ZAMOLODCHIKOV'S C FUNTION IN THE STUDY OF THE STABILITY OF TOPOLOGICAL PHASES OF MATTER

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NON-ABELIAN TOPOLOGICAL PHASES IN 2+1 D

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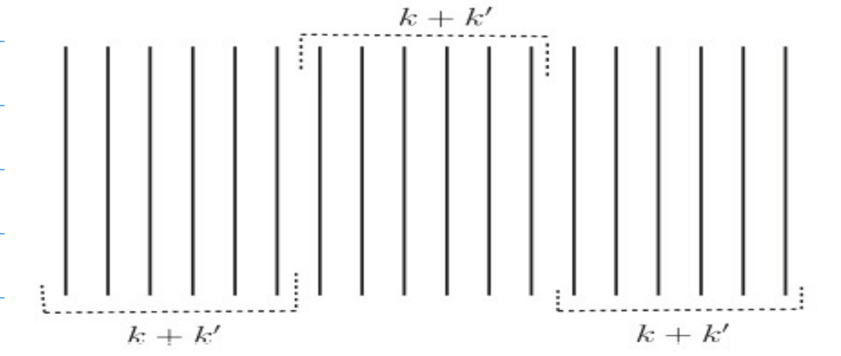
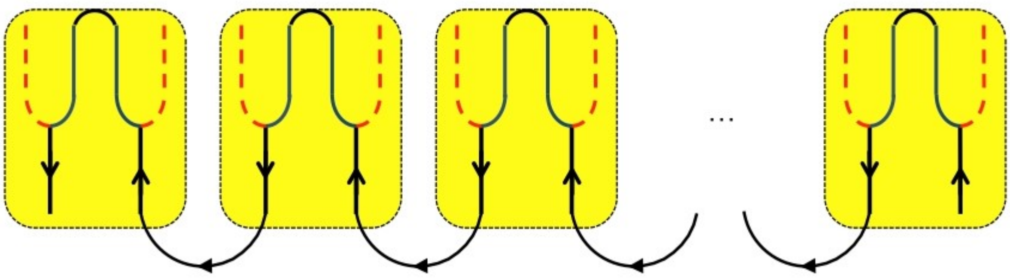


$$\mathcal{L}_0 = \sum_{I=1}^{N} \sum_{\sigma=1}^{2} \left[i\, \psi^*_{R\sigma I}\, \partial_+ \psi_{R\sigma I} + i\, \psi^*_{L\sigma I}\, \partial_- \psi_{L\sigma I} \right]$$

$$\boxed{c = 2N}$$

INTER AND INTRA WIRE INTERACTIONS



SUGAWARA'S CONSTRUCTION

$$u(2k)_1 \supset u(1) \oplus su(2)_k \oplus su(k)_2$$

$$\Rightarrow T_{R/L}[2k] = T_{R/L}[u(1)] + T_{R/L}[su(2)_k] + T_{R/L}[su(k)_2]$$

GAPPING THE $u(1)$ AND $SU(k)_2$ $(k \Leftrightarrow k')$ SECTORS

$$\mathcal{L}_{u(1)} = -g_u \prod_{i=1}^{k} \prod_{\sigma=1}^{2} \psi^*_{R,\sigma,i} \prod_{i=1}^{k} \prod_{\sigma=1}^{2} \psi_{L,\sigma,i} + [\psi^*_R \Leftrightarrow \psi_L, \; \psi_L \Leftrightarrow \psi^*_R]$$

$$\mathcal{L}_{SU(2)} = -\lambda_{SU(2)} \sum_{a=1}^{3} J^a_R J^a_L$$

$$\mathcal{L}_{SU(k)} = -\lambda_{SU(k)} \sum_{A=1}^{k^2-1} J^A_R J^A_L$$

$$J^a_{R/L} = \sum_{i=1}^{k} \sum_{\sigma,\rho=1}^{2} \psi^*_{R/L\,\sigma i} \frac{\sigma^a_{\sigma\rho}}{2} \psi_{R/L\,\rho i}$$

$$J^A_{R/L} = \sum_{i,j=1}^{k} \sum_{\sigma=1}^{2} \psi^*_{R/L\,\sigma i} T^A_{ij} \psi_{R/L\,\sigma j}$$

