Qubitization of Field Theories

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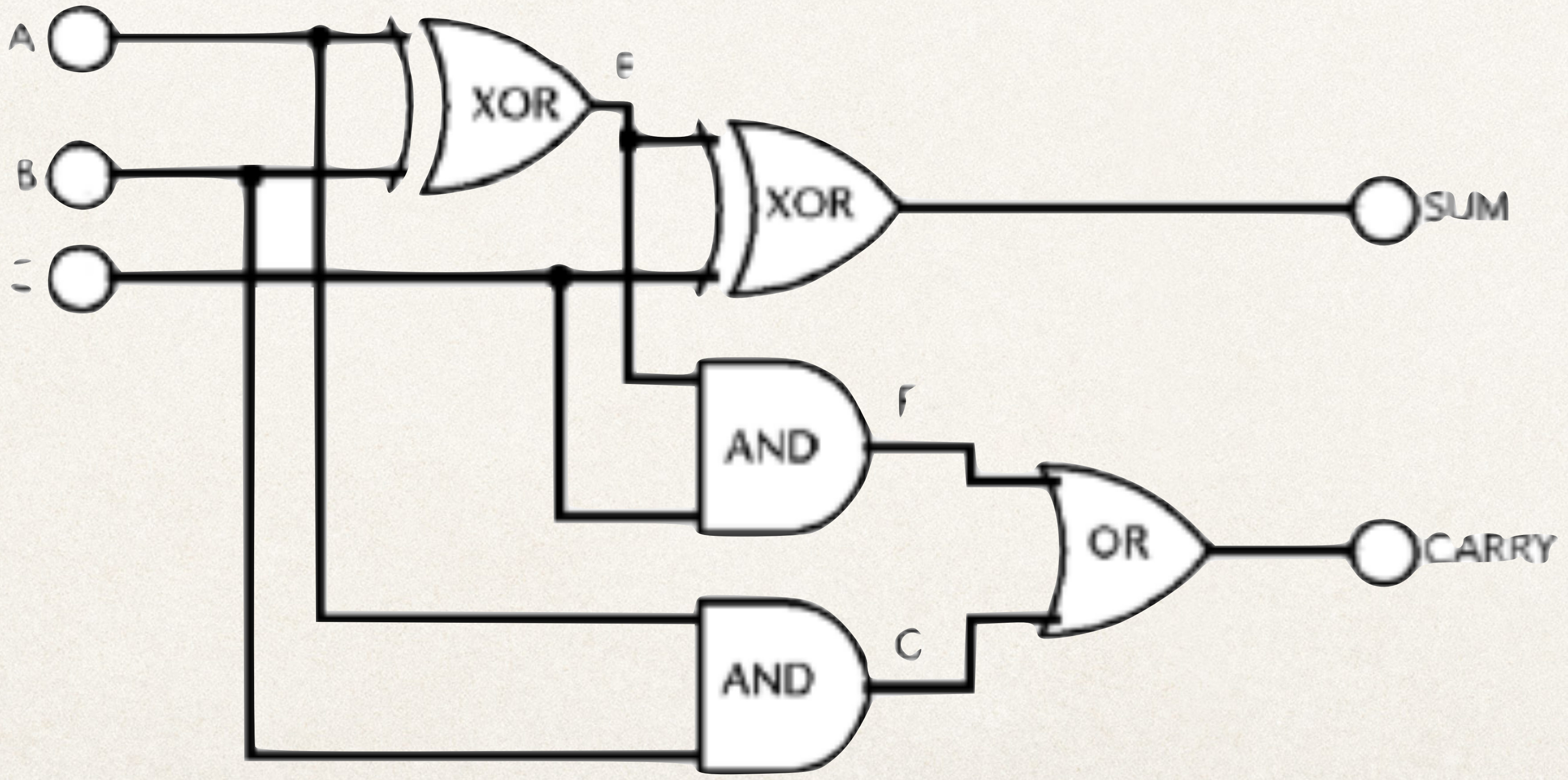
Classical numerical quantum field theory / many-body physics:

$$\langle \mathcal{O} \rangle = \frac{\int D\phi \ e^{iS[\phi]} \mathcal{O}[\phi]}{\int D\phi \ e^{iS[\phi]}}$$

Classical numerical quantum field theory / many-body physics:

$$\langle \mathcal{O} \rangle = \frac{\int D\phi \; e^{-S_E[\phi]} \mathcal{O}[\phi]}{\int D\phi \; e^{-S_E[\phi]}} \approx \frac{1}{N} \sum_{a=1}^{N} \mathcal{O}[\phi^a]$$



manipulation of
real (classical) fields

Classical numerical quantum field theory/many-body physics:

Large amount of theory and practice:

- What can and cannot be computed
- How many gates (time) is required, sometimes cost ~e^V
- Large collection of algorithms
- Memory/speed/energy trade-offs
- Most chips run videos games/cels

Classical numerical quantum field theory / many-body physics:

$$\langle \mathcal{O} \rangle = \frac{\int D\phi \; e^{iS[\phi]} \mathcal{O}[\phi]}{\int D\phi \; e^{iS[\phi]}}$$

iS is purely imaginary:

the mother of all sign problems

Classical numerical quantum field theory / many-body physics:

$$\langle \mathcal{O} \rangle = \frac{\int D\phi \ e^{-S_E[\phi]} \mathcal{O}[\phi]}{\int D\phi \ e^{-S_E[\phi]}}$$

S_E may be complex (chemical potential) probably has cost $O(e^V)$:

sign problem

Direct diagonalization:

exponential cost ~ $(2^V)^3$

Classical numerical quantum field theory/many-body physics:

$$\langle \mathcal{O} \rangle = \frac{\int D\phi \; e^{-S_E[\phi]} \mathcal{O}[\phi]}{\int D\phi \; e^{-S_E[\phi]}}$$

S_E may be complex (chemical potential) probably has cost $O(e^V)$:

QCD equation of state, Hubbard model away from 1/2-filling, …

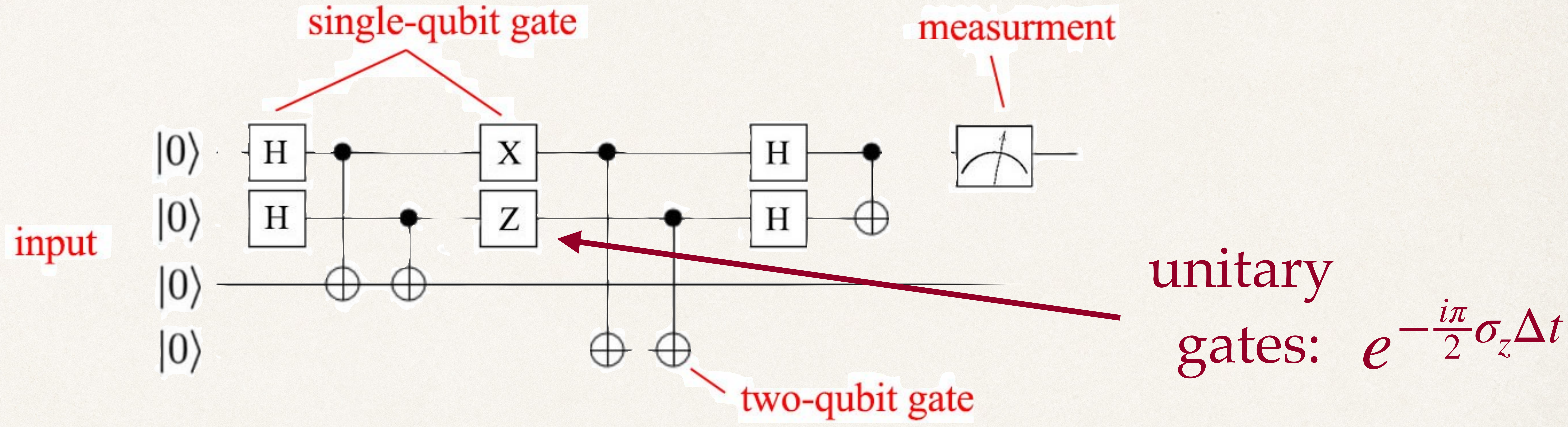
iS is purely imaginary:

transport coefficients (viscosities, heat conductivities), ν-propagation in dense/hot matter, thermalization of QGP, …

Quantum numerical quantum field theory / many-body physics:

