_H, $\sin^2\theta_W$ have theoretical uncertainties
  - Which may cancel the sensitivity to new physics
- For m_W and $\sin^2\theta_W$ today, these uncertainties are as follows

$$M_W = 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{\rm theo} = 80.359 \pm 0.011_{\rm tot}$$

Exp: 0.015

$$\sin^2\theta^\ell_{\rm eff} = 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\rm theo} = 0.23150 \pm 0.00010_{\rm tot}$$

Exp: 0.00014

- Parametric uncertainties and missing higher orders in theoretical calculations:
  - Are of the same order
  - Smaller than experimental uncertainties

Precision electroweak physics at the FCC-ee

- **Most of the parametric uncertainties will reduce at the FCC-ee**
  - **New generation of theoretical calculations is necessary to gain a factor 10 in precision**
    - **To match the precision of the direct FCC-ee measurements**

$$
\begin{aligned}
M_W &= 80.3593 \pm 0.0001_{m_t} \pm 0.0001_{M_Z} \pm 0.0003_{\Delta\alpha_{\mathrm{had}}} \\
&\quad \pm 0.0002_{\alpha_S} \pm 0.0000_{M_H} \pm 0.0040_{\mathrm{theo}} \\
&= 80.359 \pm 0.005_{\mathrm{tot}}
\end{aligned}
$$

Exp: 0.0005

$$
\begin{aligned}
\sin^2\theta^{\ell}_{\mathrm{eff}} &= 0.231496 \pm 0.000001_{m_t} \pm 0.000001_{M_Z} \pm 0.000008_{\Delta\alpha_{\mathrm{had}}} \\
&\quad \pm 0.000001_{\alpha_S} \pm 0.000000_{M_H} \pm 0.000047_{\mathrm{theo}} \\
&= 0.23150 \pm 0.00006_{\mathrm{tot}}
\end{aligned}
$$

Exp: 0.000006

  - **Will require calculations up to three or four loops to gain an order of magnitude**
    - **Might need a new paradigm in the actual computing methods**
      - **Lot of interesting work for future generations of theorists**

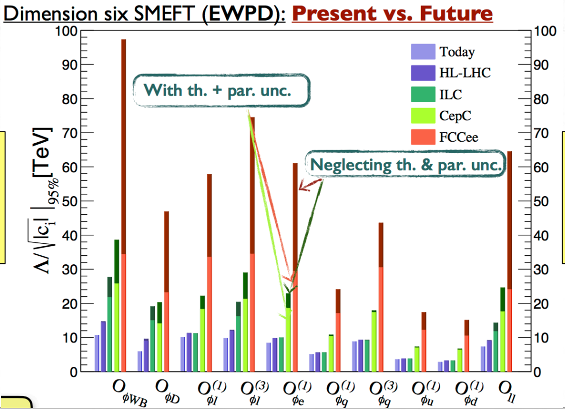
Precision electroweak physics at the FCC-ee

- **Higher-dimensional operators as a parametrization of new physics**
  - **Possible corrections to the Standard Model**
    - **Standard Model Effective Theories (SMEFT)**

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i$$

Limits on new physics scale, Λ:

Today:
Λ > 5-10 TeV



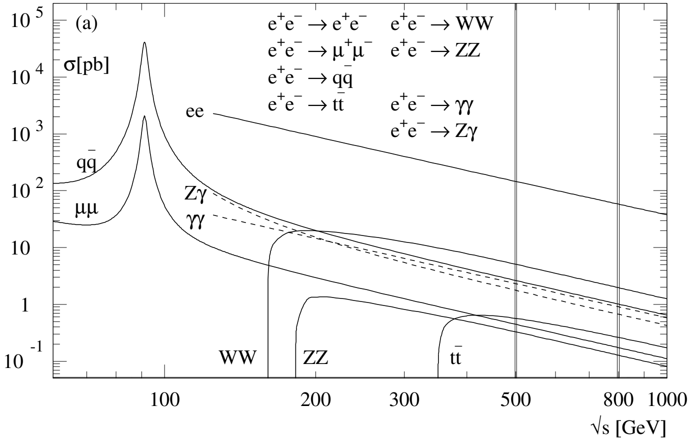
Limits on new physics scale, Λ:

After FCC-ee:
Λ > 50-100 TeV

Exploration

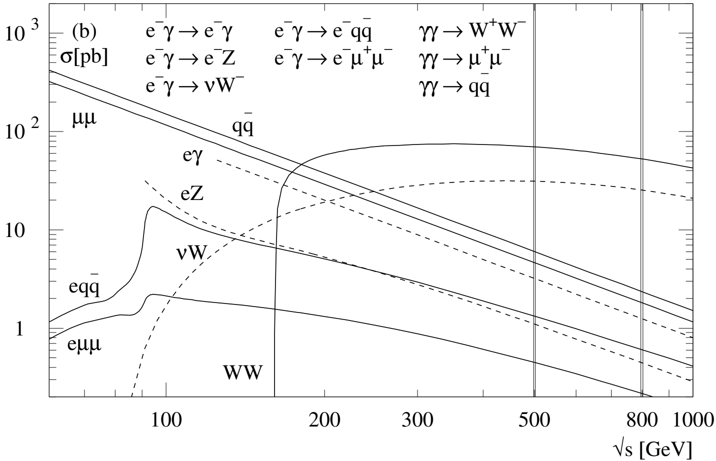
Energy	Reaction	Physics Goal	Polarization
91 GeV	$e^+e^- \to Z$	ultra-precision electroweak	Left-Right asymmetry
160 GeV	$e^+e^- \to WW$	ultra-precision W mass	Enhancement of lumi
250 GeV	$e^+e^- \to Zh$	precision Higgs couplings	
350–400 GeV	$e^+e^- \to t\bar{t}$	top quark mass and couplings	Left-Right asymmetry
	$e^+e^- \to WW$	precision W couplings	Enhancement of lumi
	$e^+e^- \to \nu\bar{\nu}h$	precision Higgs couplings	Enh. of process
500 GeV	$e^+e^- \to f\bar{f}$	precision search for Z'	Left-Right asymmetry mi
	$e^+e^- \to t\bar{t}h$	Higgs coupling to top	Enhancement of lumi
	$e^+e^- \to Zhh$	Higgs self-coupling	
	$e^+e^- \to \tilde{\chi}\tilde{\chi}$	search for supersymmetry	Suppr. of SM process
	$e^+e^- \to AH, H^+H^-$	search for extended Higgs states	
700–1000 GeV	$e^+e^- \to \nu\bar{\nu}hh$	Higgs self-coupling	Enhancement of process
	$e^+e^- \to \nu\bar{\nu}VV$	composite Higgs sector	
	$e^+e^- \to \nu\bar{\nu}t\bar{t}$	composite Higgs and top	
	$e^+e^- \to \tilde{t}\tilde{t}^*$	search for supersymmetry	Suppr. of SM process

Basic processes of SM for e^+e^- annihilation



The basic processes of the Standard Model: $e + e-$ annihilation to pairs of fermions and gauge bosons. The cross sections are given for polar angles between $10^o < \theta < 170^o$ in the final state.

Basic processes of SM for $e^{\pm}\gamma$ and $\gamma\gamma$ initial state

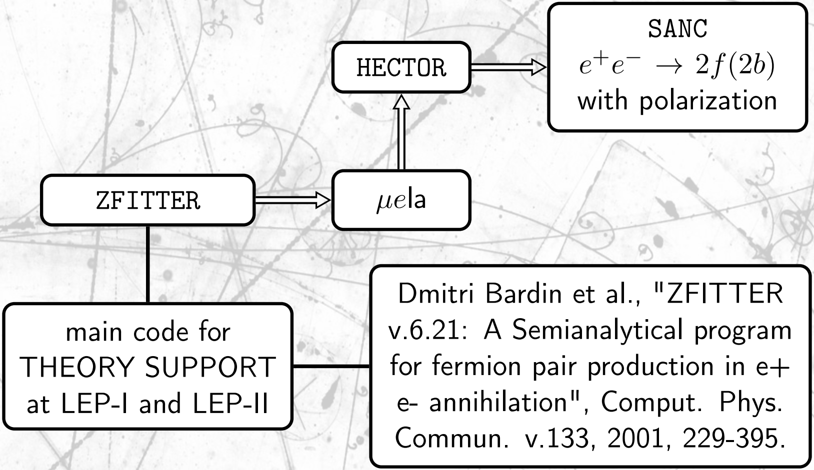


Elastic/inelastic Compton scattering and $\gamma\gamma$ reactions. $\sqrt{s}$ is the invariant $e\gamma$ and $\gamma\gamma$ energy. The polar angle of the final state particles is restricted as in (a); in addition, the invariant $\mu^+\mu^-$ and $q\overline{q}$ masses in the inelastic Compton processes are restricted to $M_{inv} > 50$ GeV.

Precision with $e+e-$ colliders: Summary

- The small mass of the Higgs boson allows two options to be contemplated
  - A 250 – 500 GeV linear collider: ILC (also CLIC at √s = 380 GeV)
  - A 88-370 GeV circular collider: FCC-ee (also CEPC at √s = 240 GeV)
- Precision measurements at the EW scale are sensitive to new physics
  - To potentially very high scales (up to ~100 TeV with FCC-ee)
    - Through a study of the Z, W, H, and top properties with unprecedented statistics
  - To potentially very small couplings (sterile neutrinos, dark matter, …)
- Understanding this physics requires an e^+e^- collider at the EW scale
  - In an ideal world, this understanding could even benefit from having two of them
- Significant synergies (detectors) and complementarities (physics)
  - Beween circular (FCC-ee, CEPC) and linear collider projects (ILC, CLIC)
    - FCC-ee offers the highest luminosities and discovery potential (Z, WW, ZH)
      - These features will remain unchallenged if a linear collider is built
    - Linear colliders can reach energies beyond 500 GeV
      - This advantage will remain unique if the FC-ee is built

SANC HISTORY OVERVIEW



HISTORY OVERVIEW

- μela

QED corrections at one loop level for polarized elastic μe scattering. Research was used for the analysis of μe scattering data from the beam polarimeter of the SMC experiment at CERN.
Dmitri Bardin et al.,hep-ph/9712310