⇒ GAPLESS SECTOR

$$g_{k,k'} = \bigoplus_{m=1}^{N} su(2)_k \oplus su(2)_{k'}$$

$$c[g_{k,k'}] = \sum_{m=1}^{N} \left(c[su(2)_k] + c[su(2)_{k'}] \right)$$

$$= N \left(\frac{3k}{k+2} + \frac{3k'}{k'+2} \right)$$

GAPPING THE BULK SECTORS AND THE DIAGONAL SUBGROUP $su(2)_{k+k'}$

$$h_{k,k'} = \bigoplus_{m=1}^{N} su(2)_{k+k'}$$

DIAGONAL GENERATORS:

$$K^{a,m}_{R/L} = J^{a,m}_{R/L} + J'^{a,m}_{R/L}$$

INTERWIRE INTERACTION

$$\mathcal{L}^{INTER}_{su(2)} = - \sum_{m=1}^{N} \sum_{a=1}^{3} \left(\lambda^a_m J^{a\,m}_L J^{a\,m+1}_R + \lambda'_a J'^{a,m}_L J'^{a,m+1}_R \right)$$

GAPPING THE DIAGONAL SECTORS

$$\mathcal{L}^{\mathrm{DIAG}}_{SU(2)} = - \sum_{m=1}^{N} \sum_{a=1}^{3} g^{a}_{m} \, K^{a,m}_{L} K^{a,m}_{R}$$

CONJECTURED TOPOLOGICAL PHASE

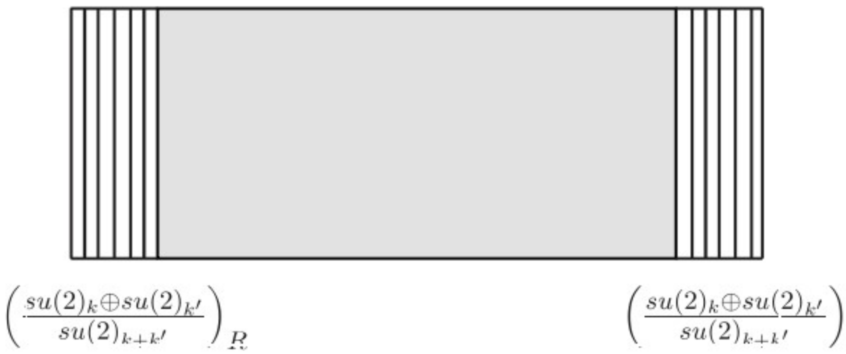
$$\left(\frac{SU(2)_k \oplus SU(2)_{k'}}{SU(2)_{k+k'}} \right)_{R/L}$$

$k'=1$ $$c(g/k) = 1 - \frac{6}{(k+2)(k+3)}$$

MINIMAL MODELS

$k'=2$ $$c(g/k) = \frac{3}{2}\left[1 - \frac{8}{(k+2)(k+4)}\right]$$

SUPERCONFORMAL MINIMAL MODELS



STABILITY

$K = J + J'$

$\Longrightarrow \quad [J, K] \neq 0$

i) $\lambda \gg g$

$\Longrightarrow$ GAPPING ALL THE $SU(2)_{K,K'}$ OF THE BULK

$\Longrightarrow$ GAPPING THE $SU(2)_{L,KK'}$ OF THE LEFT BOUNDARY AND $SU(2)_{R,KK'}$ OF THE RB.

$\Longrightarrow$ GAPPED TOPOLOGICAL PHASE

$$(SU(2)_K \oplus SU(2)_{K'})_R \qquad (SU(2)_K \oplus SU(2)_{K'})_L$$

STABLE AGAINST PERTURBATIONS OF KK INTERACTIONS

$\Longrightarrow$ NON-COSET PHASE

ii) $\lambda \ll g$

$\Longrightarrow$ GAPPING ALL $SU(2)_{k+k'}$ SECTORS

$$\Longrightarrow \left(\frac{SU(2)_k + SU(2)_{k'}}{SU(2)_{k+k'}}\right)_{L/R} \quad \text{FOR ALL BUNDLES}$$

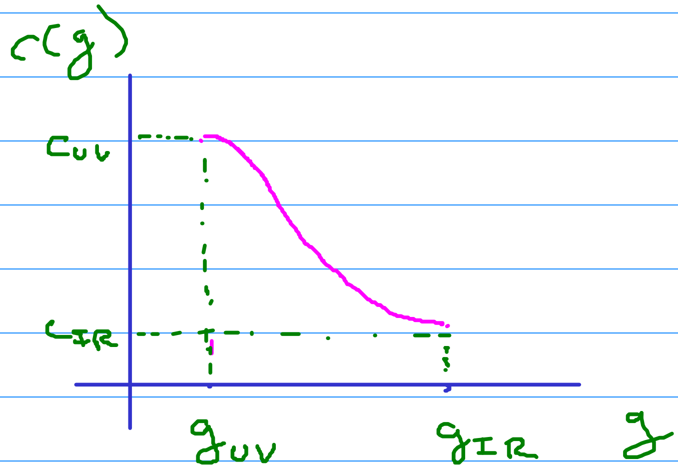
$\Longrightarrow$ UNSTABLE AGAINST JJ PERTURBATIONS

IF JJ IS RELEVANT

$\Longrightarrow$ COSET TOPOLOGICAL PHASE.