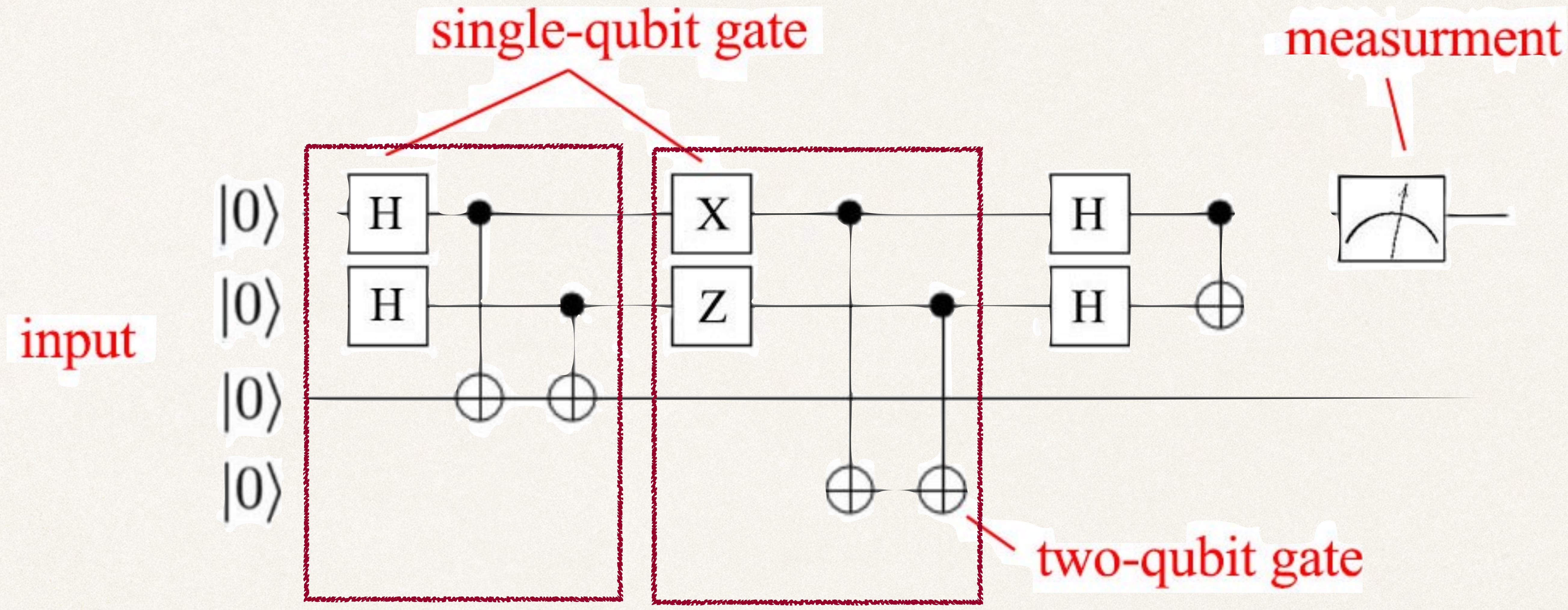
qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Quantum numerical quantum field theory/many-body physics:



Few (very clever) algorithms doing "weird" stuff

Quantum numerical quantum field theory / many-body physics:



$$|\psi(0)\rangle \quad e^{-iH\Delta t} \quad e^{-iH\Delta t} \quad \dots \qquad |\langle\psi(t)|\mathcal{O}|\psi(t)\rangle|^2$$

Troter formula

but Quantum Physics is easy !

Quantum numerical quantum field theory/many-body physics:

1. Encode the Hilbert space into qubits
2. Prepare the initial state
3. Encode the hamiltonian into quantum gates
4. Find something suitable to measure

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Field theories have infinite dimensional Hilbert spaces

but

Quantum computers have finite registers $\sim e^N$

⬇

Discretize space (lattice)

bosonic theories: discretize field space

Field theories have infinite dimensional Hilbert spaces

but

Quantum computers have finite registers ~e^N

Example: nuclear physics (protons and neutrons, spin up and down)

$|q_1\rangle$

$|q_2\rangle$

$|q_3\rangle$

$|q_4\rangle$

$|0000\rangle$ — vacuum

$|1010\rangle$ — neutron up, proton down

Example: nuclear physics (protons and neutrons, spin up and down)

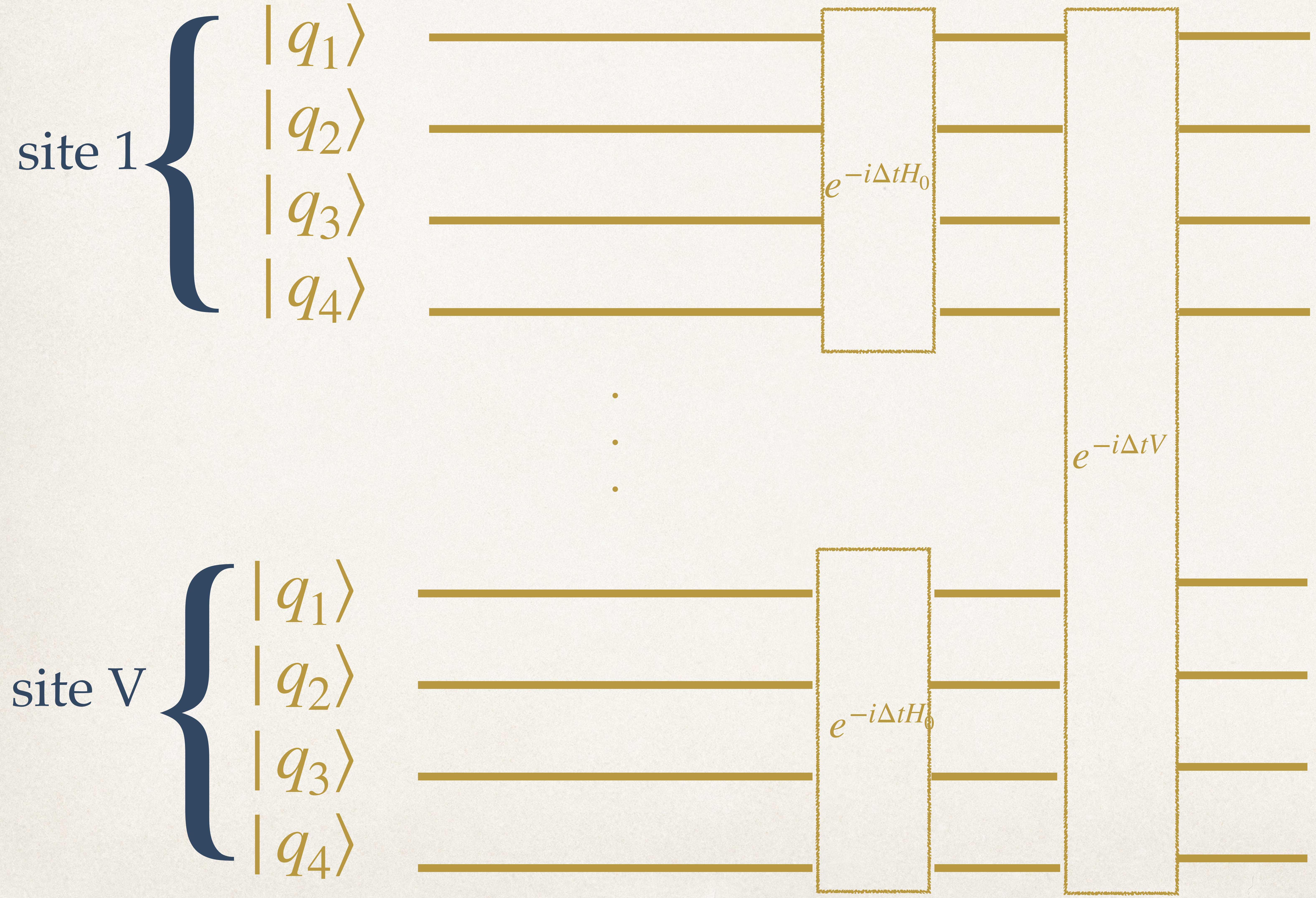
# of qubits ~ V

Hilbert space dimension 4^V

$|0000\rangle$ — vacuum

$|1010\rangle$ — neutron up, proton down

Example: nuclear physics (protons and neutrons, spin up and down)