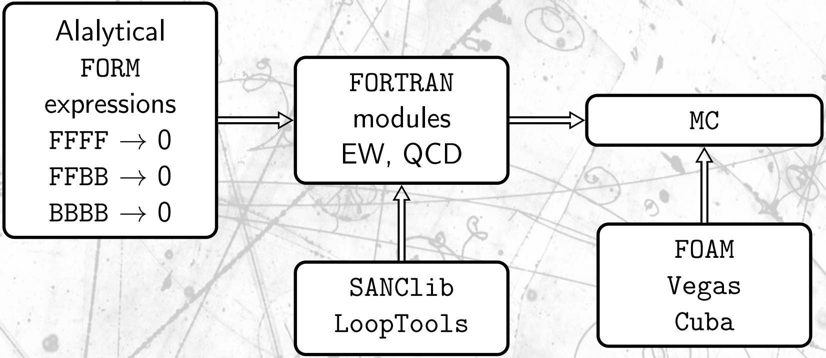
- `HECTOR 1.11`

Polarized ep scattering at HERA
Dmitri Yu. Bardin et al., "QED and electroweak corrections to deep inelastic scattering", Acta Phys. Polon.,v. B28,1997, 511-528.

- SANC

Complete one loop calculation of the EW radiative corrections for scattering e^+e^- polarized beams. The main conclusion of this study is radiative corrections as a function of the angle scattering $\cos\vartheta$ for Super Charm-Tau factory, CLIC, ILC, FCC_{ee} energy.

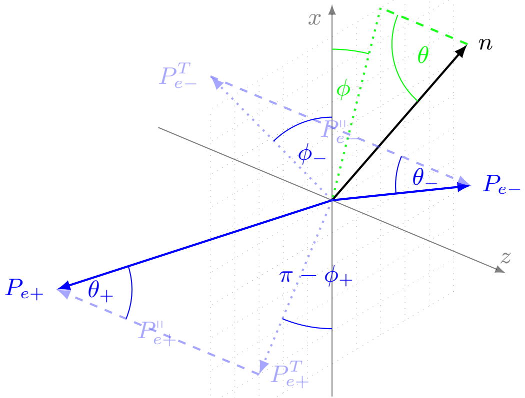
The SANC framework scheme



SANC for processes with polarized beams

- NLO EW corrections for polzrized e^+e^- scattering:
  - Bhabha scattering (arXiv:1801.00125)
  - $e^+e^- \to \mu^+\mu^-$(or $\tau^+\tau^-$) (preliminary)
  - $e^+e^- \to Z\gamma$ (preliminary)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to ZH$ (in progress)
  - $e^+e^- \to \gamma\gamma$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polzrized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

Decomposition of the $e^{\pm}$ polarization vectors



Matrix element squared

$$|\mathcal{M}|^2 = \quad \frac{1}{4}\Big\{(1-P^{||}_{e-})(1+P^{||}_{e+})|\mathcal{H}_{-+}|^2 + (1+P^{||}_{e-})(1-P^{||}_{e+})|\mathcal{H}_{-+}|^2$$

$$+(1-P^{||}_{e-})(1-P^{||}_{e+})|\mathcal{H}_{--}|^2 + (1+P^{||}_{e-})(1-P^{||}_{e+})|\mathcal{H}_{++}|^2$$

$$-2P^T_{e-}P^T_{e+}\Big[\cos(\phi_- - \phi_+)\,\mathrm{Re}(\mathcal{H}_{++}\mathcal{H}^*_{--}) + \cos(\phi_- + \phi_+ - 2\phi)\,\mathrm{Re}(\mathcal{H}_{-+}\mathcal{H}^*_{+-}$$

$$+\sin(\phi_- + \phi_+ - 2\phi)\,\mathrm{Im}(\mathcal{H}_{-+}\mathcal{H}^*_{+-}) + \sin(\phi_- - \phi_+)\,\mathrm{Im}(\mathcal{H}_{++}\mathcal{H}^*_{--})\Big]$$

$$+2P^T_{e-}\Big[\cos(\phi_- - \phi)\Big((1-P_{e+})\,\mathrm{Re}(\mathcal{H}_{+-}\mathcal{H}^*_{--}) + (1+P_{e+})\,\mathrm{Re}(\mathcal{H}_{++}\mathcal{H}^*_{-+})\Big)$$

$$-\sin(\phi_- - \phi)\Big((1-P_{e+})\,\mathrm{Im}(\mathcal{H}_{+-}\mathcal{H}^*_{--}) + (1+P_{e+})\,\mathrm{Im}(\mathcal{H}_{++}\mathcal{H}^*_{-+})\Big)\Big]$$

$$-2P^T_{e+}\Big[\cos(\phi_+ - \phi)\Big((1-P_{e-})\,\mathrm{Re}(\mathcal{H}_{-+}\mathcal{H}^*_{--}) + (1+P_{e-})\,\mathrm{Re}(\mathcal{H}_{++}\mathcal{H}^*_{+-})\Big)$$

$$-\sin(\phi_+ - \phi)\Big((1-P_{e-})\,\mathrm{Im}(\mathcal{H}_{-+}\mathcal{H}^*_{--}) + (1+P_{e-})\,\mathrm{Im}(\mathcal{H}_{++}\mathcal{H}^*_{+-})\Big)\Big]\Big\},$$

where $\mathcal{H}_{++}$, $\mathcal{H}_{--}$,$\mathcal{H}_{+-}$,$\mathcal{H}_{-+}$ — helicity amplitudes.