C-THEOREM

$\Rightarrow$ $c(g)$ MONOTONICALLY DECREASING ALONG A TRAJECTORY OF THE RG FLOW AND COINCIDES WITH THE CENTRAL CHARGE AT THE FIXED POINTS.



DEFINITION

$$c(g) = 2F - G - \frac{3}{8} H$$

$$\langle T(z,\bar{z})\, T(0,0)\rangle = \frac{F(m z\bar{z})}{z^4}$$

$$\langle T(z,\bar{z})\, \Theta(0,0)\rangle = \frac{G(m z\bar{z})}{z^3 \bar{z}}$$

$$\langle \Theta(z,\bar{z})\, \Theta(0,0)\rangle = \frac{H(m z\bar{z})}{z^2 \bar{z}^2}$$

SIMPLER MODEL

$$S_0 = \int d^2x \sum_{i,\sigma}' \left(\psi^\dagger_{Ri\sigma} \partial_z \psi_{Ri\sigma} + \psi^\dagger_{Li\sigma} \partial_{\bar z} \psi_{Li\sigma} \right)$$

$i = 1, \dots, N_c$
$\sigma = 1, \dots, N_f$

SYMMETRY GROUP: $U_R(N) \otimes U_L(N)$

$N = N_c N_f$

GAPPING THE U(1) AND SU(N_c) SECTORS

$$\mathcal{L}_U = -g_u^0 \prod_{i=1}^{N_c} \prod_{\sigma=1}^{N_f} \psi^*_{Ri\sigma} \prod_{i=1}^{N_c} \prod_{\sigma=1}^{N_f} \psi_{Li\sigma} + (L \Leftrightarrow R)$$

$$\mathcal{L}_{SU(N_c)} = -\tilde{\lambda} \sum_{a=1}^{N_c^2-1} J_R^a J_L^a$$

$$\mathcal{L}_{Th} = -g_t^0 J_R J_L$$

AUXILIARY FIELDS

$$S=\int d^2x \sum_i \psi^*_{Li\sigma}\left[\delta_{ij}(\partial_{\bar z}-B_{\bar z})-A^a_{\bar z}t^a_{ij}\right]\psi_{Lj\sigma}+$$

$$+(L\Leftrightarrow R,\ z\Leftrightarrow\bar z)+\frac{1}{\tilde\lambda}\mathrm{tr}(A_z A_{\bar z})+\frac{1}{g_z^0}B_z B_{\bar z}$$

$$+\mathcal{L}_u$$

DECOUPLING OF THE AUXILIARY FIELDS

$$\psi_R=e^{(\theta+i\phi)}\tilde\psi_R \qquad \psi^*_R=e^{-\theta-i\phi}\tilde\psi^*_R$$

$$\psi_L=e^{-\theta+i\phi}\tilde\psi_L \qquad \psi^*_L=e^{\theta-i\phi}\tilde\psi^*_L$$

$$B_z=\partial_z(\theta+i\phi) \qquad B_{\bar z}=-\partial_{\bar z}(\theta-i\phi)$$

$$J_F=e^{-\frac{N}{\pi}\int d^2x\,\partial_z\theta\,\partial_{\bar z}\theta}$$

$$J_B=\partial^2$$

$$\tilde\psi_R=M\tilde\chi_R \qquad \tilde\psi^+_R=\tilde\chi^+_R M^{-1}$$

$$\tilde\psi_L=M^{+-1}\tilde\chi_L \qquad \psi^+_L=\tilde\chi^+_L M^+$$

$$A_z=\partial_z M M^{-1} \qquad A_{\bar z}=-M^{+-1}\partial_{\bar z}M^+$$

$$J_F=e^{N_f W[M^+M]+\frac{b}{4\pi}\int d^2x\,\mathrm{tr}(M^{+-1}\partial_{\bar z}M\,\partial_z M M^{-1})}$$

$$J_B=\partial^2 e^{2N_c W[M^+M]}$$

$$Z = \int D\mu \exp - \Big[-kW[M^+] - kW[M] + \frac{k\lambda}{4\pi} \int d^2x \, \mathrm{tr}\left(M^{+-1} \partial_{\bar z} M^+ \, \partial_z M M^{-1} \right) + \frac{1}{\gamma^2} \int d^2x \, \partial_z \phi \, \partial_{\bar z} \phi + S[\tilde\chi, \theta] + S_{ghost} \Big]$$