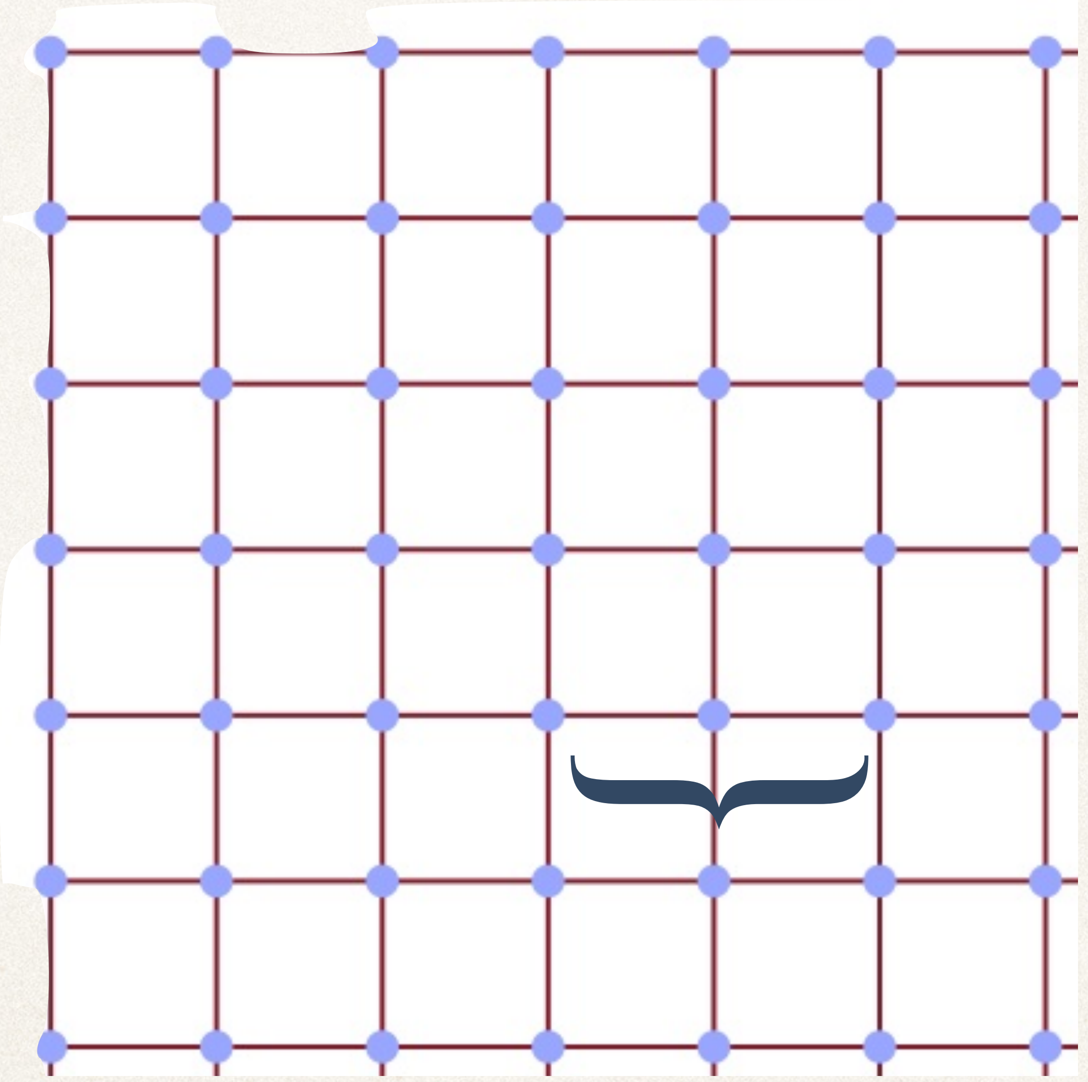
Local hamiltonian,
polynomial # of gates,
gates ~ V,
exponential gain !

This kind of encoding does not work for bosons: occupation number n=0,1,2,3, …

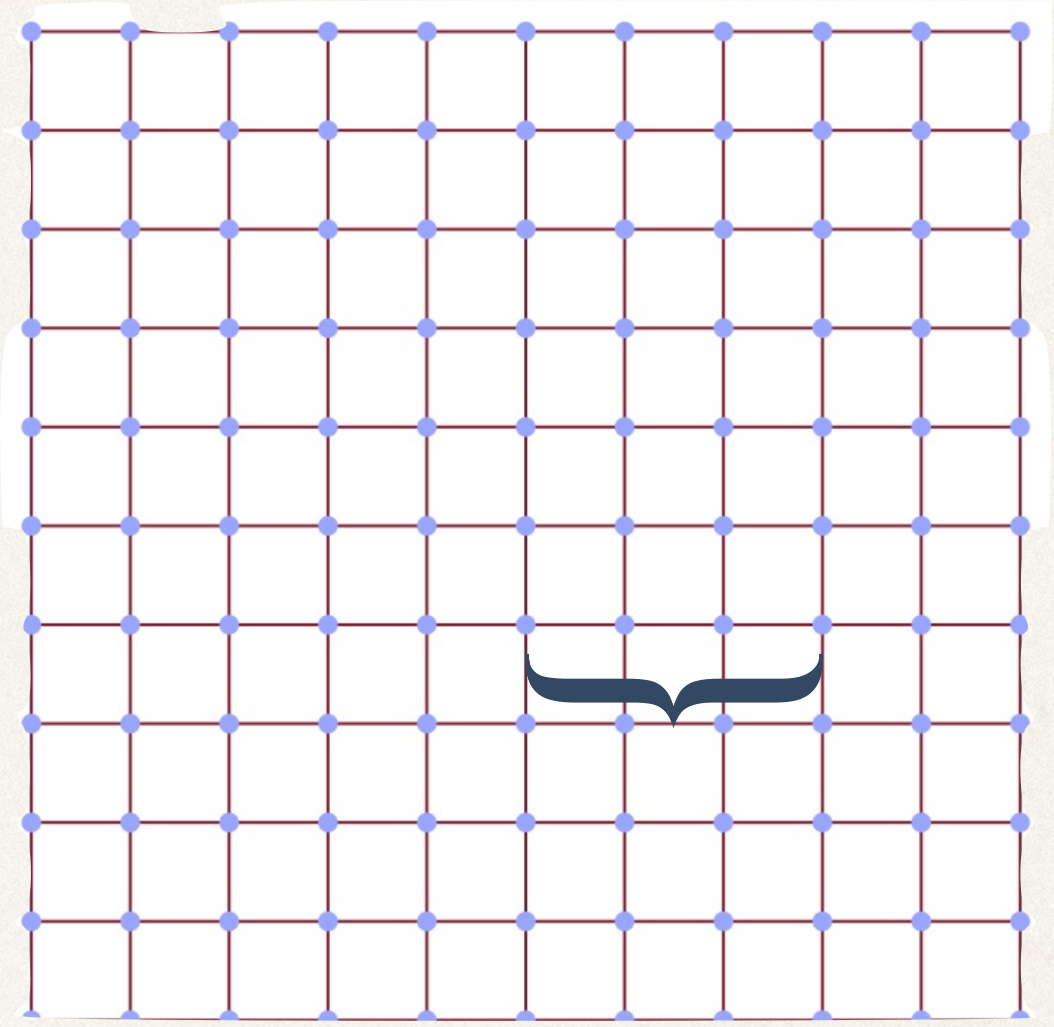
- Condensates ?
- Technical complication
- Naive truncations break symmetries of the theory: no (space) continuum limit

continuum limit:

$g_0(a), g_1(a), \ldots$



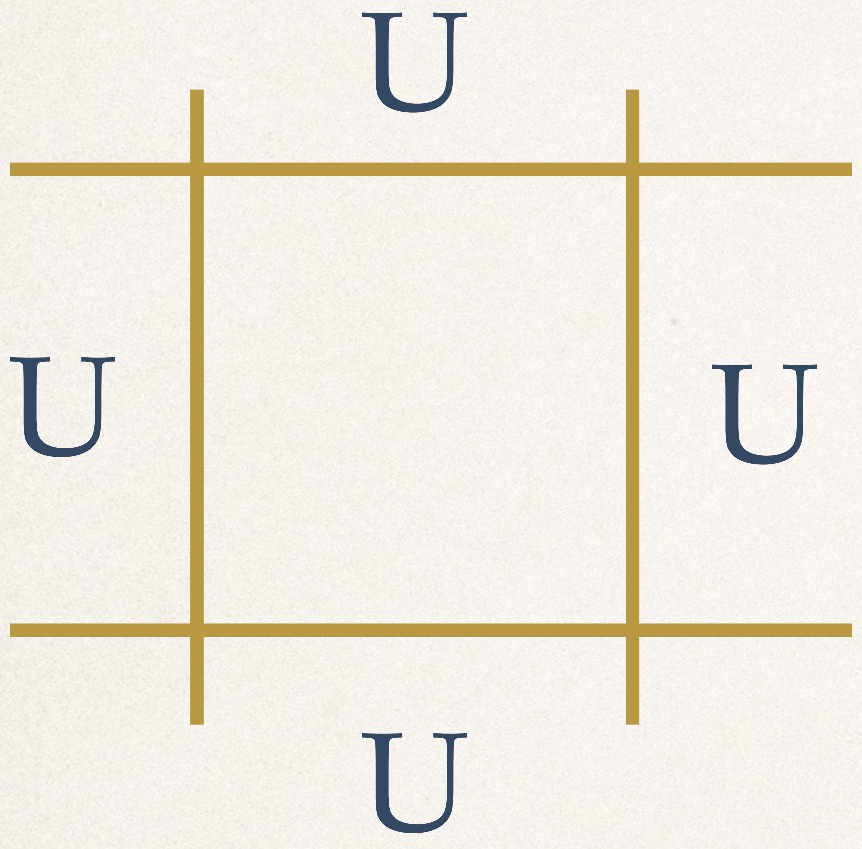
$\tilde{g}_0(a), \tilde{g}_1(a), \ldots$



$$g_0(a \to 0) \sim \frac{1}{\log(\Lambda a)}$$

$$g_4(a \to 0) \sim \frac{1}{a^4}$$

SU(3) gauge theory



at each link:

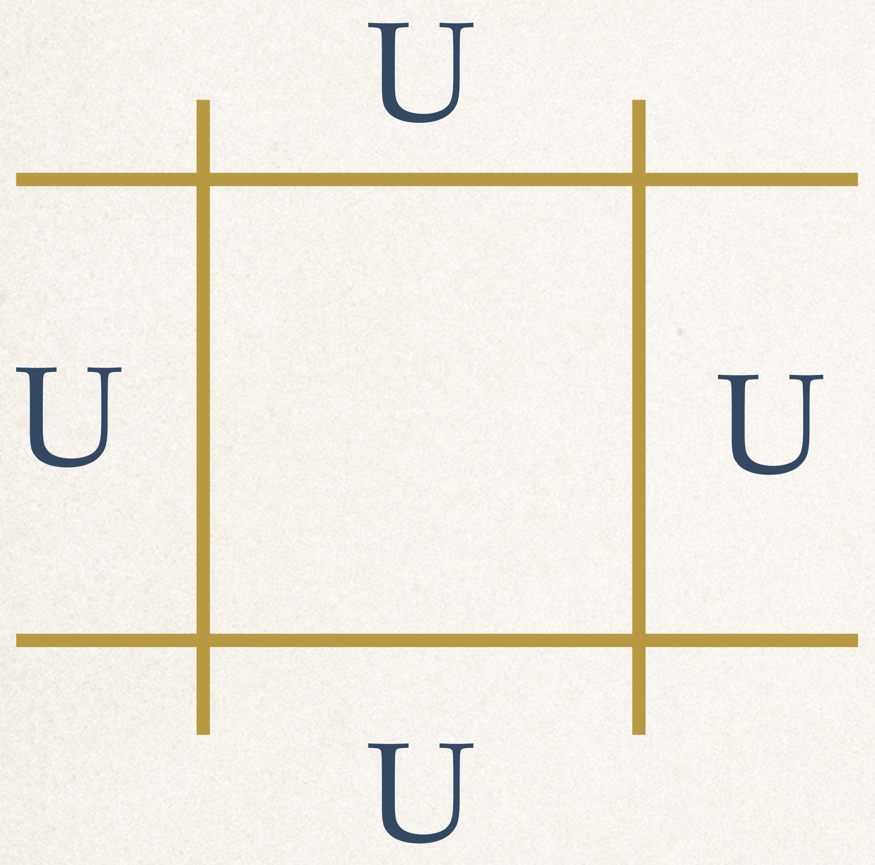
$$\Psi(U) = \Psi(e^{i\lambda^a A^a})$$

infinite-dimensional space

8-dimensional space

finest discretization: SU(3) ⟶ S(1080) (“Valentiner group”)

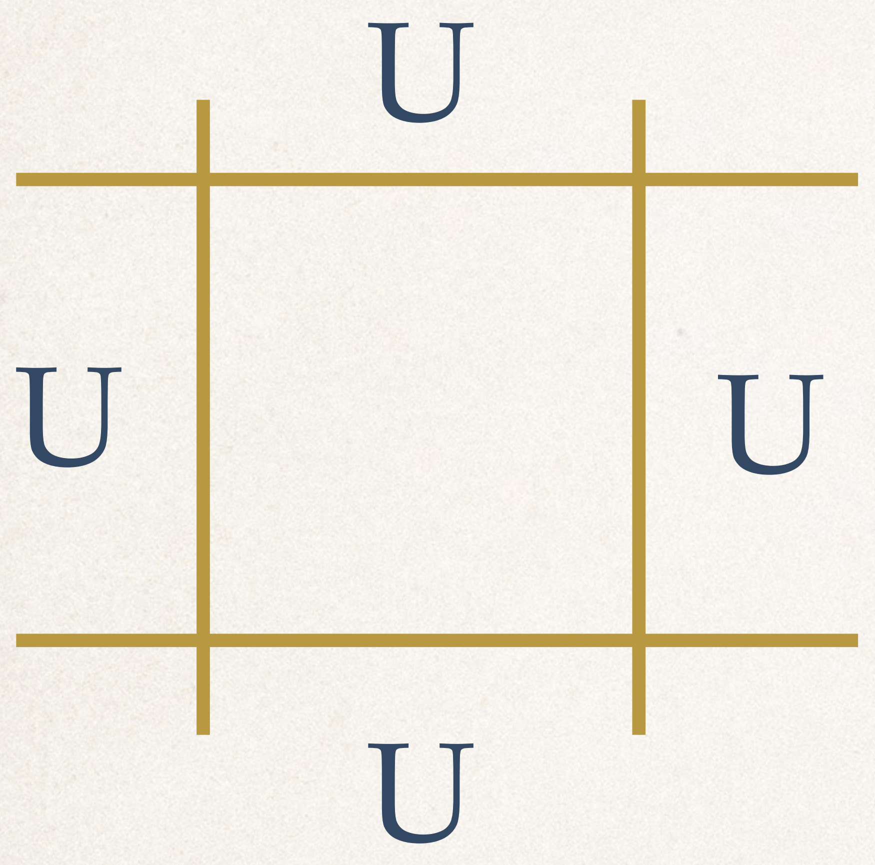
S(1080) gauge theory



$$\Psi(U) = \sum_{i=1}^{1080} \psi_i U_i$$

1080-dimensional space (11 qubits)

S(1080) gauge theory

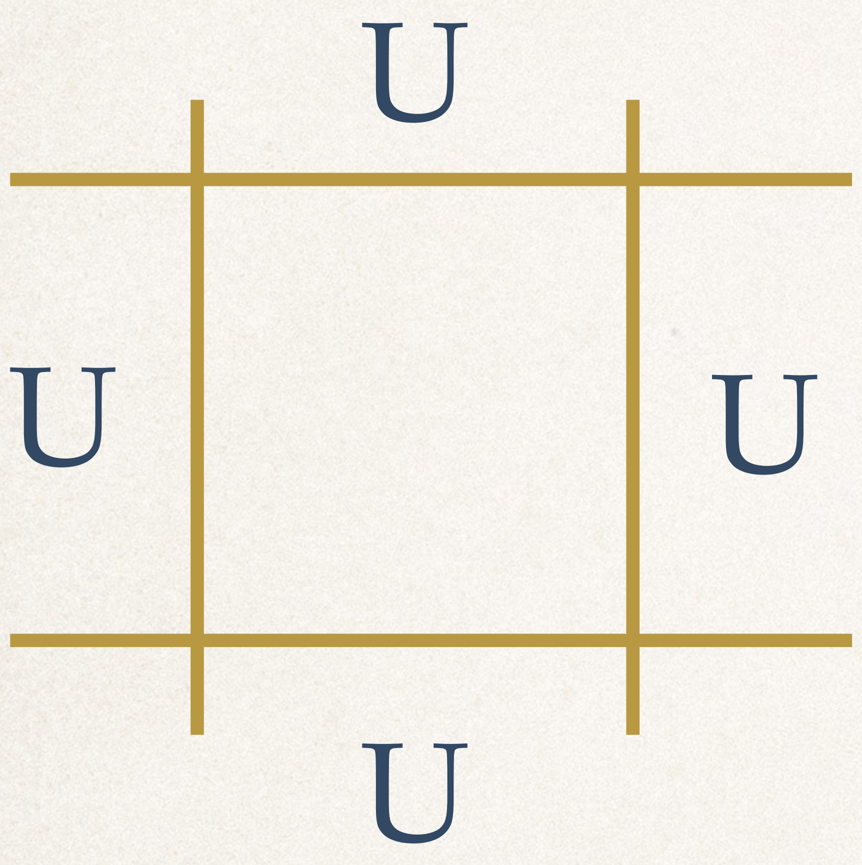


$$S = -\frac{2}{g_0^2}\sum_p \Box_p - \frac{1}{g_1^2}\sum_p \Box_p^2$$

No continuum limit.

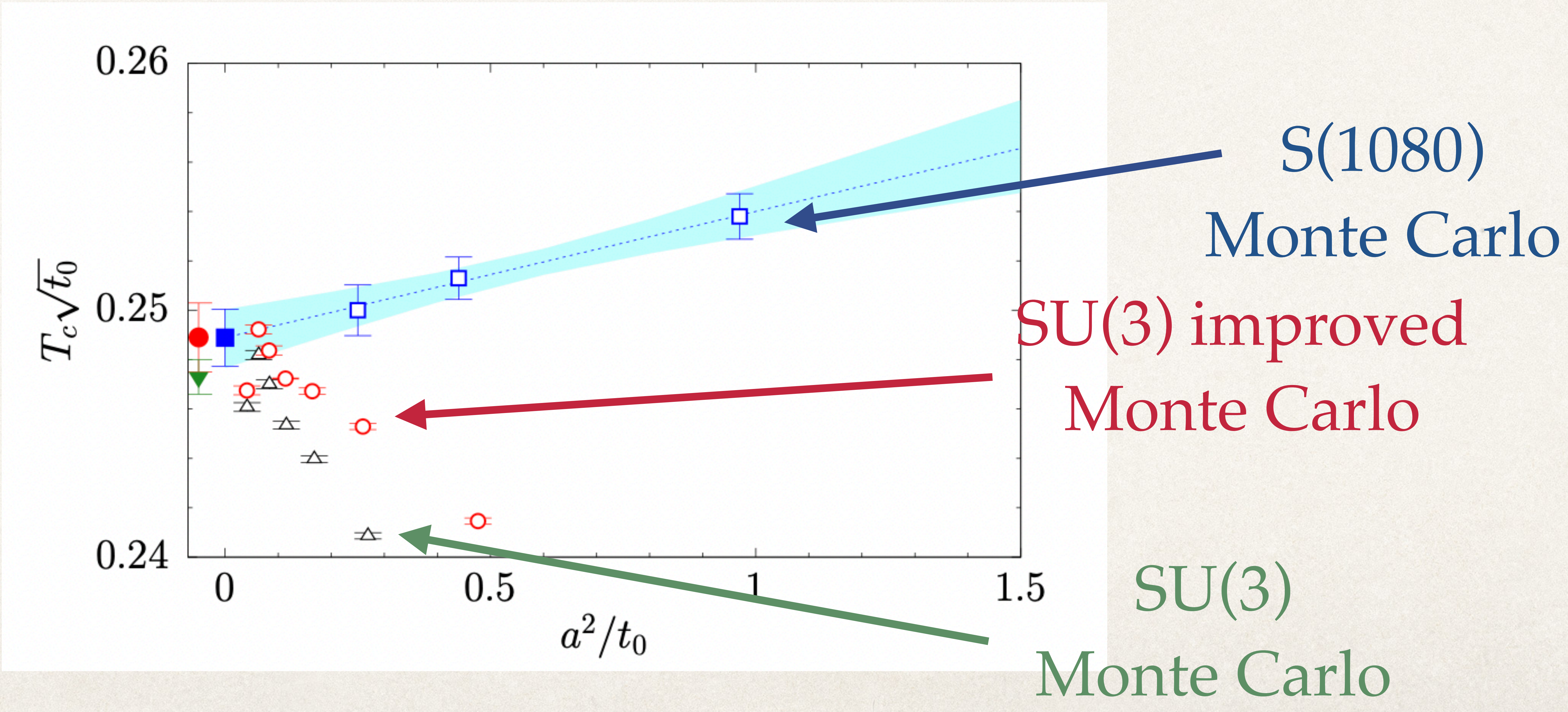
There are no g_0, g_1 for fine enough lattices