G. Moortgat-Pick et al. Phys. Rept. 460 (2008) 131–243

Polarized Bhabha scattering: notations

We consider a scattering of two polarized e^+ and e^- beams with four momentum of incoming particles p_1 and p_2, outgoing particles p_3 and p_4, in the massless case $m_e = 0$ at the one-loop EW level

$$e^+(p_1) + e^-(p_2) \longrightarrow e^+(p_3) + e^-(p_4).$$

The cross-section of this process at one-loop can be devided into four parts:

$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

where σ^{Born} — Born level cross-section, σ^{virt} — contribution of virtual(loop) corrections, σ^{soft} — contribution due to soft photon emission, σ^{hard} — contribution due to hard photon emission (with energy $E_\gamma > \omega$).
Auxiliary parameters λ ("photon mass") and ω cancel out after summation.

Bhabha: HA for Born and Virtual parts

At one-loop level we have six non-zero HAs (four independent):

$$\mathcal{H}_{++++} = \mathcal{H}_{----} = -2e^2\,\frac{s}{t}\Big[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) - \chi^t_Z\delta_e\mathcal{F}^Z_{QL}(t,s,u)\Big],$$

$$\mathcal{H}_{+-+-} = \mathcal{H}_{-+-+} = -e^2\,c_-\Big[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) - \chi^s_Z\delta_e\mathcal{F}^Z_{QL}(s,t,u)\Big],$$

$$\mathcal{H}_{+--+} = -e^2\,c_+\Big(\Big[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) + \chi^s_Z\,\big(\mathcal{F}^Z_{LL}(s,t,u) - 2\delta_e\mathcal{F}^Z_{QL}(s,t,u)\big)\Big]$$

$$+\frac{s}{t}\Big[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) + \chi^t_Z\,\big(\mathcal{F}^Z_{LL}(t,s,u) - 2\delta_e\mathcal{F}^Z_{QL}(t,s,u)\big)\Big]\Big),$$

$$\mathcal{H}_{-++-} = -e^2\,c_+\Big(\Big[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u)\Big] + \frac{s}{t}\,\Big[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u)\Big]\Big),$$

where $\quad c_+ = 1+\cos\theta, \quad c_- = 1-\cos\theta,$

$$\chi^s_Z = \frac{1}{4s^2_W c^2_W}\,\frac{s}{s-M^2_Z+iM_Z\Gamma_Z}, \quad \chi^t_Z = \frac{1}{4s^2_W c^2_W}\,\frac{t}{t-M^2_Z}, \quad \delta_e = v_e - a_e = 2s^2_W,$$

$$\mathcal{F}^{(\gamma,Z)}_{QQ}(a,b,c) = \mathcal{F}^{\gamma}_{QQ}(a,b,c) + \chi^a_Z\delta^2_e\mathcal{F}^Z_{QQ}(a,b,c).$$

We get the Born level HAs by replacing $\mathcal{F}^Z_{LL}\to 1$, $\mathcal{F}^Z_{QL}\to 1$, $\mathcal{F}^Z_{QQ}\to 1$ and $\mathcal{F}^{\gamma}_{QQ}\to 1$.

Bremsstrahlung HA

$$\mathcal{H}^{\mathsf{hard}} = \mathcal{H}^{\mathsf{isr}} + \mathcal{H}^{\mathsf{fsr}} + \mathcal{H}^{\mathsf{esr}} + \mathcal{H}^{\mathsf{psr}}$$

Crossing symmetry

$$\mathcal{H}^{\mathsf{fsr}}_{\chi_1\chi_2\chi_3\chi_4\chi_5}(p_1, p_2, p_3, p_4) = +\mathcal{H}^{\mathsf{isr}}_{-\chi_4-\chi_3-\chi_2-\chi_1\chi_5}(-p_4, -p_3, -p_2, -p_1)$$
$$\mathcal{H}^{\mathsf{esr}}_{\chi_1\chi_2\chi_3\chi_4\chi_5}(p_1, p_2, p_3, p_4) = -\mathcal{H}^{\mathsf{isr}}_{+\chi_1-\chi_3-\chi_2+\chi_4\chi_5}(+p_1, -p_3, -p_2, +p_4)$$
$$\mathcal{H}^{\mathsf{psr}}_{\chi_1\chi_2\chi_3\chi_4\chi_5}(p_1, p_2, p_3, p_4) = -\mathcal{H}^{\mathsf{isr}}_{-\chi_4+\chi_2+\chi_3-\chi_1\chi_5}(-p_4, +p_2, +p_3, -p_1)$$

CP-symmetry

$$\mathcal{H}^{\mathsf{hard}}_{\chi_1\chi_2\chi_3\chi_4\chi_5} = -\chi_1\chi_2\chi_3\chi_4\overline{\mathcal{H}}^{\mathsf{hard}}_{-\chi_1-\chi_2-\chi_3-\chi_4-\chi_5}$$
$$\text{with } D_{\chi_1,\chi_3} \to D_{-\chi_1,-\chi_3}$$

Spinor label notation

Phase fixing and notation

For any massless momentum $k_1^2 = 0$ we can solve Dirac equation $\hat{k}_1 u(k_1) = 0$ and obtain two solutions:

$$\begin{aligned} |1\rangle = u(k_1,+) = v(k_1,-) \quad [1| = \bar{u}(k_1,+) = \bar{v}(k_1,-) \\ |1] = u(k_1,-) = v(k_1,+) \quad \langle 1| = \bar{u}(k_1,-) = \bar{v}(k_1,+) \end{aligned} \tag{1}$$