$$k = 2Nc + Nf \qquad \lambda = \frac{4\pi}{k\tilde\lambda} + b - 1 \qquad \gamma^2 = \frac{g_t^{(0)}}{1 + a g_t^{(0)}/\pi}$$

$$S[\tilde\chi, \theta] = \int d^2x \sum_{i,\sigma} \tilde\chi^+_{Ri\sigma} \partial_z \tilde\chi_{Ri\sigma} + \tilde\chi^+_{Li\sigma} \partial_{\bar z} \tilde\chi_{Li\sigma} + \frac{1}{\rho^2} \partial_z \theta \, \partial_{\bar z} \theta - g_u^{(0)} \left(e^{-2N\theta} \prod \tilde\chi^+_{Ri\sigma} \prod \tilde\chi_{Li\sigma} + e^{2N\theta} (R \leftrightarrow L) \right)$$

$$S_{ghost} = \int d^2x \left[\sum_i \left(b^i_z \partial_{\bar z} \bar c^i + b^i_{\bar z} \partial_z c^i \right) + \eta_{\bar z} \partial_z \varepsilon + \eta_z \partial_{\bar z} \bar\varepsilon \right]$$

$$\rho^2 = \frac{g_t^{(0)}}{1 + (N+a) g_t^{(0)}/\pi}$$

ABELIAN BOSONIZATION

$$Z_U = \int D\tilde\chi \, D\theta \, e^{-S[\tilde\chi, \theta]}$$

$$\equiv Z_{SG} = \int D\theta \exp\left[- \int d^2x \, \partial_z \theta \, \partial_{\bar z} \theta - \frac{\alpha}{\beta} \cos(N\beta\theta) + \gamma_0 \right]$$

$$\frac{\beta^2}{4\pi} = \frac{1 + a\, g_t^{(0)}/4\pi}{1 + (N+a)\, g_t^{(0)}/\pi} \qquad \frac{\alpha}{\beta^2} = g_u (c\mu)^N$$

BOSONIZED MODEL

$$Z = \det(\partial^2)_{U(1)} \det(\partial^2)_{SU(N)} \det(\not{\partial})^N \, Z_I \, Z_{II}$$

$$Z_I = \int D\phi \, D\vartheta \, e^{-\int d^2x \left[\frac{1}{\beta^2} \partial_{\bar z}\phi \partial_z \phi + \partial_{\bar z}\vartheta \partial_z \theta - \frac{\alpha}{\beta^2} \cos(N\beta\vartheta)\right]}$$

$$Z_{II} = \int DM \, DM^{+} \exp - \Big[- K\left(W[M^{+}] + W[M]\right) +$$

$$+ \frac{K\lambda}{4\pi} \int d^2x \, \mathrm{tr}\left(M^{+-1} \partial_{\bar z} M^{+} \partial_z M M^{-1}\right) \Big]$$

FIXED POINTS

$\beta^2 \to 0 \quad , \quad \beta^2 \to \infty$

$\lambda \to 0 \quad , \quad \lambda \to -1 \quad , \quad \lambda \to \pm\infty$

$$C_{UV} = N + \frac{2 N_c (N_c^2 - 1)}{N_c + N_f} \qquad \lambda = 0$$

$$C_{free} = N \qquad \lambda = \pm\infty$$

$$C_{IR} = \frac{N_c (N_f^2 - 1)}{N_c + N_f} \qquad \lambda = -1$$

C-FUNCTION

$$e^{-S_{eff}[\gamma]} = \int DM\, DM^{+}\; e^{-S[\gamma]}$$

$$B_{\mu\nu;\rho\sigma} = -\frac{2}{\sqrt{\gamma(x)}}\frac{2}{\sqrt{\gamma(y)}}\frac{\delta^2 S_{eff}}{\delta\gamma_{\mu\nu}(x)\,\delta^2\gamma_{\rho\sigma}(y)}\bigg|_{\gamma_{\mu\nu}=\delta_{\mu\nu}}$$

$$=\langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle - \langle T_{\mu\nu}(x)\rangle\langle T_{\rho\sigma}(y)\rangle - \frac{2}{\sqrt{\gamma(x)}}\frac{2}{\gamma(y)}\left\langle\frac{\delta^2 S}{\delta\gamma_{\mu\nu}(x)\,\delta\gamma_{\rho\sigma}(y)}\right\rangle$$