S(1080) gauge theory



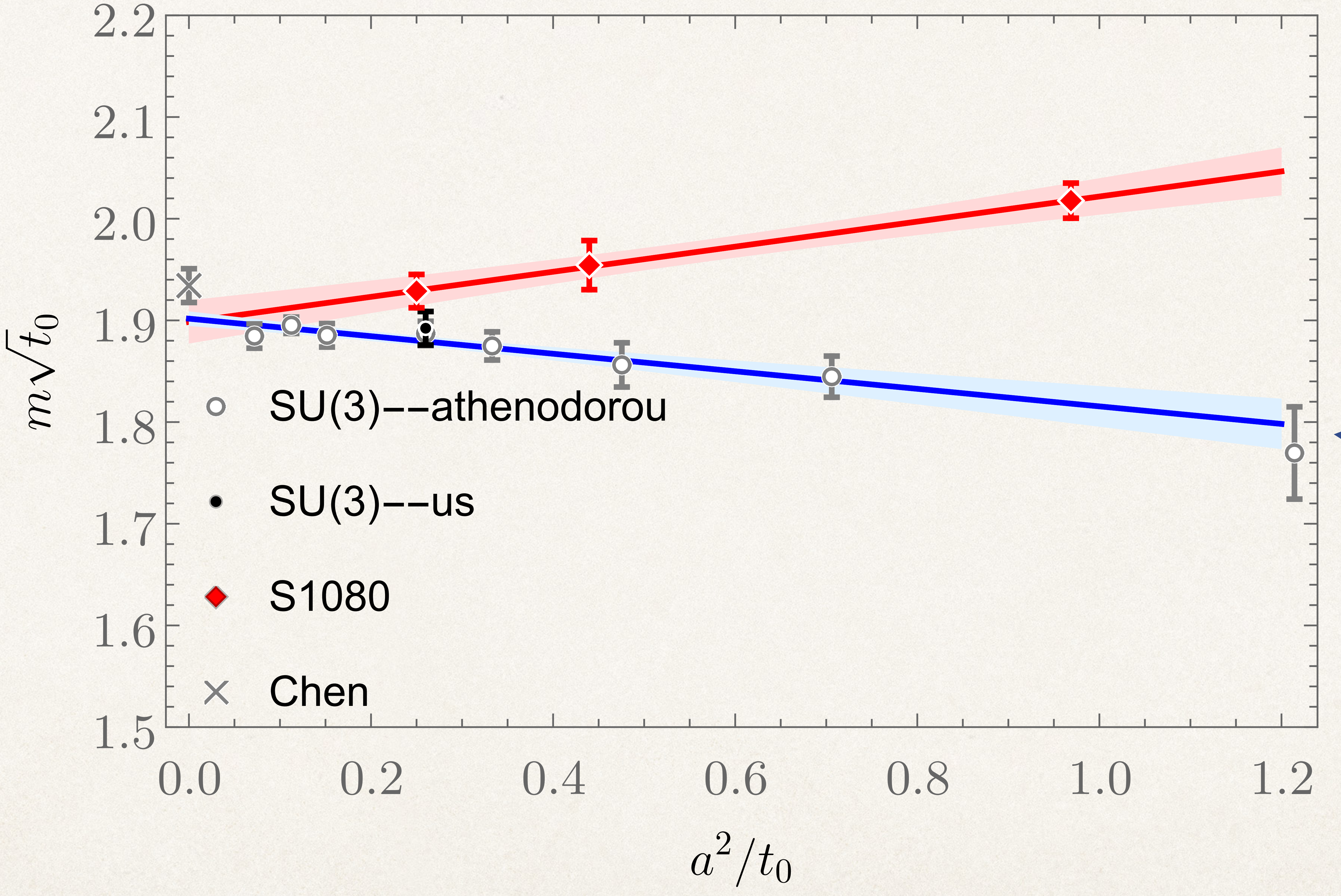
$$S = -\frac{2}{g_0^2}\sum_p \square_p - \frac{1}{g_1^2}\sum_p \square_p^2$$

extrapolate to the same continuum limit



S(1080) gauge theory

E++
glueball mass



S(1080)

SU(3)
Monte Carlo

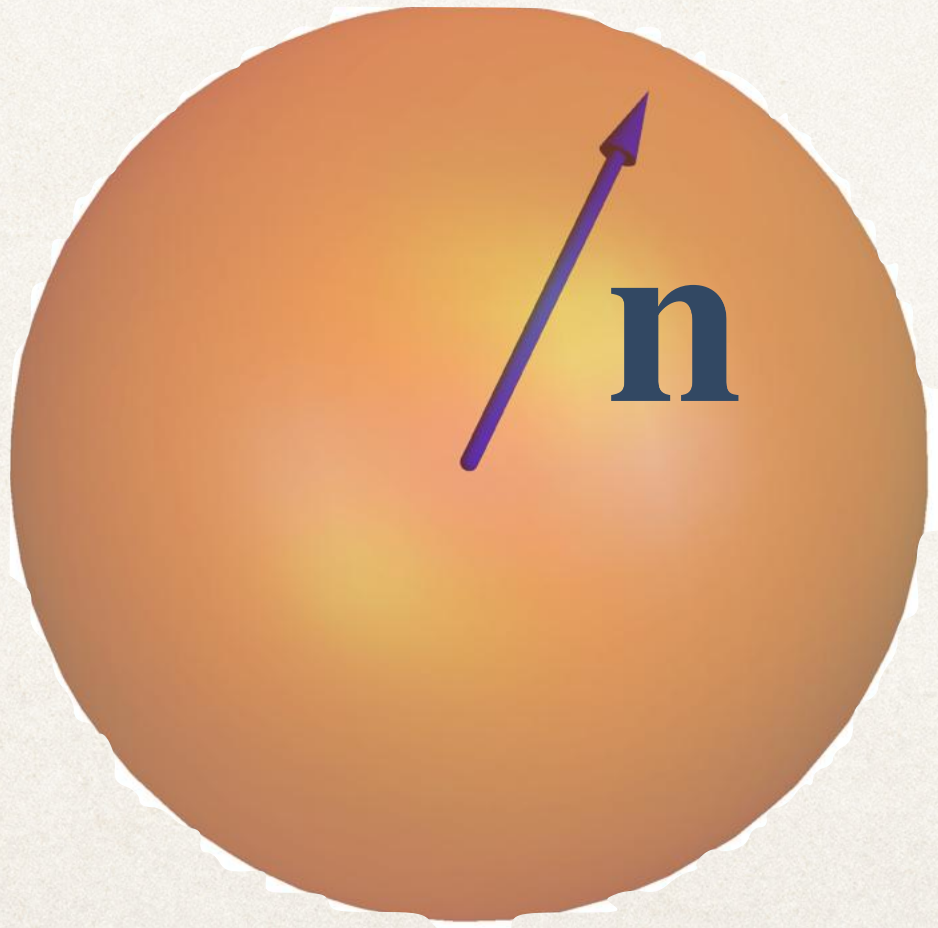
O(3) σ-model

- in 1+1D it is asymptotically free, like QCD

$$S = \frac{1}{2g^2}\int d^2x\ \partial_\mu \mathbf{n} \,.\, \partial^\mu \mathbf{n}$$

unit vector

at each point:



$$\Psi(\mathbf{n})$$

infinite-dimensional space

2-dimensional space: S^2

O(3) σ-model

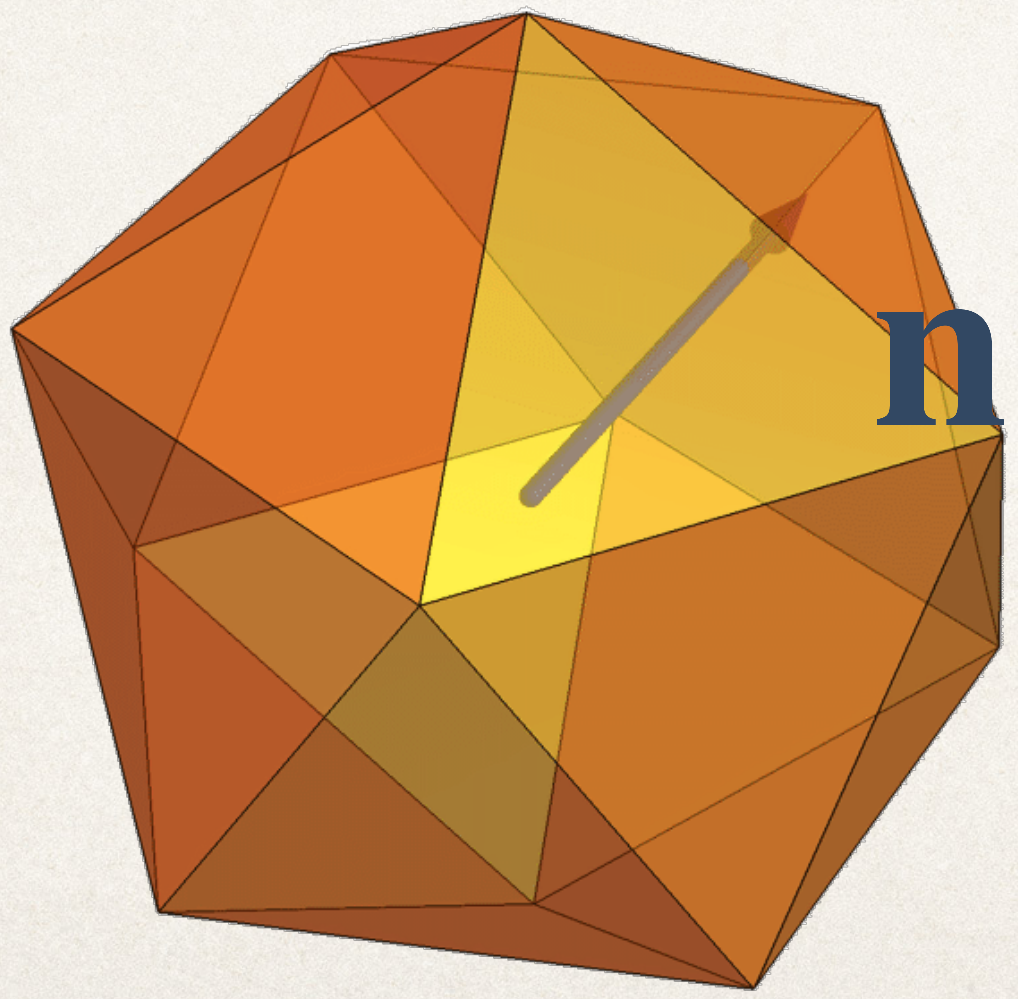
- in 2+1D it is asymptotically free, like QCD

$$S = \frac{1}{2g^2}\int d^2x\ \partial_\mu \mathbf{n}\,.\,\partial^\mu \mathbf{n}$$

unit vector

rotation invariant: O(3)

at each point:



$$\Psi(\mathbf{n})$$

20-dimensional space

discrete: 20 points

O(3) σ-model

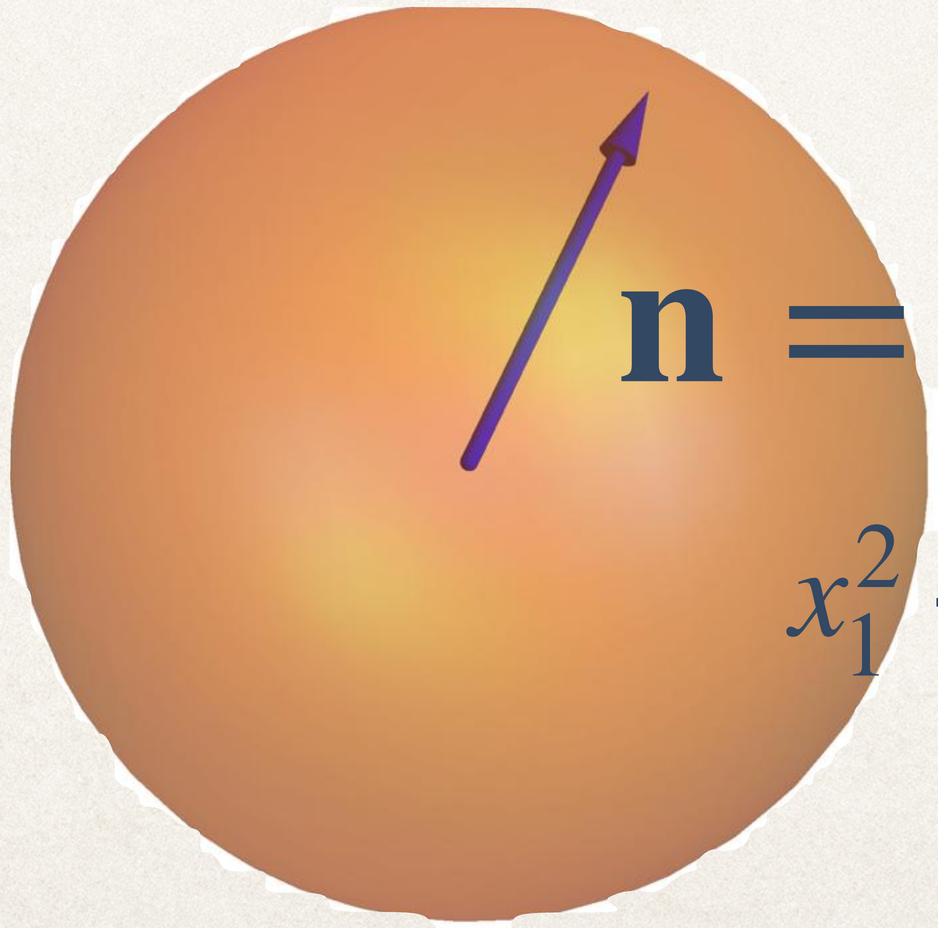
- in 2+1D it is asymptotically free, like QCD

$$S = \frac{1}{2g^2}\int d^2x\ \partial_\mu \mathbf{n} . \partial^\mu \mathbf{n}$$

unit vector

rotation invariant: O(3)

at each point:



$$\mathbf{n} = (x_1, x_2, x_3)$$

$$x_1^2 + x_2^2 + x_3^2 = 1$$

$$\Psi = \psi_0 + \psi_i x_i + \psi_{ij} x_i x_j + \cdots$$

infinite-dimensional space

S^2

O(3) σ-model

- in 2+1D it is asymptotically free, like QCD

$$S = \frac{1}{2g^2}\int d^2x\ \partial_\mu \mathbf{n} \,.\, \partial^\mu \mathbf{n}$$

unit vector

rotation invariant: O(3)

at each point:

$$\mathbf{n} = (x_1, x_2, x_3) \qquad \Psi = \psi_0 + \psi_i \sigma_i$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 3$$

4-dimensional space

4-points: "fuzzy sphere"

O(3) σ-model

- in 2+1D it is asymptotically free, like QCD

$$S = \frac{1}{2g^2}\int d^2x \, \partial_\mu \mathbf{n} \, . \, \partial^\mu \mathbf{n}$$

unit vector

rotation invariant: O(3)

at each point:

$$\mathbf{n} = (x_1, x_2, x_3) \quad \Psi = \psi_0 + \psi_i \sigma_i$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 3$$

4-dimensional space

4-points: "fuzzy sphere"

Fuzzy O(3) σ-model

Hilbert space: $(C^4)^V$

$$H\Psi = \sum_x \left[\eta g^2[\sigma_i(x), [\sigma_i(x)], \Psi] \pm \frac{\eta}{g^2}\sigma_i(x)\sigma_i(x+1)\Psi \right]$$

∇^2

$\partial\mathbf{n} \,.\, \partial\mathbf{n} \approx (\mathbf{n}_{x+1} - \mathbf{n}_x)^2 = 2 - 2\mathbf{n}_{x+1} \,.\, \mathbf{n}_x$