Spinor diada

Outer products of spinors are related to complex light-like 4-vectors:

$$|1\rangle\,[1| = \frac{1+\gamma_5}{2}\hat{k}_1 \quad |1]\,\langle 1| = \frac{1-\gamma_5}{2}\hat{k}_1$$

$$|1\rangle\,[1| + |1]\,\langle 1| = \hat{k}_1$$

$$\langle 1|\gamma^\mu|1] = [1|\gamma^\mu|1\rangle = 2k_1^\mu$$

Spinor products

Inner products of spinors are complex Lorentz invariants

$$\begin{aligned} \langle a\, b\rangle &= \langle k_a|k_b\rangle \quad \langle a\, b] = 0 \\ [a\, b] &= [k_a|k_b] \quad [a\, b\rangle = 0 \\ \langle a\, b\rangle &= -\langle b\, a\rangle \quad \langle a\, a\rangle = 0 \\ [b\, a] &= -[a\, b] \quad [a\, a] = 0 \end{aligned} \tag{2}$$

$$[b\, a] = \overline{\langle a\, b\rangle} \tag{3}$$

$$\langle a\, b\rangle [b\, a] = \left|\langle a\, b\rangle\right|^2 = 2k_a \cdot k_b = (k_a + k_b)^2 = s_{ab} \tag{4}$$

Schouten identity

$$\langle 1\, 2\rangle \langle 3\, 4\rangle = \langle 1\, 3\rangle \langle 2\, 4\rangle + \langle 1\, 4\rangle \langle 3\, 2\rangle$$

$$|2\rangle \langle 3\, 4\rangle = |3\rangle \langle 2\, 4\rangle + |4\rangle \langle 3\, 2\rangle$$

Polarization vectors

For massless vector boson with momentum k_1 in axial gauge (fixed by light-like vector k_2) we can construct polarization vectors explicitly in terms of spinor diada

$$\epsilon_\mu(k_1, +, k_2) = \frac{\langle 2|\gamma_\mu|1]}{\sqrt{2}\,\langle 2\,1\rangle}$$

$$\epsilon_\mu(k_1, -, k_2) = \frac{[2|\gamma_\mu|1\rangle}{\sqrt{2}\,[2\,1]}$$

$$\hat{\epsilon}(k_1, +, k_2) = \sqrt{2}\frac{|2\rangle[1| + |1]\langle 2|}{\langle 2\,1\rangle}$$

$$\hat{\epsilon}(k_1, -, k_2) = \sqrt{2}\frac{|2]\langle 1| + |1\rangle[2|}{[2\,1]}$$

$$\mathcal{H}^{\mathsf{isr}}_{+-+-+} = \; m_f^2 D^{\rm isr}_{+-}\mathcal{A}_{0M}\left[{}^{135}_{24}\right] + D^{\rm isr}_{++}\mathcal{A}_0\left[{}^{135}_{24}\right] + m_{f_1}^2 D^{\rm isr}_{-+}\mathcal{A}_4\left[{}^{135}_{24}\right]$$

$$\mathcal{H}^{\mathsf{isr}}_{+++++} = m_{f_1} m_f \Big[\; D^{\mathsf{isr}}_{++}\mathcal{A}_7\left[{}^{135}_{24}\right] + D^{\rm isr}_{+-}\mathcal{A}_7\left[{}^{145}_{23}\right] - D^{\rm isr}_{-+}\mathcal{A}_7\left[{}^{235}_{14}\right] - D^{\rm isr}_{--}\mathcal{A}_7\left[{}^{245}_{13}\right] \Big]$$

$$\mathcal{H}^{\mathsf{isr}}_{----+} = m_{f_1} m_f \Big[\; D^{\rm isr}_{-+}\mathcal{A}_1\left[{}^{145}_{23}\right] + D^{\rm isr}_{--}\mathcal{A}_1\left[{}^{135}_{24}\right] - D^{\rm isr}_{++}\mathcal{A}_1\left[{}^{245}_{13}\right] - D^{\rm isr}_{+-}\mathcal{A}_1\left[{}^{235}_{14}\right] \Big]$$

$$\mathcal{H}^{\mathsf{isr}}_{+++-+} = -m_{f_1} \Big[D^{\rm isr}_{-+}\mathcal{A}_2\left[{}^{235}_{14}\right] + D^{\rm isr}_{++}\mathcal{A}_2\left[{}^{135}_{24}\right] \Big]$$

$$\mathcal{H}^{\mathsf{isr}}_{-++++} = -m_f \Big[D^{\rm isr}_{-+}\mathcal{A}_5\left[{}^{245}_{13}\right] + D^{\rm isr}_{--}\mathcal{A}_5\left[{}^{235}_{14}\right] \Big]$$

$$\mathcal{H}^{\mathsf{isr}}_{+---+} = -m_f \Big[D^{\rm isr}_{++}\mathcal{A}_6\left[{}^{135}_{24}\right] + D^{\rm isr}_{+-}\mathcal{A}_6\left[{}^{145}_{23}\right] \Big]$$

$$\mathcal{H}^{\mathsf{isr}}_{--+-+} = \; m_{f_1} \Big[D^{\rm isr}_{++}\mathcal{A}_3\left[{}^{245}_{13}\right] - D^{\rm isr}_{-+}\mathcal{A}_3\left[{}^{145}_{23}\right] \Big]$$