BACKGROUND FIELD METHOD

$$M = M_0\, e^{i\rho(x)} \qquad M^{+} = M_0^{+}\, e^{i\sigma(x)}$$

TWO LOOPS

$$S_{eff}^{(2)} = \langle S_4\rangle - \frac{1}{2}\langle S_3\, S_3\rangle$$

$$S_{eff}^{(2)} = \frac{2\pi N_c(N_c^2-1)}{3k}\frac{(3\lambda^2+\lambda-1)}{(\lambda-1)^2(1+\lambda)}\int d^2x\sqrt{\gamma}\,\gamma^{\mu\nu}\left[\partial_\mu^x G\,\partial_\nu^y G - G\,\partial_\mu^x\partial_\nu^y G\right]\Big|_{x=y}$$

$$+\frac{4\pi N_c(N_c^2-1)}{3k}\frac{(3\lambda^2-3\lambda+1)}{(1-\lambda)^3}\int d^2x\, d^2y\;\varepsilon^{\mu\nu}\varepsilon^{\rho\sigma}\,G\,\partial_\mu^x\partial_\sigma^y G\,\partial_\nu^x\partial_\rho^y G\Big|_{x=y}$$

$$-\frac{2\pi\lambda^2 N_c(N_c^2-1)}{k(1-\lambda)(1+\lambda^2)}\int d^2x\, d^2y\sqrt{\gamma(x)}\sqrt{\gamma(y)}\,\gamma^{\mu\nu}(x)\gamma^{\rho\sigma}(y)\;\times$$

$$\times\left[G\,\partial_\mu^x\partial_\sigma^y G\,\partial_\nu^x\partial_\rho^y G - \partial_\mu^x G\,\partial_\sigma^y G\,\partial_\nu^x\partial_\rho^y G\right]\Big|_{x=y}$$

$$\Delta G(x,y) = \frac{1}{\sqrt{\gamma}} \delta^2(x-y)$$

$$\Delta = \frac{1}{\sqrt{\gamma}} \partial_\mu (\sqrt{\gamma}\, \gamma^{\mu\nu} \partial_\nu)$$

REGULARIZATION

IR: $\mathcal{L}_m = -m^2 \, \mathrm{tr}(M + M^+)$

UV: $G(x,x) = \frac{1}{2\pi\varepsilon} + \bar{G}(x)$

$$G(x,y) = \bar{G}(x,y) - \frac{1}{2\pi} \ln |s(x,y)|$$

$$\bar{G}(x) = \bar{G}(x,y)\Big|_{x=y}$$

INTERPOLATION BETWEEN THE FIXED POINTS

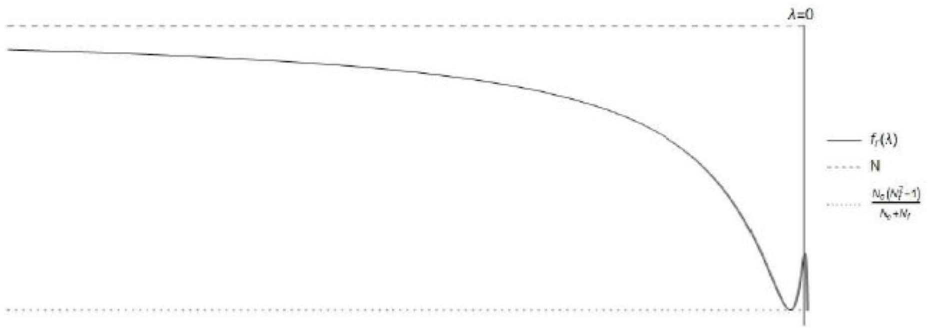
$$\Rightarrow \quad S^{(2)}_{eff} = \frac{N_c(N_c^2-1)}{k} f_r(\lambda)\, \Gamma$$

$$\Gamma = \frac{1}{96\pi} \int d^2x\, d^2y \sqrt{\gamma(x)} \sqrt{\gamma(y)}\, R(x)\, G(x,y)\, R(y)$$

ADD COUNTERTERMS FOR $f_r(\lambda)$ HAVE A MAXIMUM AT $\lambda = 0$ AND A MINIMUM AT $\lambda = -1$.

$$\Longrightarrow \quad f_r(\lambda) = \frac{4(6\lambda^2 - 3\lambda + 1)(k - N_c)}{5(\lambda - 1)^3 N_c}$$

$$c(\lambda) = N + \frac{N_c(N_c^2 - 1)}{k} f_r(\lambda)$$



CONCLUSIONS

* STABILITY OF SPIN LIQUID T.P.
(COMPETING INTERACTIONS)

REFS.

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