exact O(3) invariance

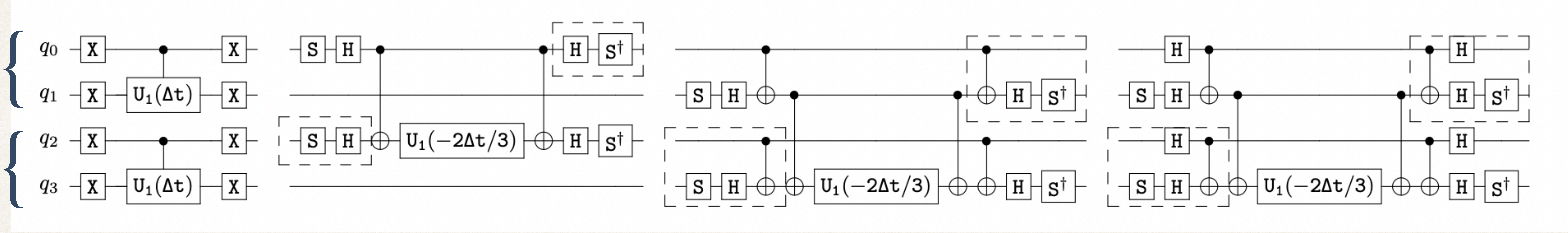
σ-model is exactly solvable

fuzzy model can be “solved” by tensor network technology

Fuzzy O(3) σ-model

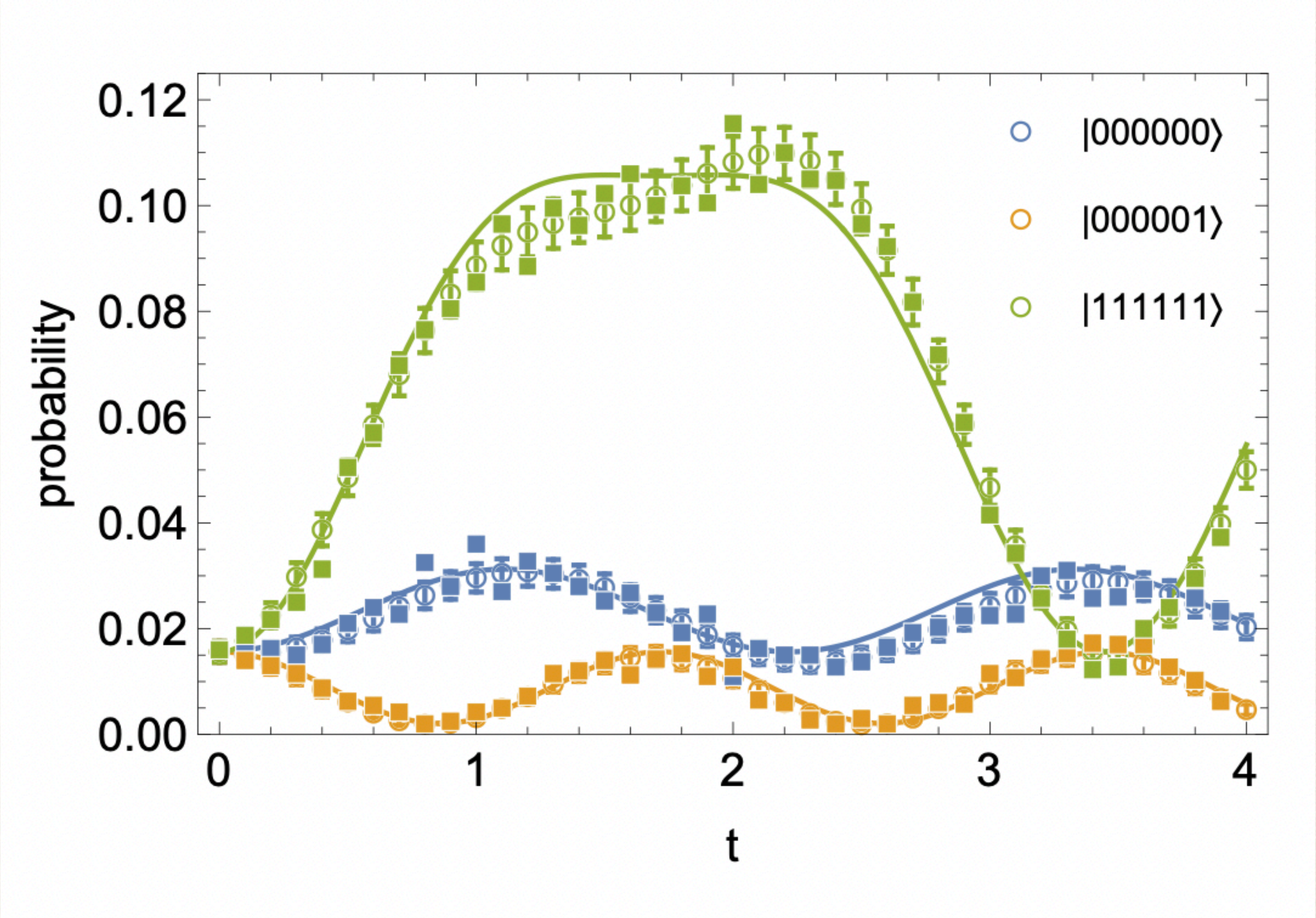
site 1

site 2



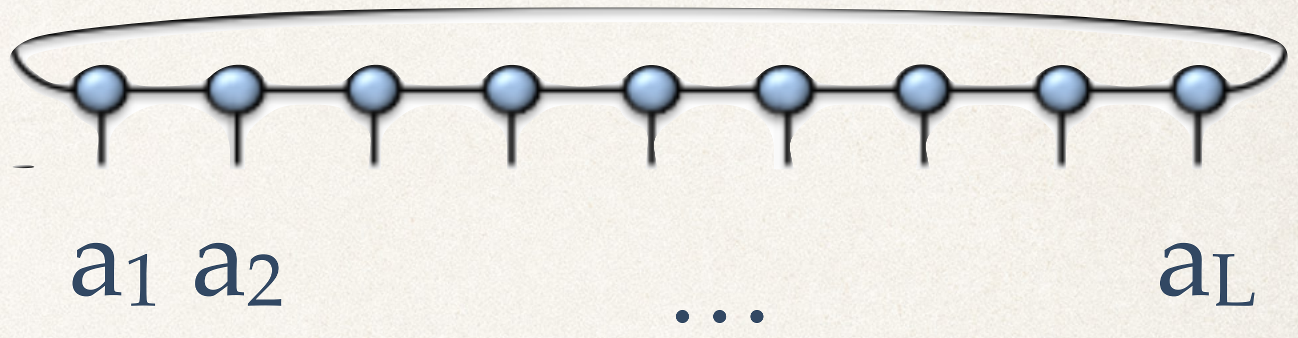
$e^{-i\Delta t K}$ $e^{-i\Delta t V_1}$ $e^{-i\Delta t V_2}$ $e^{-i\Delta t V_3}$

3-site simulation:



Fuzzy O(3) σ-model

$$|\Psi\rangle = \mathrm{tr}[A^{a_1}\cdots \ldots A^{a_N}]\,|a_1\cdots a_L\rangle$$