Abbriviations

$$D_{\chi_1,\chi_3}(s) = 2\sqrt{2}e^3 K \left[\frac{Q_e Q_t}{s} + \frac{(v_e + \chi_1 a_e)(v_\tau + \chi_3 a_\tau)}{s - M_Z^2 + M_Z \Gamma_Z} \right], \quad \chi_1, \chi_3 = \pm 1$$

$$K = 1 - \frac{m_1^2}{2p_1p_5} - \frac{m_2^2}{2p_2p_5} + \frac{m_3^2}{2p_3p_5} + \frac{m_4^2}{2p_4p_5} \quad \kappa = \frac{K-1}{K}$$

For massive particles we use variables:

$$s = (p_1 + p_2)^2 \quad t = (p_1 - p_3)^2 \quad u = (p_1 - p_4)^2$$

$$s' = (p_3 + p_4)^2 \quad t' = (p_2 - p_4)^2 \quad u' = (p_2 - p_3)^2$$

$$s + t + u + s' + t' + u' = 2\left(m_1^2 + m_2^2 + m_3^2 + m_4^2\right)$$

$$z_i = 2p_i \cdot p_5$$

$$z_1 + z_2 = z_3 + z_4 = s - s'$$

Reduced amplitudes

$$\mathcal{A}_0\left[{}^{135}_{24}\right] = \frac{\langle 1\,4\rangle^2\,[4\,3]}{\langle 1\,5\rangle\,\langle 2\,5\rangle} - \kappa\frac{\langle 1\,4\rangle\,[5\,3]}{\langle 2\,5\rangle}$$

$$\mathcal{A}_3\left[{}^{135}_{24}\right] = \frac{\langle 2\,3\rangle\,\langle 3\,5\rangle\,[4\,3]}{\langle 1\,5\rangle\,\langle 2\,5\rangle^2} - \kappa\frac{\langle 3\,5\rangle\,[5\,4]}{\langle 1\,5\rangle\,\langle 2\,5\rangle}$$

$$\mathcal{A}_{0M}\left[{}^{135}_{24}\right] = \frac{\langle 1\,2\rangle\,[5\,2]}{\langle 2\,5\rangle\,\langle 3\,5\rangle\,[5\,4]} \qquad \mathcal{A}_1\left[{}^{135}_{24}\right] = \frac{\langle 1\,2\rangle\,\langle 3\,5\rangle\,[5\,1]}{\langle 1\,5\rangle\,\langle 2\,5\rangle^2\,[5\,4]}$$

$$\mathcal{A}_2\left[{}^{135}_{24}\right] = \frac{\langle 1\,2\rangle\,\langle 1\,4\rangle\,[5\,3]}{s_{25}\,\langle 1\,5\rangle} \qquad \mathcal{A}_4\left[{}^{135}_{24}\right] = \frac{\langle 1\,2\rangle\,\langle 4\,5\rangle\,[5\,3]}{s_{15}\,\langle 2\,5\rangle^2}$$

$$\mathcal{A}_5\left[{}^{135}_{24}\right] = \frac{\langle 1\,3\rangle\,[4\,3]}{\langle 2\,5\rangle\,\langle 3\,5\rangle} \qquad \mathcal{A}_6\left[{}^{135}_{24}\right] = \frac{\langle 1\,2\rangle\,\langle 1\,4\rangle\,[5\,2]}{\langle 1\,5\rangle\,\langle 2\,5\rangle\,[5\,3]}$$

$$\mathcal{A}_7\left[{}^{135}_{24}\right] = \frac{\langle 1\,2\rangle\,[5\,3]}{s_{25}\,\langle 4\,5\rangle}$$

SANC Monte-Carlo generator for $e^+e^- \to e^+e^-$ process

We created Monte Carlo generator of unweighted events for the polarized Bhabha scattering $e^+e^- \to e^+e^-$ with complete one-loop EW corrections and with possibility to produce events in standard Les Houches format.

This generator uses adaptive algorithm `mFOAM` (CPC 177:441-458,2007) which is part of `ROOT` program.

Setup for tuned comparison

We performed a tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results WHIZARD program. The contributions of soft and virtual parts were compared with the results of ALTalk program

Initial parameters

$$\alpha^{-1}(0) = 137.03599976,$$

$$M_W = 80.451495 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV} \quad \Gamma_Z = 2.49977 \text{ GeV},$$

$$m_e = 0.5109990 \text{ MeV}, \quad m_\mu = 0.105658 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$

$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$

$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts

$$|\cos\theta| < 0.9,$$

$$E_\gamma > 1 \text{ GeV} \quad \text{(for comparison of hard Bremsstrahlung).}$$

$e^+e^- \to e^+e^-$: WHIZARD vs SANC (Born)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.677(1)	57.774(1)	56.272(1)	59.276(1)
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.677(1)	57.775(1)	56.272(1)	59.275(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.379(1)	15.030(1)	12.706(1)	17.355(1)
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.379(1)	15.030(1)	12.706(1)	17.354(1)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7756(1)
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7755(1)

$e^+e^- \to e^+e^-$: WHIZARD vs SANC (hard)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.62(1)	49.58(1)	48.74(1)	50.40(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.65(1)	49.56(1)	48.78(1)	50.44(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.14(1)	15.81(1)	13.54(1)	18.07(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.12(1)	15.79(1)	13.55(1)	18.11(2)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.693(1)	4.976(1)	3.912(1)	6.041(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.694(1)	4.975(1)	3.913(1)	6.043(1)