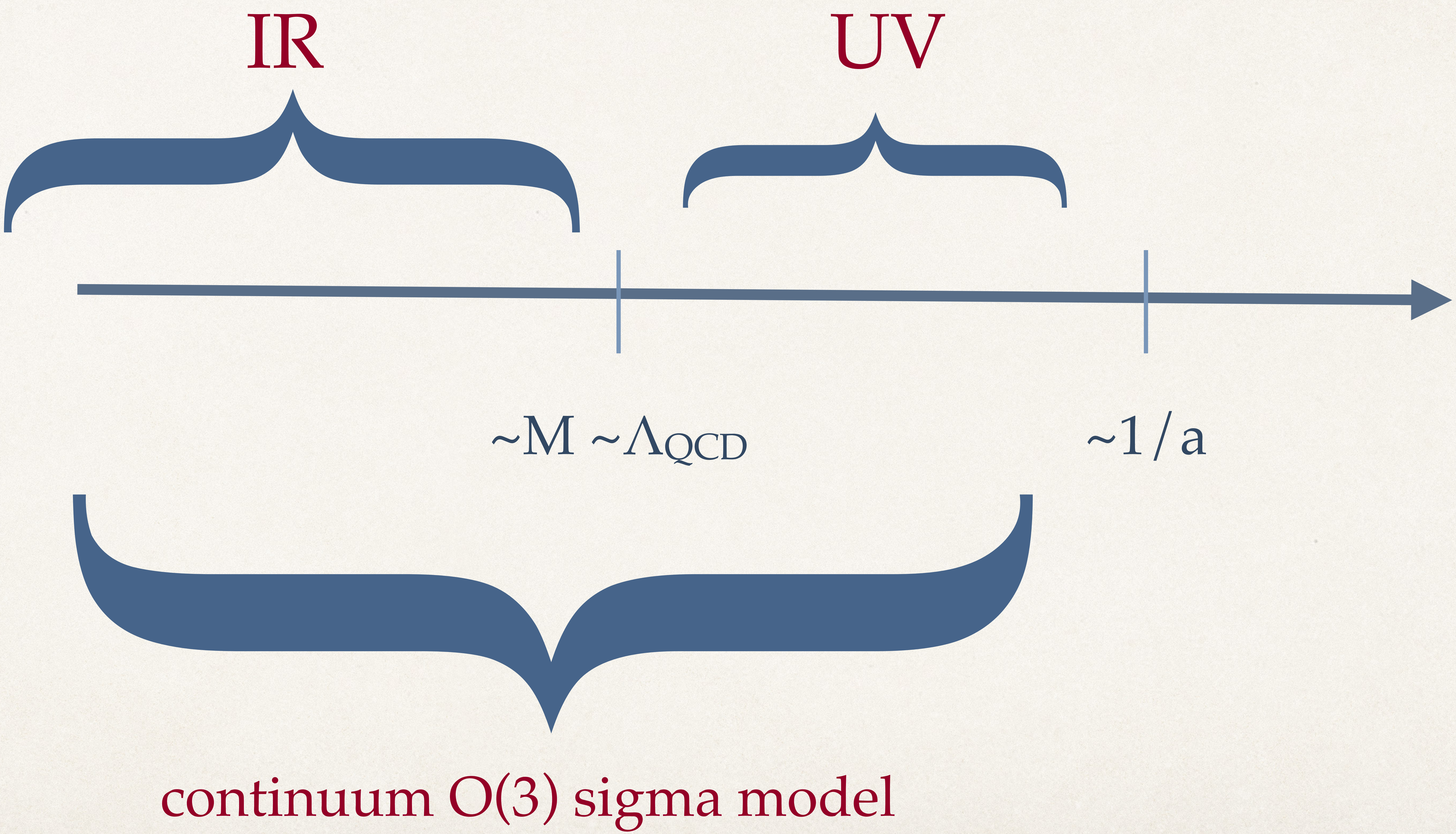


$a_i = 1,2,3,4$

variational parameters $A^{a}{}_{ij}$ $\quad i = 1,\ldots b$

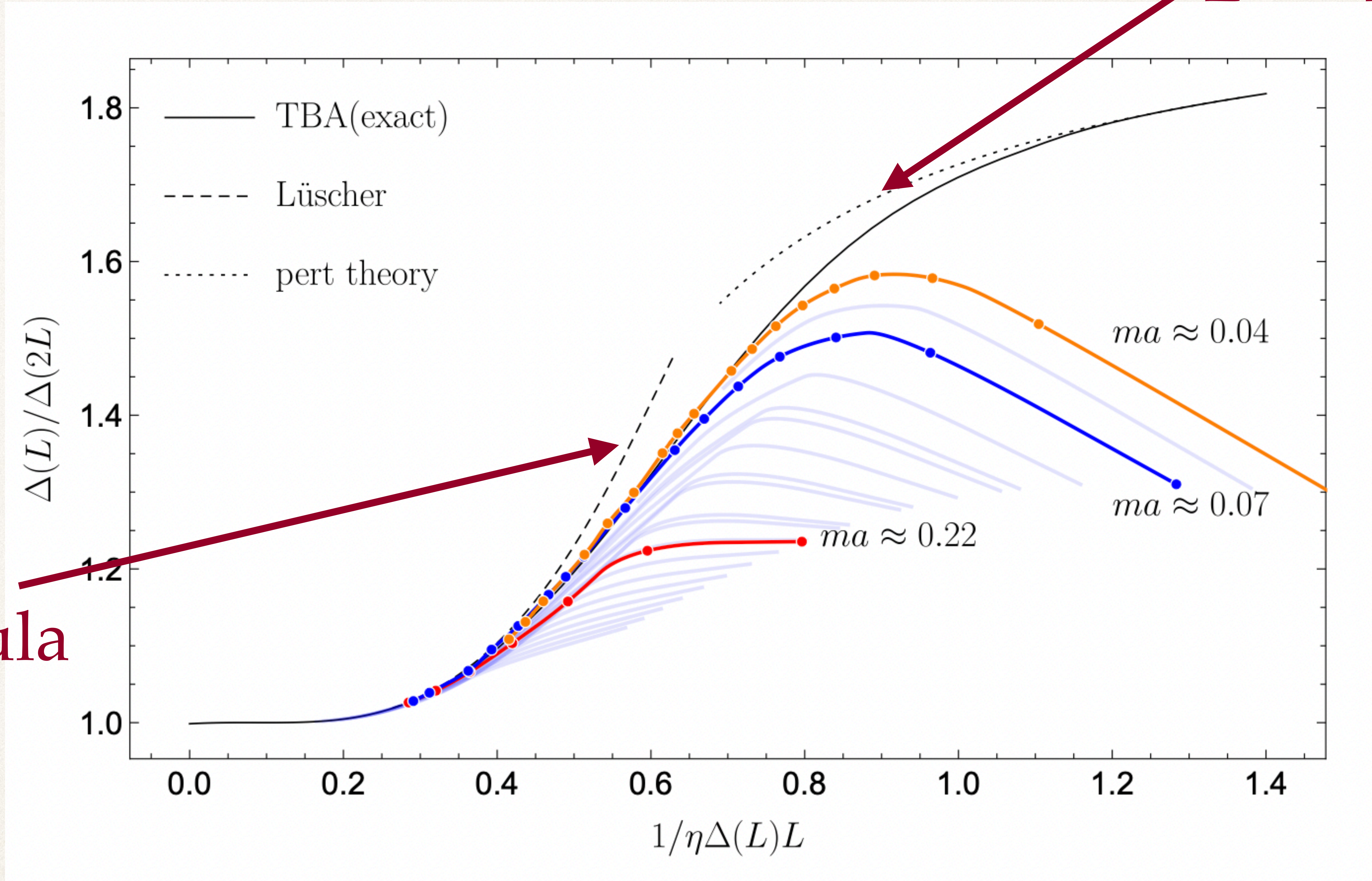
1. find energy gap Δ and correlation length 1/m
2. adjust η so Δ=m (Lorentz symmetry)
3. Δ(L) is determined by phase shifts

O(3) σ-model (asymptotically free)



Antiferromagnetic fuzzy O(3) σ-model

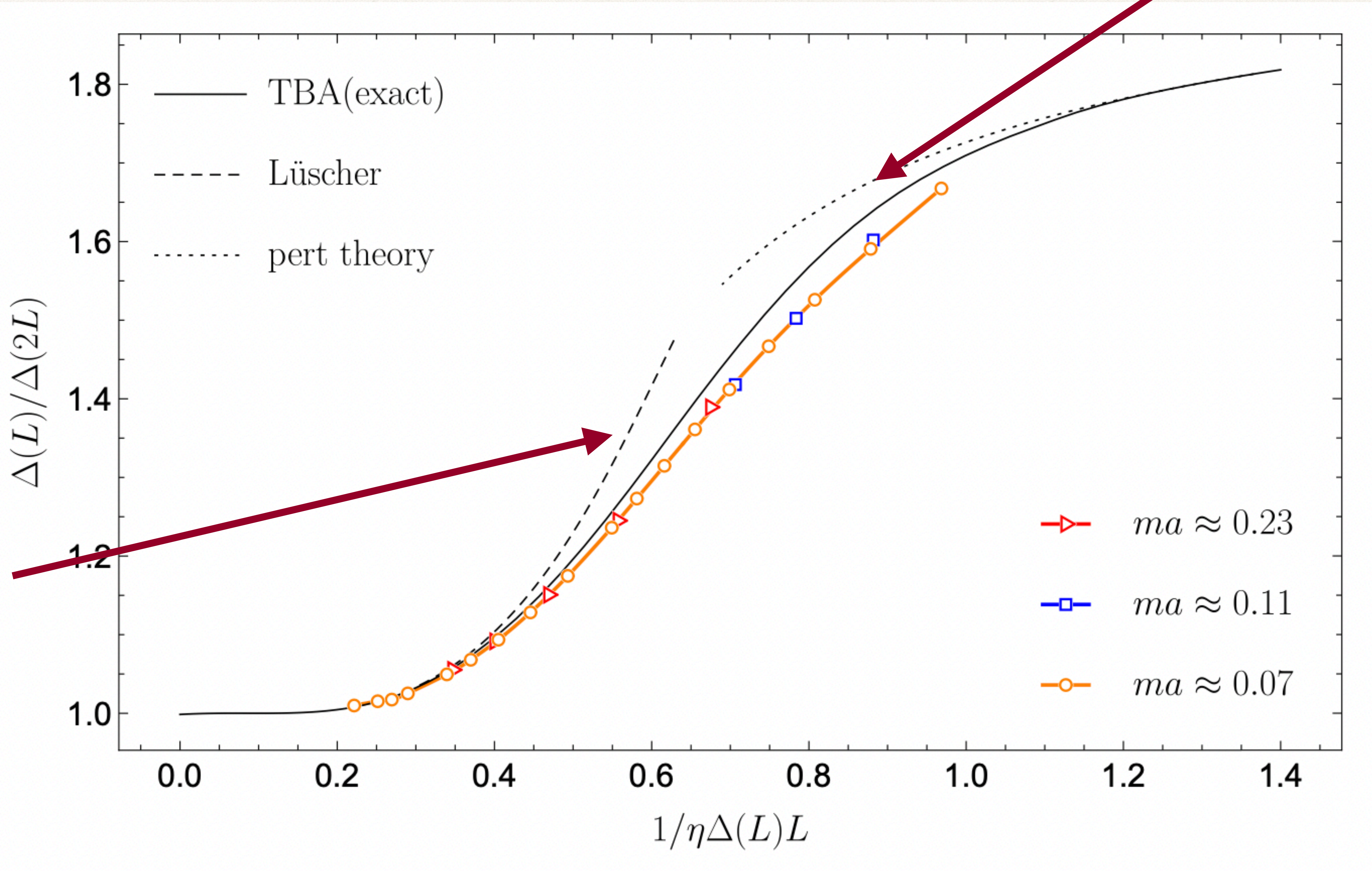
L << 1/m: pert. theory



L >> 1/m:
Luscher formula

Ferromagnetic fuzzy O(3) σ-model

L << 1/m: pert. theory



L >> 1/m:
pert. theory

Generalizations

- O(5), O(7), … are running now
- different “commutative” truncation of O(3) is running now
- O(4)=SU(2) x SU(2): chiral model
- SU(2) gauge theory is reminiscent of chiral models
- SU(3)? Quarks?

Summary

- (Trotterized) time evolution mimics real time evolution
- local hamiltonian can lead to exponential improvement on finite density/real time calculations
- encoding bosonic theories is tricky: preserve some symmetries to recover the continuum limit
- fuzzy sphere construction works for the σ-model; what about other models?