$e^+e^- \to e^+e^-$: ÅItalk vs SANC $\sqrt{s} = 500 GeV$

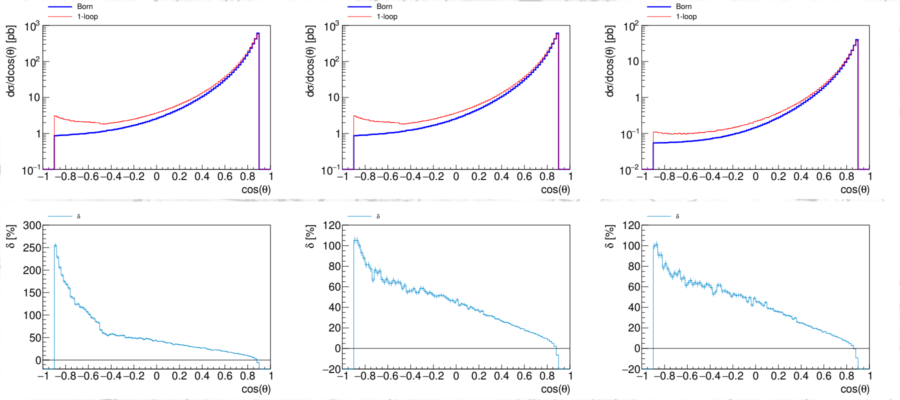
$\cos\theta$	$\sigma^{\text{Born}}_{e^+e^-}$, pb	$\sigma^{\text{Born+virt+soft}}_{e^+e^-}$, pb
-0.9	$2.16999 \cdot 10^{-1}$ $2.16999 \cdot 10^{-1}$	$1.93445 \cdot 10^{-1}$ $1.93445 \cdot 10^{-1}$
-0.5	$2.61360 \cdot 10^{-1}$ $2.61360 \cdot 10^{-1}$	$2.38707 \cdot 10^{-1}$ $2.38707 \cdot 10^{-1}$
0	$5.98142 \cdot 10^{-1}$ $5.98142 \cdot 10^{-1}$	$5.46677 \cdot 10^{-1}$ $5.46677 \cdot 10^{-1}$
$+0.5$	$4.21273 \cdot 10^{0}$ $4.21273 \cdot 10^{0}$	$3.81301 \cdot 10^{0}$ $3.81301 \cdot 10^{0}$
$+0.9$	$1.89160 \cdot 10^{2}$ $1.89160 \cdot 10^{2}$	$1.72928 \cdot 10^{2}$ $1.72928 \cdot 10^{2}$
$+0.99$	$2.06556 \cdot 10^{4}$ $2.06555 \cdot 10^{4}$	$1.90607 \cdot 10^{4}$ $1.90607 \cdot 10^{4}$
$+0.999$	$2.08236 \cdot 10^{6}$ $2.08236 \cdot 10^{6}$	$1.91624 \cdot 10^{6}$ $1.91624 \cdot 10^{6}$
$+0.9999$	$2.08429 \cdot 10^{8}$ $2.08429 \cdot 10^{8}$	$1.91402 \cdot 10^{8}$ $1.91402 \cdot 10^{8}$

$e^+e^- \to e^+e^-$: Born vs 1-loop

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.677(1)	57.775(1)	56.272(1)	59.275(1)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	61.55(1)	59.72(3)	61.02(3)	58.44(3)
δ, %	8.59(2)	3.37(5)	8.45(5)	-1.42(5)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.379(1)	15.030(1)	12.706(1)	17.354(1)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	15.436(7)	14.441(7)	13.501(6)	15.40(1)
δ, %	7.35(5)	-3.92(5)	6.26(5)	-11.29(5)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7755(1)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	3.862(2)	3.609(2)	3.148(1)	4.067(3)
δ, %	4.98(5)	-7.60(5)	3.70(5)	-14.84(6)

$e^+e^- \to e^+e^-$: **distributions on** $\cos\theta$

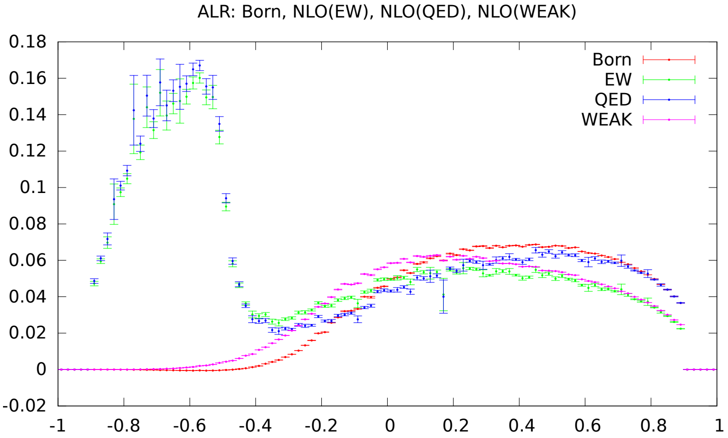
$\sqrt{s} = 250$ GeV $\qquad$ $\sqrt{s} = 500$ GeV $\qquad$ $\sqrt{s} = 1000$ GeV



$e^+e^- \to e^+e^-$: A_{LR} dependence on $\cos\theta$

$$A_{LR} = \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$

$\sqrt{s} = 250$ GeV



$e^+e^- \to \mu^+\mu^-$: SANC vs WHIZARD – Born & Hard

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, 0.6	-0.8, -0.6
$\sqrt{s} = 250$ GeV				
σ^{Born}, pb [SANC]	1.6537(1)	1.8040(1)	2.7105(1)	0.89749(1)
σ^{Born}, pb [WHIZARD]	1.6537(1)	1.8039(1)	2.7102(1)	0.89744(1)
σ^{Hard}, pb [SANC]	1.822(1)	2.034(1)	3.068(1)	1.001(1)
σ^{Hard}, pb [WHIZARD]	1.822(1)	2.034(1)	3.048(1)	1.018(1)
$\sqrt{s} = 500$ GeV				
σ^{Hard}, pb [SANC]	0.393(1)	0.426(1)	0.641(1)	0.213(1)
σ^{Hard}, pb [WHIZARD]	0.394(1)	0.428(1)	0.641(1)	0.214(1)
$\sqrt{s} = 1000$ GeV				
σ^{Hard}, pb [SANC]	0.1155(1)	0.1247(1)	0.1872(1)	0.0623(1)
σ^{Hard}, pb [WHIZARD]	0.1153(2)	0.1245(2)	0.1874(2)	0.0626(1)

$e^+e^- \to \mu^+\mu^-$: A$\mathring{\rm I}$talk vs SANC, $\sqrt{s} = 500 GeV$

	$\sigma^{\text{Born}}_{\mu^+\mu^-}$, pb	$\sigma^{\text{Born+virt+soft}}_{\mu^+\mu^-}$, pb
$\cos\vartheta = -0.9$		
[Aicalc]	0.09458936	0.09028587
[SANC]	0.09458937	0.09028587
$\cos\vartheta = -0.5$		
[Aicalc]	0.08929449	0.08468314
[SANC]	0.08929448	0.08468313
$\cos\vartheta = 0.0$		
[Aicalc]	0.1503198	0.1402075
[SANC]	0.1503199	0.1402075
$\cos\vartheta = 0.5$		
[Aicalc]	0.2865049	0.2761361
[SANC]	0.2865049	0.2761361
$\cos\vartheta = 0.9$		
[Aicalc]	0.4495681	0.4663674
[SANC]	0.4495682	0.4663675

$e^+e^- \to \mu^+\mu^-$: Preliminary SANC results for one-loop corrections

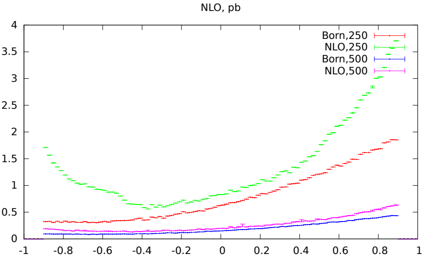
P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, 0.6	-0.8, -0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\rm Born}_{\mu^+\mu^-}$, pb	1.4174(1)	1.5462(1)	2.3231(2)	0.7690(2)
$\sigma^{\rm 1-loop}_{\mu^+\mu^-}$, pb	2.397(1)	2.614(1)	3.927(1)	1.301(1)
δ,%	69.1(1)	69.1(1)	69.1(1)	69.2(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\rm Born}_{\mu^+\mu^-}$, pb	0.34361(1)	0.37159(1)	0.55751(1)	0.18567(1)
$\sigma^{\rm 1-loop}_{\mu^+\mu^-}$, pb	0.4696(1)	0.4953(1)	0.7399(1)	0.2506(1)
δ,%	36.67(3)	33.30(2)	32.71(2)	34.98(2)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\rm Born}_{\mu^+\mu^-}$, pb	0.085354(1)	0.09213(1)	0.13818(1)	0.04608(1)
$\sigma^{\rm 1-loop}_{\mu^+\mu^-}$, pb	0.11627(2)	0.12119(2)	0.18069(3)	0.61694(1)
δ,%	36.22(2)	31.55(2)	30.78(2)	33.90(2)

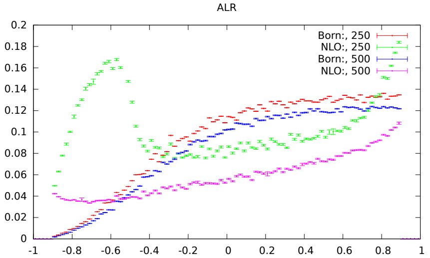
These are results. Work of comparison of our results with other groups are in progress.

$e^+e^- \to \mu^+\mu^-$: A_{LR} distributions on $\cos\theta$

$\frac{d\sigma}{d\cos\vartheta}$

$A_{LR} = \frac{\sigma_{LR}-\sigma_{RL}}{\sigma_{LR}+\sigma_{RL}}$





Conclusion

- The background for complete one loop calculation of the EW radiative corrections for scattering e^+e^- polarized (longitudional and transversal) beams is created:
  HA for virtual part & HA for Bremsstrahlung
- MC generator e^+e^- is created
- Complete $O(\alpha)$ EW corrections to polarized
  a) Bhabha scattering
  b) $e^+e^- \to \mu^+\mu^-$ $(\tau^+\tau^-)$ are computed for the first time
- Physical program of future e^+e^- colliders is under development. Many new tasks for theoreticians are there: Monte Carlo event generator(s) for experimentalists

Thank you